BST 234: Lab - 9

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Minimizing Least Squares

- Normal Equations: $A^T Ax = A^T b$
- If $\kappa(A)$ is large then $\kappa(A^T A)$ will be even larger and near singular matrix
- Hence inverting $A^T A$ becomes unstable

QR decomposition

- Given $m \times n$ A, with $m > n$, QR decomposition produce Q, an orthogonal $m \times m$ matrix and R, an $n \times n$ upper triangular matrix such that

$$A = Q \begin{bmatrix} R \\ O \end{bmatrix} \tag{1}$$

- Orthogonal matrix: $Q^T = Q^{-1}$
- Hence least square problem becomes:

$$min||r||_2^2 = min||b - Ax||_2^2 \tag{2}$$

$$= min||b - Q \begin{bmatrix} R \\ O \end{bmatrix} x||_2^2 \tag{3}$$

$$= min||Q^T b - \begin{bmatrix} R \\ O \end{bmatrix} x||_2^2 \tag{4}$$

Getting Q and R

- Givens Rotation: Method for introducing zeros in a matrix by rotating the different columns/rows in a matrix
- Householder transformation: Orthogonal reflection transformation (Implementation)

Householder transformation

- Time complexity for Householder transformation: $mn^2 - \frac{n^3}{3}$
- For Given Rotation the time complexity is : $3mn^2 - n^3$
- 50% more work to do Givens Rotation compared to Householder transformation

Solving non-linear equations - Root finding problem

- Solution only exist and is unique in very special cases
- Bisection Method: Convergence is guaranteed but extremely slow