CS340 - 2023 Quiz 2

DIVYANSH

TOTAL POINTS

14 / 20

QUESTION 1

1 Question 1 4 / 4

✓ **- 0 pts** *Correct dfa*

- 2 pts No self loops on dead state / transition to dead state not shown/No self loops on final state

- 3 pts No proper explanation

- 4 pts NFA is provided / wrong DFA

- 2 pts Start state must to be final state

- 2 pts Not explaining how dfa accepting the following Expression

QUESTION 2

2 Question 2 4 / 4

✓ **- 0 pts** *The language is $$\Sigma ^*$$ and minimal dfa will have 1 state. The explanation must consist of what language is accepted by the given NFA. can use logic / algorithm*

- 1 pts drawn the correct dfa with 2 states with correct explanation of the language $$\Sigma ^*$$

- 2 pts no explanation but correct single state well labeled DFA

- 4 pts incorrect answer/ not attempted

+ 1 pts correct dfa accepting $$\Sigma ^*$$ with explanation

- 1 pts minor mistake in notation/explanation

QUESTION 3

3 Question 3 5 / 6

- 0 pts Correct!

✓ **- 1 pts** *Minor error(s)*

- 1 pts Missing part(s) of proof

- 2 pts Correct construction, no formal proof (of equivalence)

- 4 pts Partially correct idea with partial construction

- 4 pts Correct idea, missing construction and/or proof

- 6 pts Incorrect / Unattempted / No explanation

The fact that there can be multiple final states has not been incorporated.

QUESTION 4

4 Question 4 1 / 6

- 0 pts Correct

✓ **+ 1 pts** *If written that the statement is true.*

+ 2 pts If mentioned that f(A,B) = (AB)^+.

+ 6 pts Proof with correct explanation.

✓ **- 6 pts** *Incorrect/Unattempted/No explanation.*

CS340 (2023) – Quiz 2

Duration: 35 minutes, Total marks: 20, Pages: 6.

- **Important note.** Answers without clear and concise explanations will not be graded.

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Problems

1. **(4 marks)** Let $\Sigma = \{0, 1\}$ and consider the regular expression:

$$\alpha = \epsilon + (0+1)0(0+1)^*$$

Give a DFA M with at most 4 states such that $L(M) = L(\alpha)$. Justify your answer.

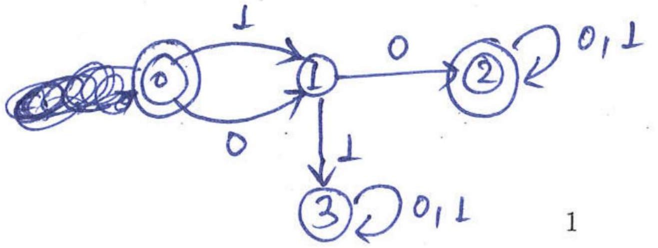
Let's first simplify the expression α. So.

$L(\alpha) = L(\epsilon) \cup L((0+1)0(0+1)^*)$ {definition of '+' on pattern}

$= L(\epsilon) \cup L((00+10)(0+1)^*)$ {concatenation of $(0+1)$ and 0}.

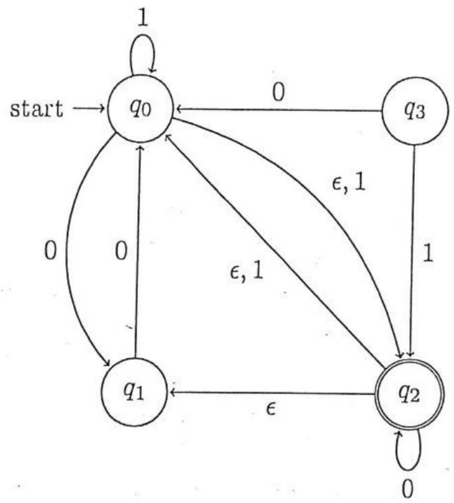
So, Now we can see that the language consist of a empty string or, $\{00, 10\}$ followed by any string over $\{0,1\}$.

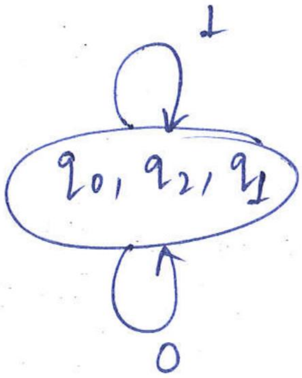
DFA.



