EE 338 :Filter Design Assignment

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Filter No. 40

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1 Filter - 1 Bandpass Filter

1.1 Un-normalized Discrete Time Filter Specifications

$B_L = 49.2kHz$ and $B_H = 59.2kHz$

- Passband: 49.2 - 59.2 kHz.
- Transition band width is 2 kHz on either side of the bandpass range.
- Stopband: 0 - 47.2 kHz and 61.2 - 160 kHz
- Tolerance in 0.15 in magnitude for both passband and stopband.
- Passband and stopband natures are both *Monotonic.*
- Discrete time sampling frequency is 320 kHz.

1.2 Normalized Digital Filter Specification

The frequency normalization using the sampling frequency $F_s = 320$ kHz is done by the following transform

$$f = \frac{F}{F_s}$$

where F is in range [-$F_s/2$,$F_s/2$] and therefore $\omega = 2\pi$f.
Specifications:

- Passband: 0.3075π to 0.37π
- Transition band width is now 0.0125π
- Stopband: 0 - 0.295π and 0.3825π - π
- Tolerance in 0.15 in magnitude for both passband and stopband.
- Passband and stopband natures are both *Monotonic.*

1.3 Analog Filter Specifications using the Bilinear Transformation

The bilinear transform $\omega = \tan(\frac{\Omega}{2})$.

ω	Ω
0	0
0.295π	0.4997
0.3075π	0.5245
0.37π	0.6569
0.3825π	0.6854
π	inf

Table 1: Bilinear Transformation

1.4 Bandpass Frequency Transformation Specifications

The bandpass transformation is given by $\Omega_L = \frac{\Omega^2 - \Omega_o}{B\Omega}$.

- $\Omega_o = \sqrt{\Omega_{p1} \times \Omega_{p2}} = 0.5869$
- B $= \Omega_{p1} - \Omega_{p2} = 0.1324$

Ω	Ω_L
0	-inf
0.4997	-1.432
0.5245	-1
0..5869	0
0.6569	1
0.6854	1.379
inf	inf

Table 2: Bandpass Frequency Transformation

1.5 Frequency Transformed Lowpass Analog Filter Specifications

- Passband edge $= \Omega_{L_p} = 1$
- Stopband edge $= \min\{|\Omega_{L_{s1}}|, |\Omega_{L_{s2}}|\} = \min\{1.4332, 1.3796\} = 1.3796$
- Tolerances $\delta_1 = \delta_2 = 0.15$ in magnitude for both Passband and Stopband
- Passband Nature is *Monotonic*
- Stopband Nature is *Monotonic*

1.6 Analog Lowpass Transfer Function Type

Since the passband specification is monotonic the filter type is *Butterworth*, order of which is given as N $\geq \frac{log(D_2/D_1)}{2 \times log(\Omega_{L_s}/\Omega_{L_p})}$.
where $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$ and $D_2 = \frac{1}{\delta_2^2} = 43.444$.

$$N \geq \frac{log(D_2/D_1)}{2 \times log(\Omega_{L_s}/\Omega_{L_p})} = 7.346$$

We choose N = 8.

