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Ans 2

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Assignment - 4

1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ Find rank and reduce to row echlon

Sol^n Performing row operations:-

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_4 \to R_4 - 2R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix} \xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_3]{R_4 \to R_4 - R_2} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here no. of non-zero rows $= 3$

$\therefore \boxed{\rho(A) = 3}$ Ans

2) According to the question
every vector space W and matrix is the forms $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$
and,

the linear Transformation $T: W \to P_2$ defined by:
$$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

Here, we have three coefficients corresponding to $1, x,$ and x^2 (k^2) $\therefore$ Dimensions $= 2+1 = \underline{3}$

Now, The standard basis for W is:-

$$E = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$ ($\because$ W is 2×2 matrix space)

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Also, T is defined as $T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = \begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix}$

So, we can represent T in terms of the standard basis elements

$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

Now, these column vectors from the matrix A

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}.$$

Now, we have to convert this into the row-echelon form.

Applying row elementary operation.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Here $\boxed{\rho(A) = 3}$ → linearly independent.

Now, $\rho(A) + N = D \rightarrow 3 + N = 3 \rightarrow \boxed{N = 0}$

Hence Rank (A) = 3, Nullity (N) = 0

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3) To get the eigen value we need to solve characteristic eq^n

det $(A - \lambda I) = 0$

$\rightarrow \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0 \longrightarrow (2-\lambda)^2 - 1 = 0$

$\therefore \lambda^2 - 4\lambda + 3 = 0$

$\therefore \boxed{\lambda = 1, 3}$ → eigen values → for $A = \lambda$, $A^{-1} = \frac{1}{\lambda}$

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for $\lambda=1$

$$[A-\lambda I][x]=0$$

$$\left(\left(\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\right)-\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\right)\begin{bmatrix}x\\ y\end{bmatrix}=0$$

$$\therefore \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\end{bmatrix}$$

$\therefore x-y=0 \quad \therefore \boxed{x=y}$

so if $x=k$ then $\boxed{x=y=k}$ eigen space $\rightarrow k\begin{bmatrix}1\\ 1\end{bmatrix}$

$\rightarrow$ Now for eigen value of $A^{-1} \rightarrow \overset{\lambda}{\left(1, \frac{1}{3}\right)}$, $A^{-1}=\begin{bmatrix}2/3 & 1/3\\ 1/3 & 2/3\end{bmatrix}$

$\rightarrow$ for $\lambda=1$

$$[A^{-1}-\lambda I][x]=0$$

$$\therefore \begin{bmatrix}2/3 & 1/3\\ 1/3 & 2/3\end{bmatrix}[x]=0$$

$\therefore \frac{2}{3}x+\frac{1}{3}y=0 \longrightarrow \boxed{x=\frac{-y}{2}}$ $\longrightarrow$ not possible

$\frac{x}{3}+\frac{2}{3}y=0 \longrightarrow \boxed{x=-2y}$

for $\lambda=3$

$$[A-3I][x]=0$$

$$\begin{bmatrix}2-3 & -1\\ -1 & 2-3\end{bmatrix}[x]=[0] \longrightarrow \begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$-x-y=0 \quad \therefore \boxed{-x=y}$

if $\boxed{x=k}$ then $\boxed{x=-y=k}$ eigen space $=\begin{bmatrix}k\\ -k\end{bmatrix}=k\begin{bmatrix}1\\ -1\end{bmatrix}$

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4) Now, using Siedel Method,

$$\begin{bmatrix} 3 & 0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

Sol^n. initial $\boxed{k=0}$ ($\because k =$ iteration)

$$\therefore (x)^{(k+1)} = \frac{1}{a_{11}} \left(b_1 - a_{12}(y)^k - a_{13} z^k\right)$$

$$y^{(k+1)} = \frac{1}{a_{22}} \left(b_2 - a_{21}(x)^{k+1} - a_{23} z^k\right)$$

$$z^{(k+1)} = \frac{1}{a_{33}} \left(b_3 - a_{31}(x)^{k+1} - a_{32}(y)^{(k+1)}\right)$$

Iteration 1st

$$x^1 = \frac{1}{3}\left(7.85 + 0.1(7) + 0.2\right) = \frac{7.85}{3} = \boxed{2.6167}$$

$$y^{(1)} = \frac{1}{7}\left(-19.3 - 0.1(x)^2 + 0.3\right) = \boxed{-2.7571}$$

$$z^1 = \frac{1}{10}\left(-71.4 - 0.1x^1 + 0.3y^1\right) = \boxed{6.7957}$$

Iteration 2nd

$$x^2 = \frac{1}{3}\left(-7.85 + 0.1y^2 + 0.2z^1\right) = \boxed{1.7908}$$

$$y^2 = \frac{1}{7}\left(-19.3 - 0.1(2.6167) + 0.3(6.7757)\right) = \boxed{-3.3377}$$

$$z^2 = \frac{1}{10}\left(71.4 - 0.3(1.7908) + 0.2(-3.3377)\right) = \boxed{7.2591}$$

