CT and MRI Image Segmentation Using Random Walk

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***Abstract*—Currently, the quantum walk is being explored as a potential method for analyzing medical images. Taking inspiration from Grady's random walk algorithm for image processing, we have developed an approach that leverages the quantum mechanism advantages inherent in quantum walk to detect and segment medical images. Furthermore, the segmented images have been evaluated in terms of clinical relevance. Theoretically, quantum walk algorithms have the potential to offer a more efficient method for medical image analysis compared to traditional methods of image segmentation, such as classical random walk, which does not rely on quantum mechanism. Within this field, there is significant potential for development, and it is of utmost importance to continue exploring and refining these methods. However, it should be noted that there is a long way to go before this becomes something that can be applied in a clinical environment.**

Keywords: Discrete-time quantum walk, medical image segmentation, random walk, quantum walk

I. Introduction

Quantum walk is currently being investigated as a potential method for medical image analysis. The idea is that quantum walk could be used to analyze medical images to detect and segment objects of interest, such as tumors or lesions. Theoretically, quantum walk algorithms could offer a more efficient method for analyzing medical images compared to traditional image segmentation methods, as well as compared to classical random walk, which is not based on quantum mechanics.

A quantum walker [14] describes the motion of a particle or object on a graph, where its position is described by a quantum mechanical wave function [2]. As in a classical random walk, the particle can take random steps in different directions, but in a quantum walk the particle is in a superposition of several positions but can also take several steps in parallel [2].

Quantum walks have proven useful for many applications in quantum computing and quantum algorithms; this applies, for example, to search algorithms, factorization of prime numbers, and representation of graphs [3].

When it comes to image segmentation, quantum walk image segmentation is a technique that uses quantum mechanical principles to segment an image into different regions [3]. The basic idea is to represent an image as a graph. The pixels of the image are the nodes of the graph and the edges between the nodes represent the similarity between the pixels. The quantum walk algorithm then performs a random walk on this graph to identify the different regions in the image.. These regions are then marked as a separate segment of the image. Quantum walk offers advantages over classical random walk, especially in areas such as speed, search efficiency, and robustness to noise [4].

In this paper, quantum walk algorithms and their applications for medical images are analyzed. To do this, a combination of analytical and numerical methods is used. We compared discrete quantum walk algorithms with classical random walk algorithms, as well as investigated their properties and how they can be used in the field of image segmentation.

The evaluation of the images took place from the perspective of clinical relevance. This is to investigate whether the method is accurate enough to be applied in a clinical environment.

II. Background

Image segmentation is used to solve everyday problems and is one of the first steps in analyzing images. Medical image segmentation involves dividing a medical image into different parts or areas, usually to identify and isolate regions of interest in the image that may contain pathological changes or other functional or anatomical structures.

What constitutes the inspiration for quantum random walk image segmentation is an algorithm presented by Grady in 2006. Grady [1] proposes the idea of performing image segmentation including classical random walk. The user defines several seeds and by this Grady means a set of pixels in an image that are assigned labels, like Figure 1a. Furthermore, he believes that these labeled pixels are usually selected by a human based on prior knowledge of the image content or by visual inspection of the image. Furthermore, he points out that the seeds are used as initial conditions for the random walk algorithm to determine the probability of each unmarked pixel belonging to a particular image segment.

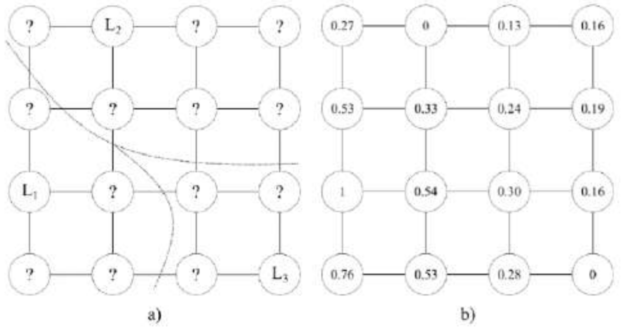

Figure 1: Grady's classical random walk – a) shows seed points with segmentation and b) shows the probability that a random walk starting from each node first reaches the seed L_1

