Q-2-(e) $f_1(t) = \begin{cases} 1 & ; \; 0 < t < 1/2 \\ -1 & ; \; 1/2 < t < 1 \end{cases}$

$$C_n = \int_0^{1/2} 1 \times e^{-in\omega_0 t}\, dt - \int_{1/2}^{1} e^{-i\omega_0 n t}\, dt \quad \{\omega_0 = 2\pi\}$$

$$= \frac{1}{in\omega_0}\left[1 - e^{-n\pi i}\right] + \frac{1}{in\omega_0}\left[e^{-in2\pi} - e^{-in\pi}\right]$$

$$= \frac{2}{in\omega_0}\left[1 - e^{in\pi}\right]$$

$$= \begin{cases} \dfrac{4}{in\omega_0} & ; \; n = \text{odd} \\ 0 & ; \; n = \text{even} \end{cases}$$

$$f(t) = \sum_{n=1,3,5} \frac{4}{in\omega_0} e^{-in\omega_0 t}$$

$$= \sum_{n=1,3,5} \frac{2}{in\pi} e^{-2\pi n t}$$

$$= \sum_{n=1,3,5} \frac{2}{in\pi}\left[\cos(2n\pi t) + i\sin(2\pi n t)\right]$$

$$= \sum_{n=1,3,5} \frac{2}{n\pi} \sin(2\pi n t)$$

$$C_k = \frac{4}{ik\omega_0} = \frac{2}{ik\pi}$$

$$a_k = 0$$

$$b_k = \frac{2}{k\pi} \; ; \; k = \text{odd}$$

$$f_2(t) = \begin{cases} \frac{1}{2} - t & ; \ 0 < t < \frac{1}{2} \\ \frac{1}{2} + t & ; \ -\frac{1}{2} < t < 0 \end{cases}$$

$= 2\pi \}$

$$C_n = \int_{-1/2}^{0} \left(\frac{1}{2}+t\right) e^{-in\omega_0 t}\, dt + \int_{0}^{1/2} \left(\frac{1}{2}-t\right) e^{-in\omega_0 t}\, dt$$

$$= \frac{-1}{2in\omega_0} - \left[\frac{e^{-in\omega_0 t}}{(-in\omega_0)^2}\right]_{-1/2}^{0} + \frac{-1}{2in\omega_0} + \left[\frac{e^{-in\omega_0 t}}{(-in\omega_0)^2}\right]_{0}^{1/2}$$

$$= \frac{-1}{in\omega_0} + \frac{2}{(-in\omega_0)^2}\left[e^{-in\pi} - 1\right]$$

$$C_K \begin{cases} \frac{-1}{in\omega_0} & ; \ n = \text{even} \\ \frac{-1}{in\omega_0} - \frac{4}{(in\omega_0)^2} & ; \ n = \text{odd} \end{cases}$$

$$f(t) = \sum_{n=\text{even}} \frac{-1}{in\omega_0}\left(\cos(\omega_0 n t) + i\sin(\omega_0 n t)\right) + \sum_{n=\text{odd}} \left(\frac{4}{n^2\omega_0^2} - \frac{1}{in\omega_0}\right)\left(\cos(\omega_0 n t) + i\sin(\omega_0 n t)\right)$$

$$C_K = \begin{cases} -1/in2\pi & ; \ n = \text{even} \\ \frac{-1}{i2\pi n} + \frac{1}{n^2\pi^2} & ; \ n = \text{odd} \end{cases}$$

$$a_K = \begin{cases} 0 & ; \ k = \text{even} \\ 1/k^2\pi^2 & ; \ k = \text{odd} \end{cases}$$

$$b_K = \left\{ \frac{-1}{2\pi K} \right.$$

$$f(t) = \sum_{n=\text{even}} -\frac{1}{2\pi n}\sin(2\pi n t) + \sum_{n=\text{odd}} \frac{1}{n^2\pi^2}\cos(2\pi n t) - \frac{1}{2\pi n}\sin(2\pi n t)$$

(g-) For a fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ to be convergent:

The periodic function $f(x)$ should be continuous in $[-l, l]$ and has continuous first & second derivatives at each point in that interval $[-l, l]$

$$f_1 = \begin{cases} 1 & ; \quad 0 \le t < \frac{1}{2} \\ -1 & ; \quad \frac{1}{2} \le t < 1 \end{cases}$$

can be written as

$$f_1 = \begin{cases} -1 & ; \quad -\frac{1}{2} \le t < 0 \\ 1 & ; \quad 0 \le t < \frac{1}{2} \end{cases}$$

[$\because$ f_1 is periodic with period 1]

Since, f_1 is discontinuous, so fourier series of f_1 is not convergent

$$f_2 = \begin{cases} \frac{1}{2} + t & ; \quad -\frac{1}{2} \le t < 0 \\ \frac{1}{2} - t & ; \quad 0 \le t < \frac{1}{2} \end{cases}$$

$$f_2 = \begin{cases} 1 & ; \quad -\frac{1}{2} \le t < 0 \\ -1 & ; \quad 0 \le t < \frac{1}{2} \end{cases}$$

Since, first derivative of f_2 is discontinuous so, fourier series of f_2 is not convergent.