

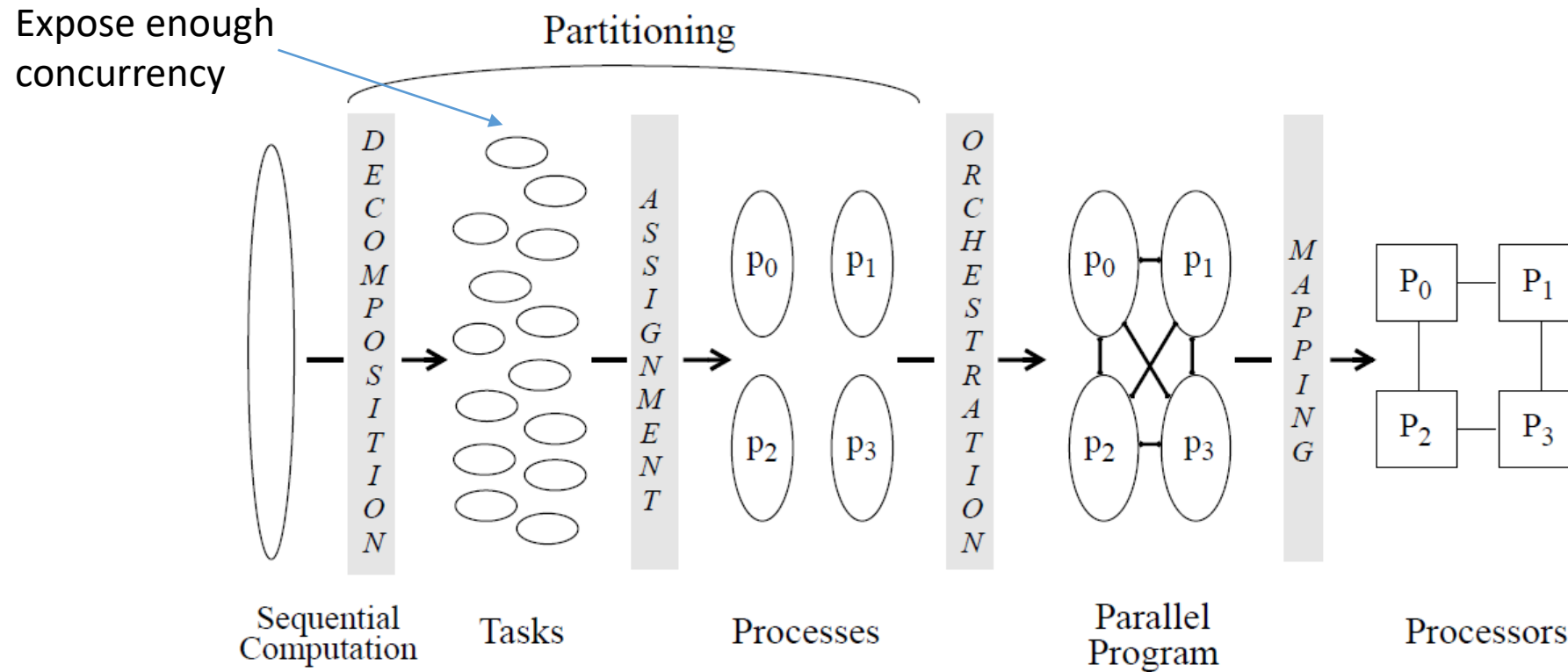
Parallelization (recap)