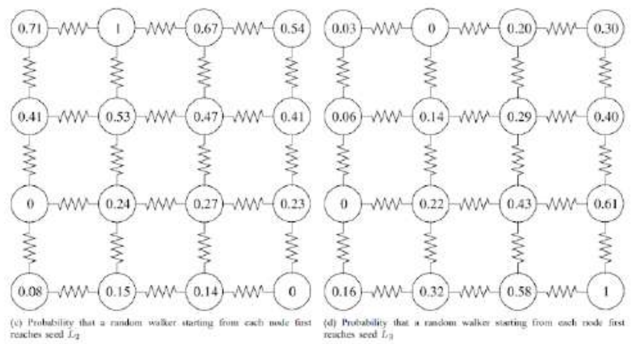

Figure 2: Grady's Classic random walk

The algorithm then tries to determine, for each unlabeled pixel, what the probabilities are of reaching each seed first by a random walk starting from the given pixel [1].

The method used for the following work is inspired by algorithms based on theory from quantum walk. The algorithm is based on similar principles as classical methods but with quantum mechanical operators [4]. One of the most famous applications of DTQW [15] is Grover's algorithm, which is used to search an unstructured database to find a specific location. Grover's algorithm can also be seen as a special case of a quantum walk. Grover's algorithm can be used for searching an unsorted database, which gives a quadratic speed improvement compared to classical algorithms [6].

A. *Medical Images*

Medical images are a central part of modern diagnostics and treatment of diseases. With advanced technology, such as X-ray, ultrasound, CT, and MRI scanning, doctors and other medical specialists can get a detailed picture of the body's internal structures and processes. These images enable doctors to detect and diagnose diseases, monitor treatments, and plan surgical procedures.

B. *Challenges of Medical Image Segmentation*

The challenges related to the segmentation of medical images are many. One of the reasons is variation in image quality and noise. It is common for the images to have different levels of noise in contrast, resolution, and lighting. The problem then becomes that it is difficult to distinguish between different tissue types and organs. According to Zhang et al [7], noise and variation in resolution and illumination are one of the most common causes of failed segmentations of brain tissue. The image quality can be affected by several factors such as imaging technology and scanning parameters, as well as by the patients' individual biological variations. Different imaging techniques can also generate images with different noise levels and resolutions [7].

Managing noise and variability in MRI images demands tailored approaches like contrast enhancement, noise reduction, or robust segmentation algorithms, ensuring accurate tissue boundary delineation.

Classical random walk is a mathematical model that describes the movement of a particle or an object in a random direction [1].. The idea is based on the particle moving randomly in one of two directions, left or right. For the classical random walk model, the particle can only move either left or right [1]. Thus, the superposition principle does not apply, which is a quantum phenomenon, which means that the particle can move in several directions at the same time. This concretely means that if we have a particle then it will move one step +1 or -1 with equal probability, as illustrated in Figure 3

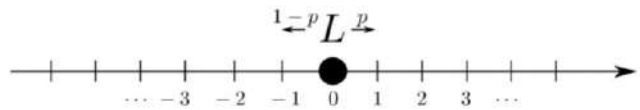

Figure 3: Classical random walk on an infinite line – a walker who at each time step moves either +1 or –1. The probability of moving left or right is 50%, so p = ½.

C. *Quantum Random Walk*

The concept of quantum random walk was first proposed in the late 1990s by Aharonov, as an extension of the classical model [9]. A further development was published in the early 2000s by J. Kempe where implementation of the model for problems in computer science was introduced [10]. The main goal in this field is to study the relationship between the classical model and its quantum counterpart and to obtain and apply universal methods to measure and quantify the performance and properties of the latter model.

D. *Discrete Time Quantum Walk*

Unlike the classical discrete model, where the walker's state is described by its position alone, its quantum counterpart consists of two subsystems [11]: One subsystem is called position space while the other subsystem is called coin space.

