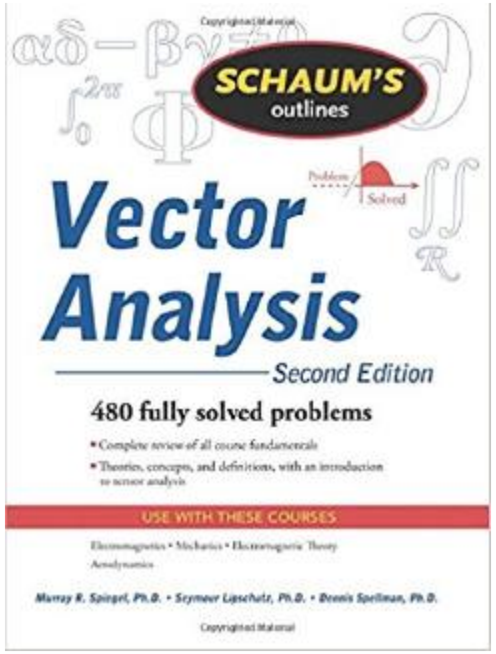
Vector Analysis

(Refresh)

Before we go ahead to understand the properties of EM waves, let us write learn to write the Laws of Electromagnetism in ***different forms (Differential and Integral forms)***



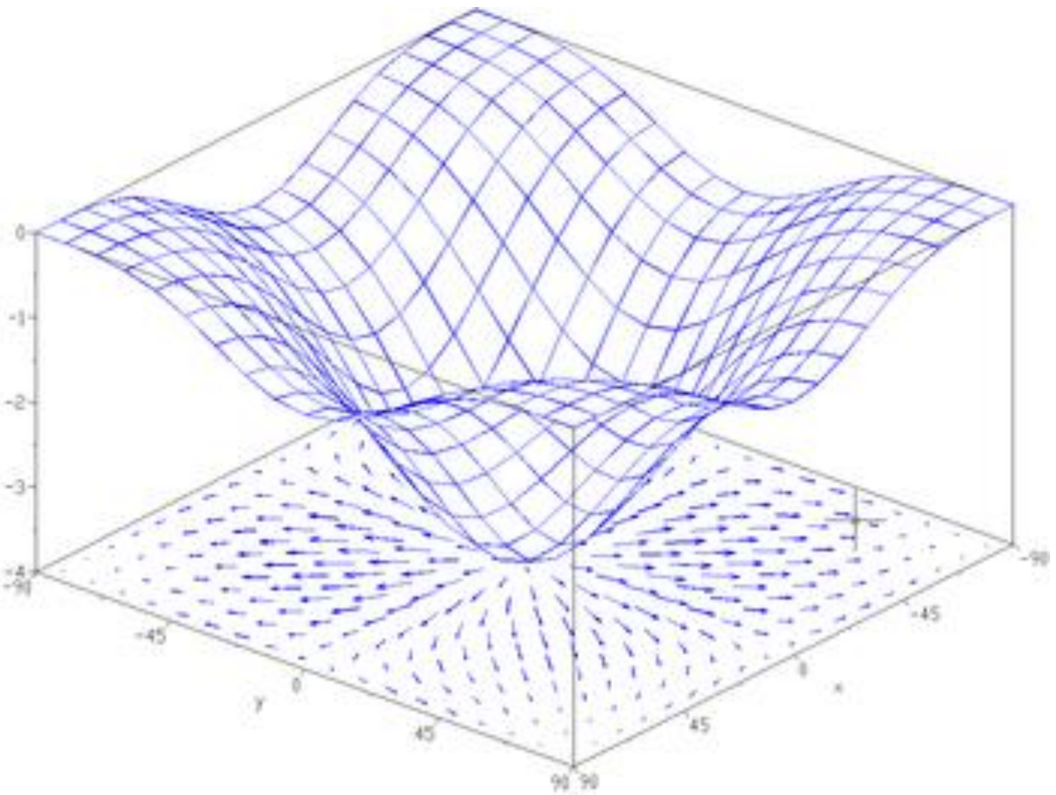
GRADIENT

For a scalar function *T* of three variable *T(x,y,z)*, the gradient of *T* is a vector quantity given by:

$$\nabla T \equiv \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

- The gradient points in the direction of the greatest rate of increase of the function,

- and its magnitude is the slope (rate of increase) of the graph in that direction.



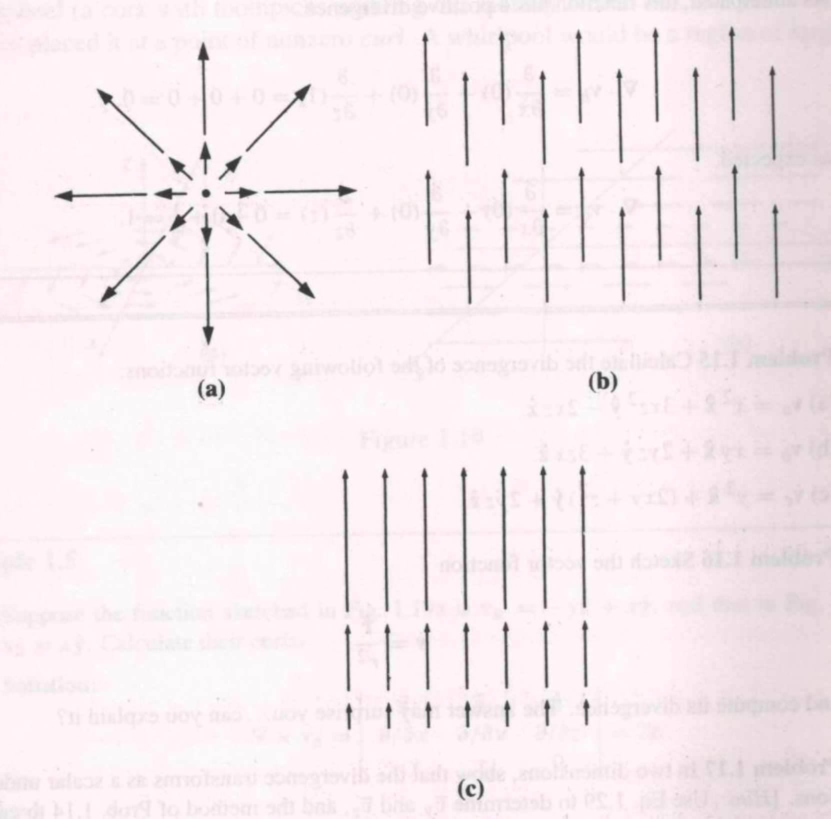
DIVERGENCE

For a vector ***T*** the divergence of ***T*** is given by:

$$\nabla \cdot \vec{T} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \cdot \left(T_x \hat{x} + T_y \hat{y} + T_z \hat{z} \right)$$

$$= \left(\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z} \right)$$

It is a measure of how much the vector ***T*** diverges / spreads out from the point in question.



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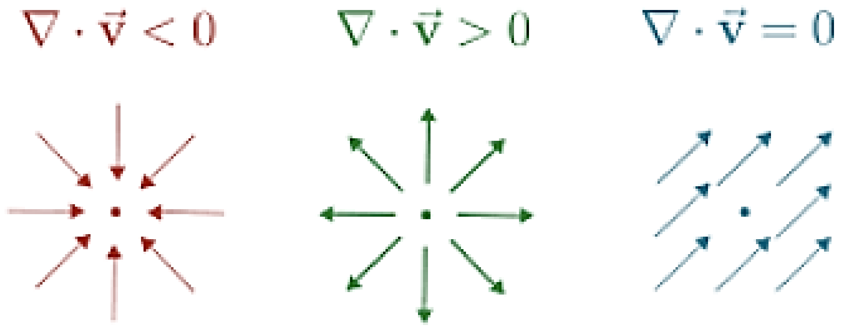
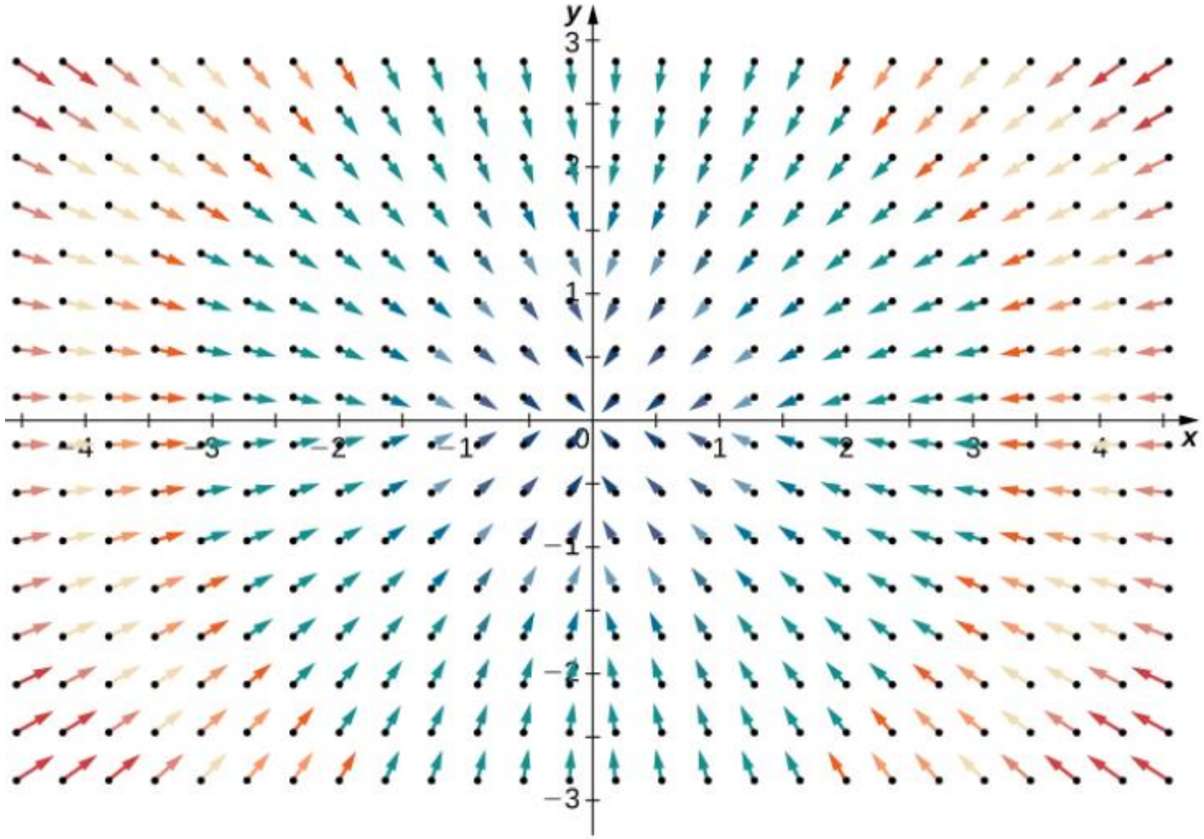
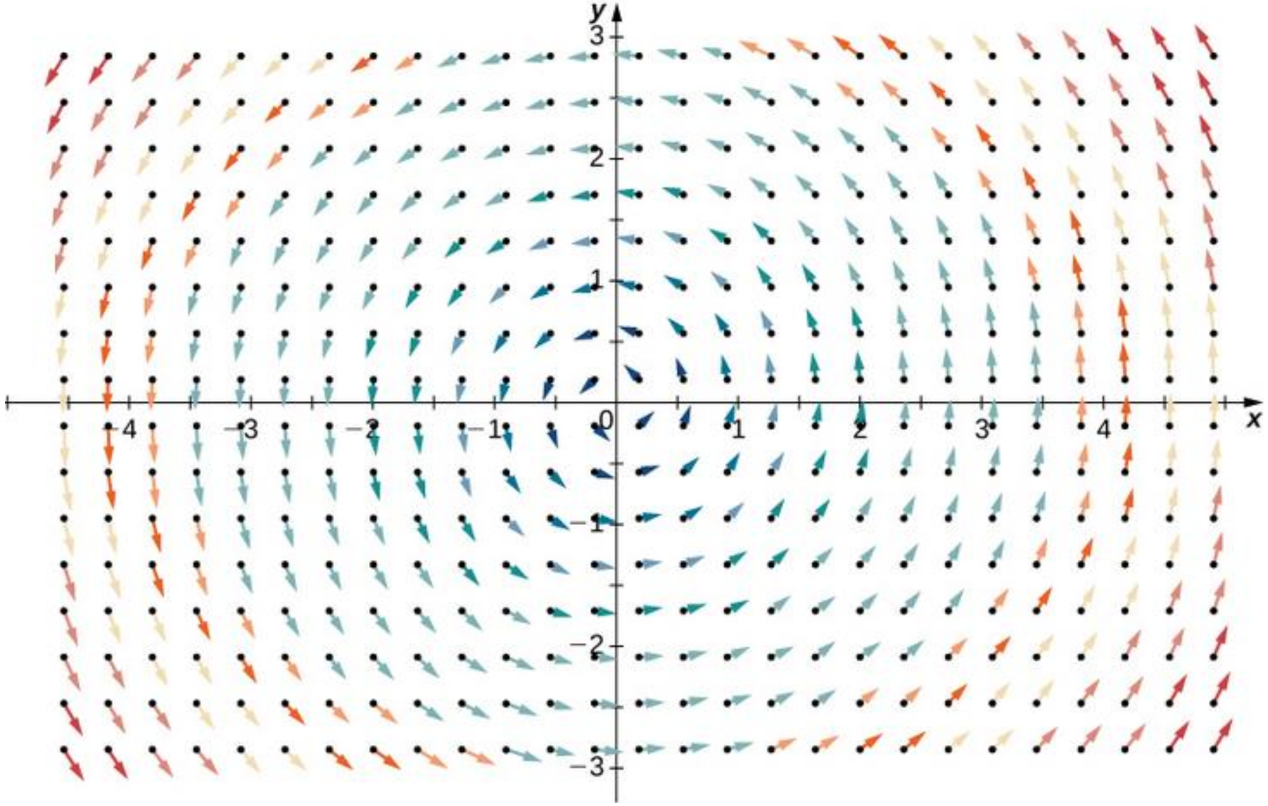


