Managing Relationships Between Restaurants and Food Delivery Platforms: Conflict, Contracts, and Coordination

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Abstract

Restaurant delivery platforms collect customer orders via the internet, transmit them to restaurants, and deliver the orders to customers. They provide value to restaurants by expanding their markets, but critics claim they destroy restaurant profits by taking a percentage of revenues and generating congestion that negatively impacts dine-in customers. We consider these tensions using a model of a restaurant as a congested service system. We find that the predominant industry contract, in which the platform takes a percentage cut of each delivery order (a "commission"), fails to coordinate the system because the platform does not internalize its effect on dine-in revenues; this leads to prices that are too low, reducing the restaurant's margins and leaving money on the table for both firms. Two commonly proposed remedies to this problem (commission caps and allowing the restaurant to set a price floor on the platform) can increase restaurant revenue but do not solve the coordination issue. We thus propose an alternative, practical coordinating contract that is a variation of the current industry standard: for each delivery order, the platform pays the restaurant a percentage revenue share *and* a fixed fee. We show that this contract, appropriately designed, coordinates the system, protects restaurant margins by ensuring a lower bound on its revenue per delivery order, and allocates revenue between the restaurant and the platform with a high degree of flexibility.

Keywords: on-demand services, delivery platforms, supply chain coordination

1 Introduction

Delivery platforms such as Grubhub, Seamless, DoorDash, Postmates, and Uber Eats are a large and rapidly growing part of the restaurant industry. These platforms are third-party service providers connecting customers and restaurants: they typically combine a convenient and easy-to-use web-based ordering system with the capability to physically deliver orders to customers. For restaurants, delivery platforms generate value by allowing them to enter the delivery market without building their own ordering platform or investing in their own delivery drivers, expanding their customer base with minimal up-front investment or fixed operating costs. Because of this, platforms often pitch themselves to restaurants as a source of "incremental" revenue, i.e., additional profit on top of their existing dine-in revenue (Meyersohn, 2018).

Despite their seeming advantages, delivery platforms are controversial with restaurant owners (Houck, 2017; Dunn, 2018; Meyersohn, 2018). Among the potential pitfalls of delivery platforms, two issues in

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particular stand out. The first derives from the fact that the standard contractual relationship offered by most platforms is one of simple revenue sharing: revenue on each order is split between the platform and restaurant according to some pre-negotiated rate. The platform's share of the revenue, also called a commission, is typically around 15-30% (Restaurant Engine, 2021), leaving the restaurant with only 70-85% of its normal revenue on each item sold.[1] Given the already thin margins in the restaurant industry, which average 4-5% or less (Lunden, 2020), relinquishing any share to a third party can drastically reduce—or eliminate—the profitability of an order.

The second key issue is that a large volume of delivery orders may place a strain on restaurant operations, adding complexity for staff and deteriorating service for dine-in customers. Restaurant staff are faced with the daunting task of juggling orders from multiple customer streams, such as dine-in, take-out, and a stream for each delivery platform that the restaurant supports. Incremental orders coming from the platform can place greater demand on the kitchen—a shared resource that serves both dine-in and delivery customers—and can lead to increased errors, longer wait times for customers, and an overall worse impression of service (Buell, 2017; Houck, 2017). In the long-run, this can harm the restaurant's reputation and cause even loyal dine-in customers to switch to delivery (where, as noted above, they generate less profit for the restaurant) or simply stop patronizing the restaurant altogether.

Thus, incremental orders generated by the delivery platform not only earn a lower profit rate than orders generated by dine-in customers, they may also deteriorate service for dine-in customers, leading to fewer (profitable) dine-in orders and more (unprofitable) delivery orders; see the case study by Buell (2017) for a vivid example. This illustrates a key source of conflict in the relationship between platforms and restaurants, one that can lead to a downward spiral for restaurants consisting of both increasing demand and decreasing profit margins, causing some to limit, or even eliminate, their partnerships with delivery platforms (Houck, 2017). As the owners of two popular New York eateries put it, "we know for a fact that as delivery increases, our profitability decreases," and as a result, "sometimes it seems like we're making food to make Seamless profitable" (Dunn, 2018).

In this paper, we investigate precisely these issues. Our chief goal is to understand how to resolve this potential source of conflict between restaurants and platforms via the use of appropriate contracts, giving customers a service that they value while avoiding the negative effects described above. To accomplish this, we model a restaurant as a congested service system. The restaurant serves two separate channels of demand: dine-in customers and delivery customers. The restaurant has full power over the price in the dine-in channel, while the platform determines the price in the delivery channel. Reflecting the issues described above, the platform keeps a share of the delivery revenue, and incremental orders from the delivery channel generate congestion that is costly for dine-in customers.

Using this model, we show that the contract that currently prevails in the industry ignores the negative

[1] Precise terms of contracts between restaurants and platforms are not usually publicly disclosed; however, Restaurant Engine (2021) describes typical commission rates around 30% for each of the four major US platforms (Grubhub and its subsidiary Seamless, DoorDash, Postmates, and UberEats), although this can vary depending on options chosen by the restaurant (e.g., how prominently to display the restaurant in search results) as well as the restaurant's negotiating power.

interaction between channels and gives both the platform and the restaurant incentive to set their price too low. As a result, joint revenue is not maximized—the system is uncoordinated—which could cause the relationship between platform and restaurant to break down. We also show that common attempts to remedy this problem, commission caps and price floors for the platform, may help the restaurant somewhat, but cannot coordinate the system, serving as a mere "band-aid" on an inadequate contract that fails to fix the underlying structural issues with simple revenue sharing.

Our central contribution is to show that a modification of the current industry standard contract, which we call generalized revenue sharing, achieves coordination with a simple and practically implementable structure. Under this contract, for each delivery order, the platform pays the restaurant both a percentage revenue share *and* a fixed fee that is independent of the delivery price. Such a contract achieves a wide (but not total) range of allocations of the delivery revenue between the parties, and ensures that for each delivery order, the restaurant receives some minimum revenue level which, under certain conditions, is at least as high as the restaurant's revenue on each dine-in order, protecting restaurant margins.

Our results thus illustrate that the popular discourse around platform-restaurant relationships, which is mostly centered on the idea that revenue sharing contracts are damaging because the commissions chosen by platforms are too high, may be focused on the wrong lever: the *structural form* of simple revenue sharing, the industry standard contract, is deficient. However, our proposed contract with a fixed fee and a percentage revenue share offers a simple and practically implementable way to alleviate a key source of conflict between restaurants and platforms, maximizing aggregate revenue and coordinating the system.

2 Related Literature

The literature on food delivery platforms is nascent but growing rapidly. A number of recent papers study operational issues faced by delivery platforms (Mao *et al.* , 2019; Oh *et al.* , 2019; Liu *et al.* , 2020; Li & Wang, 2021b; Niu *et al.* , 2021). The closest works to ours in this stream are Baron *et al.* (2019) and Chen *et al.* (2020). Baron *et al.* (2019) consider an omnichannel retail context, similar to ours, in which one channel generates a negative externality on the other; however, they do not consider a decentralized system and instead, they have a single firm that controls multiple channels. Hence, the issue of incentive conflict and miscoordination between different firms is not addressed, nor do they examine mechanisms to mitigate this (commission caps, price floors, and generalized revenue sharing) as we do.

Chen *et al.* (2020) study contracts between restaurants and delivery platforms and, like us, they find that simple revenue sharing fails to coordinate. Our models differ, however, in their assumptions about pricing power and the form of the coordinating contract. They assume that the menu price is the same in both the dine-in and delivery channels; we make no such assumption. They also find that a simple revenue sharing contract with a "price ceiling" on the delivery menu price—which, because of the identical price assumption, amounts to a constraint on the restaurant's dine-in price as well—coordinates the system. We propose a different coordinating contract that places no constraints on the restaurant's dine-in operations and involves only adjustments on payments made from the platform to the restaurant for delivery orders, which may be more appealing to some restaurateurs.

Our work is also related to the broader supply chain contracting literature, in particular work on revenue sharing contracts. Cachon & Lariviere (2005), for example, show that a revenue sharing contract where a retailer shares some of its revenue with its supplier can coordinate the supply chain and assign arbitrary fractions of revenue to each party. See Cachon (2003) and Lariviere (2015) for reviews and perspectives on the supply chain contracting literature. Our work differs from this literature because of the existence of dual sales channels—the delivery channel and the dine-in channel—and the interaction between them. As a result, while the form of our proposed coordinating contract bears some structural similarity to those proposed in this literature (e.g., Cachon & Lariviere 2005 also consider a revenue sharing contract with both a revenue share and a linear wholesale price, analogous to our fixed fee), the contract parameters play a different role and must be set in a different way in our setting; more on this distinction will be discussed in §5.

Our work is also related to models of supplier encroachment in the broader supply chain literature. Encroachment refers to a supplier, with an existing retail partner, who supplements his indirect (retail) sales with direct-to-consumer sales, and work in this literature focuses on when it is profitable to add a direct channel and how to induce the retailer to accept this. Hence, the "status quo" in encroachment models is typically the opposite of ours: in our case, the supplier (the restaurant) can always sell direct to consumers (i.e., via dine-in), and the question is whether to add the indirect (delivery) channel and how to structure the contract such that the restaurant accepts this.

Many papers in this literature do not consider coordination and focus on the impact of a supplier adding a direct sales channel on firm and supply chain profit (e.g., Arya *et al.* , 2007; Li *et al.* , 2013; Ha *et al.* , 2015). Some do propose coordinating contracts (Tsay & Agrawal, 2004; Boyaci, 2005; Cai, 2010; Xu *et al.* , 2014; Li *et al.* , 2015; Yang *et al.* , 2018); however, these proposed contracts are impractical in our setting, requiring complicated non-linear pricing schemes, sharing of direct sales channel revenue with the retailer (i.e., dine-in revenue with the platform), or removing pricing power from either the supplier or retailer. By contrast, our proposed coordinating contract requires only simple modifications to the form of the current industry standard contract, and is linear in the order volume. Hence, a key contribution of our work is showing that a simple and practically implementable contract can coordinate under the specific type of dual channel structure and interactions between channels representative of the restaurant delivery industry.

3 Model

3.1 The Restaurant and the Platform

A restaurant sells a single identical item via two channels: a *delivery channel* that serves customers via a third-party platform, and a *dine-in* channel that serves customers directly at the restaurant. The restaurant has full control over the price in the dine-in channel, p, and sets that price to maximize its revenue, all of which goes directly to the restaurant. The platform has full control over the price in the delivery channel, θ, and sets that price to maximize its revenue. The price θ is the "all-in" price for delivery channel customers, including the price of the food as well as any service or delivery fees.[2]

Motivated by practice at most restaurants, both channels are served via a shared kitchen; hence, demand from both channels generates congestion in this shared resource, which can lead to delays and poor service as described in the introduction. We ignore congestion or delays generated by either the dining room (for the dine-in channel) or delivery process (for the delivery channel). Each of these sources of congestion only affects a single channel at a time, and therefore has straightforward consequences on system performance; we focus instead on the more interesting shared step in the process that affects both channels simultaneously, i.e., the kitchen. We also assume that the capacity of the kitchen is fixed and exogenous because, in practice, during peak hours (i.e., lunch and dinner) restaurants are often fully staffed and kitchen capacity is limited by physical space; in the short- or medium-term, it will be difficult to expand capacity further. Moreover, even if staffing is a bottleneck in the kitchen, hiring quality kitchen staff at restaurants can be costly, slow, and challenging (Deliso, 2021). Lastly, as we will show, the delivery channel reduces the restaurant's margins; it can be difficult to justify capacity expansion as margins deteriorate; a first order concern will be to coordinate the system and protect the restaurant's margins, after which capacity expansion may be considered.

3.2 Customer Segments

The customer population has total size normalized to one and consists of two customer types. *Dine-in* customers only consider using the dine-in restaurant channel. These customers comprise a fraction α of the total market, and are denoted by the subscript R for "restaurant" customers. *Delivery* customers only consider using the delivery channel. These customers comprise a fraction $1 - \alpha$ of the total market, and are denoted by the subscript H for "home," since their consumption (usually) occurs at their homes. We assume $0 < \alpha < 1$ to avoid trivial cases where there is only one customer type and hence all traffic flows through a single channel. Upon arrival, customers choose whether to purchase or not, and do not consider switching between channels. Customer types are thus exogenous and derive from an earlier stage customer decision process, reflective of the fact that customer preferences for dine-in or delivery often result from factors outside of our model (e.g., a customer may be ill or tired and strongly prefer delivery, or may be planning a special evening out and strongly prefer to dine-in at the restaurant). Both dine-in and delivery customers are heterogeneous in their valuation for service on their respective channels. Dine-in customers'

[2] In practice, the delivery channel price can be set either entirely by the platform or by a combination of the restaurant and the platform (e.g., the restaurant may set the menu price of the food while the platform may set service and delivery fees). For our base model, we consider the former scenario; this reflects the fact that, even if the restaurant sets the menu price of the food, the platform has the power to both add fees and offer discounts and promotions that could effectively give it full pricing power. For example, if the restaurant sets a price of $10 for its food, the platform can set a service or delivery fee to raise the final price above $10, or can offer a promotional discount (e.g., 25% off, $5 off, free delivery, etc.) to lower the final price below $10. In §4.3, we consider the latter scenario, where the restaurant sets the price of the food in the delivery channel, effectively establishing a "price floor" on top of which the platform may raise the price further by adding service and delivery fees.

valuations are uniformly distributed on $[0, \bar{v}_R]$, with $\bar{v}_R > 0$, CDF F_R, and complementary CDF $\bar{F}_R$ (i.e., $\bar{F}_R(\cdot) = 1 - F_R(\cdot)$). Delivery customers' valuations are uniformly distributed on $[0, \bar{v}_H]$, with $\bar{v}_H > 0$, CDF F_H, and complementary CDF $\bar{F}_H$.