E. *Position Space*

Position space H_p is a so-called Hilbert space that is stretched over authorized basis vectors $|i\rangle_p$ for each attainable walker position i where each node is labeled with a consecutive integer number. The walker's position is then described according to the following two equations:

$$|\Psi\rangle_p = \sum_i \quad a_i |i\rangle_p \tag{1}$$

Where $a_i i$ represents a complex probability amplitude for the particle when it is in the state $|i\rangle_p$

$$\sum_i \quad \|a_i\|^2 = 1 \tag{2}$$

Where $\|a_i\|^2$ describes the probability of observing the particle in the state.

Coin space H_c – a subsystem that determines the direction of the walker in the position space at each time step. The dimensionality depends on the number of moves of the particle.

The complete state of the quantum walks [11] is described as follows:

$$|\Psi\rangle = |\Psi\rangle_p \otimes |\Psi\rangle_c, |\Psi\rangle \in H_p \otimes H_c \tag{3}$$

A vital component of DTQW is the use of a coin operator in the uniform evolution of the walk. The coin operator is responsible for determining the walker's path choice at each step during its quantum walk..

III. Method

The quality of the segmentation is dependent in many respects on the choice of parameters. The choice of parameters is especially important. We have first analyzed the classical random walk algorithms and quantum walk algorithms based on the theory from quantum walk. The experiments were thus performed by modifying the parameters in the code and then running the code.

A. Material

The programming language used for the coding is Python 3.11.1 (Python Software Foundation, https://www.python.org/) [13]. The program was conducted on a personal computer with Intel® Core™ i7-13700K processor with 16 cores, 24 threads and 3.4 GHz speed together with 16 GB RAM.

B. Configuration

The color coding of seeds was defined by creating two so-called lexicons in the code. The first linked color names to numerical values. The color white received the value 0 and black received the value 1. Then the colors red, green, and blue were given the values 2, 3 and 4 in that order. Similarly, the second dictionary mapped color names to RGB color values. This through an array with three integers. The first integer represented the red component, the second the green component and finally the third integer represented the blue component.

C. Graph Conversion

The first step involved converting the image of interest into a weighted graph. Each pixel in the image is supposed to correspond to a node in the graph. The images, which were M x N pixels, were converted to have M x M nodes in the graph. Between the nodes, so-called edges were created that tied the nodes of the graph together. This included horizontal and vertical edges. The edges between the nodes represent the connection between the pixels in the image and allow the walker to move from one pixel to another. In the conversion from image to weighted graph, a so-called adjacency matrix was used. The following equation, proposed by Grady [1], is applied here:

$$w_{ijkl} = \{e^{\frac{\beta d(p_{ij} p_{kl})}{d(p_{ij} p_{kl})}}, if\ p_{ij}\ and\ p_{kl}\ is\ connected \quad 0, otherwise$$

Where $d(p_{ij}, p_{kl})$ is a measure of pixel similarity, defined below, and is a free parameter responsible for highlighting the larger differences.

$$d(p_{ij}, p_{kl}) = \sum_c \left(p_{ij}[c] - p_{kl}[c]\right)$$

Where c is the successive channels of pixels, RGB.

D. Configuration

After conversion from image to graph, a step-by-step construction of the code was carried out based on the following algorithm. The mathematical equations that formed the basis of this, in addition to the weight calculation above, were presented in the background.

1. Calculate weights

2. Construct shift operator

3. Construct coin operator

4. Construct unity operator

5. Perform walkthrough for each seed

   a. Enter initial state of the seed

   b. For each $t \in T$

      i. Perform movement

      ii. Calculate the probability distribution for the walker

6. For $i = 0 \rightarrow$ till $M - 1$ make

   a. For $j = 0 \rightarrow N - 1$ make

      i. Determine seed with highest measured probability for pixel $\_ij$

      ii. Enter label

E. Clinical Relevance

Finally, segmented medical images were obtained. These were assessed based on a subjective eye assessment. This was based on how accurately the segmented images managed to depict the geometry of the pathological change in the patient.

The working process of classical random walk has already been explained in section Classical Random Walk.

F. Quantum Walk Algorithm

A quantum walk is the "quantum version" of a classical random walk. This means the coin function will be a Unitary gate (U(2)) which is non-random and reversible:

$$p + q = U \in U(2)$$

IV. RESULTS

A. Case Results

A detailed study has been conducted on Quantum Walk Algorithms. Different algorithms have been studied for different case scenarios and results were then analyzed in this section.

