INDIAN INSTITUTE OF TECHNOLOGY KANPUR

ESO 201A: Thermodynamics

(2023-24 I Semester)

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Tutorial 4

Question 1: A 1-m^3 tank containing air at 10°C and 350 kPa is connected through a valve to another tank containing 3 kg of air at 35°C and 150 kPa. Now the valve is opened, and the entire system is allowed to reach thermal equilibrium with the surroundings, which are at 20°C. Determine the volume of the second tank and the final equilibrium pressure of air.

Solution:

3-75 Two rigid tanks connected by a valve to each other contain air at specified conditions. The volume of the second tank and the final equilibrium pressure when the valve is opened are to be determined.

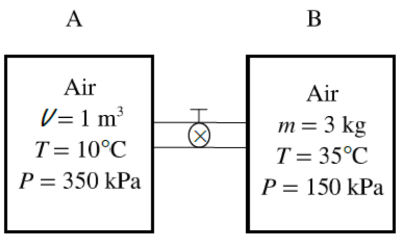
Assumptions At specified conditions, air behaves as an ideal gas.

Properties The gas constant of air is R = 0.287 kPa.m^3/kg.K (Table A-1).

Analysis Let's call the first and the second tanks A and B. Treating air as an ideal gas, the volume of the second tank and the mass of air in the first tank are determined to be

$$V_B = \left(\frac{m_1 R T_1}{P_1}\right)_B = \frac{(3\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(308\text{ K})}{150\text{ kPa}} = \mathbf{1.768\ m^3}$$

$$m_A = \left(\frac{P_1 V}{R T_1}\right)_A = \frac{(350\text{ kPa})(1.0\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(283\text{ K})} = 4.309\text{ kg}$$



Thus,

$$V = V_A + V_B = 1.0 + 1.768 = 2.768\text{ m}^3$$
$$m = m_A + m_B = 4.309 + 3 = 7.309\text{ kg}$$

Then the final equilibrium pressure becomes

$$P_2 = \frac{m R T_2}{V} = \frac{(7.309\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})}{2.768\text{ m}^3} = \mathbf{222\ kPa}$$

Question 2: Determine the specific volume of refrigerant-134a vapor at 0.9 MPa and 70°C based on (a) the ideal-gas equation, (b) the generalized compressibility chart, and (c) data from tables.

Solution:

3-82 The specific volume of R-134a is to be determined using the ideal gas relation, the compressibility chart, and the R-134a tables. The errors involved in the first two approaches are also to be determined.

Properties The gas constant, the critical pressure, and the critical temperature of refrigerant-134a are, from Table A-1,

$$R = 0.08149 \text{ kPa·m}^3\text{/kg·K}, \qquad T_{cr} = 374.2 \text{ K}, \qquad P_{cr} = 4.059 \text{ MPa}$$

Analysis (*a*) From the ideal gas equation of state,

$$\upsilon = \frac{RT}{P} = \frac{(0.08149 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(343 \text{ K})}{900 \text{ kPa}} = \mathbf{0.03105 \ m^3/kg} \qquad \mathbf{(13.3\% \ error)}$$

(*b*) From the compressibility chart (Fig. A-15),

$$\left.\begin{array}{l} P_R = \dfrac{P}{P_{cr}} = \dfrac{0.9 \text{ MPa}}{4.059 \text{ MPa}} = 0.222 \\ T_R = \dfrac{T}{T_{cr}} = \dfrac{343 \text{ K}}{374.2 \text{ K}} = 0.917 \end{array}\right\} Z = 0.894$$

R-134a
0.9 MPa
70°C

Thus,

$$\upsilon = Z\upsilon_{\text{ideal}} = (0.894)(0.03105 \text{ m}^3/\text{kg}) = \mathbf{0.02776 \ m^3/kg} \qquad \mathbf{(1.3\% \ error)}$$

(*c*) From the superheated refrigerant table (Table A-13),

$$\left.\begin{array}{l} P = 0.9 \text{ MPa} \\ T = 70°\text{C} \end{array}\right\} \upsilon = \mathbf{0.027413 \ m^3/kg}$$

Question 3: A 3.27-m^3 tank contains 100 kg of nitrogen at 175 K. Determine the pressure in the tank using (a) the ideal-gas equation, (b) the van der Waals equation, and (c) the Beattie- Bridgeman equation. Compare your results with the actual value of 1505 kPa.

Solution:

3-96 The pressure of nitrogen in a tank at a specified state is to be determined using the ideal gas, van der Waals, and Beattie-Bridgeman equations. The error involved in each case is to be determined.