As described above, greater demand placed on the shared resource of the kitchen can increase delays experienced by customers in both channels. Specifically, customers are assumed to experience a linear congestion cost proportional to the total volume of customers who purchase across both channels. Let c_t be the marginal waiting cost for customers of type $t \in \{H, R\}$, and suppose that the total mass of customers who purchase equals x. The congestion cost incurred by customers is equal to $c_t x$. A linear cost function captures the fact that customer utility is decreasing in the volume of customers who use the resource, while still maintaining expositional clarity. It is appropriate if, for example, customers arrive faster than they can be processed by the kitchen, a reasonable assumption during peak hours for restaurants, forming a queue that is proportional in length to the number of arrivals.[3] However, in §6 we extend our results to the case of general convex waiting costs.

We assume that dine-in customers should be more sensitive to congestion than delivery customers, since delivery customers wait for their food in relatively comfortable locations (e.g., their homes or hotel rooms), while dine-in customers wait in less comfortable locations (e.g., the restaurant's waiting area or on the street). Moreover, most delivery platforms allow customers to preorder food for arrival at a specific time, allowing them to avoid (or mostly avoid) waits altogether. Hence, delivery customers have *lower* marginal waiting costs than dine-in customers, i.e., $c_H < c_R$. For tractability, in our base model, we assume dine-in customers incur a strictly positive marginal cost ($c_R > 0$), while delivery customers incur zero cost ($c_H = 0$). In §6, we discuss positive marginal waiting costs for both segments, i.e., $0 < c_H < c_R$.

Let γ_R be the fraction of dine-in customers who purchase and let γ_H be the fraction of delivery customers who purchase. Given α, γ_R, and γ_H, the dine-in volume is $\alpha\gamma_R$, and the delivery volume is $(1 - \alpha)\gamma_H$. Hence, the total number of customers who purchase is $\alpha\gamma_R + (1 - \alpha)\gamma_H$, and the congestion cost incurred by dine-in customers is $c_R(\alpha\gamma_R + (1 - \alpha)\gamma_H)$. The purchase utility of a dine-in customer with valuation v_R is $U_R(p; v_R, \gamma_R, \gamma_H) = v_R - p - c_R(\alpha\gamma_R + (1 - \alpha)\gamma_H)$, while the purchase utility of a delivery customer with valuation v_H is $U_H(\theta; v_H) = v_H - \theta$. Customers from both segments are assumed to purchase if they have non-negative utility. The sequence of events is as follows. First, the restaurant publicly announces the dine-in price p. Then, the platform announces the delivery price θ. Finally, customers in both channels make their purchasing decisions simultaneously.

[3] To see this, suppose λ customers arrive uniformly over 1 hour, and the kitchen is capable of deterministically processing $\mu < \lambda$ customers per hour. Then, by the end of the 1 hour arrival window, $L = \lambda - \mu$ customers have accumulated in a queue. On average, $L/2 = (\lambda - \mu)/2$ customers are waiting in this queue. From Little's Law, since customers are departing the system at rate μ, the average wait time in the queue is $W = L/\mu = (\lambda/\mu - 1)/2$ hours. The average waiting cost for a type t customer is thus $c_t W = c_t(\lambda/\mu - 1)/2$, i.e., a linear function of the total volume of customers who seek service, λ.

3.3 Centralized System

We next analyze the centralized system, in which a single firm sets both the delivery and dine-in price to maximize total revenue across both channels. This will serve as a benchmark for comparison in our subsequent analysis. The total revenue for prices p and θ and purchase fractions γ_R and γ_H is $Z(p,\theta) = p\alpha\gamma_R + \theta(1-\alpha)\gamma_H$, where the dine-in revenue is $p\alpha\gamma_R$ and the delivery revenue is $\theta(1-\alpha)\gamma_H$. Lemmas A.1 and A.2 in §A of the online supplement derive expressions for the equilibrium purchase fractions of dine-in and delivery customers given prices in each channel. We can substitute these values into $Z(p,\theta)$ to give

$$Z(p,\theta) = p\alpha\left[\frac{\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta)}{\bar{v}_R + c_R\alpha}\right]^+ + \theta(1-\alpha)\bar{F}_H(\theta). \tag{1}$$

In general, it may not be optimal to set a price such that a strictly positive fraction of customers in both channels purchase. However, as shown in Lemma A.3 in the online supplement, a condition can be derived that guarantees this, implying that the bracketed term in (1) is strictly positive. This condition is:

Assumption 1. *The parameter values satisfy* $c_R(1-\alpha)/4 < \bar{v}_R/2 < \bar{v}_H$.

For the remainder of the paper, we assume Assumption 1 holds, since this implies a centralized firm will operate both channels simultaneously, and as a result, in a decentralized system, achieving coordination requires that the platform and restaurant coexist. Given this, we may next derive the centrally optimal prices:

Proposition 1. *The optimal price pair* (p^*, θ^*) *that maximizes total revenue* (1) *uniquely solves the system* $p^* = \left(\bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta^*)\right)/2$ *and* $\theta^* = \left(\bar{v}_H + p^* c_R\alpha/(\bar{v}_R + c_R\alpha)\right)/2$.

Proof. All proofs appear in §B of the online supplement. □

Proposition 1 will allow us to compare firm decisions in a decentralized system under various contract forms to that of a centralized system. Note that at optimality, the centralized system sets both the dine-in and delivery price to account for congestion costs generated in the dine-in channel by delivery customers, i.e., taking into account c_R. If the waiting cost is zero, i.e., $c_R = 0$, the prices are set to independently maximize the revenue in each channel. However, if $c_R > 0$, the optimal dine-in price is *lower* to account for reduced customer utility due to waiting, while the optimal delivery price is *higher* to reduce the amount of congestion—a negative externality—that the delivery channel generates for dine-in customers.

4 Simple Revenue Sharing Contracts

In this section, we analyze a decentralized system, focusing on the performance of the prevalent industry contract: a simple revenue sharing (RS) agreement in which the platform shares with the restaurant a fraction of revenue from delivery orders, while the restaurant keeps all dine-in revenue. We denote by a the share of the revenue from delivery customers that the restaurant receives. The platform's fraction of revenue, $1-a$, is often called a commission. Under RS, the restaurant's revenue for each delivery customer is $a\theta$, and the platform's revenue per delivery customer is $(1-a)\theta$. The contract parameter a is determined exogenously outside of our model, e.g., via a take-it-or-leave-it offer from the platform (as is common with smaller, independent restaurants) or a negotiation process (as is typical with larger restaurant chains).

In what follows, we consider three variations of RS, all of which can be found in practice: §4.1 analyzes a base version of the contract with no constraints placed on firm decisions, §4.2 considers a variation with a cap placed on the platform's commission, and §4.3 considers a variation in which the restaurant is allowed to set a floor on the platform's delivery channel price.

4.1 Base Contract

In a decentralized system with RS, from Lemma A.1, the platform's revenue is given by $z_{P,RS}(\theta) = (1-a)\theta(1-\alpha)\bar{F}_H(\theta)$. Maximizing this over θ gives the optimal delivery price θ_{RS}. Note that θ_{RS} will clearly be independent of the restaurant's dine-in price, p, since this appears nowhere in $z_{P,RS}(\theta)$. Hence, the restaurant can treat the platform's price (and therefore also the restaurant's total revenue from delivery customers) as constant. From Lemma A.2, the restaurant's revenue can then be written as

$$z_{R,RS}(p) = p\alpha\left[\frac{\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta_{RS})}{\bar{v}_R + c_R\alpha}\right]^+ + a\theta_{RS}(1-\alpha)\bar{F}_H(\theta_{RS}). \tag{2}$$

Maximizing $z_{P,RS}(\theta)$ and $z_{R,RS}(p)$ yields the equilibrium prices under RS:

Proposition 2. *Under RS, the platform's optimal price is $\theta_{RS} = \bar{v}_H/2$ and the restaurant's optimal price is $p_{RS} = \left(\bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta_{RS})\right)/2$.*

Observe that, since a higher delivery price reduces congestion generated in the dine-in channel, the dine-in price p_{RS} is increasing in the delivery price θ_{RS}. This observation leads to the following corollary:

Corollary 1. *Under RS, if $c_R > 0$, the delivery price, dine-in price, and dine-in purchase fraction are too low compared to the central optimum ($\theta_{RS} < \theta^*$, $p_{RS} < p^*$, and $\gamma_R^{RS} < \gamma_R^*$).*

Comparing Propositions 1 and 2, we see that under RS, if $c_R = 0$ the centralized optimum is achieved. However, if $c_R > 0$, the optimal delivery price does not take into account the negative externality it generates on the dine-in channel as in the centralized system. Instead, the platform maximizes delivery revenue ignoring its impact on dine-in congestion cost, and as a result, sets a price that is too low. Because the dine-in price is increasing in the delivery price, the restaurant also lowers its price to induce dine-in customers to purchase. Despite the fact that prices are too low, excess congestion crowds out enough dine-in customers to yield a lower dine-in volume than the centrally optimal solution. Hence, although the platform does give the restaurant access to incremental revenue from delivery orders, it both lowers the restaurant's margins and reduces its dine-in demand, consistent with the real-world examples of Dunn (2018).

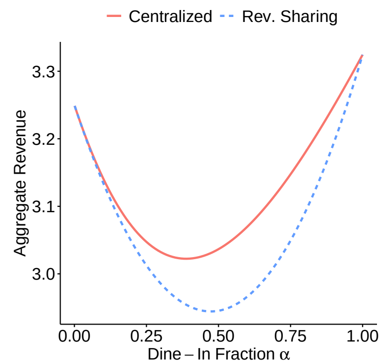


Figure 1. Revenue in the centralized system and a decentralized system with simple revenue sharing. In the example, $\bar{v}_R = 25$, $\bar{v}_H = 13$, $c_R = 22$, and $a = 0.85$.

To understand the cost of this lack of coordination, we conducted a comprehensive numerical study consisting of 30,400 parameter combinations (not counting variations in the revenue share a, which do not affect the central optimum or decentralized system revenue) including values of α from .05 to .95, values of c_R from 0 to 30, values of $\bar{v}_R$ from 0 to 40, and values of $\bar{v}_H$ from 0 to 40.[4] The percentage revenue loss under RS ranges from 0% to 26.2%, with an average of 1.1% and a median of .2%. Because the restaurant industry has very small margins, on the order of 4-5% of sales or even less (Lunden, 2020), even a single-digit percentage reduction in the top line can have an outsize impact on the bottom line. Hence, we conclude that the revenue losses associated with RS can be substantial.

We also observe that the loss under RS is greatest when the dine-in fraction α is intermediate; see Figure 1. If $\alpha \to 0$, almost all customers are in the delivery channel; any level of congestion in the dine-in channel is thus inconsequential to total revenue, and by comparing Propositions 1 and 2, we see that the platform under RS sets a price close to the central optimum. On the other hand, if $\alpha \to 1$, almost all customers are in the dine-in channel; in this case, the delivery channel cannot generate enough congestion to have a significant impact on dine-in customers, so both firms under RS set prices close to the central optimum. Intermediate α, on the other hand, is a "worst case scenario" for the restaurant and the platform: there are many potential customers on both channels, and hence they can have a significant impact on one another, causing aggregate revenue to suffer most.

This suggests that RS performs well if α is very small or very large, or if $c_R = 0$. During the COVID-19 pandemic, for example, α was reduced to essentially zero, implying that RS is sufficient to generate good performance; indeed, during the height of the pandemic, restaurants were grateful for delivery platforms as one of their only ways to generate revenue. However, as society emerges from the pandemic and α increases, the inefficiency of RS contracts will worsen, potentially increasing the conflict between platforms and restaurants as they jockey over commissions and profitability in a post-pandemic landscape (Elejalde-Ruiz, 2021).

Finally, we note that not only is RS sub-optimal in the sense that it cannot achieve the centralized optimal

[4]In 69.7% of these parameter combinations, Assumption 1 holds, while in the remainder it does not; thus, our numerical study verifies that coordination is valuable in the general case when Assumption 1 may not be satisfied.

revenue, but it can even perform worse than if the platform and restaurant used no contract at all. Absent a formal agreement, the platform can act like a regular customer, paying menu price for each order to the restaurant and then delivering to customers with a surcharge (i.e., essentially placing a take-out order on behalf of each delivery customer). This approach, which has been controversial among restaurateurs due to the fact that platforms can deliver their food without their explicit cooperation, was at one time used by Grubhub and Postmates (Saxena, 2019), but has since been outlawed in some localities such as California (Batey, 2021). We discuss this scenario in the online supplement (§C), and show that the aggregate revenue can in fact be *higher* without a contract than under RS. The reason for this is that, by forcing the platform to set the delivery price at least as high as the restaurant's dine-in price, the restaurant avoids the vicious cycle that occurs under RS wherein the platform sets too low a price for delivery, forcing the restaurant to set too low a price for dine-in. However, the no contract scenario can also be significantly worse than RS depending on the contract and problem parameters, underscoring the need for a more effective contract.

