Engineering Electromagnetism

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HA - II

1. $Q_1 = 20\mu C$

$Q_2 = -300\mu C$

$Q_1\ (0, 1, 2)$ $\quad$ $Q_2\ (2, 0, 0)$

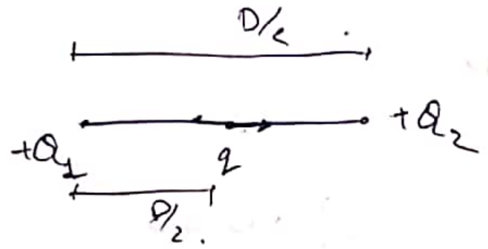
$|\vec{r}| = \sqrt{2^2 + 1^2 + 2^2} = 3$

$$|F_{12}| = \frac{K Q_1 Q_2}{|\vec{r}|^2}$$

$$= \frac{9\times10^9 \times 20\times10^{-6} \times 300\times10^{-6}}{9}$$

$|F_{12}| = 6N$ (Force of attraction)

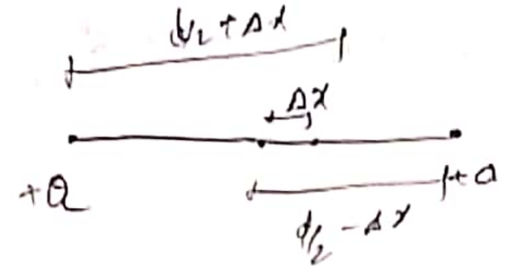
2. a)



~~$Q_1 = Q_2$~~

$F_{qQ_1} = F_{qQ_2}$ (as $Q_1 = Q_2$)

=> The force of equal magnitude is in opposite direction hence the net force is zero b)



$$\vec{F_1} = \frac{kQq}{(d/2 + \Delta x)^2} \quad , \quad \vec{F_2} = \frac{kQq}{(d/2 - \Delta x)^2}$$

$$F_{NET} = \vec{F_1} - \vec{F_2}$$

$$= \frac{kQq}{(d/2 + \Delta x)^2} - \frac{kQq}{(d/2 - \Delta x)^2}$$

$$= kQq \left[\frac{(d/2 - \Delta x)^2 - (d/2 + \Delta x)^2}{\left((d/2 + \Delta x)(d/2 - \Delta x) \right)^2} \right]$$

$$= kQq \left[\frac{\cancel{\frac{d^2}{4}} + \cancel{\Delta x^2} - d\Delta x - \cancel{\frac{d^2}{4}} - \cancel{\Delta x^2} - d\Delta x}{\left(\frac{d^2}{4} - \Delta x^2 \right)^2} \right]$$

$$= kQq \left[\frac{-2d\Delta x}{\frac{d^{4}}{16}} \right] \qquad \frac{d^2}{4} >>> \Delta x$$

$$= -\frac{kQq \times 32}{d^3} \Delta x$$

$$= -K \Delta x$$

$\therefore \boxed{\vec{F_1} \propto -\Delta x}$ => F it will exhibit S.H.M

$$T = 2\pi\sqrt{\frac{m}{k}}$$

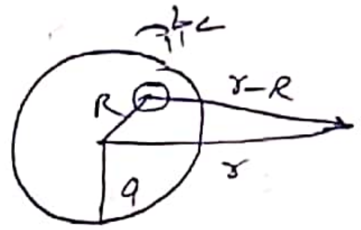
c)

$$T = 2\pi\sqrt{\frac{m \times d^3}{KQq \times 32}}$$

$$T = 2\pi\sqrt{\frac{md^2 4\pi\varepsilon_0}{Qq\,32\,8}}$$

$$\boxed{T = 2\pi\sqrt{\frac{md^2\pi\varepsilon_0}{8Qq}}} \Rightarrow \boxed{f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{8Qq}{md^2\pi\varepsilon_0}}}$$

5. $\vec{E}_{NET} = \vec{E}_{Sphere} - \vec{E}_{cavity}$



$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{|r-R|^2} \quad \text{— (1)}$$

