Question Bank Quiz-2: ESO201

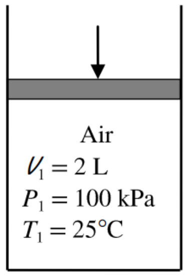
Question 1: A piston-cylinder device initially contains 2 L of air at 100 kPa and 25° C. Air is now compressed to a final state of 600 kPa and 150° C. The useful work input is 1.2 kJ. Assuming the surroundings are at 100 kPa and 25° C, determine (a) the exergy of the air at the initial and the final states, (b) the minimum work that must be supplied to accomplish this compression process, and (c) the second-law efficiency of this process. Use specific heats at the average temperature for solving this problem.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** The kinetic and potential energies are negligible.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1). The specific heats of air at the average temperature of $(298 + 423)/2 = 360$ K are $c_p = 1.009$ kJ/kg·K and $c_v = 0.722$ kJ/kg·K (Table A-2).

Analysis (a) We realize that $X_1 = \Phi_1 = \mathbf{0}$ since air initially is at the dead state. The mass of air is

$$m = \frac{P_1 V_1}{RT_1} = \frac{(100 \text{ kPa})(0.002 \text{ m}^3)}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298 \text{ K})} = 0.00234 \text{ kg}$$



Also,

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \longrightarrow V_2 = \frac{P_1 T_2}{P_2 T_1} V_1 = \frac{(100 \text{ kPa})(423 \text{ K})}{(600 \text{ kPa})(298 \text{ K})}(2 \text{ L}) = 0.473 \text{ L}$$

and

$$\begin{aligned} s_2 - s_0 &= c_{p,\text{avg}} \ln\frac{T_2}{T_0} - R\ln\frac{P_2}{P_0} \\ &= (1.009 \text{ kJ/kg}\cdot\text{K}) \ln\frac{423 \text{ K}}{298 \text{ K}} - (0.287 \text{ kJ/kg}\cdot\text{K}) \ln\frac{600 \text{ kPa}}{100 \text{ kPa}} \\ &= -0.1608 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

Thus, the exergy of air at the final state is

$$\begin{aligned} X_2 = \Phi_2 &= m\left[c_{v,\text{avg}}(T_2 - T_0) - T_0(s_2 - s_0)\right] + P_0(V_2 - V_0) \\ &= (0.00234 \text{ kg})\left[(0.722 \text{ kJ/kg}\cdot\text{K})(423\text{-}298)\text{K} - (298 \text{ K})(-0.1608 \text{ kJ/kg}\cdot\text{K})\right] \\ &\quad +(100 \text{ kPa})(0.000473 - 0.002)\text{m}^3[\text{kJ/m}^3\cdot\text{kPa}] \\ &= \mathbf{0.171 \text{ kJ}} \end{aligned}$$

(*b*) The minimum work input is the reversible work input, which can be determined from the exergy balance by setting the exergy destruction equal to zero,

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\substack{\text{Net exergy transfer} \\ \text{by heat, work, and mass}}} - \underbrace{X_{\text{destroyed}}^{\nearrow 0 \text{ (reversible)}}}_{\substack{\text{Exergy} \\ \text{destruction}}} = \underbrace{\Delta X_{\text{system}}}_{\substack{\text{Change} \\ \text{in exergy}}}$$

$$\begin{aligned} W_{\text{rev,in}} &= X_2 - X_1 \\ &= 0.171 - 0 = \mathbf{0.171 \text{ kJ}} \end{aligned}$$

(c) The second-law efficiency of this process is

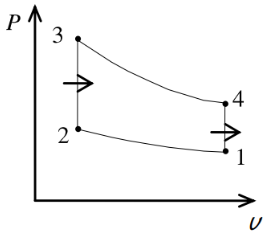
$$\eta_{\text{II}} = \frac{W_{\text{rev,in}}}{W_{\text{u,in}}} = \frac{0.171 \text{ kJ}}{1.2 \text{ kJ}} = \mathbf{14.3\%}$$

