EE2100: Matrix Theory

Quiz - 3

Group 1

(One among the following set of questions)

1. (3 points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{4\times 4}$ given by

$$\mathbf{A} = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Let $\mathbf{B} \in \mathcal{R}^{4\times 4}$ denote the inverse of a matrix. Compute the entry B_{13} [Hint: use the idea of inverse linear transformation].

2. (3 points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{4\times 4}$ given by

$$\mathbf{A} = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Let $\mathbf{B} \in \mathcal{R}^{4\times 4}$ denote the inverse of a matrix. Compute the entry B_{14} [Hint: use the idea of inverse linear transformation].

Group 2

(One among the following set of questions)

1. (1 point) If $\forall \mathbf{v} \in \mathcal{R}^n$ $\mathbf{Av} = \mathbf{Bv}$ (where $\mathbf{A} \in \mathcal{R}^{m\times n}$, $\mathbf{B} \in \mathcal{R}^{m\times n}$), then $\mathbf{A} = \mathbf{B}$.

   A. True

   B. False

2. (1 point) If $\exists \mathbf{v} \in \mathcal{R}^n$ such that $\mathbf{Av} = \mathbf{Bv}$ (where $\mathbf{A} \in \mathcal{R}^{m\times n}$, $\mathbf{B} \in \mathcal{R}^{m\times n}$), then $\mathbf{A} = \mathbf{B}$.

   A. True

   B. False

Group 3 (All the questions)

1. (2 points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{4\times 4}$ whose column vectors are given by

$$\begin{aligned}\mathbf{a}_1 =& \mathbf{e}_1 \\ \mathbf{a}_2 =& \mathbf{e}_3 + x\mathbf{e}_2 \\ \mathbf{a}_3 =& x\mathbf{e}_3 - \mathbf{e}_2 \\ \mathbf{a}_4 =& \mathbf{e}_4\end{aligned} \tag{9}$$

   The rank of the matrix is

2. (2 points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{4\times 4}$ whose column vectors are given by

$$\begin{aligned}\mathbf{a}_1 =& \mathbf{e}_1 \\ \mathbf{a}_2 =& \mathbf{e}_3 + x\mathbf{e}_2 \\ \mathbf{a}_3 =& x\mathbf{e}_3 - \mathbf{e}_2 \\ \mathbf{a}_4 =& \mathbf{e}_4\end{aligned} \tag{10}$$

   The nullity of the matrix is

3. (4 points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{4\times 4}$ whose column vectors are given by

$$\begin{aligned}\mathbf{a}_1 =& \mathbf{e}_1 \\ \mathbf{a}_2 =& \mathbf{e}_3 + x\mathbf{e}_2 \\ \mathbf{a}_3 =& x\mathbf{e}_3 - \mathbf{e}_2 \\ \mathbf{a}_4 =& \mathbf{e}_4\end{aligned} \tag{11}$$

   The second entry of the vector $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} = \mathbf{b}$ (where $\mathbf{b} = [p, q, r, s]^t$ is

4. (4 points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{4\times 4}$ whose column vectors are given by

$$\begin{aligned}\mathbf{a}_1 =& \mathbf{e}_1 \\ \mathbf{a}_2 =& \mathbf{e}_3 + x\mathbf{e}_2 \\ \mathbf{a}_3 =& x\mathbf{e}_3 - \mathbf{e}_2 \\ \mathbf{a}_4 =& \mathbf{e}_4\end{aligned} \tag{14}$$

   Let $\mathbf{C} = \mathbf{A}^T\mathbf{A}$. The entry $C_{2,2}$ is

5. (4 points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{4\times 4}$ whose column vectors are given by

$$\begin{aligned} \mathbf{a}_1 &= \mathbf{e}_1 \\ \mathbf{a}_2 &= \mathbf{e}_3 + x\mathbf{e}_2 \\ \mathbf{a}_3 &= x\mathbf{e}_3 - \mathbf{e}_2 \\ \mathbf{a}_4 &= \mathbf{e}_4 \end{aligned} \tag{16}$$

Let $\mathbf{C}$ be the inverse of $\mathbf{A}$. The entry $C_{2,2}$ is