PROBABILITY

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16.4.10 [1]**The random variable X has a probability distribution P(X) of the following form.where k is some number:**

$$\Pr\left(\left(\right)X\right)=\begin{cases}k, & \text{if x=0}\\ 2k, & \text{if x=1}\\ 3k, & \text{if x=2}\\ 0, & \text{otherwise}\end{cases} \tag{16.4.10.1}$$

a) Determine the value of k

b) Find P(X < 2),P(X ≤ 2),P(X ≥ 2)

Solution If we expand the probabilities given further more by substituting the value of x and only considering 0 to 4 hours as the probability of studying in the remaining hours is zero, we get

x	0	1	2
$\Pr\left(X=x\right)$	k	2k	3k

we also know that,

$$\sum_{k=0}^{2}\Pr\left(X=k\right)=1 \tag{16.4.10.2}$$

By substituting the probabilities in (16.4.10.2)

$$\implies k+2k+3k=1 \tag{16.4.10.3}$$

$$\implies 6k=1 \tag{16.4.10.4}$$

$$k=0.167 \tag{16.4.10.5}$$

x	0	1	2
$\Pr\left\{X=x\right\}$	0.167	0.334	0.501

(16.4.10.6)

[1]Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

We know that, Cumulative Distributive Function (CDF)

$$F(x) = \Pr\left(X \leq x\right) \tag{16.4.10.7}$$

x	0	1	2
$F(X)$	0.167	0.501	1.00

And also,

$$\Pr\left\{x < X \leq y\right\} = F\left\{y\right\} - F\left\{x\right\} \tag{16.4.10.8}$$

(a) P(X < 2)

$$\implies \sum_{k=0}^{1} \Pr\left(X = k\right) = \Pr\left(X \geq 2\right) \tag{16.4.1.9}$$

$$\implies \Pr\left(0 < X \leq 1\right) \tag{16.4.1.10}$$

$$= F(1) \tag{16.4.1.11}$$

$$= 0.501 \tag{16.4.1.12}$$

(b) P(X ≤ 2)

$$\implies \sum_{k=0}^{2} \Pr\left(X = k\right) = \Pr\left(X \leq 2\right) \tag{16.4.2.13}$$

$$= F(2) \tag{16.4.2.14}$$

$$= 1 \tag{16.4.2.15}$$

(c) P(X ≥ 2)

$$\implies \Pr\left(1 < X \leq 2\right) \tag{16.4.3.16}$$

$$= F(2) - F(1) \tag{16.4.3.17}$$

$$= 1.002 - 0.501 \tag{16.4.3.18}$$

$$= 0.501 \tag{16.4.3.19}$$