$$1 + (\frac{s}{j\Omega_c})^{16} = 0$$

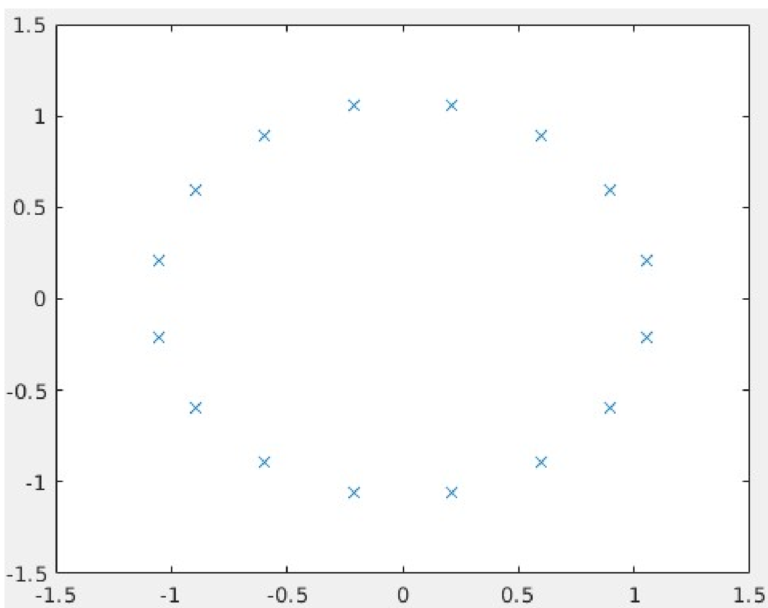

Figure 1: Roots of the above equation

By selecting the left half planes as the poles of the analog filter we get the transfer function $H_analog(s_L)$ as

$$H_{analog}(s_L) = \frac{\Omega_c^N}{s_L^8 + 5.514s_L^7 + 15.2s_L^6 + 27.2s_L^5 + 34.4s_L^4 + 31.48s_L^3 + 20.36s_L^2 + 8.547s_L + 1.794}$$

where $\Omega_c = (\frac{\Omega_{Lp}}{D1^{1/2N}} + \frac{\Omega_{Ls}}{D2^{1/2N}})/2$.

1.7 Analog Bandpass Transfer Function

The bandpass filter is given by the transformation:

$$s_L = \frac{s^2 + \Omega_o^2}{Bs}$$
$$s_L = \frac{s^2 + (0.5869)^2}{0.1324s}$$

Substituting s_L in $H_{analog}(s_L)$ we get H_{analog}(s).

1.8 discrete time filter transfer function

Using the bilinear transform again we get the discrete time filter using the transformation s = $\frac{1-z^-1}{1+z^{-1}}$.
H(z) = $\frac{N(z)}{D(z)}$
N(z) = $9.234e-09-7.387e-08z^{-2}+2.585e-07z^{-4}-5.171e-07z^{-6}+6.464e-07z^{-8}-5.171e-07z^{-10}+2.585e-07z^{12}-7.387e-08z^{-14}+9.234e-09z^{-16}$.

D(z) = $1-7.273z^{-1}+30.08z^{-2}-86.25z^{-3}+189.5z^{-4}-333z^{-5}+481.5z^{-6}-581.3z^{-7}+591.3z^{-8}-507.6z^{-9}+367.2z^{-10}-221.7z^{-11}+110.2z^{-12}-43.79z^{-13}+13.33z^{-14}-2.815z^{-15}+0.338z^{-16}$.

