# INDIAN INSTITUTE OF TECHNOLOGY KANPUR

## ESO 201A: Thermodynamics

(2023-24 I Semester)

**Instructor:** Dr Avinash Kumar Agarwal

## Tutorial 2

**Question 1:** Consider a U-tube whose arms are open to the atmosphere. Now water is poured into the U-tube from one arm, and light oil ($\rho$ = 790 kg/m$^3$) from the other. One arm contains 70-cm-high water, while the other arm contains both fluids with an oil-to-water height ratio of 4. Determine the height of each fluid in that arm.

**(Ans: water height = 0.168m, oil height = 0.673m)**

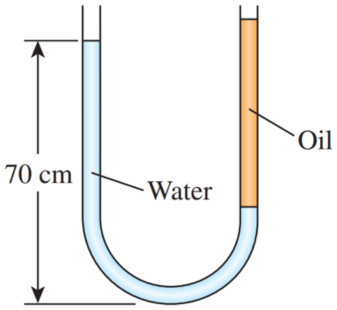


**Fig 1**

**Solution:**

***Assumptions*** Both water and oil are incompressible substances.

***Properties*** The density of oil is given to be $\rho$ = 790 kg/m$^3$. We take the density of water to be ρ =1000 kg/m$^3$.

***Analysis*** The height of water column in the left arm of the monometer is given to be $h_{w1}$ = 0.70 m. We let the height of water and oil in the right arm to be $h_{w2}$ and $h_a$, respectively. Then, $h_a = 4h_{w2}$. Noting that both arms are open to the atmosphere, the pressure at the bottom of the U-tube can be expressed as

$$P_{\text{bottom}} = P_{\text{atm}} + \rho_w g h_{w1} \quad \text{and} \quad P_{\text{bottom}} = P_{\text{atm}} + \rho_w g h_{w2} + \rho_a g h_a$$

Setting them equal to each other and simplifying,

Noting that $h_a = 4h_{w2}$, the water and oil column heights in the second arm are determined to be

$$0.7\,\text{m} = h_{w2} + (790/1000)\,4h_{w2} \quad \rightarrow \quad h_{w2} = \mathbf{0.168m}$$

$$0.7\,\text{m} = 0.168\,\text{m} + (790/1000)h_a \quad \rightarrow \quad h_a = \mathbf{0.673m}$$

***Discussion*** Note that the fluid height in the arm that contains oil is higher. This is expected since oil is lighter than water.

**Question 2:** It is well-known that the temperature of the atmosphere varies with altitude. In the troposphere, which extends to an altitude of 11 km, for example, the variation of temperature can be approximated by $T = T_0 - \beta z$ , where $T_0$ is the temperature at sea level, which can be taken to be 288.15 K, and $b$ = 0.0065 K/m. The gravitational acceleration also changes with altitude as $g(z) = g_0/\left(1 + \frac{1}{6370320}z\right)^2$ where $g_0$ =9.807 m/s² and $z$ is the elevation from sea level in m. Obtain a relation for the variation of pressure in the troposphere (a) by ignoring and (b) by considering the variation of g with altitude.

**Ans: (a)** $P = P_0\left(1 - \frac{\beta z}{T_0}\right)^{\frac{g}{\beta R}}$

**(b)** $P = P_0 \exp\left[-\frac{g_0}{R(\beta+kT_0)}\left(\frac{1}{1+1/kz} + \frac{1}{1+kT_0/\beta}\ln \quad \frac{1+kz}{1-\beta z/T_0}\right)\right]$

**Solution:**

***Assumptions*** The air in the troposphere behaves as an ideal gas.

***Analysis*** (*a*) Pressure change across a differential fluid layer of thickness *dz* in the vertical *z* direction is

$$dP = -\rho g dz$$

From the ideal gas relation, the air density can be expressed as $\rho = \frac{P}{RT} = \frac{P}{R(T_0 - \beta z)}$ . Then,

$$dP = -\frac{P}{R(T_0 - \beta z)}g dz$$

Separating variables and integrating from $z = 0$ where $P = P_0$ to $z = z$ where $P = P$,

$$\int_{P_0}^{P}\frac{dP}{P} = -\int_0^z \frac{g dz}{R(T_0 - \beta z)}$$

Performing the integrations.