Derived Datatypes

Vector Variants

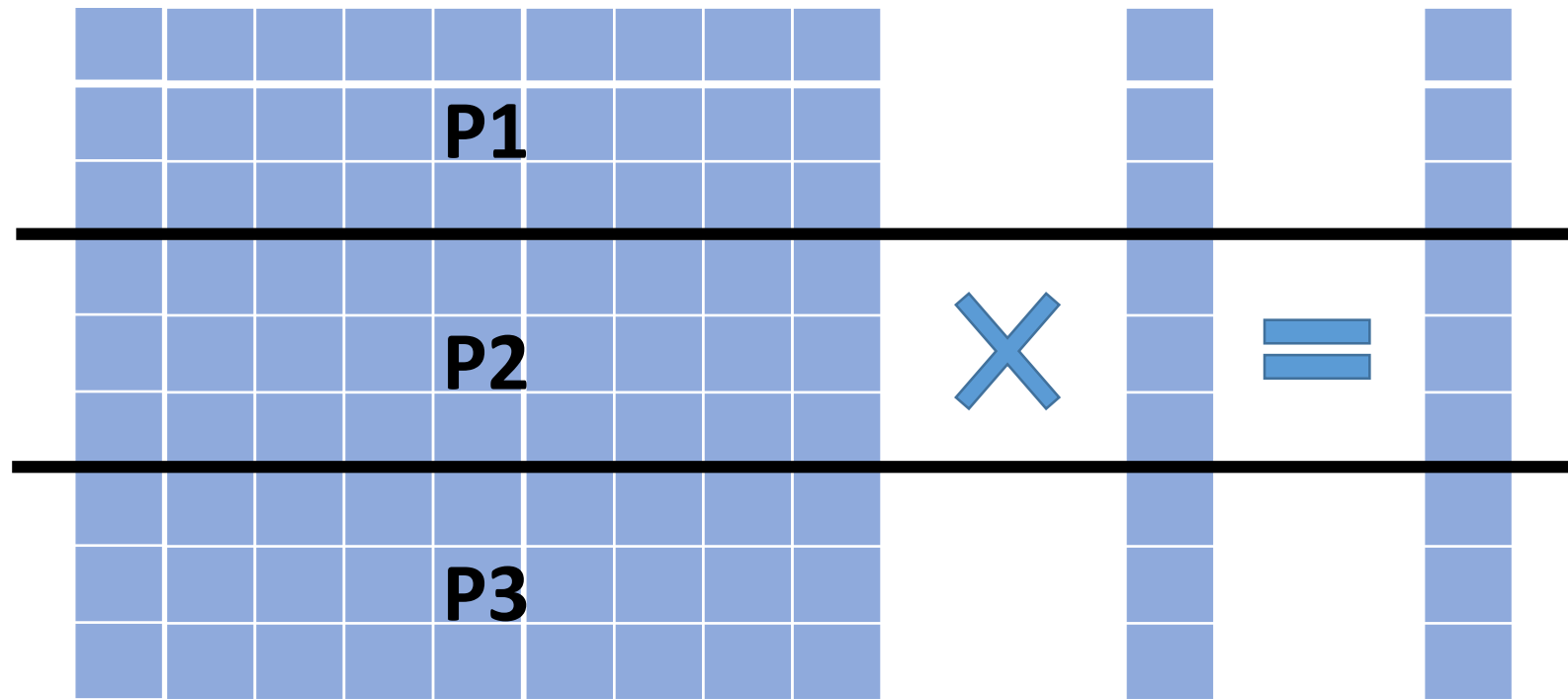
Lecture 13
March 4, 2024

Parallelization Steps

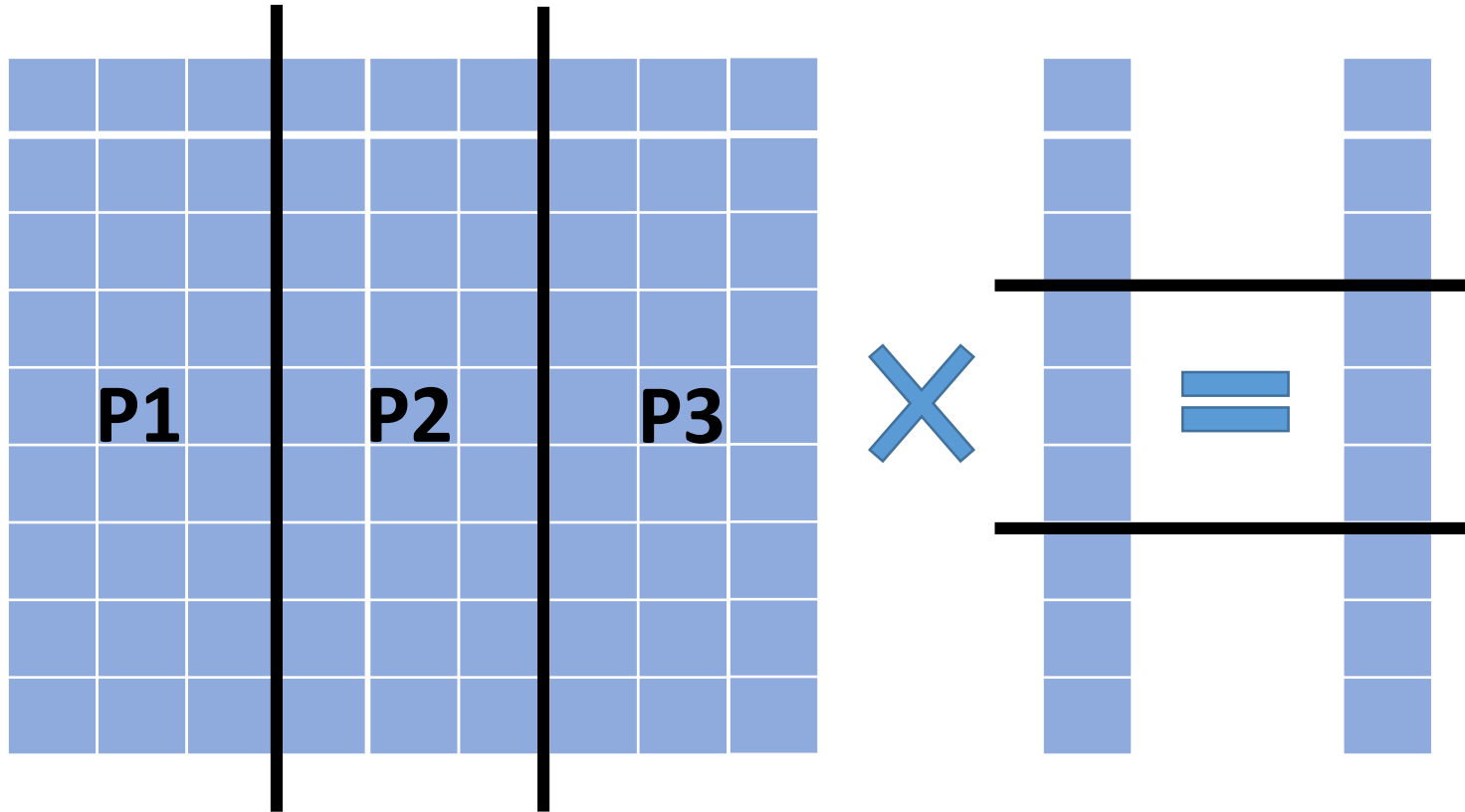


Source: Culler et al.

Matrix Vector Multiplication – Row-wise Decomposition

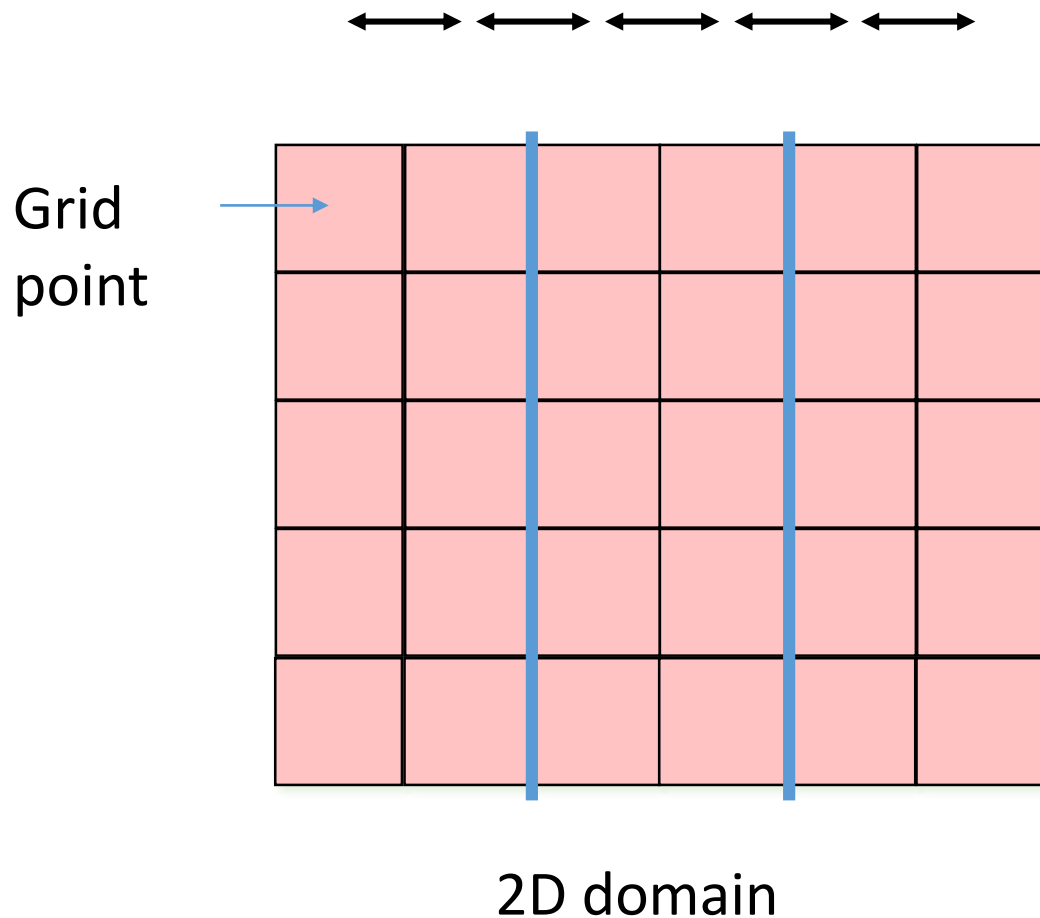


Matrix Vector Multiplication – Column-wise Decomposition



Row-wise vs. column-wise partitioning

1D Domain Decomposition



N grid points

P processes

N/P points per process

#Communications?