$$\operatorname{div}(\vec{\mathbf{R}}) = \frac{\partial}{\partial x}(-x) + \frac{\partial}{\partial y}(-y) = -2.$$



This vector field has negative divergence.

$$\operatorname{div}(\langle -y, x\rangle) = \frac{\partial}{\partial x}(-y) + \frac{\partial}{\partial y}(x) = 0.$$



Vector field $\langle -y, x\rangle$ also has zero divergence.

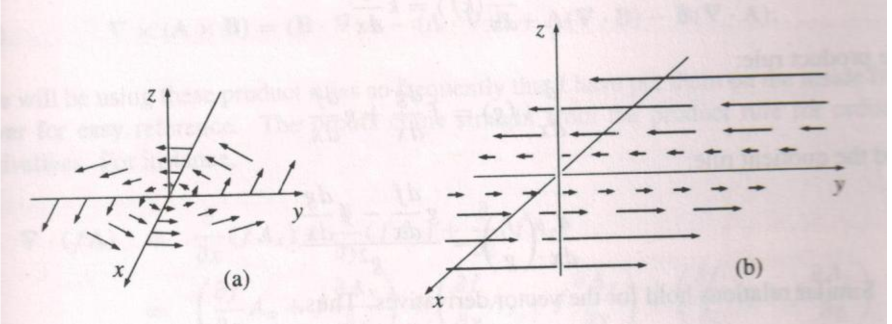
CURL

For a vector ***T*** the Curl of ***T*** is given by:

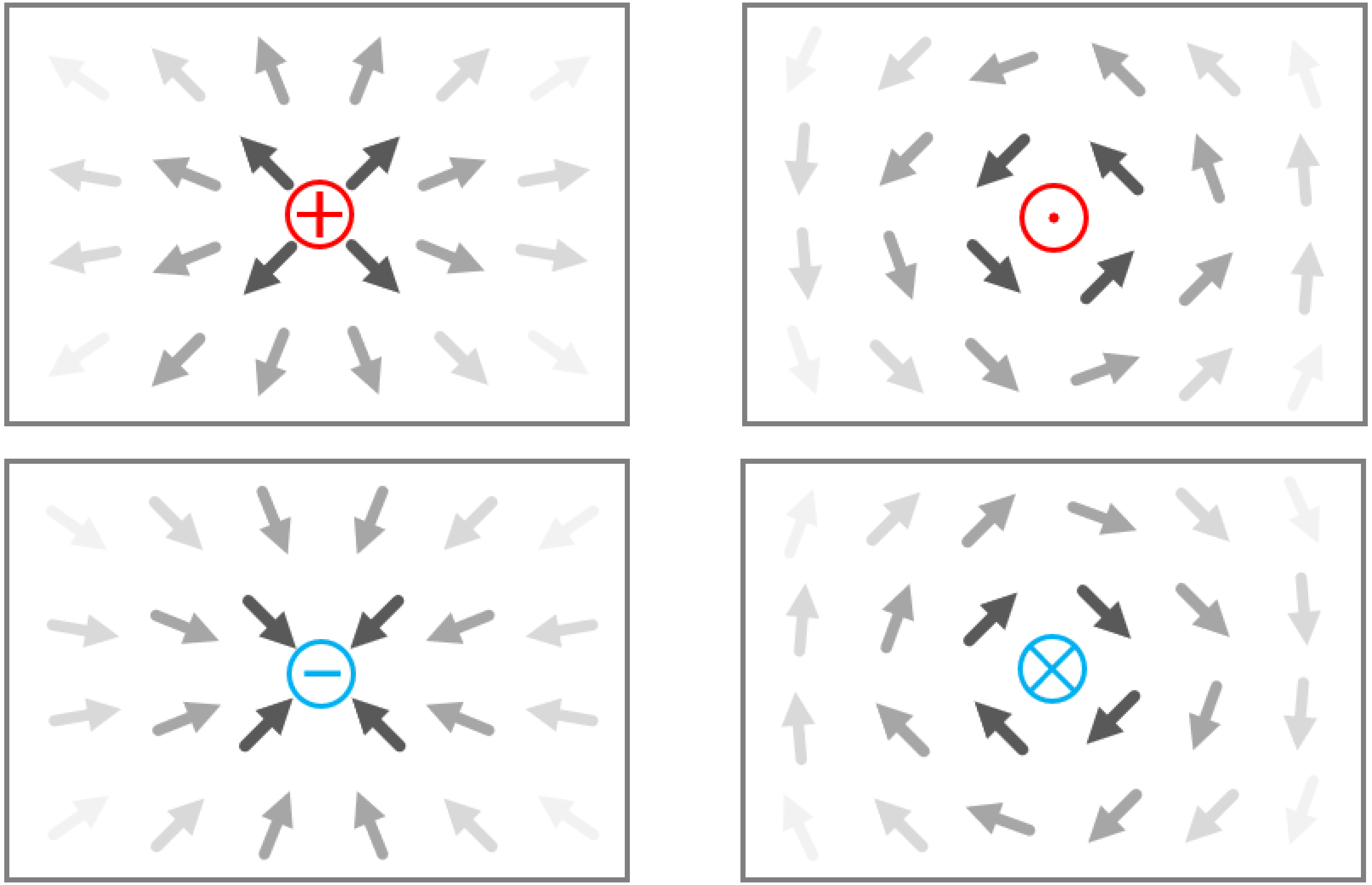
$$\nabla \times \vec{T} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \times \left(T_x\hat{x} + T_y\hat{y} + T_z\hat{z} \right)$$

$$= \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ T_x & T_y & T_z \end{pmatrix}$$

It is a measure of how much the vector ***T*** curls around the point in question.



