INDIAN INSTITUTE OF TECHNOLOGY KANPUR

ESO 201A: Thermodynamics

(2023-24 I Semester)

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Tutorial 11

Question 1:

The gas-turbine portion of a combined gas–steam power plant has a pressure ratio of 16. Air enters the compressor at 300 K at a rate of 14 kg/s and is heated to 1500 K in the combustion chamber. The combustion gases leaving the gas turbine are used to heat the steam to 400°C at 10 MPa in a heat exchanger. The combustion gases leave the heat exchanger at 420 K. The steam leaving the turbine is condensed at 15 kPa. Assuming all the compression and expansion processes to be isentropic, determine the mass flow rate of the steam, **(Ans. 1.275 kg/s)**

Solution:

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $c_p = 1.005$ kJ/kg·K and $k = 1.4$ (Table A-2).

Analysis (*a*) The analysis of gas cycle yields

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = (300\text{ K})(16)^{0.4/1.4} = 662.5\text{ K}$$

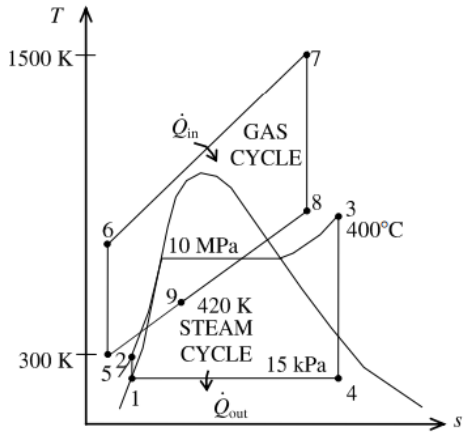
$$\dot{Q}_{\text{in}} = \dot{m}_{\text{air}}(h_7 - h_6) = \dot{m}_{\text{air}}c_p(T_7 - T_6)$$
$$= (14\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(1500 - 662.5)\text{K} = 11{,}784\text{ kW}$$

$$\dot{W}_{C,\text{gas}} = \dot{m}_{\text{air}}(h_6 - h_5) = \dot{m}_{\text{air}}c_p(T_6 - T_5)$$
$$= (14\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(662.5 - 300)\text{K} = 5100\text{ kW}$$

$$T_8 = T_7\left(\frac{P_8}{P_7}\right)^{(k-1)/k} = (1500\text{ K})\left(\frac{1}{16}\right)^{0.4/1.4} = 679.3\text{ K}$$

$$\dot{W}_{T,\text{gas}} = \dot{m}_{\text{air}}(h_7 - h_8) = \dot{m}_{\text{air}}c_p(T_7 - T_8)$$
$$= (14\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(1500 - 679.3)\text{ K} = 11{,}547\text{ kW}$$

$$\dot{W}_{\text{net, gas}} = \dot{W}_{T,\text{gas}} - \dot{W}_{C,\text{gas}} = 11{,}547 - 5{,}100 = 6447\text{ kW}$$



From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f\,@\,15\text{ kPa}} = 225.94\text{ kJ/kg}$$
$$v_1 = v_{f\,@\,15\text{ kPa}} = 0.001014\text{ m}^3\text{/kg}$$

$$w_{\text{pI, in}} = v_1(P_2 - P_1) = (0.001014\text{ m}^3\text{/kg})(10{,}000 - 15\text{ kPa})\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) = 10.12\text{ kJ/kg}$$
$$h_2 = h_1 + w_{\text{pI, in}} = 225.94 + 10.13 = 236.06\text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 10\text{ MPa} \\ T_3 = 400°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3097.0\text{ kJ/kg} \\ s_3 = 6.2141\text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 15\text{ kPa} \\ s_4 = s_3 \end{array}\right\} \begin{array}{l} x_4 = \dfrac{s_4 - s_f}{s_{fg}} = \dfrac{6.2141 - 0.7549}{7.2522} = 0.7528 \\ h_4 = h_f + x_4 h_{fg} = 225.94 + (0.7528)(2372.3) = 2011.8\text{ kJ/kg} \end{array}$$