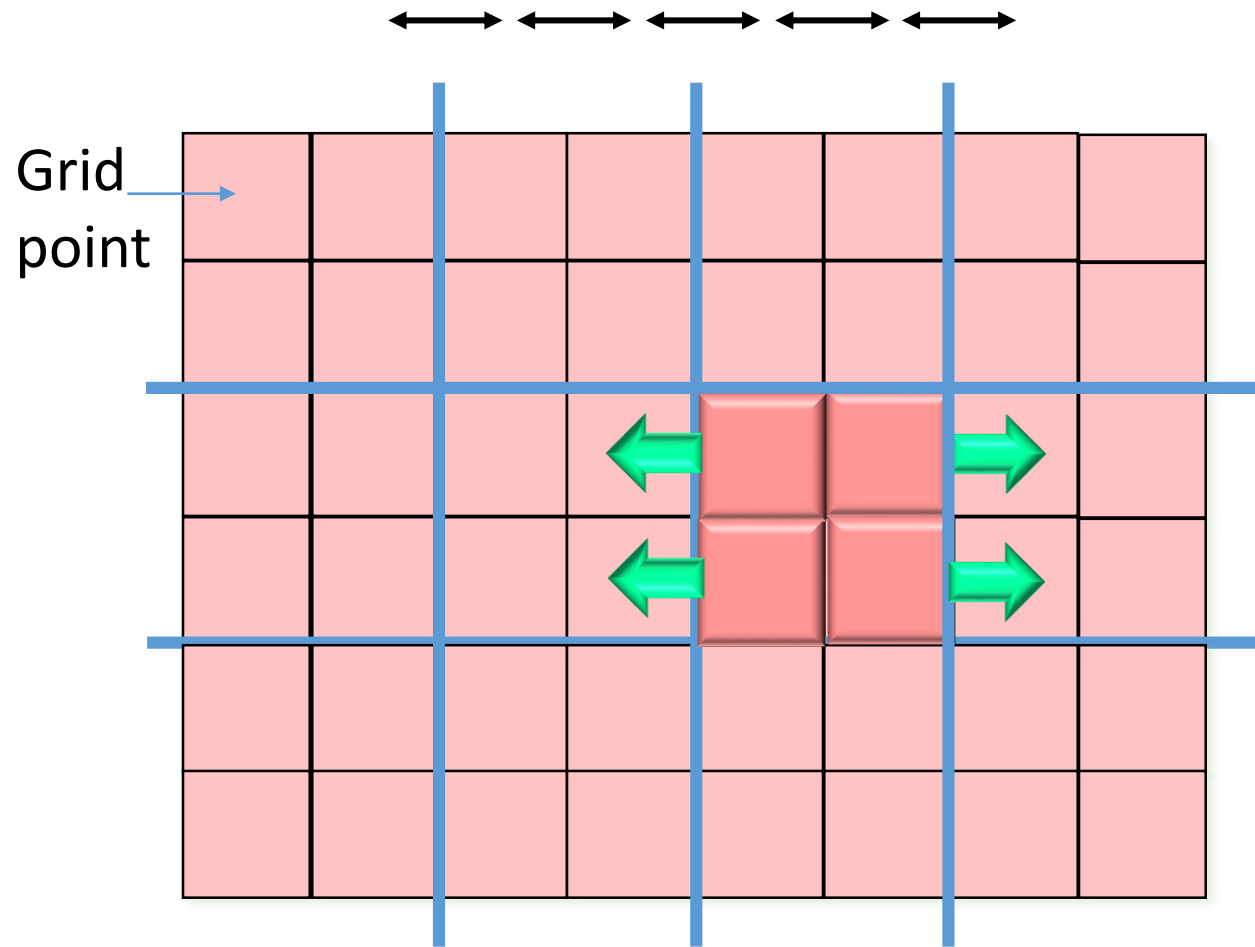
$2\sqrt{N}$ (assuming square grid)

#Computations?

N/P (assuming square grid)

Communication to computation ratio=?

2D Domain decomposition



2 Sends()
2 Recvs()

#Communications?

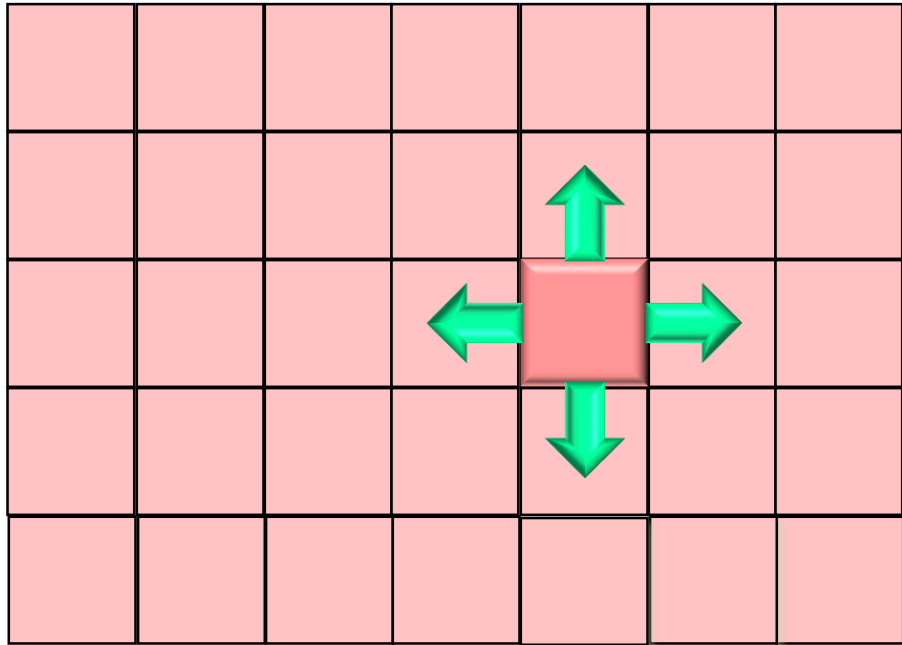
$2\sqrt{N}/\sqrt{P}$ (assuming square grid)

#Computations?

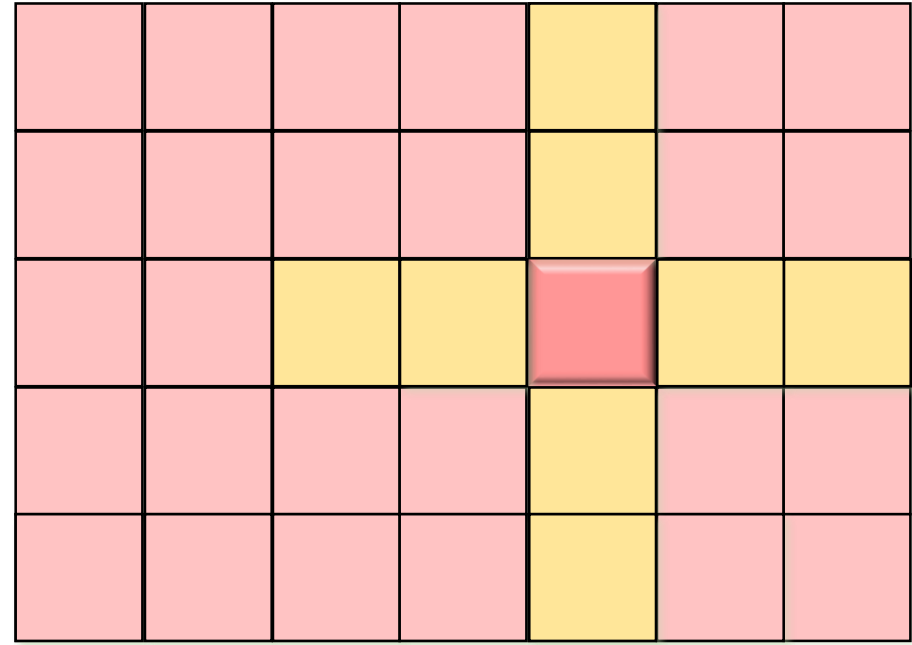
N/P (assuming square grid)

Communication to computation ratio=?

