$A \leq_p SAT$: on input $w \xrightarrow{f} \varphi$ formula.

Let $C = Q \cup \Gamma \cup \{\#\}$.

Propositions in ϕ:

For each cell (i,j) and each $s \in C$ there is a proposition $x_{i,j,s}$.

Interpretation. if $x_{i,j,s}$ is true then cell (i,j) contains s in T.

Total number of cells in T : $(n^k)^2$.

To construct φ s.t
φ is satisfiable iff there is an accepting tableau for N on w.

$$\varphi = (\varphi_{cell} \wedge \varphi_{start} \wedge \varphi_{accept} \wedge \varphi_{move})$$

$$\varphi_{cell} = \bigwedge_{1 \leq i,j \leq n^k} \left[\underbrace{\left(\bigvee_{s \in C} x_{i,j,s} \right)}_{\text{At least one Variable is True}} \wedge \underbrace{\left(\bigwedge_{\substack{s,t \in C \\ s \neq t}} \left(\overline{x_{i,j,s}} \vee \overline{x_{i,j,t}} \right) \right)}_{\text{At most one Variable is True.}} \right]$$

$$\varphi_{accept} = \bigvee_{1 \leq i,j \leq n^k} x_{i,j,t}$$

$$\varphi_{move} = \bigwedge_{1 \le i,j \le n^k} (\text{the } (i,j) \text{ window is consistent}).$$

a	q_1	b
q_2	a	c

a	q_1	b
a	a	q_2

$$\delta(q_1, b) = \{(q_2, c, L), (q_2, a, R)\}$$

a	a	q_1
a	a	b

$$\varphi_{move} = \bigwedge_{1 \le i,j \le n^k} (\underbrace{\text{the } (i,j) \text{ window is consistent}}) .$$

$$\bigvee_{\substack{a_1 \dots a_6 \\ \text{Consistent}}} \left(x_{i,j-1,a_1} \wedge x_{i,j,a_2} \wedge x_{i,j+1,a_3} \wedge x_{i+1,j-1,a_4} \wedge x_{i+1,j-1,a_5} \wedge x_{i+1,j+1,a_6} \right)$$

$$\varphi_{move} = \bigwedge_{1 \le i,j \le n^k} (\text{the } (i,j) \text{ window is consistent}).$$

Complexity of the reduction:

Suppose $|C| = \ell$, it depends on TM N and not the input

So total number of variables: $\ell \cdot n^{2k} = O(n^{2k})$.

Size of φ_{cell} is $O(n^{2k})$

Size of φ_{start} is $O(n^k)$.

Size of φ_{move} and φ_{accept} is $O(n^{2k})$.

So size of $\underbrace{\varphi \text{ is } O(n^{2k})}_{\text{Size of } \varphi \text{ is polynomial in } n.}$.

$CLIQUE = \{ G\#k \mid G$ is an undirected graph with a k-Clique $\}$.

Theorem. CLIQUE is NP-Complete.

3CNF - all clauses have 3 literals

$(x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_2 \vee \overline{x_4} \vee x_3)$

$3SAT = \{ \alpha \mid \alpha$ is a satisfiable 3CNF formula $\}$.

Theorem. 3SAT is NP-Complete.

Theorem. CLIQUE is NP-Complete.

$3SAT \leq_p CLIQUE$.

$$\varphi \xrightarrow{f} (G, k)$$