$$\ln\frac{P}{P_0} = \frac{g}{R\beta}\ln\frac{T_0 - \beta z}{T_0}$$

Rearranging, the desired relation for atmospheric pressure for the case of constant *g* becomes

$$P = P_0\left(1 - \frac{\beta z}{T_0}\right)^{\frac{g}{\beta R}}$$

(b) When the variation of *g* with altitude is considered, the procedure remains the same but the expressions become more complicated,

$$dP = -\frac{P}{R(T_0 - \beta z)}\frac{g_0}{(1+z/6{,}370{,}320)^2}dz$$

Separating variables and integrating from $z = 0$ where $P = P_0$ to $z = z$ where $P = P$,

$$\int_{P_0}^{P}\frac{dP}{P} = -\int_0^z \frac{g_0 dz}{R(T_0 - \beta z)(1+z/6{,}370{,}320)^2}$$

Performing the integrations,

$$\ln P\Big|_{P_0}^{P} = \frac{g_0}{R\beta}\left|\frac{1}{(1+kT_0/\beta)(1+kz)} - \frac{1}{(1+kT_0/\beta)^2}\ln\frac{1+kz}{T_0-\beta z}\right|_0^z$$

where $R = 287$ J/kg·K = 287 m$^2$/s$^2$·K is the gas constant of air. After some manipulations, we obtain

$$P = P_0 \exp\left[-\frac{g_0}{R(\beta + kT_0)}\left(\frac{1}{1+1/kz} + \frac{1}{1+kT_0/\beta}\ln\frac{1+kz}{1-\beta z/T_0}\right)\right]$$

where $T_0 = 288.15$ K, $\beta = 0.0065$ K/m, $g_0 = 9.807$ m/s$^2$, $k = 1/6{,}370{,}320$ m$^{-1}$, and $z$ is the elevation in m.

***Discussion***: When performing the integration in part (b), the following expression from integral tables is used, together with a transformation of variable $x = T_0 - \beta z$,

$$\int\frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2}\ln\frac{a+bx}{x}$$

Also, for $z = 11{,}000$ m, for example, the relations in (*a*) and (*b*) give 22.62 and 22.69 kPa, respectively.

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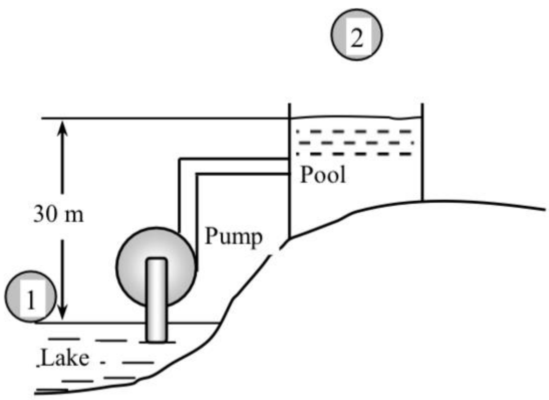
**Question 3:** A water pump that consumes 2 kW of electric power when operating is claimed to take in water from a lake and pump it to a pool whose free surface is 30 m above the free surface of the lake at a rate of 50 L/s. Determine if this claim is reasonable.

**(Ans: Claim is false)**

**Solution:**

A water pump is claimed to raise water to a specified elevation at a specified rate while consuming electric power at a specified rate. The validity of this claim is to be investigated.

***Assumptions 1*** The water pump operates steadily. 2 Both the lake and the pool are open to the atmosphere, and the flow velocities in them are negligible.



