Deterministic Finite Automata (DFA)

$$M = (Q, \Sigma, s, \delta, F)$$

Q - finite set of states.

Σ - input alphabet (a finite set)

$s \in Q$: Start state of M

$F \subseteq Q$: Set of Final/accept states of M.

$\delta : Q \times \Sigma \longrightarrow Q$ is a transition function.

M is in some state q and reads input "a"

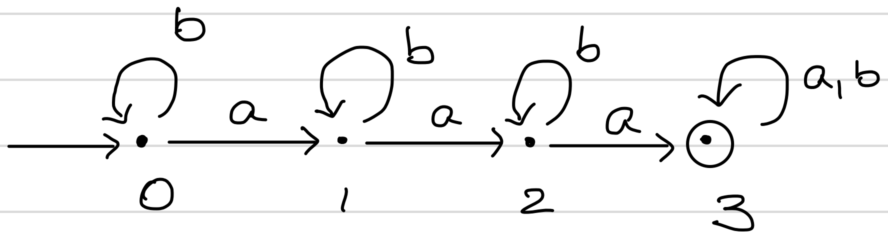
$\delta(q, a) = q'$; M moves to state q'

Example:

$Q = \{0, 1, 2, 3\}$; $\Sigma = \{a, b\}$; $s = 0$; $F = \{3\}$

Transition Function :

$$\delta(0, a) = 1 \qquad \delta(2, a) = \delta(3, a) = 3$$
$$\delta(1, a) = 2 \qquad \delta(q, b) = q \quad q \in \{0, 1, 2, 3\}.$$



$\longrightarrow \cdot$: Start state.

$\odot$: Final State

DFA $M = (Q, \Sigma, S, \delta, F)$

Input $x \in \Sigma^*$. M runs on input x.

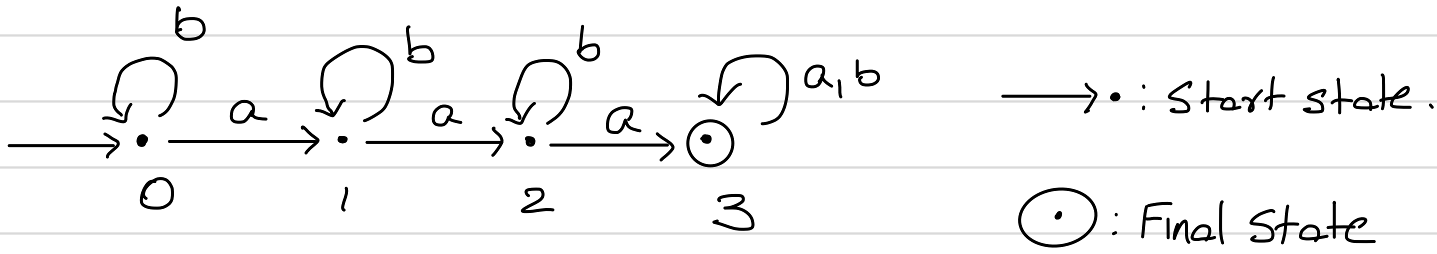
How does M process the input string x

- M starts in the initial state δ.
- Scan x from left to right reading one symbol at a time
- Suppose the first symbol in x is $a \in \Sigma$.
- M reads a and changes its state to $q = \delta(\delta, a)$
- Repeat the process for each subsequent symbols.

How does M decides whether to accept or reject x

- After reading all symbols in x, M is in some state $q \in Q$.
- if $q \in F$ then M accepts x
- if $q \notin F$ then M rejects x

Example



Input $x = aabba$

M accepts x since $3 \in F$.

Input $y = abbab$

After reading y, M is in state 2 and $2 \notin F$.

So M rejects y.

$$L(M) = \{ x \in \{a,b\}^* \mid x \text{ contains at least } 3 \text{ a's} \}.$$

DFA $M = (Q, \Sigma, \delta, s, F)$ where. $\delta : Q \times \Sigma \to Q$

We can "lift" δ as a transition function over strings.

$\hat{\delta} : Q \times \Sigma^* \to Q$.

Defining $\hat{\delta}$ inductively.

$$\hat{\delta}(q, \epsilon) = q$$

$$\hat{\delta}(q, xa) = \delta(\underbrace{\hat{\delta}(q, x)}_{\text{well defined by induction}}, a)$$

Claim 1. We can show that $\hat{\delta}(q, a) = \delta(q, a)$

$$\hat{\delta}(q, a) = \hat{\delta}(q, \epsilon a) \quad [\text{Since } a = \epsilon a]$$

$$= \delta(\hat{\delta}(q, \epsilon), a)$$

$$= \delta(q, a)$$

x is accepted by M if $\hat{\delta}(s, x) \in F$
x is rejected by M if $\hat{\delta}(s, x) \notin F$

DFA $M = (Q, \Sigma, \delta, s, F)$ where. $\delta : Q \times \Sigma \to Q$

We can "lift" δ as a transition function over strings.

$\hat{\delta} : Q \times \Sigma^* \to Q$.

Defining $\hat{\delta}$ inductively.

$$\hat{\delta}(q, \epsilon) = q$$

$$\hat{\delta}(q, xa) = \delta(\underbrace{\hat{\delta}(q, x)}_{\text{well defined by induction}}, a)$$

Claim 1. We can show that $\hat{\delta}(q, a) = \delta(q, a)$

$$\hat{\delta}(q, a) = \hat{\delta}(q, \epsilon a) \quad [\text{Since } a = \epsilon a]$$

$$= \delta(\hat{\delta}(q, \epsilon), a)$$

$$= \delta(q, a)$$

x is accepted by M if $\hat{\delta}(s, x) \in F$
x is rejected by M if $\hat{\delta}(s, x) \notin F$

Language of the DFA M.

$$L(M) = \{x \in \Sigma^* \mid \hat{\delta}(s, x) \in F\}$$

$A \subseteq \Sigma^*$ is regular if $A = L(M)$ for some DFA M.