CIRCLE ASSIGNMENT

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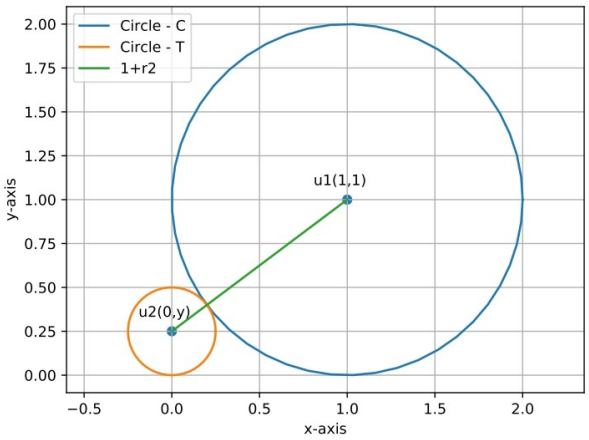
FWC22094 IITH Future Wireless Communication (FWC) MATRICES

Problem Statement – **Let C be the circle with center at (1,1) and radius is 1.If T is the circle centered at (0,y) passing through origin and touching the circle C externally,then the radius of T is equal to:**



Construction

The input parameters are as follows

Symbol	Value	Description
r_1	1	radius
r_2	y	radius
u_1	$\begin{pmatrix}1\\1\end{pmatrix}$	center
u_2	$\begin{pmatrix}0\\y\end{pmatrix}$	center

solution

step 1

The general equation of the circle is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where V is the identity matrix
Let the equation of the circle C with radius $\mathbf{r_1}$ and center $\mathbf{u_1}$

$$\mathbf{x}^\top \mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0 \tag{2}$$

Equation of the circle T with center $\mathbf{u_2}$ and radius $\mathbf{r_2}$

$$\mathbf{x}^\top \mathbf{x} + 2\mathbf{u_2}^\top \mathbf{x} + f_2 = 0 \tag{3}$$

Radius of circle T is the distance between origin and center,

$$\mathbf{r_2} = \|\mathbf{u_2} - \mathbf{O}\| \tag{4}$$

$$r_2 = \sqrt{\begin{pmatrix}0 & y\end{pmatrix}\begin{pmatrix}0\\y\end{pmatrix}} \tag{5}$$

$$\boxed{r_2 = y} \tag{6}$$

Distance between $\mathbf{u_1}$ and $\mathbf{u_2}$:

$$\mathbf{d} = \|\mathbf{u_1} - \mathbf{u_2}\| \tag{7}$$

$$\mathbf{d} = 1 + r_2 \tag{8}$$

$$r_1 + r_2 = \|\mathbf{u_1} - \mathbf{u_2}\| \tag{9}$$

$$(r_1 + r_2)^2 = \|\mathbf{u_1} - \mathbf{u_2}\|^2 \tag{10}$$

$$r_1^2 + r_2^2 + 2r_1r_2 = \|\mathbf{u_1}\|^2 + \|\mathbf{u_2}\|^2 - 2u_1^\top u_2 \tag{11}$$

$$1 + r_2^2 + 2r_2 = 2 + r_2^2 - 2\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}0\\r_2\end{pmatrix} \tag{12}$$

$$1 + 2r_2 = 2 - 2r_2 \tag{13}$$

$$4r_2 = 1 \tag{14}$$

$$\boxed{r_2{=}1/4} \tag{15}$$