(a) if $A \to \alpha B \beta$ and $B \to \epsilon$ are in $\hat{P}$ then $A \to \alpha\beta \in \hat{P}$

(b) if $A \to B$ and $B \to \gamma$ are in $\hat{P}$ then $A \to \gamma \in \hat{P}$.

Example 1: $\{a^n b^n \mid n \geq 0\} - \{\epsilon\} = \{a^n b^n \mid n \geq 1\}$.

$S \to aSb \mid \epsilon$

$S \to aSb \mid ab$

Add nonterminals A, B

$S \to ASB \mid AB \quad A \to a \quad B \to b$

Add nonterminal C, replace $S \to ASB$ with
$S \to AC$, $C \to SB$

G': $S \to AB \mid AC$, $C \to SB$, $A \to a$, $B \to b$

Balanced Parenthesis $S \to [S] \mid SS \mid \epsilon$

$S \to [S] \mid SS \mid [\,]$

Add new nonterminals A, B

$S \to ASB \mid SS \mid AB$, $A \to [$, $B \to]$

Add a new nonterminal C. Replace $S \to ASB$ with
$S \to AC$ and $C \to SB$.

G': $S \to AB \mid AC \mid SS$, $C \to SB$, $A \to [$, $B \to]$

Linear Grammar.

A CFG G is right linear if all productions are of the form

$$A \to xB, \quad A \to x \quad \text{for } A, B \in N, \ x \in \Sigma^*.$$

At most one nonterminal appears on the RHS. That nonterminal must be the rightmost symbol.

A CFG G is left linear if all productions are of the form

$$A \to Bx \quad A \to x \quad \text{for } A, B \in N, \ x \in \Sigma^*.$$

At most one nonterminal appears on the LHS. That nonterminal must be the leftmost symbol.

A regular grammar is one that is either right linear or left linear.

Example 1. $G_1 = (\{S\}, \{a,b\}, P_1, S)$ with $\underbrace{P_1 : S \to abS \mid a}_{\text{Right linear}}$

$L(G_1) = L((ab)^*a)$

Example 2. $G_2 = (\{S, S_1, S_2\}, \{a,b\}, P_2, S)$ with

$P_2 : S \to S_1ab, \ S_1 \to S_1ab \mid S_2, \ S_2 \to a$

left linear

$L(G_2) = L(a(ab)^*)$

Both G_1 and G_2 are regular grammars.

Example 3. $G_3 = (\{S, A, B\}, \{a, b\}, P_3, S)$ where

P_3 : $\underbrace{S \to A, \quad A \to aB \mid \epsilon, \quad B \to Ab}_{\text{not a regular grammar}}$

Every production is left or right linear but the the grammar is neither left linear nor right linear.

A linear grammar is a grammar in which at most one nonterminal can occur on the RHS of any production irrespective of the position.

Note. A regular grammar is linear. Not all linear grammars are regular.

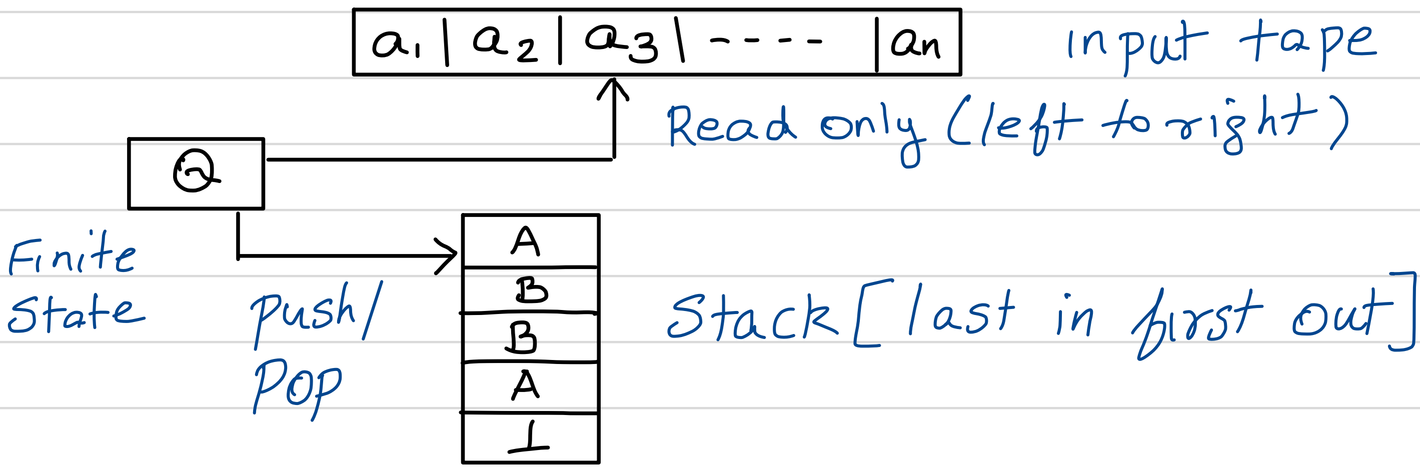
Theorem 1. Let G be a right linear grammar, then $L(G)$ is regular.

Theorem 2. Let $A \subseteq \Sigma^*$ be a regular set then there exists a right linear grammar G s.t $A = L(G)$.

Theorem 3. $A \subseteq \Sigma^*$ is regular iff there exists a regular grammar G s.t $L(G) = A$.

Non deterministic pushdown automata (NPDA)

Finite automata + a stack.



Working of the machine

- pops the top symbol off the stack.
- Makes a transition based on the top of the stack, input symbol and current state.

Transition: push a sequence of symbols onto the stack, change state, move the read head one cell to the right

ϵ-transitions are allowed: Machine can pop and push without reading an input symbol or moving the input head pointer.

Stack can store unbounded information but access is limited

Definition of a nondeterministic PDA.

$$M = (Q, \Sigma, \Gamma, \delta, s, \perp, F)$$

Q - finite set of states, Σ - finite set: input alphabet

$s \in Q$ - start state, $F \subseteq Q$: set of final / accept States.

Γ - finite set: Stack alphabet

$\perp$ - initial stack symbol.

$$\delta \subseteq (Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma) \times (Q \times \Gamma^*)$$

$((p, a, A), (q, B_1, \ldots B_k)) \in \delta$: Example 1

B_1: last, B_k: First

$((p, \epsilon, A), (q, B_1 \ldots B_k)) \in \delta$: Example 2