1.9 Realization using Direct Form II

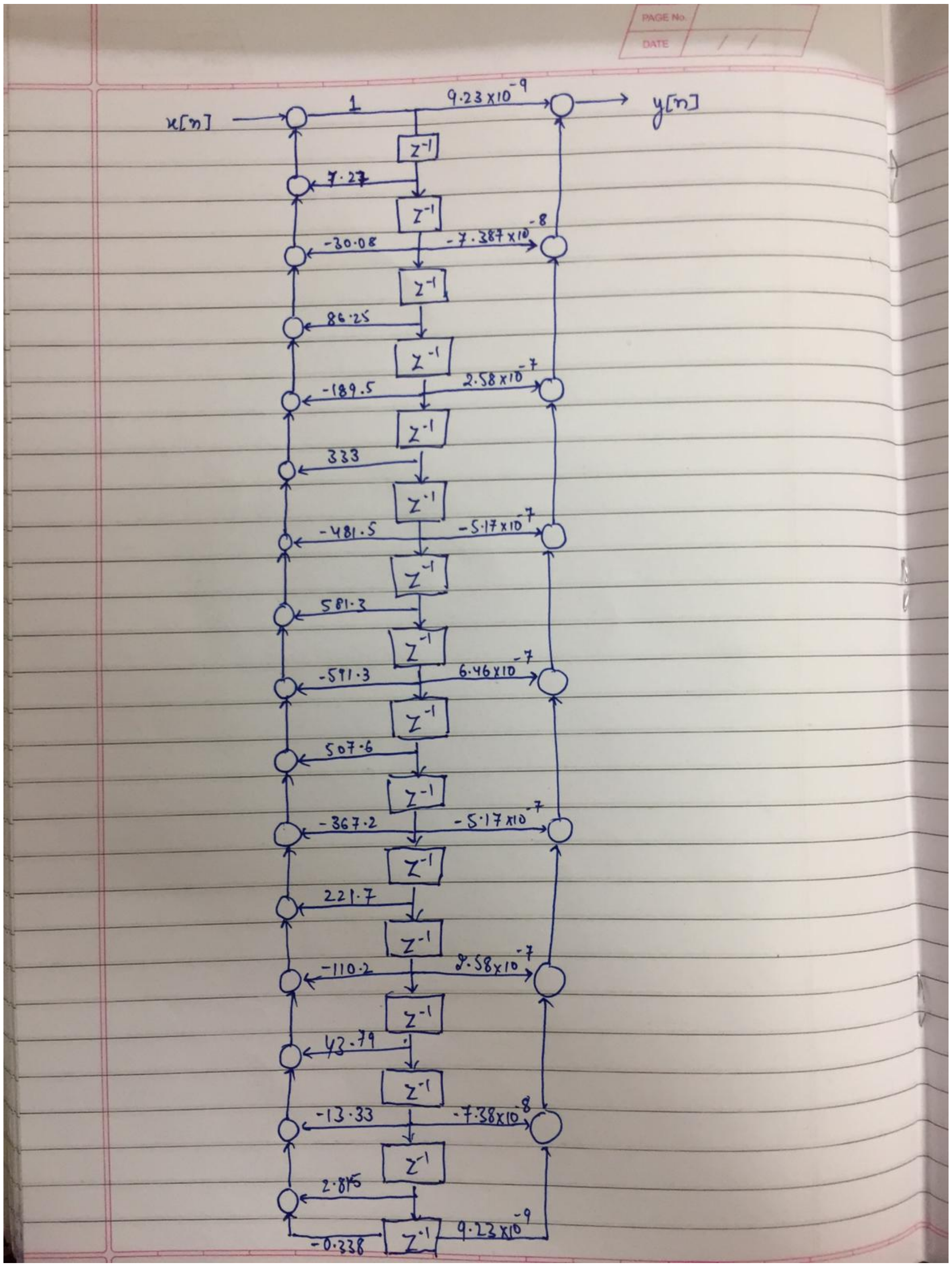

Figure 2: Direct Form II of the bandpass filter

1.10 FIR Bandpass Filter Transfer Function Using Kaiser Window

The formulae for designing a Kaiser window can be found in the documentation of the matlab functions "kaiser" and "kaiserord".
The tolerance is 0.15 for both passband and stopband so parameter α is calculated as

$$\alpha = -20log_{10}(0.15) = 16.48dB$$

Now depending on the value of α the parameter β is chosen. For $\alpha < 21$, $\beta = 0$.
Now the order of the kaiser window is given as

$$N \geq \frac{\alpha - 7.95}{2.285 \times \Delta\omega} = 95.04$$

$\Delta\omega = 0.0393$.
Using the same order(96) and cutoff frequencies the tolerance levels don't meet the requirements in the passband, so a higher order about 131 and a weighted shift to the cutoff fulfills the requirements.

2 Filter - II Bandstop Filter

2.1 Un-normalized Discrete Time Filter Specifications

$B_L = 33.6kHz$ and $B_H = 39.6kHz$

- Passband: 0 - 31.6 kHz and 41.6 to 125 kHz.
- Transition band width is 2 kHz on either side of the stopband range.
- Stopband: 33.6 - 39.6 kHz
- Tolerance in 0.15 in magnitude for both passband and stopband.
- Passband nature is *Equiripple* and stopband nature is *Monotonic.*
- Discrete time sampling frequency is 250 kHz.