The results of the work will be presented below with three medical images of the same brain, with three separate ways of producing them. The first is a CT image, the second a T_1-weighted MRI image, and the third and final image is a T_2-weighted MRI image. Initially, an original image is presented. Next to this original image, the placement of seeds is reported. The coordinates of these seeds were extracted to get a clearer visual perception of geometries. These seeds have been placed manually and selected based on premises that were presented in the method. Segmentation of CT Image

Initially, a gray-scale CT image was used on a patient with a hypodensity in the left frontal lobe that can be observed in figure 20. The figure also shows an image of seed placement with four assorted colors, namely black, white, green and red. The number of black and white seeds is 4 each, while the number of red and green seeds is more.

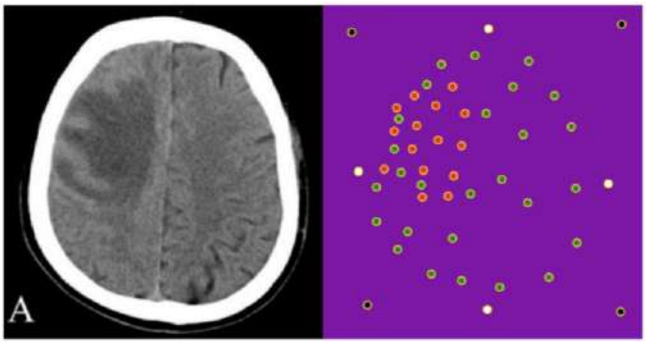

Figure 20: The left image shows a CT image of a brain showing hypodensity in the left frontal lobe. The right picture shows placement of seeds.

The segmentation of the CT image in Figure 20 can be observed in Figure 21. Here, three different images with the same T value (60) and three different beta values, namely 50, 100 and 150 are presented from left to right.

Figure 21: Segmentation of CT image above with the following parameter values from left to right, beta = 50, 100, 150 and T = 60 for all

B. Segmentation of T1-Weighted MRI Image

Next, a T1-weighted MRI image in grayscale, on the same patient as mentioned before, can be observed in figure 22. In the figure, a similar image of seed placement is also seen before, with the exception that there are now five distinct colors, namely black, white, green, red, and blue. The number of black seeds is the same in number, but the number of white seeds is more. The blue seeds dominate in number.

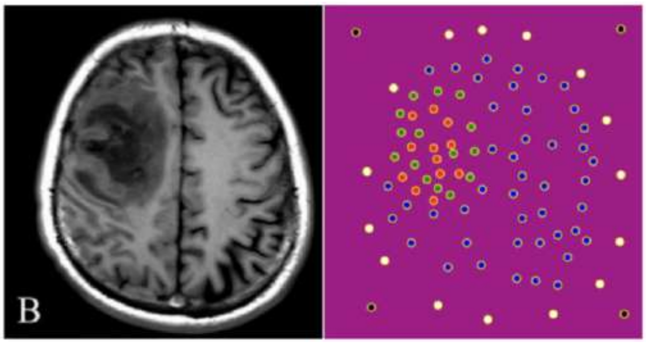

Figure 22: The left image shows a T1-weighted MRI image of the same brain as previously mentioned. The right picture shows placement of seeds

The segmentation of the T1-weighted MRI image in Figure 22 can be observed in Figure 23. Three different images with the same parameter values as before are presented here. Something that can be observed is how the white areas dominate for the lower beta value.

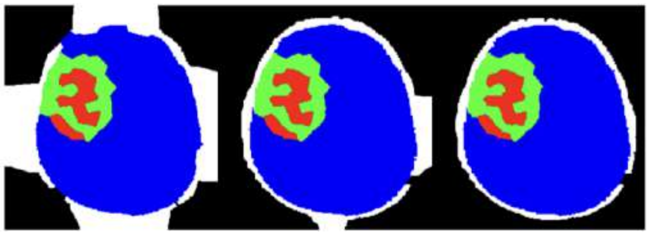

Figure 23: Segmentation of T1 image above with the following parameter values from left to right, beta = 50, 100, 150 and T = 60 for all

C. Segmentation of T2-Weighted MRI Image

Finally, a T2-weighted grayscale MRI image, on the same patient as mentioned earlier, can be observed in figure 24. The figure also shows seed placement like previous images, with the exception that it is now three distinct colours, namely black, green and red. The number of black seeds is more in number than before. The seeds are scattered sporadically and, as before, a purple background was chosen to make it easier to distinguish the seed location.