{ $\{0,2\}$ are final states }

2. (4 marks) Consider the NFA N given below. Construct a DFA M with minimal number of states such that $L(M) = L(N)$. Clearly justify your answer.



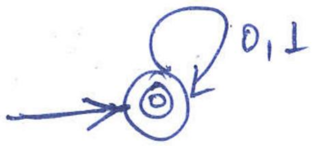


→ If we map q_0, q_1, q_2 to a single state (as ~~Ø~~ in the NFA it can do the transition to all 3 with ϵ).

→ Finally any incoming string will also ~~to~~ lead to some state in $\{q_0, q_1, q_2\}$ of NFA hence the only o state of DFA.

→ Also, q_3 is not reachable as it has no incoming edge.

So, DFA



This is the final DFA.

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3. (6 marks) Let Σ be some alphabet set. For a string $x = a_1 a_2 \cdots a_{k-1} a_k \in \Sigma^*$, let $rev(x) = a_k a_{k-1} \cdots a_2 a_1$. For $A \subseteq \Sigma^*$, let $rev(A) = \{rev(x) \mid x \in A\}$. Is the following statement true? Clearly justify your answer.

Statement. If A is regular then then $rev(A)$ is also regular.

My claim is that rev(A) is regular.

Proof:- Since A is regular let M be the finite automata such that $L(M) = A$. $M(Q, \Sigma, \delta, s, F)$.

Let us define a NFA, N as $N(Q, \Sigma, \Delta, S, F')$ such that $S = F$, $F' = \{s\}$ and $\Delta(\{u\}, a) = v$ such that $\delta(v, a) = u$ {i.e. reverse direction of δ}. for $u, v \in Q$ and $a \in \Sigma$.

I will show, $\hat{\Delta}(\{u\}, x) = v$ such that $\hat{\delta}(v, x) = u$ by induction of length of x.

→ base case covered in definition

→ $\hat{\Delta}(\{u\}, xa) = \Delta(\hat{\Delta}(\{u\}, x), a)$ {definition of $\hat{\Delta}$}

$= \Delta(v', a)$ for $\hat{\delta}(v', x) = u$ {induction Hypothesis}

$= v$ for $\delta(v, a) = v'$ {definition of Δ}.

So, we can take u in S and $v \in \{s\}$ to create NFA accepting rev(A). Hence proved

4. (6 marks) Given two sets $A \subseteq \Sigma^*$ and $B \subseteq \Sigma^*$ we define:

$$f(A,B) = \{w \in \Sigma^* \mid w = \overset{x_0, y_0}{\wedge} x_1 y_1 x_2 y_2 \ldots x_k y_k, \text{ where } k \geq 0,$$
$$\text{for all } i : 1 \leq i \leq k, x_i \in A \text{ and } y_i \in B\}.$$

Is the following statement true? Clearly justify your answer.

Statement. If A and B are regular then $f(A,B)$ is also regular.

Proof:- ~~I will define a function $g(A,B)$ such that~~

~~$g(A,B) = \{w \in \Sigma^* \mid w = xy$, where $x \in A$ and $y \in B\}$.~~

~~It is clear that $g(A,B)$ is basically concatenation of A and B. Since we know that sets are closed over concatenation so $g(A,B)$ is also regular.~~

We ~~can~~ can do induction on K.

for $K=0$, (Base case)

$f(A,B) = x_0 y_0 \mid x_0 \in A$ and $y_0 \in B$

So $f(A,B)$ is regular as set are closed over concatenation.

Induction step.

$$f(A,B) = \underbrace{x_0 y_0 x_1 y_2 \ldots x_{k-1} y_{k-1}} \; \underbrace{x_k y_K}$$

So, left part is regular from IH and right part (x_y, y_k) is regular by concatenation. So whole set is regular as concatenation.

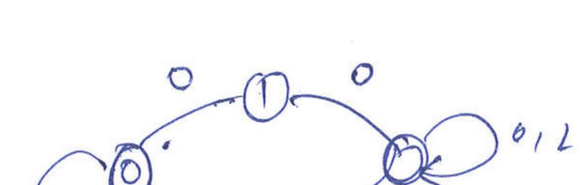
$\delta(u, a) = v$

$\alpha = \epsilon + (0+1) \, 0 \, (0+1)^*$

$\{\epsilon\} \quad \{0,1\} \quad \{0\} \quad \{0,1\}^*$

$\{0,1\} \cdot \{0\} \cdot \{0,1\}^*$

$\{00, 10\} \cdot \{0,1\}^*$



q_1, q_2