INDIAN INSTITUTE OF TECHNOLOGY KANPUR

ESO 201A: Thermodynamics

(2023-24 I Semester)

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Tutorial 10

Question 1: An ideal diesel engine has a compression ratio of 20 and uses air as the working fluid. The state of air at the beginning of the compression process is 95 kPa and 20°C. If the maximum temperature in the cycle is not to exceed 2200 K, determine (a) the thermal efficiency and (b) the mean effective pressure. Assume constant specific heats for air at room temperature.

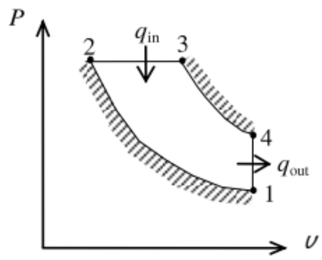
Solution:

9-57 An ideal diesel engine with air as the working fluid has a compression ratio of 20. The thermal efficiency and the mean effective pressure are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $c_p = 1.005$ kJ/kg·K, $c_v = 0.718$ kJ/kg·K, $R = 0.287$ kJ/kg·K, and $k = 1.4$ (Table A-2).

Analysis (*a*) Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (293\text{ K})(20)^{0.4} = 971.1\text{ K}$$

Process 2-3: P = constant heat addition.

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2200\text{ K}}{971.1\text{ K}} = 2.265$$

Process 3-4: isentropic expansion.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{2.265 v_2}{v_4}\right)^{k-1} = T_3\left(\frac{2.265}{r}\right)^{k-1} = (2200\text{ K})\left(\frac{2.265}{20}\right)^{0.4} = 920.6\text{ K}$$

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2) = (1.005\text{ kJ/kg}\cdot\text{K})(2200 - 971.1)\text{K} = 1235\text{ kJ/kg}$$

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1) = (0.718\text{ kJ/kg}\cdot\text{K})(920.6 - 293)\text{K} = 450.6\text{ kJ/kg}$$

$$w_{net,out} = q_{in} - q_{out} = 1235 - 450.6 = 784.4\text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{784.4\text{ kJ/kg}}{1235\text{ kJ/kg}} = \mathbf{63.5\%}$$

(*b*)

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})}{95\text{ kPa}} = 0.885\text{ m}^3/\text{kg} = v_{max}$$

$$v_{min} = v_2 = \frac{v_{max}}{r}$$

$$\text{MEP} = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1(1 - 1/r)} = \frac{784.4\text{ kJ/kg}}{(0.885\text{ m}^3/\text{kg})(1 - 1/20)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{933\text{ kPa}}$$

Question 2: A gas-turbine power plant operates on the simple Brayton cycle between the pressure limits of 100 and 1600 kPa. The working fluid is air, which enters the compressor at 40°C at a rate of 850 m^3/min and leaves the turbine at 650°C. Assuming a compressor isentropic efficiency of 85 percent and a turbine isentropic efficiency of 88 percent, determine (a) the net power output, (b) the back work ratio, and (c) the thermal efficiency. Use constant specific heats with $c_v = 0.821$ kJ/kg·K, $c_p = 1.108$ kJ/kg·K, and $k = 1.35$.