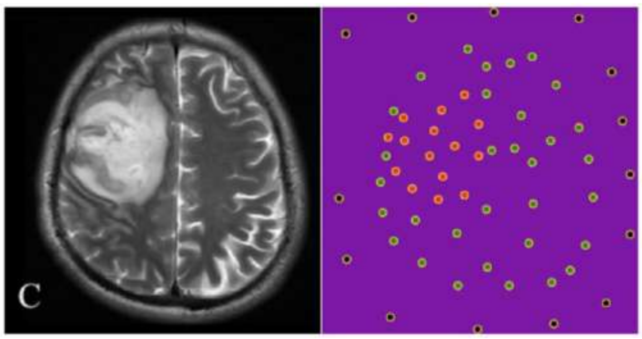

Figure 24: The left image shows a T2-weighted MRI image of the same brain as previously mentioned. The right picture shows placement of seeds

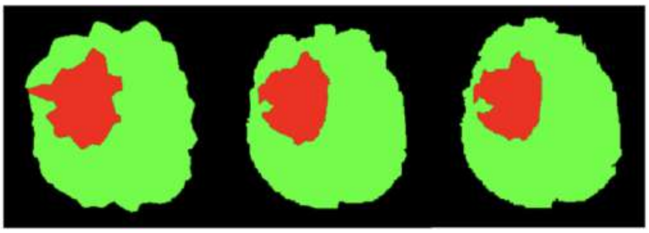

Figure 25: Segmentation of T_2 image above with the following parameter values from left to right, beta = 50, 100, 150 and T = 60 for all

D. Result of Image Segmentation Using Classical Random Walk Algorithm

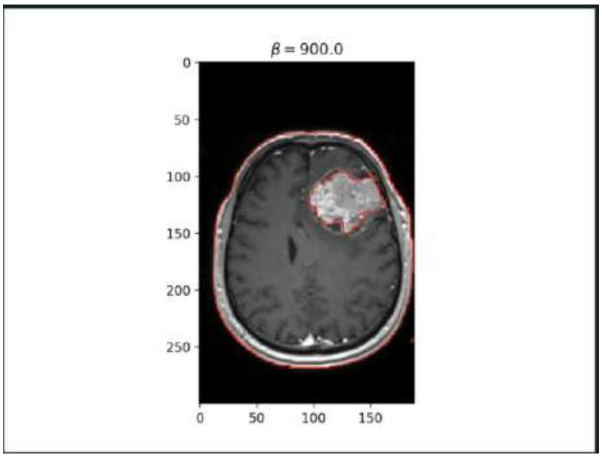

Figure 28: Contour image as output

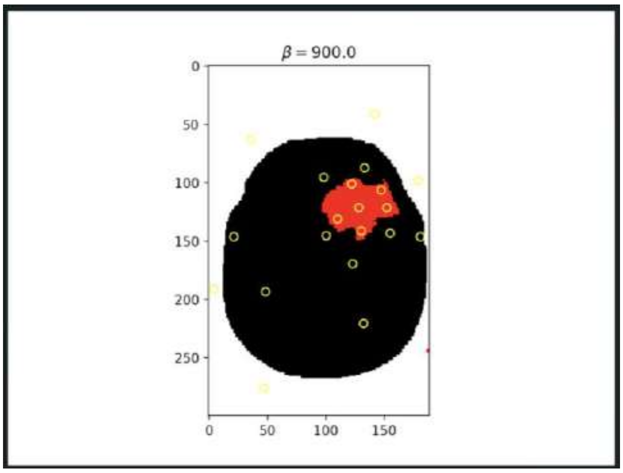

Figure 29: Segmented image as output using random walk algorithm

V. Discussion

The paper's results were presented in their entirety with figures in the form of medical images with associated segmentation of these images. The images used are of the same brain where the patient had hypodensity in the left frontal lobe. Hypodensity is a medical term used to describe a reduced density in a specific tissue or region of the body. This may have been caused due to several reasons. In this case, the patient in question has melioidosis.

