CS340 - 2023 Mid-semester exam

DIVYANSH

TOTAL POINTS

10 / 50

QUESTION 1

1 Question 1 **2 / 5**

✓ **+ 0 pts** *Incorrect*

+ 2 *Point adjustment*

Equivalence of states isn't just determined by the action of transition function on a single input symbol. You need to better justify why the states cannot be merged. You also need to argue why this means there is no other clever DFA with fewer states.

QUESTION 2

Question 2 6 pts

2.1 Part A **0 / 2**

✓ **- 2 pts** *Wrong Answer/Wrong Explanation/No Explanation*

2.2 Part B **2 / 2**

✓ **- 0 pts** *Correct*

2.3 Part C **2 / 2**

✓ **+ 2 pts** *Correct*

QUESTION 3

Question 3 8 pts

3.1 Part A **4 / 4**

✓ **- 0 pts** *Correct*

3.2 Part B **0 / 4**

✓ **- 4 pts** *Incorrect Language (other than {a^n|n>0 ,n!=3} or a+) or the answer is wrong*

It is a Regular Language {a^n|n>0,n!=3}

QUESTION 4

4 Question 4 **0 / 8**

✓ **- 8 pts** *If mentioned false or proved not regular.*

QUESTION 5

5 Question 5 **0 / 6**

✓ **- 6 pts** *Incorrect/ Unattempted/ Not Clear*

QUESTION 6

Question 6 8 pts

6.1 Part A **0 / 4**

✓ **- 4 pts** *Incorrect*

6.2 **0 / 4**

✓ **- 4 pts** *Not attempted*

QUESTION 7

7 Question 7 **0 / 9**

✓ **+ 0 pts** *Incorrect*

Incorrect proof. There is no odd position in a

DFA

CS340 (2023) – Mid-semester Exam

Duration: 120 minutes, Total marks: 50, Pages: 12.

- **Important note.** Answers without clear and concise explanations will not be graded.

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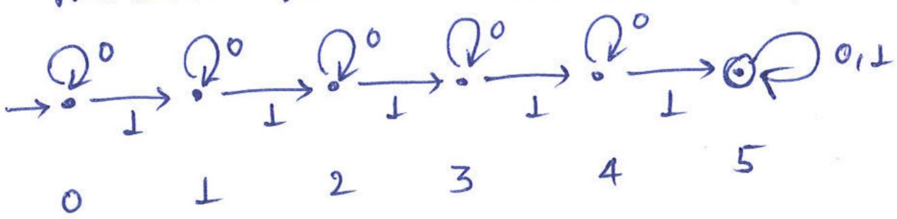
Roll No: 210355

Problems

1. **(5 marks)** Let $\Sigma = \{0, 1\}$ and let $A = \{w \mid w \text{ contains at least five 1s}\}$. Is the following statement true? Precisely justify your answer.

   Statement. Any DFA M where $L(M) = A$ has at least 6 states.

This state is true. Consider the DFA,



Now, given the DFA of 6 states $\{0,1,2,3,4,5\}$ there for any ~~it not~~ two states p_1 and p_2

$\delta(p_1, a) \neq \delta(p_2, a)$ for $a \in \Sigma$. ← eq(i)

and hence no two state is equivalent and can't be collapsed.

eq(i) is valid because '0' alphabet retains the same state.

(visible from the DFA shown).

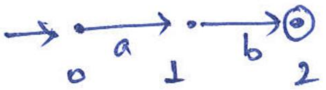
2. **(6 marks)** Are the following statements true? Clearly justify your answer.

(a) Let N be an arbitrary finite automata (NFA or DFA). Let N' be the automaton obtained by swapping the final states and non-final states of N.
Statement 1. $L(N')$ is the complement of $L(N)$.

This statement is true. Let us assume two sets L_1 and L_2 where L_1 is the language of N, and $L_2 = \Sigma^* - L_1$.
Now $\forall x \in L_1$, x reached the final state which is accepted.
But after ~~ch~~ swapping the final states, the non-reaching will be accepted which L_2. Hence $L(N')$ is complement of $L(N)$.

(b) **Statement 2.** If $A \subseteq \Sigma^*$ is not a regular set and $B \subseteq A$ then B is not a regular set.

This statement is False. I can show a counter example, Let's say $\Sigma = \{a, b\}$ and $A = \{a^n b^n, n \geq 0\}$.
We have seen in class that A is not a regular set.
Now, Let's consider this set $B = \{ab\}$. Clearly $B \subseteq A$.
And we can obviously construct a finite automata which accepts B



(c) **Statement 3.** If $A_1 \subseteq \Sigma^*$ is regular, $A_2 \subseteq \Sigma^*$ is not regular, and $A_1 \cap A_2$ is regular, then $A_1 \cup A_2$ is not regular.

~~This true~~ This is true.

Let us assume $A_1 \cup A_2$ is regular.

