

# CIRCLE ASSIGNMENT

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MATRICES

**Problem Statement** – Let  $C$  be the circle with center at  $(1,1)$  and radius is 1. If  $T$  is the circle centered at  $(0,y)$  passing through origin and touching the circle  $C$  externally, then the radius of  $T$  is equal to:

Radius of circle  $T$  is the distance between origin and center,

$$r_2 = \|u_2 - O\| \quad (4)$$

$$r_2 = \sqrt{\begin{pmatrix} 0 & y \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix}} \quad (5)$$

$$r_2 = y \quad (6)$$

Distance between  $u_1$  and  $u_2$ :

$$d = \|u_1 - u_2\| \quad (7)$$

$$d = 1 + r_2 \quad (8)$$

$$r_1 + r_2 = \|u_1 - u_2\| \quad (9)$$

$$(r_1 + r_2)^2 = \|u_1 - u_2\|^2 \quad (10)$$

## Construction

The input parameters are as follows

Symbol	Value	Description
$r_1$	1	radius
$r_2$	$y$	radius
$u_1$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	center
$u_2$	$\begin{pmatrix} 0 \\ y \end{pmatrix}$	center

$$r_1^2 + r_2^2 + 2r_1r_2 = \|u_1\|^2 + \|u_2\|^2 - 2u_1^T u_2 \quad (11)$$

$$1 + r_2^2 + 2r_2 = 2 + r_2^2 - 2\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ r_2 \end{pmatrix} \quad (12)$$

$$1 + 2r_2 = 2 - 2r_2 \quad (13)$$

$$4r_2 = 1 \quad (14)$$

$$r_2 = 1/4 \quad (15)$$

## solution

### step 1

The general equation of the circle is

$$x^T V x + 2u^T x + f = 0 \quad (1)$$

where  $V$  is the identity matrix

Let the equation of the circle  $C$  with radius  $r_1$  and center  $u_1$

$$x^T x + 2u_1^T x + f_1 = 0 \quad (2)$$

Equation of the circle  $T$  with center  $u_2$  and radius  $r_2$

$$x^T x + 2u_2^T x + f_2 = 0 \quad (3)$$