Divergence | Curl

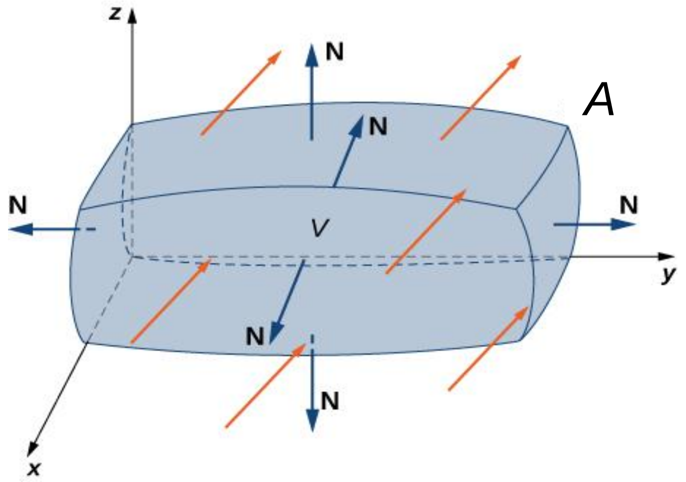


div (vector field) = scalar | *curl (vector field) = vector*

DIVERGENCE THEOREM

/ Green's Theorem / Gauss's Theorem

$$\int_V (\nabla \cdot \vec{E}) d\tau = \oint_A \vec{E} \cdot d\vec{a}$$

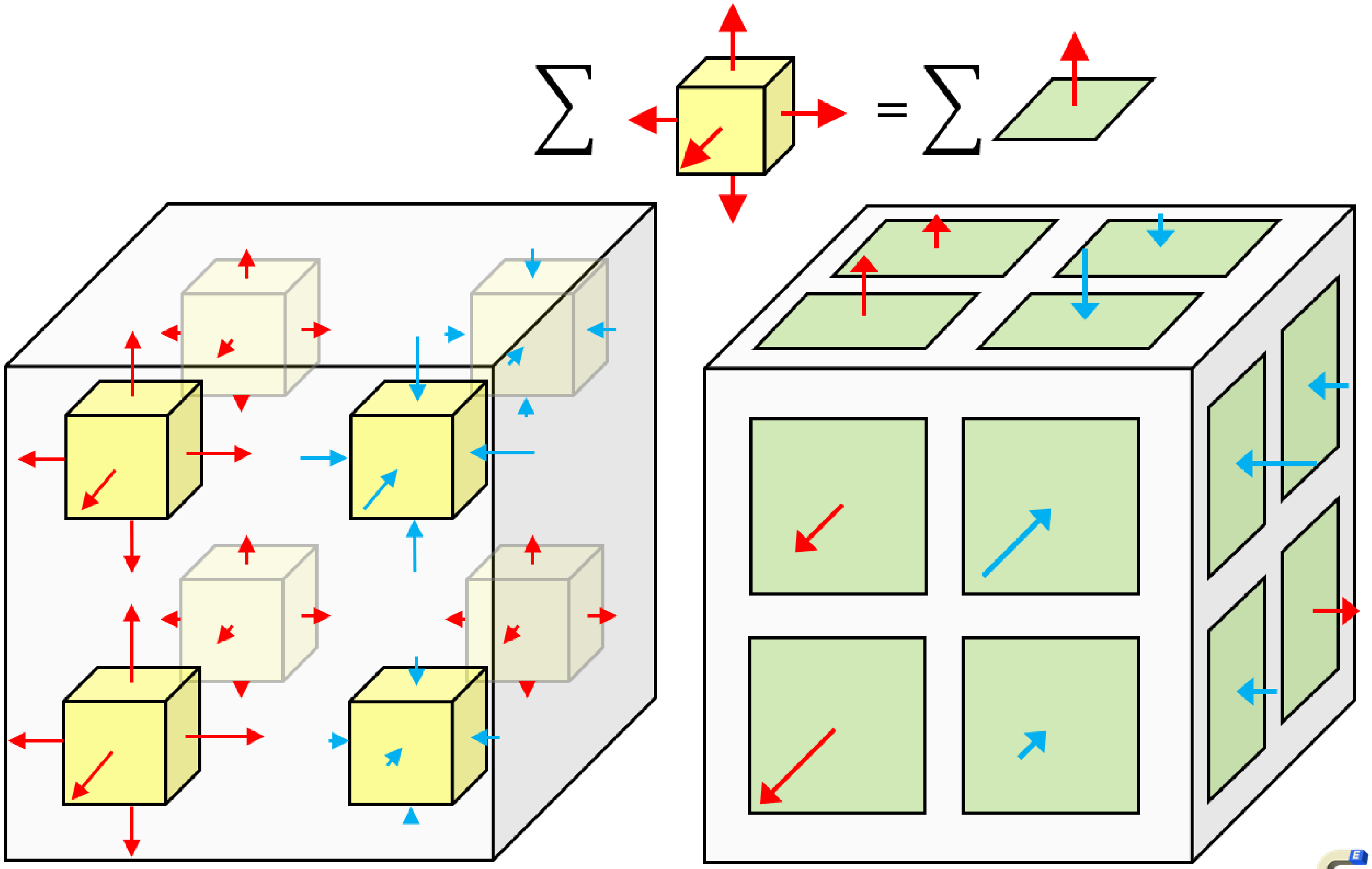


Integral of a derivative (in this case the divergence) over a volume is equal to the value of the function at the surface that bounds the volume.

$$\int_V (\text{Faucets within the volume}) = \oint_A (\text{Flow out through the surface})$$

DIVERGENCE THEOREM

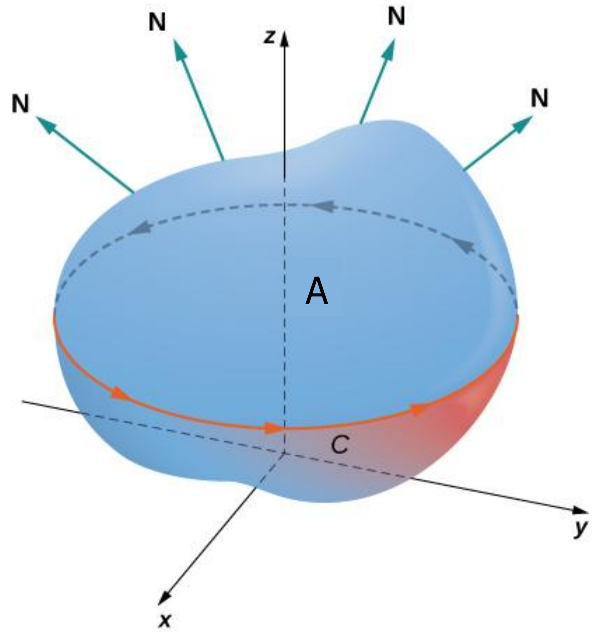
/ Green's Theorem / Gauss's Theorem



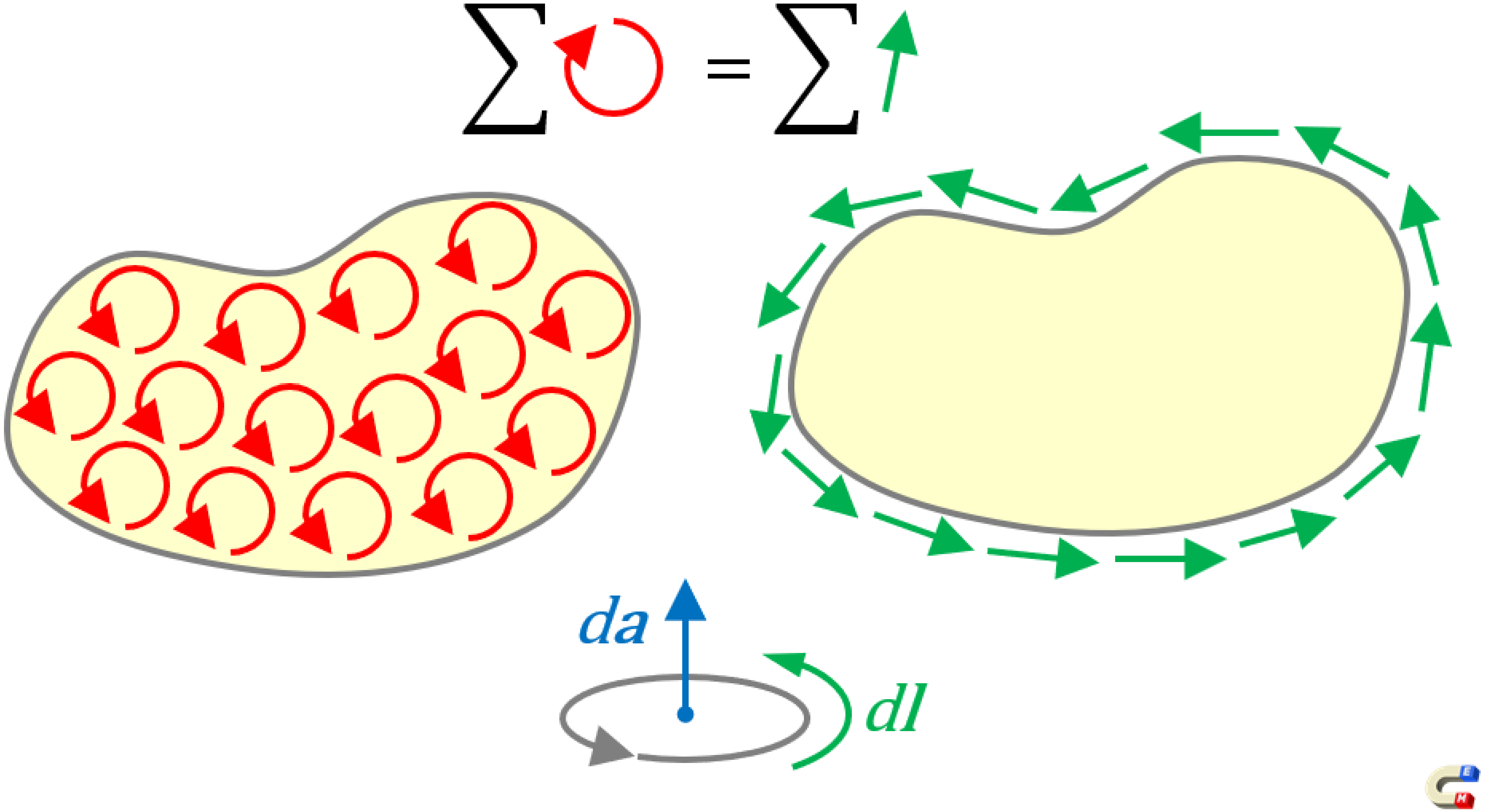
STOKES' THEOREM

$$\int_A (\nabla \times \vec{E}) \cdot d\vec{a} = \oint_C \vec{E} \cdot dl$$

Integral of a derivative (in this case the curl) over a patch of surface is equal to the value of the function at the boundary (perimeter of the patch).



STOKES' THEOREM



Eelectromagnetic waves

Dipole Radiation

The correct formula for the electric field

$$\mathbf{E} = \frac{-q}{4\pi\epsilon_0}\left[\frac{\mathbf{e}_{\mathbf{r}'}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\mathbf{e}_{\mathbf{r}'}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2\mathbf{e}_{\mathbf{r}'}}{dt^2}\right]$$

$e_{r'}$:unit vector directed from q to P at earlier time

Important features

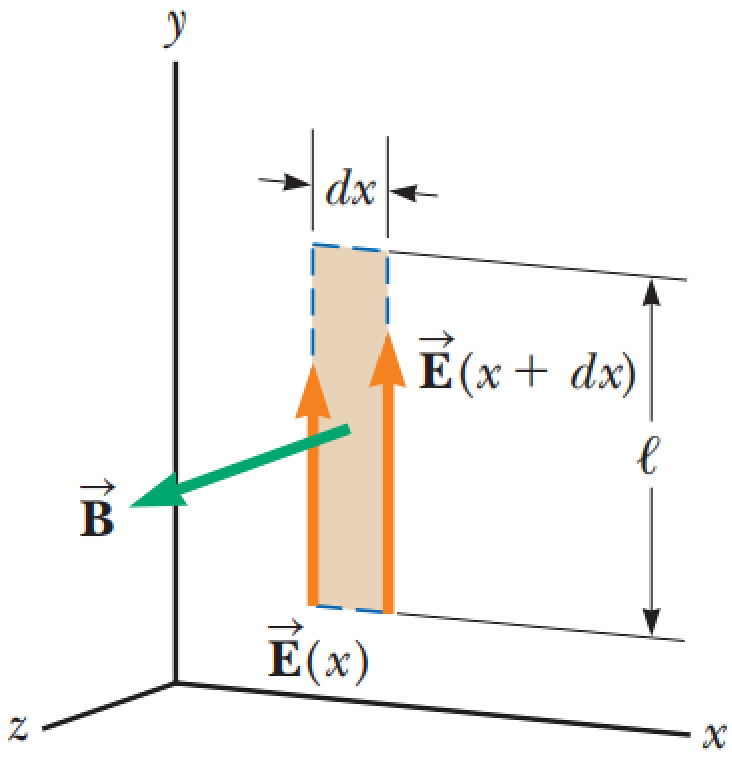
1. No information can propagate instantaneously

2. The electric field at the time t is determined by the position of the charge at an earlier time, when the charge was at r', the retarded position.

3. First two terms falls off as $1/r'^2$ and hence are of no interest at large distances

Faraday's Law

The induced electromotive force in any closed circuit is equal to the negative of the time rate of change of the magnetic flux through the circuit.



$$\text{Voltage generated} = -N\frac{\Delta BA}{\Delta t}$$

N: Number of turns
B: External magnetic field
A: Area of coil

We know that Faraday´s law in the integral form in given as:

$$\oint_C \vec{E}\,.\,\vec{dl} = -\frac{\partial}{\partial t}\iint_S \vec{B}\,.\,\vec{ds}$$

where C is the rectangle in the XY plane of length l, width Δx, and S is the open surface spanning the contour C

Using the Faraday´s law on the contour C, we get:

$$\left[E_y(x+\Delta x) - E_y(x)\right]l = -\frac{\partial B_z}{\partial t}l\Delta x$$

this implies...

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

Keep this in mind...