Iteration 3rd

$$x^3 = \frac{1}{3}\left(7.85 + 0.1(-3.3377) + 0.2(7.2591)\right) = \boxed{1.6792}$$

$$y^3 = \frac{1}{7}\left(-19.3 - 0.1(1.6792) + 0.3(7.2591)\right) = \boxed{-3.2744}$$

$$z^3 = \frac{1}{10}\left(71.4 - 0.3(1.6792) + 0.2(-3.2744)\right) = \boxed{7.3651}$$

sol^n 5) Here, in the form of $AX = B$.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $B = 0$ $\therefore$ Our system of eq^n is consistent (Homogeneous)
Now, let's convert this into row echlon form

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}]{R_4 \to R_4 - R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & -14 & 2 \end{bmatrix} \xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, $(\rho(A) = 2) < (n = 3)$.
$\therefore$ This system of eq^n has ∞ sol^n

Now we have $1x + 3y + 2z = 0$ — (1)
$-7y - z = 0$ — (2) $\therefore \boxed{z = 7y}$

from (1) $\boxed{x = -17y}$

if $\boxed{y = k}$, $x = -17k$, $z = 7k$ $\therefore X = \begin{bmatrix} -17k \\ k \\ 7k \end{bmatrix} = k \begin{bmatrix} -17 \\ 1 \\ 7 \end{bmatrix}$

eigen vector

Sol^n 6 To check if T is a linear Transformation, we need to verify 2 properties.
(1) Additivity $\to$ $T(u + v) = T(u) + T(v)$ (u, v are polynomial
(2) Homogenity $T(cu) = cT(u)$ c scalar)

Let's Check
(1) Additivity $\to$ $T((a_1 + b_1x + c_1x^2) + (a_2 + b_2x + c_2x^2))$
$= T((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2))$
$= (a_1 + a_2 + 1) + (b_1 + b_1 + 1) + (c_1 + c_1 + 2)x^2 = L.H.S$

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Now, $T(a_1+b_1x+c_1x^2) + T(a_2+b_2x+c_2x^2))$ (can take any constant to add not necessary 1)
$= (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$

$= (a_1+a_2+2) + (b_1+b_2+2)x + (c_1+c_2+2)x^2 = R.H.S$

$\therefore$ L.H.S $\neq$ R.H.S $\Rightarrow$ T is not additive

(2) Homogenity $T(cu) = T(c(a+bx+cx^2)) = (ac+1) + (cb+1)x + (cc+1)x^2$ → L.H.S

& R.H.S $\exists c$ $T(u) = cT(a+bx+cx^2) = c((a+1) + (b+1)x + (c+1)x^2)$
$= (ca+c) + (cb+c)x + (cc+c)x^2$.

$\therefore$ L.H.S $\neq$ R.H.S
$\therefore$ T is not homogeneous

Therefore T is not a Linear Transformation

Solⁿ 7 To determine if the set is a basis for $V_3(\mathbb{R})$ we need to check 2 things:-

1) Linear Dependence: Confirm that none of the vector in S can be written as a Linear Combination of the other.

2) Spanning: Verify that the set S spans $V_3(R)$, meaning that any vector in $V_3(R)$ can be expressed as linear combination of vector.

Now, form a matrix with vectors of S as its column and row reduce to check for linear independence

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \xrightarrow{R_3 \to R_3 - \frac{9}{5}R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

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Now $\rho(A) = 2 < (n=3)$

so, it is infinite, linearly Dependent, Hence don't span $(R)V_3$ and not basis

Next, Let's Determine dimension & basis of subspace spanned by S

Dimension → no. of linearly independent vectors in S. From above, we see that the first and second row are L.I but row 3 not.

∴ Dimension = 2, and basis for the subspace spanned by S is given by LI vectors = {(1,2,5) (3,1,0)}

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sol 9 Application :- Image Transformation through affine transformation

affine transformation involve translation, rotation, scaling and shearing of images, and these operations can be efficiently represented by using matrices.

example - Scaling Operation
Let's take 2D image represented as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

To scale the image by a factor of (s) in the x-direction and (t) in the y-direction the transformation matrix T is :

$$T = \begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix}$$

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For eg:- We have image represented by $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
We want to scale it by a factor of 2 in x-dirⁿ and 3 in y-dirⁿ the transformation would be:-

$$T = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

The resulting transformation image 'I' would be:-

$$T' = \begin{bmatrix} 2 & 4 \\ 9 & 12 \end{bmatrix}$$

- This demonstrates how matrix operation facilitates efficient and systematic manipulation of images.

Solⁿ 10 Linear transformation play a crucial role in computer vision, particularly in the context of rotating 2D images. A linear transformation can be presented by a matrix that operates on the coordinates of each pixel in the image, producing a transformed image. For rotation specifically, the rotation matrix is employed.

Rotation matrix for 2D image $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

If I = original image matrix, after multiply I with R, we obtain the rotated image 'I', reflecting the rotation.

This linear transformation is fundamental in computer vision application allowing for efficient manipulation and analysis of images, including rotation to correct orientation or align objects in the image.

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