Stencils

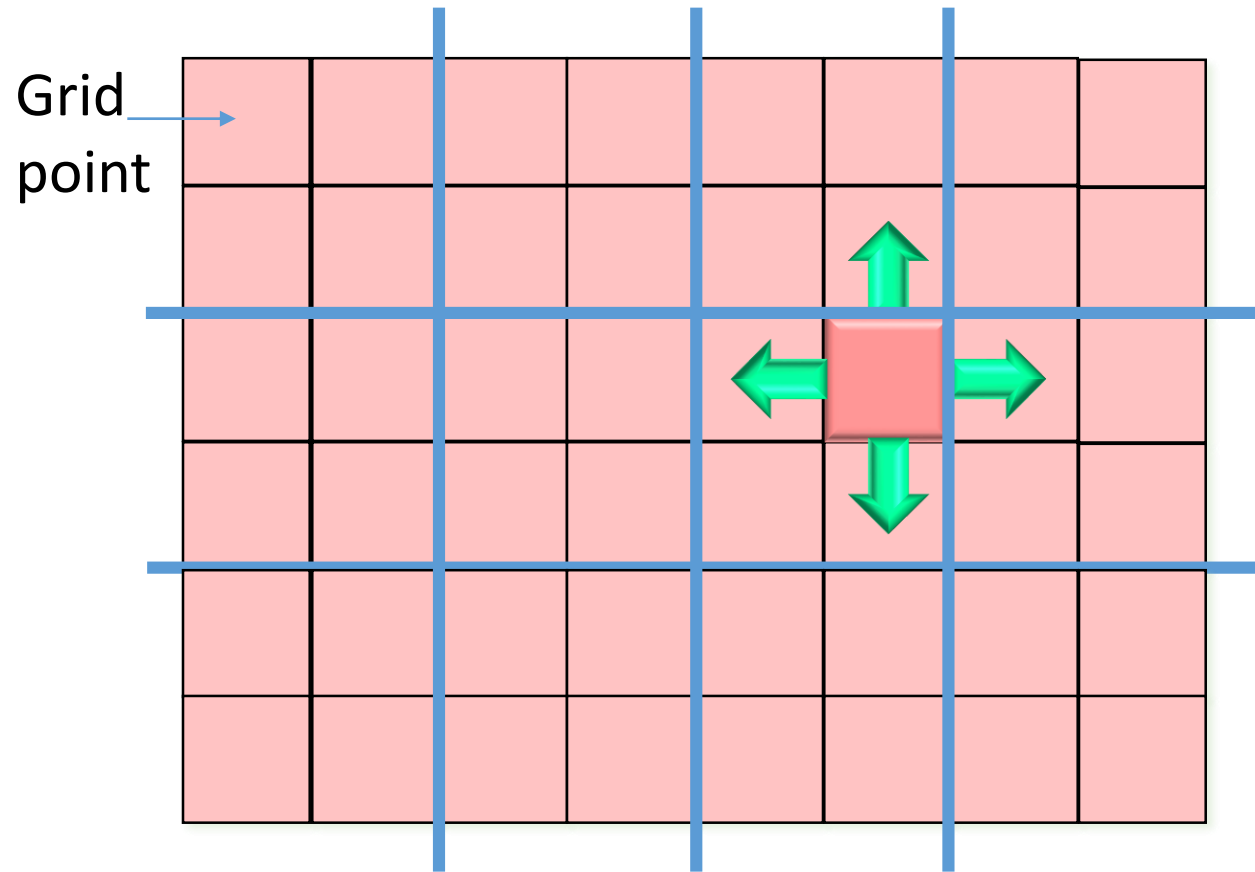


Five-point stencil



Nine-point stencil

2D Domain decomposition



4 Sends()
4 Recvs()

N grid points ($\sqrt{N} \times \sqrt{N}$ grid)
P processes ($\sqrt{P} \times \sqrt{P}$ grid)
N/P points per process

#Communications?

$4\sqrt{N}/\sqrt{P}$ (assuming square grid)

#Computations?

N/P (assuming square grid)

Communication to computation ratio=?

Send / Recv

| | | | | | | | |
|----|----|----|----|--|--|--|--|
| 0 | 1 | 2 | 3 | | | | |
| 4 | 5 | 6 | 7 | | | | |
| 8 | 9 | 10 | 11 | | | | |
| 12 | 13 | 14 | 15 | | | | |
| | | | | | | | |
| | | | | | | | |

MPI_Send

MPI_Recv

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|

Send / Recv

| | | | | | | | |
|----|----|----|----|--|--|--|--|
| 0 | 1 | 2 | 3 | | | | |
| 4 | 5 | 6 | 7 | | | | |
| 8 | 9 | 10 | 11 | | | | |
| 12 | 13 | 14 | 15 | | | | |
| | | | | | | | |
| | | | | | | | |

MPI_Pack (buf)
MPI_Send (buf)

MPI_Recv (buf)
MPI_Unpack (buf)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|

MPI_Pack

```
int MPI_Pack (const void *inbuf, int incount, MPI_Datatype datatype,  
void *outbuf, int outsize, int *position, MPI_Comm comm)
```

```
MPI_Pack (&num1, 1, MPI_INT, buffer, 1000, &position, MPI_COMM_WORLD);  
MPI_Pack (&num2, 1, MPI_INT, buffer, 1000, &position, MPI_COMM_WORLD);  
MPI_Send (buffer, position, MPI_PACKED, dest, 0, MPI_COMM_WORLD);
```

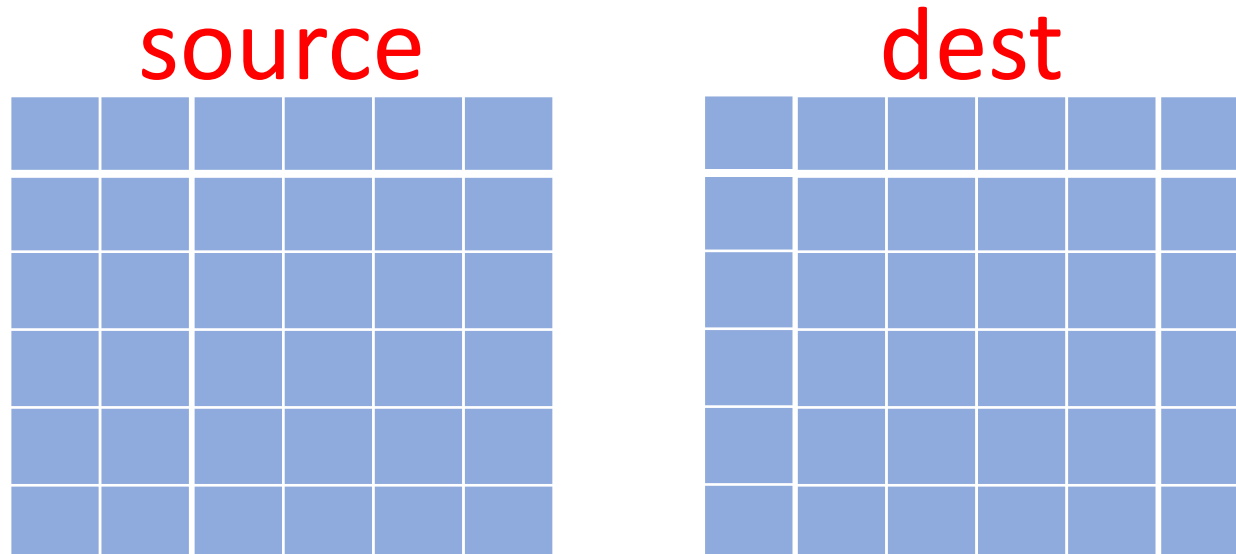
```
MPI_Recv (recvbuf, 2, MPI_INT, source, 0, MPI_COMM_WORLD)
```