$\overline{A_1 \cap A_2} = \overline{A_1} \cup \overline{A_2}$ is also regular, and $A_1 \cap A_2$ being regular shows that A_2 is regular, which is a contradiction.

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3. **(8 marks)** Let $\Sigma = \{a\}$ and $A = \{a^{2^n} \mid n \geq 0\}$. Are the following statements true?

(a) **Statement 1.** A^* is regular.
If "True", construct an automaton N such that $L(N) = A^*$. Clearly justify your construction. If "False", prove using pumping lemma that A^* is not regular.

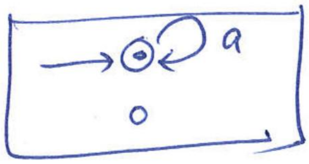
This statement is true. We have, $A = \{a, a^2, a^4, a^8, \ldots, a^{2^i}\}$

Now, $A^* = A^0 \cup A^1 \cup A^2 \cup A^3 \ldots$

We can have, $A = \{a^{2^{i_1} + 2^{i_2} + 2^{i_3} + \ldots 2^{i_k}} \mid k \geq 0\}$.

Hence, $A^* = \{a\}^*$

So, the automata is



(b) **Statement 2.** $A\,\overline{A}$ is regular. (Recall that $A\,\overline{A}$ denotes the concatenation of A with the complement of A).
If "True", construct an automaton N such that $L(N) = A\,\overline{A}$. Clearly justify your construction. If "False", prove using pumping lemma that $A\,\overline{A}$ is not regular.

This statement is false.

$A\bar{A}$ will be of form $\{a^{2^i + j} \mid j \neq 2^k \text{ for any } k\}$.

Let say xyz such that y is a^j

we can have v, vw as y such that $|v| > 0$

Now, we can do v^i such that uv^iw is power of 2.

This will be accepted in lemma but not in $A\bar{A}$.

4. **(8 marks)** Let $\Sigma = \{a, b\}$ and consider the set $A = \{xww^r y \mid x, y \in \Sigma^*, w \in \Sigma^+\}$, where w^r denotes the reverse of the string w. Is the following statement true?

Statement. A is regular.

If "True", construct an automaton N such that $L(N) = A$ and clearly justify your construction.
If "False", prove using pumping lemma that A is not regular.

This statement is False. I will prove this with pumping lemma.

Let as assume A is regular, hence there exists a automata (with let's say K states) which accepts A.

~~consider $xww^r y$ of form xzy where $\boxed{z = ww^r}$. Now we can have z of form $p_1 p_2 p_3$ such that p_1 is ϵ, p_2 is w and p_3 is w^r so $|p_2| > 0$ as $w \in \Sigma^+$.~~

Let consider the strings for which we have $|w| > k$, now,

$$x\, \underbrace{w_0\, w_1\, w_2}_{p_1} - \underbrace{w_i \; w_{n-1}}_{p_2}\, \underbrace{w_n}_{p_3}\; w_n w_{n-1} - w_1 w_0 y$$

since $|w| > k$, there will repetition states and let's say that it is captured in p_2 and rest of w is p_1 and p_3.

Now, all the forms $x p_1 p_2^i p_3 w^r z$ will accepted in the automata but won't be in A since $p_1 p_2^i p_3$ can't be reverse of w^r for all i. (contradiction)

Hence, A is not regular.

5. **(6 marks)** Let $h : \Sigma^* \to \Gamma^*$ be a homomorphism and let $A \subseteq \Sigma^*$. Is the following statement true? Precisely justify your answer.

Statement. If $h(A)$ is regular then A is regular.

This statement is true.

We know that $h(A) = \{ h(x) \mid x \in A \}$. let's say $h(A) = B$.

So given B is regular, we need to show $h^{-1}(B)$ is regular.

Let's the automata accepting B is M. $(Q, \Gamma, \delta, s, F)$ with $L(M) = B$. We need to construct an automata M' such that $L(M') = A$.

We define M' as $(Q', \Sigma, \delta', s', F')$ with $\underline{Q' = Q}$.

$$\delta'(q, a) = \hat{\delta}(q, h(a)) \quad \text{— eq(1)}$$

$$s' = h(\epsilon), \quad F' = F.$$

We can define $\hat{\delta}'$ inductively, by induction on input size $|x|$. base case in eq(1).

Induction step,

$$\hat{\delta}'(q, xa) = \delta'(\hat{\delta}'(q, x), a)$$

(definition of $\hat{\delta}'$)

$$= \hat{\delta}(\hat{\delta}(q, h(x)), h(a)) \quad \text{(Induction hypothesis}$$

$$= \hat{\delta}(q, h(xa))$$

To show language of M' as A, we have,

$x \in L(M') \Rightarrow \hat{\delta}'(s', x) \in F' \Rightarrow \hat{\delta}(s, h(x)) \in F$ (defined above)

(definition)

$\Rightarrow h(x) \in B \Rightarrow \underline{\underline{x \in A}}$.

