Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 87 (2016) 184 – 189

Procedia
Computer Science

4th International Conference on Recent Trends in Computer Science and Engineering

Unknown radio source localization based on a modified closed form solution using TDOA measurement technique

D.Eswara Chaitanya[a]*, G.Sasibhushana Rao[b]

[a]Dept. of E.C.E, ANITS College of Engineering,Visakhapatnam, Andhra Pradesh,India.
[b]Dept.of.E.C.E,Andhra University College of Engineering, Visakhapatnam, Andhra Pradesh,India.

Abstract

There have been huge advances in the development of positioning applications after the success of GPS. Accurate positioning capability of a radio source is in great demand in the fields of medicine, defence, automation etc. Especially in the Indian subcontinent, localization of an unknown enemy Radio Source such as an enemy radar system, tracking of Unmanned Aerial Vehicle (UAV) etc are essential capabilities for the defence wing. The time difference of arrival (TDOA) technique used for location determination makes use of the method of hyperbolic multilateration. Position accuracy is affected by various factors such as choice of number of sensors, measurement noise, algorithm employed for positioning and sensor geometry. A number of positioning algorithms are available for this purpose. These include iterative algorithms, evolutionary algorithms and many others that are not particularly feasible as the run time of the hardwired code is a major constraint in real time applications. A closed form algorithm proposed in the literature achieves considerable speed in implementation but falters in the resolution of the altitudinal estimate. Hence, a modified algorithm is proposed in this paper that resolves the altitude of the unknown radio source. For better understanding, an example of localizing an unknown radio source using the proposed algorithm is explained.

*Keywords:*Radio Source Localization, TDOA, Sensor geometry, Direct Solution.

1. Introduction

The precise location of any fixed or moving object/radiating source is the primary concern of a number of fields like medicine, automation, Electronic Warfare etc. In order to locate moving sources, a number of position location techniques are employed and hyperbolic position location technique[1] is one among them. It is also known as

* Corresponding author. Tel.: +91-9705817692
*E-mail address:*d.e.chaitanya@gmail.com

Peer-review under responsibility of the Organizing Committee of ICRTCSE 2016
doi:10.1016/j.procs.2016.05.146

Hyperbolic Multilateration or time difference of arrival (TDOA)[2,3] position determination method. The TDOA based positioning method is hugely popular because of its relatively high accuracy despite the fact that it does not require information about the transmitted signal. However, a number of choices with respect to design are to be made during implementation of any positioning technique. The first is the choice of arrangement or deployment of sensors i.e. sensor geometry. The second is the number of sensors to be deployed. Further, the choice of reference sensor is also significant. However, a major design issue is the positioning algorithm to be used or developed.

This paper is divided into 6 sections. The second section introduces the method of TDOA positioning. The third section details the proposed algorithm for hyperbolic multilateration. The working methodology used for carrying out the required analysis is presented in section 4. Section 5 details the results obtained. In section 6, the conclusions pertaining to the discussion that follows are presented.

2. TDOA positioning technique

Among the various positioning technologies available, Hyperbolic Multilateration (TDOA positioning) is most widely used for Automatic Source/Object Location. Hyperbolic position location estimation is accomplished in two stages[4]. The first stage involves estimation of the time difference of arrival (TDOA) between receivers through the use of time delay estimation techniques. The estimated TDOAs are then transformed into range difference measurements between base stations, resulting in a set of nonlinear hyperbolic range difference equations. The second stage utilizes efficient algorithms to produce an unambiguous solution to these nonlinear hyperbolic equations. The solution produced by these algorithms result in the estimated position location of the source.