2.2 Normalized Digital Filter Specification

The frequency normalization using the sampling frequency $F_s = 250$ kHz is done by the following transform

$$f = \frac{F}{F_s}$$

where F is in range [-$F_s/2$,$F_s/2$] and therefore $\omega = 2\pi$f.
Specifications:

- Passband: 0 - 0.2528π and 0.3328π to π
- Transition band width is now 0.0125π
- Stopband: 0.2688π and 0.3168π - π
- Tolerance in 0.15 in magnitude for both passband and stopband.
- Passband nature is *Equiripple* and stopband nature is *Monotonic.*

2.3 Analog Filter Specifications using the Bilinear Transformation

The bilinear transform $\omega = \tan(\frac{\Omega}{2})$.

ω	Ω
0	0
0.2528π	0.4193
0.2688π	0.4492
0.3168π	0.5432
0.3328π	0.5762
π	inf

Table 3: Bilinear Transformation

2.4 Bandpass Frequency Transformation Specifications

The bandpass transformation is given by $\Omega_L = \frac{B\Omega}{\Omega^2 - \Omega_o}$.

- $\Omega_o = \sqrt{\Omega_{p1} \times \Omega_{p2}} = 0.4196$
- B $= \Omega_{p1} - \Omega_{p2} = 0.1569$

Ω	Ω_L
0	0^-
0.4193	-1
0.4492	-1.769
$0.4916^-(\Omega_o)$	-inf
$0.4916^+(\Omega_o)$	inf
0.5432	1
0.5762	1.596
inf	0^+

Table 4: Bandstop Frequency Transformation

2.5 Frequency Transformed Lowpass Analog Filter Specifications

- Passband edge = $\Omega_{L_p} = 1$
- Stopband edge = $\min\{|\Omega_{L_{s1}}|, |\Omega_{L_{s2}}|\} = \min\{1.769, 1.596\} = 1.596$
- Tolerances $\delta_1 = \delta_2 = 0.15$ in magnitude for both Passband and Stopband
- Passband Nature is *Equiripple*
- Stopband Nature is *Monotonic*

2.6 Analog Lowpass Transfer Function Type

Since the passband specification is equiripple the filter type is *Chebyschev*, order of which is given as N $\geq \frac{cosh^{-1}(D_2/D_1)}{cosh^{-1}(\Omega_{L_s}/\Omega_{L_p})}$.
where $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$ and $D_2 = \frac{1}{\delta_2^2} = 43.444$.

$$N \geq \frac{cosh^{-1}(D_2/D_1)}{cosh^{-1}(\Omega_{L_s}/\Omega_{L_p})} = 4.3034$$

We choose N = 5, but it doesn't meet the amplitude requirement in the passband due to odd order, so we choose N = 6.

$$1 + \epsilon^2 \times cos^2(6cos^{-1}(\frac{s}{jw_p})) = 0$$

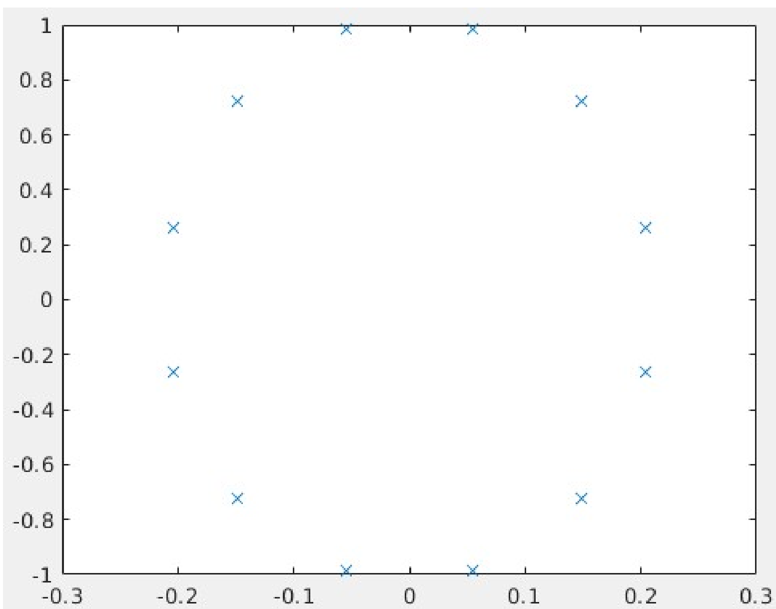

Figure 3: Roots of the above equation

We choose the left half poles for stability.
The obtained transfer function is given as

