DFA $\quad M = (Q, \Sigma, s, \delta, F)$

$\delta : Q \times \Sigma \to Q \quad s \in Q.$

$L(M) = \{x \in \Sigma^* \mid M \text{ accepts } x\}$

NFA $\quad N = (Q, \Sigma, \Delta, S, F)$

$\Delta : Q \times \Sigma \longrightarrow 2^Q \quad ; \quad S \subseteq Q.$

$L(N) = \{x \in \Sigma^* \mid N \text{ accepts } x\}.$

Regular sets.

$A \subseteq \Sigma^*$ is regular if $\exists$ a DFA M s.t $L(M) = A$.

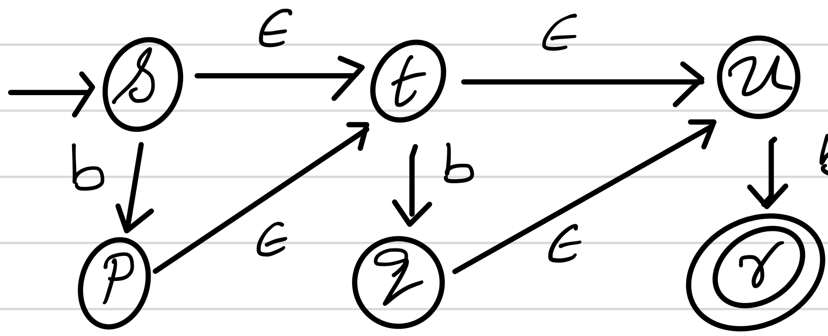
For every NFA N $\exists$ a DFA M s.t $L(M) = L(N)$

$A \subseteq \Sigma^*$ is regular if $\exists$ a NFA s.t $L(N) = A$.

NFA with ϵ-transitions

$p \xrightarrow{\epsilon} q$: the automaton can take an ϵ-transition any time without reading an input symbol

Example - N:



if N is in state s and the next symbol is b then M can do the following:

- Read b and move to state p.
- Move to t without reading any symbol and then read b and move to q
- Move to t (on ϵ), move to u (on ϵ), read b and move to r.

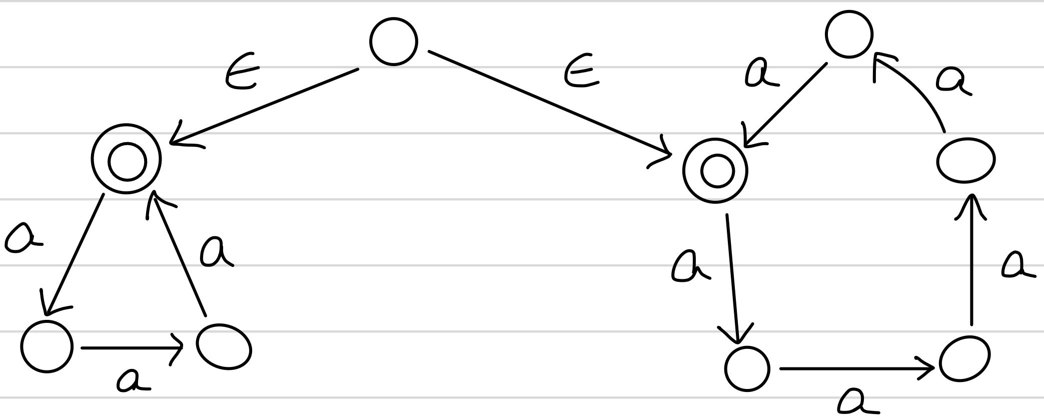
$L(N) = \{b, bb, bbb\}$

Example $\Sigma = \{a\}$

$A = \{w \in \Sigma^* \mid |w| \text{ is divisible by } 3\}$

$B = \{w \in \Sigma^* \mid |w| \text{ is divisible by } 5\}$

$C = \{w \in \Sigma^* \mid |w| \text{ is divisible by } 3 \text{ or } 5\}$



Theorem. For every ε-NFA N, there exists a DFA M s.t $L(M) = L(N)$.

Conclusion: ε-transitions do not add expressive power, it adds convenience.