In order to solve for the unknown source position in the space, three coordinates i.e. x, y and z are required to be determined with the help of four receivers that could be either fixed or mobile. In this paper, the location of the four fixed receivers i, j, k and l has been taken as $(x_i, y_i, z_i), (x_j, y_j, z_j), (x_k, y_k, z_k)$ and (x_l, y_l, z_l) respectively and by considering the speed of EM waves as ‘c’, the distances of the receivers from the unknown source will be as follows:

$$d_i = ct_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}$$

$$d_j = ct_j = \sqrt{(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2}$$

$$d_k = ct_k = \sqrt{(x_k - x)^2 + (y_k - y)^2 + (z_k - z)^2}$$

$$d_l = ct_l = \sqrt{(x_l - x)^2 + (y_l - y)^2 + (z_l - z)^2}$$

The above equations are a set of Time of Arrival (TOA) values multiplied by the velocity of light, c to form the corresponding range difference values. They can be combined so as to form expressions for the corresponding Range difference of arrival (RDOA = TDOA*c) equations as follows:-

$$d_{ab} = d_a - d_b = \sqrt{(x_a - x)^2 + (y_a - y)^2 + (z_a - z)^2} - \sqrt{(x_b - x)^2 + (y_b - y)^2 + (z_b - z)^2}$$

where $(a, b) = \{(i, j), (i, k), (i, l), (j, k), (j, l), (k, l)\}$

Each TDOA equation will represent a hyperboloid (as shown in Fig. 1) indicating the presence of the target anywhere on this hyperboloid. The above equation is nonlinear and can be solved using a closed-form solution or can be linearised and approximated to 1st order Taylor’s series[5]. A major advantage of this method is that it does not require knowledge of the transmit time from the source, as do the Time of Arrival (TOA) method which is the principle behind the development of the Global Positioning System (GPS). Consequently, strict clock synchronization between the source and receiver is not required. As a result, hyperbolic position location techniques do not require additional hardware or software implementation within the mobile unit.

However, clock synchronization is required of all receivers used for the position location estimate. Furthermore, unlike TOA methods, the hyperbolic position location method is able to reduce or eliminate common errors experienced at all receivers due to the channel.

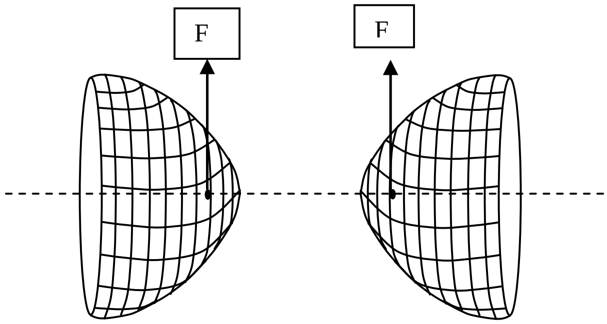

Figure 1 Hyperboloid with its two foci

3. Proposed algorithm

Ezzat's approach[6] gives a unique solution to the 3D problem from the TDOA perspective. It does not depend upon range data. It is necessary to know the time of transmission and the time of arrival to get the amount of time travelled from the transmitter to the receiver. For the most part, it is impossible to know the time of transmission, which can be defined as t_0. Many 3D solutions currently available are based on range data[7-9]. Some other solutions in the literature are in 2D and can't be extended to 3D[10]. The TDOA method is the only technique that can be used without the range data. Thus, even without t_0, it is possible to find the location of the emitter. The closed-form solution presented does not require the calculation of range data and does not depend on the availability of any information other than the times of arrival. The basic form of time of arrival equation is as follows

$$t_i = t_0 + D_i/c$$

where
t_i is the time of arrival at receiver i ,
D_i is the distance between the emitter and the receiver,
c is the speed of light.
t_0 is the time of the transmission

