INDIAN INSTITUTE OF TECHNOLOGY KANPUR

ESO 201A: Thermodynamics

(2023-24 I Semester)

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Tutorial 7

Question 1: A well-insulated rigid tank contains 3 kg of a saturated liquid–vapor mixture of water at 200 kPa. Initially, three quarters of the mass is in the liquid phase. An electric resistance heater placed in the tank is now turned on and kept on until all the liquid in the tank is vaporized. Determine the entropy change of the steam during this process. **(Ans: 11.1 kJ/K)**

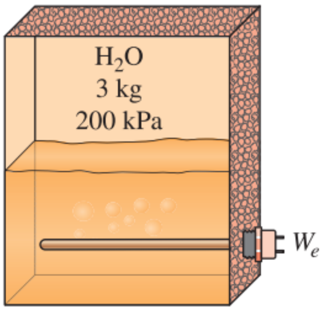
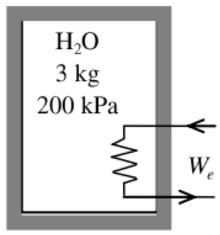


Fig. 1

Analysis From the steam tables (Tables A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 200 \text{ kPa} \\ x_1 = 0.25 \end{array}\right\} \begin{array}{l} \upsilon_1 = \upsilon_f + x_1\upsilon_{fg} = 0.001061 + (0.25)(0.88578 - 0.001061) = 0.22224 \text{ m}^3/\text{kg} \\ s_1 = s_f + x_1 s_{fg} = 1.5302 + (0.25)(5.5968) = 2.9294 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} \upsilon_2 = \upsilon_1 = 0.22224 \text{ m}^3/\text{kg} \\ x_2 = 1 \text{ (sat. vapor)} \end{array}\right\} s_2 = 6.6335 \text{ kJ/kg}\cdot\text{K}$$



Then the entropy change of the steam becomes

$$\Delta S = m(s_2 - s_1) = (3 \text{ kg})(6.6335 - 2.9294) \text{ kJ/kg}\cdot\text{K} = \mathbf{11.1 \text{ kJ/K}}$$

Question 2: A piston–cylinder device contains 5 kg of steam at 100°C with a quality of 50 percent. This steam undergoes two processes as follows:

1-2: Heat is transferred to the steam in a reversible manner while the temperature is held constant until the steam exists as a saturated vapor.

2-3: The steam expands in an adiabatic, re versible process until the pressure is 15 kPa.

(a) Sketch these processes with respect to the saturation lines on a single T-s diagram.

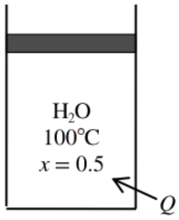
(b) Determine the heat transferred to the steam in process 1-2, in kJ.

(c) Determine the work done by the steam in process 2-3, in kJ.

(Ans: (b) 5641 kJ (c) 1291 kJ)

Assumptions **1** The kinetic and potential energy changes are negligible. **2** The thermal energy stored in the cylinder itself is negligible. **3** Both processes are reversible.

Analysis (b) From the steam tables (Tables A-4 through A-6),



$$\left.\begin{array}{l} T_1 = 100°\text{C} \\ x = 0.5 \end{array}\right\} h_1 = h_f + xh_{fg} = 419.17 + (0.5)(2256.4) = 1547.4 \text{ kJ/kg}$$

$$\left.\begin{array}{l} T_2 = 100°\text{C} \\ x_2 = 1 \end{array}\right\} \begin{array}{l} h_2 = h_g = 2675.6 \text{ kJ/kg} \\ u_2 = u_g = 2506.0 \text{ kJ/kg} \\ s_2 = s_g = 7.3542 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_3 = 15 \text{ kPa} \\ s_3 = s_2 \end{array}\right\} u_3 = 2247.9 \text{ kJ/kg}$$

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

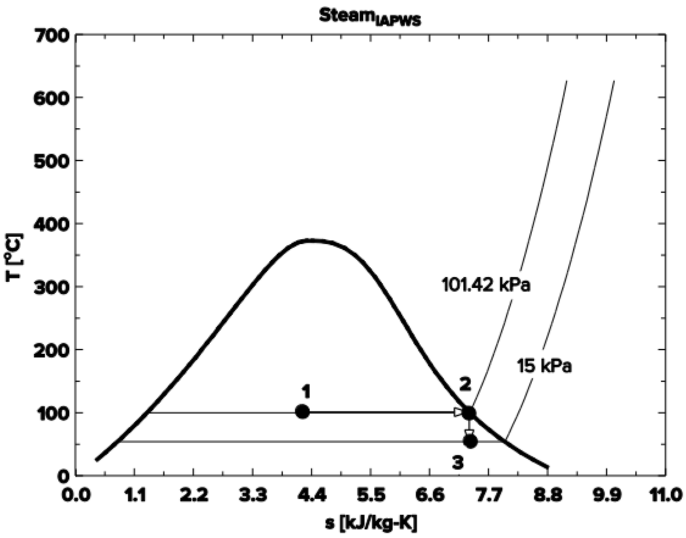
$$Q_{\text{in}} - W_{\text{b,out}} = \Delta U = m(u_2 - u_1)$$