Question 2: Someone has suggested that the air-standard Otto cycle is more accurately represented if polytropic processes replace the two isentropic processes with a polytropic exponent of n = 1.3. Consider such a cycle when the compression ratio is 8, P_1 = 95 kPa, T_1 = 15° C, and the maximum cycle temperature is 1200° C. Determine (a) the heat input to the cycle, (b) the heat rejected from the cycle, and (c) the thermal efficiency of the cycle. Use constant specific heats at a room temperature of 27° C to solve this problem.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $R = 0.287$ kPa·m³/kg·K, $c_p = 1.005$ kJ/kg·K, $c_\upsilon = 0.718$ kJ/kg·K, and $k = 1.4$ (Table A-2a).

Analysis The temperature at the end of the compression is



$$T_2 = T_1\left(\frac{\upsilon_1}{\upsilon_2}\right)^{n-1} = T_1 r^{n-1} = (288\text{ K})(8)^{1.3-1} = 537.4\text{ K}$$

And the temperature at the end of the expansion is

$$T_4 = T_3\left(\frac{\upsilon_3}{\upsilon_4}\right)^{n-1} = T_3\left(\frac{1}{r}\right)^{n-1} = (1473\text{ K})\left(\frac{1}{8}\right)^{1.3-1} = 789.4\text{ K}$$

The integral of the work expression for the polytropic compression gives

$$w_{1-2} = \frac{RT_1}{n-1}\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^{n-1} - 1\right] = \frac{(0.287\text{ kJ/kg}\cdot\text{K})(288\text{ K})}{1.3-1}(8^{1.3-1} - 1) = 238.6\text{ kJ/kg}$$

Similarly, the work produced during the expansion is

$$w_{3-4} = -\frac{RT_3}{n-1}\left[\left(\frac{\upsilon_3}{\upsilon_4}\right)^{n-1} - 1\right] = -\frac{(0.287\text{ kJ/kg}\cdot\text{K})(1473\text{ K})}{1.3-1}\left[\left(\frac{1}{8}\right)^{1.3-1} - 1\right] = 654.0\text{ kJ/kg}$$

Application of the first law to each of the four processes gives

$$q_{1-2} = w_{1-2} - c_\upsilon(T_2 - T_1) = 238.6\text{ kJ/kg} - (0.718\text{ kJ/kg}\cdot\text{K})(537.4 - 288)\text{K} = 59.53\text{ kJ/kg}$$

$$q_{2-3} = c_\upsilon(T_3 - T_2) = (0.718\text{ kJ/kg}\cdot\text{K})(1473 - 537.4)\text{K} = 671.8\text{ kJ/kg}$$

$$q_{3-4} = w_{3-4} - c_\upsilon(T_3 - T_4) = 654.0\text{ kJ/kg} - (0.718\text{ kJ/kg}\cdot\text{K})(1473 - 789.4)\text{K} = 163.2\text{ kJ/kg}$$

$$q_{4-1} = c_\upsilon(T_4 - T_1) = (0.718\text{ kJ/kg}\cdot\text{K})(789.4 - 288)\text{K} = 360.0\text{ kJ/kg}$$

The head added and rejected from the cycle are

$$q_{\text{in}} = q_{2-3} + q_{3-4} = 671.8 + 163.2 = \mathbf{835.0\text{ kJ / kg}}$$
$$q_{\text{out}} = q_{1-2} + q_{4-1} = 59.53 + 360.0 = \mathbf{419.5\text{ kJ / kg}}$$

The thermal efficiency of this cycle is then

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{419.5}{835.0} = 0.498 = \mathbf{49.8\%}$$

Question 3: A gas turbine power plant operates on a modified Brayton cycle, shown in the figure, with an overall pressure ratio of 8. Air enters the compressor at 0° C and 100 kPa. The maximum cycle temperature is 1500 K. The compressor and the turbines are isentropic. The high-pressure turbine develops just enough power to run the compressor. Assume constant properties for air at 300 K with c_v = 0.718 kJ/kgK, c_p = 1.005 kJ/kgK, R = 0.287 kJ/kgK, k = 1.4.