4.2 Commission Caps

In an effort to protect restaurant margins, recent regulations in some US cities and states have capped the commission charged by delivery platforms at 15% or similar (Lucas, 2020), initially in response to the COVID-19 pandemic but potentially becoming permanent in some municipalities (Forman, 2021). It is not clear, however, that these caps have their intended effect, namely protecting small independent restaurants, and recent findings in Li & Wang (2021a) have shown that commission caps help larger chain restaurants more than smaller ones. Below, we discuss the implications of commission caps in the context of our model.

Commission caps are a special case of simple revenue sharing in which the commission $1 - a$ paid to the platform for delivery orders is limited below a maximum value, or equivalently, there is a lower bound $\underline{a}$ on the restaurant's share. In the context of our model, a commission cap could potentially benefit the restaurant by increasing its share of the revenue above the level that emerges from the initial (not modeled) contract determination stage. However, because Corollary 1 holds for any revenue share a, we have the following:

Corollary 2. *Under RS with a commission cap, for any cap $\underline{a} < 1$, the equilibrium prices in both channels and the equilibrium dine-in volume are strictly less than centrally optimal.*

In other words, because the underlying contract is still RS, both parties still price too low under a commission cap, and the restaurant still sees less dine-in volume than centrally optimal. Because of this, while a commission cap can help allocate a larger share of the system revenue to the restaurant, aggregate revenue will still be lower than in a centralized system, i.e., money is still left on the table. Combined with the insight that the efficiency of RS is greatest when $\alpha \to 0$ or 1, this suggests that commission caps may be effective in protecting restaurant revenues when almost all customers come from one channel; however, commission caps fail to fix the underlying problem with RS, namely that the platform does not account for the congestion effects of delivery customers on dine-in revenue and, as a result, cannot help increase system profit for intermediate α.

4.3 Delivery Price Floor

We next consider another variation of RS that introduces a potential remedy to the problem of delivery prices that are too low. In practice, platforms sometimes allow restaurants to set the menu price for delivery orders, on top of which the platform adds its own fees: this changes the structure of RS contracts by allowing the restaurant to set two separate menu prices, one for dine-in customers and another for delivery customers. Since the platform can further raise the price by adding its own fees, this amounts to allowing the restaurant to set a price floor in the delivery channel.

To model this scenario, we suppose that the restaurant sets a floor $\underline{\theta}$ for the delivery channel price, and the platform subsequently chooses a delivery price $\theta \geq \underline{\theta}$. Other than the price floor, the setting is equivalent to RS, i.e., the restaurant receives a fraction a of delivery revenues.[5] From Proposition 2, we know that under RS, the platform's revenue is unimodal in θ and that the optimal θ does not depend on p. Hence, with a price floor $\underline{\theta}$ determined by the restaurant, the equilibrium delivery price chosen by the platform will be $\max\{\theta_{RS}, \underline{\theta}\}$. The restaurant thus chooses a price floor that is not binding in equilibrium (in which case the outcome is identical to Proposition 2), or one that binds in equilibrium, in which case the restaurant effectively chooses the price in both channels. We focus on the latter scenario, as this turns out to be always optimal. The restaurant receives all of the dine-in revenue and a fraction a of the delivery revenue, hence its revenue function is the same as equation (2), but now θ is a decision variable, i.e.,

$$z_{R,PF}(p,\theta) = p\alpha\left[\frac{\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta)}{\bar{v}_R + c_R\alpha}\right]^+ + a\theta(1-\alpha)\bar{F}_H(\theta). \tag{3}$$

This is similar to the centralized system revenue function in (1), with one difference: because the second term of (3) is multiplied by $a < 1$, the restaurant will, relative to the centralized system, overvalue dine-in revenue and undervalue delivery revenue. This will cause the platform to set a delivery channel price that is too high relative to the centralized optimum, because it feels the full cost of increased delivery volume (reflected in dine-in congestion cost) but only part of the benefit of increased delivery volume (reflected in its $a < 1$ share of delivery revenue). The dine-in price will also be too high, since the amount of congestion generated in the dine-in channel is less than centrally optimal. This is formalized in the following proposition, in which the equilibrium prices with a price floor are denoted by p_{PF} and θ_{PF}:

Proposition 3. *Under RS with a delivery price floor, for $0 \leq a < 1$, the equilibrium prices in both channels are higher than the central optimum ($\theta_{PF} > \theta^*$ and $p_{PF} > p^*$).*

[5] We note that, sometimes in practice, the platform only splits revenue of the menu price with the restaurant, i.e., the platform keeps 100% of the customer delivery fees. In other words, for a delivery fee f, menu price θ', and restaurant revenue share a', platform revenue is $f + (1-a')\theta'$ while restaurant revenue is $a'\theta'$. Our formulation is equivalent to this if we define a new menu price $\theta \equiv f + \theta'$ and a new revenue share $a \equiv a'\theta'/(f+\theta')$, i.e., for any f, a', and θ', we can provide a θ and a such that our model results in equivalent payments.

Having the power to set a delivery price floor leads the restaurant to set a price floor that is higher than the centrally optimal delivery price, which leads to delivery and dine-in prices that are higher than centrally optimal. This implies that, under revenue sharing with a price floor, customers in *both* channels pay higher prices than under simple revenue sharing or at the central optimum; indeed, it is possible for the restaurant to set the price floor so high that no customers purchase from the platform, causing the delivery channel to shut down. Hence, allowing the restaurant to set a price floor for the delivery channel does give it the power to protect its margins, although it is perhaps too effective in doing so: the result is that the restaurant maximizes its own revenue to the detriment of the entire system, failing to achieve coordination.

5 Coordination with Generalized Revenue Sharing

In this section, we consider how to remedy the problem with RS and coordinate the system. We first note that an obvious way to coordinate the system is for the restaurant to share dine-in revenue with the platform, in addition to the platform sharing delivery revenue with the restaurant, i.e., a "two-way revenue sharing" contract. It is straightforward to show (details omitted) that with appropriately chosen parameters, such a contract coordinates the system because it makes both revenue functions into affine transformations of the aggregate revenue (see Xu *et al.* , 2014). However, requiring the restaurant to share *dine-in* revenue with the platform is impractical, and restaurants are unlikely to agree to this.[6] On the other hand, in the literature on supplier encroachment several contract types have been shown to coordinate a system with direct and indirect channels wherein competition generates negative externalities between channels. These mechanisms include pricing constraints on one or both parties (e.g., Cai, 2010) and non-linear pricing (e.g., Tsay & Agrawal, 2004); as noted earlier, these are impractical for our setting.

Because of this, we turn next to devising a simple and practically implementable coordinating contract. Ideally, such a contract will cause the platform to appropriately account for the congestion that its customers create in a way that only affects payments between the platform and restaurant on delivery orders. In addition, such a contract should avoid non-linear payment schemes or constraints placed upon the pricing power of either firm. Finally, it is also desirable to protect restaurant revenue (i.e., to ensure such revenue is not too low) on delivery orders, and to allocate revenue between the restaurant and platform with a high degree of flexibility. In this section, we propose just such a contract, which we call *generalized revenue sharing* (GRS), and demonstrate that it accomplishes all of these goals.

In a GRS contract, for each order, the platform pays the restaurant a fixed fee τ and a percentage a of the delivery price θ. Under this contract, τ is analogous to a linear wholesale price paid on each order, and a is the restaurant's share of delivery revenue. Using Lemma A.1, the platform's revenue is $z_{P,G}(p,\theta) = \Big((1-a)\theta - \tau\Big)\bar{F}_H(\theta)(1-\alpha)$, where the subscript G denotes the GRS contract. Using Lemma

[6] Two-way revenue sharing also has a verifiability problem: the restaurant has no incentive to accurately disclose its dine-in revenue to the platform, i.e., it prefers to under-report dine-in revenue. Note that the reverse cannot happen: the platform cannot under-report delivery volume or revenue to the restaurant, since the restaurant prepares each meal that the platform delivers.

A.2, the restaurant's revenue is

$$z_{R,G}(p,\theta) = p\alpha\left[\frac{\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta)}{\bar{v}_R + c_R\alpha}\right]^+ + (a\theta+\tau)(1-\alpha)\bar{F}_H(\theta).$$

Even though platform revenue does not directly include the dine-in congestion cost, an appropriately chosen τ can induce the platform to choose the centrally optimal delivery price, as the following result shows.

Proposition 4. *With a generalized revenue sharing contract, for $0 \le a < 1$, if $\tau = (1-a)p^*\Big(c_R\alpha/(\bar{v}_R+c_R\alpha)\Big)$, the platform sets $\theta = \theta^*$ and the restaurant sets $p = p^*$, i.e., the system is coordinated.*

GRS coordinates the system because τ charges the platform for the marginal negative externality generated by a delivery order on the dine-in channel. With linear waiting costs, the coordinating τ takes an appealingly simple form: it is a $1-a$ share of a fraction $c_R\alpha/(\bar{v}_R + c_R\alpha) < 1$ of the centrally optimal dine-in price. This represents the marginal loss in revenue that occurs from an incremental increase in dine-in congestion, multiplied by the platform's share of the delivery revenue, $1-a$. By charging this amount to the platform for each delivery order, the platform will both enjoy a $1-a$ share of the marginal delivery revenue *and* incur a $1-a$ share of the marginal congestion cost it generates for the dine-in channel, and hence choose the centrally optimal delivery price; anticipating this, the restaurant chooses the centrally optimal dine-in price as well.

The coordinating contract is, essentially, a linear wholesale price contract plus a revenue share. This resembles the revenue sharing contract for physical inventories discussed in Cachon & Lariviere (2005), but the contract works in a different way, and the wholesale price plays a different role, in our setting. In Cachon & Lariviere (2005), incentive misalignment arises because of double marginalization, and the coordinating contract is a profit sharing contract because the retailer pays the same fraction of the production cost as the fraction of revenue it retains, i.e., the channel cost is shared in the same proportion as revenue. In our setting, incentive misalignment does not occur because of double marginalization within a single channel, but rather due to a negative externality *between* channels arising from the specific features of the restaurant industry. Because of this, the coordinating revenue sharing contract in Cachon & Lariviere (2005) would assign a per-order fee of zero when applied to our setting (since we have assumed production cost is zero) and would not coordinate our system. By contrast, τ in our coordinating contract depends not on marginal production cost but on the drivers of the negative externality, specifically customer valuations, waiting costs, and the relative sizes of the dine-in and delivery channels.

The simplicity of the GRS contract—observe that it only involves adjustments made to the payment between the platform and the restaurant for each delivery order, and moreover, the resulting payment is linear in the number of orders—means that it is practically implementable in our setting. As discussed previously, it is also desirable to ensure restaurant revenues are not too low on delivery orders, and to flexibly allocate revenue between parties. The next result shows that GRS possesses both properties:

Proposition 5. *Under a coordinating generalized revenue sharing contract:*
(i) For each delivery order, the restaurant receives at least $p^*(c_R\alpha)/(\bar{v}_R + c_R\alpha)$. If $\bar{v}_R < \bar{v}_H$ and $a \ge \bar{v}_R/\bar{v}_H$,

the restaurant earns at least as much for a delivery order as for a dine-in order, i.e., $a\theta^* + \tau \geq p^*$.

(ii) By varying a*, the contract can allocate to the platform any fraction between 0 and* δ *of the delivery revenue, where* $\delta \geq 1 - (\bar{v}_R/\bar{v}_H)(c_R\alpha)/(\bar{v}_R + c_R\alpha)$*. Moreover, if* $c_R\alpha \leq \bar{v}_R \leq \bar{v}_H$*, then* $\delta \geq 1/2$*.*

Proposition 5(i) gives a lower bound on the revenue per order that a restaurant can expect under a GRS contract, and shows this is a fraction of the dine-in price that is increasing in the congestion cost (c_R) and dine-in fraction (α). This shows that if congestion is very costly to the restaurant (c_R is high or there are many dine-in customers), a GRS contract ensures the restaurant receives significant revenue on each delivery order. In fact, in some cases—if delivery customers have sufficiently high valuations, and if the restaurant's share of delivery revenue is high enough—GRS results in a *higher* revenue per order to the restaurant for delivery orders than for dine-in orders. In an industry with many small, independent restaurants with little bargaining power, these are highly desirable features, protecting restaurant revenue (like commission caps or price floors) while *also* coordinating the system.