For process 1-2, it reduces to

$$Q_{12,\text{in}} = m(h_2 - h_1) = (5 \text{ kg})(2675.6 - 1547.4)\text{kJ/kg} = \mathbf{5641 \text{ kJ}}$$

(c) For process 2-3, it reduces to

$$W_{23,\text{b,out}} = m(u_2 - u_3) = (5 \text{ kg})(2506.0 - 2247.9)\text{kJ/kg} = \mathbf{1291 \text{ kJ}}$$



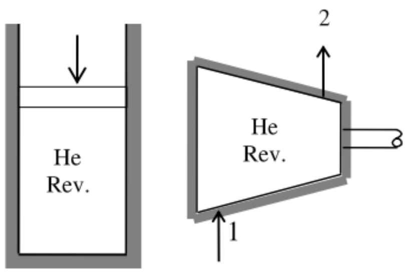
Question 3: Helium gas is compressed from 90 kPa and 30°C to 450 kPa in a reversible, adiabatic process. Determine the final temperature and the work done, assuming the process takes place (a) in a piston–cylinder device and (b) in a steady-flow compressor. **(Ans: (a) 576.9 K, 853.4 kJ/kg (b) 1422.3 kJ/kg)**

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** The process is given to be reversible and adiabatic, and thus isentropic. Therefore, isentropic relations of ideal gases apply.

Properties The specific heats and the specific heat ratio of helium are c_v = 3.1156 kJ/kg.K, c_p = 5.1926 kJ/kg.K, and k = 1.667 (Table A-2).

Analysis (*a*) From the ideal gas isentropic relations,

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (303\text{ K})\left(\frac{450\text{ kPa}}{90\text{ kPa}}\right)^{0.667/1.667} = \mathbf{576.9\text{ K}}$$



(*a*) We take the air in the cylinder as the system. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{\text{in}} = \Delta U = m(u_2 - u_1) \cong mc_v(T_2 - T_1)$$

Thus,

$$w_{\text{in}} = c_v(T_2 - T_1) = (3.1156\text{ kJ/kg}\cdot\text{K})(576.9 - 303)\text{K} = \mathbf{853.4\text{ kJ/kg}}$$

(*b*) If the process takes place in a steady-flow device, the final temperature will remain the same but the work done should be determined from an energy balance on this steady-flow device,

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) \cong \dot{m}c_p(T_2 - T_1)$$

Thus,

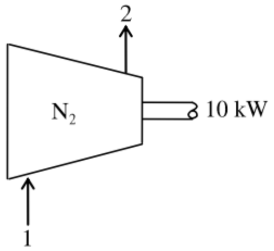
$$w_{\text{in}} = c_p(T_2 - T_1) = (5.1926\text{ kJ/kg}\cdot\text{K})(576.9 - 303)\text{K} = \mathbf{1422.3\text{ kJ/kg}}$$

Question 4: Nitrogen gas is compressed from 80 kPa and 27°C to 480 kPa by a 10-kW compressor. Determine the mass flow rate of nitrogen through the compressor, assuming the compression process to be (a) isentropic, (b) polytropic with n = 1.3, (c) isothermal, and (d) ideal two-stage polytropic with n = 1.3. **(Ans: (a) 0.048 kg/s (b) 0.051 kg/s (c) 0.063 kg/s (d) 0.056 kg/s)**

Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** The process is reversible. **3** Kinetic and potential energy changes are negligible.

Properties The gas constant of nitrogen is R = 0.297 kJ/kg.K (Table A-1). The specific heat ratio of nitrogen is k = 1.4 (Table A-2).

Analysis (*a*) Isentropic compression:



$$\dot{W}_{\text{comp, in}} = \dot{m}\frac{kRT_1}{k-1}\left\{\left(P_2/P_1\right)^{(k-1)/k} - 1\right\}$$

or,

$$10 \text{ kJ/s} = \dot{m}\frac{(1.4)(0.297 \text{ kJ/kg}\cdot\text{K})(300 \text{ K})}{1.4-1}\left\{\left(480 \text{ kPa}/80 \text{ kPa}\right)^{0.4/1.4} - 1\right\}$$

It yields

$$\dot{m} = \mathbf{0.048 \text{ kg/s}}$$

(*b*) Polytropic compression with n = 1.3:

$$\dot{W}_{\text{comp, in}} = \dot{m}\frac{nRT_1}{n-1}\left\{\left(P_2/P_1\right)^{(n-1)/n} - 1\right\}$$

or,

$$10 \text{ kJ/s} = \dot{m}\frac{(1.3)(0.297 \text{ kJ/kg}\cdot\text{K})(300 \text{ K})}{1.3-1}\left\{\left(480 \text{ kPa}/80 \text{ kPa}\right)^{0.3/1.3} - 1\right\}$$