Properties The gas constant, molar mass, critical pressure, and critical temperature of nitrogen are (Table A-1)

$$R = 0.2968 \text{ kPa·m}^3\text{/kg·K}, \quad M = 28.013 \text{ kg/kmol}, \quad T_{cr} = 126.2 \text{ K}, \quad P_{cr} = 3.39 \text{ MPa}$$

Analysis The specific volume of nitrogen is

$$\upsilon = \frac{V}{m} = \frac{3.27 \text{ m}^3}{100 \text{ kg}} = 0.0327 \text{ m}^3\text{/kg}$$

N_2
$0.0327 \text{ m}^3\text{/kg}$
175 K

(*a*) From the ideal gas equation of state,

$$P = \frac{RT}{\upsilon} = \frac{(0.2968 \text{ kPa}\cdot\text{m}^3\text{/kg}\cdot\text{K})(175 \text{ K})}{0.0327 \text{ m}^3\text{/kg}} = \mathbf{1588 \text{ kPa} \ (5.5\% \text{ error})}$$

(*b*) The van der Waals constants for nitrogen are determined from

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} = \frac{(27)(0.2968 \text{ kPa}\cdot\text{m}^3\text{/kg}\cdot\text{K})^2(126.2 \text{ K})^2}{(64)(3390 \text{ kPa})} = 0.175 \text{m}^6\cdot\text{kPa/kg}^2$$

$$b = \frac{RT_{cr}}{8P_{cr}} = \frac{(0.2968 \text{ kPa}\cdot\text{m}^3\text{/kg}\cdot\text{K})(126.2 \text{ K})}{8\times 3390 \text{ kPa}} = 0.00138 \text{ m}^3\text{/kg}$$

Then,

$$P = \frac{RT}{\upsilon - b} - \frac{a}{\upsilon^2} = \frac{0.2968\times 175}{0.0327 - 0.00138} - \frac{0.175}{(0.0327)^2} = \mathbf{1495 \text{ kPa} \ (0.7\% \text{ error})}$$

(*c*) The constants in the Beattie-Bridgeman equation are

$$A = A_o\left(1 - \frac{a}{\bar{\upsilon}}\right) = 136.2315\left(1 - \frac{0.02617}{0.9160}\right) = 132.339$$

$$B = B_o\left(1 - \frac{b}{\bar{\upsilon}}\right) = 0.05046\left(1 - \frac{-0.00691}{0.9160}\right) = 0.05084$$

$$c = 4.2\times 10^4 \text{ m}^3\cdot\text{K}^3\text{/kmol}$$

since $\bar{\upsilon} = M\upsilon = (28.013 \text{ kg/kmol})(0.0327 \text{ m}^3\text{/kg}) = 0.9160 \text{ m}^3\text{/kmol}$. Substituting,

$$P = \frac{R_uT}{\bar{\upsilon}^2}\left(1 - \frac{c}{\bar{\upsilon}T^3}\right)(\bar{\upsilon} + B) - \frac{A}{\bar{\upsilon}^2}$$

$$= \frac{8.314\times 175}{(0.9160)^2}\left(1 - \frac{4.2\times 10^4}{0.9160\times 175^3}\right)(0.9160 + 0.05084) - \frac{132.339}{(0.9160)^2}$$

$$= \mathbf{1504 \text{ kPa} \ (0.07\% \text{ error})}$$

Question 4: Calculate the total work, in kJ, for process 1–3 shown in Fig. 2 when the system consists of 2 kg of nitrogen.

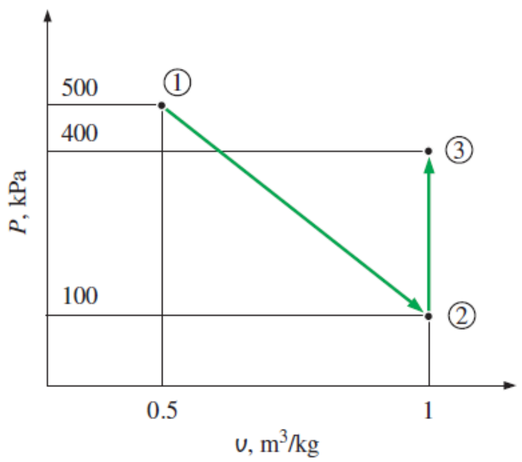


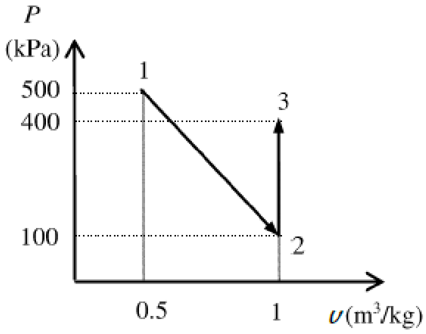
Fig. 1

Solution:

4-4 The boundary work done during the process shown in the figure is to be determined.

Assumptions The process is quasi-equilibrium.

Analysis No work is done during the process 2-3 since the area under process line is zero. Then the work done is equal to the area under the process line 1-2:



$$W_{b,\text{out}} = \text{Area} = \frac{P_1 + P_2}{2} m(\upsilon_2 - \upsilon_1)$$

$$= \frac{(100+500)\text{kPa}}{2}(2\text{ kg})(1.0-0.5)\text{m}^3/\text{kg}\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{300\ kJ}$$

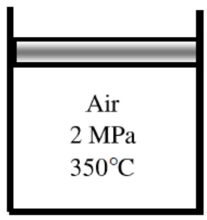
Question 5: A piston–cylinder device contains 0.15 kg of air initially at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle.