Part (ii) also shows that while GRS cannot achieve full flexibility in allocating delivery revenue, it does achieve a wide range of allocations. (Note it is impossible to offer full flexibility while also protecting restaurant revenue.) In particular, when dine-in customers and delivery customers value the respective services highly enough, the contract can achieve anywhere from a 50-50 split of delivery revenue (at one extreme, a wholesale price contract with $a = 0$) to full allocation to the restaurant (the other extreme with $a = 1$). Hence, while the precise contract terms are a matter of negotiation between restaurant and platform, GRS possesses the flexibility to accommodate a wide range of scenarios while also achieving coordination.

In short, GRS contracts simultaneously protect the restaurant and increase aggregate revenue, while ensuring a wide range of revenue allocations are possible. The existence of a simple coordinating contract that is practically implementable in this setting is good news for restaurants and platforms. In addition, policymakers interested in regulating this market can take note. After commission caps were instituted in some cities in 2020, platforms disputed them with legal action and lobbying efforts, and Uber Eats even tacked on an additional fee for customers to offset lost commissions (Carson, 2020; Brown, 2020). Our results show that rather than attempting to adjust the parameters of an inherently flawed contract—simple revenue sharing—policymakers and firms alike might consider a different contractual form, generalized revenue sharing, that can mitigate the incentive conflict between platforms and restaurants and maximize joint revenue.

6 Robustness

6.1 Positive Delivery Channel Waiting Costs

In our base model, we assumed that congestion was costly for dine-in customers (i.e., $c_R > 0$) but costless for delivery customers ($c_H = 0$). This assumption simplifies the analysis and allows us to derive clean insights; however, in §D of the online supplement, we numerically relax this assumption to consider the case where delivery customers experience positive waiting costs, i.e., $0 < c_H < c_R$. When $c_H > 0$, a second negative externality is generated: dine-in customers cause delays that are costly to delivery customers. We find that, as one might expect, RS continues to be incapable of achieving coordination in this case, since the platform still fails to account for the negative externality it generates for dine-in customers; hence, our insights about the detrimental effects of RS persist.

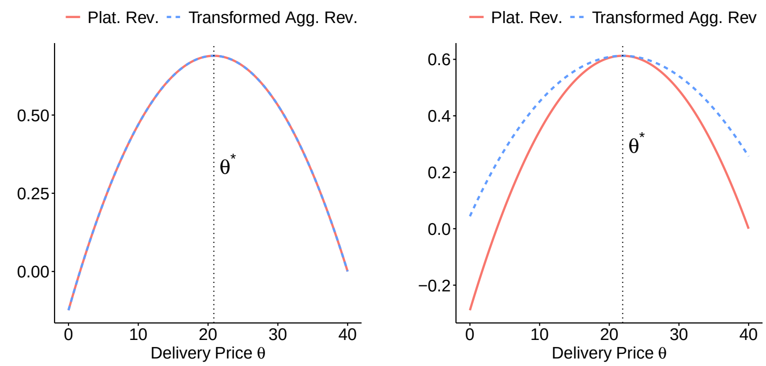
While it can be shown that the GRS contract proposed in Proposition 4 (i.e., assuming $c_H = 0$) does not always coordinate when $c_H > 0$, we numerically observe that it performs exceptionally well, even when using contract parameters optimized for the $c_H = 0$ case as in the proposition: in 2,700 parameter combinations, we find that the average revenue loss relative to the central optimum is just 0.006% (compared to an average loss of 1.2% with RS) while the maximum loss is 0.3% (compared to a maximum of 11.1% with RS). GRS performs extremely well because the restaurant, in receiving a share of the delivery revenue, already has a strong incentive to account for the second externality it generates for the delivery channel; consequently, we conclude that our key insights are robust to this assumption, and even under the more general setting in which *both* delivery and dine-in customers are congestion sensitive, the GRS contract we propose exhibits excellent performance.

6.2 Non-Linear Waiting Costs

Our base model also assumes waiting costs for dine-in customers are linear and equal to $c_R x$, where x is the total purchase volume. Linearity is not necessary to generate our key insights, as shown in §E of the online supplement. Specifically, we show that if dine-in customers have a waiting cost function $W(x)$ that is increasing and convex in x, it remains true that RS cannot coordinate, and moreover, GRS coordinates the system if the platform pays the restaurant a fraction a of delivery revenue and a fixed fee of $\tau = (1-a)\alpha\bar{F}_R(v_R^*)W'(x^*)$ on each delivery order, where v_R^* is the centrally optimal dine-in valuation threshold. The quantity $\alpha\bar{F}_R(v_R^*)$ is the centrally optimal dine-in purchase volume, and $W'(x^*)$ is the marginal congestion cost of a dine-in customer at the centrally optimal total purchase volume x^*. Hence, the coordinating τ is the platform's share of the marginal congestion cost that would be added to the system by an additional delivery customer, although it is no longer a simple fraction of the centrally optimal dine-in price as in the linear congestion cost case.

This shows that for important practical classes of cost functions, such as quadratic or queueing-derived congestion costs, our insights continue to hold. The fact that a contract with a linear per-order fee coordinates even when waiting costs are non-linear is not *a priori* obvious. Figure 2(a) and 2(b) illustrate why linear fees work with non-linear costs.[7] With linear waiting costs (a), the linear fee essentially makes the platform's revenue function an affine transformation of the centralized system revenue at every point, ensuring the

[7] In the figure, the curve labeled "Transformed Agg. Rev." represents a particular affine transformation of the aggregate revenue function, while the curve labeled "Plat. Rev." represents the platform's revenue function under a coordinating generalized revenue sharing contract. In both (b) and (c), the specific non-linear congestion cost function used derives from the time in queue for an $M/M/1$ system, i.e., $W(x) = c_R(1/(\mu - \lambda x) - 1/\mu)$, where $\mu = \lambda = 1$ and x equals the total purchase fraction.



(a) Linear Congestion Costs **(b)** $M/M/1$ Congestion Costs

Figure 2. Platform revenue and an affine transformation of centralized revenue for $c_R = 8$, $\bar{v}_R = 30$, $\bar{v}_H = 40$, $\alpha = .5$, and $a = .85$.

platform chooses the centrally optimal delivery channel price; because of this, the curves in (a) overlap. With non-linear costs (b), a linear fee can no longer accomplish this. However, it can make the platform's revenue function *tangent* to an affine transformation of the centralized system revenue, precisely at the centrally optimal delivery price; hence, the curves in (b) touch only at the centrally optimal price. This induces the platform to choose the proper delivery price, and conditional on this, the restaurant will choose the centrally optimal dine-in price. Consequently, the structure of the coordinating GRS contract—a linear fee for each order plus a fraction of delivery revenues—is the same even with non-linear costs.

7 Conclusion

In this paper, we have considered how to best structure relationships between food delivery platforms and the restaurants with whom they partner. Our results show that the most common contractual relationship between platforms and restaurants has inherent flaws. Simple revenue sharing does not appropriately charge the platform for the negative externality it generates on the dine-in channel, and as a result, cannot coordinate the system and can reduce the profitability of restaurants, many of whom already experience very thin margins. Neither commission caps nor delivery channel price floors can remedy this shortcoming.

This supports the popular view (Houck, 2017; Dunn, 2018; Meyersohn, 2018) that delivery platforms may *not* benefit restaurants, and partnering with a third-party delivery service can lead to a vicious cycle in which service quality and profitability deteriorate despite the increase in volume. On the other hand, a generalized revenue sharing contract can coordinate the system using a simple and practically implementable contract form involving shared delivery order revenue and a fixed fee per order paid by the platform to the restaurant, in addition to protecting restaurant revenues and flexibly allocating delivery revenue.

Given the relative novelty of this business model and the rapidity with which it is evolving, there are many aspects of the relationship between platforms and restaurants that warrant further study. Future work could incorporate other factors that may influence the profitability of a delivery platform or the restaurants that partner with it, such as competition between restaurants or platforms, or the capacity decisions of restaurants (e.g., should a restaurant add more shared kitchen capacity or should it invest in dedicated delivery kitchen capacity, a so-called "ghost kitchen" model, see Hawley, 2020). More broadly, as delivery platforms continue to grow in popularity, it will become increasingly important for them to effectively manage relationships with restaurants in order to avoid conflict that threatens the viability of their business model. Our work illustrates some of the important issues that can arise in these relationships, and offers a simple and practically implementable way to alleviate them and improve coordination of the food delivery supply chain.

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Online Supplement to "Managing Relationships Between Restaurants and Food Delivery Platforms: Conflict, Contracts, and Coordination"

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A Supporting Results

In this section we derive the customer equilibrium purchase volume on each channel for given prices, and provide a sufficient condition for the centralized system to operate both channels.

Lemma A.1. *The equilibrium delivery purchase fraction is given by* $\hat{\gamma}_H = \bar{F}_H(\theta) = \left[(\bar{v}_H - \theta) / \bar{v}_H \right]^+$.

Proof. First, we note that the utility for a delivery customer with realized valuation v_H who decides to purchase is $U_H(\theta; v_H) = v_H - \theta$. Hence, the fraction that purchases is exactly the fraction of customers whose valuation exceeds the delivery price θ, namely $\bar{F}_H(\theta)$. □

Lemma A.2. *The equilibrium dine-in purchase fraction is given by*

$$\hat{\gamma}_R = \left[\frac{\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta)}{\bar{v}_R + c_R\alpha} \right]^+. \tag{A.1}$$

Proof. It is optimal for a given dine-in customer to purchase if and only if $U_R(p, \theta; \gamma_R, \hat{\gamma}_H) \geq 0$. By Lemma A.1, we can replace $\hat{\gamma}_H$ with $\bar{F}_H(\theta)$. So, a dine-in customer with a realized valuation v_R should purchase if and only if $v_R - p - c_R\big(\gamma_R\alpha + (1-\alpha)\bar{F}_H(\theta)\big) \geq 0$. Note that if $\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta) \leq 0$, then the system will be too congested with delivery customers for any dine-in customers to purchase because the net utility is negative for any possible dine-in valuation and positive purchase fraction due to the congestion from delivery customers. We thus must have $\hat{\gamma}_R = 0$, and indeed, taking the positive part of the expression in equation (A.1) gives 0 in this case. Otherwise, i.e., if

$$\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta) > 0, \tag{A.2}$$

then the system can support some dine-in customers. Clearly, for any purchase fraction γ_R, if a customer with valuation v_R purchases, then it must also be optimal for a customer with valuation $v'_R > v_R$ to purchase. Hence, the equilibrium has a threshold structure in which customers with valuations above some threshold purchase, and the others do not. Our requirement for equilibrium is thus to find a threshold valuation v_R that solves the equation $v_R = p + c_R\alpha\bar{F}_R(v_R) + c_R(1-\alpha)\bar{F}_H(\theta)$. This equation has a strictly positive solution

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because the left-hand side (LHS) is smaller than the right-hand side (RHS) for $v_R = 0$, while the LHS is larger than the RHS for $v_R = \bar{v}_R$ by equation (A.2) and the fact that $\bar{F}_R(\bar{v}_R) = 0$. The solution is unique because the LHS is increasing and the RHS is decreasing in v_R, and the purchase fraction $\hat{\gamma}_R = \bar{F}_R(\hat{v}_R)$ will be strictly between 0 and 1 because the threshold $\hat{v}_R$ is strictly between 0 and v_R. Substituting the uniform complementary CDF $\bar{F}_R(v_R) = 1 - \hat{v}_R/\bar{v}_R$ and isolating $\hat{v}_R$ gives the equilibrium threshold $\hat{v}_R$, yielding the corresponding purchase fraction $\hat{\gamma}_R = \bar{F}_R(\hat{v}_R)$ given in equation (A.1). □

Importantly, the customer equilibria of Lemmas A.1 and A.2 hold for any given dine-in and delivery prices, regardless of the contract (or lack thereof) between the restaurant and the delivery platform that precipitated those prices. Our next result provides a sufficient condition for a centralized firm to operate both channels:

Lemma A.3. *If $c_R(1-\alpha)/4 < \bar{v}_R/2 < \bar{v}_H$, then it is centrally optimal to operate both channels.*

Proof. The delivery revenue is concave quadratic in θ and is maximized at $\theta = \bar{v}_H/2$ .. Under our assumption that $\bar{v}_R > c_R(1-\alpha)/2 = c_R(1-\alpha)\bar{F}_H(\bar{v}_H/2)$, equation (A.1) implies that there exists a dine-in price $p > 0$ such that some dine-in customers are willing to purchase with the congestion from delivery customers induced by delivery price θ_{RS}, which will achieve the same delivery revenue along with positive dine-in revenue. So, operating both channels dominates operating just the delivery channel, and any prices resulting in zero dine-in volume are strictly sub-optimal.

