

Lecture 15 - Some examples of regular sets.

Let $A, B \subseteq \Sigma^*$. Let

$$\text{Shuffle-1}(A, B) = \{w \mid w = a_1 b_1 \dots a_k b_k \text{ where} \\ a_1 a_2 \dots a_k \in A \text{ and } b_1 b_2 \dots b_k \in B \text{ and} \\ \forall i, a_i b_i \in \Sigma\}.$$

Claim 1. if A and B are regular then $\text{Shuffle-1}(A, B)$ is regular.

$$\text{Shuffle-2}(A, B) = \{w \mid w = a_1 b_1 \dots a_k b_k \text{ where} \\ a_1 a_2 \dots a_k \in A \text{ and } b_1 b_2 \dots b_k \in B \text{ and} \\ \forall i, a_i b_i \in \Sigma^*\}$$

Claim 2. if A and B are regular then $\text{Shuffle-2}(A, B)$ is regular.

For $A \subseteq \Sigma^*$ let $\text{cycle}(A) = \{yx \mid xy \in A\}$.

claim. if A is regular then $\text{cycle}(A)$ regular