Noting that $\dot{Q} \cong \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0$ for the heat exchanger, the steady-flow energy balance equation yields

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta\dot{E}_{\text{system}}^{\nearrow 0\text{ (steady)}} = 0 \longrightarrow \dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e \longrightarrow \dot{m}_s(h_3 - h_2) = \dot{m}_{\text{air}}(h_8 - h_9)$$

$$\dot{m}_s = \frac{h_8 - h_9}{h_3 - h_2}\dot{m}_{\text{air}} = \frac{c_p(T_8 - T_9)}{h_3 - h_2}\dot{m}_{\text{air}} = \frac{(1.005\text{ kJ/kg}\cdot\text{K})(679.3 - 420)\text{ K}}{(3097.0 - 236.06)\text{ kJ/kg}}(14\text{ kg/s}) = \mathbf{1.275\text{ kg/s}}$$

Question 2: A refrigerator uses refrigerant-134a as the working fluid and operates on the vapor-compression refrigeration cycle. The evaporator and condenser pressures are 200 kPa and 1400 kPa, respectively. The isentropic efficiency of the compressor is 88 percent. The refrigerant enters the compressor at a rate of 0.025 kg/s superheated by 10.1°C and leaves the condenser subcooled by 4.4°C. Determine (a) the rate of cooling provided by the evaporator, the power input, and the COP. Determine (b) the same parameters if the cycle operated on the ideal vapor-compression refrigeration cycle between the same pressure limits. **(Ans.(a) 3.317 kW, 1.218 kW and 2.723 (b) 2.931 kW, 1.016 kW and 2.885)**

Solution:

A vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The rate of cooling, the power input, and the COP are to be determined. Also, the same parameters are to be determined if the cycle operated on the ideal vapor-compression refrigeration cycle between the same pressure limits.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (a) From the refrigerant-134a tables (Tables A-11 through A-13)

$$T_{\text{sat@200 kPa}} = -10.1°\text{C}$$

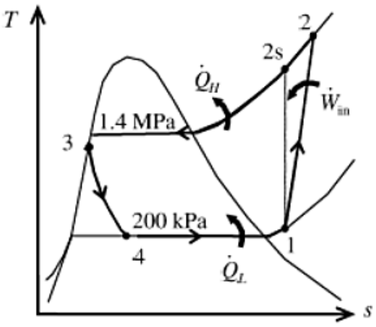
$$\left.\begin{array}{l} P_1 = 200 \text{ kPa} \\ T_1 = -10.1 + 10.1 = 0°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 253.07 \text{ kJ/kg} \\ s_1 = 0.9699 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 1400 \text{ kPa} \\ s_2 = s_1 \end{array}\right\} h_{2s} = 295.95 \text{ kJ/kg}$$

$$T_{\text{sat@1400 kPa}} = 52.4°\text{C}$$

$$\left.\begin{array}{l} P_3 = 1400 \text{ kPa} \\ T_3 = 52.4 - 4.4 = 48°\text{C} \end{array}\right\} h_3 \cong h_{f@48°\text{C}} = 120.41 \text{ kJ/kg}$$

$$h_4 = h_3 = 120.41 \text{ kJ/kg}$$



$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$0.88 = \frac{295.95 - 253.07}{h_2 - 253.07} \longrightarrow h_2 = 301.80 \text{ kJ/kg}$$

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.025 \text{ kg/s})(253.07 - 120.41) = \mathbf{3.317 \text{ kW}}$$

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.025 \text{ kg/s})(301.80 - 120.41) = \mathbf{4.535 \text{ kW}}$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.025 \text{ kg/s})(301.80 - 253.07) = \mathbf{1.218 \text{ kW}}$$

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{3.317 \text{ kW}}{1.218 \text{ kW}} = \mathbf{2.723}$$