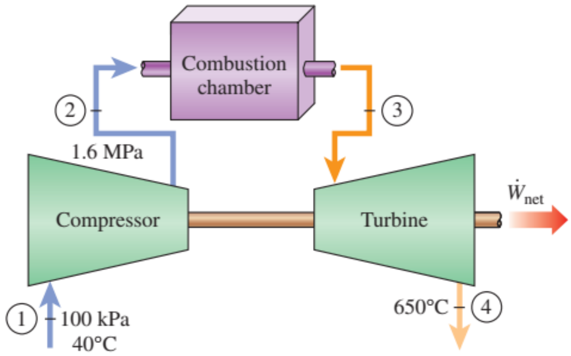


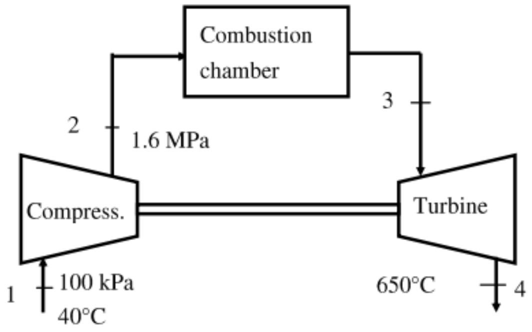
Fig. 1

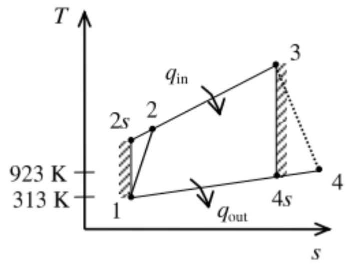
Solution:

9-95 A gas-turbine plant operates on the simple Brayton cycle. The net power output, the back work ratio, and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The gas constant of air is $R = 0.287$ kJ/kg·K (Table A-1). Other properties of air are given as $c_p = 1.108$ kJ/kg.K, $c_v = 0.821$ kJ/kg.K, and $k = 1.35$.





Analysis (a) For the compression process,

$$r_p = \frac{P_2}{P_1} = \frac{1600 \text{ kPa}}{100 \text{ kPa}} = 16$$

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (313 \text{ K})(16)^{0.35/1.35} = 642.3 \text{ K}$$

$$\eta_C = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \longrightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_C} = 313 + \frac{642.3-313}{0.85} = 700.4 \text{ K}$$

For the expansion process,

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = T_3\left(\frac{1}{16}\right)^{0.35/1.35} = 0.4873 T_3$$

$$\eta_T = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \longrightarrow \begin{aligned} T_4 &= T_3 - \eta_T(T_3 - T_{4s}) \\ 923 &= T_3 - (0.88)(T_3 - 0.4873T_3) \\ T_3 &= 1682 \text{ K} \end{aligned}$$

The mass flow rate is determined from

$$\dot{m} = \frac{P_1\dot{V}_1}{RT_1} = \frac{(100 \text{ kPa})(850/60 \text{ m}^3\text{/s})}{(0.287 \text{ kPa}\cdot\text{m}^3\text{/kg}\cdot\text{K})(40+273 \text{ K})} = 15.77 \text{ kg/s}$$

The net power output is

$\dot{W}_{C,in} = \dot{m}c_p(T_2 - T_1) =$ **(15.77 kg/s) (1.108 kJ/kg.K) (700.4-313)K = 6769 kW**

$\dot{W}_{T,out} = \dot{m}c_p(T_3 - T_4) =$ **(15.77 kg/s) (1.108 kJ/kg.K) (1682-923)K = 13,262 kW**

$\dot{W}_{net} = \dot{W}_{T,out} - \dot{W}_{C,in} =$ **(13262-6769) = 6493 kW**

(*b*) The back work ratio is

$$r_{bw} = \frac{\dot{W}_{C,in}}{\dot{W}_{T,out}} = \frac{\mathbf{6769\ kW}}{\mathbf{13262\ kW}} = \mathbf{0.51}$$

(*c*) The rate of heat input and the thermal efficiency are

$\dot{Q}_{in} = \dot{m}c_p(T_3 - T_2) =$ **(15.77 kg/s) (1.108 kJ/kg.K) (1682-700.4)K = 17151.6 kW**

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\mathbf{6493\ kW}}{\mathbf{17151.6\ kW}} = \mathbf{0.38 = 38\%}$$

Question 3: A gas turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa and 20°C. The compressor pressure ratio is 8, the maximum cycle temperature is 800°C, and the cold airstream leaves the regenerator 10°C cooler than the hot airstream at the inlet of the regenerator. The cycle produces 150 kW. The compressor isentropic efficiency is 87 percent, and the turbine isentropic efficiency is 93 percent. Determine the exergy destruction for each of the processes of the cycle. The temperature of the hot reservoir is the same as the maximum cycle temperature, and the temperature of the cold reservoir is the same as the minimum cycle temperature. Use constant specific heats at room temperature.

