

EE3025 Presentation

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Given Problem

Compute

$$X(k) = \Delta \sum_{n=1}^{N-1} x(n) e^{-j2\pi kn/N} \quad k = 0, 1, \dots, N-1$$

and $H(k)$ using $h(n)$

Solution

Sol: Given,

$$y(n) + (1/2)y(n-1) = x(n) + x(n-2) \quad \dots(1)$$

Where, $x(n) = \{1, 2, 3, 4, 2, 1\}$

Now, For $H(k)$ we need $h(n)$

Taking Z-Transform,

$$Y(z) = \frac{2(z^2 + 1)}{z(2Z + 1)}X(z)$$

and

$$H(z) = \frac{2(z^2 + 1)}{z(2Z + 1)}$$

$$H(z) = \left[\frac{1}{1 + (1/2)z^{-1}} + \frac{z^{-2}}{1 + (1/2)z^{-1}} \right] z^{-1}$$

Taking Inverse-Z transform

$$h(n) = \left[\frac{-1}{2} \right]^{n-2} u(n-2) + \left[\frac{-1}{2} \right]^n u(n)$$

Now, We know that

$$H(k) = \sum_{n=1}^{N-1} h(n)e^{-j2\pi kn/N}$$

Where, $k = 0, 1, \dots, N-1$

and

$$X(k) = \sum_{n=1}^{N-1} x(n)e^{-j2\pi kn/N}$$

Where, $k = 0, 1, \dots, N-1$

Plots:

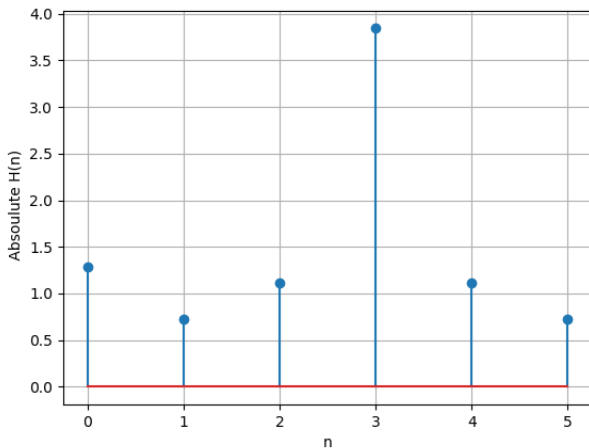


Figure: Absolute $H(K)$

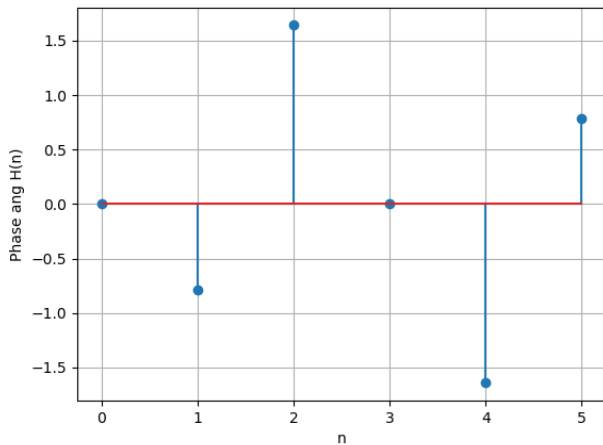


Figure: Angle $H(K)$

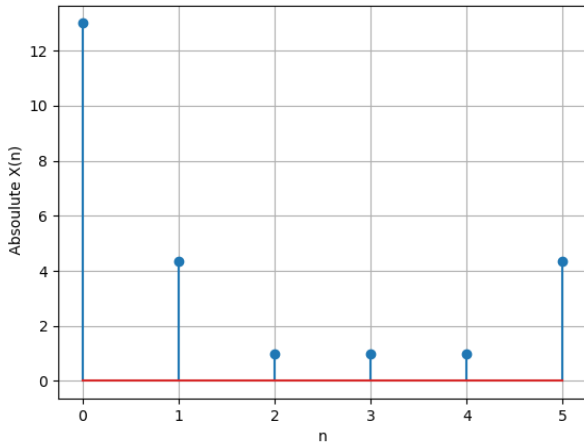


Figure: Absolute $X(K)$

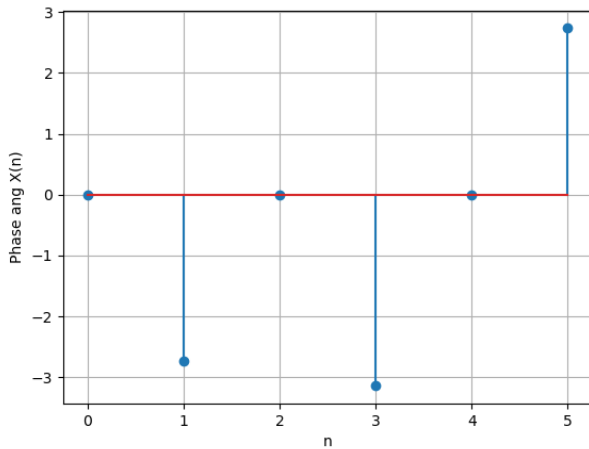


Figure: Angle $X(K)$

Now, Let's take $e^{-j2\pi kn/N} = W^{nk}$

Expressing (1) in the form of DDT Matrix, N=6

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W & . & . & . & W^5 \\ 1 & W^2 & . & . & . & W^{10} \\ 1 & W^3 & . & . & . & W^{15} \\ 1 & W^4 & . & . & . & W^{20} \\ 1 & W^5 & . & . & . & W^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

We know that $x(n) = \{ 1, 2, 3, 4, 2, 1 \}$

putting the value of $x(n)$ and after solving We get

$$X(0) = 13 + 0j$$

$$X(1) = -4 - 1.73j$$

$$X(2) = 1 + 0j$$

$$X(0) = -1 + 0j$$

$$X(0) = 1 + 0j$$

$$X(0) = -4 + 1.73j$$

Which matches with The plots of $X(n)$

Similarly for $H(k)$, Let's Take $N=6$ too

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W & . & . & . & W^5 \\ 1 & W^2 & . & . & . & W^{10} \\ 1 & W^3 & . & . & . & W^{15} \\ 1 & W^4 & . & . & . & W^{20} \\ 1 & W^5 & . & . & . & W^{25} \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \\ h(4) \\ h(5) \end{bmatrix}$$

We know that

$$h(n) = \left[\frac{-1}{2} \right]^{n-2} u(n-2) + \left[\frac{-1}{2} \right]^n u(n)$$

putting the value of $h(n)$ and after solving We get,

$$H(0) = 1.29 + 0j$$

$$H(1) = 0.54 - 0.51j$$

$$H(2) = -1.1 + 1.53j$$

$$H(3) = -3.8 + 0j$$

$$H(4) = -1.1 - 1.53j$$

$$H(5) = 0.54 + 0.51j$$

Which matches with The plots of $H(n)$