CS345A: Design and Analysis of Algorithms

Mock Quiz

Marks = 11 **Date: 14 August 2023**

NAME: ROLL No:

Attempt any one of the following problems.

1. **Two versions of Gayle and Shapley algorithm**(6 marks)
   Provide an instance of stable matching problem on a set of 3 men and 3 women such that the output of 'man proposing' version of Gayle Shapley is totally different from the output of 'woman proposing' version.
   Note: You just need to provide only the following details in your answer.
   1. The preference lists of 3 men and 3 women.
   2. Output of the two versions of Gayle Shapley algorithm.

   Answer: Let W_1, W_2, and W_3 be three women and M_1, M_2, and M_3 be three men.

   The preference lists of women is the following.

	1	2	3
W_1	M_1	M_2	M_3
W_2	M_2	M_1	M_3
W_3	M_3	M_1	M_2

   The preference lists of men is the following.

	1	2	3
M_1	W_2	W_1	W_3
M_2	W_3	W_1	W_2
M_3	W_1	W_3	W_2

   The stable matching produced by the 'man proposing' version of Gayle Shapley is $\{(W_2, M_1), (W_3, M_2), (W_1, M_3)\}$.

   The stable matching produced by the 'woman proposing' version is $\{(W_1, M_1), (W_2, M_2), (W_3, M_3)\}$.

2. **Non-dominated points in higher dimensions** (11 marks)
You are given a set P of n points in 3-dimensions. A point $q \in P$ is non-dominated if there is no point $r \in P \backslash \{q\}$ that dominates q in each dimension. Assume without loss of generality that no two points in P have the same x-coordinates or y-coordinates or z-coordinates. Design an $O(n \log n)$ time algorithm that computes all non-dominated points in P.
Hint: Process the points in the decreasing order of their z-coordinates. Under what conditions will ith point in this order be a non-dominated point ? In order to achieve efficiency you might also like to make use of some well known data structure you learnt in ESO207 .
Note: You just need to describe the algorthm. There is no need to analyse its time complexity or prove its correctness.

Answer:
Let L be the list of points P sorted in the decreasing order of their z-coordinates.
Let p_i be the point at ith place in the list L.
Let P_i denote the set of first i points in the list L.
Let Proj(i) be the projection of points of set P_i on (x, y) plane, and let ND(i) be the set of non-dominated points of set Proj(i). We keep a height balanced Binary Search Tree T that, at the end of processing P_i, stores ND(i) using x-coordinate of points as the key. T initially stores point p_1. Note that p_1 is surely a non-dominated point. We process points p_i with $i > 1$ in the increasing value of i using the following procedure.
Process(p_i):

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q ← successor(p_i,T);
if q = null then
    Print(p_i is a non-dominated point);
    Insert(p_i, T);
else
    if (y(q) < y(p_i)) then
        Print(p_i is a non-dominated point);
        q ← predecessor(p_i,T);
        while (q ≠ null and (y(q) < y(p_i)) do
            Delete q from T;
            q ←predecessor(p_i,T);
        end while
        Insert(p_i, T);
    end if
end if
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Note: I have not added comments to the pseudocode above. For knowing proper comments, and explanation of the algorithm, please do attend the doubt clearing session at 12:00 noon on 19th August in RM101. You may write algorithm in English as well. However, it must be complete and unambiguous.