6. **(8 marks)** Let $\Sigma = \{a, b\}, \Gamma = \{0, 1\}$ and h be a homomorphism defined as follows: $h(a) = 01$, $h(b) = 0$.

$(10+1)^*$

(a) Let $\alpha = (10 \cup 1)^*$ be a regular expression. What is $h^{-1}(L(\alpha))$? Clearly explain your answer.

Given α, $(10+1)^*$, we have $L(\alpha) = \{1, 10\}^*$

(b) For a string $z \in \Gamma^*$ and $c \in \Gamma$, let $\#_c(z)$ denote the number of occurrence of c in z. Let $B = \{w \in \Gamma^* \mid \#_0(w) = \#_1(w)\}$. That is, the set of all strings with equal number of 0s and 1s. What is $h^{-1}(B)$? Clearly explain your answer.

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7. **(9 marks)** For a string $w \in \Sigma^*$, let $even(w)$ denote the string obtained by deleting all symbols that occur in the odd position of w. For example, $even(a) = \epsilon$, $even(ab) = b$, $even(abc) = b$, $even(abcd) = bd$. For $A \subseteq \Sigma^*$, let $even(A) = \{even(w) \mid w \in A\}$. Is the following statement true?

Statement. If A is regular then $even(A)$ is regular.

If "True", construct an automaton N such that $L(N) = even(A)$ and clearly justify your construction. If "False", prove using pumping lemma that $even(A)$ is not regular.

~~This statement is true.~~

~~Given that A is regular~~

~~This statement is false.~~

This statement is true.

If A is regular, ~~what~~ we have the automata M which accepts it as $(Q, \Sigma, \delta, s, F)$. Now construct a automata same M just the transition state δ' will have flag.

$\delta'((q,0), a) = \epsilon$ (epsilon transition for odd position)

$\delta'((q,1), 0) = \delta(q,a)$ (even)

Rough Work

$A = \{ \underline{x w w^r y} \rightarrow \quad x \underbrace{w w^r}_{y} z$

$\underline{w_0\ w_1\ w_2 \ldots}\ \ \underline{w_n}\ \ w_n\ w_{n-1} \ldots w_0$

$\alpha = \underline{(10 \cup 1)}^*$

$h : \Sigma \rightarrow \Gamma^*$

$\underline{\{10\}} \quad \{1\}.$

$\{1, 10\}^*$

b a b ~~b~~ a

$a^{2^i} \cdot a^{2^j}$

a^{2^n} $a^{2^i + 2^j}$

$A = [a, a^2, a^4, a^8, a^{16}, a^{32} \ldots]$

$A^* = A^0 \cup A^1 \cup A^2$

$= [\epsilon, a, a^2, a^3,$

$h(A) = \{ h(x) \mid x \in A \}$

a b c d $\rightarrow$ b d

$A = \{ a, a^2, a^4, a^8 \ldots \}$

$A^0 = \{\epsilon\}$

$A^2 = \{ a^2, a^3,$

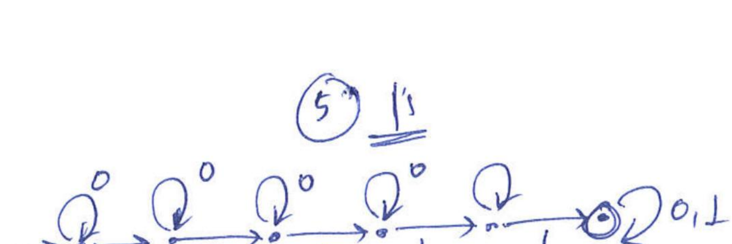
$A = \{a, a^2, a^4, a^8, a^{16} \ldots \}$

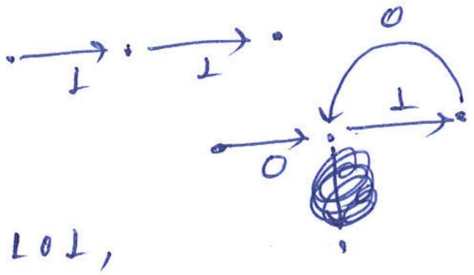
$\bar{A} \; \{ a^3, a^5, a^6, a^7, a^9, a^{10}, a^{11} \ldots \}.$

a a a b a b

L(M) = ene A

5 1's





$\{1, 10\}^*$

$\{\epsilon, 1, 10, 110, 101,$

111 1011...

$\cancel{a^n b^n}, n \geq 0$

$A_1 \subseteq \Sigma^* \quad A_2 \subseteq \Sigma^*$

$$\overline{(A_1 \cap A_2)} = \overline{A_1} \cup \overline{A_2}$$

$$\overline{(A_1 \cup A_2)} = \overline{A_1} \cap \overline{A_2}$$