It yields

$$\dot{m} = \mathbf{0.051 \text{ kg/s}}$$

(*c*) Isothermal compression:

$$\dot{W}_{\text{comp, in}} = \dot{m}RT\ln\frac{P_1}{P_2} \longrightarrow 10 \text{ kJ/s} = \dot{m}(0.297 \text{ kJ/kg}\cdot\text{K})(300 \text{ K}) \ \ln\left(\frac{480 \text{ kPa}}{80 \text{ kPa}}\right)$$

It yields

$$\dot{m} = \mathbf{0.063 \text{ kg/s}}$$

(*d*) Ideal two-stage compression with intercooling (n = 1.3): In this case, the pressure ratio across each stage is the same, and its value is determined to be

$$P_x = \sqrt{P_1P_2} = \sqrt{(80 \text{ kPa})(480 \text{ kPa})} = 196 \text{ kPa}$$

The compressor work across each stage is also the same, thus total compressor work is twice the compression work for a single stage:

$$\dot{W}_{\text{comp,in}} = 2\dot{m}w_{\text{comp, I}} = 2\dot{m}\frac{nRT_1}{n-1}\left\{\left(P_x/P_1\right)^{(n-1)/n} - 1\right\}$$

or,

$$10 \text{ kJ/s} = 2\dot{m}\frac{(1.3)(0.297 \text{ kJ/kg}\cdot\text{K})(300 \text{ K})}{1.3-1}\left\{\left(196 \text{ kPa}/80 \text{ kPa}\right)^{0.3/1.3} - 1\right\}$$

It yields

$$\dot{m} = \mathbf{0.056 \text{ kg/s}}$$

Question 5: Hot combustion gases enter the nozzle of a turbojet engine at 260 kPa, 747°C, and 80 m/s, and they exit at a pressure of 85 kPa. Assuming an isentropic efficiency of 92 percent and treating the combustion gases as air, determine (a) the exit velocity and (b) the exit temperature. **(Ans: (a) 728 m/s (b) 786 K)**

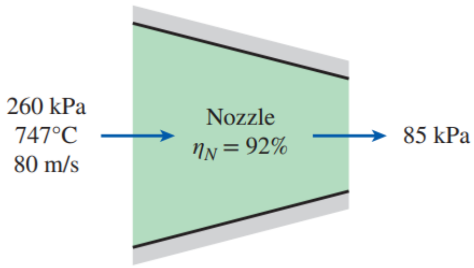
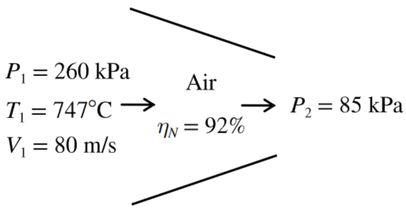


Fig. 2

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Combustion gases can be treated as air that is an ideal gas with variable specific heats.

Analysis From the air table (Table A-17),



$$T_1 = 1020 \text{ K} \longrightarrow h_1 = 1068.89 \text{ kJ/kg}, \quad P_{r_1} = 123.4$$

From the isentropic relation,

$$P_{r_2} = \left(\frac{P_2}{P_1}\right) P_{r_1} = \left(\frac{85 \text{ kPa}}{260 \text{ kPa}}\right)(123.4) = 40.34 \longrightarrow h_{2s} = 783.92 \text{ kJ/kg}$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system for the isentropic process can be expressed as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}{}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_{2s} + V_{2s}^2/2) \quad (\text{since } \dot{W} = \dot{Q} \cong \Delta\text{pe} \cong 0)$$

$$h_{2s} = h_1 - \frac{V_{2s}^2 - V_1^2}{2}$$

Then the isentropic exit velocity becomes

$$V_{2s} = \sqrt{V_1^2 + 2(h_1 - h_{2s})} = \sqrt{(80 \text{ m/s})^2 + 2(1068.89 - 783.92) \text{ kJ/kg} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 759.2 \text{ m/s}$$

Therefore,

$$V_{2a} = \sqrt{\eta_N} V_{2s} = \sqrt{0.92}(759.2 \text{ m/s}) = 728.2 \text{ m/s} \cong \mathbf{728\ m/s}$$

The exit temperature of air is determined from the steady-flow energy equation,

$$h_{2a} = 1068.89 \text{ kJ/kg} - \frac{(728.2 \text{ m/s})^2 - (80 \text{ m/s})^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 806.95 \text{ kJ/kg}$$

From the air table we read

$$T_{2a} = \mathbf{786\ K}$$