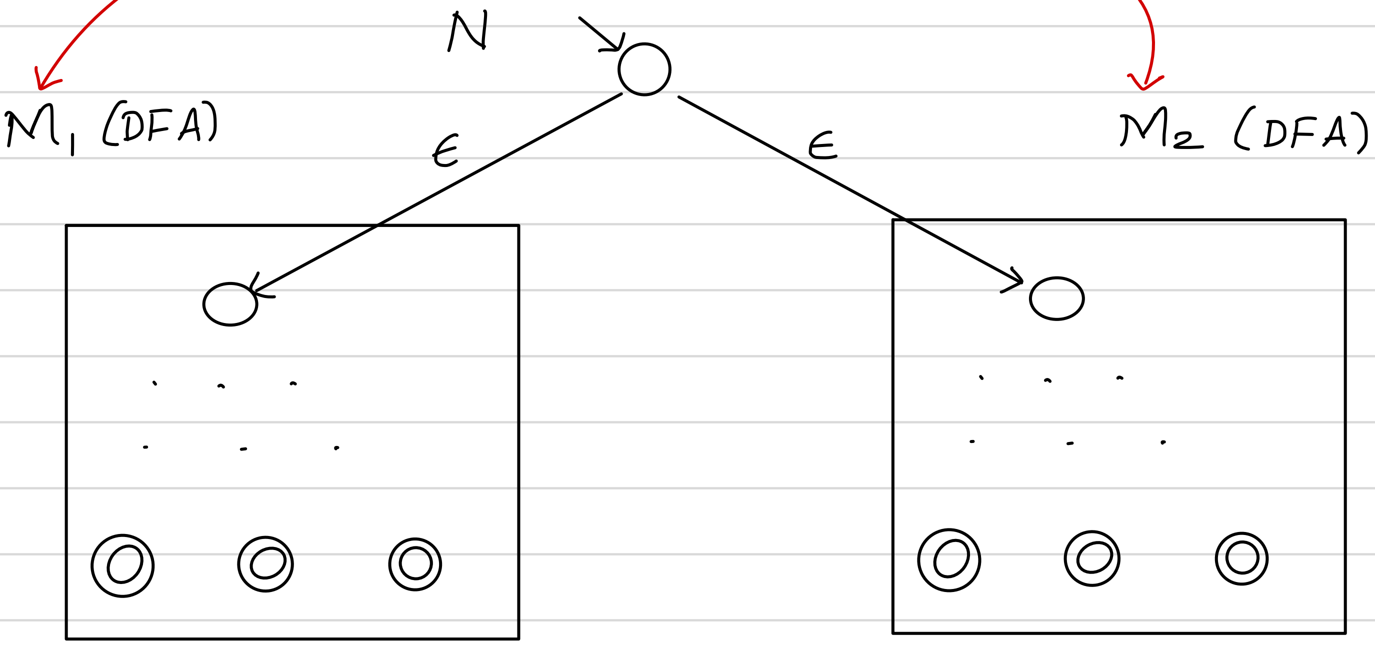
Closure properties (Revisited)

- Regular sets are closed under intersection & union (the product construction).

Union

- De Morgan's Law
- Product Construction
- A simpler construction using an NFA?

$\underbrace{A \cup B}_{N}$ where A and B are regular



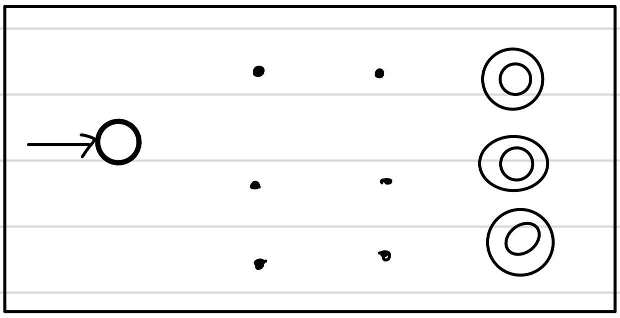
$AB = \{xy \mid x \in A \text{ and } y \in B\}$

If A and B are regular then AB is regular

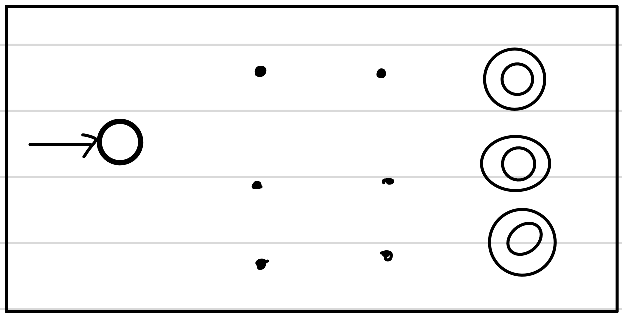
$\exists$ DFA M_1 s.t $L(M_1) = A$ and $\exists$ DFA M_2 s.t $L(M_2) = B$

To construct N s.t $L(N) = AB$
↳ an ϵ-NFA.

