CS425: Computer Networks
Homework-2

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Solution

problem 1

The only code file attached is `run.py`. To run the file, simply execute the following command:

```
python run.py
```

On running the above command, two choices is provided as shown below:

```
--------------------Program Started---------------------
[1] Generate n-bits CRC message for transmission
[2] Generate random k-bits msg and check the  CRC on received end
Enter your choice [1]/[2]:
```

The choice `[1]`, is the default task which asks for the message to be sent, the CRC pattern `P` and then returns the `n-bit` message will be transmitted.

On the other hand, choice `[2]` asks for number of bits `k`, the length of message, then generates a random message of `k` length. Further it asks for the CRC pattern `P` and the finally after adding a random error it runs the CRC again to check if the message received has any errors or not.

problem 2

Let us the consider the scenario wherer the window size is 2^k. Now, the transmitter starts sending the frames in the order $(0, 1, 2...n-1)$ and all n of the frames will be sent at once. If for some reason the acknowledgment for those frames is lost, the transmitter will transmit all the n frames $(0, 1, 2...n)$ while the receiver will be expecting the frames from the next window $(0, 1, 2...n)$. This will lead to repetition of frames at receiver's end and hence error.
On the other hand if we have window size limited to $2^k - 1$, for the scenario discussed above, the receiver will be expecting frame number n instead of 0 and will accept the repeated frames if transmitter sends them.

problem 3

If we are using a sequence of k-bits we can have a maximum of 2^k messages. Now, for the Selective-Reject ARQ mechanism uppose we have two consecutive windows W_1 and W_2. If we have overlap among these two windows, we can frame numbers for them as $W_1 = 0, 1, 2, 3 ... w - 1$ and $W_1 = w, w + 1, ..0, ...$ So, for those scenarios where the sender fails to receive the acknowledgment for frames from 1^{st} window, it will transmit them again but the receiver which is now expecting frames from 2^{nd} window will consider the newly transmitted frames as of 2^{nd} window's, and will consider that some frames of that window is lost. Here, we got into a problem.
So our window size should restrict no overlap between any two consecutive windows. Now, if the two windows don't overlap anywhere, they will have $2w$ (w for each window) distinct frames and we can have at max 2^k frames. Hence, we have $2w \leq 2^k \implies w \leq 2^{k-1}$.
Thus, a k-bits sequence numbers in Selective-Reject ARQ mechanism have 2^{k-1} window limit.

problem 4

According to the question, the efficiency (U) will be at least 50%. Now, we also have $U = \frac{1}{1+2a}$. So, we have

$$U = \frac{1}{1+2a} \geq \frac{1}{2}$$
$$2 \geq 1 + 2q$$
$$a \leq \frac{1}{2}$$

We also have,

$$a = \frac{propagation\ delay}{transmission\ time} = \frac{t_{prop}}{t_{frame}}$$

According to the question, $t_{prop} = 20$ ms. So, as per the inequalit of a derived above, we have $\frac{20\ ms}{t_{frame}} \leq \frac{1}{2} \implies t_{frame} \geq 40$ ms. For, n-bits frame size we can say

$$\frac{n}{4\ kbps} = t_{frame} \geq 40\ ms$$
$$n \geq 160\ bits$$

So, In stop-and-wait we need the frame size to be at least 160-bits in order to have an efficiency of at leat 50%.

problem 5

Let the probability of error of any bit be $p(= 10^{-3})$. Also, the event of error at each of the 4 bits be denoted by E_1, E_2, E_3 and E_4. Hence,

$$P(E_i) = p$$

(a)

The probability the received frame has no erros $= P(\bar{E_1}).P(\bar{E_2}).P(\bar{E_3}).P(\bar{E_4})$. Which equals to

$$P(\bar{E_1}).P(\bar{E_2}).P(\bar{E_3}).P(\bar{E_4}) = (1 - 10^{-3})^4$$
$$\simeq 0.996$$

(b)

Probability that receiver frame contains atleast 1 error:

$$1 - P(getting\ no\ error) \tilde{=} 1 - 0.996 = 0.004$$

(c)

Since we are sending an additional parity, transmitted bits are 5 in total now. Now, we not that in case of parity check sum, the error is not detected when the number of errors in even. We have,

$$P(error\ not\ detected) = P(\#errors\ is\ even)$$

$$= P(\#errors\ is\ 2) + P(\#errors\ is\ 4)$$

Now we know that,

$$P(\#errors\ is\ i) = \binom{5}{i} P(error\ in\ 1\ bit)^i P(no\ error\ in\ any\ bit)^{5-i}$$

Hence,

$$P(error\ not\ detected) =$$

$$\binom{5}{2} P(error\ in\ 1\ bit)^2 P(no\ error\ in\ any\ bit)^3 + \binom{5}{4} P(error\ in\ 1\ bit)^4 P(no\ error\ in\ any\ bit)^1$$

$$= \binom{5}{2}(0.001)^2(0.999)^3 + \binom{5}{4}(0.001)^4(0.999)^1$$

$$= 0.00000997003 \tilde{=} 9.97 * 10^{-6}$$

problem 6

We have 6 bits in the CRC pattern P hence we first add (#P-1) $0's$ at the end of the message M and then we do the division process to get the remainder.