MPI_Unpack

```
int MPI_Unpack (const void *inbuf, int insize, int *position, void  
*outbuf, int outcount, MPI_Datatype datatype, MPI_Comm comm)
```

```
for (int r=0; r<6; r++)  
    MPI_Pack (&array[r][5], 1, MPI_INT, buffer, 1000, &position, MPI_COMM_WORLD);  
MPI_Send (buffer, position, MPI_PACKED, dest, 0, MPI_COMM_WORLD);  
  
MPI_Recv (recvBuf, count, MPI_PACKED, source, 0, MPI_COMM_WORLD, &status);  
for (int r=0; r<6; r++)  
    MPI_Unpack (recvBuf, 1000, &position, &recvArr[r][0], 1, MPI_INT, MPI_COMM_WORLD);
```

MPI_PACK Example



```
for (int r=0; r<6; r++)  
    MPI_Pack (&array[r][5], 1, MPI_INT, buffer, 1000, &position, MPI_COMM_WORLD);  
MPI_Send (buffer, position, MPI_PACKED, dest, 0, MPI_COMM_WORLD);  
  
MPI_Recv (recvColumn, count, MPI_INT, source, 0, MPI_COMM_WORLD, &status);
```

MPI_Pack

pack.c

```
MPI_Init(&argc, &argv);

MPI_Comm_rank(MPI_COMM_WORLD, &myrank) ;
MPI_Comm_size(MPI_COMM_WORLD, &size);

// initialize data
for (int i=0; i<M; i++)
    for (int j=0; j<N; j++)
        array2D[i][j] = myrank+i+j;

sTime = MPI_Wtime();
if (myrank == 0) {
    // pack the last element of every row (N ints)
    for (int j=0; j<N; j++) {
        MPI_Pack (&array2D[j][M-1], 1, MPI_INT, buffer, 400, &position, MPI_COMM_WORLD);
        printf ("packed %d %d\n", j, position);
    }
    MPI_Send (buffer, position, MPI_PACKED, 1, 1, MPI_COMM_WORLD);
}
else {
    // receive N ints
    if (myrank == 1)
        MPI_Recv (buffer, count, MPI_INT, 0, 1, MPI_COMM_WORLD, &status);
    // verify
    MPI_Get_count (&status, MPI_INT, &count);
}
eTime = MPI_Wtime();
time = eTime - sTime;

printf ("%lf\n", time);
```

int MPI_Pack (const void
*inbuf, int incount,
MPI_Datatype datatype,
void *outbuf, int outsize,
int *position, MPI_Comm
comm)

Halo Exchange

Sub-domain

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

Sub-domain

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

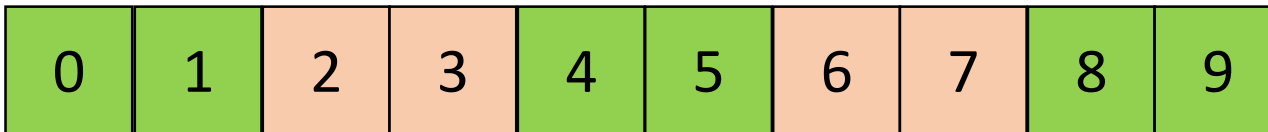
| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

- Every time step
 - Stencil computation
 - $Val_{t+1} = \text{Average of } Val_t \text{ (4 neighboring points)}$
 - Communicate halo regions
 - Multiple MPI_Sends
 - MPI_Pack + MPI_Send
 - MPI Derived datatype + MPI_Send

Derived Datatypes

MPI_Type_vector



count = #blocks

blocklength = #elements in each block

stride = #elements between start of each block

count = 3, blocklength = 1, stride = 4

count = 3, blocklength = 2, stride = 4

MPI_Type_vector (count, blocklength, stride, oldtype, newtype)

Code

```
int N = atoi (argv[1]);
int count = atoi (argv[2]);
int blocklen = atoi (argv[3]);
int stride = atoi (argv[4]);
int numVectors = atoi (argv[5]);
int data[N];

MPI_Type_vector (count, blocklen, stride, MPI_INT, &newvtype);
MPI_Type_commit (&newvtype);

//initialize data
for (int i=0; i<N; i++)
    data[i]=0;

if (myrank == 0)    /* code for process 0 */
{
    for (int i=0; i<N; i++)
        data[i]=i;
    MPI_Send(data, numVectors, newvtype, 1, 99, MPI_COMM_WORLD);
}
else if (myrank == 1) /* code for process 1 */
{
    printf("\n");
    MPI_Recv(data, numVectors, newvtype, 0, 99, MPI_COMM_WORLD, &status);
    MPI_Get_count (&status, MPI_INT, &recvcount);
    for (int i=0; i<N; i++)
        printf ("%d ", data[i]);
    printf("\n\n");
}

MPI_Type_free (&newvtype);
```

vector.c

mpirun -np 2
./a.out 10 5 1 2 1
1 0 3 0 5 0 7 0 9 0

mpirun -np 2
./a.out 20 5 1 2 2
1 0 3 0 5 0 7 0 9
10 0 12 0 14 0 16 0 18 0

Code – Send Selected Columns

vector2D.c

```
int N = atoi (argv[1]);
int column = atoi (argv[2]);
int count = atoi (argv[3]);
int blocklen = atoi (argv[4]);
int stride = atoi (argv[5]);
int data[N][N], received[N*blocklen];

MPI_Type_vector (count, blocklen, stride, MPI_INT, &newvtype);
MPI_Type_commit (&newvtype);

//initialize data
for (int i=0; i<N; i++)
    for (int j=0; j<N; j++)
        data[i][j]=0;

if (myrank == 0)    /* code for process 0 */
{
    for (int i=0; i<N; i++)
        for (int j=0; j<N; j++)
            data[i][j]=column+i+j;
    MPI_Send(
        , 1, newvtype, 1, 99, MPI_COMM_WORLD);
}
else if (myrank == 1) /* code for process 1 */
{
    printf ("\n");
    MPI_Recv(received,
        MPI_INT, 0, 99, MPI_COMM_WORLD, &status);
    for (int i=0; i<count*blocklen; i++)
        printf ("%d ", received[i]);
    printf ("\n\n");
}
```

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

?

?