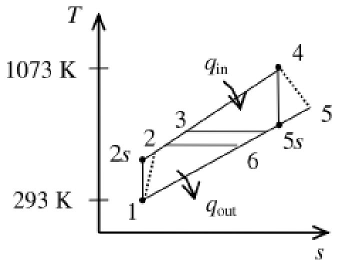
Solution:

9-147 The exergy loss of each process for a regenerative Brayton cycle is to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg.K and k = 1.4 (Table A-2a).

Analysis For the compression and expansion processes we have



$$T_{2s} = T_1 r_p^{(k-1)/k} = (293\text{ K})(8)^{0.4/1.4} = 530.8\text{ K}$$

$$\eta_C = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \longrightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_C} = 293 + \frac{530.8-293}{0.87} = 566.3\text{ K}$$

$$T_{5s} = T_4\left(\frac{1}{r_p}\right)^{(k-1)/k} = (1073\text{ K})\left(\frac{1}{8}\right)^{0.4/1.4} = 592.3\text{ K}$$

$$\eta_T = \frac{c_p(T_4-T_5)}{c_p(T_4-T_{5s})} \longrightarrow T_5 = T_4 - \eta_T(T_4-T_{5s}) = 1073-(0.93)(1073-592.3) = 625.9\text{ K}$$

When the first law is applied to the heat exchanger, the result is

$$T_3 - T_2 = T_5 - T_6$$

while the regenerator temperature specification gives

$$T_3 = T_5 - 10 = 625.9 - 10 = 615.9\text{ K}$$

The simultaneous solution of these two results gives

$$T_6 = T_5 - (T_3 - T_2) = 625.9 - (615.9 - 566.3) = 576.3\text{ K}$$

Application of the first law to the turbine and compressor gives

$$\begin{aligned} w_{\text{net}} &= c_p(T_4-T_5) - c_p(T_2-T_1) \\ &= (1.005\text{ kJ/kg}\cdot\text{K})(1073-625.9)\text{ K} - (1.005\text{ kJ/kg}\cdot\text{K})(566.3-293)\text{ K} \\ &= 174.7\text{ kJ/kg} \end{aligned}$$

The heat input and heat output are

$$q_{\text{in}} = c_p(T_4-T_3) = (1.005\text{ kJ/kg}\cdot\text{K})(1073-615.9)\text{K} = 459.4\text{ kJ/kg}$$
$$q_{\text{out}} = c_p(T_6-T_1) = (1.005\text{ kJ/kg}\cdot\text{K})(576.3-293)\text{K} = 284.7\text{ kJ/kg}$$

The exergy destruction during a process of a stream from an inlet state to exit state is given by

$$x_{\text{dest}} = T_0 s_{\text{gen}} = T_0\left(s_e - s_i - \frac{q_{\text{in}}}{T_{\text{source}}} + \frac{q_{\text{out}}}{T_{\text{sink}}}\right)$$

Application of this equation for each process of the cycle gives

$$x_{\text{dest, 1-2}} = T_0\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right) = (293)\left[(1.005)\ln\frac{566.3}{293} - (0.287)\ln(8)\right] = \mathbf{19.2\ kJ/kg}$$

$$x_{\text{dest, 5-3}} = T_0\left(c_p \ln\frac{T_3}{T_5} - R\ln\frac{P_3}{P_5} - \frac{q_{in}}{T_{source}}\right) = (293)\left[(1.005)\ln\frac{1073}{615.9} - 0 - \frac{459.4}{1073}\right] = \mathbf{38.0\ kJ/kg}$$