Two or more receivers are needed to be able to calculate the TDOA as mentioned before. When this condition is satisfied, it is possible to eliminate t_0 from any pair of two equations, which results in the TDOA equation

$$t_2 - t_1 = \frac{D_2 - D_1}{c}$$

A propagation mode between any two points in which the path of the signal is not a straight line will be mathematically equivalent to propagation along a straight line but with a velocity that is less than c, as the time of arrival is important. The TDOA equation for the case in which the path of the signal is nonlinear will be written as follows after adding the path delay:

$$(t_2 - t_1) = \frac{D_2}{\alpha_2 c} - \frac{D_1}{\alpha_1 c} = \frac{1}{c}\left[\frac{D_2}{\alpha_2} - \frac{D_1}{\alpha_1}\right]$$

where,
α_i represents path delay coefficients whose value is less than or equal to 1. α is one when there is no path effect on

the propagating signal like in the case where the signal propagates through air. In this paper, all the path delay coefficients are assumed to be one. A minimum of five receivers are required for position computation. This approach involves mathematics of substitution and direct computation by elimination which finally result in equations of the following form.

$$a_{11}x + a_{12}y + a_{13}z = b_1$$

$$a_{21}x + a_{22}y + a_{23}z = b_2$$

$$a_{31}x + a_{32}y + a_{33}z = b_3 \tag{1}$$

where

$$a_{11} = \frac{1}{(t_2-t_1)}\left(\frac{x_2}{\alpha_2{}^2} - \frac{x_1}{\alpha_1{}^2}\right) - \frac{1}{(t_3-t_1)}\left(\frac{x_3}{\alpha_3{}^2} - \frac{x_1}{\alpha_1{}^2}\right)$$

$$a_{12} = \frac{1}{(t_2-t_1)}\left(\frac{y_2}{\alpha_2{}^2} - \frac{y_1}{\alpha_1{}^2}\right) - \frac{1}{(t_3-t_1)}\left(\frac{y_3}{\alpha_3{}^2} - \frac{y_1}{\alpha_1{}^2}\right)$$

$$a_{13} = \frac{1}{(t_2-t_1)}\left(\frac{z_2}{\alpha_2{}^2} - \frac{z_1}{\alpha_1{}^2}\right) - \frac{1}{(t_3-t_1)}\left(\frac{z_3}{\alpha_3{}^2} - \frac{z_1}{\alpha_1{}^2}\right)$$

$$b_1 = \frac{1}{(t_2-t_1)}\left(\frac{x_2{}^2+y_2{}^2+z_2{}^2}{\alpha_2{}^2} - \frac{x_1{}^2+y_1{}^2+z_1{}^2}{\alpha_1{}^2}\right) - \frac{1}{(t_3-t_1)}\left(\frac{x_3{}^2+y+z^2}{\alpha_3{}^2} - \frac{x_1{}^2+y_1{}^2+z_1{}^2}{\alpha_1{}^2}\right)$$

The remaining coefficients can also be determined from similar equations. Equations in (1) can be expressed as AX = B and can be solved using least square estimation as shown

$$\mathrm{X} = (A^T A)^{-1} A^T * B$$

However, the above approach doesn't obtain good resolution in the altitudinal estimate. This is of major concern. Hence, the above approach is modified accordingly to obtain unknown source location estimate in three dimensions. The TDOA location in three dimensions can be derived/extended from the solution obtained in 2D. Now that the emitter can be located in a plane with the x and y coordinates available, it is the altitude that has to be estimated. The maximum possible search space with respect to the altitude is usually pre decided.

In the proposed algorithm, the altitude is estimated by a search process in which the altitude is varied in steps of a step size which can be pre-determined considering various parameters such as accuracy of the estimate required, computation time etc. For each value of z considered, the sum of squares of residuals obtained by the difference of true and calculated rdoas is determined. The 'z' coordinate estimate for which the lowest value is obtained is chosen to be the altitude of the emitter.

4. Methodology

In order to analyse the performance of the proposed modified closed form algorithm with regards efficacy and convergence, the following methodology is used. The entire coding process required for this work is done in MATLAB. The first assumption is to randomly choose the location of the unknown source. Here, two such locations are chosen at random. This approach requires no initial guess. The number of receivers used is fixed at 5. The sensors are assumed to be arranged in Pentagon configuration.