Examples

```
int N = atoi (argv[1]);  
int column = atoi (argv[2]);  
int count = atoi (argv[3]);  
int blocklen = atoi (argv[4]);  
int stride = atoi (argv[5]);  
int data[N][N], received[N*blocklen];
```

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 5 |
| 3 | 4 | 5 | 6 |

```
class $ mpirun -np 2 ./vector2D 4 0 4 1 4
```

```
class $ mpirun -np 2 ./vector2D 4 0 4 2 4
```

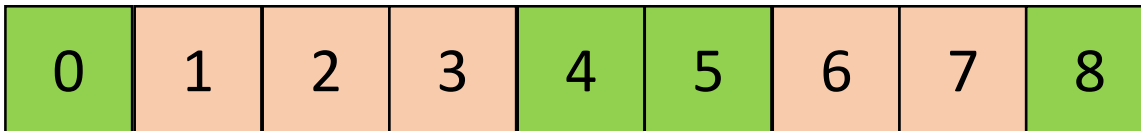
| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 5 |
| 3 | 4 | 5 | 6 |

MPI_Type_indexed

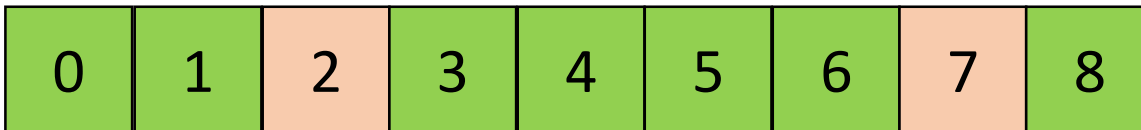
count = #blocks

blocklengths = #elements in each block

displacements = displacement of start of each block



count = 3, blocklengths = 1,2,1
displacements = 0,4,8



count = 3, blocklengths = 2,4,1
displacements = 0,3,8

MPI_Type_indexed (count, blocklengths,
displacements, oldtype, newtype)

Code

```
int N = atoi (argv[1]);
int numElements = atoi (argv[2]);
int count = 3;
int blocklengths[] = {1, 2, 3};
int displacements[] = {0, 3, 6};
int data[N];

MPI_Type_indexed (count, blocklengths, displacements, MPI_INT, &newtype);
MPI_Type_commit (&newtype);

//initialize data
for (int i=0; i<N; i++)
    data[i]=0;

if (myrank == 0)    /* code for process 0 */
{
    for (int i=0; i<N; i++)
        data[i]=i;
    MPI_Send(data, numElements, newtype, 1, 99, MPI_COMM_WORLD);
}

/* code for process 1 */

MPI_Recv(data, numElements, newtype, 0, 99, MPI_COMM_WORLD, &status);
```

```
class $ mpirun -np 2 ./indexed 30 1
```

Lower Triangular Matrix

```
int N = atoi (argv[1]);
int data[N][N];
int count = N;
int blocklengths[N], displacements[N];
```

```
MPI_Type_indexed (count, blocklengths, displacements, MPI_INT, &newtype);
MPI_Type_commit (&newtype);
```

```
//initialize data
```

```
for (int i=0; i<N; i++)
  for (int j=0; j<N; j++)
    data[i][j]=0;
```

```
if (myrank == 0)    /* code for process 0 */
{
  for (int i=0; i<N; i++)
    for (int j=0; j<N; j++)
      data[i][j]=i+j;
  MPI_Send(data, 1, newtype, 1, 99, MPI_COMM_WORLD);
}
```

mpirun -np 2 ./indexed2D 4

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 5 |
| 3 | 4 | 5 | 6 |

Summary

MPI_Datatype newtype

- MPI_Type_vector (count, blocklength, stride, oldtype, newtype)
- MPI_Type_indexed (count, blocklengths, displacements, oldtype, newtype)
- MPI_Type_create_subarray (ndims, array_of_sizes, array_of_subsizes, array_of_starts, order, oldtype, newtype)

MPI_Type_commit (newtype)

.....

MPI_Type_free (newtype)

MPI Collectives Variants

Collectives – Variable Data

- Communicate unequal amount of data to/from each process involved in the collective function call

| | | |
|-----------|--|--|
| 2B | | |
| 2B | | |
| 2B | | |

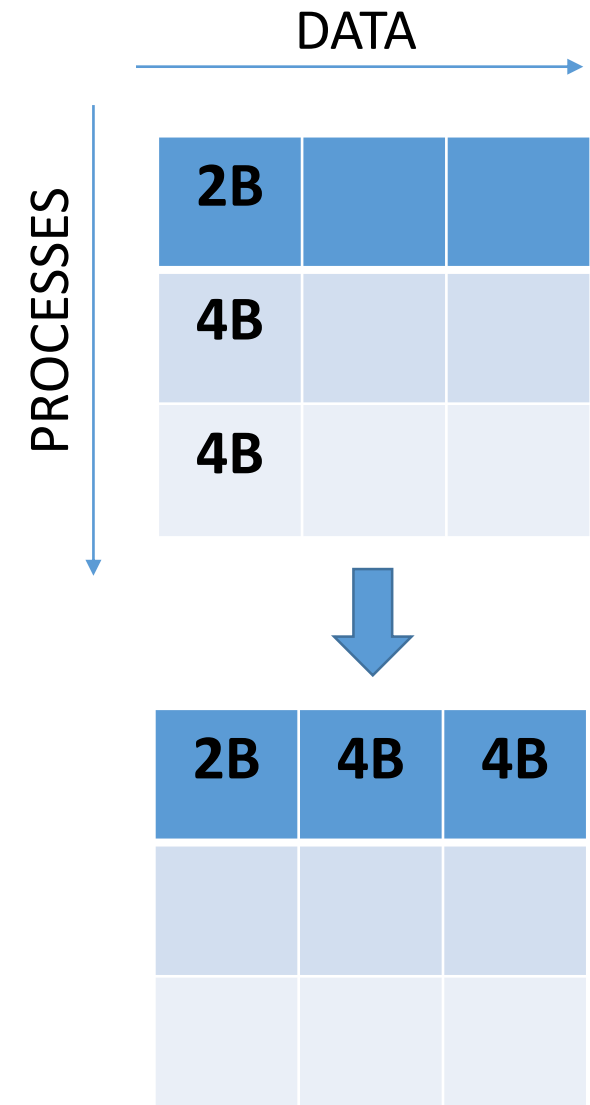
| | | |
|-----------|--|--|
| 2B | | |
| 3B | | |
| 4B | | |

Gatherv

- Root gathers different amounts of data from the other processes
- int `MPI_Gatherv` (sendbuf, sendcount, sendtype, *recvbuf*, recvcounts, displs, recvtype, root, comm)
- recvcounts – Number of elements to be received from each process
- displs – Displacement at which to place received data

`MPI_Recv` (`recvbuf+displs[i]`, `recvcounts[i]`, `recvtype`, `i`, `i`, `comm`, `&status`) at root for i^{th} process

`MPI_Send` at non-root



```

int message[arrSize];
int countArray[numtasks], displArray[numtasks];

int displ = 0;           // note that root process is 0 here

// this information is needed by the root
if (!rank)
    for (i = 0; i < numtasks; i++) {
        countArray[i] = arrSize*(i+1);           // depends on the counts, root may need to get it from the processes
        displArray[i] = displ;
        displ += countArray[i];
        printf ("%d %d %d\n", i, countArray[i], displArray[i]);
    }

if (!rank)
    printf ("\n");

// every process initializes their local array
srand(time(NULL));
for (i = 0; i < arrSize; i++) {
    message[i] = i; // (double)rand() / (double)RAND_MAX;
}

int recvMessage[displ];    // significant at the root process

// receive different counts of elements from different processes
MPI_Gatherv (message, arrSize, MPI_INT, recvMessage, countArray, displArray, MPI_INT, 0, MPI_COMM_WORLD);

if (!rank)
    for (i = 0; i < displ; i++) {
        printf ("%d %d\n", i, recvMessage[i]);
    }