***Properties*** We take the density of water to be $\rho = 1000$ kg/m$^3$ = 1 kg/L.

***Analysis*** For a control volume that encloses the pump-motor unit, the energy balance can be written as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{system} / dt^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{in}} + \dot{m}\text{pe}_1 = \dot{m}\text{pe}_2 \quad \rightarrow \quad \dot{W}_{\text{in}} = \dot{m}\Delta\text{pe} = \dot{m}g(z_2 - z_1)$$

since the changes in kinetic and flow energies of water are negligible. Also,

$$\dot{m} = \rho\dot{V} = (1\text{ kg/L})(50\text{ L/s}) = 50\text{ kg/s}$$

Substituting, the minimum power input required is determined to be

$$\dot{W}_{\text{in}} = \dot{m}g(z_2 - z_1) = (50\text{ kg/s})(9.81\text{ m/s}^2)(30\text{ m})\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 14.7\text{ kJ/s} = \mathbf{14.7\,kW}$$

which is much greater than 2 kW. Therefore, the claim is **false.**

***Discussion*** The conservation of energy principle requires the energy to be conserved as it is converted from one form to another, and it does not allow any energy to be created or destroyed during a process. In reality, the power required will be considerably higher than 14.7 kW because of the losses associated with the conversion of electrical-to-mechanical shaft and mechanical shaft-to-potential energy of water.

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**Question 4:** The driving force for fluid flow is the pressure difference, and a pump operates by raising the pressure of a fluid (by converting the mechanical shaft work to flow energy). A gasoline pump is measured to consume 3.8 kW of electric power when operating. If the pressure differential between the outlet and inlet of the pump is measured to be 7 kPa and the changes in velocity and elevation are negligible, determine the maximum possible volume flow rate of gasoline.

**(Ans: 0.543 m$^3$/s)**

**Solution:** A gasoline pump raises the pressure to a specified value while consuming electric power at a specified rate. The maximum volume flow rate of gasoline is to be determined.

***Assumptions 1*** The gasoline pump operates steadily. 2 The changes in kinetic and potential energies across the pump are negligible. Analysis For a control volume that encloses the pump-motor unit, the energy balance can be written as:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt^{\nearrow 0\ (\text{steady})}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0 \quad \rightarrow \quad \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}(P\boldsymbol{v})_1 = \dot{m}(P\boldsymbol{v})_2 \quad \rightarrow \quad \dot{W}_{in} = \dot{m}(P_2 - P_1)\boldsymbol{v} = \dot{\boldsymbol{V}}\,\Delta P$$

since $\dot{m} = \dot{\boldsymbol{V}}/\boldsymbol{v}$ and the changes in kinetic and potential energies of gasoline are negligible, Solving for volume flow rate and substituting, the maximum flow rate is determined to be

$$\dot{\boldsymbol{V}}_{max} = \frac{\dot{W}_{in}}{\Delta P} = \frac{3.8\ \text{kJ/s}}{7\ \text{kPa}}\left(\frac{1\ \text{kPa}\cdot\text{m}^3}{1\ \text{kJ}}\right) = \mathbf{0.543\ m^3/s}$$

***Discussion*** The conservation of energy principle requires the energy to be conserved as it is converted from one form to another, and it does not allow any energy to be created or destroyed during a process. In reality, the volume flow rate will be less because of the losses associated with the conversion of electrical-to-mechanical shaft and mechanical shaft-to-flow energy.

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**Question 5:** Two sites are being considered for wind power generation. In the first site, the wind blows steadily at 7 m/s for 3000 hours per year, whereas in the second site the wind blows at 10 m/s for 1500 hours per year. Assuming the wind velocity is negligible at other times for simplicity, determine which is a better site for wind power generation. (Hint: Note that the mass flow rate of air is proportional to wind velocity).

**(Ans: Second site is better)**

**Solution:**

Two sites with specified wind data are being considered for wind power generation. The site better suited for wind power generation is to be determined.

***Assumptions 1.*** The wind is blowing steadily at specified velocity during specified times. ***2.*** The wind power generation is negligible during other times.