Similarly, it is straightforward to show that the optimal dine-in price for a system with no delivery customers is $p = \bar{v}_R/2$, so the maximum possible dine-in revenue that can be achieved is $Z(\bar{v}_R/2, \bar{v}_H)$ (when $\theta = \bar{v}_H$, by Lemma A.1 we have $\hat{\gamma}_H = \bar{F}_H(\bar{v}_H) = 0$). The partial derivative of the aggregate revenue with respect to θ, for values of θ for which the dine-in channel is active, is

$$\frac{\partial Z}{\partial \theta} = (1-\alpha)\Big[f_H(\theta)\Big(\frac{c_R\alpha p}{\bar{v}_R + c_R\alpha} - \theta\Big) + \bar{F}_H(\theta)\Big]. \tag{A.3}$$

For $p = \bar{v}_R/2$ and substituting the density f_H and complementary CDF $\bar{F}_H$, the corresponding FOC, which has a unique solution, is $\theta = \frac{\bar{v}_H}{2} + \frac{1}{2}\left(\frac{\bar{v}_R}{2}\right)\left(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\right) < \bar{v}_H$, where the inequality holds by our assumption that $\bar{v}_R/2 < \bar{v}_H$. At a delivery price $\theta = \bar{v}_H$ and dine-in price $p = \bar{v}_R/2$, no delivery customers will purchase, and positive dine-in volume will be achieved (this corresponds to the optimal solution of the dine-in-only system). Since the unique solution to the FOC is strictly less than $\bar{v}_H$, and because the derivative is moving from positive to negative at this solution (the quantity in equation A.3 is strictly positive at $\theta = 0$), the partial derivative $\partial Z/\partial\theta$ is strictly negative at $\theta = \bar{v}_H$ for $p = \bar{v}_R/2$.

Strictly speaking, the expression for the partial derivative given in (A.3) does not apply at $p = \bar{v}_R/2$ and $\theta = \bar{v}_H$, as the function is not differentiable at this point because delivery volume exactly reaches zero there, creating a kink. So, we define the single-variable function $Z_{\bar{v}_R/2}(\theta)$, which is the aggregate revenue as a function of θ, given a dine-in price of $\bar{v}_R/2$. This function coincides with Z for all price pairs with $p = \bar{v}_R/2$. The function $Z_{\bar{v}_R/2}$ is left-differentiable for $\theta \le \bar{v}_H$, and the left derivative is equal to the expression on the RHS of equation (A.3), with the appropriate prices substituted. By the argument in the preceding paragraph, this left derivative is strictly negative at $\theta = \bar{v}_H$. Hence, there exists $\epsilon > 0$ such that $Z(\bar{v}_R/2, \bar{v}_H) = Z_{\bar{v}_R/2}(\bar{v}_H) < Z_{\bar{v}_R/2}(\bar{v}_H - \epsilon) = Z(\bar{v}_R/2, \bar{v}_H - \epsilon)$. Since the prices $p = \bar{v}_R/2$ and $\theta = \bar{v}_H$ achieve the maximum possible dine-in revenue, the above implies that we can achieve strictly higher aggregate revenue by operating both channels than by operating only dine-in. Hence, any prices resulting in zero delivery volume are strictly sub-optimal.

Having established that it is strictly sub-optimal to operate either channel without the other, we conclude that it is centrally optimal to make both channels active. □

B Proofs of Main Results

Proof of Proposition 1. Differentiating the aggregate revenue (1) yields the first-order conditions (FOC's)

$$p^* = \frac{\bar{v}_R}{2} - \frac{c_R(1-\alpha)\bar{F}_H(\theta^*)}{2} \tag{B.1}$$

and

$$\theta^* = \frac{\bar{v}_H}{2} + \frac{p^*}{2}\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big). \tag{B.2}$$

Solving the system (B.1)-(B.2) yields the unique solution (p^*, θ^*). The expressions can be derived in closed form but are uninformative and we omit them for brevity, but it is important to note that under the assumptions of Lemma A.3, it can be shown that we have $p^* > 0$, which in turn implies that $\theta^* > \bar{v}_H/2$. Substituting from equation (B.1), we have

$$\begin{aligned}
\gamma_R^* &= \frac{[\bar{v}_R - p^* - c_R(1-\alpha)\bar{F}_H(\theta^*)]^+}{\bar{v}_R + c_R\alpha} \\
&= \frac{1}{\bar{v}_R + c_R\alpha}\Big[\bar{v}_R - \frac{1}{2}(\bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta^*)) - c_R(1-\alpha)\bar{F}_H(\theta^*)\Big]^+ \\
&= \frac{1}{2(\bar{v}_R + c_R\alpha)}\Big[\bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta^*)\Big]^+ \\
&> \frac{1}{2(\bar{v}_R + c_R\alpha)}\Big(\bar{v}_R - c_R(1-\alpha)\bar{F}_H(\frac{\bar{v}_H}{2})\Big) > 0.
\end{aligned} \tag{B.3}$$

where the first inequality holds because $\theta^* > \bar{v}_H/2$ and the second inequality holds by our assumption that $\bar{v}_R > c_R(1-\alpha)/2 = c_R(1-\alpha)\bar{F}_H(\bar{v}_H/2)$. Thus, there is strictly positive dine-in volume at the prices (p^*, θ^*). Moreover, we have $p^* \leq \bar{v}_R/2 < \bar{v}_H$ by inspection of equation (B.1) and our assumption that $\bar{v}_R/2 < \bar{v}_H$. From equation (B.2), we then have

$$\theta^* = \frac{\bar{v}_H}{2} + \frac{p^*}{2}\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big) < \frac{\bar{v}_H}{2} + \frac{\bar{v}_H}{2}\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big) < \bar{v}_H,$$

which implies that $\bar{F}_H(\theta^*) > 0$, i.e., there is strictly positive delivery volume at the prices (p^*, θ^*). Thus, both channels are active at (p^*, θ^*), implying that the expressions for the partial derivatives are valid at these prices and that (p^*, θ^*) is indeed a critical point of the aggregate revenue function.

We can restrict our attention to prices $p \in [0, \bar{v}_R]$ and $\theta \in [0, \bar{v}_H]$ without loss of optimality. Consider any prices (p, θ) in this rectangle other than the pair (p^*, θ^*) (here we allow $p = p^*$ or $\theta = \theta^*$, but not both). If one or both channels is not active at (p, θ), then these prices are strictly sub-optimal by Lemma A.3. On the other hand, if both channels are active, then the expressions for the partial derivatives that lead to the FOC's (B.1) and (B.2) are valid in a neighborhood around (p, θ). That these prices fail to solve one or both of the FOC's then implies that there is a strictly improving direction for the aggregate revenue, and hence, the price pair cannot be optimal. Thus, all price pairs other than (p^*, θ^*) are strictly sub-optimal. By the extreme value theorem, the aggregate revenue function Z must achieve a global maximum on the rectangle $[0, \bar{v}_R] \times [0, \bar{v}_H]$ because it is a continuous function on a closed and bounded set. We conclude that the price pair (p^*, θ^*) uniquely achieves the global maximum of the aggregate revenue. $\square$

Proof of Proposition 2. After substituting the uniform complementary CDF $\bar{F}_H(\theta) = 1 - \theta/\bar{v}_H$, the platform's revenue $z_{P,RS}(\theta)$ is concave quadratic in θ with the unique global maximizer $\theta_{RS} = \bar{v}_H/2$, which as noted does not depend on the dine-in price. For restaurant revenue, Assumption 1 implies that $\bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta_{RS}) > 0$, so positive dine-in revenue is achievable given the platform's equilibrium delivery price. We can thus restrict our attention to prices with strictly positive dine-in volume, i.e., $p < \bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta_{RS})$. For the fixed delivery volume $\hat{\gamma}_H = \bar{F}_H(\theta_{RS})$, the restaurant's revenue is concave quadratic in p on the relevant interval, and the first-order condition is sufficient for a global maximum. The FOC is

$$p_{RS} = \frac{\bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta_{RS})}{2} < \bar{v}_R - c_R(1-\alpha)\bar{F}_H(\theta_{RS}), \tag{B.4}$$

implying that the solution to the FOC falls into the appropriate interval and is the globally optimal price. This FOC is replicated in the statement of the proposition. $\square$

Proof of Corollary 1. The comparison for delivery and dine-in prices follows because the FOC (B.1) for the centrally optimal dine-in price p in terms of θ is the same as the restaurant's FOC for RS (equation B.4); the solution to this FOC is increasing in the delivery price; and we have $\theta_{RS} < \theta^*$ if $c_R > 0$. The comparison for the dine-in purchase volume follows from equation (B.3) in the proof of Proposition 1 (to see this, observe that the last non-zero expression in equation (B.3) is equal to the dine-in purchase volume under RS; this follows after substituting the equilibrium dine-in price p_{RS}). $\square$

Proof of Proposition 3. Assuming $\hat{\gamma}_R > 0$, differentiating (3) with respect to p gives $\frac{\partial z_{R,PF}}{\partial p} = \frac{\alpha}{\bar{v}_R + c_R\alpha}\Big(\bar{v}_R - 2p - c_R(1-\alpha)\bar{F}_H(\theta)\Big)$, which is the same FOC (B.1) as in the centralized system's problem. Similarly, for

$\hat{\gamma}_R > 0$, differentiating with respect to θ gives $\frac{\partial z_{R,PF}}{\partial \theta} = (1-\alpha)\Big(a\bar{F}_H(\theta) - (a\theta - \frac{pc_R\alpha}{\bar{v}_R + c_R\alpha})f_H(\theta)\Big)$ and the corresponding FOC $\frac{pc_R\alpha}{a(\bar{v}_R+c_R\alpha)} = \theta - \frac{\bar{F}_H(\theta)}{f_H(\theta)}$. After rearranging, the FOC's (B.1) and (B.2) for maximizing the aggregate revenue can be expressed equivalently as

$$p = \frac{\bar{v}_R}{2} - \frac{c_R(1-\alpha)\bar{F}_H(\theta)}{2} =: g(\theta) \tag{B.5}$$

and

$$p = \Big(\frac{\bar{v}_R + c_R\alpha}{c_R\alpha}\Big)(2\theta - \bar{v}_H) =: h(\theta), \tag{B.6}$$

respectively. The solution (p^*, θ^*) to this system of equations can be viewed as the intersection point of two increasing functions $g(\theta)$ and $h(\theta)$, as defined above. The function $g(\theta)$ is linearly increasing in θ because $\bar{F}_H(\theta)$ is linearly decreasing in θ, and $h(\theta)$ is also linearly increasing in θ. In the price floor setting in which the restaurant sets both prices, the first FOC is the same as equation (B.5). The second FOC becomes

$$p = a\Big(\frac{\bar{v}_R + c_R\alpha}{c_R\alpha}\Big)(2\theta - \bar{v}_H) =: \tilde{h}(\theta). \tag{B.7}$$

Now, suppose that $\bar{v}_R/2 < a\bar{v}_H$. In this case, similar analysis to that in the proof of Proposition 1 implies that a unique solution (p_{PF}, θ_{PF}) exists to the system (B.5) and (B.7), with $p_{PF}, \theta_{PF} > 0$; that this solution is the global maximizer of $z_{R,PF}(p,\theta)$; and that both channels are active at these prices. Any solution to the FOC (B.7) with a positive dine-in price must clearly have $\theta > \bar{v}_H/2$, so we can restrict our attention to $\theta > \bar{v}_H/2$. We have $h(\bar{v}_H/2) = \tilde{h}(\bar{v}_H/2) = 0 < g(\bar{v}_H/2)$ (where $g(\bar{v}_H/2) > 0$ by the condition in Assumption 1 that $\bar{v}_R/2 > c_R(1-\alpha)/4$), implying that h increases to meet g at (p^*, θ^*), while $\tilde{h}$ increases to meet g at $(\tilde{p}, \tilde{\theta})$.

Observe that for $\theta > \bar{v}_H/2$, we have $h(\theta) > ah(\theta) = \tilde{h}(\theta) > 0$. Thus, the function $\tilde{h}$ is strictly below g at θ^*, implying that the intersection point of the increasing functions g and $\tilde{h}$ must be at a larger value of θ than the intersection of g and h, i.e., we have $\theta_{PF} > \theta^*$. Moreover, that the function g is increasing in θ implies that also $p_{PF} = g(\theta_{PF}) > g(\theta^*) = p^*$.

Finally, it is indeed optimal to operate both channels if $\bar{v}_R/2 < a\bar{v}_H$ by an analogous result to Lemma A.3 for the restaurant's revenue. In revenue sharing with a price floor, because $\theta_{PF} > \theta_{RS}$, if the restaurant sets $\underline{\theta} = \theta_{PF}$, then the platform's best response is to set $\theta = \max\{\theta_{RS}, \theta_{PF}\} = \theta_{PF}$. So, the restaurant can implement its own optimal solution by setting $\underline{\theta} = \theta_{PF}$, which is therefore its equilibrium strategy (along with $p = p_{PF}$). Thus, the equilibrium with a simple price floor is the same as the outcome when the restaurant sets both prices, namely $p = p_{PF}$ and $\theta = \theta_{PF}$.

On the other hand, if $a\bar{v}_H < \bar{v}_R/2 < \bar{v}_H$, then the restaurant may or may not prefer to operate the delivery channel. If operating the delivery channel is optimal, then the result follows from the same logic as above (we ignore pathological parameter combinations such that $g(\theta)$ and $\tilde{h}(\theta)$ happen to be identical lines; note that this is not possible if $\bar{v}_R/2 < a\bar{v}_H$, and is ruled out for the centralized problem by Assumption 1).