Now

$q\alpha$ $\quad q = \sigma 4\pi b^2$

~~Multiplying~~

$$= \frac{Q\,a^2}{4\pi\varepsilon_0 a^2 r^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{|r-R|^2}$$

$$= \frac{\sigma a^2}{\varepsilon_0 r^2} - \frac{\sigma \cancel{4\pi} b^2}{\cancel{4\pi}\varepsilon_0 |r-R|^2}$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{r^2} - \frac{b^2}{|r-R|^2}\right] = \frac{\sigma}{\varepsilon_0}\cancel{\left[\frac{a^2(r-R)^2 - r^2 b^2}{r^2 - (r-R)^2}\right]}$$

as $|r-R| >>> 1$

$\therefore \dfrac{b}{|r-R|} <<<< 1$ and can be neglected

$$\therefore \boxed{\vec{E}_{NET} = \frac{\sigma a^2}{\varepsilon_0 r^2}} \Rightarrow$$

$$|E_{NET}| \propto \frac{1}{r^2}$$

3. E is a vector field therefore by Stokes theorem we get

$$\Rightarrow \int (\nabla \times \vec{E}) \cdot ds = \oint \vec{E} \cdot dl$$

$$\Rightarrow -\int (\nabla \times \nabla V) \cdot ds = \oint \vec{E} \cdot dl \qquad (E = -\nabla V)$$

$\Rightarrow$ Now we know that curl of a gradient is zero.

$$\therefore -\int (\nabla \times \nabla V) \cdot ds = \oint \vec{E} \cdot dl = 0$$

$\therefore \oint E \cdot dl = 0$ therefore workdone by the electric field is path independent therefore it is conservative.

4. Given

$$\lambda \approx \frac{-|e|}{1.7A^\circ}$$

$$\oint E \cdot ds = \frac{q}{\varepsilon_0} \quad \text{(Gauss's law)}$$

$$\lambda = \frac{q}{dr} \Rightarrow q = \lambda dr$$

$$\Rightarrow \oint \vec{E} \cdot ds = \frac{\lambda dr}{\varepsilon_0}$$

$\Rightarrow$ As $\vec{E}$ is uniform & parallel to $\vec{ds}$

$$\Rightarrow E \times 2\pi r \cdot dr = \frac{\lambda \cdot dr}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

$$\lambda = \frac{-|e|}{1.7A^\circ} \Rightarrow E = \frac{-|e|}{2\pi r \varepsilon_0 \times 1.7}$$

$$\boxed{E = \frac{-17}{r}}$$

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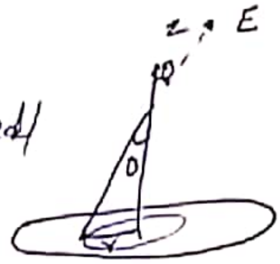
b. $(0, \phi, h)$

Let's consider electric field due to a ring



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \frac{Kqh}{(h^2+R^2)^{3/2}}$$

$q = \sigma 2\pi r dr$

$$dE = E\cos\theta$$

$$= \frac{Kqh}{(h^2+r^2)^{3/2}} \frac{h}{\sqrt{h^2+r^2}}$$

$$dE = \frac{Kqh^2}{(h^2+r^2)^2}$$

$$= \frac{K\sigma 2\pi r dr\, h^2}{(h^2+r^2)^2}$$

$\Rightarrow$ for the whole disc $E = \int_0^a \frac{1}{2\cancel{4\pi}\varepsilon_0}\frac{\sigma \cancel{2\pi} r dr h^2}{(h^2+r^2)^2}$

$$= \int_0^a \frac{\sigma r h^2 dr}{2\varepsilon_0 (h^2+r^2)^2} = \frac{\sigma h^2}{2\varepsilon_0}\int_0^a \frac{r dr}{(h^2+r^2)^2}$$

$$= \frac{\sigma h^2}{2\varepsilon_0}\left[-\frac{1}{2(h^2+r^2)}\right]_0^a$$

$$\boxed{= \frac{\sigma h^2}{4\varepsilon_0}\left[\frac{1}{h^2} - \frac{1}{h^2+a^2}\right]}$$

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