Conic section Assignment

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November 2022

Problem Statement **- If the line $2x+\sqrt{6}y=2$ touches the hyperbola $x^2-2y^2=4$,then the point of contact is**

Solution

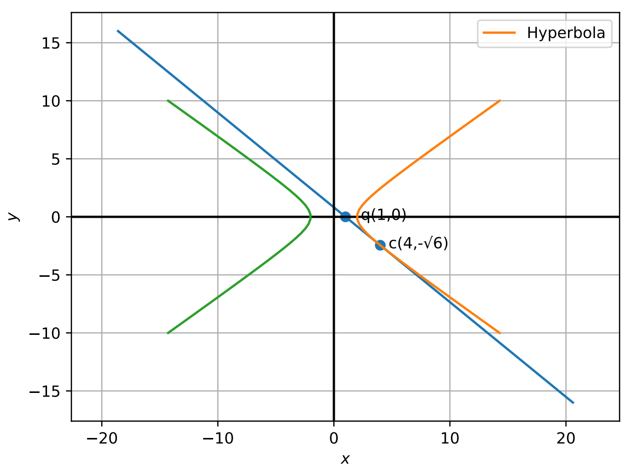

Figure 1:

The given equation of hyperbola $x^2-2y^2=4$ can be written in the general quadratic form as

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{1}$$

where

$$\mathbf{V}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{2}$$

$$\mathbf{u}=\begin{pmatrix}0\\0\end{pmatrix}, \tag{3}$$

$$f=-4 \tag{4}$$

The points of intersection of the line

$$L: \quad \mathbf{x}=\mathbf{q}+\mu\mathbf{m} \quad \mu\in\mathbf{R} \tag{5}$$

with the conic section are given by

$$\mathbf{x}_i=\mathbf{q}+\mu_i\mathbf{m} \tag{6}$$

where

$$\mu_i=\frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right.\\ \left.\pm\sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right]^2-\left(\mathbf{q}^T\mathbf{V}\mathbf{q}+2\mathbf{u}^T\mathbf{q}+f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\right) \tag{7}$$

From the line $2x+\sqrt{6}y=2$ the vectors q,m are taken,

$$\mathbf{q}=\begin{pmatrix}1\\0\end{pmatrix} \tag{8}$$

$$\mathbf{m}=\begin{pmatrix}-\sqrt{6}\\2\end{pmatrix} \tag{9}$$

by substituting eq(2),(3),(4),(8),(9) in eq(7)

$$\mu=\frac{\sqrt{6}}{2} \tag{10}$$

substituting eq(8),(9),(10) in eq(6) the point of contact is

$$\mathbf{c}=\mathbf{q}+\mu\mathbf{m} \tag{11}$$

Construction

Points	intersection points
c	$\begin{pmatrix}4\\-\sqrt{6}\end{pmatrix}$
q	$\begin{pmatrix}1\\0\end{pmatrix}$