Deterministic Turing Machine

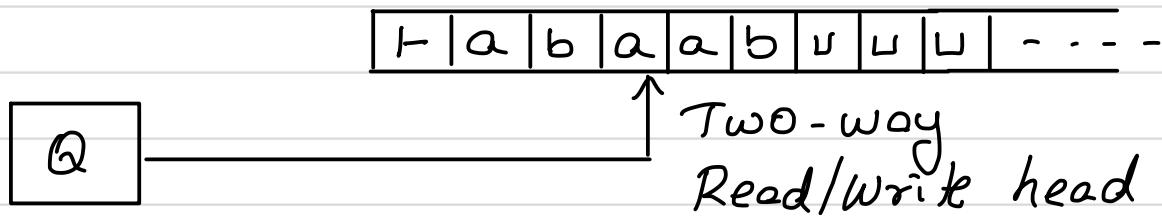| ⊢	a	b	a	a	b	␣	␣	␣	- · - · -

Two-way
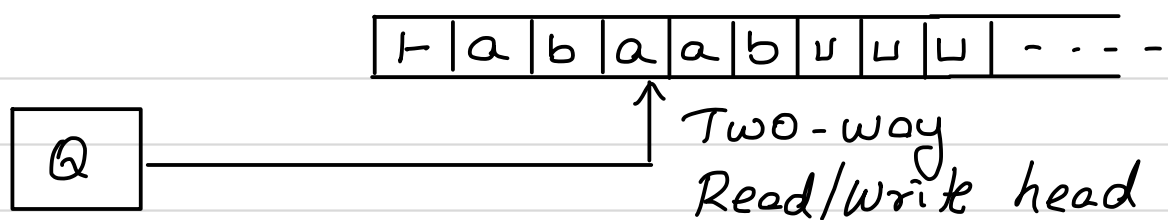Read/Write head

Q

Finite set of States + Semi-infinite tape

- infinite to the right
- Delimited on the left with an end marker ⊢

Head — Can move left or right over the tape, can read and write symbols on the tape.

Infinitely many cells to the right of the input String contains a special blank symbol ␣

Operation of a Turing Machine.

- Start in the start state s with head scanning the left end marker ⊢
- Each step, read the symbol on the tape under its head
- Depending on the symbol and current state, write a new symbol on that tape cell.

- Move the head left or right and change to a new state
- Accept input by entering a special accept state.
- Reject input by entering a special reject state.

| ⊢ | a | b | a | a | b | ⊔ | ⊔ | ⊔ | - · - - |

Q

↑ Two-way
Read/Write head

Formal definition of a Turing machine

$$M = (Q, \Sigma, \Gamma, \vdash, \sqcup, s, \delta, t, r)$$

Σ : finite input alphabet

Γ : finite tape alphabet $\quad [\Sigma \subseteq \Gamma]$

$\sqcup \in \Gamma - \Sigma$: blank symbol
$\vdash \in \Gamma - \Sigma$: left end marker
$s \in Q$: start state.

$t \in Q$: Accept state, $\quad r \in Q$: Reject state

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$$

$$\delta(p, a) = (q, b, d)$$
$$\hookrightarrow \in \{L, R\}$$

Assumptions:

- $\forall p \in Q, \exists q \in Q$ such that $\delta(p, \vdash) = \delta(q, \vdash, R)$

- $\forall b \in \Gamma, \exists c, c' \in \Gamma$ and $\exists d, d' \in \{L, R\}$ such that
$$\delta(t, b) = (t, c, d)$$
$$\delta(r, b) = (r, c', d')$$

Once M enters accept/reject state it never leaves it.

Example. A TM that accepts $\{a^n b^n c^n \mid n \geq 0\}$

1. Start in s, scans to the right over the input once to ensure the input is of the form $a^* b^* c^*$
   - writes to the tape the same symbol it reads
   - writes an end marker at the end of the input.

2. Scan the input over rounds to match one a, b and c and erase these symbols.

Example: $a^4 b^4 c^4$

⊢	a	a	a	a	b	b	b	b	c	c	c	c	⊔	⊔	-	-	-

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