If the restaurant prefers not to operate the delivery channel, then it sets the price floor at $\underline{\theta} = \bar{v}_H > \theta^*$, in which case its optimal dine-in price is $p = \bar{v}_R/2 > p^*$; thus, in this case also both prices are higher than their centrally optimal counterparts. □

Proof of Proposition 4. Substituting $\tau = (1-a)c_R\alpha p^*/(\bar{v}_R + c_R\alpha)$ into the platform's revenue function $z_{P,G}(p, \theta)$ and differentiating with respect to θ gives $\frac{\partial z_{P,G}}{\partial \theta} = (1-a)(1-\alpha)\Big(\bar{F}_H(\theta) - \Big(\theta - \frac{c_R\alpha p^*}{\bar{v}_R + c_R\alpha}\Big) f_H(\theta)\Big)$. Evaluating this expression at $\theta = c_R\alpha p^*/(\bar{v}_R + c_R\alpha)$ (a lower price would earn the platform strictly negative net revenue) gives

$$\frac{\partial z_{P,G}}{\partial \theta}\Big|_{\theta = p^*} = (1-a)(1-\alpha)\bar{F}_H\Big(p^*\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big)\Big) > 0, \tag{B.8}$$

where the inequality holds because $p^* < \bar{v}_H$ (see the proof of Proposition 1) and the fraction it is multiplied with is less than 1. The FOC has the same solution θ^* as the central planner's optimal delivery price (equation B.2). Being the unique solution to the FOC and a local maximum (derivative is transitioning from positive to negative by equation B.8), this solution is the global maximizer of the platform's revenue. We conclude that the platform sets the centrally optimal delivery price θ^*.

Given $\tau = (1-a)p^*\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big)$ and anticipating that the platform will set $\theta = \theta^*$, the restaurant's revenue is

$$z_{R,G}(p, \theta^*) = p\alpha\left[\frac{\bar{v}_R - p - c_R(1-\alpha)\bar{F}_H(\theta^*)}{\bar{v}_R + c_R\alpha}\right]^+ + (a\theta^* + \tau)\bar{F}_H(\theta^*)(1-\alpha). \tag{B.9}$$

Because the platform's price will be θ^* regardless of the dine-in price, and because τ and a are given, the second term in this expression is independent of p. So, the restaurant sets its price to maximize the total dine-in revenue (the first term in (B.9)). Differentiating the revenue (B.9) gives the same FOC as the central optimum, namely equation (B.1). This is also the same as the restaurant's FOC (B.4) under simple revenue sharing (replacing θ_{RS} with θ^*). Analogous arguments to those in the proof of Proposition 2 imply that the unique solution to this FOC is optimal for the restaurant (and results in positive dine-in volume with delivery price θ^* by the proof of Proposition 1), and the resulting solution is the centrally optimal restaurant price p^*. We conclude that under a generalized revenue sharing contract with $\tau = (1-a)p^*\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big)$ for $0 \le a < 1$, the equilibrium prices are (p^*, θ^*) and the maximum possible aggregate revenue is achieved. □

Proof of Proposition 5. (i) Under a coordinating generalized revenue sharing contract, the platform's payment to the restaurant for each delivery order is

$$a\theta^* + (1-a)p^*\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big) = a(\bar{v}_H - \theta^*) + p^*\Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\Big), \tag{B.10}$$

The first statement in part (i) follows by inspection of equation (B.10): for any value of a, the restaurant's payment on each delivery order is at least $p^*(c_R\alpha)/(\bar{v}_R + c_R\alpha)$. For the second statement in part (i), if

$\bar{v}_R < \bar{v}_H$, then it can be deduced that $a\theta^* + \tau \geq p^*$ if and only if

$$a \geq \bar{v}_R \Big(\frac{2\bar{v}_R - c_R(1-\alpha)}{2\bar{v}_H\bar{v}_R + c_R\alpha(2\bar{v}_H - \bar{v}_R)} \Big). \tag{B.11}$$

If $\bar{v}_R < \bar{v}_H$, then the RHS of equation (B.11) is bounded above by $\bar{v}_R/\bar{v}_H$. The inequality therefore holds by our assumption that $a \geq \bar{v}_R/\bar{v}_H$, completing the proof of (i).

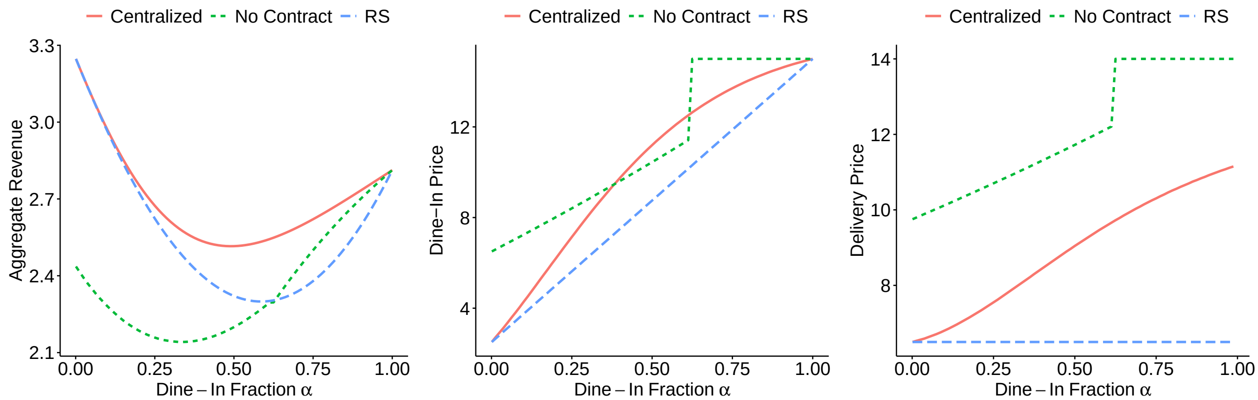
(ii) An allocation of zero to the platform can be approached by setting $a \approx 1$. On the other hand, the maximum allocation that the delivery platform can receive under a coordinating generalized revenue sharing contract is with $a = 0$. In that case, the platform's net revenue per order is $\theta^* - p^* \Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \Big)$. Dividing this quantity by θ^* gives the fraction κ of delivery revenue kept by the platform, namely $\kappa = 1 - \frac{p^*}{\theta^*} \Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \Big)$. From equations (B.1) and (B.2), respectively, we have $p^* \leq \bar{v}_R/2$ and $\theta^* \geq \bar{v}_H/2$. These bounds imply $\frac{1}{2} \leq 1 - \frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \leq 1 - \frac{\bar{v}_R/2}{\bar{v}_H/2} \Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \Big) \leq 1 - \frac{p^*}{\theta^*} \Big(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \Big) = \kappa$; the third inequality and the equality establish the first statement of (ii), while the second inequality, which holds under the additional assumption $c_R\alpha \leq \bar{v}_R \leq \bar{v}_H$, establishes the second statement of (ii). This completes the proof. □

C No Contract Case

Another possible financial arrangement between restaurant and platform is the lack of any formal contract: we call this the "no contract" case. In this case, the platform pays the dine-in menu price for each delivery order, like any other customer, and sets its own markup on top of this price. In this appendix, we briefly discuss this arrangement. The sequence of events is the same as in the rest of the paper, and the equilibrium purchase fractions are again determined from Lemmas A.1 and A.2. Given a dine-in price p, the platform's net revenue from charging a delivery price θ is given by $z_{P,NC}(p, \theta) = (1 - \alpha)(\theta - p)\hat{\gamma}_H$. The platform clearly should set $\theta \geq p$, as otherwise it loses money on every order. So, if $p \geq \bar{v}_H$, then the platform is priced out of the market: any price that achieves positive net revenue for the platform will result in zero delivery volume. If $p < \bar{v}_H$, then the platform can induce positive volume at a money-making price. In this setup, the restaurant earns p for each order, whether dine-in or delivery. So, the restaurant's revenue function is $z_{R,NC}(p, \theta) = p(\alpha\hat{\gamma}_R + (1 - \alpha)\hat{\gamma}_H)$. Solving the game by backward induction, we have the equilibrium prices given in the following proposition. The constants δ_1 and δ_2 depend on $\bar{v}_R$, $\bar{v}_H$, and α.

Proposition C.1. *If $\bar{v}_R/2 < \bar{v}_H \leq 2\bar{v}_R$ and $c_R \leq \min\{\delta_1, \delta_2\}$, then both channels operate in equilibrium, and the dine-in price p_{NC} and delivery price θ_{NC} are given by $p_{NC} = \frac{\bar{v}_H\bar{v}_R(1+\alpha)}{2(2\bar{v}_H\alpha + \bar{v}_R(1-\alpha))}$ and $\theta_{NC} = \frac{\bar{v}_H + p_{NC}}{2}$.*

We omit the proof of Proposition C.1 for brevity. Figure C.1 illustrates that the aggregate revenue without any formal contract can be higher than the aggregate revenue under simple revenue sharing. In the figure, these scenarios are for larger α. That the complete absence of a contract can outperform simple revenue sharing again highlights the problems of the latter. However, the no-contract arrangement is not always an improvement over simple revenue sharing, and sometimes can perform much worse, which we observe in the



(a) Aggregate Revenue **(b)** Dine-In Price **(c)** Delivery Price

Figure C.1. Aggregate revenue and price comparison including no-contract case ($\bar{v}_R = 30, \bar{v}_H = 13, c_R = 50$)

figure for smaller α. Neither contract dominates the other, and both are sub-optimal, unlike generalized revenue sharing.

D Positive Delivery Channel Waiting Costs

In our base model, we assumed that delivery customers are not sensitive to congestion—i.e., they have zero waiting cost, $c_H = 0$—in order to reflect that they are *less* congestion-sensitive than dine-in customers in a tractable way. In this appendix, we relax the assumption by numerically considering the case in which delivery customers have a positive waiting cost $c_H > 0$.

First, consider fixed prices p and θ. The equilibrium dine-in threshold v_R and delivery threshold v_H (derived for the base model in Lemmas A.1 and A.2) are found from the simultaneous solution to the equations

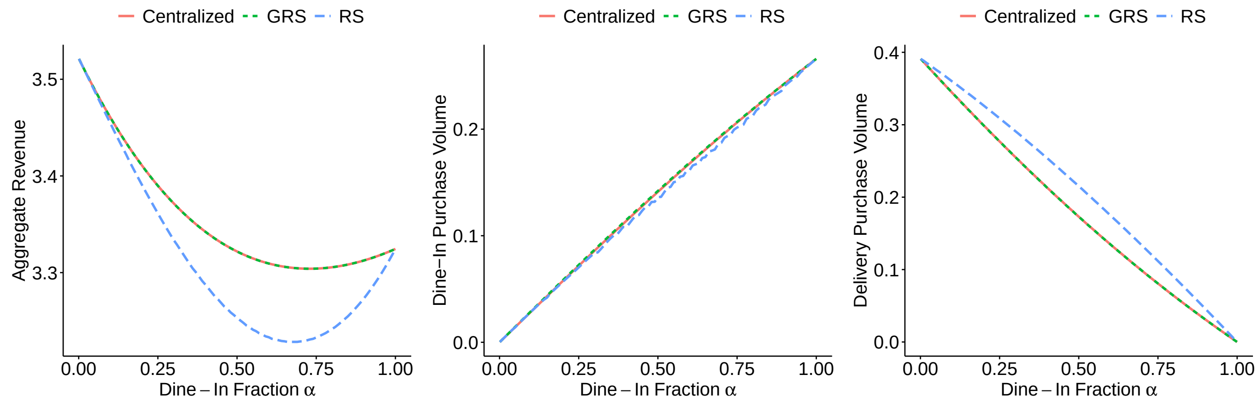
$$v_R = p + c_R(\alpha \bar{F}_R(v_R) + (1-\alpha)\bar{F}_H(v_H)) \tag{D.1}$$

and

$$v_H = \theta + c_H(\alpha \bar{F}_R(v_R) + (1-\alpha)\bar{F}_H(v_H)), \tag{D.2}$$

if this solution results in positive volume on both channels. Equations (D.1) and (D.2) form a linear system with two unknowns, and the system has a unique solution that can be found in closed form.

If the solution does *not* give positive volume on both channels, then in equilibrium at most one channel is active. A dine-in-only equilibrium must have a dine-in threshold that satisfies $v_R = p + c_R\alpha\bar{F}_R(v_R)$. If, for a dine-in purchase fraction determined by the above threshold, there is no delivery volume, then the dine-in-only equilibrium exists. Analogous arguments reveal conditions for a delivery-only equilibrium to exist. It is possible that multiple equilibria exist (i.e., both channels active, and only one channel active); in our numerical analysis, our selection rule is to begin by searching for a multi-channel equilibrium, then



(a) Aggregate Revenue **(b)** Dine-In Volume **(c)** Delivery Volume

Figure D.1. Revenues and purchase volumes under different contracts ($\bar{v}_R = 25, \bar{v}_H = 18, c_R = 22, c_H = 5$)

search for a dine-in-only equilibrium if the multi-channel fails to exist, and finally search for a delivery-only equilibrium if both the previous two equilibria fail to exist. If none of the equilibria exist, then we say that the equilibrium purchase fractions are zero on both channels for the given prices.