$$H_{analog} = \frac{(-1)^N |p_1 p_2 p_3 p_4 p_5 p_6|}{\sqrt{1+\epsilon^2}(s-p_1)(s-p_2)(s-p_3)(s-p_4)(s-p_5)(s-p_6)}$$

$$H_{analog} = \frac{0.05932}{1.176s^6 + 0.9586s^5 + 2.155s^4 + 1.211 * s^3 + 0.9982s^2 + 0.3011s + 0.0698}$$

2.7 Analog Bandstop Transfer Function

The bandstop filter is given by the transformation:

$$s_L = \frac{Bs}{s^2 + \Omega_o^2}$$
$$s_L = \frac{0.1569s}{s^2 + (0.4916)^2}$$

Substituting s_L in $H_{analog}(s_L)$ we get H_{analog}(s).

2.8 discrete time filter transfer function

Using the bilinear transform again we get the discrete time filter using the transformation s = $\frac{1-z^-1}{1+z^{-1}}$. H(z) = $\frac{N(z)}{D(z)}$
N(z) = $0.4679 - 3.429z^{-1} + 13.28z^{-2} - 34.2z^{-3} + 64.53z^{-4} - 93.09z^{-5} + 105z^{-6} - 93.09z^{-7} + 64.53z^{-8} - 34.2z^{-9} + 13.28z^{-10} - 3.429z^{-11} + 0.4679z^{-12}$.

D(z) = $1 - 6.562z^{-1} + 22.7z^{-2} - 52.66z^{-3} + 89.6z^{-4} - 117.1z^{-5} + 120.3z^{-6} - 97.69z^{-7} + 62.48z^{-8} - 30.81z^{-9} + 11.23z^{-10} - 2.755z^{-11} + 0.3609z^{-12}$.