(a) Sketch the T-s diagram for the cycle. Label the data states.

(b) Determine the temperature and pressure at state 4, the exit of the high-pressure turbine.

(c) If the net power output is 200 MW, determine the mass flow rate of the air into the low-pressure turbine (in kg/s).

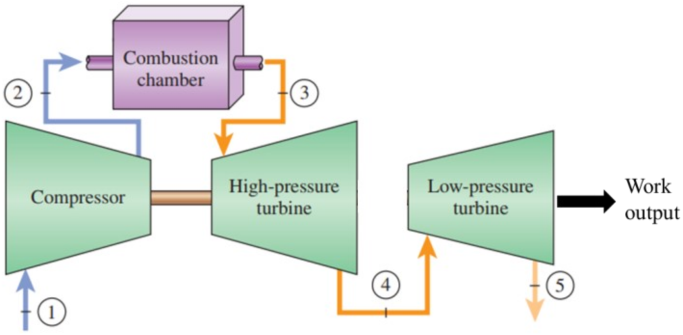


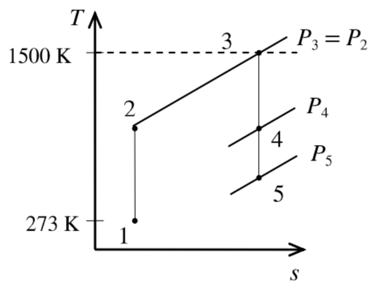
Fig. 2

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air are given as c_v = 0.718 kJ/kg·K, c_p = 1.005 kJ/kg·K, R = 0.287 kJ/kg·K, k = 1.4.

Analysis (b) For the compression process,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (273\text{ K})(8)^{0.4/1.4} = 494.5\text{ K}$$



The power input to the compressor is equal to the power output from the high-pressure turbine. Then,

$$\dot{W}_{\text{Comp, in}} = \dot{W}_{\text{HP Turb, out}}$$

$$\dot{m}c_p(T_2 - T_1) = \dot{m}c_p(T_3 - T_4)$$

$$T_2 - T_1 = T_3 - T_4$$

$$T_4 = T_3 + T_1 - T_2 = 1500 + 273 - 494.5 = \mathbf{1278.5\ K}$$

The pressure at this state is

$$\frac{P_4}{P_3} = \left(\frac{T_4}{T_3}\right)^{k/(k-1)} \longrightarrow P_4 = rP_1\left(\frac{T_4}{T_3}\right)^{k/(k-1)} = 8(100\text{ kPa})\left(\frac{1278.5\text{ K}}{1500\text{ K}}\right)^{1.4/0.4} = \mathbf{457.3\ kPa}$$

(c) The temperature at state 5 is determined from

$$T_5 = T_4\left(\frac{P_5}{P_4}\right)^{(k-1)/k} = (1278.5\text{ K})\left(\frac{100\text{ kPa}}{457.3\text{ kPa}}\right)^{0.4/1.4} = 828.1\text{ K}$$

The net power is that generated by the low-pressure turbine since the power output from the high-pressure turbine is equal to the power input to the compressor. Then,

$$\dot{W}_{\text{LP Turb}} = \dot{m}c_p(T_4 - T_5)$$

$$\dot{m} = \frac{\dot{W}_{\text{LP Turb}}}{c_p(T_4 - T_5)} = \frac{200{,}000\text{ kW}}{(1.005\text{ kJ/kg}\cdot\text{K})(1278.5 - 828.1)\text{K}}$$

$$= \mathbf{441.8\ kg/s}$$