Ans-1

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# Assignment - 2

\# Following sets linearly independent or not?

1) [1, 0, 0] , [1, 1, 0] , [1, 1, 1]

Sol$^n$ We know if $\boxed{c_1 v_1 + c_2 v_2 + c_3 v_3 - - - - + c_n v_n = 0}$

Independent if $c_1 = c_2 = c_3 = 0 = c_n$

Dependent: when all $c_i$'s are not zero. atleast one is non zero

→ To check dependancy of above vector

let $0 = c_1(1\ 0\ 0) + c_2(1, 1, 0) + c_3(1, 1, 1)$

∴ if there exist $c_1$ or $c_2$ or $c_3$ non zero then its Dependent

∴ if non-trivial solution is there ∴ it will be dependent

$$c_1 + c_2 + c_3 = 0$$
$$0c_1 + c_2 + c_3 = 0$$
$$0c_1 + 0c_2 + c_3 = 0$$

$\longrightarrow$ $AX = B$

⇒ To check ~~con~~ if it has non trivial sol$^n$.

we can do it by row echlon method (Gauss Jordan)

or if det(A) ≠ 0 then sol$^n$ is ~~non~~-trivial solution only

$$A:B = \left|\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right|$$

det(A) = 1

∴ It's ~~doesnot~~ have non-trivial solution

∴ only solution is $c_1, c_2, c_3 = (0, 0, 0)$

∴ it is linearly independent.

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2) $[7 \quad -3 \quad 11 \quad -6]$ , $[-56 \quad 24 \quad -88 \quad 48]$

Sol$^n$ $c_1 v_1 + c_2 v_2 = 0$

$\therefore$ $c_1(7 \quad -3 \quad 11 \quad -6) + c_2(-56 \quad 24 \quad -88 \quad 48) = 0$

M-1 Here second is multiple of $1^{st}$ vector then $\det(A) = 0$
$\therefore$ non trivial

M-2
$$\begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ -6 & 48 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using gaussian elimination

$7c_1 - 56c_2 = 0$
$-3c_1 + 24c_2 = 0$
$11c_1 - 88c_2 = 0$
$-6c_1 + 48c_2 = 0$

solving this we get.

that $(c_1, c_2) = (8, 1)t$

$\therefore$ non-trivial solution of $c_1, c_2$ exist

$\therefore$ these sets are linearly dependent

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3) $[-1 \quad 5 \quad 0]$, $[16 \quad 8 \quad -3]$ , $[-64 \quad 56 \quad 9]$

Sol$^n$ $c_1 v_1 + c_2 v_2 + c_3 v_3 = \{0, 0, 0\}$ $\rightarrow$ if $c_1 = c_2 = c_3 = 0$ $\rightarrow$ linearly independent
$\rightarrow$ else dependent

$v_1 = (-1, 5, 0)$ $v_2 = (16, 8, -3)$ $v_3 = (-64, 56, 9)$

$\therefore$ $c_1(-1 \quad 5 \quad 0) + c_2(16 \quad 8 \quad -3) + c_3(-64 \quad 56 \quad 9) = (0, 0, 0)$

$-c_1 + 16c_2 + (-64)c_3 = 0$
$5c_1 + 8c_2 + 56c_3 = 0$
$0c_1 - 3c_2 + 9c_3 = 0$.

If it has $\infty$ sol$^n$.
it is dependent.

To check it has non-trivial sol$^n$. of $c_3, c_2, c_1$ we will check by gauss elimination

Ans 3
Ans 4

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$$A:B = \left[\begin{array}{ccc|c} -1 & 16 & -64 & 0 \\ 5 & 8 & 56 & 0 \\ 0 & -3 & 9 & 0 \end{array}\right] \quad R_2 \to R_2 + 5R_1$$

$$\therefore A:B = \left[\begin{array}{ccc|c} -1 & 16 & -64 & 0 \\ 0 & 88 & -264 & 0 \\ 0 & -3 & 9 & 0 \end{array}\right]$$

$$A:B = \left[\begin{array}{ccc|c} -1 & 16 & -64 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -1 & 3 & 0 \end{array}\right] \quad R_3 \to R_3 + R_2$$

$$A:B = \left[\begin{array}{ccc|c} -1 & 16 & -64 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \rho(A) = \rho(A:B) \neq n$$

$\therefore \infty$ sol$^n$ → non-trivial solution.