2.9 Realization using Direct Form II

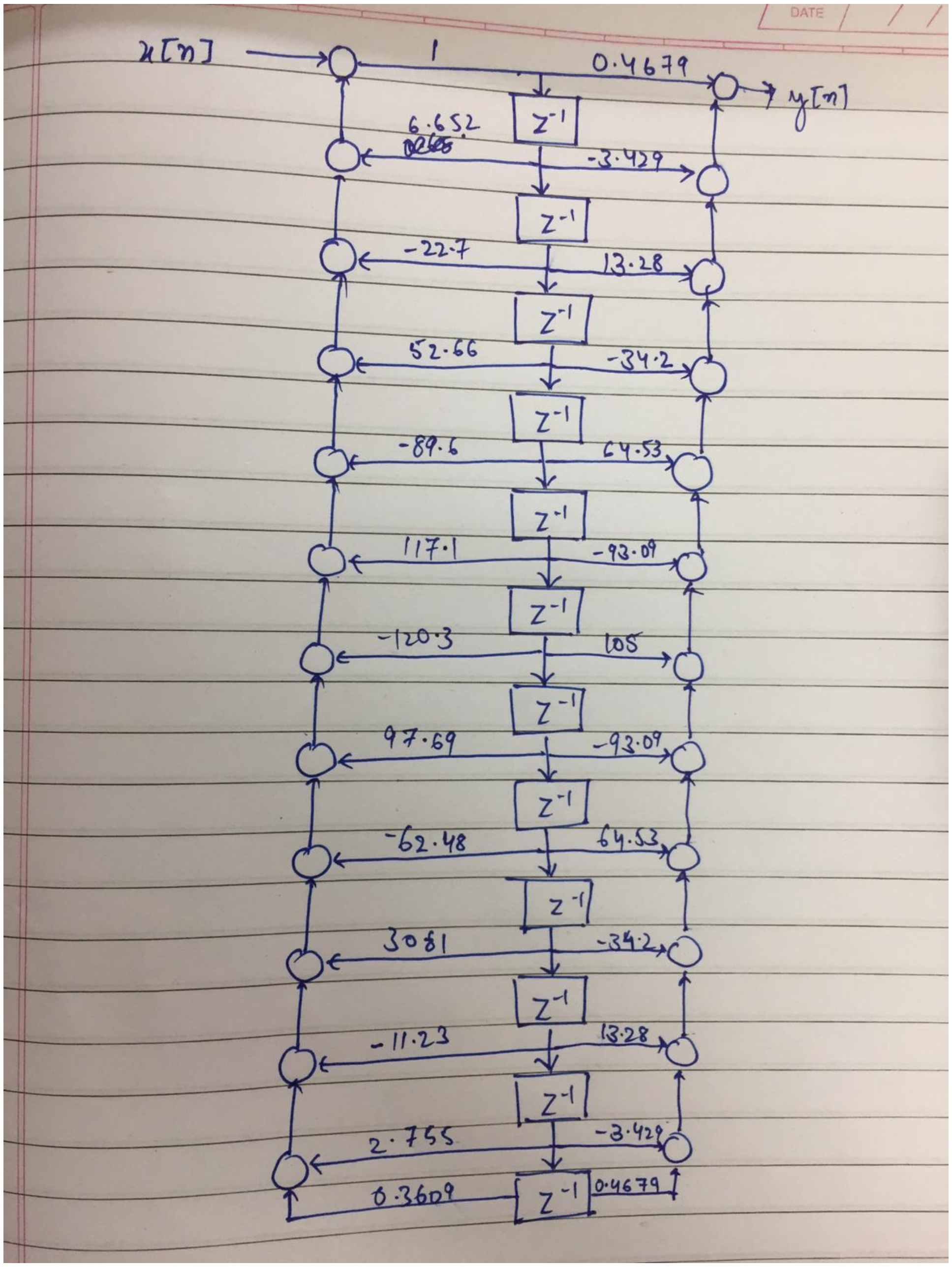

Figure 4: Direct Form II of the bandstop filter

2.10 FIR Bandpass Filter Transfer Function Using Kaiser Window

The formulae for designing a Kaiser window can be found in the documentation of the matlab functions "kaiser" and "kaiserord".
The tolerance is 0.15 for both passband and stopband so parameter α is calculated as

$$\alpha = -20log_{10}(0.15) = 16.48dB$$

Now depending on the value of α the parameter β is chosen. For $\alpha < 21$, $\beta = 0$.
Now the order of the kaiser window is given as

$$N \geq \frac{\alpha - 7.95}{2.285 \times \Delta\omega} = 74.25$$

$\Delta\omega = 0.0503$.
Using the same order(75) and cutoff frequencies the tolerance levels don't meet the requirements in the passband, so a higher order about 105 and a weighted shift to the cutoff fulfills the requirements.

3 Matlab Simulation Plots for Specification Verification

The IIR filters are direct and don't require any parameter tuning to satisfy the filter requirements the plots are shown below.

