MA41031: Stochastic Processes in Finance

<u>Implementation of Black Scholes Model</u>

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Introduction

Black-Scholes Model is used to price options to eliminate arbitrage. It was originally intended for use in pricing options for the European market. This model remains very important because it removes the effect of uncertainties making the result easier to compute. In the following project, we implement the Black-Scholes considering an underlying stock in the Indian stock market.

Assumptions

To determine the price of vanilla European options, several assumptions are made:

- European options can only be exercised at expiration
- No dividends are paid during the option's life
- Market movements cannot be predicted
- The risk-free rate and volatility are constant
- Follows a lognormal distribution

Model

Option prices depend on various factors such as current stock price, intrinsic value, time to expiration or time value, and volatility. Black-Scholes Model is one of the most popular models to price options.

In Black-Scholes formulas, the following parameters are defined:

- S, the spot price of the asset at time 0
- T, the maturity of the option. Time to maturity is defined as T (assuming initial as zero)
- K, strike price of the option
- r, the risk-free interest rate, assumed to be constant between 0 and T
- σ, volatility of underlying asset, the standard deviation of the asset returns

The Black-Scholes model is based on the following partial differential equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

We are looking for V(S,t) where t denotes time and S shows the price of the underlying.

Solving the equation, we get

$$C_0 = \exp(-rT)E_Q[(S_T - K)_+] = S_0\Phi(d_1) - K\exp(-rT)\Phi(d_2)$$

$$d_1 = \frac{log(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{log(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

C_0 denotes the price for a call option.

Methodology

According to the Black-Scholes model, following functions give the price of call and put options respectively:

$c = S_0e^{-qT}N(d_1) - Ke^{-rT}N(d_2)$
$p = Ke^{-rT}N(-d_2) - S_0e^{-rT}N(-d_1)$

Where

$d_1 = \{ \ln(S_0/K)+(r-q+\sigma^2/2)T \}/ \sigma \sqrt{T}$
$d_2 = \{ \ln(S_0/K)+(r-q-\sigma^2/2)T \}/ \sigma \sqrt{T} = d_1 - \sigma \sqrt{T}$

Code in R

```
Call <- function(S, K, r, T, sigma) {
  d1<- (log(S/K) + (r + sigma^2/2)*T) / (sigma*sqrt(T))
  d2<- d1 - sigma*sqrt(T)
  S * pnorm(d1) -  K*exp(-r*T)*pnorm(d2)
}

Put  <- function(S, K, r, T, sigma) {
  d1<- (log(S/K) + (r + sigma^2/2)*T) / (sigma*sqrt(T))
  d2<- d1 - sigma*sqrt(T)
  -S * pnorm(-d1) -  K*exp(-r*T)*pnorm(-d2)
}
```

Let us consider the following example :

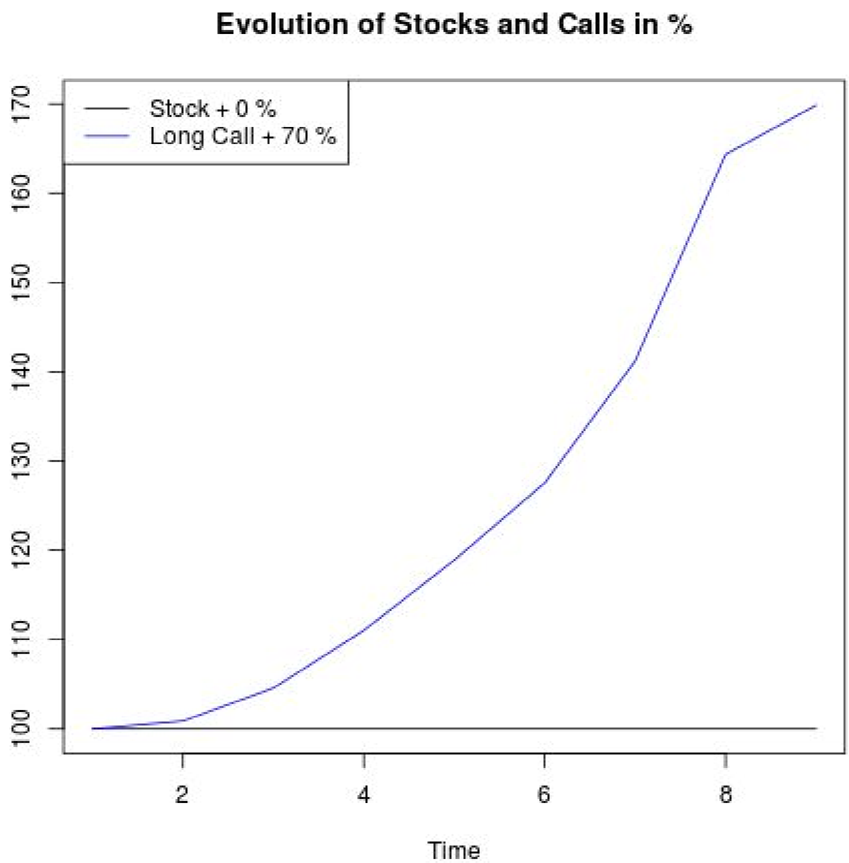
```
#Parameters
S        <- 100   # Spot Price
K         <- 70     # Strike Price
r        <- 0      # risk-free Interest rate
T        <- 1       # Time to Maturity
Sigma <- c(.16, .21, .3, .4, .5, .6, .75, 1, 1.06)     #Implied annual volatility
```

