Assignment-1 (EE3025)

EE18BTECH11013 - Divyansh Maduriya

Download all python codes from

https://github.com/Divyansh−28/EE3025−EE18BTECH11013/tree/master/codes

and latex-tikz codes from

https://github.com/Divyansh−28/EE3025−EE18BTECH11013

1 Problem

Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.0.1}$$

and $H(k)$ using h(n).

2 Solution

we know that

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2.0.1}$$

$$Where, x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{2.0.2}$$

Taking Z-transform

$$Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{2.0.3}$$

and

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{2.0.4}$$

$$H(z) = \left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right]z^{-1} \tag{2.0.5}$$

Taking Inverse Z-Transform,

$$h(n) = \left[\frac{-1}{2}\right]^{n-2} u(n-2) + \left[\frac{-1}{2}\right]^{n} u(n) \tag{2.0.6}$$

From equation 1.0.1

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.0.7}$$

and

$$H(k) = \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.0.8}$$

Plots:

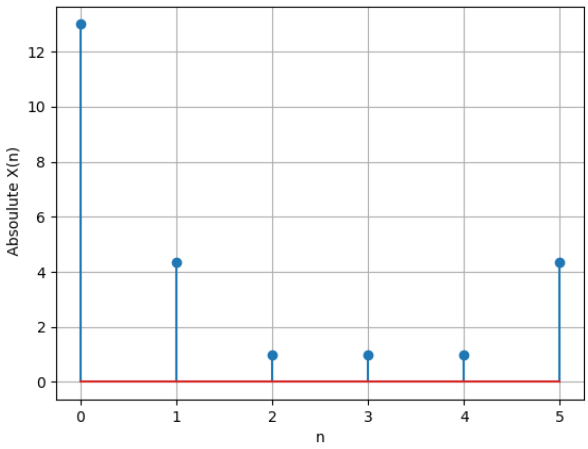

Fig. 0: Abs X(n)

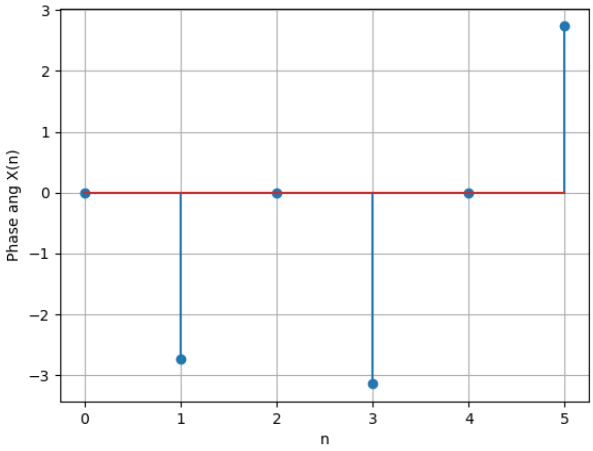

Fig. 0: angle X(n)

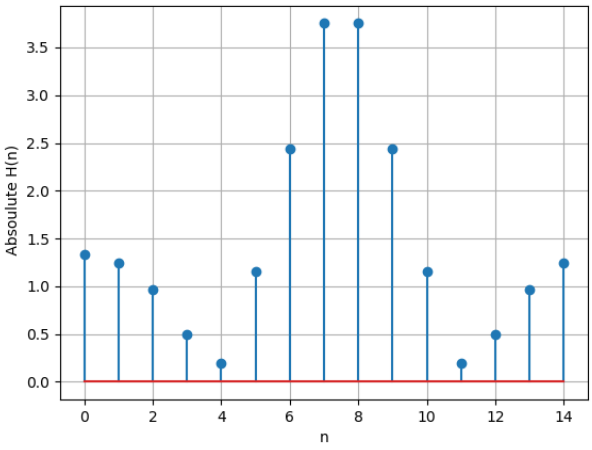

Fig. 0: Abs H(n)

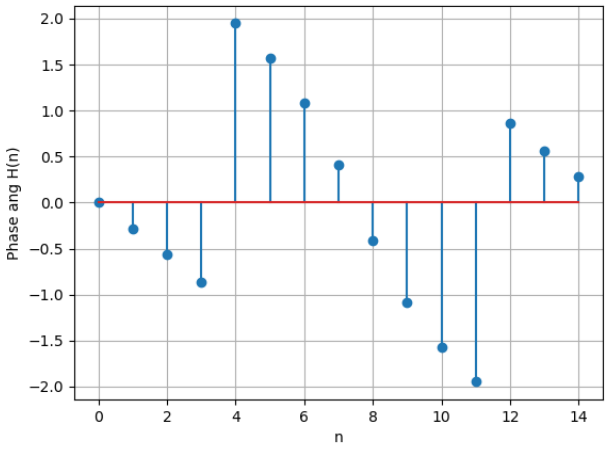

Fig. 0: Phase of h(k)

3 DFT Matrix

Now, Let's take $e^{-j2\pi kn/N} = W^{nk}$
Expressing (2.08) in the form of DFT Matrix, N=6

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W & . & . & . & W^5 \\ 1 & W^2 & . & . & . & W^{10} \\ 1 & W^3 & . & . & . & W^{15} \\ 1 & W^4 & . & . & . & W^{20} \\ 1 & W^5 & . & . & . & W^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

We know that x(n)= { 1,2,3,4,2,1 }
putting the value of x(n) and after solving We get

X(0) = 13 + 0j
X(1) = -4 -2.73j
X(2) = 1 + 0j
X(3) = -1 - 3.2j
X(4) = 1 + 0j
X(5) = -4 + 2.73j

Which matches with The plots of X(n)

Similarly for H(k), Let's Take N=6 too

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W & . & . & . & W^5 \\ 1 & W^2 & . & . & . & W^{10} \\ 1 & W^3 & . & . & . & W^{15} \\ 1 & W^4 & . & . & . & W^{20} \\ 1 & W^5 & . & . & . & W^{25} \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \\ h(4) \\ h(5) \end{bmatrix}$$

We know that

$$h(n) = \left[\frac{-1}{2}\right]^{n-2} u(n-2) + \left[\frac{-1}{2}\right]^{n} u(n)$$

putting the value of h(n) and after solving We get,
H(0) = 1.29 + 0j
H(1) = 0.54 -0.51j
H(2) = -1.1 + 1.53j
H(3) = -3.8 + 0j
H(4) = -1.1 - 1.53j
H(5) = 0.54 + 0.51j

Which matches with The plots of H(n)