Ans 1

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Assignment - 1

$\rho(A) \neq \rho(A:B)$ — no solution
$\rho(A) = \rho(A:B) =$ no. of unknown — unique solⁿ
$\rho(A) = \rho(A:B) \neq$ no. of unknown — ∞ solⁿ

Q1 Test for consistency and solve?–

a) $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$

Solⁿ

$$A:B = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 13 \\ 2 & 10 & -47 & : & 32 \end{bmatrix} \qquad AX = B$$

→ Converting it to row echlon form

⇒ $R_2 \Rightarrow 2R_2 - 3R_1$, $R_3 \rightarrow R_3 - R_1$

$$A:B = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 11 & -27 & : & 11 \\ 0 & 22 & -54 & : & 27 \end{bmatrix} \longrightarrow R_3 \rightarrow R_3 - 2R_2$$

∴

$$A:B = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 11 & -27 & : & 11 \\ 0 & 0 & 0 & : & 5 \end{bmatrix} \qquad \begin{aligned} \rho(A) &= 2 \\ \rho(A:B) &= 3 \end{aligned}$$

∴ $\rho(A) \neq \rho(A:B)$ ⟶ no solution.

b) $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$

Solⁿ $AX = B$

∴
$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$\therefore A:B = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \longrightarrow R_1 \leftrightarrow R_2$$

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$$A:B = \begin{bmatrix} 1 & -2 & -1 & : & -4 \\ 2 & -1 & 3 & : & 8 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 3R_1$
$R_2 \rightarrow R_2 - 2R_1$

$$\therefore A:B = \begin{bmatrix} 1 & -2 & -1 & : & -4 \\ 0 & 3 & 5 & : & 16 \\ 0 & 7 & -1 & : & 12 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 - 7R_2$

$$A:B = \begin{bmatrix} 1 & -2 & +1 & : & -4 \\ 0 & 3 & 5 & : & 16 \\ 0 & 0 & -38 & : & -76 \end{bmatrix}$$

Using back substitution gaussian elimination

$38z = 76$

$\therefore \boxed{z = 2}$

$\rho(A) = \rho(B) = 2.$

$\therefore$ solution $= (x, y, z) = (2, 2, 2)$

$\therefore 3y + 5z = 16.$

$\therefore \boxed{y = 2}$

$x - 2y - z = -4.$

$\boxed{x = 2}$

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c) $4x - y = 12$, $-x + 5y - 2z = 0$, $-2x + 4z = 8$.

Sol^n $A:B = \begin{bmatrix} 4 & -1 & 0 & : & 12 \\ -1 & 5 & -2 & : & 0 \\ -2 & 0 & 4 & : & 8 \end{bmatrix}$

$AX = B$

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$$

→ To find X we need to check consistency for consistency $\rho(A) = \rho(A:B)$

→ for finding rank we will find row echlon form

→ $A:B = \begin{bmatrix} -1 & 5 & -2 & : & 0 \\ 4 & -1 & 0 & : & 12 \\ -2 & 0 & 4 & : & 8 \end{bmatrix}$ $(R_2 \leftrightarrow R_1)$

Now,

$R_2 \rightarrow R_2 - 4R_1$
$R_3 \rightarrow R_3 + 2R_1$

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Ans 1

As 2

$$A:B = \left[\begin{array}{ccc:c} 1 & -5 & 2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & -10 & 8 & 8 \end{array}\right]$$

$R_3 \longrightarrow R_3 + \frac{10}{19} R_2$

$$\therefore A:B = \left[\begin{array}{ccc:c} 1 & -5 & 2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & 0 & \frac{80}{19} & \frac{80}{19} \end{array}\right]$$

Rank = no. of non zero Rows

$$\rho(A) = \rho(A:B) = 3$$

$\Rightarrow$ Using back Substitution (Gaussian Elimination)

$$\frac{80}{19} z = \frac{80}{19}$$

$$\therefore \boxed{z = 1}$$

$$19y - 8z = 12$$

$$19y - 8 = 12$$

$$y = \frac{20}{19}$$

$$\Rightarrow x = 5y - 2z$$

$$x = 5\left(\frac{20}{19}\right) - 2 \qquad \therefore \boxed{x = \frac{62}{19}}$$

Ans $x, y, z = \left(\frac{62}{19}, \frac{20}{19}, 1\right)$

Q2. values of λ, μ for which $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$

has i) no solution

ii) unique solution

iii) infinite solution

sol^n $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$

no. of unknown = 3.

Ans 2

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$\therefore$ for no solution $\rightarrow \rho(A) \neq \rho(A:B)$

for unique solution $\rightarrow \rho(A) = \rho(A:B) =$ no. of unknown

for ∞ solution $\rightarrow \rho(A) = \rho(A:B) \neq$ no. of unknown

$\rightarrow$ So firstly we need rank of A and A:B we will do row echloen substitution

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_2 - R_1$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & \lambda - 1 & \mu - 6 \end{bmatrix}$$

if $\rho(A) \neq \rho(A:B) \rightarrow$ no solution.

$\hookrightarrow \boxed{\lambda = 1, \quad \mu \neq 6.}$

if $\rho(A) = \rho(A:B) \rightarrow$ consistent

$\therefore \lambda - 1 = \mu - 6 = 0$

$\therefore$ for consitency

$\rightarrow$ for unique sol^n rank must be 3.