$y - 3z = 0$

$y = 3z$

Let $z = t$, $y = 3t$, $x = -16t$

$\therefore (c_1, c_2, c_3) = t(-16, 3, 1)$

$\therefore$ sets are linearly dependent

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4) $[1 \; -1 \; 1]$, $[1 \; 1 \; -1]$, $[-1 \; 1 \; 1]$, $[0, 1, 0]$

sol$^n$ $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$

$c_1(1 \; -1 \; 1) + c_2(1 \; 1 \; -1) + c_3(-1 \; 1 \; 1) + c_4(0, 1, 0) = (0, 0, 0)$

$$c_1 + c_2 - c_3 + 0c_4 = 0$$
$$-c_1 + c_2 + c_3 + c_4 = 0$$
$$c_1 - c_2 + c_3 + 0c_4 = 0$$

Ans 4
Ans 5

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$$A:B = \begin{bmatrix} 1 & 1 & -1 & 0 & : & 0 \\ -1 & 1 & 1 & 1 & : & 0 \\ 1 & 7 & 1 & 0 & : & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$
$R_2 \rightarrow R_2 + R_1$

$$A:B = \begin{bmatrix} 1 & 1 & -1 & 0 & : & 0 \\ 0 & 2 & 0 & 1 & : & 0 \\ 0 & 0 & 2 & 1 & : & 0 \end{bmatrix}$$

Rank $= 3$.

$\therefore \rho(A) = \rho(A:B) = 3 =$ no. of unknown.

$\therefore$ trivial solution only.

$\therefore$ sets are linearly independent

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5) $[2\ -4]$, $[1, 9]$, $[3, 5]$.

Sol$^n$ $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$\therefore c_1(2\ -4) + c_2(1\ 9) + c_3(3\ 5) = (0, 0)$

$\therefore$

$2c_1 + c_2 + 3c_3 = 0$ $\longrightarrow$ Two equation
$-4c_1 + 9c_2 + 5c_3 = 0$ 3 unknown.

$[A:B] \Rightarrow \begin{bmatrix} 2 & 1 & 3 & : & 0 \\ -4 & 9 & 5 & : & 0 \end{bmatrix}$ $R_2 \rightarrow R_2 + 2R_1$

$\Rightarrow [A:B] = \begin{bmatrix} 2 & 1 & 3 & : & 0 \\ 0 & 11 & 11 & : & 0 \end{bmatrix}$ $\rho(A) = \rho(A:B) = 2 \neq n$.

$\therefore$ using back substitution.

~~let $z = t$~~
~~$\therefore y = t$~~

~~$c_2 + c_3 = 0$~~

let $c_3 = t$

$c_2 + c_3 = 0$
$2c_1 + c_2 + 3c_3 = 0$
$c_1 = -2t$

$\therefore c_2 = -t$
$c_1 = -2t$

$c_1, c_2, c_3 = t(-2, -1, 1)$

$\therefore \infty$ sol$^n$ $\therefore$ set is linearly dependent

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6) [3 -2 0 4], [5 0 0 1], [-6 1 0 1], [2 0 0 ...

Sol$^n$ $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$
if ($c_1 = c_2 = c_3 = c_4 = 0$ ∴ linearly independent)
else dependent.