Aside from the introduction of $c_H > 0$, the model is otherwise identical to our base model. To analyze this model, we numerically determined the optimal delivery and dine-in prices under both simple revenue sharing (as in §4) and for a centralized system (as in §3.3), and compared both the prices and aggregate revenue for these two different cases. We also calculated the prices and revenue under generalized revenue sharing with terms set based on the analysis in §5 (i.e., the contract terms are determined assuming $c_H = 0$).

Figure D.1 presents the results of this analysis for a representative example. The results suggested in the figure align qualitatively with our previous findings. Namely, (i) simple revenue sharing fails to coordinate because the platform sets the delivery price too low under simple revenue sharing, crowding out dine-in customers and reducing the aggregate revenue, and (ii) generalized revenue sharing achieves near-optimal aggregate revenue. Indeed, in the figure, the outcomes under generalized revenue sharing and the central optimum appear extremely close, perhaps even identical. However, as we next prove via a counterexample, generalized revenue sharing with the parameters from §5 may not, in general, perfectly coordinate the system for $c_H > 0$:

Proposition D.1. *For $c_H > 0$, a generalized revenue sharing contract with parameters defined in Proposition 4 (i.e., assuming $c_H = 0$) does not always achieve the maximum aggregate revenue.*

Proof. Consider an instance of our model with $\bar{v}_R = \bar{v}_H = 1$, $c_R = c_H = 1$, and $\alpha = 1/2$. In this case, for prices for which both channels are active, the simultaneous solution of equations (D.1) and (D.2) gives $\hat{\gamma}_R = \frac{1}{2} + \frac{\theta}{4} - \frac{3p}{4}$ and $\hat{\gamma}_H = \frac{1}{2} + \frac{p}{4} - \frac{3\theta}{4}$. Substituting these values into the aggregate revenue function and solving the first-order conditions yields the prices $p^* = \theta^* = 1/2$, achieving aggregate revenue of $1/4$, and indeed both channels are active at these prices with equilibrium purchase fractions $\hat{\gamma}_R = \hat{\gamma}_H = 1/4$. Moreover, the function is concave (the Hessian matrix is easily verified to be negative definite), so these prices are indeed optimal among prices that induce both channels to be active. Finally, it is optimal to induce both channels to be active because solving the problem for a single channel yields a maximum revenue of 1/6. Thus, $p^* = \theta^* = 1/2$ are the centrally optimal prices for this instance.

Now, consider our generalized revenue sharing contract with the terms found in §5. Namely, for a revenue share a, we let the per-order fee $\tau = (1-a)p^*(c_R\alpha/(\bar{v}_R + c_R\alpha)) = (1-a)/6$. For a dine-in price $p = p^* = 1/2$, we have $\hat{\gamma}_R = \frac{1}{2} + \frac{\theta}{4} - \frac{3}{8} = \frac{1}{8} + \frac{\theta}{4} > 0$, so, in the neighborhood of this dine-in price, there will be positive dine-in volume no matter the delivery price, meaning that the formulas above for $\hat{\gamma}_R$ and $\hat{\gamma}_H$ are valid in this neighborhood (we can ignore prices θ high enough that there is no delivery volume, as these are clearly sub-optimal for the platform). Substituting $\hat{\gamma}_R$, $\hat{\gamma}_H$, and $\tau = (1-a)/6$ into the platform's generalized revenue sharing revenue function and differentiating gives the first-order condition $\theta = 5/12 + p/6$, which is the platform's optimal price as a function of the dine-in price and will also yield positive delivery volume for any dine-in price. We then substitute this expression for θ into the restaurant's revenue function under generalized revenue sharing, which eliminates θ and yields a function only of the dine-in price p. Differentiating this revenue function for the restaurant gives the derivative $\frac{30-68p+a(3+2p)}{48}$. Then, substituting $p = 1/2$ into the derivative gives $-(1-a)/12 < 0$, implying that there is a strictly improving direction for the restaurant. Thus, the restaurant will not set $p = 1/2$, implying that generalized revenue sharing does not achieve the optimal aggregate revenue. $\square$

Because the generalized revenue sharing contract with terms set assuming $c_H = 0$ does not always perfectly coordinate, we conducted a comprehensive numerical study to evaluate its performance. We considered 2,700 parameter combinations (21,600 combinations if we account for variations in the revenue share a) and determined the aggregate revenue under the centralized solution, simple revenue sharing, and generalized revenue sharing using the contract terms from §5. We considered values of α from .1 to .9, values of c_R from 10 to 30, values of c_H from 2 to 8, values of $\bar{v}_R$ from 8 to 40, values of $\bar{v}_H$ from 8 to 40, and values of the restaurant's delivery revenue share a from .6 to .95.

We observed that the percentage revenue loss under simple revenue sharing ranges from 0% to 11.1%, with an average of 1.2% and a median of .6%. By contrast, under generalized revenue sharing, the percentage revenue loss ranges from 0% to 0.3%, with an average of .006% and a median of .0013%. Thus, encouragingly, in line with Figure D.1, we see from the comprehensive study that generalized revenue sharing achieves near-optimal performance over a wide range of parameters. These findings suggest that generalized revenue sharing is robust to delivery customers who exhibit some degree of congestion-sensitivity. A key reason for this is that, since the restaurant receives a share of the platform revenue (as well as a fixed fee for each delivery order), the restaurant when setting its price already internalizes some (perhaps even most) of the negative externality that dine-in customers impose on congestion-sensitive delivery customers.

E General Convex Waiting Costs

In this section we extend our model to incorporate general convex increasing waiting costs. §E.1 contains the main results, and §E.2 contains supporting results.

E.1 Main Results

Suppose dine-in customers have a congestion cost function $W(x)$, where x is the combined purchase volume across both channels and $W(0) = 0$, $W'(x) > 0$, and $W''(x) > 0$. Under this assumption, given purchase fractions γ_R and γ_H, the utility of a dine-in customer with valuation v_R is $U_R(p, \theta) = v_R - p - W\Big(\alpha\gamma_R + (1-\alpha)\gamma_H\Big)$. The utility of delivery customers is identical to the base model. Because of this, as with linear congestion costs, the equilibrium delivery purchase fraction is $\hat{\gamma}_H = \bar{F}_H(\theta)$ (Lemma A.1 continues to hold). The unique dine-in equilibrium is described by a threshold valuation $\hat{v}_R$ such that only customers with valuations above $\hat{v}_R$ purchase, as shown in the following lemma:

Lemma E.1. *With general convex waiting costs for dine-in customers, if $p + W\big((1-\alpha)\bar{F}_H(\theta)\big) < \bar{v}_R$, then a unique dine-in equilibrium exists, with dine-in valuation threshold $\hat{v}_R$ and corresponding dine-in purchase fraction $\hat{\gamma}_R := \bar{F}_R(\hat{v}_R)$, such that customers purchase whose valuations exceed $\hat{v}_R$. The equilibrium threshold $\hat{v}_R < \bar{v}_R$ uniquely solves*

$$\hat{v}_R = p + W\Big(\alpha\bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta)\Big). \tag{E.1}$$

Otherwise, i.e., if $p + W\big((1-\alpha)\bar{F}_H(\theta)\big) \geq \bar{v}_R$, the equilibrium dine-in volume $\hat{\gamma}_R = 0$.

Proof. We have $\hat{\gamma}_H := \bar{F}_H(\theta)$ by Lemma A.1. Thus, if a fraction γ_R of dine-in customers purchase, then for prices p and θ, the purchase utility of a dine-in customer with realized valuation v_R is $U_R(p, \theta) = v_R - p - W\Big(\alpha\gamma_R + (1-\alpha)\bar{F}_H(\theta)\Big)$. If $p + W((1-\alpha)\bar{F}_H(\theta)) \geq \bar{v}_R$, then dine-in customers are priced out of the market and the equilibrium dine-in purchase volume $\hat{\gamma}_R = 0$; by convention we let $\hat{v}_R = \bar{v}_R$ in this case.[2] Alternatively, if $p + W((1-\alpha)\bar{F}_H(\theta)) < \bar{v}_R$, then positive dine-in volume can be supported, and the threshold structure can be seen by inspection of the utility. Equation (E.1) must hold in equilibrium, as otherwise some customers would be acting sub-optimally. This equation has a unique solution, namely the equilibrium threshold $\hat{v}_R$. □

The aggregate revenue is $Z(p, \theta) = p\alpha\hat{\gamma}_R + \theta(1-\alpha)\hat{\gamma}_H$, where $\hat{\gamma}_R = \bar{F}_R(\hat{v}_R)$. Using (E.1), we can eliminate

[2] Dine-in customers are also priced out of the market if W is not defined for $x \geq (1-\alpha)\bar{F}_H(\theta)$ (e.g., if this would overload the queue in the $M/M/1$ setting). Technically, equation (E.1) is not valid in this case. However, the only points where this is relevant have zero dine-in volume, i.e., only the delivery channel is active. Under our assumptions in Lemmas E.2 and E.3, positive dine-in volume is achievable at the delivery-revenue-maximizing price θ_{RS}. Thus, it cannot be centrally optimal to have only the delivery channel be active because the dine-in revenue can be improved (from zero) without harming the delivery revenue (nor can it be an equilibrium under simple revenue sharing, for similar reasons). We can thus restrict our attention to pairs $(\hat{v}_R, \theta)$ with positive dine-in volume without affecting the equilibrium or the optimal solution; for such pairs, equation (E.1) applies.

p and express the aggregate revenue as a function only of $\hat{v}_R$ and θ by

$$Z(\hat{v}_R,\theta) = \alpha\bar{F}_R(\hat{v}_R)\Big(\hat{v}_R - W\big(\alpha\bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta)\big)\Big) + \theta(1-\alpha)\bar{F}_H(\theta). \tag{E.2}$$

Lemmas E.2 and E.3 in §E.2 give sufficient conditions for concavity of the revenue function in the relevant range and for it to be optimal to operate both channels; henceforth, we assume that these conditions hold. We now solve the centralized problem.

Proposition E.1. *With general convex waiting costs for dine-in customers, the unique optimal solution satisfies $(v_R^*,\theta^*) \in (\bar{v}_R/2,\bar{v}_R)\times(\bar{v}_H/2,\bar{v}_H)$, and it is given by the unique simultaneous solution on this set of the system*

$$v_R^* = \frac{\bar{v}_R}{2} + \frac{W(x^*) + \alpha\bar{F}_R(v_R^*)W'(x^*)}{2} \tag{E.3}$$

and

$$\theta^* = \frac{\bar{v}_H}{2} + \frac{\alpha\bar{F}_R(v_R^*)W'(x^*)}{2}, \tag{E.4}$$

where $x^* := \alpha\bar{F}_R(v_R^*) + (1-\alpha)\bar{F}_H(\theta^*)$.

Proof. Equations (E.3) and (E.4) are the FOC's found by differentiating the aggregate revenue (E.2) with respect to $\hat{v}_R$ and θ, respectively. These equations are necessary for optimality other than on the boundaries, which are sub-optimal by Lemma E.3 (in addition to the upper boundaries where $\hat{v}_R = \bar{v}_R$ or $\theta = \bar{v}_H$, the lower boundaries where $\hat{v}_R = 0$ or $\theta = 0$—and corner points where $(\hat{v}_R,\theta) = (0,\bar{v}_H)$ or $(\bar{v}_R, 0)$—are also trivially sub-optimal). Furthermore, inspection of these equations reveals that any solution to them must satisfy $\hat{v}_R \geq \bar{v}_R/2$ and $\theta \geq \bar{v}_H/2$.

We can therefore restrict our attention to pairs $(\hat{v}_R,\theta)$ with $\bar{v}_R/2 \leq \hat{v}_R \leq \bar{v}_R$ and $\bar{v}_H/2 \leq \theta \leq \bar{v}_H$. For some $\bar{v}_R/2 \leq \hat{v}_R \leq \bar{v}_R$, consider the FOC (E.4) for θ. At $\theta = \bar{v}_H/2$, the LHS is either equal to the RHS if $\hat{v}_R = \bar{v}_R$ or strictly less than the RHS if $\hat{v}_R < \bar{v}_R$. In the former case, the unique solution is $\theta = \bar{v}_H/2$. In the latter case, at $\theta = \bar{v}_H$, the LHS is strictly greater than the RHS by the assumption in Lemma E.3 that $\alpha W'(\alpha/2) < 2\bar{v}_H$. Therefore, by the intermediate value theorem, the FOC (E.4) has a solution between $\bar{v}_H/2$ and $\bar{v}_H$. Moreover, the LHS of this equation is increasing in θ, while the RHS is decreasing in θ by the convexity of W. Thus the solution to equation (E.4) is unique for each $\hat{v}_R$, continuous in $\hat{v}_R$, and within the appropriate range, and we can express it as a function $\theta^*(\hat{v}_R)$ that satisfies $\bar{v}_H/2 \leq \theta^*(\hat{v}_R) < \bar{v}_H$.

