Ans 1

Divya Musahib
2301010036

Page No.: 1
Date:

Assignment - 3

⇒ Find eigen values and eigen vector.

1) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Solⁿ For finding eigen value we need λ and for eigen vector $[X]$ we know

$[A - \lambda I][X] = 0$

$\therefore |A - \lambda I| = 0$

Solⁿ

$|A - \lambda I| = 0 = \begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix}$

$[A][x] = \lambda[x]$
$[A][x] = \lambda[I][x]$
$[A - \lambda I][x] = 0$

$= (2+\lambda)(-\lambda(1-\lambda) - 12) - 2(-2\lambda - 6) - 3(-4 + (1-\lambda)) = 0$

Calculating it $\quad (\lambda+3)(\lambda-5)(\lambda+3) = 0$

$\therefore \lambda_{1,2,3} = -3, -3, 5$
↳ eigen values

$\therefore$ For finding eigen vector we will put λ in $[A - \lambda I][x] = 0$

For $\lambda = -3$:

$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Apply row echlon form ~~Substitution~~

$\therefore \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad \rho(A) = 2$. ∞ solⁿ.

$x_1 + 2x_2 - 3x_3 = 0$

Let $x_2 = k_2$, $x_3 = k_3$

$\therefore$ Eigen vector $= X = \begin{bmatrix} -2k_1 + 3k_2 \\ k_1 \\ k_2 \end{bmatrix}$

$\therefore X = k_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

~~Ans 2~~ Divya Musahib
~~Ans 1~~ 2301010036

Page No:
Date 2

~~For $\lambda = 5$~~

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad n(A) = 1$$

let $x_3 = k$, $-x_1 - 2x_2 - 5x_3 = 0$

$-8x_2 \quad -16x_3 = 0$

$-x_1 = k.$ $\qquad \boxed{x_2 = -2k}$

$\therefore$ Eigen vector $\Rightarrow X = \begin{bmatrix} -K \\ -2K \\ K \end{bmatrix} = K \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

---

2) $\begin{bmatrix} +4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \Rightarrow [A - \lambda I][X] = 0$

Soln

$$[A - \lambda I] = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$\therefore$

$$4-\lambda\left((1-\lambda)^2\right) + 1\left(-2 + 2(1-\lambda)\right) = 0$$

$$\therefore (\lambda-1)(\lambda-3)(\lambda-2) = 0$$

$$\lambda_1 = 1, \ \lambda_2 = 3, \ \lambda_3 = 2$$

~~For $\lambda = 1$~~

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{let } x_2 = k.$$

$\therefore 3x_1 + x_3 = 0$

$2x_3 = 0 \qquad \Rightarrow x_1 = 0, \ x_2 = k, \ x_3 = 0.$

eigen vector $X = \begin{bmatrix} 0 \\ K \\ 0 \end{bmatrix} = K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

~~Ans 2~~

Ans 3

Divya Musahib

2301010036

for $\lambda = 3$
$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad r(A) = 1$$

$x_1 + x_3 = 0$ let $x_3 = k$

$-2x_2 + 2x_3 = 0$

$\therefore x_1 = -k$, $x_2 = k$

eigen vector $x = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} k$

$\lambda = 3$
$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + x_3 = 0$ let $x_3 = k$

$-x_2 + x_3 = 0$

$\therefore x_2 = k$

$x_1 = -k/2$

$\therefore$ eigen vector $x = \begin{bmatrix} -k/2 \\ k \\ k \end{bmatrix} = k\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$

3) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$ It is a lower triangular matrix so, diagonal elements will be the eigen values.

Hence, $\lambda_1 = 5$, $\lambda_2 = 0$, $\lambda_3 = 3$

for $\lambda = 5$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 - 2x_3 = 0$ Let $x_3 = k$

$-5x_2 + 3x_3 = 0$ $x_1 = -2k$

$x_2 = 3k/5$

Divya Musahib
2301010036.

eigen vector $\Rightarrow X = k \begin{bmatrix} -2 \\ 3/5 \\ 1 \end{bmatrix}$

for $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 = 0$ Let $x_2 = k$.

$-x_1 + 3x_3 = 0$ $x_1 = 0$, $x_3 = 0$

$\therefore$ eigen vector $= k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$

$-3x_2 = 0$

Let $x_3 = k$.

$x_2 = 0$, $x_3 = 0$

$\therefore$ eigen vector $\Rightarrow k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

---

4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix} \rightarrow$ Eigen values will be

$\lambda_1 = 0$, $\lambda_2 = 3$, $\lambda_3 = 2$

$\hookrightarrow$ triangular matrix.

for $\lambda = 0$

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_3 = 0$

$3x_2 + 4x_3 = 0$

Let $x_1 = k$

$\xrightarrow{f}$ Ans 4

Ans 5

Divya Musahib

2301010036

let $x_1 = k$, $x_2 = 0$, & $x_3 = 0$

$\therefore$ eigen vector $\longrightarrow$ $X = k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

~~Sol for $\lambda = 3$~~

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 = 0$ let $x_2 = k$

$4x_3 = 0$

$5x_3 = 0$ $\therefore$ eigen vector $X = k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

~~for $\lambda = -2$~~

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$ let $x_3 = k$

$5x_2 + 4x_3 = 0$ $\therefore x_1 = 0$, $x_2 = -\frac{4}{5}k$

$\therefore$ eigen vector $\Rightarrow$ $X = k\begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$

5) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$ Proving without calculation eigen values are 0's.

Soln

Because there are 3 identical rows in this matrix. So 2 eigen values must be 0