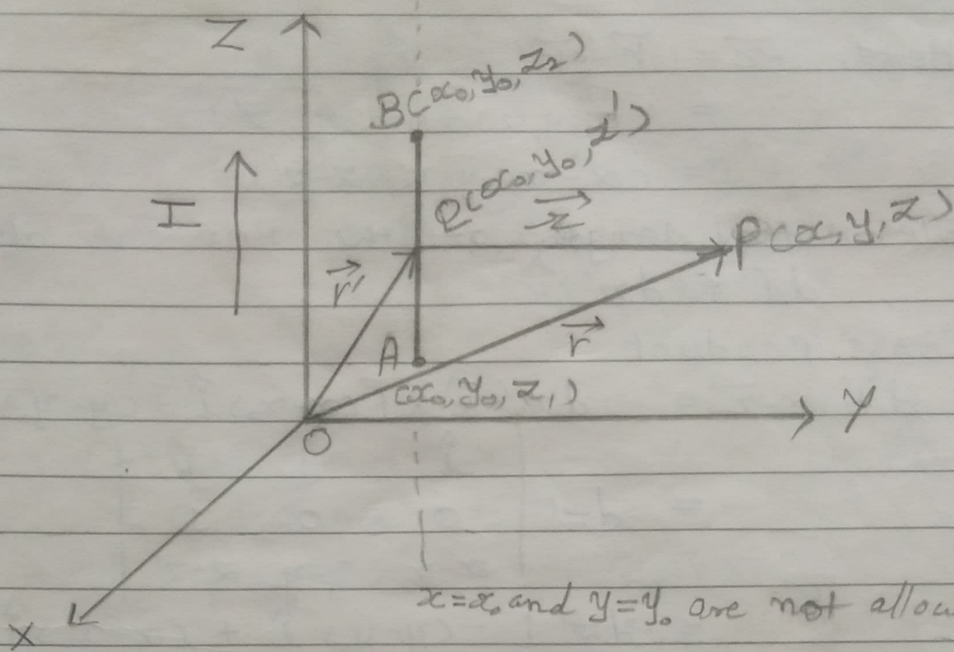
1 # Magnetic field due to straight conducting filament



$x = x_0$ and $y = y_0$ are not allowed

→ A current carrying straight filament is placed parallel to z-axis.

→ Here,

(1) source point , $Q(x_0, y_0, z')$

(2) observation Point, $P(x, y, z)$

(3) lower end of the filament , $A(x_0, y_0, z_1)$

(4) upper end of the filament, $B(x_0, y_0, z_2)$

(5) The direction of current I is from A to B

→ The magnetic field $\vec{B}$ at P can be written as,

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_A^B \frac{d\vec{l'} \times \vec{r}}{|\vec{r}|^3}$$

$$= \frac{\mu_0 I}{4\pi} \vec{H}$$

→ Where,

$$\vec{H} = \int_A^B \frac{d\vec{l'} \times \vec{r}}{|\vec{r}|^3} \qquad (1)$$

→ Here, $\vec{\imath} = \vec{r} - \vec{r}'$

$= (x, y, z) - (x_0, y_0, z')$

$= (x - x_0, y - y_0, z - z')$

→ Elementary length of the filament along z-axis,

$d\vec{l} = dz' \hat{k}$

→ Cross product,

$$d\vec{l} \times \vec{\imath} = dz' \hat{k} \times [(x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z')\hat{k}]$$

$$= dz' \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ x - x_0 & y - y_0 & z - z' \end{vmatrix}$$

$$= dz' [-(y - y_0)\hat{i} + (x - x_0)\hat{j} + 0\hat{k}]$$

→ $|\vec{\imath}| = [(x - x_0)^2 + (y - y_0)^2 + (z - z')^2]^{1/2}$

→ Now, sub. all these values in Eq^n (1)

$$\therefore \vec{H} = \int_{z_1}^{z_2} \frac{[-(y - y_0)\hat{i} + (x - x_0)\hat{j} + 0\hat{k}]\, dz'}{[(x - x_0)^2 + (y - y_0)^2 + (z - z')^2]^{3/2}}$$

→ The components of $\vec{H}$ are,

$$H_x = -\int_{z_1}^{z_2} \frac{(y - y_0)\, dz'}{[(x - x_0)^2 + (y - y_0)^2 + (z - z')^2]^{3/2}}$$

$$H_y = \int_{z_1}^{z_2} \frac{(x - x_0)\, dz'}{[(x - x_0)^2 + (y - y_0)^2 + (z - z')^2]^{3/2}}$$

$$H_z = 0$$

→ Let, $u = x - x_0$ ⇒ $dw' = -dz'$

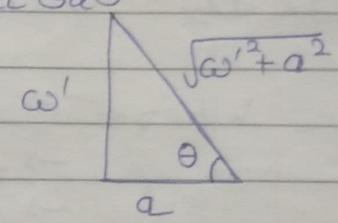
$v = y - y_0$ → $w_1 = z - z_1$

$w' = z - z'$ $\quad w_2 = z - z_2$

(1) $$H_x = -\int_{\omega_1}^{\omega_2} \frac{v\,(-d\omega')}{[u^2+v^2+\omega'^2]^{3/2}}$$

$$= v\int_{\omega_1}^{\omega_2} \frac{d\omega'}{[a^2+\omega'^2]^{3/2}}, \quad a^2 = u^2+v^2 \text{ (constant)}$$

→ take $\omega' = a\tan\theta$, $\Rightarrow d\omega' = a\sec^2\theta\, d\theta$



$$\int \frac{d\omega'}{[a^2+\omega'^2]^{3/2}}$$

$$= \int \frac{a\sec^2\theta\, d\theta}{[a^2+a^2\tan^2\theta]^{3/2}}$$

$$= \frac{1}{a^2}\int \cos\theta\, d\theta$$

$$= \frac{1}{a^2}\sin\theta$$

$$= \frac{1}{a^2}\frac{\omega'}{\sqrt{\omega'^2+a^2}}$$

$$\therefore H_x = \frac{v}{a^2}\left[\frac{\omega'}{\sqrt{a^2+\omega'^2}}\right]_{\omega_1}^{\omega_2}$$

$$= \frac{v}{a^2}\sum_{i=1}^{2}(-1)^i \frac{\omega_i}{\sqrt{a^2+\omega_i^2}}$$

→ Similarly we can find,

(2) $$H_y = -\frac{u}{a^2}\sum_{i=1}^{2}(-1)^i\frac{\omega_i}{\sqrt{a^2+\omega_i^2}}$$

(3) $H_z = 0$

→ So, the field components for a straight current carrying filament are,

(1) $B_x = \frac{\mu_0 I}{4\pi}\left(\frac{v}{a^2}\right)\sum_{i=1}^{2}(-1)^i \frac{\omega_i}{\sqrt{a^2+\omega_i^2}}$

(2) $B_y = -\frac{\mu_0 I}{4\pi}\left(\frac{u}{a^2}\right)\sum_{i=1}^{2}(-1)^i \frac{\omega_i}{\sqrt{a^2+\omega_i^2}}$

(3) $B_z = 0.0$

Where, → $a^2 = u^2 + v^2$,

→ $u = x - x_0$ → $v = y - y_0$,

→ $\omega_1 = z - z_1$ → $\omega_2 = z - z_2$

→ $\mu_0 = 4\pi \times 10^{-7}$ SI

→ I is flowing from lower to upper end

* References :-

1 Analytical derivation of magnetic field and vector potential due to different current carrying conducting geometries, Mrityunjay Kundu & Shishir P. Deshpande, IPR
2 Special thanks to Gattu Ramesh Sir.