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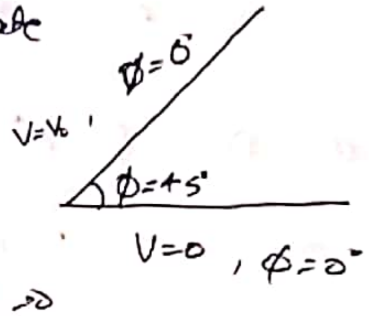
EE18BTECH11013

Engineering Electromagnetism

Assignment - III

1 a) Using Cylindrical Coordinate System we get,



$$\frac{\rho\,\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{\partial}{\partial\phi}\left(\frac{\partial V}{\partial\phi}\right) \to 0$$

$$\Rightarrow V = \left(Ae^{k} + Be^{-k}\right)\left(C\sin k\phi + D\cos k\phi\right)$$

at $\phi = 0$, $V = 0$

$\therefore D = 0$

$\phi \to \infty$, $V = 0 \Rightarrow A = 0$

$$V = Ce^{-k}\sin k\phi$$

at $\phi = \pi/4 \Rightarrow V = V_0$, hence $C = \dfrac{V_0 \rho^{k}}{\left(\sin k\pi/4\right)}$

$$\Rightarrow V(\rho, \phi) = V_0 \frac{\sin k\phi}{\sin\left(\frac{k\pi}{4}\right)}$$

(b) Surface Charge density (σ):

$$\sigma = (\varepsilon_0 \vec{E}) = -\varepsilon_0(\nabla V) = \frac{-\varepsilon_0 1}{\rho}\left(\frac{\partial V}{\partial\phi}\right)$$

$$\Rightarrow \sigma = -\frac{\varepsilon_0}{\sigma}\frac{\partial}{\partial\phi}\left(V_0\frac{\sin k\phi}{\sin k\pi/4}\right)$$

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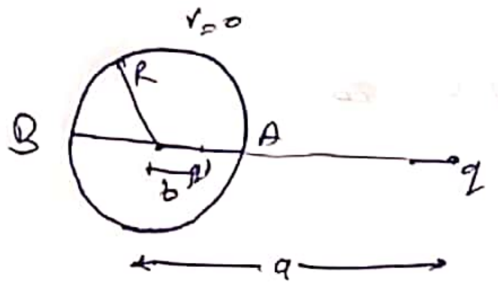
$$\Rightarrow \quad \sigma = \frac{-V_0 \varepsilon K}{\sigma \sin(K\pi/r)} \left(\cos K\phi\right)$$

$$\Rightarrow \quad \sigma = \frac{-KV_0\varepsilon_0}{\sigma} \left(\frac{\cos K\phi}{\sin K\pi/r}\right)$$

~~at~~ $\phi = 0^\circ$

we have $\boxed{\sigma = \dfrac{-KV_0\varepsilon_0}{\sigma \sin K\pi/r}}$

3.



let q' is placed at distance b from the centre

$$V_A = \frac{Kq}{a-R} + \frac{Kq'}{R-b} \quad \text{—} ①$$

$$\& \quad V_B = \frac{Kq}{a+R} + \frac{Kq'}{R+b} \quad \text{—} ②$$

and from ① & ② we have

as the surface is equipotential

$$\therefore V_A = V_B$$

$$\frac{Kq}{a-R} + \frac{Kq'}{R-b} = \frac{Kq}{a+R} + \frac{Kq'}{R+b}$$

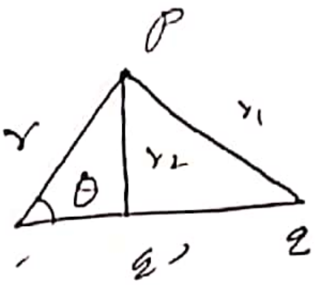
we get

$$\boxed{q' = -\left(\frac{R}{a}\right)q}, \quad \boxed{b = R^2/a}$$

Now

$V_p = V(r, \theta)$

$= V_q + \vec{V}_{q'}$



$= \frac{Kq}{r_1} + \frac{Kq'}{r_2}$

$r_1 = \sqrt{r^2+a^2-2ar\cos\theta}$

$= Kq\left[\frac{1}{\sqrt{r^2+a^2-2ar\cos\theta}} - \frac{R/a}{\sqrt{r^2+b^2-2rb\cos\theta}}\right]$ $r_2 = \sqrt{r^2+b^2-2rb\cos\theta}$

$V_{P(r,\theta)} = Kq\left[\frac{1}{\sqrt{r^2+a^2-2ar\cos\theta}} - \frac{1}{\sqrt{R^2+\left(\frac{ra}{R}\right)^2-2ar\cos\theta}}\right]$

4. i) $V = x^2+y^2$

$\nabla^2(x^2+y^2)$

$2+2=4$

$\nabla^2 V \neq 0$

It doesn't satisfies Laplace Equation

(ii) $V = x^2 - y^2$

$\nabla^2(x^2-y^2)$

$2-2=0$

$\nabla^2 = 0$

It satisfies Laplace Equation.