(1) Given:

- test to detect a disease that 0.1% have population have.
- test is 99% effective in detecting an infected person.
- false +ve rate is 0.5% of cases.

Let E1 and E2

E1 → disease

E2 → no disease.

E1 & E2 are complimentary to each other.

$$P(E_1) + P(E_2) = 1$$

$$P(E_2) = 1 - P(E_1) = 1 - 0.001$$

$$= 0.999$$

Let A be the event that blood test result is positive.

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$$

$P(A|E_1) = P \Rightarrow 99\% \Rightarrow 0.99$ disease

$P(A|E_2) = P \quad 0.5\% \Rightarrow 0.005$ no disease

Probability of person has disease and test result is +ve

$$P(E_1|A)$$

By Bayes theorem.

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$\Rightarrow \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$$

$$\Rightarrow \frac{0.00099}{0.00099 + 0.004995}$$

$$\Rightarrow \frac{0.00099}{0.005985}$$

$$\Rightarrow \frac{990}{5985}$$

$$\Rightarrow \frac{110}{665}$$

$$\Rightarrow \frac{110}{665}$$

$$\Rightarrow \frac{22}{133}$$

$$\Rightarrow \boxed{0.165}$$

(2) Given:

Production line 1

$n_1 \rightarrow$ no. of parts $= 1000$

defective $d_1 \rightarrow$ no. of defective $= 100$

Production line 2

$n_2 =$ no. of parts $= 2000$

$d_2 =$ no. of defectives $= 150$

Total no. of parts $(n) = n_1 + n_2$

$= 1000 + 2000$

$= 3000$.

Probability of defectives $P(d) = \frac{d_1 + d_2}{n_1 + n_2}$

$= \frac{100 + 150}{1000 + 2000}$

$= \frac{250}{3000} \Rightarrow 0.0833$.

Probability of defective in lines is

$$P(\text{line}_1 | d) = \frac{P(\text{line}_1 \cap d)}{P(\text{line}_1 \cap d) + P(\text{line}_2 \cap d)}$$

$$= \frac{\frac{\cancel{3000}}{3} \quad \frac{100}{1000}}{\frac{100}{3000} + \frac{150}{3000}}$$

$$\Rightarrow \frac{10}{25} \Rightarrow \boxed{0.4}$$

③ Given:

~~Ther~~ store Owner opens a Large shipment of shirts of Various colours.

- 56% of blue shirts.
- store randomly selects five shirts.

$$P(x) = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

$x \rightarrow$ no. of successes
$n \rightarrow$ no. of trails.
$p \rightarrow$ no. of success in a trail.
$q \rightarrow$ no. of failures in a trail.

The Probability may be found by determining there is '0' blue shirt in selection. Subtracting it from 1.

$x = 0$, $n = 5$, $p = 0.56$ $q = 0.44$

$$P(0 \text{ blue shirt}) = \frac{5!}{5!\,0!}(0.56)^0(0.44^5)$$

$$= (1)(1)(0.0165)$$

$$\Rightarrow 0.0165$$

$$P(\text{at least 1 blue}) = 1 - P(0 \text{ blue shirt})$$

$$= 1 - 0.0165$$

$$= 0.9835$$

$$= \boxed{98.35\%}$$