So following is the further calculation steps :

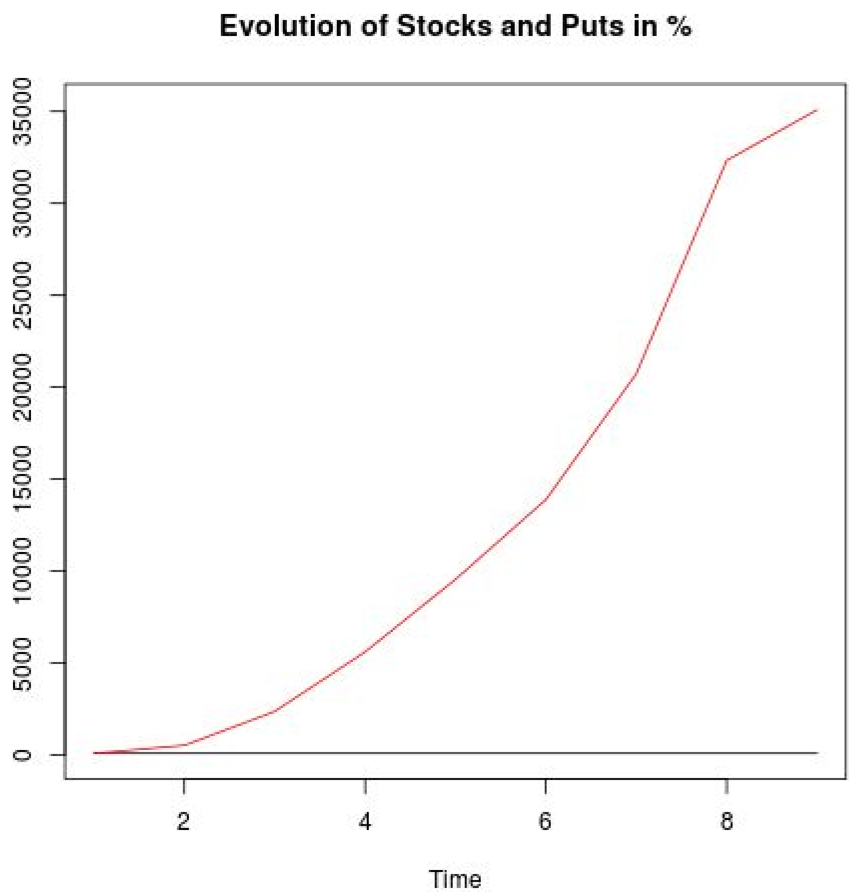
```
d1  <-  (log(S/K) + (r + sigma^2/2)* T) / (sigma*sqrt(T))
d2  <-  d1 - sigma*sqrt(T)
p   <-  cbind(S, K, T, sigma, p = S * pnorm(-d1) -  K*exp(-r*T)*pnorm(d2))
p   <-  cbind(S, K, T, sigma, p = S * pnorm(-d1) -  K*exp(-r*T)*pnorm(d2))
```

Plotting θ(time value of an option) :

```
cprct  <-  c[,c(1,5)] / matrix(rep(c[1,c(1,5)],each=length(c[,5]) ),ncol = 2) * 100
ts.plot(cprct, col = c(1,4), main = "Evolution of Stocks and Calls in %")
legend('topleft', c(paste("Stock +",tail(cprct,1)[1]-100, "%") ,
                     paste("Long Call +", round(tail(cprct,1)[2])-100, "%")),
         col=c(1,4), lty=1)
```




```
pprct <- p[,c(1,5)] / matrix(rep(p[1,c(1,5)],each=length(c[,5]) ),ncol = 2) * 100
ts.plot(pprct, col=1:2, main = "Evolution of Stocks and Puts in %")
```



Real-Time Example

We used the day wise stock price collected from IBM. These prices formed the underlying. We set the strike price of 70 and spot price to be 100 . The risk free rate of return is taken to be 0.02 and the time until expiration is taken to be 1.

R Code

```
library(quantmod)
getSymbols("IBM")
head(IBM)
library(magrittr)
dts = IBM %>% as.data.frame %>% row.names
dts %>% head %>% print
dts %>% length %>% print
stkp = as.matrix(IBM$IBM.Adjusted)
rets = diff(log(stkp))
dts = as.Date(dts)
```

```
sigma_daily = sd(rets)
sigma_annual = sigma_daily*sqrt(252)
print(sigma_annual)
sigma = sigma_annual
S       <- 100   # Spot Price
K       <- 70    # Strike Price
r       <- 0.02          # risk-free Interest rate
T       <- 1     # Time to Maturity
d1  <-  (log(S/K) + (r + sigma^2/2)*T) / (sigma*sqrt(T))
d2  <-  d1 - sigma*sqrt(T)
p = -S * pnorm(-d1) + K*exp(-r*T)*pnorm(-d2)
c =  S * pnorm(d1)  - K*exp(-r*T)*pnorm(d2))
```

Results

After running the code, we get the price of a call option price as 0.33 and put option price as 31.72.

References

1. "Understanding How Options are Priced", Mitchell Grant, *Investopedia*
2. "Black Scholes Model Definition", Will Kention, *Investopedia*
3. Math 425 (Partial Differential equations), Solving Black-Scholes PDE, Prof Dennis DeTurck, *University of Pennsylvania*
4. "Black-Scholes option pricing", Steve Riedo, *RPubs*