Detection of Lead-Lag relationships

In [309]:

```
#Importing Libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

Agriculture Index

In [310]:

```
df = pd.read_csv("AGRICULTURE.csv", index_col='DATE', parse_dates=True)
```

In [311]:

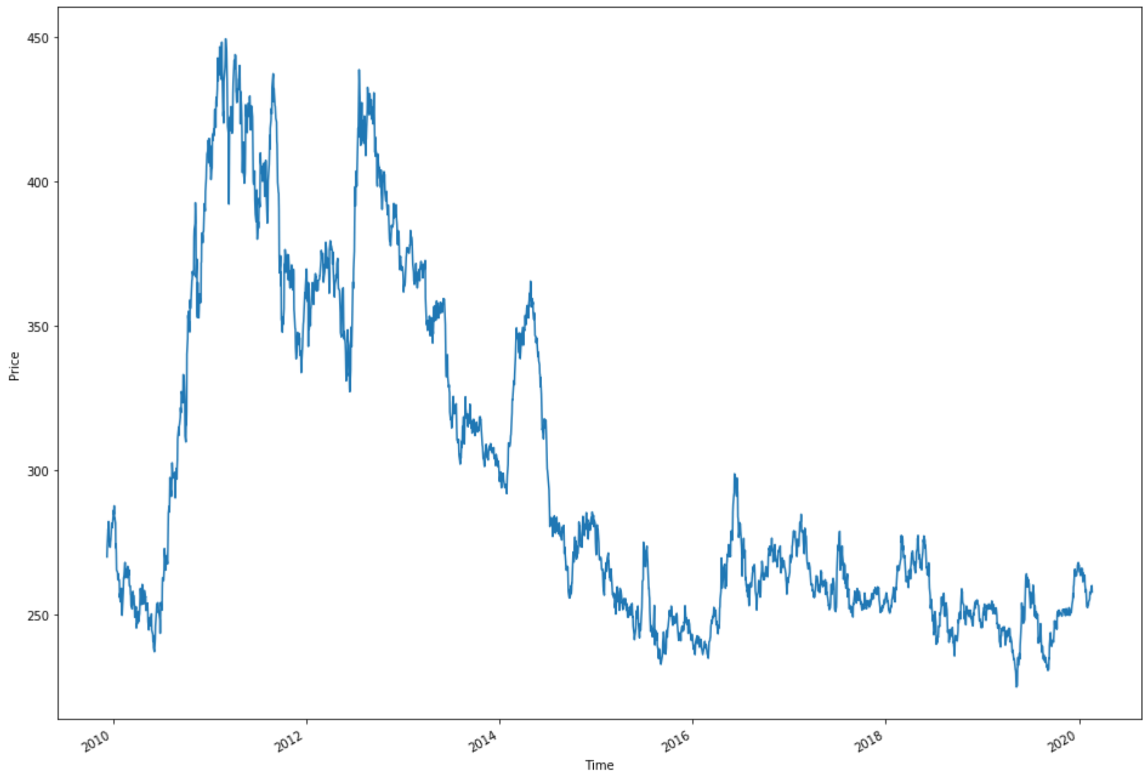
```
df.head()
```

Out[311]:

	SPOT	FUTURE
DATE		
2009-12-10	270.1053	126.8045
2009-12-11	273.7968	128.4607
2009-12-14	279.7481	131.1803
2009-12-15	278.2661	130.4855
2009-12-16	282.3329	132.3927

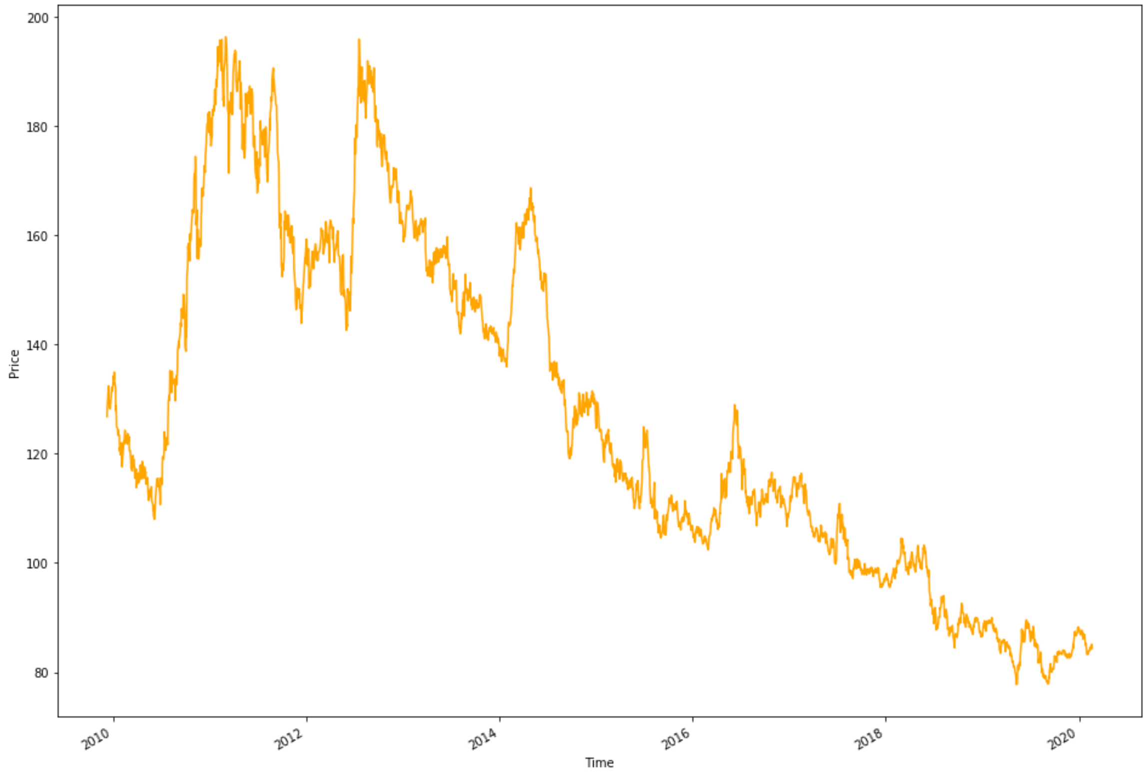
In [312]:

```
df['SPOT'].plot(figsize=(16,12))
plt.xlabel('Time')
plt.ylabel('Price');
```



In [313]:

```
df['FUTURE'].plot(figsize=(16,12), c='orange')
plt.xlabel('Time')
plt.ylabel('Price');
```

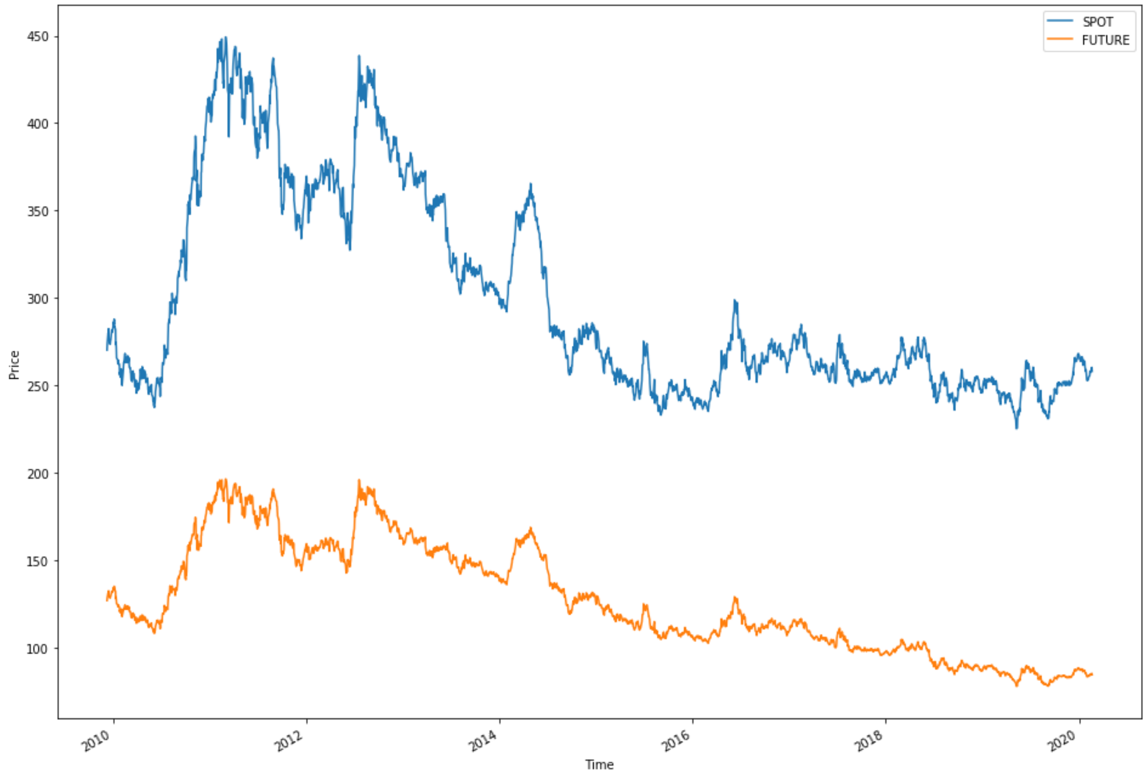


In [314]:

```
df.plot(figsize=(16,12))
plt.xlabel('Time')
plt.ylabel('Price')
```

Out[314]:

```
Text(0,0.5,'Price')
```



STATIONARITY TEST

In [315]:

```
from statsmodels.tsa.stattools import adfuller
```

In [316]:

```
#ON SPOT PRICE
adfuller(df['SPOT'])
```

Out[316]:

```
(-1.5108152003856385,
 0.5281545911336507,
 1,
 2567,
 {'1%': -3.432900000469521,
  '5%': -2.8626665895880508,
  '10%': -2.567369725077316},
 13557.035863748632)
```

In [317]:

```
df['SPOT Difference']=df['SPOT']-df['SPOT'].shift(1)
adfuller(df['SPOT Difference'].dropna())
```

Out[317]:

```
(-49.105108554868515,
 0.0,
 0,
 2567,
 {'1%': -3.432900000469521,
  '5%': -2.8626665895880508,
  '10%': -2.567369725077316},
 13553.042843639545)
```

In [318]:

```
#ON FUTURE PRICE
adfuller(df['FUTURE'])
```

Out[318]:

```
(-0.9362975103280478,
 0.7757842976422112,
 5,
 2563,
 {'1%': -3.4329039841780644,
  '5%': -2.8626683488356117,
  '10%': -2.5673706617172343},
 9219.330858001931)
```

In [319]:

```
df['FUTURE Difference']=df['FUTURE']-df['FUTURE'].shift(1)
adfuller(df['FUTURE Difference'].dropna())
```

Out[319]:

```
(-21.506530804569486,
 0.0,
 4,
 2563,
 {'1%': -3.4329039841780644,
  '5%': -2.8626683488356117,
  '10%': -2.5673706617172343},
 9215.508134106149)
```

In [320]:

```
df.head()
```

Out[320]:

	SPOT	FUTURE	SPOT Difference	FUTURE Difference
DATE				
2009-12-10	270.1053	126.8045	NaN	NaN
2009-12-11	273.7968	128.4607	3.6915	1.6562
2009-12-14	279.7481	131.1803	5.9513	2.7196
2009-12-15	278.2661	130.4855	-1.4820	-0.6948
2009-12-16	282.3329	132.3927	4.0668	1.9072

The Augmented Dickey-Fuller(ADF) unit root test shows that the order of integration of both the time series is 1

Selection of Lag Order

In [321]:

```
from statsmodels.tsa.vector_ar.vecm import select_order
```

In [322]:

```
lags = select_order(df.iloc[:,0:2],maxlags=4,deterministic='co')
```

```
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
```

In [323]:

```
lag.ics
```

Out[323]:

```
defaultdict(list,
            {'aic': [0.21481885765020273,
              0.1700199596430323,
              0.16402455232867902,
              0.14868554216991547,
              0.14640060953972608],
             'bic': [0.23306947798730276,
              0.19739589014868233,
              0.2005257930028791,
              0.19431209301266558,
              0.2011524705510262],
             'hqic': [0.22143618925865835,
              0.1799459570557157,
              0.17725921554559027,
              0.16522887119105453,
              0.16625260436509295],
             'fpe': [1.2396373320845766,
              1.1853285300706864,
              1.178243291072056,
              1.1603081556622221,
              1.1576600229930996]})
```

In [324]:

```
lag.selected_orders
```

Out[324]:

```
{'aic': 4, 'bic': 3, 'hqic': 3, 'fpe': 4}
```

In [325]:

```
lag.vecm
```

Out[325]:

```
True
```

As per the above information criteria, it shows that the model is a VECM model with optimal lag length of 4

COINTEGRATION TESTS

In [326]:

```
#Checking for cointegration between the two variables
from statsmodels.tsa.vector_ar.vecm import coint_johansen
'''https://towardsdatascience.com/vector-autoregressions-vector-error-correction-multiv
ariate-model-a69daf6ab618'''
```

Out[326]:

```
'https://towardsdatascience.com/vector-autoregressions-vector-error-correc
tion-multivariate-model-a69daf6ab618'
```

In [327]:

```
'''#Checking for cointegration using ADFuller
from statsmodels.regression.linear_model import OLS
#import statsmodels.tools as sm
#x = sm.add_constant(df['FUTURE'])
res = OLS(df['SPOT'],df['FUTURE']).fit()
res.summary()'''
```

Out[327]:

```
"#Checking for cointegration using ADFuller \nfrom statsmodels.regression.
linear_model import OLS\n#import statsmodels.tools as sm\n#x = sm.add_cons
tant(df['FUTURE'])\nres = OLS(df['SPOT'],df['FUTURE']).fit()\nres.summary
()"
```

In [328]:

```
'''adfuller(res.resid)'''
```

Out[328]:

```
'adfuller(res.resid)'
```

In [329]:

```
'''res.resid.plot()'''
```

Out[329]:

```
'res.resid.plot()'
```

In [330]:

```
result = coint_johansen(endog = df.iloc[:,0:2], det_order = 0 , k_ar_diff=4)
```

''' Read output of coint_johansen: https://kite.com/python/docs/statsmodels.tsa.vector_ar.vecm.coint_johansen (https://kite.com/python/docs/statsmodels.tsa.vector_ar.vecm.coint_johansen) '''

In [331]:

```
result.lr1 # Trace statistic
```

Out[331]:

```
array([6.38714619, 0.00917241])
```

In [332]:

```
result.cvt #Shows critical values for trace statistics
```

Out[332]:

```
array([[13.4294, 15.4943, 19.9349],
       [ 2.7055,  3.8415,  6.6349]])
```

In [333]:

```
result.lr2 #Eigen value statistic
```

Out[333]:

```
array([6.37797378, 0.00917241])
```

In [334]:

```
result.cvm #Critical Values for Eigen value statistic
```

Out[334]:

```
array([[12.2971, 14.2639, 18.52  ],
       [ 2.7055,  3.8415,  6.6349]])
```

In [335]:

```
result.evec
```

Out[335]:

```
array([[ 0.02892297, -0.04578122],
       [-0.02391151,  0.09764633]])
```

As per the above Johansen's cointegration test it shows that the variables are not cointegrated

VECM Model

In [336]:

```
#VECM Model
from statsmodels.tsa.vector_ar.vecm import VECM
```

In [337]:

```
model = VECM(endog=df.iloc[:,0:2],k_ar_diff = 4, coint_rank=1, deterministic='co').fit
()
```

```
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
```

In [338]:

```
model.summary()
```

Out[338]:

Det. terms outside the coint. relation & lagged endog. parameters for equation SPOT

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.5960	0.390	1.529	0.126	-0.168	1.360
L1.SPOT	-0.3263	0.095	-3.440	0.001	-0.512	-0.140
L1.FUTURE	0.8693	0.222	3.922	0.000	0.435	1.304
L2.SPOT	0.2291	0.096	2.388	0.017	0.041	0.417
L2.FUTURE	-0.5749	0.224	-2.565	0.010	-1.014	-0.136
L3.SPOT	0.1777	0.096	1.852	0.064	-0.010	0.366
L3.FUTURE	-0.3562	0.224	-1.590	0.112	-0.795	0.083
L4.SPOT	-0.1077	0.095	-1.135	0.256	-0.294	0.078
L4.FUTURE	0.2126	0.222	0.959	0.337	-0.222	0.647

Det. terms outside the coint. relation & lagged endog. parameters for equation FUTURE

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.1687	0.167	1.011	0.312	-0.158	0.496
L1.SPOT	-0.0520	0.041	-1.280	0.201	-0.132	0.028
L1.FUTURE	0.1762	0.095	1.857	0.063	-0.010	0.362
L2.SPOT	0.0553	0.041	1.346	0.178	-0.025	0.136
L2.FUTURE	-0.1549	0.096	-1.615	0.106	-0.343	0.033
L3.SPOT	0.0162	0.041	0.394	0.693	-0.064	0.097
L3.FUTURE	-0.0232	0.096	-0.242	0.809	-0.211	0.165
L4.SPOT	-0.0733	0.041	-1.806	0.071	-0.153	0.006
L4.FUTURE	0.1574	0.095	1.660	0.097	-0.028	0.343

Loading coefficients (alpha) for equation SPOT

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-0.0031	0.002	-1.574	0.116	-0.007	0.001

Loading coefficients (alpha) for equation FUTURE

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-0.0010	0.001	-1.129	0.259	-0.003	0.001

Cointegration relations for loading-coefficients-column 1

	coef	std err	z	P>|z|	[0.025	0.975]

beta.1	1.0000	0	0	0.000	1.000	1.000
beta.2	-0.8267	0.437	-1.892	0.058	-1.683	0.030

CONCLUSION

Coefficient of lagged values of FUTURE in the regression of SPOT: significant at 5% level
Coefficient of lagged values of SPOT in the regression of FUTURE: not significant at 5% level
Coefficient of loading coeffcient in the regression of SPOT: not significant at 5% level
Coefficient of loading coeffcient in the regression of FUTURE: not significant at 5% level

Hence, based on the above results we can say there is a short-run unidirectional causality running from FUTURE to SPOT and there is no long term causality running between the two variables.

Live Stock Index

In [339]:

```
df = pd.read_csv("LIVE STOCK.csv", index_col='DATE', parse_dates=True)
```

In [340]:

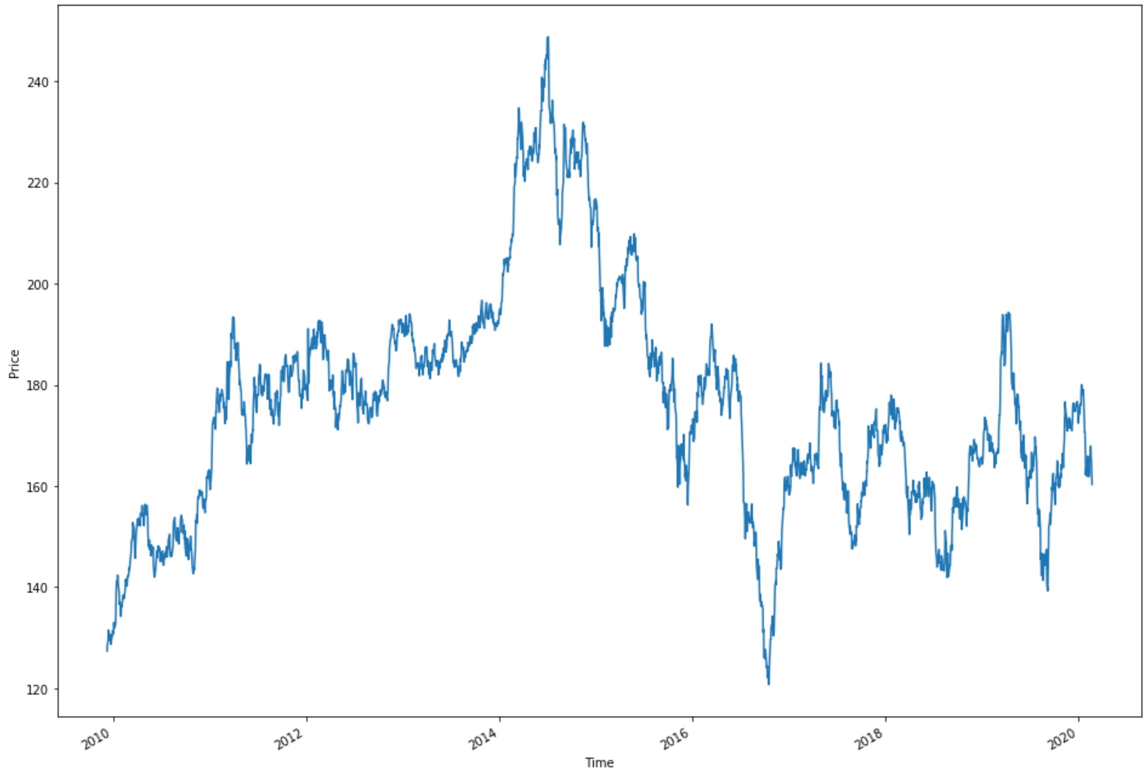
```
df.head()
```

Out[340]:

	SPOT	FUTURE
DATE		
2009-12-10	127.3650	66.9102
2009-12-11	128.4296	67.4695
2009-12-14	129.4118	67.9858
2009-12-15	131.4888	69.0770
2009-12-16	131.0259	68.8339

In [341]:

```
df['SPOT'].plot(figsize=(16,12))
plt.xlabel('Time')
plt.ylabel('Price');
```



In [342]:

```
df['FUTURE'].plot(figsize=(16,12),color = 'orange')
plt.xlabel('Time')
plt.ylabel('Price');
```

In [343]:

```
df.plot(figsize=(16,12))
plt.xlabel('Time')
plt.ylabel('Price')
```

Out[343]:

Text(0,0.5,'Price')

STATIONARITY TEST

In [344]:

```
from statsmodels.tsa.stattools import adfuller
```

In [345]:

```
#ON SPOT PRICE
adfuller(df['SPOT'])
```

Out[345]:

```
(-2.6009450561177263,
 0.09280210083008011,
 3,
 2568,
 {'1%': -3.4328990064834404,
  '5%': -2.8626661506329856,
  '10%': -2.5673694913735794},
 10053.544161022579)
```

In [346]:

```
df['SPOT Difference']=df['SPOT']-df['SPOT'].shift(1)
adfuller(df['SPOT Difference'].dropna())
```

Out[346]:

```
(-26.784864768702366,
 0.0,
 2,
 2568,
 {'1%': -3.4328990064834404,
  '5%': -2.8626661506329856,
  '10%': -2.5673694913735794},
 10054.662471479684)
```

In [347]:

```
#ON FUTURE PRICE
adfuller(df['FUTURE'])
```

Out[347]:

```
(-1.5502558007809173,
 0.5085204177870704,
 0,
 2571,
 {'1%': -3.432896029169223,
  '5%': -2.862664835817767,
  '10%': -2.5673687913539416},
 4802.425783417375)
```

In [348]:

```
df['FUTURE Difference']=df['FUTURE']-df['FUTURE'].shift(1)
adfuller(df['FUTURE Difference'].dropna())
```

Out[348]:

```
(-49.77417134338738,
 0.0,
 0,
 2570,
 {'1%': -3.432897020834196,
  '5%': -2.8626652737482425,
  '10%': -2.5673690245121046},
 4802.955402459513)
```

In [349]:

```
df.head()
```

Out[349]:

	SPOT	FUTURE	SPOT Difference	FUTURE Difference
DATE				
2009-12-10	127.3650	66.9102	NaN	NaN
2009-12-11	128.4296	67.4695	1.0646	0.5593
2009-12-14	129.4118	67.9858	0.9822	0.5163
2009-12-15	131.4888	69.0770	2.0770	1.0912
2009-12-16	131.0259	68.8339	-0.4629	-0.2431

The Augmented Dickey-Fuller(ADF) unit root test shows that the order of integration of both the time series is 1

Selection of Lag Order

In [350]:

```
from statsmodels.tsa.vector_ar.vecm import select_order
```

In [351]:

```
lags = select_order(df.iloc[:,0:2],maxlags=4,deterministic='co')
```

```
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
```

In [352]:

```
lag.ics
```

Out[352]:

```
defaultdict(list,
            {'aic': [0.21481885765020273,
              0.1700199596430323,
              0.16402455232867902,
              0.14868554216991547,
              0.14640060953972608],
             'bic': [0.23306947798730276,
              0.19739589014868233,
              0.2005257930028791,
              0.19431209301266558,
              0.2011524705510262],
             'hqic': [0.22143618925865835,
              0.1799459570557157,
              0.17725921554559027,
              0.16522887119105453,
              0.16625260436509295],
             'fpe': [1.2396373320845766,
              1.1853285300706864,
              1.178243291072056,
              1.1603081556622221,
              1.1576600229930996]})
```

In [353]:

```
lag.selected_orders
```

Out[353]:

```
{'aic': 4, 'bic': 3, 'hqic': 3, 'fpe': 4}
```

In [354]:

```
lag.vecm
```

Out[354]:

```
True
```

As per the above information criteria, it shows that the model is a VECM model with optimal lag length of 4

COINTEGRATION TESTS

In [355]:

```
#Checking for cointegration between the two variables
from statsmodels.tsa.vector_ar.vecm import coint_johansen
'''https://towardsdatascience.com/vector-autoregressions-vector-error-correction-multiv
ariate-model-a69daf6ab618'''
```

Out[355]:

```
'https://towardsdatascience.com/vector-autoregressions-vector-error-correc
tion-multivariate-model-a69daf6ab618'
```

In [356]:

```
'''#Checking for cointegration using ADFuller
from statsmodels.regression.linear_model import OLS
#import statsmodels.tools as sm
#x = sm.add_constant(df['FUTURE'])
res = OLS(df['SPOT'],df['FUTURE']).fit()
res.summary()'''
```

Out[356]:

```
"#Checking for cointegration using ADFuller \nfrom statsmodels.regression.
linear_model import OLS\n#import statsmodels.tools as sm\n#x = sm.add_cons
tant(df['FUTURE'])\nres = OLS(df['SPOT'],df['FUTURE']).fit()\nres.summary
()"
```

In [357]:

```
'''adfuller(res.resid)'''
```

Out[357]:

```
'adfuller(res.resid)'
```

In [358]:

```
'''res.resid.plot()'''
```

Out[358]:

```
'res.resid.plot()'
```

In [359]:

```
result = coint_johansen(endog = df.iloc[:,0:2], det_order = 0 , k_ar_diff=4)
```

''' Read output of coint_johansen:
https://kite.com/python/docs/statsmodels.tsa.vector_ar.vecm.coint_johansen
(https://kite.com/python/docs/statsmodels.tsa.vector_ar.vecm.coint_johansen) '''

In [360]:

```
result.lr1 # Trace statistic
```

Out[360]:

```
array([10.59146604,  2.25174744])
```

In [361]:

```
result.cvt #Shows critical values for trace statistics
```

Out[361]:

```
array([[13.4294, 15.4943, 19.9349],
       [ 2.7055,  3.8415,  6.6349]])
```

In [362]:

```
result.lr2 #Eigen value statistic
```

Out[362]:

```
array([8.3397186 , 2.25174744])
```

In [363]:

```
result.cvm #Critical Values for Eigen value statistic
```

Out[363]:

```
array([[12.2971, 14.2639, 18.52  ],
       [ 2.7055,  3.8415,  6.6349]])
```

In [364]:

```
result.evec
```

Out[364]:

```
array([[ 0.05458684, -0.01792027],
       [-0.06153226,  0.16062588]])
```

As per the above Johansen's cointegration test it shows that the variables are not cointegrated

VECM Model

In [365]:

```
#VECM Model
from statsmodels.tsa.vector_ar.vecm import VECM
```

In [366]:

```
model = VECM(endog=df.iloc[:,0:2],k_ar_diff = 4, coint_rank=1, deterministic='co').fit
()
```

```
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
```

In [367]:

```
model.summary()
```

Out[367]:

Det. terms outside the coint. relation & lagged endog. parameters for equation SPOT

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.4418	0.191	2.311	0.021	0.067	0.817
L1.SPOT	0.1443	0.067	2.170	0.030	0.014	0.275
L1.FUTURE	-0.3351	0.186	-1.806	0.071	-0.699	0.029
L2.SPOT	0.2508	0.067	3.753	0.000	0.120	0.382
L2.FUTURE	-0.6573	0.186	-3.528	0.000	-1.023	-0.292
L3.SPOT	0.1137	0.067	1.702	0.089	-0.017	0.245
L3.FUTURE	-0.2214	0.186	-1.191	0.234	-0.586	0.143
L4.SPOT	0.0308	0.066	0.463	0.643	-0.100	0.161
L4.FUTURE	-0.0691	0.185	-0.373	0.709	-0.432	0.294

Det. terms outside the coint. relation & lagged endog. parameters for equation FUTURE

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.1121	0.069	1.635	0.102	-0.022	0.246
L1.SPOT	-0.0006	0.024	-0.026	0.979	-0.047	0.046
L1.FUTURE	0.0187	0.067	0.281	0.779	-0.112	0.149
L2.SPOT	-0.0041	0.024	-0.171	0.864	-0.051	0.043
L2.FUTURE	0.0068	0.067	0.102	0.919	-0.124	0.138
L3.SPOT	0.0039	0.024	0.163	0.871	-0.043	0.051
L3.FUTURE	0.0178	0.067	0.267	0.789	-0.113	0.148
L4.SPOT	0.0267	0.024	1.119	0.263	-0.020	0.073
L4.FUTURE	-0.0693	0.066	-1.046	0.296	-0.199	0.061

Loading coefficients (alpha) for equation SPOT

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-0.0044	0.002	-2.371	0.018	-0.008	-0.001

Loading coefficients (alpha) for equation FUTURE

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-0.0012	0.001	-1.775	0.076	-0.002	0.000

Cointegration relations for loading-coefficients-column 1

	coef	std err	z	P>|z|	[0.025	0.975]

beta.1	1.0000	0	0	0.000	1.000	1.000
beta.2	-1.1272	0.846	-1.333	0.183	-2.785	0.530

CONCLUSION

Coefficient of lagged values of FUTURE in the regression of SPOT: significant at 5% level.
Coefficient of lagged values of SPOT in the regression of FUTURE: not significant at 5% level
Coefficient of loading coeffcient in the regression of SPOT: not significant at 5% level
Coefficient of loading coeffcient in the regression of FUTURE: not significant at 5% level

Hence, based on the above results we can say there is a short-run unidirectional causality running from FUTURE to SPOT and there is no long term causality running between the two variables.

Precious Metals Index

In [368]:

```
df = pd.read_csv("PRECIOUS METALS.csv", index_col='DATE', parse_dates=True)
```

In [369]:

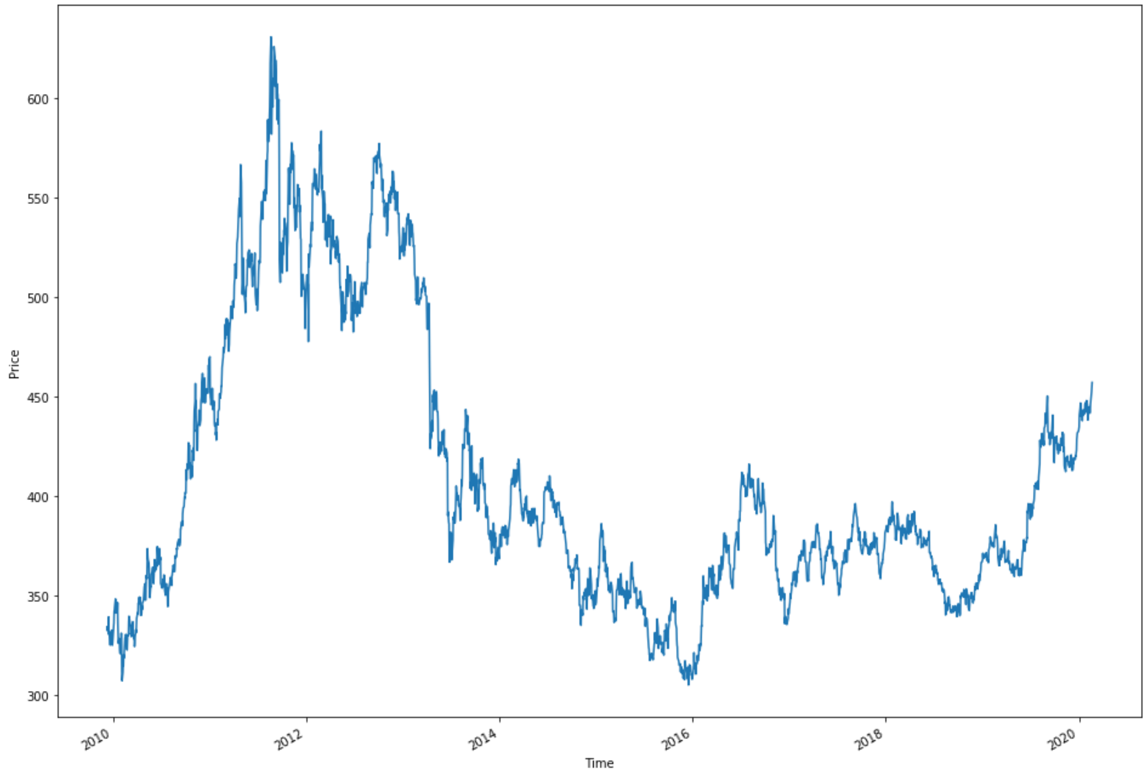
```
df.head()
```

Out[369]:

	SPOT	FUTURE
DATE		
2009-12-10	334.3950	330.8218
2009-12-11	332.5137	328.9610
2009-12-14	334.7776	331.2021
2009-12-15	335.2786	331.6981
2009-12-16	339.4102	335.7859

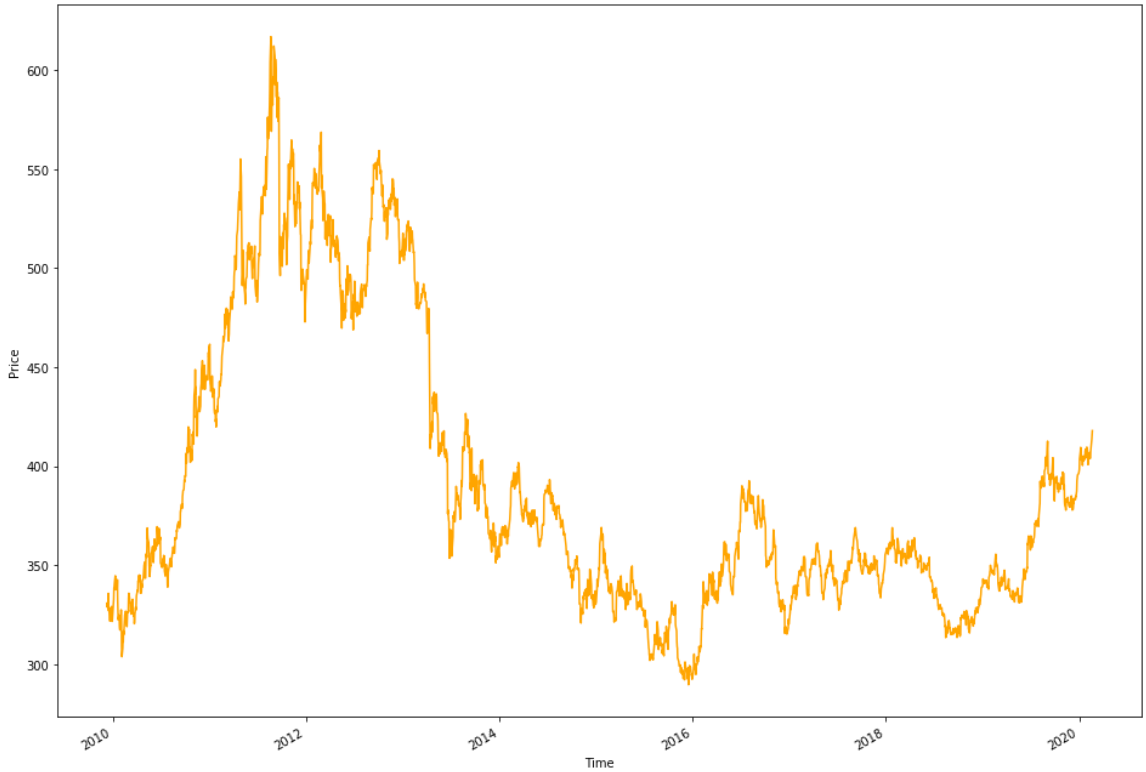
In [370]:

```
df['SPOT'].plot(figsize=(16,12))
plt.xlabel('Time')
plt.ylabel('Price');
```



In [397]:

```
df['FUTURE'].plot(figsize=(16,12),color='orange')
plt.xlabel('Time')
plt.ylabel('Price');
```

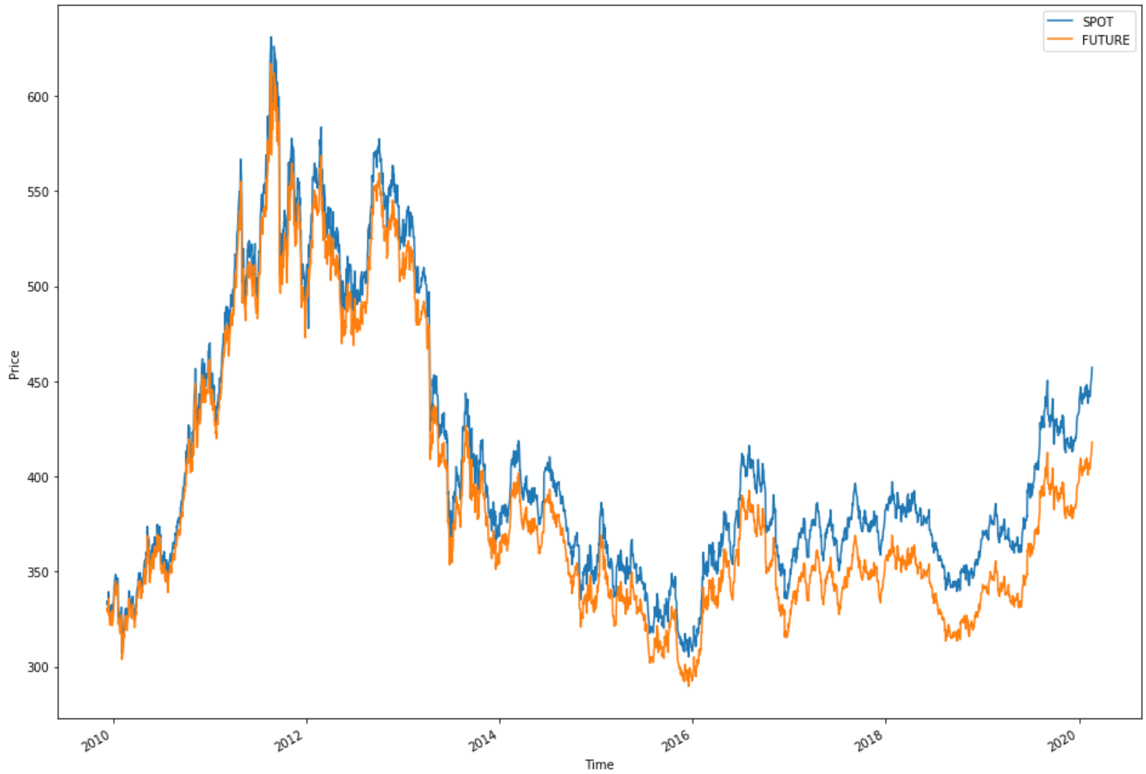


In [372]:

```
df.plot(figsize=(16,12))
plt.xlabel('Time')
plt.ylabel('Price')
```

Out[372]:

```
Text(0,0.5,'Price')
```



STATIONARITY TEST

In [373]:

```
from statsmodels.tsa.stattools import adfuller
```

In [374]:

```
#ON SPOT PRICE
adfuller(df['SPOT'])
```

Out[374]:

```
(-1.977330628130098,
 0.2966004183641978,
 9,
 2560,
 {'1%': -3.4329069801374077,
  '5%': -2.862669671881199,
  '10%': -2.5673713661193847},
 15523.10659408757)
```

In [375]:

```
df['SPOT Difference']=df['SPOT']-df['SPOT'].shift(1)
adfuller(df['SPOT Difference'].dropna())
```

Out[375]:

```
(-15.978085359558444,
 6.884561565751759e-29,
 8,
 2560,
 {'1%': -3.4329069801374077,
  '5%': -2.862669671881199,
  '10%': -2.5673713661193847},
 15518.061816683363)
```

In [376]:

```
#ON FUTURE PRICE
adfuller(df['FUTURE'])
```

Out[376]:

```
(-1.839941732983566,
 0.3608196718566172,
 9,
 2560,
 {'1%': -3.4329069801374077,
  '5%': -2.862669671881199,
  '10%': -2.5673713661193847},
 15253.427698165557)
```

In [377]:

```
df['FUTURE Difference']=df['FUTURE']-df['FUTURE'].shift(1)
adfuller(df['FUTURE Difference'].dropna())
```

Out[377]:

```
(-16.035185188253703,
 5.996990709845228e-29,
 8,
 2560,
 {'1%': -3.4329069801374077,
  '5%': -2.862669671881199,
  '10%': -2.5673713661193847},
 15247.927160892903)
```

In [378]:

```
df.head()
```

Out[378]:

	SPOT	FUTURE	SPOT Difference	FUTURE Difference
DATE				
2009-12-10	334.3950	330.8218	NaN	NaN
2009-12-11	332.5137	328.9610	-1.8813	-1.8608
2009-12-14	334.7776	331.2021	2.2639	2.2411
2009-12-15	335.2786	331.6981	0.5010	0.4960
2009-12-16	339.4102	335.7859	4.1316	4.0878

The Augmented Dickey-Fuller(ADF) unit root test shows that the order of integration of both the time series is 1

Selection of Lag Order

In [379]:

```
from statsmodels.tsa.vector_ar.vecm import select_order
```

In [380]:

```
lags = select_order(df.iloc[:,0:2],maxlags=4,deterministic='co')
```

```
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
```

In [381]:

```
lag.ics
```

Out[381]:

```
defaultdict(list,
            {'aic': [0.21481885765020273,
              0.1700199596430323,
              0.16402455232867902,
              0.14868554216991547,
              0.14640060953972608],
             'bic': [0.23306947798730276,
              0.19739589014868233,
              0.2005257930028791,
              0.19431209301266558,
              0.2011524705510262],
             'hqic': [0.22143618925865835,
              0.1799459570557157,
              0.17725921554559027,
              0.16522887119105453,
              0.16625260436509295],
             'fpe': [1.2396373320845766,
              1.1853285300706864,
              1.178243291072056,
              1.1603081556622221,
              1.1576600229930996]})
```

In [382]:

```
lag.selected_orders
```

Out[382]:

```
{'aic': 4, 'bic': 3, 'hqic': 3, 'fpe': 4}
```

In [383]:

```
lag.vecm
```

Out[383]:

```
True
```

As per the above information criteria, it shows that the model is a VECM model with optimal lag length of 4

COINTEGRATION TESTS

In [384]:

```
#Checking for cointegration between the two variables
from statsmodels.tsa.vector_ar.vecm import coint_johansen
'''https://towardsdatascience.com/vector-autoregressions-vector-error-correction-multiv
ariate-model-a69daf6ab618'''
```

Out[384]:

```
'https://towardsdatascience.com/vector-autoregressions-vector-error-correc
tion-multivariate-model-a69daf6ab618'
```

In [385]:

```
'''#Checking for cointegration using ADFuller
from statsmodels.regression.linear_model import OLS
#import statsmodels.tools as sm
#x = sm.add_constant(df['FUTURE'])
res = OLS(df['SPOT'],df['FUTURE']).fit()
res.summary()'''
```

Out[385]:

```
"#Checking for cointegration using ADFuller \nfrom statsmodels.regression.
linear_model import OLS\n#import statsmodels.tools as sm\n#x = sm.add_cons
tant(df['FUTURE'])\nres = OLS(df['SPOT'],df['FUTURE']).fit()\nres.summary
()"
```

In [386]:

```
'''adfuller(res.resid)'''
```

Out[386]:

```
'adfuller(res.resid)'
```

In [387]:

```
'''res.resid.plot()'''
```

Out[387]:

```
'res.resid.plot()'
```

In [388]:

```
result = coint_johansen(endog = df.iloc[:,0:2], det_order = 0 , k_ar_diff=4)
```

''' Read output of coint_johansen: https://kite.com/python/docs/statsmodels.tsa.vector_ar.vecm.coint_johansen (https://kite.com/python/docs/statsmodels.tsa.vector_ar.vecm.coint_johansen) '''

In [389]:

```
result.lr1 # Trace statistic
```

Out[389]:

```
array([4.53108969, 0.01443324])
```

In [390]:

```
result.cvt #Shows critical values for trace statistics
```

Out[390]:

```
array([[13.4294, 15.4943, 19.9349],
       [ 2.7055,  3.8415,  6.6349]])
```

In [391]:

```
result.lr2 #Eigen value statistic
```

Out[391]:

```
array([4.51665646, 0.01443324])
```

In [392]:

```
result.cvm #Critical Values for Eigen value statistic
```

Out[392]:

```
array([[12.2971, 14.2639, 18.52  ],
       [ 2.7055,  3.8415,  6.6349]])
```

In [393]:

```
result.evec
```

Out[393]:

```
array([[ 0.07794125, -0.10503688],
       [-0.06442988,  0.11014795]])
```

As per the above Johansen's cointegration test it shows that the variables are not cointegrated

VECM Model

In [394]:

```
#VECM Model
from statsmodels.tsa.vector_ar.vecm import VECM
```

In [395]:

```
model = VECM(endog=df.iloc[:,0:2],k_ar_diff = 4, coint_rank=1, deterministic='co').fit
()
```

```
C:\Users\Divyam Jain\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_
model.py:225: ValueWarning: A date index has been provided, but it has no
associated frequency information and so will be ignored when e.g. forecast
ing.
  ' ignored when e.g. forecasting.', ValueWarning)
```

In [396]:

```
model.summary()
```

Out[396]:

Det. terms outside the coint. relation & lagged endog. parameters for equation SPOT

	coef	std err	z	P>|z|	[0.025	0.975]
const	1.4654	0.681	2.151	0.032	0.130	2.801
L1.SPOT	-0.7691	0.115	-6.681	0.000	-0.995	-0.543
L1.FUTURE	0.7746	0.120	6.434	0.000	0.539	1.011
L2.SPOT	-0.5750	0.139	-4.137	0.000	-0.847	-0.303
L2.FUTURE	0.5923	0.145	4.085	0.000	0.308	0.877
L3.SPOT	-0.1582	0.139	-1.139	0.255	-0.430	0.114
L3.FUTURE	0.1684	0.145	1.159	0.247	-0.116	0.453
L4.SPOT	-0.1487	0.115	-1.294	0.196	-0.374	0.077
L4.FUTURE	0.1369	0.121	1.133	0.257	-0.100	0.374

Det. terms outside the coint. relation & lagged endog. parameters for equation FUTURE

	coef	std err	z	P>|z|	[0.025	0.975]
const	1.4004	0.652	2.149	0.032	0.123	2.677
L1.SPOT	-0.0174	0.110	-0.158	0.874	-0.233	0.198
L1.FUTURE	-0.0060	0.115	-0.052	0.958	-0.232	0.220
L2.SPOT	-0.0408	0.133	-0.307	0.759	-0.301	0.220
L2.FUTURE	0.0367	0.139	0.265	0.791	-0.235	0.308
L3.SPOT	0.1775	0.133	1.336	0.182	-0.083	0.438
L3.FUTURE	-0.1789	0.139	-1.287	0.198	-0.451	0.094
L4.SPOT	0.0188	0.110	0.172	0.864	-0.197	0.234
L4.FUTURE	-0.0366	0.116	-0.317	0.752	-0.263	0.190

Loading coefficients (alpha) for equation SPOT

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-0.0162	0.008	-2.072	0.038	-0.031	-0.001

Loading coefficients (alpha) for equation FUTURE

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	-0.0159	0.007	-2.123	0.034	-0.030	-0.001

Cointegration relations for loading-coefficients-column 1

	coef	std err	z	P>|z|	[0.025	0.975]

beta.1	1.0000	0	0	0.000	1.000	1.000
beta.2	-0.8266	0.084	-9.858	0.000	-0.991	-0.662

CONCLUSION

Coefficient of lagged values of FUTURE in the regression of SPOT: significant at 5% level
Coefficient of lagged values of SPOT in the regression of FUTURE: not significant at 5% level
Coefficient of loading coeffcient in the regression of SPOT: significant at 5% level
Coefficient of loading coeffcient in the regression of FUTURE: significant at 5% level

Hence, based on the above results we can say there is a short-run unidirectional causality running from FUTURE to SPOT and there is a bidirectional long term causality running between the two variables.