For the given sensor configuration, the RDOA values were calculated with respect to the pre decided source location. These values were then input to the code. However, since we are not working with real time data, errors are introduced into the TDOA values by the addition of Gaussian noise. These errors reflect in the RDOA values in terms of range difference errors. The estimated source position is then estimated for all the configurations and related observations are made. The measured parameters are location coordinates of the unknown source in Cartesian coordinate system i.e. x, y and z. However, since only the x and y estimates obtained from the closed form solution are close to the true values, the altitude is now estimated using the following procedure.

The height of the source is estimated from a search method that minimizes the sum of squares of differences in observed and calculated estimates. Since, the x-y planar location of the unknown source is now available, what remains is the height of the source from the reference zero level. Depending on the possible unknown sources that are to be identified, an estimate of the maximum height can be determined. The application for which the algorithm is intended will decide the accuracy of altitudinal estimate required. The maximum possible height of the unknown source is divided into steps of a pre decided step size. Since the observed RDOA values are available, at each step (altitudinal value), a new set of RDOAs are calculated. That value of altitudinal choice which minimizes the sum of squares of the differences in observed and calculated RDOAs is our height estimate. This above process can be regarded as minimization of the following function OF (Optimization function)

$$OF = \sum_{i=2}^{no\ of\ sensors} (RDOA_{Observed_{ref,i}} - \left(\sqrt{\left(x_{ref} - x\right)^2 + \left(y_{ref} - y\right)^2 + \left(z_{ref} - z\right)^2} - \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}\right))^2$$

Where (x, y, z) is the estimate, (x_{ref}, y_{ref}, z_{ref}) is the reference sensor and (x_i, y_i, z_i) are the other sensors.

5. Results and discussion

Two different unknown source locations are used in this section. The simulation constraints used in evaluating the proposed algorithm are given below.

Sensor Coordinates in meters:

X Coordinates = [0, -19021, -11756, 11756, 19021]

Y Coordinates = [-20000, -6180, 16180, 16180, -6180]

Z Coordinates = [25, 35, 40, 30, 45]

Reference Sensor Coordinates = [0, -20000, 25]

Assumed Source Location 1 = [5000, 15000, 7000]

Assumed Source Location 2 = [20000, -15000, 7000]

Step size (for resolution in ‘z’) = 1000m

The noise to be introduced in the TDOA measurements is a random choice of either 2ns or 4ns. The height of the source is however fixed at 7kms. Table 1 lists the errors in location estimate of assumed unknown source 1. The variation can be observed in the altitudinal estimate of the proposed algorithm. Similarly, Table 2 lists the errors in location estimate of assumed unknown source 2. From Table 1 and 2, it is evident that the error in z is zero since we have opted for a true value of ‘z’ which is a multiple of 1000 (7000 in our example). Any other value of source height, say 7250m would indicate an error of 250m for each TDOA error combination since our assumed step size was 1000m. Similarly, if the assumed true value of source height is 7700m, the error indicated in the results would have been 200m and 300m if the step size is 500m and 1000m respectively.

Table. 1 Performance analysis of proposed algorithm for Assumed Unknown Source Location 1

Error in TDOA1 (ns)	Error in TDOA2 (ns)	Error in TDOA3 (ns)	Error in TDOA4 (ns)	Error in X (mts)	Error in Y (mts)	Error in Z (mts) (Direct soln)	Error in Z (mts) (Proposed algorithm)
2	2	2	2	2.88	8.49	4163.45	0
2	2	2	4	2.87	9.36	5049.19	0
2	2	4	2	3.42	10.10	4519.66	0
2	2	4	4	3.41	10.98	5405.28	0
2	4	2	2	0.97	2.61	2478.45	0
2	4	2	4	0.97	3.48	3364.58	0
2	4	4	2	1.51	4.23	2835.23	0
4	2	2	2	7.14	19.47	7879.73	0
4	2	2	4	7.13	20.34	8764.80	0