$$
\begin{array}{l}
\qquad\quad\; 10110110 \\
110011 \,\big)\, \overline{1110001100000} \\
\qquad\quad\; \underline{110011} \\
\qquad\qquad 010111 \\
\qquad\qquad \underline{000000} \\
\qquad\qquad\; 101111 \\
\qquad\qquad\; \underline{110011} \\
\qquad\qquad\quad 111000 \\
\qquad\qquad\quad \underline{110011} \\
\qquad\qquad\quad\; 010110 \\
\qquad\qquad\quad\; \underline{000000} \\
\qquad\qquad\qquad 101100 \\
\qquad\qquad\qquad \underline{110011} \\
\qquad\qquad\qquad\; 111110 \\
\qquad\qquad\qquad\; \underline{110011} \\
\qquad\qquad\qquad\quad 011010 \\
\qquad\qquad\qquad\quad \underline{000000} \\
\qquad\qquad\qquad\qquad 11010
\end{array}
$$

So, the remainder we have is 11010, hence the CRC to be transmitted will be 1110001111010.

problem 7

According to the question, the message to encode is 10010011011. Representing this message, M, as polynomial we have M(x) = $x^{10} + x^7 + x^4 + x^3 + x + 1$. Modifying M(x) by multiplying it by $x^{degree(p)} = x^4$ to get dividend = $x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4$
And we have the divisor as, P(x) = $x^4 + x + 1$

$$
\begin{array}{l}
\qquad\qquad x^{10} + x^6 + x^4 + x^2 \\
x^4 + x + 1 \,\big)\, \overline{x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4} \\
\qquad\qquad\; \underline{x^{14} + x^{11} + x^{10}} \\
\qquad\qquad\quad x^{10} + x^8 + x^7 \\
\qquad\qquad\quad \underline{x^{10} + x^7 + x^6} \\
\qquad\qquad\qquad\quad x^8 + x^6 + x^5 \\
\qquad\qquad\qquad\quad \underline{x^8 + x^5 + x^4} \\
\qquad\qquad\qquad\qquad\quad x^6 + x^4 + x^4 \\
\qquad\qquad\qquad\qquad\quad \underline{x^6 + x^3 + x^2} \\
\qquad\qquad\qquad\qquad\qquad\quad x^3 + x^2
\end{array}
$$

Decoding the remainder $x^3 + x^2$ we have the CRC = 1100. Encoded message to be transmitted will be 10010011011_1100

(b)

Error pattern introduced as per the question 100010000000000, So the message received on the receiver side will be `XOR` of `encoded message` and `error pattern`. Hence, the message received will be

$$100100110111100 \ xor \ 100010000000000 \ = 000110110111100$$

Checking of the presence of error will be checked by dividing the received message by the CRC pattern P. If the remainder is zero then there is no error and if the remainder is nonzero, then the error is detected. Polynomial representation of received message is $x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2$

$$
\begin{array}{r l}
 & x^7 + x^6 + x^3 + x^2 + x \\
\cline{2-2}
x^4 + x + 1 \,\big) & x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2 \\
 & x^{11} + x^8 + x^7 \\
\cline{2-2}
 & \quad x^{10} + x^7 + x^7 \\
 & \quad x^{10} + x^7 + x^6 \\
\cline{2-2}
 & \qquad\quad x^6 + x^5 + x^4 + x^3 + x^4 \\
 & \qquad\quad x^6 + x^3 + x^2 \\
\cline{2-2}
 & \qquad\qquad\quad x^5 + x^2 + x^3 \\
 & \qquad\qquad\quad x^5 + x^2 + x \\
\cline{2-2}
 & \qquad\qquad\qquad\quad x^3 + x + x^2
\end{array}
$$

Remainder is $x^3 + x + x^2 = x^3 + x^2 + x$
Since the remainder is not zero we say we have detected the presence of error.

(c)

This time the error pattern is 100110000000000. Similar to the above part, message received will be

$$100100110111100 \ xor \ 100110000000000 \ = \ 000010110111100$$

Polynomial representation received message is $x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2$

$$
\begin{array}{r l}
 & x^6 + x^4 + x^2 \\
\cline{2-2}
x^4 + x + 1 \,\big) & x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2 \\
 & x^{10} + x^7 + x^6 \\
\cline{2-2}
 & \quad x^8 + x^6 + x^5 \\
 & \quad x^8 + x^5 + x^4 \\
\cline{2-2}
 & \qquad\quad x^6 + x^4 + x^4 \\
 & \qquad\quad x^3 + x^2 \\
\cline{2-2}
 & \qquad\qquad\quad x^3 + x^2 + x^3 \\
 & \qquad\qquad\quad x^2 + x^2 \\
\cline{2-2}
 & \qquad\qquad\qquad\quad 0
\end{array}
$$

Here, we got the remainder to be zero hence the error was not detected.