Now, consider the FOC (E.3) for the dine-in threshold $\hat{v}_R$, and substitute the optimal delivery price function $\theta^*(\hat{v}_R)$; we have just seen that this function is well-defined and continuous. Specifically, we have

$$\hat{v}_R = \frac{\bar{v}_R}{2} + \frac{W(\alpha\bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta^*(\hat{v}_R))) + \alpha\bar{F}_R(\hat{v}_R)W'(\alpha\bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta^*(\hat{v}_R)))}{2} \tag{E.5}$$

At $\hat{v}_R = \bar{v}_R/2$, the LHS of equation (E.5) is strictly less than the RHS. At $\hat{v}_R = \bar{v}_R$, the LHS is strictly greater than the RHS by the assumption in Lemma E.3 that $W((1-\alpha)/2) < \bar{v}_R$. Thus, by the intermediate value theorem, equation (E.5) has a solution, which is also a solution to the system (E.3) and (E.4) because $\theta^*(\hat{v}_R)$ solves equation (E.4) by definition. Such a solution must then satisfy $\bar{v}_R/2 < \hat{v}_R < \bar{v}_R$ and also $\bar{v}_H/2 < \theta < \bar{v}_H$ (with the first inequality in the latter chain becoming strict because $\hat{v}_R < \bar{v}_R$).

By the above arguments, we can further restrict our focus to $(\hat{v}_R, \theta)$ in the open set $(\bar{v}_R/2, \bar{v}_R) \times (\bar{v}_H/2, \bar{v}_H)$, on which the aggregate revenue Z is strictly concave by Lemma E.2. Considering the function Z with this set as its domain, we have a twice continuously differentiable, strictly concave function on a convex open set, which can have at most one optimal point—see, e.g., Boyd & Vandenberghe (2004, Chapter 9). Thus, since a solution to the FOC's exists and is therefore optimal by the concavity of Z, on this set it is both the only solution to the FOC's and the unique maximizer of the aggregate revenue. Because points outside of the set are strictly sub-optimal, we conclude that the unique solution to the FOC's—defined by equations (E.3) and (E.4)—is the global optimum of the centralized problem. □

We note that the optimal dine-in price p^* can be immediately calculated from (v_R^*, θ^*) using equation (E.1). Next, we show that just as with linear congestion costs, with general convex congestion costs, simple revenue sharing results in strictly sub-optimal aggregate revenue, and moreover, that generalized revenue sharing coordinates:

Proposition E.2. *With general convex waiting costs for dine-in customers:*
(i) The aggregate revenue under simple revenue sharing is strictly sub-optimal for any $0 \leq a < 1$*, and the delivery price* $\theta_{RS} < \theta^*$*.*
(ii) A generalized revenue sharing contract with $0 \leq a < 1$ *and* $\tau = (1-a)\alpha \bar{F}_R(v_R^*) W'(x^*)$ *induces the centrally optimal prices* (p^*, θ^*) *in equilibrium, i.e., it coordinates the system.*

Proof. (i) The platform's price under simple revenue sharing is $\theta_{RS} = \bar{v}_H/2$, as in the linear case, because the platform's revenue function under simple revenue sharing is the same regardless of the form of the congestion cost function W. By Proposition E.1, we have $\theta^* > \bar{v}_H/2$. Thus, θ_{RS} cannot be part of the optimal solution, so the aggregate revenue under simple revenue sharing must be strictly sub-optimal.

(ii) By Proposition E.1, the optimal solution (v_R^*, θ^*) is the unique solution of the system of FOC's (E.3) and (E.4), and this solution defines an optimal price pair (p^*, θ^*). Lemma E.4 implies that we can implement θ^* on the delivery channel under a generalized revenue sharing contract with the parameters in the proposition statement.

Substituting out the price p using equation (E.1) in Lemma E.1 with $\theta = \theta^*$, the restaurant's revenue $z_{R,G}$ under generalized revenue sharing can be expressed in terms of $\hat{v}_R$ as

$$z_{R,G}(\hat{v}_R) = \alpha \bar{F}_R(\hat{v}_R)\Big(\hat{v}_R - W\big(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta^*)\big)\Big) + (a\theta^* + \tau(a, \theta^*))(1-\alpha)\bar{F}_H(\theta^*). \tag{E.6}$$

Differentiating gives

$$\begin{aligned} z'_{R,G}(\hat{v}_R) = & -\alpha f_R(\hat{v}_R)\big(\hat{v}_R - W(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta^*))\big) \\ & + \alpha \bar{F}_R(\hat{v}_R)\big(1 + \alpha f_R(\hat{v}_R) W'(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta^*))\big), \end{aligned} \tag{E.7}$$

which yields the FOC

$$\hat{v}_R - W(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta^*)) = \frac{\bar{F}_R(\hat{v}_R)}{f_R(\hat{v}_R)}\big(1 + \alpha f_R(\hat{v}_R) W'(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta^*))\big). \tag{E.8}$$

The LHS is strictly increasing in $\hat{v}_R$, while the RHS is decreasing in $\hat{v}_R$ because W' is increasing by convexity and the reciprocal failure rate is decreasing for the uniform distribution. Thus, the equation has exactly one solution because the LHS is smaller than the RHS for $\hat{v}_R = 0$, and larger than the RHS for $\hat{v}_R = \bar{v}_R$ by the fact that $\bar{F}_H(\theta^*) < 1/2$ and the assumption in Lemma E.3 that $W((1-\alpha)/2)) < \bar{v}_R$. We have $z'_{R,G}(0) > 0$, so this solution is the global maximizer of the restaurant's revenue.[3] Moreover, when evaluated at $\theta = \theta^*$ and after simplification, the FOC (E.3) for the centrally optimal dine-in threshold is identical to equation (E.8). Both equations thus have the same unique solution, which by Proposition E.1 is v_R^*. To induce the dine-in valuation threshold v_R^* under the delivery price θ^*, the restaurant must set the dine-in price at p^* by definition. Thus, generalized revenue sharing induces the centrally optimal solution. □

E.2 Supporting Results for General Convex Costs

Lemma E.2. *If (i) the function W and its first two derivatives are well-defined for $x \leq 1/2$ and (ii) $(1-\alpha)W'(1/2) \leq 4\bar{v}_H$, then the aggregate revenue* (E.2) *is jointly (and strictly) concave in $(\hat{v}_R, \theta)$ on the open rectangle $(\bar{v}_R/2, \bar{v}_R) \times (\bar{v}_H/2, \bar{v}_H)$.*

Proof. We will show the convexity of the negative of the aggregate revenue, i.e., of $-Z(\hat{v}_R, \theta)$. Straightforward manipulations reveal that the second partial derivatives satisfy $-Z_{\hat{v}_R \hat{v}_R} > 0$ and $-Z_{\theta\theta} > 0$. The convexity thus depends on the sign of the determinant of the Hessian matrix. This determinant (which includes $W''(x)$, hence the need for this to be well-defined in the relevant range) has nonnegative terms—including a strictly positive term—and a single term of ambiguous sign, which after omitting strictly positive factors is

$$4\bar{v}_H - (1-\alpha)W'(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta)). \tag{E.9}$$

For $\hat{v}_R$ and θ in the ranges in the lemma statement, we have $\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta) < 1/2$, and thus the quantity in equation (E.9) is positive by our assumption that $(1-\alpha)W'(1/2) \leq 4\bar{v}_H$ and the fact that $W'(x)$

[3] If the cost function is unbounded and $\hat{v}_R = 0$ is infeasible at $\theta = \theta^*$ (e.g., if the cost is the $M/M/1$ delay and $\hat{v}_R = 0$ would overload the queue), then we can use the infimum of the set of achievable values of $\hat{v}_R$ (because it is optimal to operate both channels, the set of achievable values of $\hat{v}_R$ is non-empty and extends strictly below $\bar{v}_R$). Within some $\epsilon > 0$ of this infimum, the congestion cost is arbitrarily large and thus the LHS of equation (E.8) is strictly negative, while the RHS is strictly positive. For $\hat{v}_R = \bar{v}_R$, the LHS is again larger than the RHS, so we can similarly conclude that the equation has exactly one solution. Also, the derivative (E.7) of the restaurant's revenue is strictly positive within ϵ of the infimum for the same reason that W is arbitrarily large; thus, the solution to equation (E.8) maximizes the restaurant's revenue.

is increasing in x by the convexity of W. Hence, the determinant of the Hessian is strictly positive. Since the diagonal entries are also strictly positive, the Hessian of $-Z$ is positive definite. Thus, the function $-Z(\hat{v}_R, \theta)$ is strictly convex, so the aggregate revenue $Z(\hat{v}_R, \theta)$ is strictly concave. $\square$

Lemma E.3. *If $\alpha W'(\alpha/2) < 2\bar{v}_H$ and $W((1-\alpha)/2) < \bar{v}_R$, then higher revenue can be achieved with both channels active than with only one channel active.*

Proof. The partial derivative of the aggregate revenue in θ is

$$\frac{\partial Z}{\partial \theta} = (1-\alpha)\Big[\bar{F}_H(\theta) - f_H(\theta)\big(\theta - \alpha \bar{F}_R(\hat{v}_R) W'(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)\bar{F}_H(\theta))\big)\Big]. \tag{E.10}$$

Let $\tilde{v}_R$ be the dine-in valuation threshold that maximizes dine-in revenue with zero delivery volume (i.e., with $\theta = \bar{v}_H$); it is straightforward to verify that we have $\bar{F}_R(\tilde{v}_R) < \bar{F}_R(\bar{v}_R/2) = 1/2$. Combined with the convexity of W and our assumption that $\alpha W'(\alpha/2) < 2\bar{v}_H$, this implies

$$\alpha \bar{F}_R(\tilde{v}_R) W'(\alpha \bar{F}_R(\tilde{v}_R)) < \frac{\alpha W'(\alpha/2)}{2} < \bar{v}_H. \tag{E.11}$$

Substitution into equation (E.10) then implies that $\partial Z/\partial\theta|_{\hat{v}_R = \tilde{v}_R, \theta = \bar{v}_H} < 0$ (note that we could define an auxiliary function fixing $\hat{v}_R = \tilde{v}_R$ and take its left derivative in θ to account for the fact that the functional form changes at $\theta = \bar{v}_H$, but we omit these steps for brevity). Thus, the aggregate revenue can be improved from the maximum dine-in-only revenue by recruiting some delivery customers, and operating both channels is strictly better than operating only dine-in.

Next, setting $\theta = \theta_{RS} = \bar{v}_H/2$ maximizes delivery revenue, and we have $W((1-\alpha)\bar{F}_H(\bar{v}_H/2)) = W((1-\alpha)/2)$. Under our assumption that $W((1-\alpha)/2) < \bar{v}_R$, Lemma E.1 thus implies that at $\theta = \theta_{RS}$, there exists $\hat{v}_R < \bar{v}_R$ with corresponding dine-in price $p = \hat{v}_R - W(\alpha \bar{F}_R(\hat{v}_R) + (1-\alpha)/2) > 0$, i.e., positive dine-in volume is achievable with a strictly positive dine-in price. It is thus possible to achieve the maximum possible delivery revenue and positive dine-in revenue simultaneously, so operating both channels dominates operating just the delivery channel. We conclude that under the assumptions in the lemma statement, higher revenue can be achieved with positive volume on both channels than with only one channel active. $\square$

Lemma E.4. *For any $0 \le a < 1$ and $0 < \tilde{\theta} < \bar{v}_H$, a generalized revenue sharing contract with parameters a and $\tau = \tau(a, \tilde{\theta}) = (1-a)(2\tilde{\theta} - \bar{v}_H)$ will induce the platform to choose the delivery price $\tilde{\theta}$. Moreover, the optimal delivery price θ^* can be induced by setting $\tau = \tau(a, \theta^*) = (1-a)\alpha \bar{F}_R(v_R^*) W'(x^*)$.*

Proof. Under a generalized revenue sharing contract with parameters a and τ, the platform's revenue $z_{P,G}$ is given by

$$z_{P,G}(\theta) = \big((1-a)\theta - \tau\big)(1-\alpha)\bar{F}_H(\theta). \tag{E.12}$$

As $\bar{F}_H(\theta) = 1 - \theta/\bar{v}_H$ for $\theta \le \bar{v}_H$, the function $z_{P,G}$ is concave quadratic in θ for fixed a and τ. The FOC, namely $\theta = \frac{\bar{v}_H}{2} + \frac{\tau}{2(1-a)}$, is thus sufficient for a global maximum. Setting the RHS equal to $\tilde{\theta}$ and isolating τ

gives $\tau = (1-a)(2\tilde{\theta} - \bar{v}_H)$. Thus, under the τ that satisfies this equation, it is optimal for the platform to set its price at $\tilde{\theta}$, as long as $0 < \tilde{\theta} < \bar{v}_H$, which holds by assumption. Note that for $\theta < \bar{v}_H/2$, the required τ is negative, but the coordinating τ for any a is always positive because $\theta^* > \bar{v}_H/2$ by Proposition E.1. Finally, because also the optimal price $\theta^* < \bar{v}_H$ by Proposition E.1, the arguments above imply that θ^* is implementable under generalized revenue sharing for $0 \leq a < 1$ by setting $\tau = (1-a)(2\theta^* - \bar{v}_H)$. Substituting equation (E.4) for θ^* into this formula for τ yields the equation for $\tau(a, \theta^*)$ given in the lemma statement. $\square$

References

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