(b) Ideal vapor-compression refrigeration cycle solution

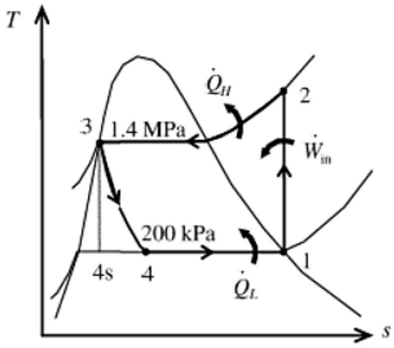
From the refrigerant-134a tables (Tables A-11 through A-13)

$$\left.\begin{array}{l} P_1 = 200 \text{ kPa} \\ x_1 = 1 \end{array}\right\} \begin{array}{l} h_1 = 244.50 \text{ kJ/kg} \\ s_1 = 0.93788 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 1400 \text{ kPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 285.13 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 1400 \text{ kPa} \\ x_3 = 0 \end{array}\right\} h_3 = 127.25 \text{ kJ/kg}$$

$$h_4 = h_3 = 127.25 \text{ kJ/kg}$$



$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.025 \text{ kg/s})(244.50 - 127.25) = \mathbf{2.931 \text{ kW}}$$

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.025 \text{ kg/s})(285.13 - 127.25) = \mathbf{3.947 \text{ kW}}$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.025 \text{ kg/s})(285.13 - 244.50) = \mathbf{1.016 \text{ kW}}$$

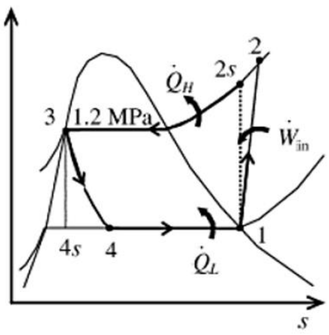
$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{2.931 \text{ kW}}{1.016 \text{ kW}} = \mathbf{2.885}$$

Question 3: A room is kept at −5°C by a vapor-compression refrigeration cycle with R-134a as the refrigerant. Heat is rejected to cooling water that enters the condenser at 20°C at a rate of 0.13 kg/s and leaves at 28°C. The refrigerant enters the condenser at 1.2 MPa and 50°C and leaves as a saturated liquid. If the compressor consumes 1.9 kW of power, determine (a) the COP, (b) the second-law efficiency of the refrigerator. Take T_0 = 20°C and C_p, water = 4.18 kJ/kg·°C. **(Ans. (a) 1.29 (b) 12%)**

Solution:

A vapor-compression refrigeration cycle is used to keep a space at a low temperature. The mass flow rate of R-134a, the COP, The exergy destruction in each component and the exergy efficiency of the compressor, the second-law efficiency, and the exergy destruction are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.



Analysis (a) The properties of R-134a are (Tables A-11 through A-13)

$$\left.\begin{array}{l} P_2 = 1.2 \text{ MPa} \\ T_2 = 50°\text{C} \end{array}\right\} \begin{array}{l} h_2 = 278.28 \text{ kJ/kg} \\ s_2 = 0.9268 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_3 = 1.2 \text{ MPa} \\ x_3 = 0 \end{array}\right\} \begin{array}{l} h_3 = 117.79 \text{ kJ/kg} \\ s_3 = 0.4245 \text{ kJ/kg}\cdot\text{K} \end{array}$$

The rate of heat transferred to the water is the energy change of the water from inlet to exit

$$\dot{Q}_H = \dot{m}_w c_p (T_{w,2} - T_{w,1}) = (0.13 \text{ kg/s})(4.18 \text{ kJ/kg}\cdot°\text{C})(28-20)°\text{C} = 4.347 \text{ kW}$$

The energy decrease of the refrigerant is equal to the energy increase of the water in the condenser. That is,

$$\dot{Q}_H = \dot{m}_R (h_2 - h_3) \longrightarrow \dot{m}_R = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{4.347 \text{ kW}}{(278.28 - 117.79) \text{ kJ/kg}} = 0.02709 \text{ kg/s}$$