A. Analysis of Method

There were three configurable parameters – seeds, beta, and T. All these parameters were chosen experimentally, i.e., determined based on which values generated the most optimal segmentation, based on a subjective eye assessment. An important aspect to point out regarding the parameter choices is that they have been adjusted based on what is optimal for medical images and that the conditions regarding other images may vary. One such aspect could be color, for example.

Identification of object boundaries is important in beta parameter selection. If beta is too low, slight differences in pixel intensity are not considered, which leads to too great a generalization. For example, there is a substantial difference between a dark shade of gray and a light shade of gray, but this risk being depicted in the same way. If the beta parameter is too high, regions risk being over segmented due to minor differences in pixel intensity.

B. Advantages of Quantum Walk Algorithm

Like all algorithms, DTQW for image processing has its advantages and disadvantages. One of the main advantages is its flexibility regarding modification of parameters. By being able to adjust various parameters, we can fulfill different segmentation needs and characteristics of images of interest.. Another advantage to point out is the quantitative nature of the algorithm. Since the algorithm is based on the probability distribution of pixels, this can provide a more accurate representation than other methods such as GrabCut and Interactive Level Sets.

C. Analysis of Results

Purely visually, it is possible to conclude that the characteristics of the CT image, i.e., figure 20, differs from the MRI images from figure 22 and figure 24. In general, an advantage of CT images is that they effectively distinguish between soft tissues and bone tissues, which can be analyzed in the shade of the gray scale. However, figure 20 is noisier than other CT images, which is not preferable for image segmentation as it is more difficult to distinguish different regions. CT usually has a higher resolution compared to MRI. Since the images in this study have the same resolution, the results are not affected by this fact.

When comparing the different images in Figure 21, the contour of the head is not clearly defined at low values of beta. The skull, which is clearly elliptical in nature, makes up a large part of Figure 20 compared to the MRI images, where the outline is thinner. The skull is instead represented by four small disjointed white segments. Instead, the green area, i.e., the part that should represent the brain's normal tissue, is overrepresented. With an increasing beta value, a much better segmentation of the white region is obtained, which can be seen in Figure 21.

Regarding the region of interest, i.e., the hypodensity in the left frontal lobe, a satisfactory segmentation is not obtained. The segmentation is not close to making an accurate representation of the region, but the geometry differs significantly relative to the original image in Figure 20. This is related to noise, image resolution and contrast. Regardless of the value of the beta parameter, it is difficult to distinguish the different regions as there are exceedingly small color differences.

The image segmentation becomes much better in the case of the T1-weighted MRI image in Figure 22. This is due to factors mentioned earlier; such as noise, image resolution and contrast.

Furthermore, the region of interest is evaluated, i.e., the hypodensity in the left frontal lobe. There are clear contrasts in the area. Centrally it is black and with increasing radius it becomes darker gray. A low value of beta does a surprisingly decent job of rendering the area with high precision. When compared with the images that have a higher beta value, no clear differences are seen, but the geometry is intact. Upon careful analysis, we see that the highest beta value does not have equally sharp edges in several places when it comes to the green and red area. This means that there has been an improvement.

The outline of the head in Figure 24 is represented as a thin dark gray strip in the middle between two black regions. This contour is difficult to image, as can be observed in Figure 25. Like the previous image (Figure 23), weak pixel differences mean that the walker cannot make a distinction between different intensity values. The difference here, however, is that there is no case where the contour is included, whereas in the previous image (Figure 23) and the first example (Figure 21), it helped to increase the beta parameter to get a more accurate representation.

VI. Conclusion

Implementation of Classical Random Walk and quantum walk has been successfully done and studied. It is theoretically possible to use a quantum walk inspired algorithm to segment medical images. In terms of clinical relevance, however, the result is not satisfactory. Best results were achieved for the T1-weighted MRI image. Significant quality challenges were encountered in the production of the CT image and the T2-weighted MRI image.

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