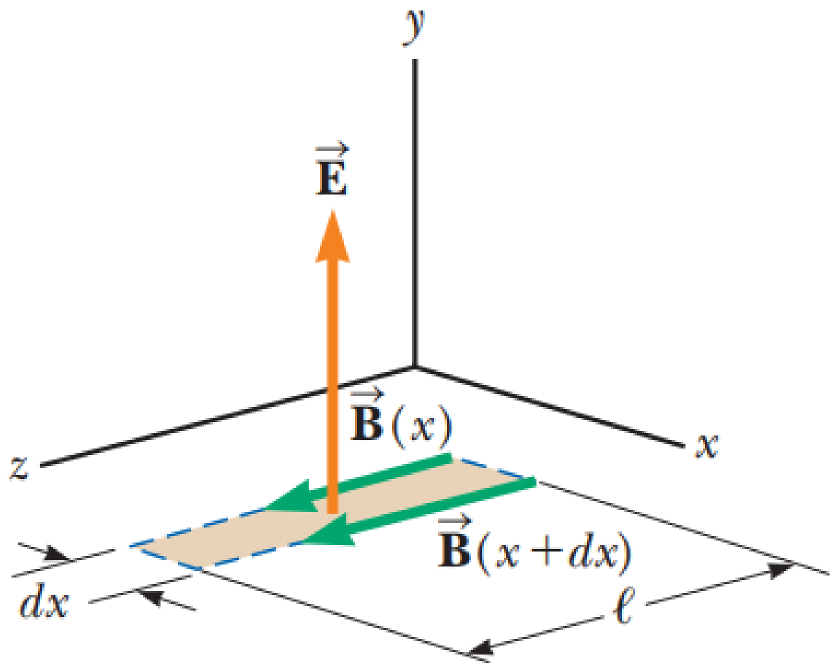
Ampere's Law

Ampère's circuital law relates the integrated magnetic field around a closed loop to the electric current passing through the loop.

The Ampere´s law with displacement current term can be written as:

$$\oint_C \vec{B} \, . \, \vec{dl} = \varepsilon_o \mu_o \frac{\partial}{\partial t} \iint_s \vec{E} \, . \, \vec{ds}$$

In free space, the displacement current is related to the time rate of change of electric field.



Using Ampere´s law, for the contour C, we get:

$$[-B_z(x + \Delta x) + B_z(x)]l = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t} l \Delta x$$

this implies...

$$-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$$

Outcome of Faraday's and Ampere's laws

Using the eqns. obtained earlier:

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

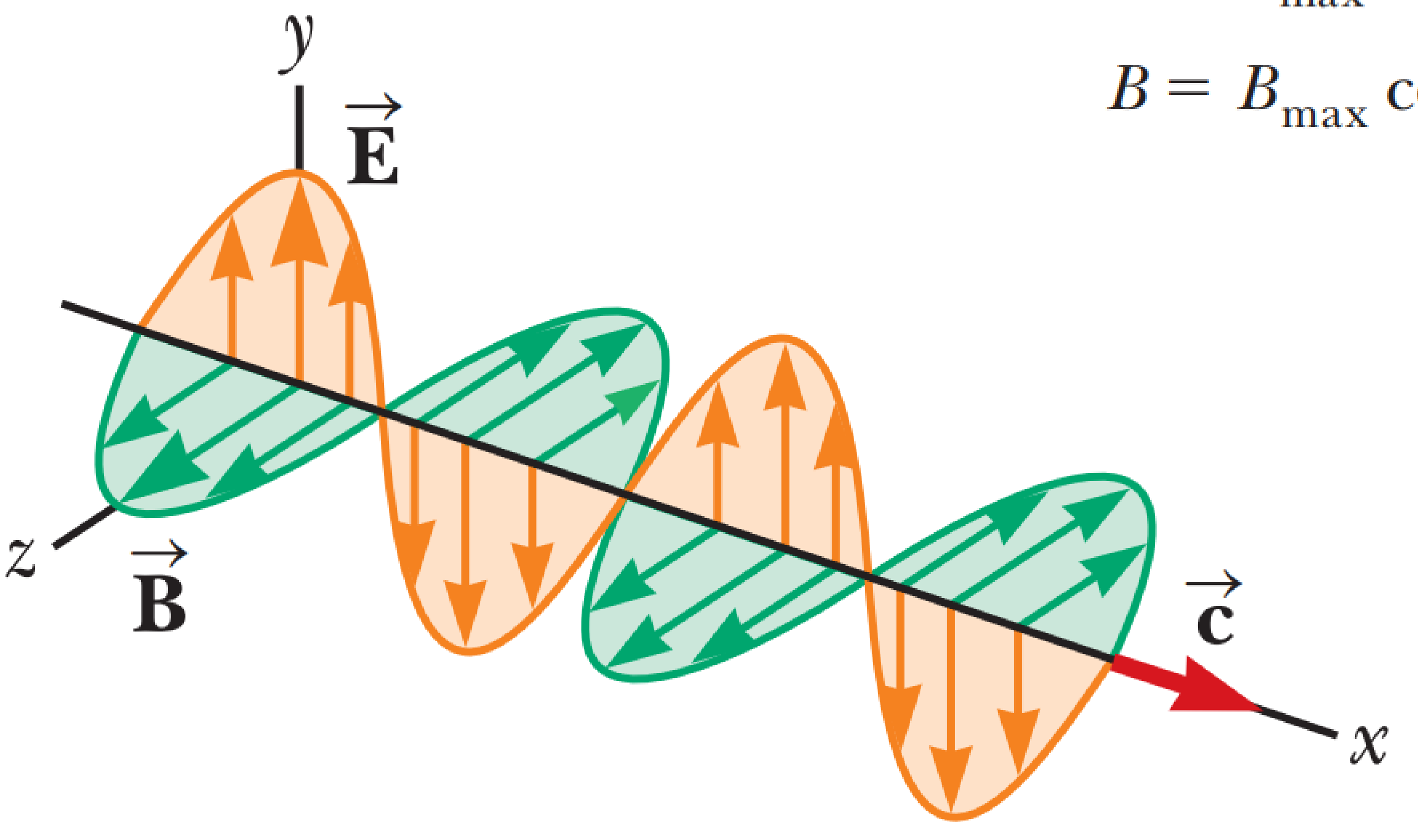
$$\boxed{-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}}$$

$$\boxed{\begin{gathered}\frac{\partial^2 E_y}{\partial t^2} = c^2 \frac{\partial^2 E_y}{\partial x^2} \\ where \\ c^2 = \frac{1}{\varepsilon_0 \mu_0}\end{gathered}}$$

Form of wave equation

Note: Similar Equation can be derived for B_z

Solution of EM wave



$$E = E_{max} \cos(kx - \omega t)$$

$$B = B_{max} \cos(kx - \omega t)$$

Wave Speed

$$\boxed{\frac{\omega}{k} = c}$$

$$\frac{\partial E}{\partial x} = -kE_{max} \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{max} \sin(kx - \omega t)$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\frac{E_{max}}{B_{max}} = \frac{E}{B} = c$$

Electric and Magnetic field are related at every point.

Laws of Electromagnetism

Laws of Electromagnetism in **Integral and Differential** forms:

Formulation in SI units

Name	Integral equations	Differential equations	Meaning
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0} \iiint_{\Omega} \rho \, \mathrm{d}V$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	The electric field leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$	$\nabla \cdot \mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_{\Sigma} \mathbf{B} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	The voltage accumulated around a closed circuit is proportional to the time rate of change of the magnetic flux it encloses.
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \iint_{\Sigma} \mathbf{J} \cdot \mathrm{d}\mathbf{S} + \mu_0 \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \iint_{\Sigma} \mathbf{E} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.

https://i.stack.imgur.com/2QoMD.png

EM Wave Equation (3D)

Electromagnetic waves

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

for E field

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

for B field

In general, electromagnetic waves

$$\boxed{\nabla^2 \psi = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}}$$

Where ψ represents E or B or their components

Laws of Electromagnetism

Put together, these are "Maxwell's equations' in vacuum

Gauss's laws

$$\nabla \cdot \mathbf{E} = 0 \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2)$$

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

Ampere's law (modified)

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (4)$$

Wave Equation (in vacuum)

Use the following identity

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{T}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{T}) - \nabla^2 \vec{T}$$

Start from Maxwell's 3rd Law:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial(\vec{\nabla} \times \vec{B})}{\partial t}$$

$$\Downarrow$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial(\vec{\nabla} \times \vec{B})}{\partial t}$$

Use Maxwell's 4th Law:

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\frac{1}{c^2} = \mu_0 \varepsilon_0$$

For E-field

Wave Equation (in vacuum)

Use the following identity

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{T})$$
$$= \vec{\nabla}(\vec{\nabla} \cdot \vec{T}) - \nabla^2 \vec{T}$$

Start from Maxwell's 4th Law:

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t}$$

$$\Downarrow$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t}$$

Use Maxwell's 3rd Law:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

$$\frac{1}{c^2} = \mu_0 \varepsilon_0$$

Plane EM waves in vacuum

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}\exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t))$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}\exp(i(k_x x + k_y y + k_z z - \omega t))$$

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}\exp(i(k_x x + k_y y + k_z z - \omega t))$$

Plane EM waves in vacuum

$$\mathbf{k}\cdot\mathbf{k} = k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2/c^2$$

$$\nabla\cdot\mathbf{E}(\mathbf{r},t) = \mathbf{k}\cdot\mathbf{E} = 0$$

Wave vector **k** is perpendicular to **E**

$$\nabla\cdot\mathbf{B}(\mathbf{r},t) = \mathbf{k}\cdot\mathbf{B} = 0$$

Wave vector **k** is perpendicular to **B**

Plane EM waves in vacuum

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}$$

$$\nabla \times \mathbf{B}(\mathbf{r},t) = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t}$$

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$$

$$\mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2}\mathbf{E}$$

$$\hat{\mathbf{k}} \times \mathbf{E} = \frac{\omega}{k}\mathbf{B} = c\mathbf{B}$$

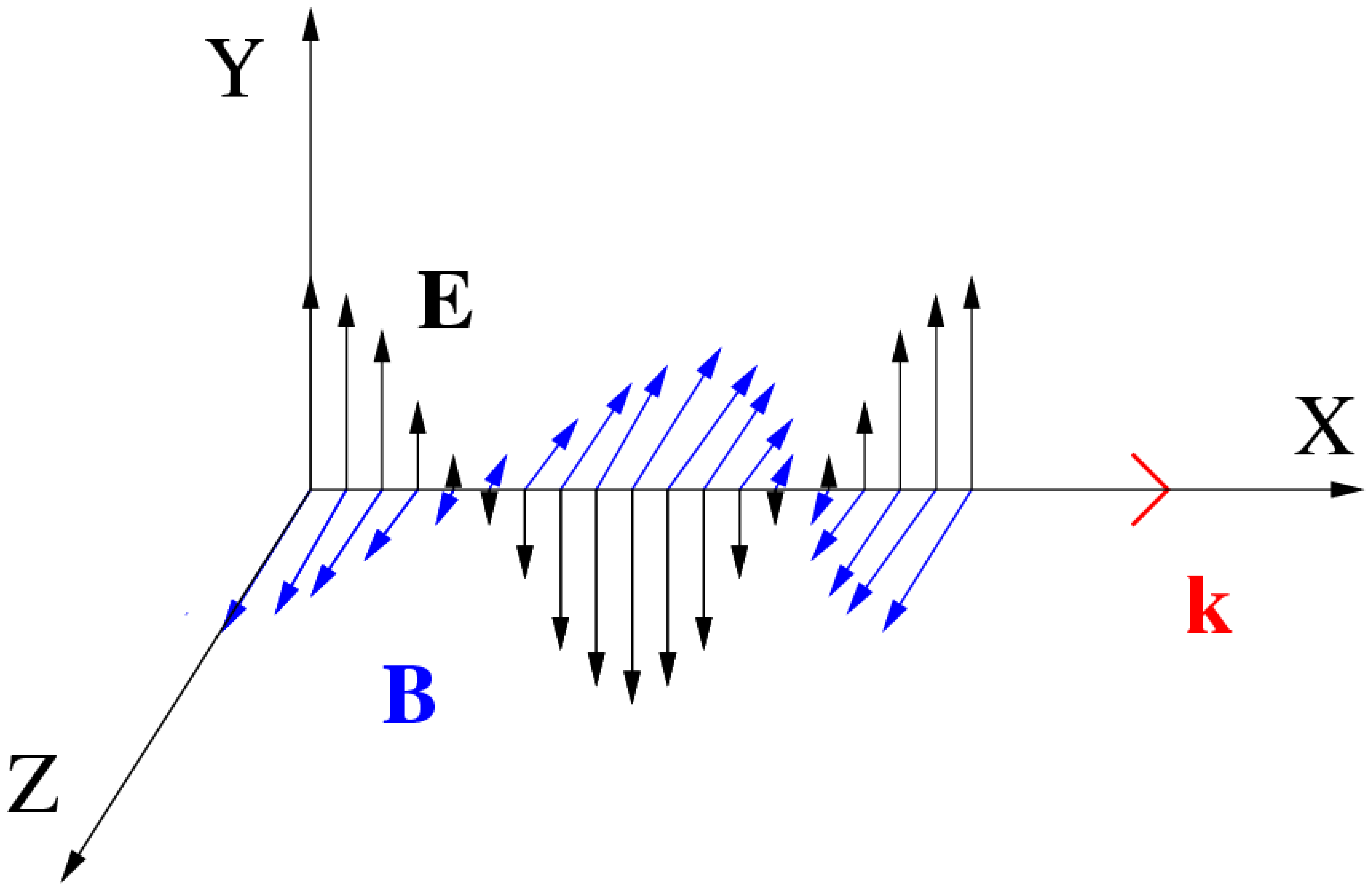
$$\mathbf{B} \times \hat{\mathbf{k}} = \frac{\omega}{kc^2}\mathbf{E} = \frac{1}{c}\mathbf{E}$$