The refrigeration load is

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{in} = 4.347 - 1.9 = 2.447 \text{ kW}$$

The COP of the refrigerator is determined from its definition,

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{2.447 \text{ kW}}{1.9 \text{ kW}} = \mathbf{1.29}$$

(*b*) The COP of a reversible refrigerator operating between the same temperature limits is

$$\text{COP}_{\text{Carnot}} = \frac{T_L}{T_H - T_L} = \frac{-5+273}{(20+273)-(-5+273)} = 10.72$$

The minimum power input to the compressor for the same refrigeration load would be

$$\dot{W}_{\text{in, min}} = \frac{\dot{Q}_L}{\text{COP}_{\text{Carnot}}} = \frac{2.447 \text{ kW}}{10.72} = 0.2283 \text{ kW}$$

The second-law efficiency of the cycle is

$$\eta_{II} = \frac{\dot{W}_{\text{in, min}}}{\dot{W}_{in}} = \frac{0.2283}{1.9} = 0.120 = \mathbf{12.0\%}$$

Question 4: An ideal vapor-compression refrigeration cycle that uses refrigerant-134a as its working fluid maintains a condenser at 800 kPa and the evaporator at −20°C. Determine this system's COP and the amount of power required to service a 150 kW cooling load. **(Ans. 3.83 and 39.15 kW)**

Solution:

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

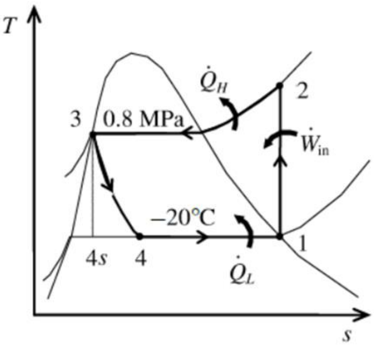
Analysis In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-11, A-12, and A-13),

$$\left.\begin{array}{l} T_1 = -20°\text{C} \\ \text{sat. vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g@-20°\text{C}} = 238.43 \text{ kJ/kg} \\ s_1 = s_{g@-20°\text{C}} = 0.94575 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_2 = 0.8 \text{ MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 275.74 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 0.8 \text{ MPa} \\ \text{sat. liquid} \end{array}\right\} h_3 = h_{f@0.8\text{ Mpa}} = 95.48 \text{ kJ/kg}$$

$$h_4 \cong h_3 = 95.48 \text{ kJ/kg (throttling)}$$



The mass flow rate of the refrigerant is

$$\dot{Q}_L = \dot{m}(h_1 - h_4) \longrightarrow \dot{m} = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{150 \text{ kJ/s}}{(238.43 - 95.48) \text{ kJ/kg}} = 1.049 \text{ kg/s}$$

The power requirement is

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (1.049 \text{ kg/s})(275.74 - 238.43) \text{ kJ/kg} = \mathbf{39.15 \text{ kW}}$$

The COP of the refrigerator is determined from its definition,

$$\text{COP}_\text{R} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{150 \text{ kW}}{39.15 \text{ kW}} = \mathbf{3.83}$$

Question 5: Consider air at 350 K and 0.75 m^3/kg. Determine the change in pressure corresponding to an increase of (a) 1 percent in temperature at constant specific volume,(b) 1 percent in specific volume at constant temperature, and (c) 1 percent in both the temperature and specific volume. **(Ans. (a) 1.339 kPa, (b) -1.339 kPa, (c) 0 kPa)**

Solution:

Air at a specified temperature and specific volume is considered. The changes in pressure corresponding to a certain increase of different properties are to be determined.

Assumptions Air is an ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa·m³/kg·K (Table A-1).