***Analysis*** Kinetic energy is the only form of mechanical energy the wind possesses, and it can be converted to work entirely. Therefore, the power potential of the wind is its kinetic energy, which is $V^2/2$ per unit mass, and $\dot{m}V^2/2$ for a given mass flow rate. Considering a unit flow area ($A = 1\ \text{m}^2$), the maximum wind power and power generation becomes

$$e_{mech,1} = ke_1 = \frac{V_1^2}{2} = \frac{(7\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.0245\ \text{kJ/kg}$$

$$e_{mech,2} = ke_2 = \frac{V_2^2}{2} = \frac{(10\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.050\ \text{kJ/kg}$$

$$\dot{W}_{max,1} = \dot{E}_{mech,1} = \dot{m}_1 e_{mech,1} = \rho V_1 A ke_1 = (1.25\ \text{kg/m}^3)(7\ \text{m/s})(1\ \text{m}^2)(0.0245\ \text{kJ/kg}) = 0.2144\ \text{kW}$$

$$\dot{W}_{max,2} = \dot{E}_{mech,2} = \dot{m}_2 e_{mech,2} = \rho V_2 A ke_2 = (1.25\ \text{kg/m}^3)(10\ \text{m/s})(1\ \text{m}^2)(0.050\ \text{kJ/kg}) = 0.625\ \text{kW}$$

since 1 kW = 1 kJ/s. Then the maximum electric power generations per year become

$$E_{max,1} = \dot{W}_{max,1}\Delta t_1 = (0.2144 \text{ kW})(3000 \text{ h/yr}) = \mathbf{643 \text{ kWh/yr}} \text{ (per m}^2 \text{ flow area)}$$

$$E_{max,2} = \dot{W}_{max,2}\Delta t_2 = (0.625 \text{ kW})(1500 \text{ h/yr}) = \mathbf{938 \text{ kWh/yr}} \text{ (per m}^2 \text{ flow area)}$$

Therefore, **second site** is a better one for wind power generation.

***Discussion*** Note the power generation of a wind turbine is proportional to the cube of the wind velocity, and thus the average wind velocity is the primary consideration in wind power generation decisions.

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**Question 6:** Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in Fig. 2. Determine the pressure difference between the two pipelines. Take the density of seawater at that location to be $\rho$ = 1035 kg/m$^3$ . Can the air column be ignored in the analysis?

**(Ans: 3.39 kPa, Air column can be ignored)**

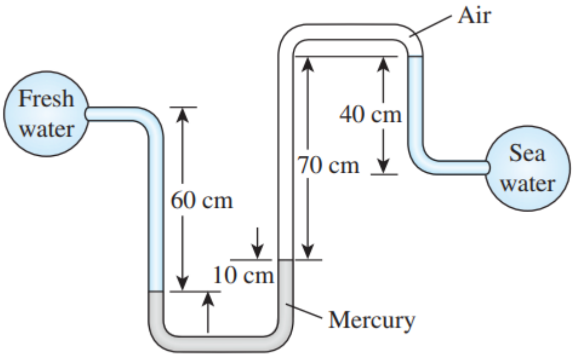
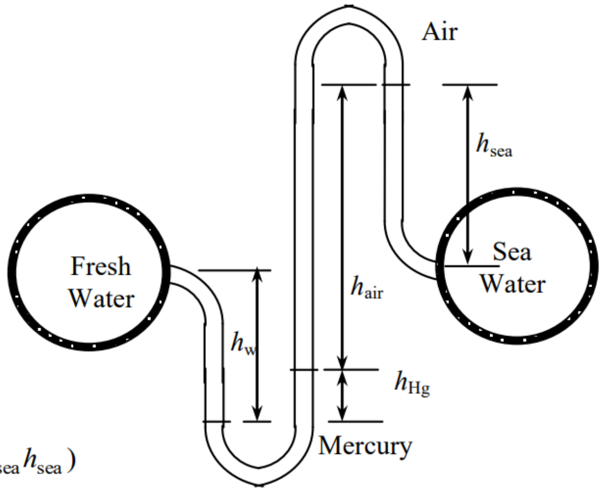



**Fig. 2**

**Solution:**

Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

***Assumptions*** **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.




***Properties*** The densities of seawater and mercury are given to be $\rho_{sea}$ = 1035 kg/m$^3$ and $\rho_{Hg}$ = 13,600 kg/m$^3$. We take the density of water to be $\rho_w$ =1000 kg/m$^3$.