∴ $c_1(3\ -2\ 0\ 4) + c_2(5\ 0\ 0\ 1) + c_3(-6\ 1\ 0\ 1) + c_4(2\ 0\ 0\ 3) = (0, 0\ 0\ 0)$

∴ $3c_1 + 5c_2 - 6c_3 + 2c_4 = 0$
$-2c_1 + 0c_2 + 1c_3 + 0c_4 = 0$
$0c_1 + 0c_2 + 0c_3 + 0c_4 = 0$
$4c_1 + 1c_2 + 1c_3 + 3c_4 = 0$

$AC = B \rightarrow A:B = \left[\begin{array}{cccc|c} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \end{array}\right]$

$R_4 \leftrightarrow R_3$
$R_3 \rightarrow R_3 + 2R_2$

∴ $A:B = \left[\begin{array}{cccc|c} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $R_2 \rightarrow 3R_2 + 2R_1$

∴ $A:B = \left[\begin{array}{cccc|c} 3 & 5 & -6 & 2 & 0 \\ 0 & 10 & -9 & 4 & 0 \\ 0 & 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\rho(A) = \rho(A:B) = 3 \neq n$
∴ non trivial sol$^n$. ~~only~~.

n no of unk...

∴ Set are Linearly Dependent

7) [3 4 7], [2 0 3], [8, 2, 3], [5 5 6]

Sol$^n$: $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_2 = 0$.

∴ $c_1(3\ 4\ 7) + c_2(2\ 0\ 3) + c_3(8\ 2\ 3) + c_4(5\ 5\ 6) = ($

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Ans 7

Ans 8

$$A:B = \left[\begin{array}{cccc|c} 3 & 2 & 8 & 5 & 0 \\ 4 & 0 & 2 & 5 & 0 \\ 7 & 3 & 3 & 6 & 0 \end{array}\right]$$

$R_3 \to 3R_3 - 7R_1$

$R_2 \to 3R_2 - 4R_1$

$$A:B = \left[\begin{array}{cccc|c} 3 & 2 & 8 & 5 & 0 \\ 0 & -8 & -26 & -4 & 0 \\ 0 & -5 & -47 & -2 & 0 \end{array}\right]$$

$\therefore \rho(A) = \rho(A:B) = 3 = n$

$\therefore$ Trivial solution

$\therefore$ sets are linearly independent.

8) [6 0 3 1 4 2], [0 -1 2 7 0 5], [12 3 0 -19 8 -11]

Soln $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$AC = B$.

$$\therefore A:B = \left[\begin{array}{ccc|c} 6 & 0 & 12 & 0 \\ 0 & -1 & 3 & 0 \\ 3 & 2 & 0 & 0 \\ 1 & 7 & -19 & 0 \\ 4 & 0 & 8 & 0 \\ 2 & 5 & -11 & 0 \end{array}\right] \xrightarrow{R_4 \leftrightarrow R_1} = \left[\begin{array}{ccc|c} 1 & 7 & -19 & 0 \\ 6 & 0 & 12 & 0 \\ 0 & -1 & 3 & 0 \\ 3 & 2 & 0 & 0 \\ 4 & 0 & 8 & 0 \\ 2 & 5 & -11 & 0 \end{array}\right]$$

$R_2 \to R_2 - 6R_1$

$R_4 \to R_4 - 3R_1$

$R_5 \to R_5 - 4R_1$

$R_6 \to R_6 - 2R_1$

$$A:B = \left[\begin{array}{ccc|c} 1 & 7 & -19 & 0 \\ 0 & -42 & 126 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -19 & 57 & 0 \\ 0 & -28 & 84 & 0 \\ 0 & -9 & 27 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 7 & -19 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \end{array}\right]$$

Taking common out

$$A:B = \left[\begin{array}{ccc|c} 1 & 7 & -19 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A:B) = 2 \neq$ number of unknowns

$\therefore \infty$ solution $\therefore$ non trivial exist

$\therefore -y + 3z = 0$

or

$-c_2 + 3c_3 = 0$

$\therefore 3c_3 = c_2$

Let $c_3 = t$, $c_2 = 3t$

$c_1 = -2t$.

$\therefore c_1\, c_2\, c_3 = t(-2, 3, 1)$

$\therefore$ sets are linearly dependent