$DFA = (M, \Sigma, S, \delta, F)$ $NFA = (M, \Sigma, \Delta, S, F)$

- $A \subseteq \Sigma^*$ is regular if $\exists$ DFA M s.t $L(M) = A$.

- For every NFA N, $\exists$ DFA M s.t $L(M) = L(N)$
  (Subset Construction)

- $A \subseteq \Sigma^*$ is regular if $\exists$ NFA N s.t $L(N) = A$.

Pattern Matching - Classify Strings.

> ls $\underbrace{*.pdf}_{\text{pattern } \gamma}$ - represents files with pdf extension.

$$L(\gamma) = \{x \mid x \text{ matches } \gamma\}$$

> ls ab*.pdf

Atomic Pattern

Compound pattern.

Σ - alphabet set

Atomic Patterns.

Syntax	Semantics : $L(\gamma) \subseteq \Sigma^*$.
$a \in \Sigma$	$L(a) = \{a\}$
ϵ	$L(\epsilon) = \{\epsilon\}$.
ϕ	$L(\phi) = \phi$ - matches nothing.
#	$L(\#) = \Sigma$
@	$L(@) = \Sigma^*$

Compound Patterns.

Syntax.	Semantics
$\alpha + \beta$	$L(\alpha+\beta) = L(\alpha) \cup L(\beta)$
$\alpha \cap \beta$	$L(\alpha \cap \beta) = L(\alpha) \cap L(\beta)$
$\alpha\beta$	$L(\alpha\beta) = L(\alpha)L(\beta)$

$$= \{xy \mid x \in L(\alpha) \text{ and } y \in L(\beta)\}.$$

α^*

$$L(\alpha^*) = L(\alpha)^0 \cup L(\alpha)^1 \cup L(\alpha)^2 \cup \cdots$$

$$= \{x_1 x_2 \cdots x_n \mid n \geq 0;\ x_i \in L(\alpha),\ 1 \leq i \leq n\}$$

α^+

$$L(\alpha^+) = L(\alpha)^1 \cup L(\alpha)^2 \cup \cdots$$

$$= L(\alpha)^+$$

$\neg\alpha$

$$L(\neg\alpha) = \overline{L(\alpha)} = \Sigma^* - L(\alpha)$$

Some questions .

- Given a string x and a pattern α, how hard is it to determine if $x \in L(\alpha)$.

- Can you represent every set by some pattern?

- Patterns α and β are equivalent if $L(\alpha) = L(\beta)$
  Is it possible to check equivalence of patterns?

- $\epsilon \equiv \neg(\# @)$

- $@ \equiv \#^*$

- $\alpha^+ \equiv \alpha\alpha^*$

- $\# \equiv a_1 + a_2 + \cdots + a_n$ where $\Sigma = \{a_1, a_2, \cdots a_n\}$.

By De Morgan's law $\alpha \cap \beta \equiv \neg(\neg\alpha + \neg\beta)$

Regular Expression

Atomic Patterns.

$a \in \Sigma$

ϵ

ϕ

Compound Patterns

$+$

$\cdot$

$*$

Patterns constructed from the above syntax are called Regular Expressions.

Precedence : $* > \cdot > +$ (left to right)

Ex: $\alpha + \beta\gamma \longrightarrow \alpha + (\beta\gamma)$ and not $(\alpha+\beta)\gamma$

$\alpha\beta^* \longrightarrow \alpha(\beta^*)$ and not $(\alpha\beta)^*$

Note: use parantesis.

Theorem. Let $A \subseteq \Sigma^*$. The following statements are equivalent.

1. A is regular. $\exists$ a finite automaton M s.t $L(M) = A$.

2. $A = L(\alpha)$ for some pattern α.

3. $A = L(\alpha)$ for some regular expression α.

Proof. $3 \Rightarrow 2$ is trivial.

$2 \Rightarrow 1$

$1 \Rightarrow 3$: To convert an automata M to an equivalent regular expression

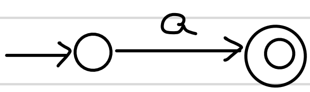
Theorem. Let $A \subseteq \Sigma^*$. The following statements are equivalent.

1. A is regular. $\exists$ a finite automaton M s.t. $L(M) = A$.
2. $A = L(\alpha)$ for some pattern α.
3. $A = L(\alpha)$ for some regular expression α.

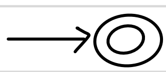
Proof $2 \Rightarrow 1$.

Atomic pattern	Compound pattern
$a \in \Sigma$	$\beta + \gamma$
ϵ	$\beta \cap \gamma$
ϕ	$\beta\gamma$
	β^*
$\left.\begin{array}{l}\# \\ @\end{array}\right\}$ Redundant	β^+
	$\neg\beta$

$a \in \Sigma \quad L(a) = \{a\}$

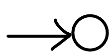


$\epsilon: L(\epsilon) = \{\epsilon\}$



Regular

$\phi: L(\phi) = \phi$



$\beta+\gamma$: $L(\beta+\gamma) = L(\beta) \cup L(\gamma)$ (Definition)

By IH $L(\beta)$ is regular and $L(\gamma)$ is regular.

Regular sets are closed under union
So $L(\beta+\gamma)$ is regular.

$\beta \cap \gamma$: $L(\beta \cap \gamma) = L(\beta) \cap L(\gamma)$ (Definition)

By IH $L(\beta)$ is regular and $L(\gamma)$ is regular

Regular sets are closed under intersection
So $L(\beta \cap \gamma)$ is regular

$\beta\gamma$: $L(\beta\gamma) = L(\beta) L(\gamma)$ (Definition)

By IH, $L(\beta)$ and $L(\gamma)$ are regular.

Regular sets are closed under concatenation
So $L(\beta\gamma)$ is regular.

β^* : $L(\beta^*) = L(\beta)^*$ (Defn.)
By IH, $L(\beta)$ is regular; Regular sets are closed under *
So $L(\beta^*)$ is regular.

$\neg\beta$: $L(\neg\beta) = \overline{L(\beta)}$ (Defn)
By IH $L(\beta)$ is regular.
Regular sets are closed under complementation
So $L(\neg\beta)$ is regular.