Solution:

4-20 A piston-cylinder device contains air gas at a specified state. The air undergoes a cycle with three processes. The boundary work for each process and the net work of the cycle are to be determined.

Properties The properties of air are R = 0.287 kJ/kg.K, k = 1.4 (Table A-2a).

Analysis For the isothermal expansion process:



$$V_1 = \frac{mRT}{P_1} = \frac{(0.15\text{ kg})(0.287\text{ kJ/kg.K})(350+273\text{ K})}{(2000\text{ kPa})} = 0.01341\text{ m}^3$$

$$V_2 = \frac{mRT}{P_2} = \frac{(0.15\text{ kg})(0.287\text{ kJ/kg.K})(350+273\text{ K})}{(500\text{ kPa})} = 0.05364\text{ m}^3$$

$$W_{b,1-2} = P_1V_1\ln\left(\frac{V_2}{V_1}\right) = (2000\text{ kPa})(0.01341\text{ m}^3)\ln\left(\frac{0.05364\text{ m}^3}{0.01341\text{ m}^3}\right) = \mathbf{37.18\ kJ}$$

For the polytropic compression process:

$$P_2V_2^n = P_3V_3^n \longrightarrow (500\text{ kPa})(0.05364\text{ m}^3)^{1.2} = (2000\text{ kPa})V_3^{1.2} \longrightarrow V_3 = 0.01690\text{ m}^3$$

$$W_{b,2-3} = \frac{P_3V_3 - P_2V_2}{1-n} = \frac{(2000\text{ kPa})(0.01690\text{ m}^3) - (500\text{ kPa})(0.05364\text{ m}^3)}{1-1.2} = \mathbf{-34.86\ kJ}$$

For the constant pressure compression process:

$$W_{b,3-1} = P_3(V_1 - V_3) = (2000\text{ kPa})(0.01341 - 0.01690)\text{m}^3 = \mathbf{-6.97\ kJ}$$

The net work for the cycle is the sum of the works for each process

$$W_{\text{net}} = W_{b,1-2} + W_{b,2-3} + W_{b,3-1} = 37.18 + (-34.86) + (-6.97) = \mathbf{-4.65\ kJ}$$

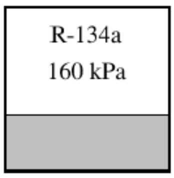
Question 6: A 0.5-m³ rigid tank contains refrigerant-134a initially at 160 kPa and 40 percent quality. Heat is now transferred to the refrigerant until the pressure reaches 700 kPa. Determine (a) the mass of the refrigerant in the tank and (b) the amount of heat transferred. Also, show the process on a P-v diagram with respect to saturation lines.

Solution:

4-31 A rigid tank is initially filled with superheated R-134a. Heat is transferred to the tank 4-31 until the pressure inside rises to a specified value. The mass of the refrigerant and the amount of heat transfer are to be determined, and the process is to be shown on a P-υ diagram.

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.

Analysis (a) We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as



$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

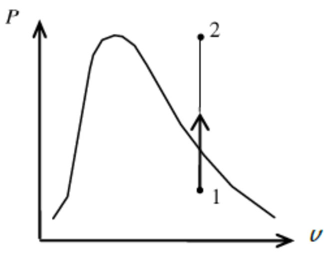
$$Q_{\text{in}} = \Delta U = m(u_2 - u_1) \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$

Using data from the refrigerant tables (Tables A-11 through A-13), the properties of R-134a are determined to be

$$\left.\begin{array}{l} P_1 = 160 \text{ kPa} \\ x_1 = 0.4 \end{array}\right\} \begin{array}{ll} \upsilon_f = 0.0007435, & \upsilon_g = 0.12355 \text{ m}^3/\text{kg} \\ u_f = 31.06, & u_{fg} = 190.31 \text{ kJ/kg} \end{array}$$

$$\upsilon_1 = \upsilon_f + x_1\upsilon_{fg} = 0.0007435 + 0.4(0.12355 - 0.0007435) = 0.04987 \text{ m}^3/\text{kg}$$
$$u_1 = u_f + x_1 u_{fg} = 31.06 + 0.4(190.31) = 107.19 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_2 = 700 \text{ kPa} \\ (\upsilon_2 = \upsilon_1) \end{array}\right\} u_2 = 377.23 \text{ kJ/kg} \quad \text{(Superheated vapor)}$$



Then the mass of the refrigerant is determined to be

$$m = \frac{V_1}{\upsilon_1} = \frac{0.5 \text{ m}^3}{0.04987 \text{ m}^3/\text{kg}} = \mathbf{10.03 \text{ kg}}$$

(b) Then the heat transfer to the tank becomes

$$\begin{aligned} Q_{\text{in}} &= m(u_2 - u_1) \\ &= (10.03 \text{ kg})(377.23 - 107.19) \text{ kJ/kg} \\ &= \mathbf{2708 \text{ kJ}} \end{aligned}$$