INDIAN INSTITUTE OF TECHNOLOGY KANPUR

ESO 201A: Thermodynamics

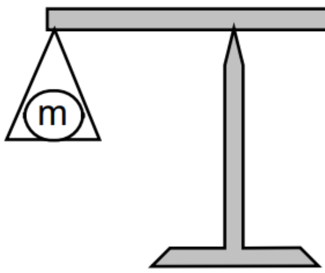
(2023-24 I Semester)

Instructor: Dr Avinash Kumar Agarwal

Tutorial -1

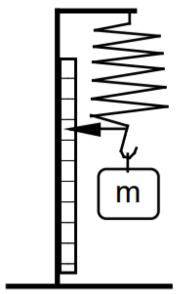
Question 1: On the moon, the gravitational acceleration is approximately one-sixth that on the surface of the earth. A 5-kg mass is "weighed" with a beam balance on the surface of the moon. What is the expected reading? If this mass is weighed with a spring scale that reads correctly for standard gravity on earth, what is the reading?

Solution: A beam scale compares masses and thus is not affected by the variations in gravitational acceleration. The beam scale will read what it reads on earth,



Hence, beam balance reading will be 5 kg.

A spring scale measures weight, which is the local gravitational force applied on a body:



Spring balance reading (in kg units) $\propto F \propto g$

Hence spring balance reading will be 5/6 kg.

Question 2: During a heating process, the temperature of a system rises by 10°C. Express this rise in temperature in K, °F, and R.

Solution:

The temperature rise of a system is to be expressed in different units.

Analysis: This problem deals with temperature changes, which are identical in Kelvin and Celsius scales. Then,

$$\Delta T(\mathrm{K}) = \Delta T(^\circ\mathrm{C}) = 10\mathrm{K}$$

The temperature changes in Fahrenheit and Rankine scales are also identical and are related to the changes in Celsius and Kelvin scales through:

$$T(\mathrm{R}) = 1.8T(\mathrm{K}) \text{ and } \mathrm{T}(^\circ\mathrm{F}) = 1.8\mathrm{T}(^\circ\mathrm{C}) + 32$$

Hence

$$\Delta T(\mathrm{R}) = 1.8\,\Delta T(\mathrm{K}) = (1.8)(10)\mathrm{R} = 18\ \mathrm{R}$$

and

$$\Delta T(^\circ\mathrm{F}) = \Delta T(\mathrm{R}) = 18^\circ\mathrm{F}$$

Discussion: Note that the units °C and K are interchangeable when dealing with temperature differences.

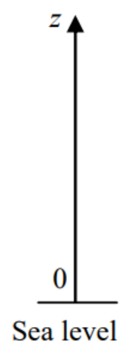
Question 3: At 45° latitude, the gravitational acceleration as a function of elevation z above sea level is given by $g = a - bz$,where a = 9.807 m/s² and b = $3.32 \times 10^{-6}\mathrm{s}^{-2}$.

(a) Determine the height above sea level where the weight of an object will decrease by 0.3 percent.

(b) By what percentage is the weight of an airplane reduced when it cruises at 11000 m?

Solution:

(a) The variation of gravitational acceleration above the sea level is given as a function of altitude. The height at which the weight of a body will decrease by 0.3% is to be determined.



The weight of a body at the elevation z can be expressed as:

$$W = mg = m(9.807 - 3.32 \times 10^{-6}z)$$

In our case,

$$W = (1 - 0.3/100)W_s = 0.997W_s = 0.997mg_s = 0.997(m)(9.807)$$

Substituting,

$$0.997(9.807) = (9.807 - 3.32 \times 10^{-6} z)$$

Hence, $\boldsymbol{z}$ **= 8862 m**

(b) For standard gravity, $g_s = 9.807\ \mathrm{ms^{-2}}$

At height H, $g_H = 9.807 - 3.32 \times 10^{-6} \times 11000 = 9.7705\ \mathrm{ms^{-2}}$

$W_s = mg_s$

$W_H = mg_H$

Percentage reduction in weight = $((W_s - W_H)/W_s) \times 100 = ((g_s - g_H)/g_s) \times 100$

Substituting the values we get, % reduction = 0.37%

Question 4: The density of mercury changes approximately linearly with temperature as $\rho_{Hg} = 13595 - 2.5T$ kg/m³ (T in Celsius). If a pressure difference of 100 kPa is measured in the summer at 35°C and in the winter at −15°C, what is the difference in column height between the two measurements?