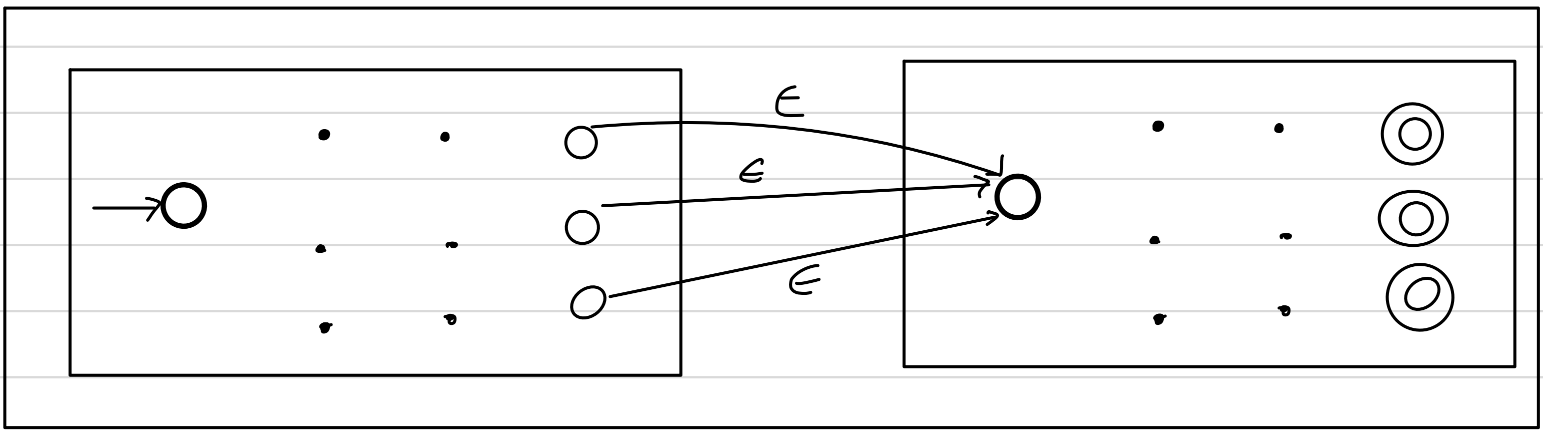
M_1 (DFA)



M_2 (DFA)



NFA: N



$L(N) = AB$

$M_1 = (Q_1, \Sigma, \Delta_1, S_1, F_1)$ $\quad$ $M_2 = (Q_2, \Sigma, \Delta_2, S_2, F_2)$

$L(M_1) = A$ $\quad$ $L(M_2) = B$

$N = (Q, \Sigma, \Delta, S, F)$ st $L(N) = AB$

$Q = Q_1 \cup Q_2$ $\quad$ $S = S_1$ $\quad$ and $\quad$ $F = F_2$

Defining the transition function Δ

$$\Delta(q, a) = \begin{cases} \Delta_1(q, a) & q \in Q_1 \text{ and } q \notin F_1 \\ \Delta_1(q, a) & q \in F_1 \text{ and } a \neq \epsilon \\ \Delta_1(q, a) \cup S_2 & q \in F_1 \text{ and } a = \epsilon \\ \Delta_2(q, a) & q \in Q_2. \end{cases}$$