B is perpendicular to **E**

$$c\mathbf{B} \times \hat{\mathbf{k}} = \mathbf{E}$$

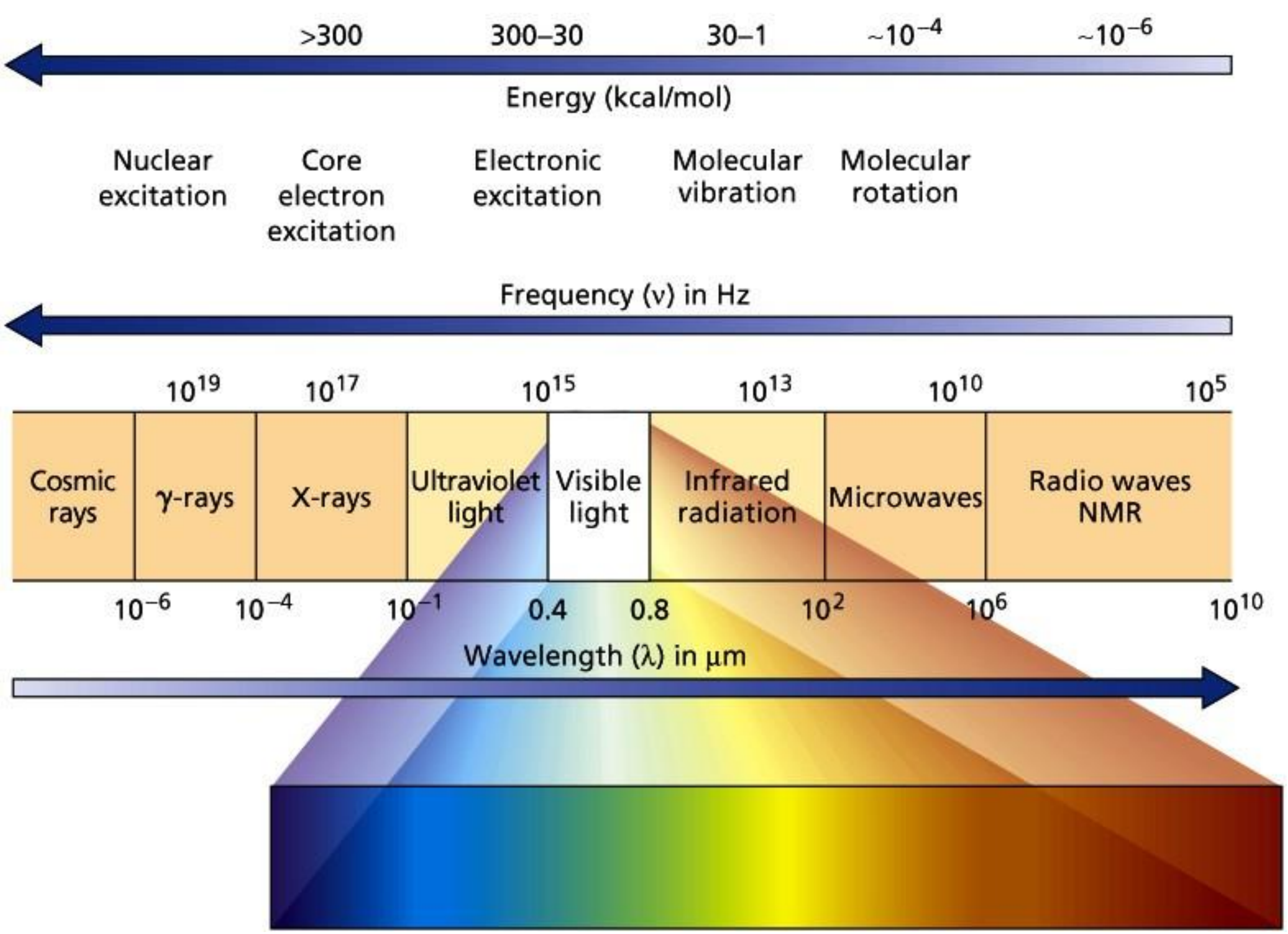
B, k and E make a right handed Cartesian co-ordinate system

Plane EM waves in vacuum



Energy and Momentum (EM waves)

Electromagnetic Spectrum



EM Waves Transport Energy and Momentum

The energy density of the E field (between the plates of a charged capacitor):

$$u_E = \frac{1}{2}\varepsilon_o E^2$$

Similarly, the energy density of the B field (within a current carrying toroid):

$$u_B = \frac{1}{2\mu_o} B^2$$

Using: $\boldsymbol{E = cB}$ and $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$ $\quad u_E = u_B$

The energy streaming through space in the form of EM wave is shared equally between constituent electric and magnetic fields.

EM Waves Transport Energy and Momentum

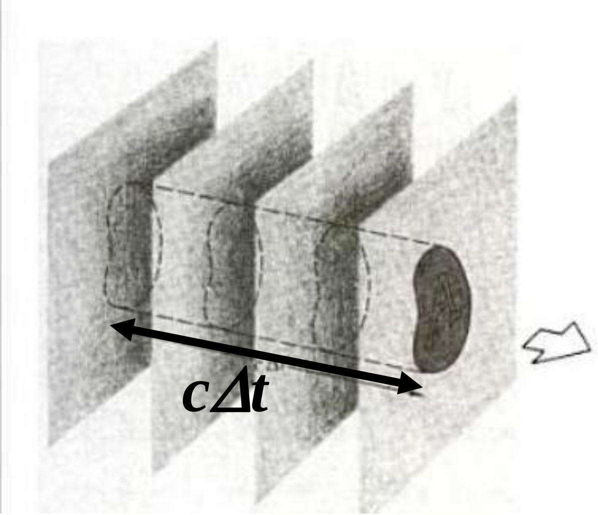
Total energy density of the EM field: $u = u_E + u_B = \varepsilon_o E^2 = \frac{1}{\mu_o} B^2$

S represents the flow of electromagnetic energy associated with a traveling wave.

S symbolizes transport of energy per unit time across a unit area: **Poynting Vector**

$$S = \frac{uc\Delta t A}{\Delta t A} = uc$$

$$S = \frac{1}{\mu_o} EB$$



Assume that the energy flows in the direction of the propagation of wave (in isotropic media)

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B} \qquad S = c^2 \varepsilon_o \vec{E} \times \vec{B}$$

The magnitude of **S** is the power per unit area crossing a surface whose normal is parallel to **S**.

EM Waves Transport Energy and Momentum

Given: $$\vec{E} = \vec{E}_o \cos(k \cdot \vec{r} - \omega t)$$

$$\vec{B} = \vec{B}_o \cos(\vec{k} \cdot \vec{r} - \omega t)$$

Instantaneous flow of energy per unit area per unit time

$$S = c^2 \varepsilon_o \vec{E}_o \times \vec{B}_o \cos^2(k \cdot \vec{r} - \omega t)$$

Time averaged value of the magnitude of the Poynting vector

$$\langle S \rangle = \frac{c^2 \varepsilon_o}{2} \left| \vec{E}_o \times \vec{B}_o \right|$$

The Irradiance is proportional to the square of the amplitude of the electric field:

$$I \equiv \langle S \rangle = \frac{c \varepsilon_o}{2} E_o^2 \qquad I = c \varepsilon_o \langle E^2 \rangle$$

EM Waves Transport Energy and Momentum

EM waves transport energy:

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$$

EM wave transport momentum:

$$p = U / c$$

U: Energy of the EM wave
c: Speed of the EM wave

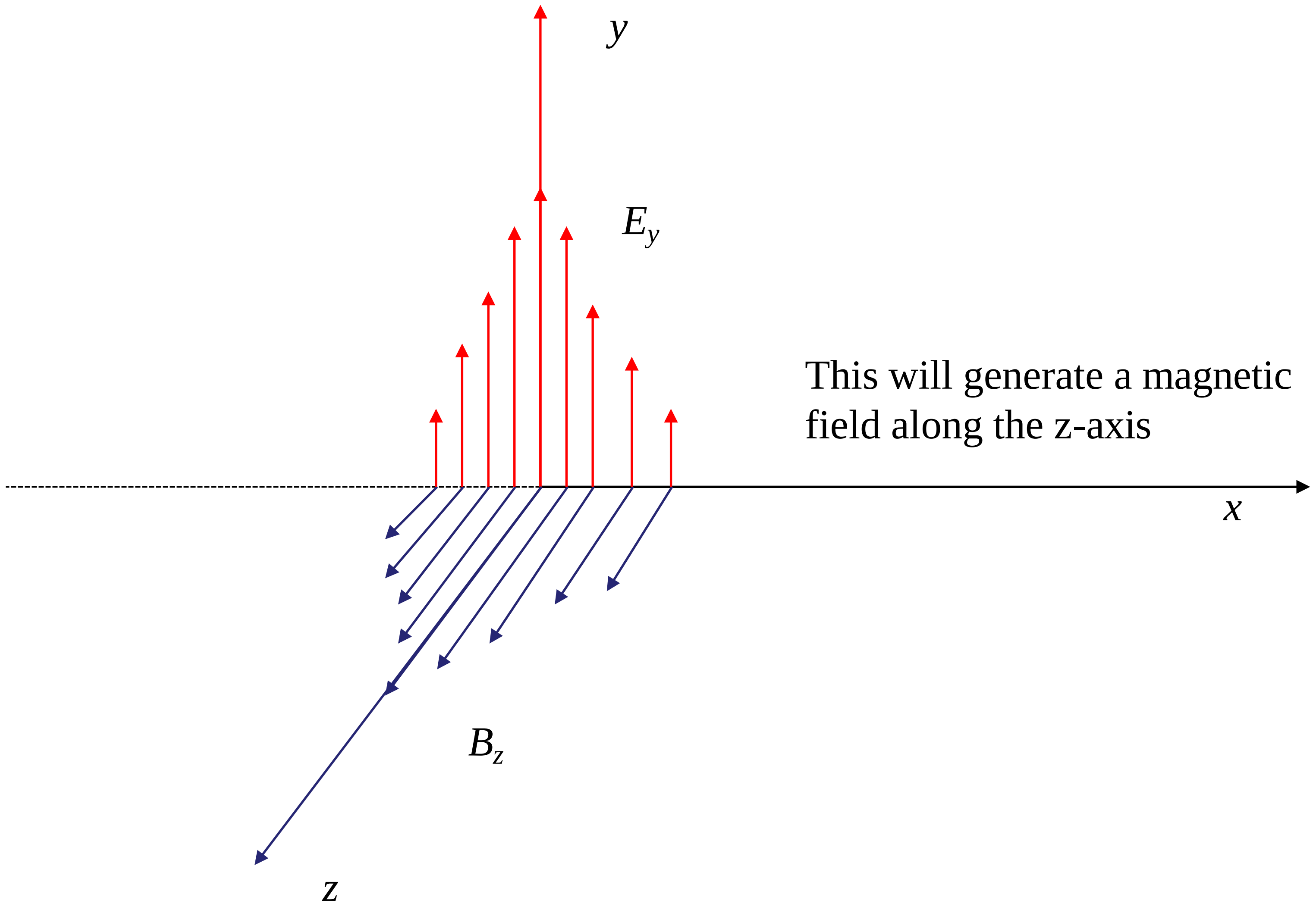
They exert a pressure:

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{cA}\frac{dU}{dt} = \frac{S}{c}$$

Alternative Approach to Derive the EM-wave equation (1D)

EM Waves

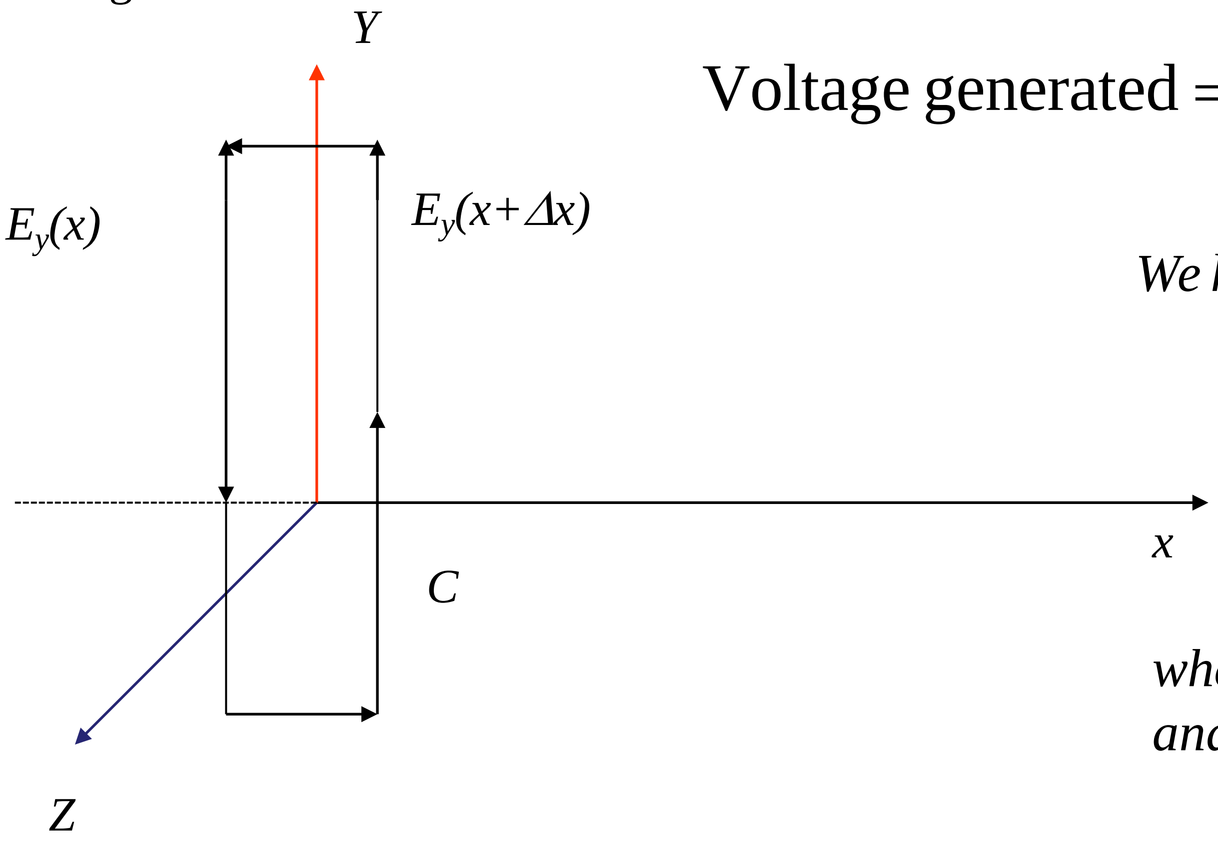
Consider an oscillating electric field E_y



This will generate a magnetic field along the z-axis

Faraday's Law

The induced electromotive force in any closed circuit is equal to the negative of the time rate of change of the magnetic flux through the circuit.



$$\text{Voltage generated} = -N\,\frac{\Delta BA}{\Delta t}$$

N: Number of turns
B: External magnetic field
A: Area of coil

We know that Faraday´s law in the integral form in given as:

$$\oint_C \vec{E}\,.\,\vec{dl} = -\frac{\partial}{\partial t}\iint_s \vec{B}\,.\,\vec{ds}$$

where C is the rectangle in the XY plane of length l, width Δx, and S is the open surface spanning the contour C

Using the Faraday´s law on the contour C, we get:

$$\left[E_y(x+\Delta x) - E_y(x)\right]l = -\frac{\partial B_z}{\partial t}\,l\Delta x$$

this implies...

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

Keep this in mind...

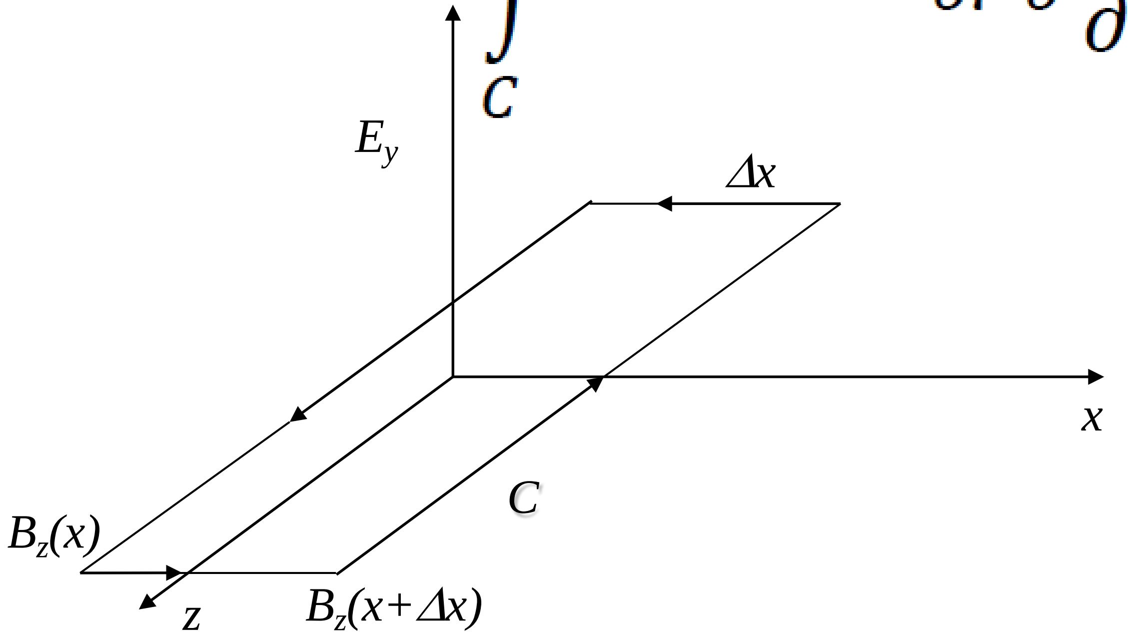
Ampere's Law

Ampère's circuital law relates the integrated magnetic field around a closed loop to the electric current passing through the loop.

The Ampere´s law with displacement current term can be written as:

$$\oint_C \vec{B} \, . \, \vec{dl} = \varepsilon_o \mu_o \frac{\partial}{\partial t} \iint_S \vec{E} \, . \, \vec{ds}$$

In free space, the displacement current is related to the time rate of change of electric field.



Using Ampere´s law, for the contour C, we get:

$$[-B_z(x + \Delta x) + B_z(x)]l = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t} l \Delta x$$

this implies...

$$-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$$

Outcome of Faraday's and Ampere's laws

Using the eqns. obtained earlier:

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

$$\boxed{-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}}$$

$$\boxed{\begin{gathered}\frac{\partial^2 E_y}{\partial t^2} = c^2 \frac{\partial^2 E_y}{\partial x^2} \\ where \\ c^2 = \frac{1}{\varepsilon_0 \mu_0}\end{gathered}}$$

Form of wave equation

Note: Similar Equation can be derived for B_z

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Solution of 3D wave equation

A plane wave satisfies wave equation in Cartesian coordinates

A spherical wave satisfies wave equation in spherical polar coordinates

A cylindrical wave satisfies wave equation in cylindrical coordinates

In Cartesian coordinates

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

Separation of variables $\psi(x, y, z, t) = X(x)Y(y)Z(z)T(t)$

Substituting for ψ we obtain $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \frac{1}{c^2}\left(\frac{1}{T}\frac{\partial^2 T}{\partial t^2}\right)$

Variables are separated out. Each variable-term independent, and must be a constant

Solution of 3D wave equation

So we may write $$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2;\quad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2;$$

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -k_z^2;\quad \left(\frac{1}{T}\frac{\partial^2 T}{\partial t^2}\right) = -\omega^2$$

where we use
$$\omega^2/c^2 = k_x^2 + k_y^2 + k_z^2 = k^2$$

Solutions are then $$X(x) = e^{\pm i k_x x};\quad Y(y) = e^{\pm i k_y y};$$

$$Z(z) = e^{\pm i k_z z};\quad T(t) = e^{\pm i\omega t}$$

Total Solution is $$\psi(x,y,z,t) = X(x)Y(y)Z(z)T(t)$$

$$= Ae^{i[\omega t \mp (k_x x + k_y y + k_z z)]}$$

$$= Ae^{i[\omega t \mp \vec{k}.\vec{r}]}$$ **Plane wave**

3D Plane waves

$$\psi(\mathbf{r}) = A\sin(\mathbf{k}\cdot\mathbf{r})$$

$$\psi(\mathbf{r}) = B\cos(\mathbf{k}\cdot\mathbf{r})$$

$$\psi(\mathbf{r}) = C\exp(i\mathbf{k}\cdot\mathbf{r})$$

The surface of constant phase is:

$$\mathbf{k}\cdot\mathbf{r} = \phi_c$$

$$k_x x + k_y y + k_z z = \phi_c$$

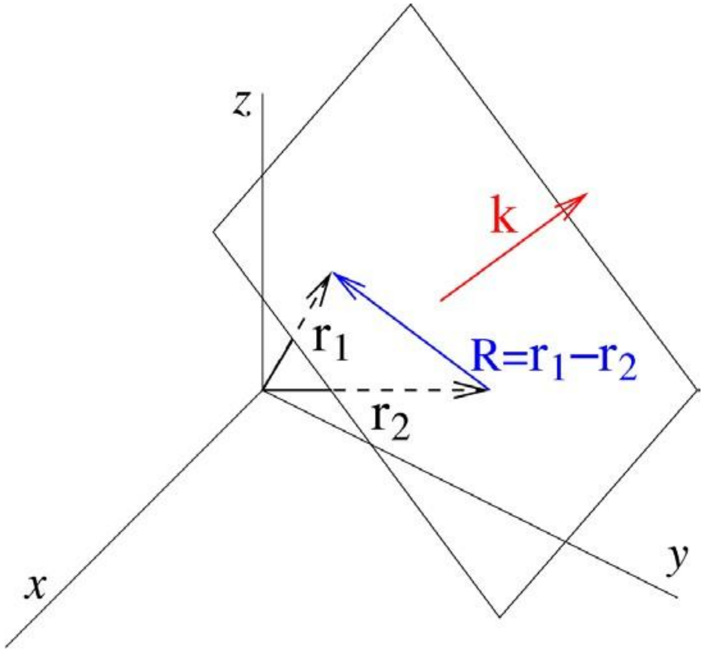
$$\mathbf{r_1}, \quad \mathbf{r_2}$$

$$\mathbf{k}\cdot\mathbf{r_1} = \mathbf{k}\cdot\mathbf{r_2} = \phi_c$$

$$\mathbf{k}\cdot(\mathbf{r_1} - \mathbf{r_2}) = 0$$

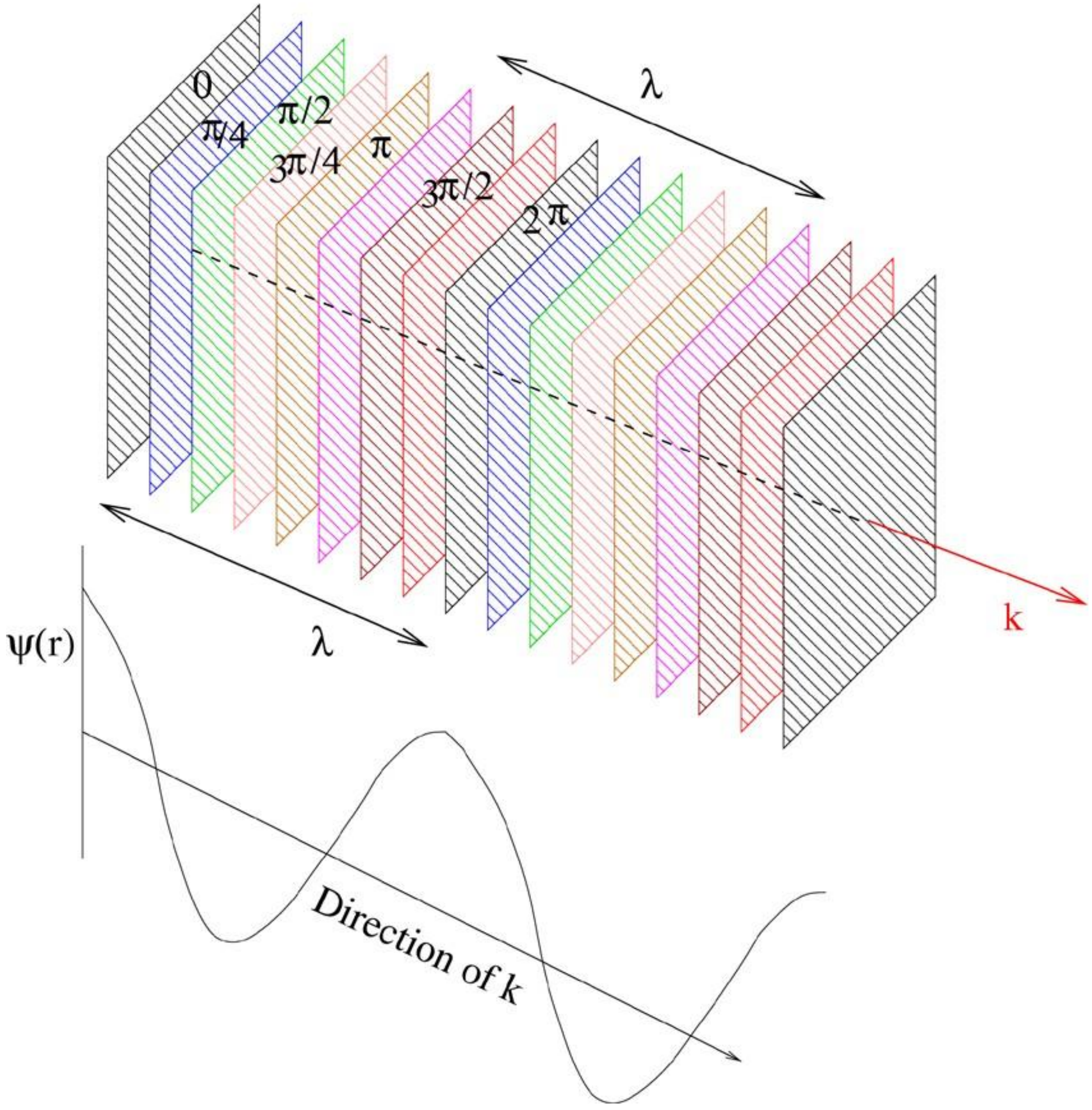
$$\mathbf{r_1} - \mathbf{r_2} = \mathbf{R}$$

$$\mathbf{k}\cdot\mathbf{R} = 0$$



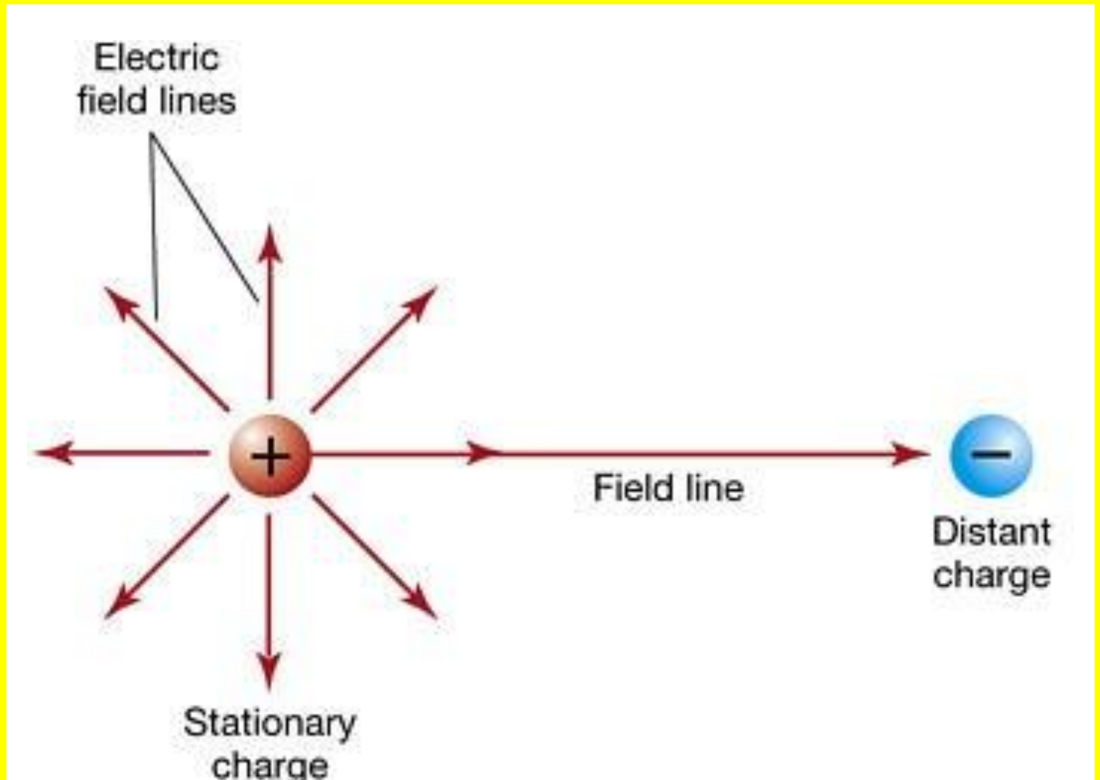
Vectors **k** and **R** are orthogonal to each other. So, the surface swapped by a constant phase is a two dimensional plane and the vector **k** is normal to that plane.

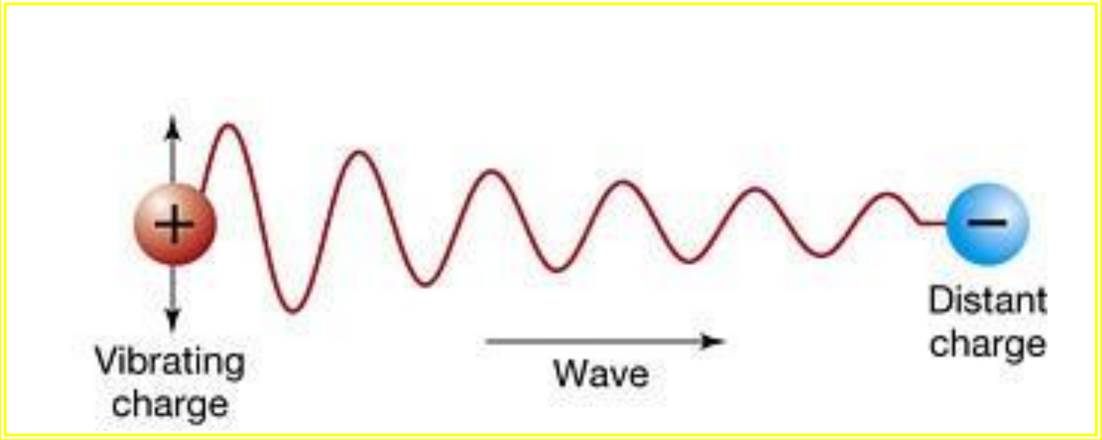
3D Plane waves



Dipole Oscillation, EM-Radiation.

Dipole Radiation





What is the electric field produced at a point *P* by a charge *q* located at a distance *r*?

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\hat{e}_r}{r^2}$$

Where, $\hat{e}_r$ is an unit vector from *P* to the position of the charge.

If a charge moves non-uniformly, it radiates

Dipole Radiation

The correct formula for the electric field

$$\mathbf{E} = \frac{-q}{4\pi\epsilon_0}\left[\frac{\mathbf{e}_{\mathbf{r}'}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\mathbf{e}_{\mathbf{r}'}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2\mathbf{e}_{\mathbf{r}'}}{dt^2}\right]$$

$e_r, e_{r'}$:unit vector directed from q to P at earlier time

Important features

1. No information can propagate instantaneously

2. The electric field at the time t is determined by the position of the charge at an earlier time, when the charge was at r', the retarded position.

3. First two terms falls off as $1/r'^2$ and hence are of no interest at large distances

Dipole Radiation

Correct Expression

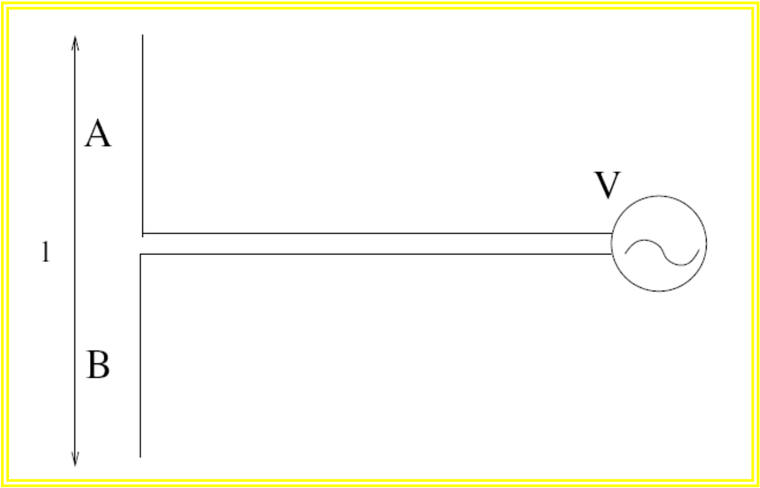
(at large distances)

$$\mathbf{E} = \frac{-q}{4\pi\epsilon_0 c^2}\left[\frac{d^2\mathbf{e}_{\mathbf{r}'}}{dt^2}\right]$$

This is electro-magnetic radiation or simply radiation.