$$x_{\text{dest, 3-4}} = T_0\left(c_p \ln\frac{T_4}{T_3} - R\ln\frac{P_4}{P_3}\right) = (293\text{ K})\left[(1.005)\ln\frac{625.9}{1073} - (0.287)\ln\left(\frac{1}{8}\right)\right] = \mathbf{16.1\ kJ/kg}$$

$$x_{\text{dest, 6-1}} = T_0\left(c_p \ln\frac{T_1}{T_6} - R\ln\frac{P_1}{P_6} + \frac{q_{out}}{T_{\text{sink}}}\right) = (293)\left[(1.005)\ln\frac{293}{576.3} - 0 + \frac{284.7}{293}\right] = \mathbf{85.5\ kJ/kg}$$

$$x_{\text{dest,regen}} = T_0(\Delta s_{2-5} + \Delta s_{4-6} = T_0\left(c_p \ln\frac{T_5}{T_2} + c_p \ln\frac{T_6}{T_4}\right)$$
$$= (293)\left[(1.005)\ln\frac{615.9}{566.3} + (1.005)\ln\frac{576.3}{625.9}\right] = \mathbf{0.32\ kJ/kg}$$

Question 4: Consider a 210-MW steam power plant that operates on a simple ideal Rankine cycle. Steam enters the turbine at 10 MPa and 500°C and is cooled in the condenser at a pressure of 10 kPa. Show the cycle on a T-s diagram with respect to saturation lines, and determine (a) the quality of the steam at the turbine exit, (b) the thermal efficiency of the cycle, and (c) the mass flow rate of the steam.

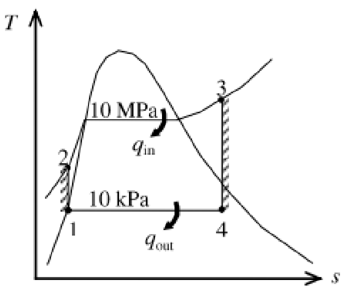
Solution:

10-19 A steam power plant that operates on a simple ideal Rankine cycle is considered. The quality of the steam at the turbine exit, the thermal efficiency of the cycle, and the mass flow rate of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f\,@\,10\text{ kPa}} = 191.81\text{ kJ/kg}$$
$$v_1 = v_{f\,@\,10\text{ kPa}} = 0.00101\text{ m}^3\text{/kg}$$



$$w_{p,\text{in}} = v_1(P_2 - P_1)$$
$$= (0.00101\text{ m}^3\text{/kg})(10{,}000 - 10\text{ kPa})\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right)$$
$$= 10.09\text{ kJ/kg}$$
$$h_2 = h_1 + w_{p,\text{in}} = 191.81 + 10.09 = 201.90\text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 10\text{ MPa} \\ T_3 = 500\ ^\circ\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3375.1\text{ kJ/kg} \\ s_3 = 6.5995\text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 10\text{ kPa} \\ s_4 = s_3 \end{array}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.5995 - 0.6492}{7.4996} = \mathbf{0.7934}$$
$$h_4 = h_f + x_4 h_{fg} = 191.81 + (0.7934)(2392.1) = 2089.7\text{ kJ/kg}$$

(*b*)
$$q_{\text{in}} = h_3 - h_2 = 3375.1 - 201.90 = 3173.2\text{ kJ/kg}$$
$$q_{\text{out}} = h_4 - h_1 = 2089.7 - 191.81 = 1897.9\text{ kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 3173.2 - 1897.9 = 1275.4\text{ kJ/kg}$$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1275.4\text{ kJ/kg}}{3173.2\text{ kJ/kg}} = \mathbf{40.2\%}$$

(*c*)
$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{210{,}000\text{ kJ/s}}{1275.4\text{ kJ/kg}} = \mathbf{164.7\ kg/s}$$

Question 5: An ideal reheat Rankine cycle with water as the working fluid operates the boiler at 15,000 kPa, the reheater at 2000 kPa, and the condenser at 100 kPa. The temperature is 450°C at the entrance of the high-pressure and low-pressure turbines. The mass flow rate through the cycle is 1.74 kg/s. Determine the power used by pumps, the power produced by the cycle, the rate of heat transfer in the reheater, and the thermal efficiency of this system.

