EE2100: Matrix Theory

Quiz - 2

Please Note :

1. If there are any inconsistencies in grading, please use the google form shared with you.

Group 1

(One among the following set of questions)

1. (3 points) Consider the problem of computing the orthonormal basis of the subspace using the Grahm Schmidt Approach. Let $\mathbb{W}$ denote the subspace spanned by the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} \in \mathcal{R}^3$ where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ a \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} \text{ and, } \mathbf{v}_3 = \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

The first entry of the third orthonormal basis vector for the subspace $\mathbb{W}$ is (Please Note: Apply the Grahm Schmit algorithm by considering vectors in the following order, i.e., $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$)

2. (3 points) Consider the problem of computing the orthonormal basis of the subspace using the Grahm Schmidt Approach. Let $\mathbb{W}$ denote the subspace spanned by the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} \in \mathcal{R}^3$ where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ a \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} \text{ and, } \mathbf{v}_3 = \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

The second entry of the third orthonormal basis vector for the subspace $\mathbb{W}$ is (Please Note: Apply the Grahm Schmit algorithm by considering vectors in the following order, i.e., $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$)

Group 2

(One among the following set of questions)

1. (3 points) Consider the problem of expressing the vector $\mathbf{v} \in \mathcal{R}^3$ as a linear combination of combination of the basis vectors $\left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_1}, \underbrace{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}_{\mathbf{b}_2}, \underbrace{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_3} \right\}$ i.e., $\mathbf{v} = \sum_{i=1}^{3} \alpha_i \mathbf{b}_i$. Let $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Compute the coefficient of linear combination α_3.

2. (3 points) Consider the problem of expressing the vector $\mathbf{v} \in \mathcal{R}^3$ as a linear combination of combination of the basis vectors $\left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_1}, \underbrace{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}_{\mathbf{b}_2}, \underbrace{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_3} \right\}$ i.e., $\mathbf{v} = \sum_{i=1}^{3} \alpha_i \mathbf{b}_i$. Let $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Compute the coefficient of linear combination α_1.

3. (3 points) Consider the problem of expressing the vector $\mathbf{v} \in \mathcal{R}^3$ as a linear combination of combination of the basis vectors $\left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_1}, \underbrace{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}_{\mathbf{b}_2}, \underbrace{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_3} \right\}$ i.e., $\mathbf{v} = \sum_{i=1}^{3} \alpha_i \mathbf{b}_i$. Let $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Compute the coefficient of linear combination α_2.

Group 3

(One among the following set of questions)

1. (1 point) For some $\mathbf{a}$,$\mathbf{b} \in \mathcal{R}^n$, that satisfy $\mathbf{b} \neq \alpha\mathbf{a}$, $\mathbf{Proj_a b} = \mathbf{0}$

   A. True

   B. False

2. (1 point) For all $\mathbf{a}$,$\mathbf{b} \in \mathcal{R}^n$, that satisfy $\mathbf{b} \neq \alpha\mathbf{a}$, $\mathbf{Proj_a b} = \mathbf{0}$

   A. True

   B. False

Group 4

(One among the following set of questions)

1. (1 point) Consider all possible sets of 3 linearly independent vectors (denoted by say, $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathcal{R}^3$) such that $\mathbf{b} \perp \mathbf{a}$. Let $\mathbf{d} = \mathbf{c} - \mathbf{Proj_b c} - \mathbf{Proj_a c}$. Then, $\mathbf{d} \perp \mathbf{b}$ and $\mathbf{d} \perp \mathbf{a}$.

   A. True

   B. False

2. (1 point) Consider all possible sets of 3 linearly independent vectors (denoted by say, $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathcal{R}^3$) . Let $\mathbf{d} = \mathbf{c} - \mathbf{Proj_b c} - \mathbf{Proj_a c}$. Then, $\mathbf{d} \perp \mathbf{b}$ and $\mathbf{d} \perp \mathbf{a}$.

   A. True

   B. False

Group 5

1. Consider all possible $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathcal{R}^3$ such that $\mathbf{a} \perp \mathbf{b}$ and $\mathbf{b} \perp \mathbf{c}$. Then $\mathbf{a} \perp \mathbf{c}$.

   A. True

   B. False

Group 6

(One among the following set of questions)

1. (3 points) The coordinate vector of $\mathbf{x} \in \mathcal{R}^3$ in basis $\mathcal{B} = \left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_1}, \underbrace{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}_{\mathbf{b}_2}, \underbrace{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_3} \right\}$ is $\mathbf{x}_{\mathbb{B}}$. If $\mathbf{x}_{\mathbb{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, the first entry of $\mathbf{x}$ in the standard basis is

2. (3 points) The coordinate vector of $\mathbf{x} \in \mathcal{R}^3$ in basis $\mathcal{B} = \left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_1}, \underbrace{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}_{\mathbf{b}_2}, \underbrace{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_3} \right\}$ is $\mathbf{x}_{\mathbb{B}}$. If $\mathbf{x}_{\mathbb{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, the second entry of $\mathbf{x}$ in the standard basis is

3. (3 points) The coordinate vector of $\mathbf{x} \in \mathcal{R}^3$ in basis $\mathcal{B} = \left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_1}, \underbrace{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}_{\mathbf{b}_2}, \underbrace{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{b}_3} \right\}$ is $\mathbf{x}_{\mathbb{B}}$. If $\mathbf{x}_{\mathbb{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, the third entry of $\mathbf{x}$ in the standard basis is