Solution:

Mercury thermometer measures temperature by measuring the volume expansion of a fixed mass of liquid mercury due to a change in density.

The manometer fluid reading from the given formula gives:

Density in summer, $\rho_{su} = 13595 - 2.5 \times 35 = 13507.5\ \mathrm{kg/m^3}$

Density in winter, $\rho_w = 13595 - 2.5 \times (-15) = 13632.5\ \mathrm{kg/m^3}$

The manometer reading h relates to the pressure difference as

$$\Delta P = \rho L g \quad \Rightarrow \quad L = \frac{\Delta P}{\rho g}$$

The two different heights that we will measure become

$$L_{su} = \frac{100 \times 10^3}{13507.5 \times 9.807} \frac{\mathrm{kPa\ (Pa/kPa)}}{\mathrm{(kg/m^3)\ m/s^2}} = 0.7549\ \mathrm{m}$$

$$L_w = \frac{100 \times 10^3}{13632.5 \times 9.807} \frac{\mathrm{kPa\ (Pa/kPa)}}{\mathrm{(kg/m^3)\ m/s^2}} = 0.7480\ \mathrm{m}$$

$$\Delta L = L_{su} - L_w = \mathbf{0.0069\ m = 6.9\ mm}$$

Question 5: The density of liquid water is $\rho = 1008 - T/2$ [kg/m³] with T in °C. If the temperature increases 10°C, how much deeper does a 1m layer of water become?

Solution:

The density change for a change in temperature of 10°C becomes

$$\Delta\rho = -\Delta T/2 = -5 \text{ kg/m}^3$$

from an ambient density of

$$\rho = 1008 - T/2 = 1008 - 25/2 = 995.5 \text{ kg/m}^3$$

Assume the area is the same and the mass is the same $m = \rho V = \rho AH$, then we have

$$\Delta m = 0 = V\Delta\rho + \rho\Delta V \quad \Rightarrow \quad \Delta V = - V\Delta\rho/\rho$$

and the change in the height is:

$$\Delta H = \frac{\Delta V}{A} = \frac{H\Delta V}{V} = \frac{-H\Delta\rho}{\rho} = \frac{-1 \times (-5)}{995.5} = \mathbf{0.005\ m}$$

barely measurable!

Question 6: The atmosphere becomes colder at higher elevations. As an average, the standard atmospheric absolute temperature can be expressed as $T_{atm} = 288 - 6.5\times10^{-3}\, z$, where z is the elevation in meters. How cold is it outside an airplane cruising at 12 000 m, expressed in degrees Kelvin and Celsius?

Solution:

For an elevation of $z = 12\,000$ m we get

$$T_{atm} = 288 - 6.5 \times 10^{-3}\, z = \mathbf{210\ K}$$

To express that in degrees Celsius we get

$$T_C = T - 273.15 = \mathbf{-63.15^oC}$$

Question 7: Using the freezing and boiling point temperatures for water on both the Celsius and Fahrenheit scales, develop a conversion formula between the scales. Find the conversion formula between the Kelvin and Rankine temperature scales.

Solution:

Assuming linear expansion (hence linear relation between scales), let

$F = a\,°C + b$

a and b are to be calculated:

Freezing of water:

$32 = a\,0 + b$

Hence, $b = 32$.

So now we have,

$° F = a C + 32$

From boiling of water:

$212 = a\ 100 + 32$

Hence,

$a = (212-32)/100$

$a = 9/5$

Hence,

$° F = ° C (9/5) + 32$