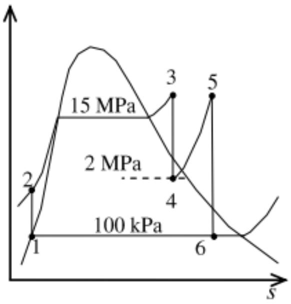
Solution:

10-36 An ideal reheat steam Rankine cycle produces 2000 kW power. The mass flow rate of the steam, the rate of heat transfer in the reheater, the power used by the pumps, and the thermal efficiency of the cycle are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis From the steam tables (Tables A-4, A-5, and A-6 or EES),

$$h_1 = h_{f\,@100\text{ kPa}} = 417.51 \text{ kJ/kg}$$
$$v_1 = v_{f\,@100\text{ kPa}} = 0.001043 \text{ m}^3\text{/kg}$$



$$\begin{aligned} w_{\text{p, in}} &= v_1(P_2 - P_1) \\ &= (0.001043 \text{ m}^3\text{/kg})(15000 - 100)\text{kPa}\left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3}\right) \\ &= 15.54 \text{ kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{\text{p, in}} = 417.51 + 15.54 = 433.05 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 15{,}000 \text{ kPa} \\ T_3 = 450°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3157.9 \text{ kJ/kg} \\ s_3 = 6.1434 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 2000 \text{ kPa} \\ s_4 = s_3 \end{array}\right\} \begin{array}{l} x_4 = \dfrac{s_4 - s_f}{s_{fg}} = \dfrac{6.1434 - 2.4467}{3.8923} = 0.9497 \\ h_4 = h_f + x_4 h_{fg} = 908.47 + (0.9497)(1889.8) = 2703.3 \text{ kJ/kg} \end{array}$$

$$\left.\begin{array}{l} P_5 = 2000 \text{ kPa} \\ T_5 = 450°\text{C} \end{array}\right\} \begin{array}{l} h_5 = 3358.2 \text{ kJ/kg} \\ s_5 = 7.2866 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_6 = 100 \text{ kPa} \\ s_6 = s_5 \end{array}\right\} \begin{array}{l} x_6 = \dfrac{s_6 - s_f}{s_{fg}} = \dfrac{7.2866 - 1.3028}{6.0562} = 0.9880 \\ h_6 = h_f + x_6 h_{fg} = \textbf{417.51+(0.988)(2257.5)=2648 kJ/kg} \end{array}$$

Thus,

$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4) = 3157.9 - 433.05 + 3358.2 - 2703.3 = 3379.8 \text{ kJ/kg}$$
$$q_{\text{out}} = h_6 - h_1 = 2648.0 - 417.51 = 2230.5 \text{ kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = \textbf{3379.8-2230.5 = 1149.3 kJ/kg}$$

The power produced by the cycle is

$$\dot{W}_{\text{net}} = \dot{m}w_{\text{net}} = (1.74 \text{ kg/s})(1149.2 \text{ kJ/kg}) = \textbf{2000 kW}$$

The rate of heat transfer in the rehetaer is

$$\dot{Q}_{\text{reheater}} = \dot{m}(h_5 - h_4) = (1.740 \text{ kg/s})(3358.2 - 2703.3) \text{ kJ/kg} = \textbf{1140 kW}$$
$$\dot{W}_{\text{P, in}} = \dot{m}w_{\text{P, in}} = (1.740 \text{ kg/s})(15.54 \text{ kJ/kg}) = \textbf{27 kW}$$

and the thermal efficiency of the cycle is

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2230.5}{3379.8} = \textbf{0.340}$$