Table. 2 Performance analysis of proposed algorithm for Assumed Unknown Source Location 2

Error in TDOA1 (ns)	Error in TDOA2 (ns)	Error in TDOA3 (ns)	Error in TDOA4 (ns)	Error in X (mts)	Error in Y (mts)	Error in Z (mts) (Direct soln)	Error in Z (mts) (Proposed algorithm)
2	2	2	2	3.33	2.49	2060.07	0
2	2	2	4	3.62	2.72	2422.43	0
2	2	4	2	7.38	5.10	2919.25	0
2	2	4	4	7.67	5.33	3281.79	0
2	4	2	2	6.07	5.16	1283.80	0
2	4	2	4	5.78	4.93	921.89	0
2	4	4	2	2.03	2.56	425.38	0
4	2	2	2	11.44	9.58	5883	0
4	2	2	4	11.73	9.81	6245.78	0

6. Conclusions

This paper presented the TDOA method of obtaining the position fix of any unknown source. The direct solution approach available in literature was presented and its disadvantage in obtaining the altitudinal estimate was illustrated with two clear examples. The main advantage of the direct solution approach lies in its ability to converge to a solution in a single iteration. The proposed algorithm tries to keep the advantage of this approach intact while trying to obtain a proper altitudinal estimate of the unknown radio source. Though the speed of the closed form approach was sacrificed to an extent and the estimation of unknown source height requires more than a single iteration, the proposed work is of great use in applications that involve accurate 3D estimate of the unknown source. The trade-off between speed of implementation and accuracy is for the designer to choose. The number of iterations required by the proposed algorithm to estimate the height can be reduced by a proper choice of the step size depending on the altitudinal resolution required.

References

1. Seyed A. (Reza) Zekavat and R.Michael Buehrer, “Hand book of Position Location-Theory, Practice, and Advances”, IEEE Press, John Wiley & Sons, Inc., Publications, New Jersey-54, Edition- 2012.
2. Gaarder, N. T., “On estimating the location of a signal source.”, *IEEE Transactions on Information Theory*, **IT-15**, 5 (1969)
3. Torrieri, D. J., “Statistical theory of passive location systems.”, *IEEE Transactions on Aerospace and Electronic Systems*, **AES-20**, 2 (1984).
4. D.Eswara Chaitanya, M.N.V.S.S.Kumar, G.Sasibhushana Rao and Rajkumar Goswami, “Convergence Issues of Taylor Series Method in Determining Unknown Target Location Using Hyperbolic Multilateration”, ICSEMR 2014, ISBN:978-1-4799-7613-3/14/IEEE-2014.
5. G. Shen, R. Zetik, and R. Thoma, “Performance comparison of toa and tdoa based location estimation algorithms in los environment”, In Proceedings of Positioning, Navigation and Communication, WPNC 2008, pages 71-78, IEEE, 2008
6. Ezzat G.Bakhoum, “Closed-Form Solution of Hyperbolic Geolocation Equations”, IEEE Transactions on Aerospace and Electronic Systems Vol.42, No.4, Oct-2006.
7. Schau, H. C., and Robinson, A. Z., “Passive source localization employing intersecting spherical surfaces from time-of-arrival differences.”, *IEEE Transactions on Acoustics, Speech and SignalProcessing*, **ASSP-35**, 8 (1987).
8. Smith, J. O., and Abel, J. S., “Closed-form least squares source location estimation from range-difference measurements.” *IEEE Transactions on Acoustics, Speech and SignalProcessing*, **ASSP-35**, 12 (1987).
9. Chan, Y. T., and Ho, K. C., “A simple and efficient estimator for hyperbolic location.”, IEEE Trans. on Signal Processing, V. 42, 8, 1994.
10. Schmidt, R. O. “A new approach to geometry of range difference location”, *IEEE Transactions on Aerospace and Electronic Systems*, **AES-8**, 6 (1972).