3.1 Butterworth Bandpass Filter

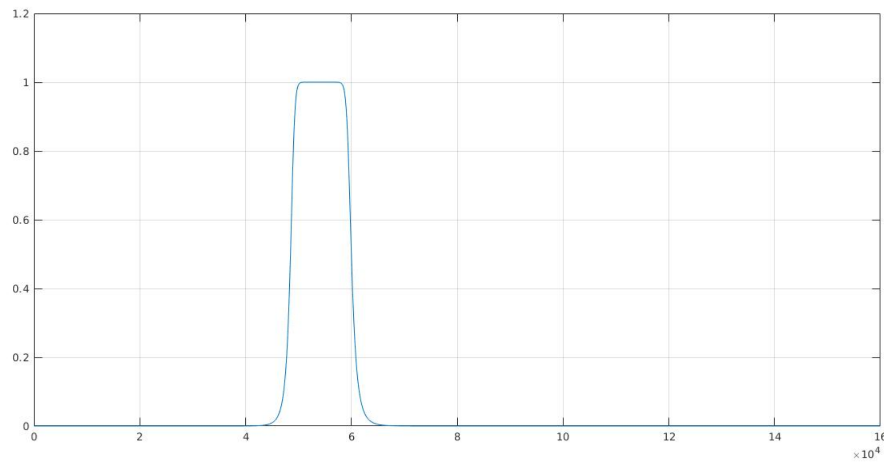

Figure 5: IIR bandpass filter

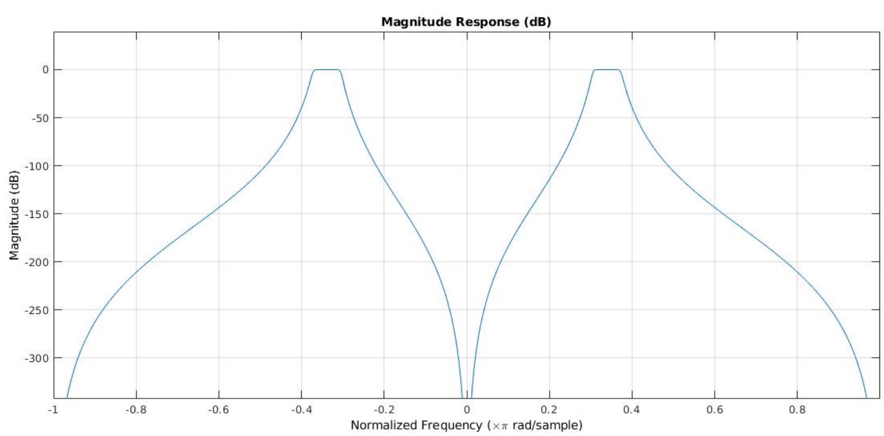

Figure 6: IIR bandpass filter on log scale

3.2 Chebyschev Bandstop Filter

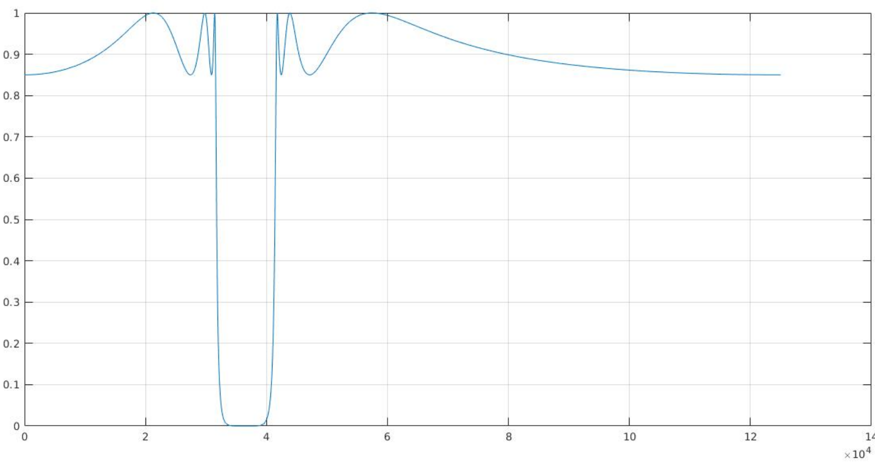

Figure 7: IIR bandstop filter

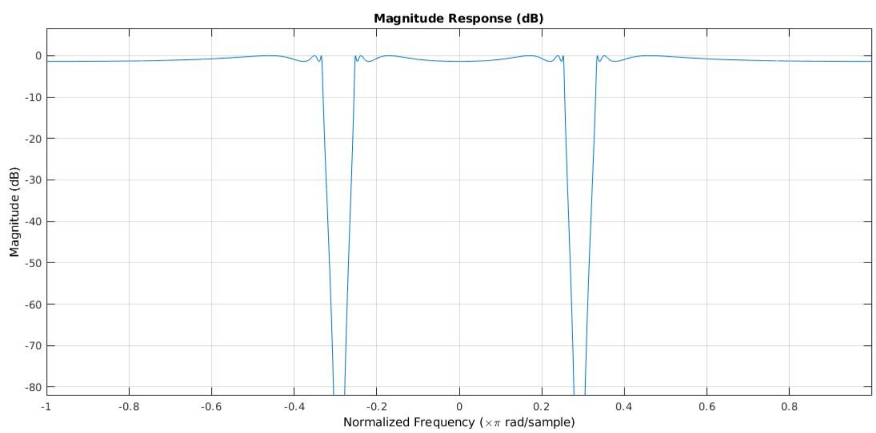

Figure 8: IIR bandstop filter on log scale

3.3 FIR Bandpass Filter