$\therefore \lambda - 1 \neq 0, \quad \mu - 6 \neq 0.$

$\therefore \lambda \neq 1, \quad \mu \neq 6.$

$\rightarrow$ for ∞ sol^n $\rightarrow$ rank must be 2

$\therefore \lambda - 1 = 0, \quad \mu = 6$

$\therefore \lambda = 1, \mu = 6$

Ans $\rightarrow$ For no solution $\rightarrow \lambda = 1, \mu \neq 6$

$\rightarrow$ For unique solution $\rightarrow \lambda \neq 1, \mu \neq 6.$

$\rightarrow$ for ∞ solution $\rightarrow \lambda = 1, \mu = 6.$

Ans 3

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Q3 value of λ for which eq^n $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ has solution.

Sol^n $\rightarrow$ we need to find λ so that it is consistent

$\therefore \rho(A) = \rho(A:B)$

$$A:B = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$$

$AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

we will apply row echloen substitutions to obtain rank.

$R_3 \rightarrow R_3 - R_1$

$R_2 \rightarrow R_2 - R_1$

$$\Rightarrow A:B = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{array}\right]$$

$R_3 \rightarrow R_3 - 3R_2$

$$\Rightarrow A:B = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-(3\lambda-3) \end{array}\right]$$

$\rho(A) = 2$ $\therefore$ $\rho(A:B)$ must be 2

$\therefore$ $\lambda^2-1-(3\lambda-3) = 0$ should be true.

$\therefore$ $\lambda^2-1-3\lambda+3 = 0$

$\lambda^2-3\lambda+2 = 0$

$(\lambda-2)(\lambda-1) = 0$ $\therefore$ either $\lambda = 2$, or $\lambda = 1$

for consistency

$\rightarrow$ we need to find solution of X for both λ

Ans 3

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let $d = 1$

$\therefore A : B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

using back substitution (Gaussian elimination)

$\therefore 0x + y + 0z = 0$

$\therefore \boxed{y = 0}$

$\rightarrow x + y + z = 1$

$\rightarrow x + z = 1$

$x = 1 - z$. let $z = t$.

$\therefore \boxed{x = 1 - t . \quad y = 0 , \quad z = t}$ will be the solution

and there will be infinite sol^n.

similarly for $d = 2$

$\therefore A : B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$y = 1$

$x + y + z = 1$

$\therefore x + z = 0$.

Let $z = t$.

$\therefore x = -t$.

$\therefore \boxed{x = -t , y = 1, z = t}$

↳ will be the solution

$\therefore \infty$ solution for $d = 2$. also

final ans :

$\therefore$ d should be 2 or 1 to have solution of the eq^n.

if $d = 1$ solution will be $[(1-k), (0), k]$

if $d = 2$ solution will be $(-k, 1, k)$

k is any variable.

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Ans 4

Q 4 Find solution of $x+3y-2z=0$, $2x-y+4z=0$
$x-11y+14z=0$.

Sol^n $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

We will find solution using gaussian elimination

step 1 → converting to row echl.
step 2 Back substitution

$\therefore A:B = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$ $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$\therefore A:B = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{array}\right]$ $R_3 \to R_3 - 2R_2$

$\therefore A:B = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\therefore x + 3y - 2z = 0$
$-7y + 8z = 0$

$\boxed{\text{let } z = t}$

$y = \frac{8t}{7}$

$\therefore x = 2t - 3\left(\frac{8t}{7}\right)$

$x = \frac{-10t}{7}$

$\therefore$ solution will be $(x,y,z) = \left(\frac{-10t}{7}, \frac{8t}{7}, t\right)$

$\therefore$ As solution of the form $(x,y,z) = K(-10, 8, 7)$

Ans

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Ans 5

Q5 values of λ for which non trivial sol^n and solve them

$$3x + y - \lambda z = 0$$
$$4x - 2y - 3z = 0$$
$$2\lambda x + 4y + \lambda z = 0$$

Sol^n ~~Method-1~~ ~~Method 2~~

For non-trivial solution $\longrightarrow$ it should be ∞ solution

condition for ∞ solution is $\rho(A) = \rho(A:B) \neq$ no. of unknowns

$$\therefore A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow A:B = \left[\begin{array}{ccc:c} 3 & 1 & -\lambda & 0 \\ 4 & -2 & -3 & 0 \\ 2\lambda & 4 & \lambda & 0 \end{array}\right]$$

$R_3 \longrightarrow 3R_3 - 2\lambda R_1$

$R_2 \longrightarrow 3R_2 - 4R_1$

$$\therefore A:B = \left[\begin{array}{ccc:c} 3 & 1 & -\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 0 & 10 & 5\lambda & 0 \end{array}\right]$$

$R_3 \longrightarrow R_3 + R_2$

$$\therefore A:B = \left[\begin{array}{ccc:c} 3 & 1 & -\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 0 & 0 & 9\lambda-9 & 0 \end{array}\right]$$

for ∞ sol^n $\rho(A) = \rho(A:B) \neq n$

$9\lambda - 9 = 0$

$\therefore \lambda = 1$

$\therefore$ if $\lambda = 1$

$$\therefore A:B = \left[\begin{array}{ccc:c} 3 & 1 & -1 & 0 \\ 0 & -10 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

doing back substitution

$-10y - 5z = 0$ $\therefore$ let $y = t$, $z = -2t$, $x = -t$

$3x + y - z = 0$

$\therefore$ Sol^n is of the form $(x, y, z) = t(-1, 1, -2)$