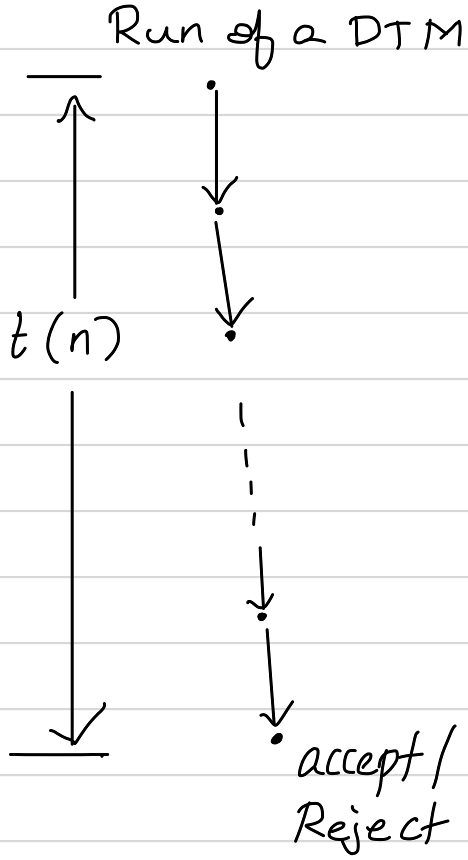
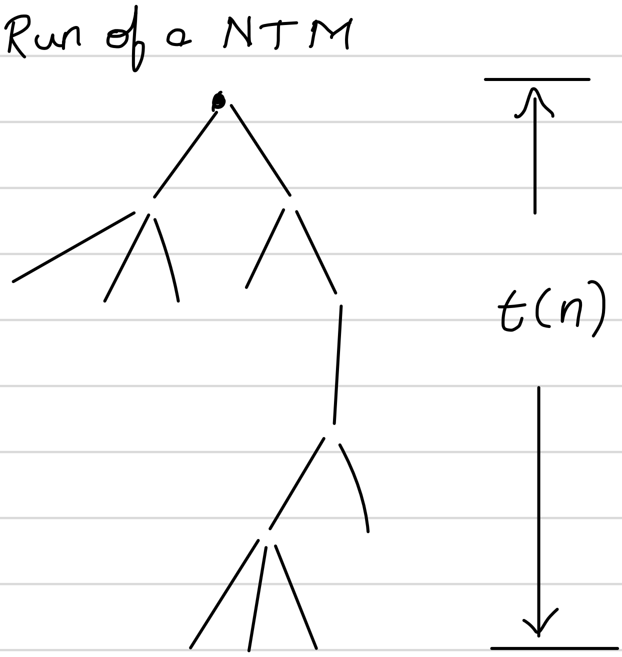




The running time of an NTM is $t : \mathbb{N} \to \mathbb{R}^+$ if $\forall x \in \Sigma^*$

M halts in atmost $O(t(|x|))$ Steps.
The maximum number of steps M uses on any branch of its computation on x is $O(t(|x|))$ Steps

Theorem. Every $t(n)$ time ND Single tape TM has an equivalent $2^{O(t(n))}$ time deterministic single tape TM.

Theorem. Every $t(n)$ time multitape DTM has an equivalent $O(t^2(n))$ time single tape DTM.

NP-completeness.

Cook-Levin Theorem. SAT $\in$ P iff P=NP.

Polynomial time reduction.

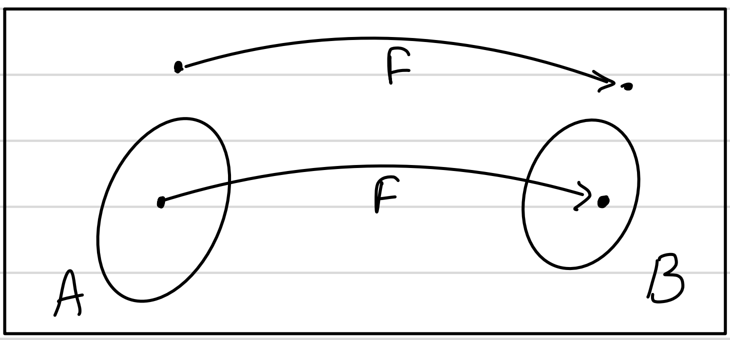
A function $f: \Sigma^* \longrightarrow \Sigma^*$ is a polynomial time computable function if some polynomial time TM exist that given x writes $f(x)$ on its tape.

A is polynomial time reducible to B : $A \leq_p B$

if $\exists$ a polynomial time computable function $f: \Sigma^* \rightarrow \Sigma^*$ such that

for every w,

$w \in A$ iff $f(w) \in B$.



Theorem. if $A \leq_p B$ and $B \in P$ then $A \in P$.

Proof. Let M be the polynomial time TM s.t $L(M) = A$.

Construct a polynomial time TM N as follows:

On input w:

1. Write $f(w)$ on the tape.
2. Run M on $f(w)$ and accept if M accepts.

$w \in A$ iff $f(w) \in B$ iff $f(w) \in L(M)$ iff $w \in L(N)$.

N runs in polynomial time since steps 1 and 2 runs in polynomial time

NP-completeness.

Language A is NP-complete if it satisfies:

1. $A \in NP$
2. every B is NP is polynomial time reducible to A.

Theorem. if A is NP-Complete and $A \in P$ then $P = NP$.