```

Demo

```
class $ mpirun -np 2 ./gatherv 2  
0 2 0  
1 4 2
```

```
0 0  
1 1  
2 0  
3 1  
4 2  
5 3
```

```
class $ mpirun -np 3 ./gatherv 2  
0 2 0  
1 4 2  
2 6 6
```

```
0 0  
1 1  
2 0  
3 1  
4 2  
5 3  
6 0  
7 1  
8 2  
9 3  
10 4  
11 5
```

Bug

```
Fatal error in PMPI_Gatherv: Message truncated, error stack:
PMPI_Gatherv(435).....: MPI_Gatherv failed(sbuf=0x7ffd379c4250, scount=2, MPI_INT, rbuf=0x7ffd379c4210, rcnts=0x7ffd379c4240,
displs=0x7ffd379c4230, MPI_INT, root=0, MPI_COMM_WORLD) failed
MPIR_Gatherv_impl(235).....:
MPIR_Gatherv(151).....:
MPIDI_CH3_PktHandler_EagerShortSend(363): Message from rank 1 and tag 4 truncated; 16 bytes received but buffer size is 8
class $ vi gatherv.c
class $ !mpicc
mpicc -o gatherv gatherv.c
class $ !mpirun
mpirun -np 2 ./gatherv 2
0 2 0
1 4 2
class $
```

```
class $ time mpirun -np 3 ./gatherv 2

real    0m0.007s
user    0m0.004s
sys     0m0.010s
class $ time mpirun -np 30 -hosts csews4:10,csews2:10,csews3:10 ./gatherv 200

real    0m0.695s
user    0m0.026s
sys     0m0.006s
class $ time mpirun -np 30 -hosts csews4:10,csews2:10,csews3:10 ./gatherv 20000

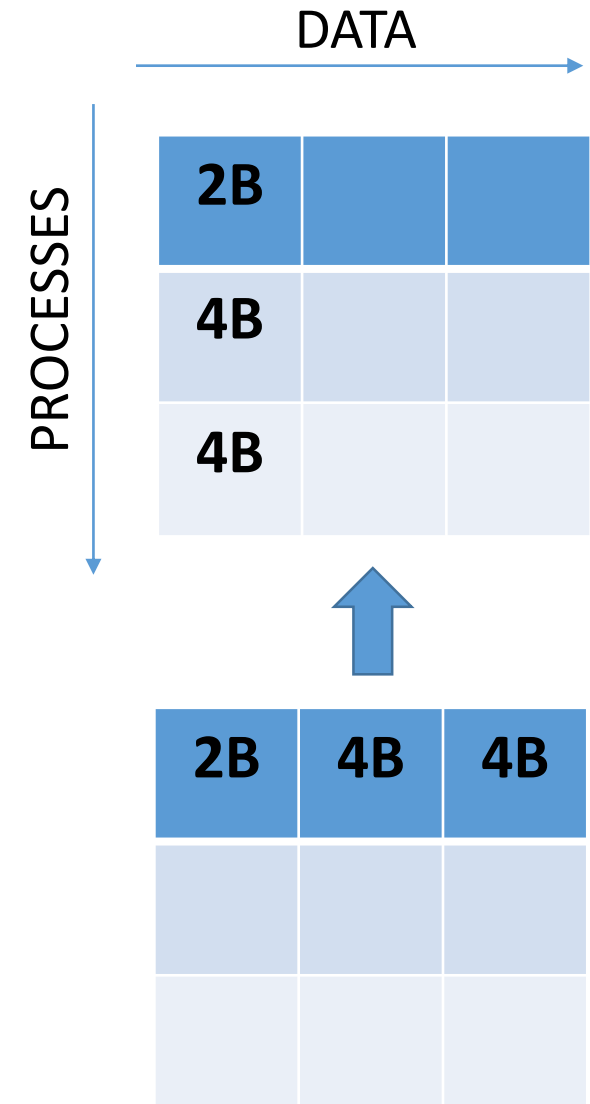
real    0m1.857s
user    0m0.033s
sys     0m0.002s
class $ time mpirun -np 60 -hosts csews4:10,csews2:10,csews3:10,csews5:10,csews6:10,csews7:10 ./gatherv 20000

real    0m2.965s
user    0m12.571s
sys     0m1.559s
class $ time mpirun -np 60 -hosts csews4:10,csews2:10,csews3:10,csews5:10,csews6:10,csews7:10 ./gatherv 800000

real    1m5.621s
user    7m2.519s
sys     1m2.687s
class $ █
```

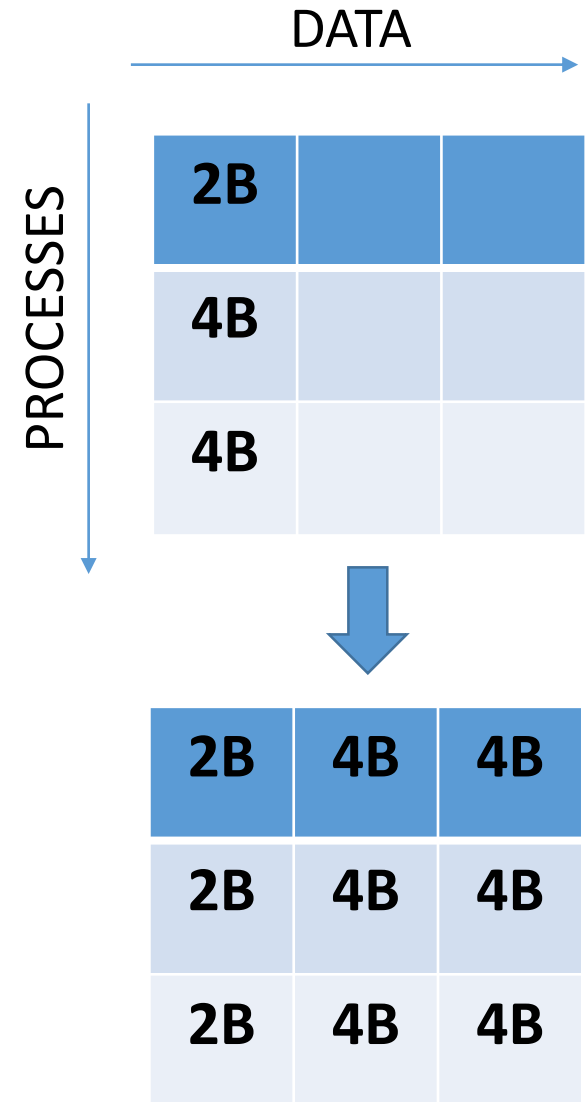
MPI_Scatterv

- Root scatters different amounts of data to the other processes
- int `MPI_Scatterv` (const void *sendbuf, const int *sendcounts, const int *displs, MPI_Datatype sendtype, void *recvbuf, int recvcount, MPI_Datatype recvtype, int root, MPI_Comm comm)
- sendcounts – Number of elements to be sent to each process
- displs – Displacement (relative to sendbuf) at which the data to be sent resides