To prove all tolerance and bandwidth requirement the plots are given below.

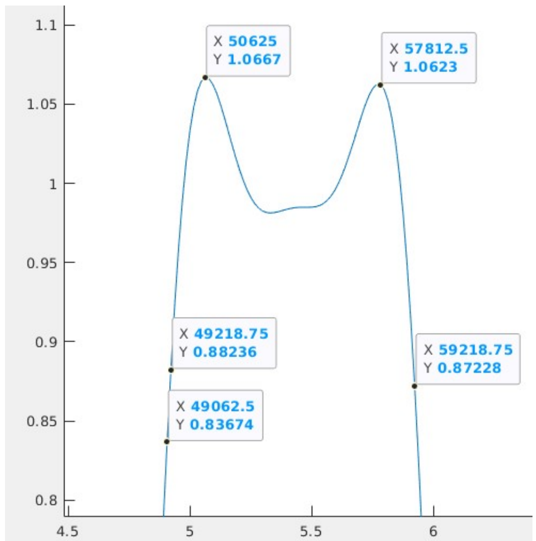

Figure 9: FIR bandpass filter

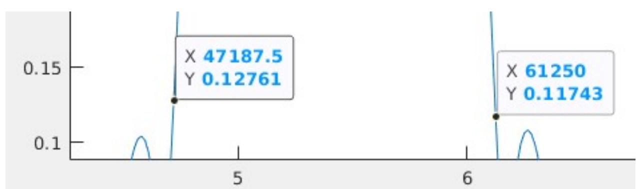

Figure 10: FIR bandpass filter

3.4 FIR Bandstop Filter

To prove all tolerance and bandwidth requirement the plots are given below.

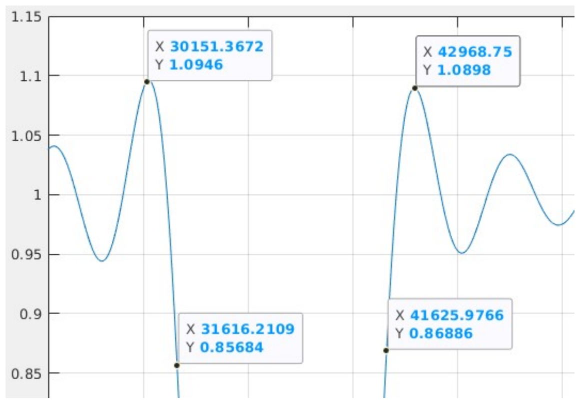

Figure 11: FIR bandstop filter

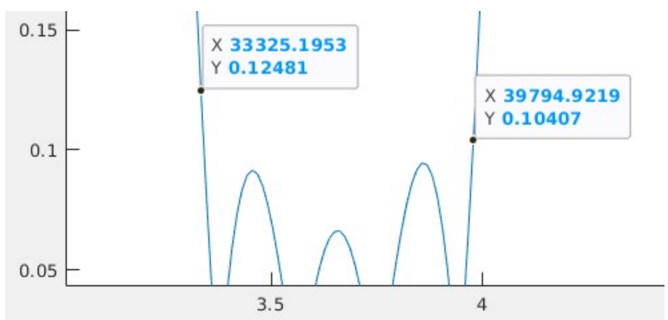

Figure 12: FIR bandstop filter