Analysis An ideal gas equation can be expressed as $P = RT/\upsilon$. Noting that R is a constant and $P = P(T, \upsilon)$,

$$dP = \left(\frac{\partial P}{\partial T}\right)_\upsilon dT + \left(\frac{\partial P}{\partial \upsilon}\right)_T dv = \frac{RdT}{\upsilon} - \frac{RT\,d\upsilon}{\upsilon^2}$$

(*a*) The change in T can be expressed as $dT \cong \Delta T = 350 \times 0.01 = 3.5$ K. At υ = constant,

$$(dP)_\upsilon = \frac{RdT}{\upsilon} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(3.5\ \text{K})}{0.75\ \text{m}^3/\text{kg}} = \mathbf{1.339\ kPa}$$

(*b*) The change in υ can be expressed as $d\upsilon \cong \Delta\upsilon = 0.75 \times 0.01 = 0.0075$ m³/kg. At T = constant,

$$(dP)_T = -\frac{RT\,d\upsilon}{\upsilon^2} = -\frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(350\ \text{K})(0.0075\ \text{m}^3/\text{kg})}{(0.75\ \text{m}^3/\text{kg})^2} = \mathbf{-1.339\ kPa}$$

(*c*) When both υ and T increases by 1%, the change in P becomes

$$dP = (dP)_\upsilon + (dP)_T = 1.339 + (-1.339) = \mathbf{0}$$

Thus the changes in T and υ balance each other.

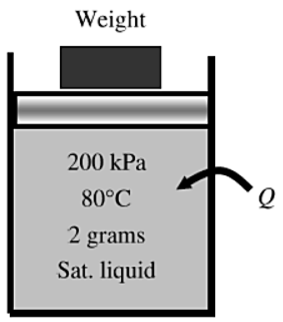
Question 6: Two grams of a saturated liquid are converted to a saturated vapor by being heated in a weighted piston–cylinder device arranged to maintain the pressure at 200 kPa. During the phase conversion, the volume of the system increases by 1000 cm³, 5 kJ of heat is required, and the temperature of the substance stays constant at 80°C. Estimate the saturation pressure P_{sat} of this substance when its temperature is 100°C. **(Ans. 483.2 kPa)**

Solution:

A substance is heated in a piston-cylinder device until it turns from saturated liquid to saturated vapor at a constant pressure and temperature. The saturation pressure of this substance at a different temperature is to be estimated.

Analysis From the Clapeyron equation,

$$\left(\frac{dP}{dT}\right)_{\text{sat}} = \frac{h_{fg}}{T\upsilon_{fg}} = \frac{(5\ \text{kPa}\cdot\text{m}^3)/(0.002\ \text{kg})}{(353\ \text{K})(1\times10^{-3}\ \text{m}^3)/(0.002\ \text{kg})} = 14.16\ \text{kPa/K}$$



Using the finite difference approximation,

$$\left(\frac{dP}{dT}\right)_{\text{sat}} \approx \left(\frac{P_2 - P_1}{T_2 - T_1}\right)_{\text{sat}}$$

Solving for P_2,

$$P_2 = P_1 + \frac{dP}{dT}(T_2 - T_1) = 200\ \text{kPa} + (14.16\ \text{kPa/K})(373 - 353)\text{K} = \mathbf{483.2\ kPa}$$

Question 7: Consider a gas whose equation of state is P(v – a) = RT, where a is a positive constant. Is it possible to cool this gas by throttling? **(Ans. No)**

Solution:

The equation of state of a gas is given to be $P(\upsilon-a) = RT$. It is to be determined if it is possible to cool this gas by throttling.

Analysis The equation of state of this gas can be expressed as

$$\upsilon = \frac{RT}{P} + a \longrightarrow \left(\frac{\partial \upsilon}{\partial T}\right)_P = \frac{R}{P}$$

Substituting into the Joule-Thomson coefficient relation,

$$\mu = -\frac{1}{c_p}\left(\upsilon - T\left(\frac{\partial \upsilon}{\partial T}\right)_P\right) = -\frac{1}{c_p}\left(\upsilon - T\frac{R}{P}\right) = -\frac{1}{c_p}(\upsilon - \upsilon + a) = -\frac{a}{c_p} < 0$$

Therefore, this gas **cannot** be cooled by throttling since μ is always a negative quantity.