***Analysis*** Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach the sea water pipe (point 2), and setting the result equal to $P_2$ gives

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{air} g h_{air} + \rho_{sea} g h_{sea} = P_2$$

Rearranging and neglecting the effect of air column on pressure,

$$P_1 - P_2 = -\rho_w g h_w + \rho_{Hg} g h_{Hg} - \rho_{sea} g h_{sea} = g(\rho_{Hg} h_{Hg} - \rho_w h_w - \rho_{sea} h_{sea})$$

Substituting,

$$\begin{aligned} P_1 - P_2 &= (9.81\,\text{m/s}^2)[(13600\,\text{kg/m}^3)(0.1\,\text{m}) \\ &\quad - (1000\,\text{kg/m}^3)(0.6\,\text{m}) - (1035\,\text{kg/m}^3)(0.4\,\text{m})]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) \\ &= 3.39\,\text{kN/m}^2 = \mathbf{3.39\,kPa} \end{aligned}$$

Therefore, the pressure in the fresh water pipe is 3.39 kPa higher than the pressure in the sea water pipe.

***Discussion*** A 0.70-m high air column with a density of 1.2 kg/m$^3$ corresponds to a pressure difference of 0.008 kPa. Therefore, its effect on the pressure difference between the two pipes is negligible.

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**Question 7:** A pressure cooker cooks a lot faster than an ordinary pan by maintaining a higher pressure and temperature inside. The lid of a pressure cooker is well sealed, and steam can escape only through an opening in the middle of the lid. A separate metal piece, the petcock, sits on top of this opening and prevents steam from escaping until the pressure force overcomes the weight of the petcock. The periodic escape of the steam in this manner prevents any potentially dangerous pressure buildup and keeps the pressure inside at a constant value. Determine the mass of the petcock of a pressure cooker whose operation pressure is 100 kPa gage and has an opening cross-sectional area of 4 mm$^2$ . Assume an atmospheric pressure of 101 kPa, and draw the free-body diagram of the petcock.

**(Ans: 40.8 g)**

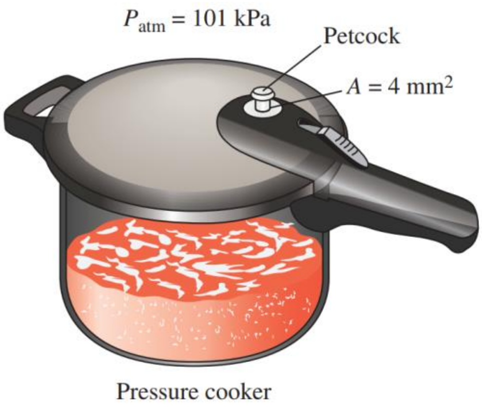


**Fig. 3**

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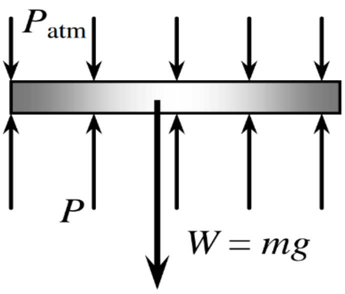
**Solution:** The gage pressure in a pressure cooker is maintained constant at 100 kPa by a petcock. The mass of the petcock is to be determined.




***Assumptions*** There is no blockage of the pressure release valve.

***Analysis*** Atmospheric pressure is acting on all surfaces of the petcock, which balances itself out. Therefore, it can be disregarded in calculations if we use the gage pressure as the cooker pressure. A force balance on the petcock ($\Sigma F_y = 0$) yields

$$W = P_{gage}A$$

$$m = \frac{P_{gage}A}{g} = \frac{(100\ \text{kPa})(4\times10^{-6}\ \text{m}^2)}{9.81\ \text{m/s}^2}\left(\frac{1000\ \text{kg/m}\cdot\text{s}^2}{1\ \text{kPa}}\right)$$

$$= \mathbf{0.0408\ kg}$$

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