It is also to be noted that only accelerating charges produce radiation.

Electric Dipole Oscillator



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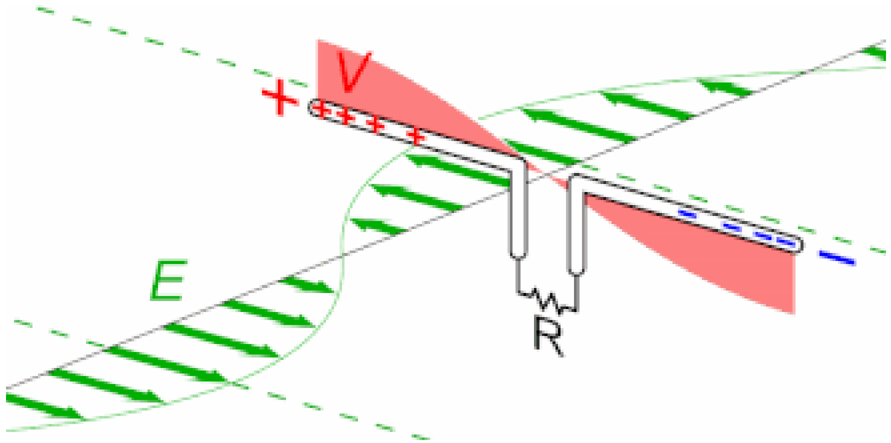
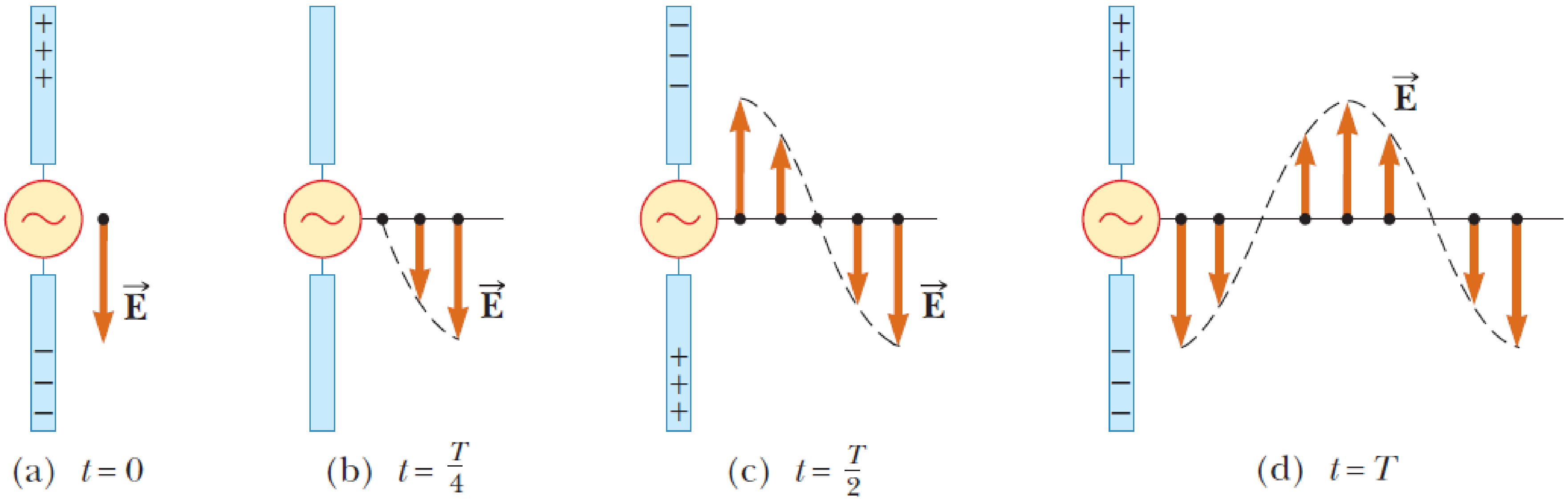
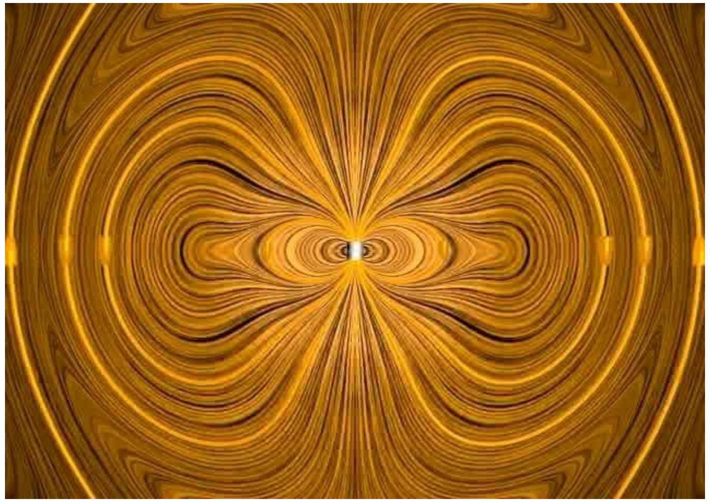


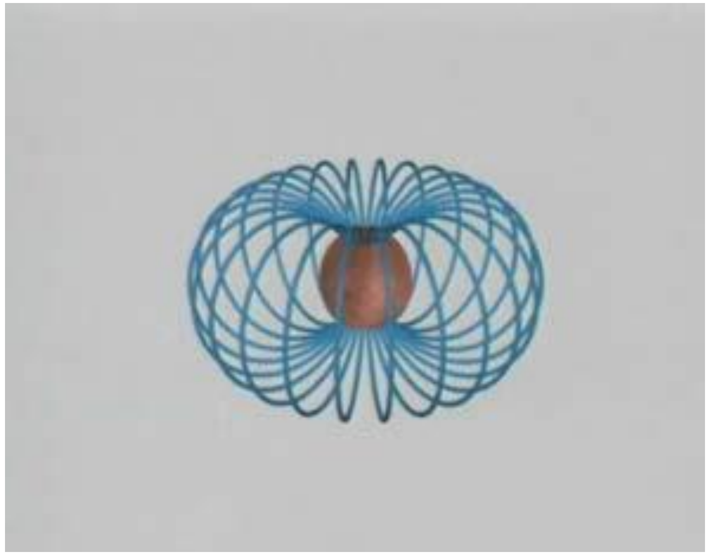
Courtesy: Wikipedia

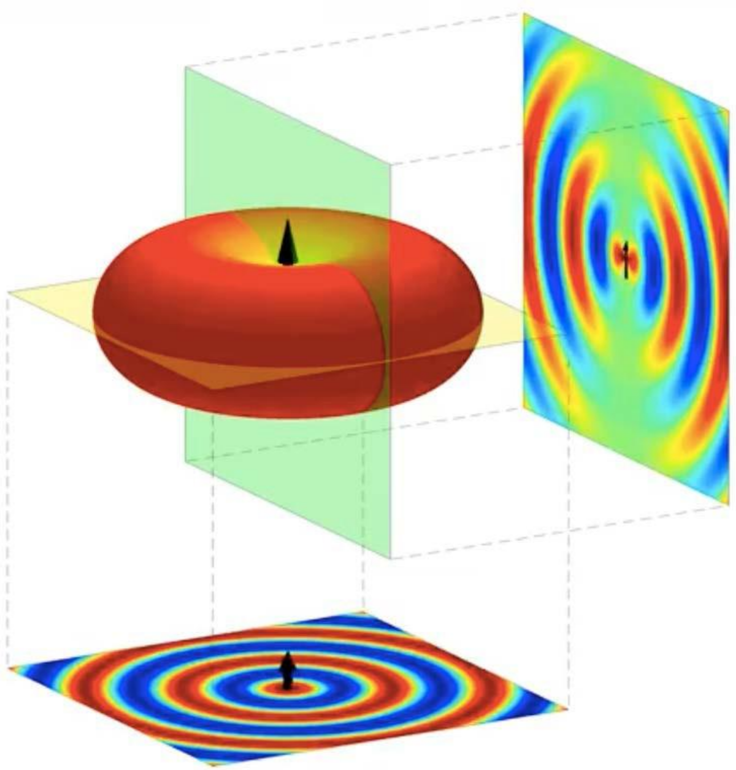
Radio-wave transmission



Dipole Radiation Pattern



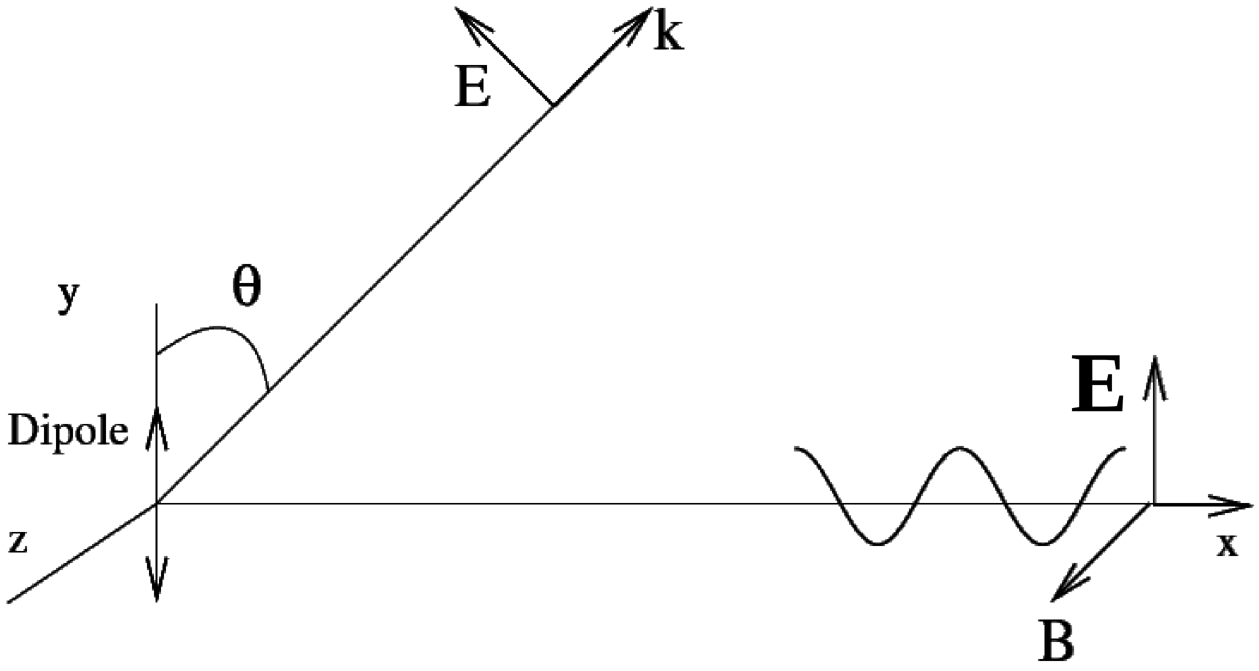




https://youtu.be/UOVwjKi4B6Y

http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/01-DipoleRadiation/01-Dipole_320.html

Electromagnetic Waves



At a given point *x*: $y(t) = y_0 \cos(\omega t)$