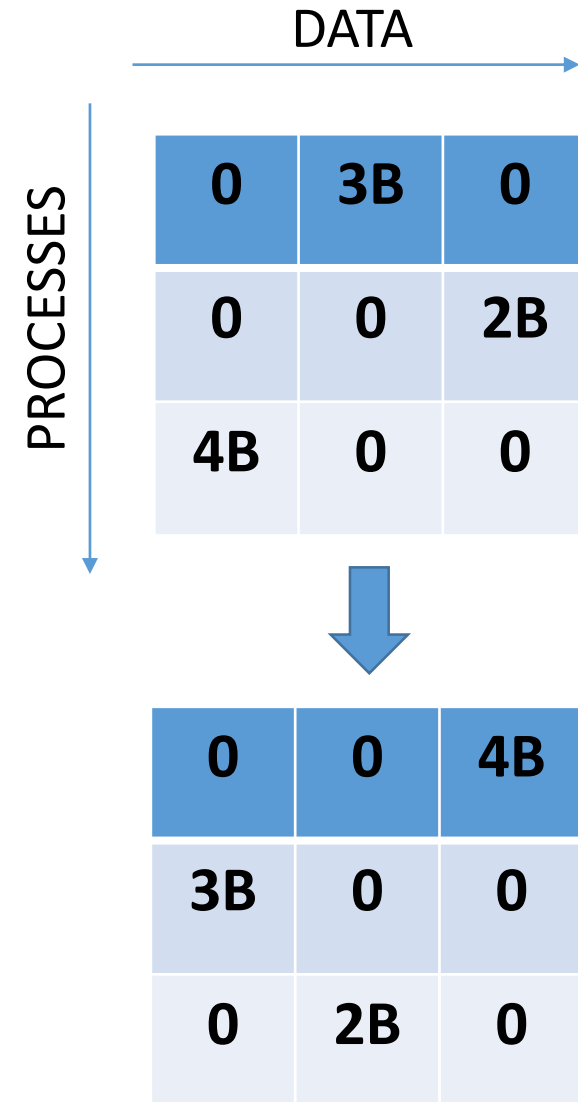
Allgatherv

- All processes gather values of different lengths from all processes
- int `MPI_Allgatherv` (sendbuf, sendcount, sendtype, *recvbuf*, recvcounts, displs, recvtype, comm)
- *recvcounts* – Number of elements to be received from each process
- *displs* – Displacement at which to place received data



Alltoallv

- Every process sends data of different lengths to other processes
- int `MPI_Alltoallv` (sendbuf, sendcount, sdispls, sendtype, *recvbuf*, recvcount, rdispls, recvtype, comm)
- Output parameter – *recvbuf*
- It's not necessary to receive some data from all processes, i.e. some entries of count and displs may be 0



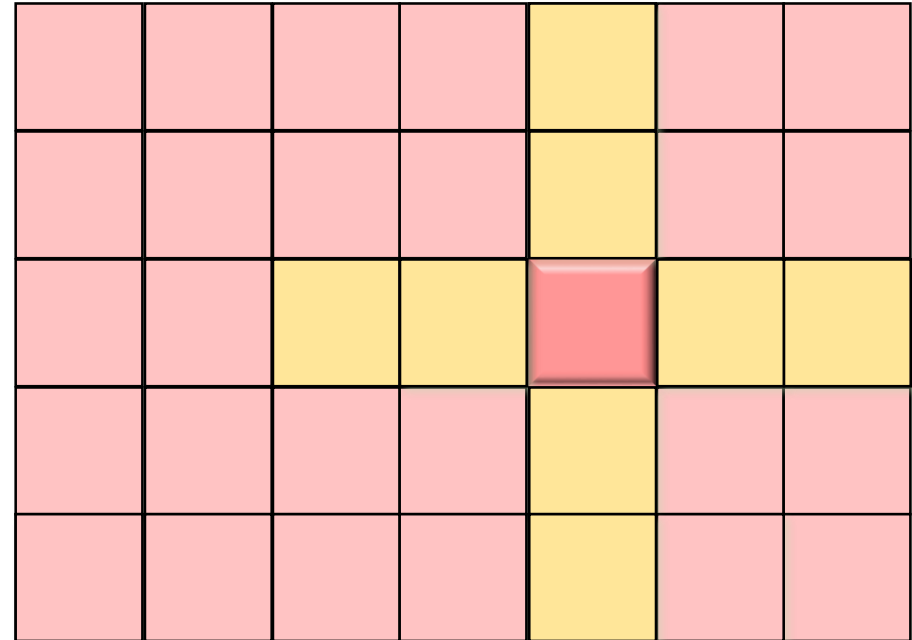
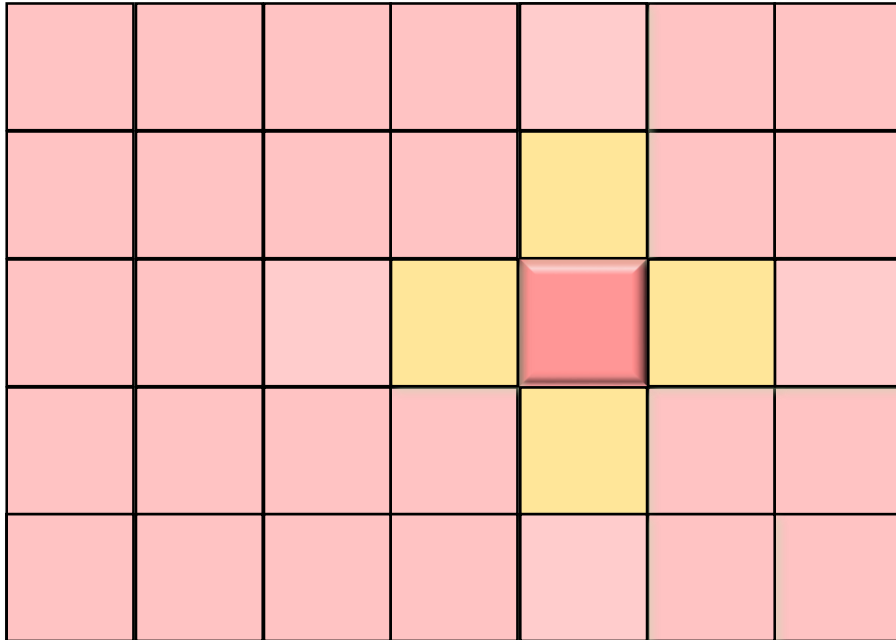
Non-blocking Collectives

- Introduced in MPI-3
- Benefit of non-blocking point-to-point
- Overlap communication and computation
- Reduce synchronization
- Improve performance for overlapping communicators
- How do we ensure completion?
 - `MPI_Wait` (request, status)

Non-blocking Collectives

- `MPI_Ibcast` (buffer, count, datatype, root, comm, request)
- `MPI_Igather` (sendbuf, sendcount, sendtype, recvbuf, recvcount, recvtype, root, comm, request)
- `MPI_Igatherv` (sendbuf, sendcount, sendtype, recvbuf, recvcounts, displs, recvtype, root, comm, request)
- `MPI_Ialltoall` (sendbuf, sendcount, sendtype, recvbuf, recvcount, recvtype, comm, request)
- ...

Assignment 1 (Compare 5- and 9-point Stencil)



Halo Exchange (5- and 9-point Stencil)

Sub-domain

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

Sub-domain

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

| | | | |
|----|----|----|----|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

- Every time step t
 - Communicate halo regions
 - MPI_Pack + send
 - recv + MPI_Unpack
 - Stencil computation
 - $Val_{t+1} = \text{Average of } Val_t \text{ (x neighboring points) and itself}$
[x= 4 or 8]