Let $A \subseteq \Sigma^*$

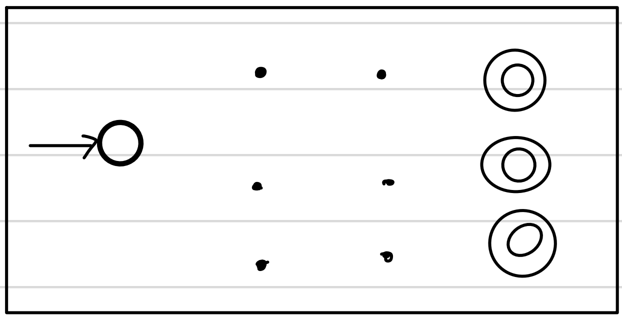
$$A^* = \{x_1 x_2 \ldots x_n \mid n \geq 0 \text{ and } x_i \in A,\ 1 \leq i \leq n\}.$$

$$= \{\epsilon\} \cup A \cup A^2 \cup \cdots$$

Statement. if A is regular then A^* is regular

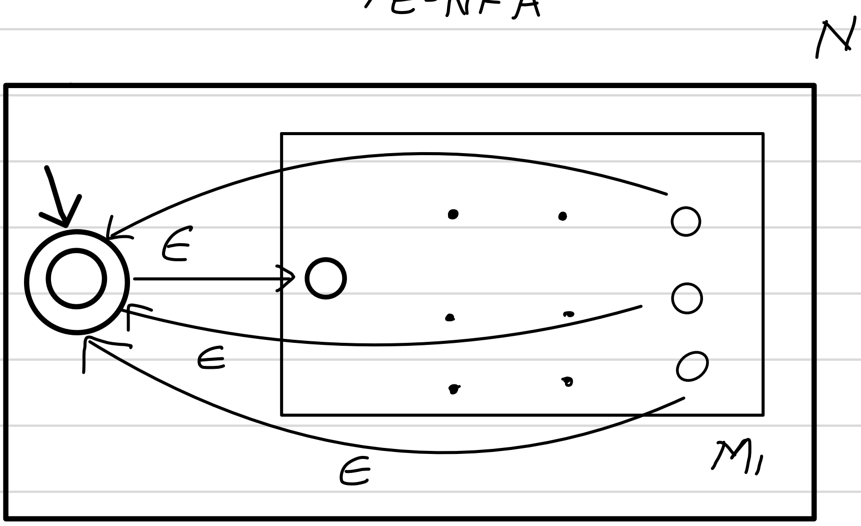
$\exists$ a DFA M_1 s.t $L(M_1) = A$.

M_1 (DFA)



To construct an NFA N s.t $L(N) = A^*$

↳ ϵ-NFA

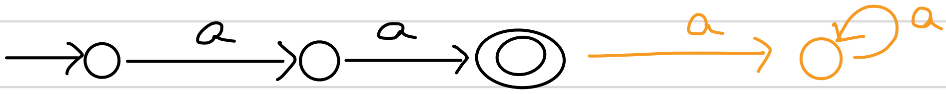


$L(N) = A^*$.

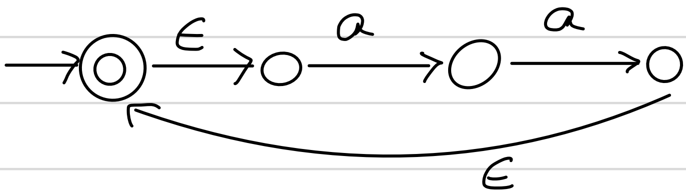
Example

$A = \{aa\}$ $\Sigma = \{a\}$

M:



N s.t $L(N) = A^*$



Suppose $A \subseteq \Sigma^*$ is regular then

$\exists\ M_1 = (Q_1, \Sigma, \Delta_1, S_1, F_1)$ s.t $L(M_1) = A$

Construct $N = (Q, \Sigma, \Delta, S, F)$ s.t $L(N) = A^*$

$Q = Q_1 \cup \{s_0\}$ $\quad S = \{s_0\}$ $\quad F = \{s_0\}$

Transition function Δ

$$\Delta(q,a) = \begin{cases} \Delta_1(q,a) & \text{if } q \in Q_1 \text{ and } q \notin F_1 \\ \Delta_1(q,a) & \text{if } q \in F_1 \text{ and } a \neq \epsilon \\ \Delta_1(q,a) \cup \{s_0\} & \text{if } q \in F_1 \text{ and } a = \epsilon \\ S_1 & \text{if } q = s_0 \text{ and } a = \epsilon \\ \phi & \text{if } q = s_0 \text{ and } a \neq \epsilon. \end{cases}$$