Theorem. if A is NP-Complete and $A \leq_p B$ for B in NP, then B is NP-Complete.

Proof. Polynomial time reductions Compose.

Since A is NP-complete $C \leq_p A$ $\forall C \in NP$.

By assumption $A \leq_p B$.

Thus $C \leq_p B$ $\forall C \in NP$. So C is NP-Complete.

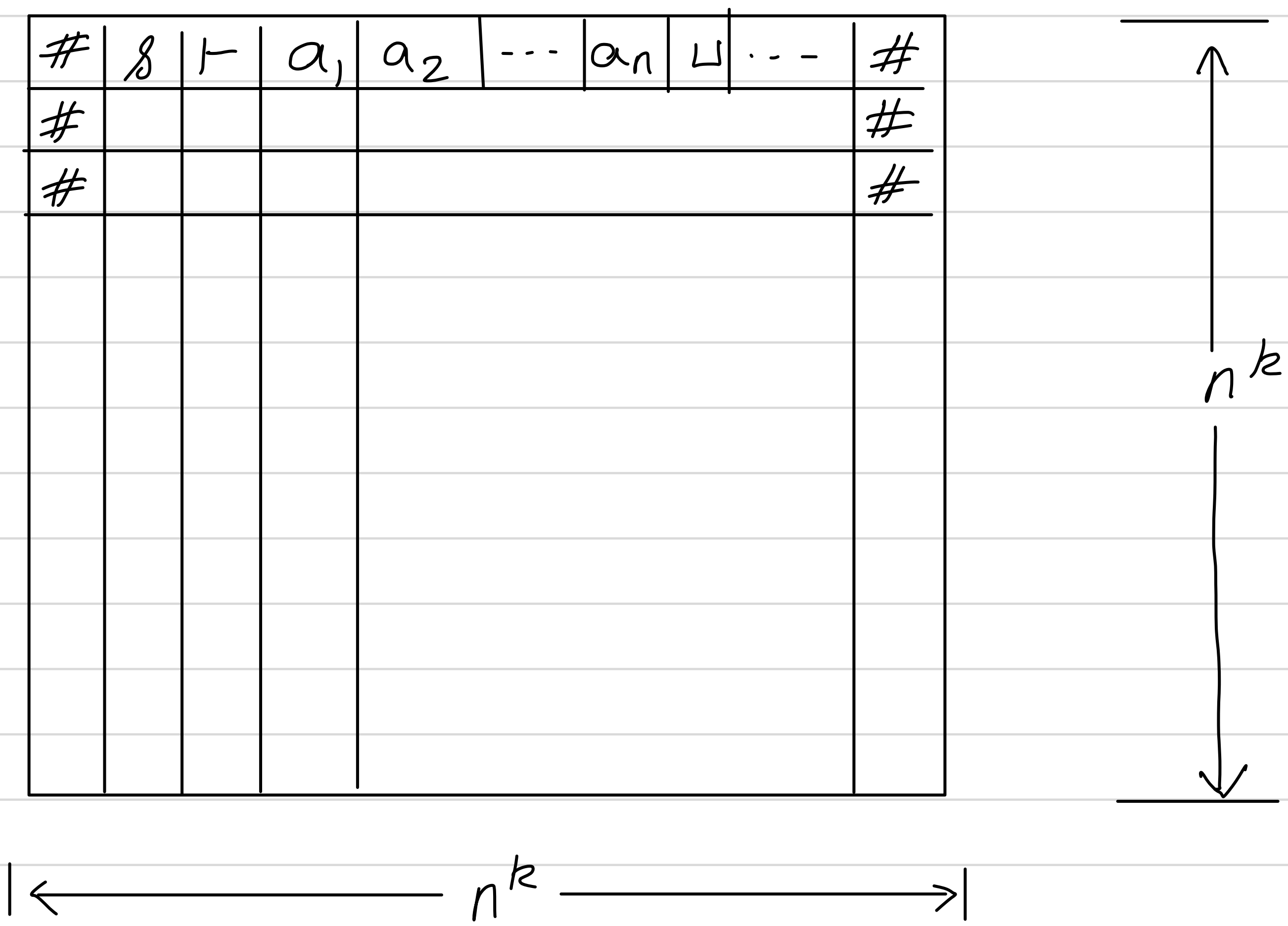
Theorem. SAT is NP-Complete.

1. SAT $\in$ NP - we already saw this.

2. Any language in NP is polynomial time reducible to SAT.

Let N be a NTM s.t $L(N) = A$ and N runs in time n^k.

Build a tableau for N - $n^k \times n^k$ table - rows are configuration



1^{st} row: initial configuration.

T is accepting if any row of T is an accepting configuration.

$A \leq_p SAT$: on input $w \xrightarrow{f} \varphi$ formula.

Let $C = Q \cup \Gamma \cup \{\#\}$.

Propositions in ϕ:

For each cell (i,j) and each $s \in C$ there is a proposition $x_{i,j,s}$.

Interpretation. if $x_{i,j,s}$ is true then cell (i,j) contains s in T.

Total number of cells in T : $(n^k)^2$.

To construct φ s.t
φ is satisfiable iff there is an accepting tableau for N on w.

$$\varphi = (\varphi_{cell} \wedge \varphi_{start} \wedge \varphi_{accept} \wedge \varphi_{move})$$