Portfolio Optimisation Using Deep Learning

A project report submitted in partial fulfillment of the requirements for B.Tech. Project

IPG-MBA

in

Information Technology

by

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(2019IMG-018)

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2022

CANDIDATES DECLARATION

I hereby certify that the work, which is being presented in the report, entitled **Portfolio Optimisation Using Deep Learning** , in partial fulfilment of the requirement for the award of the Degree of **Bachelor of Technology** and submitted to the institution is an authentic record of my own work carried out during the period *June 2022* to *September 2022* under the supervision of **Prof. Rajendra Sahu**. I have also cited the references about the text(s)/figure(s)/table(s) from where they have been taken.

Date: Signatures of the Candidates

This is to certify that the above statement made by the candidates is correct to the best of my knowledge.

Date: Signatures of the Research Supervisor

ACKNOWLEDGEMENTS

I'm grateful to **Prof. Rajendra Sahu** for allowing me to operate independently and experiment with ideas. I would like to take this moment to express my sincere gratitude to him for his support and guidance, as well as for his personal concern for my report and active support. The sessions he held to boost my confidence and inspire me, were proved to very successful and were crucial in giving me a feeling of self-assurance and confidence. His helpful direction, recommendations, insightful judgement, constructive criticism, and an eye for excellence have all contributed to the current work's fostering and flourishing. My mentor constantly answered all of my questions with smiling graciousness and enormous patience, never making me feel inexperienced by always listening to my ideas, praising and refining them, and allowing me a free hand in my report. Only due of his enormous attention and helpful attitude has the current effort progressed to this point.

Finally, I'd want to thank our Institution and colleagues, whose continual encouragement helped me revive my spirit, concentrate my time and focus, and assist me in carrying out this task.

ABSTRACT

Predicting future prices for stocks and their price movement patterns is challenging. As a result, it is far more difficult to create a portfolio of financial assets using projected prices to optimise its risk and returns. This study analysed the historical price time series of 28 stocks from various Indian stock market sectors randomly chosen and were part of the NIFTY 100 Index from January 1, 2000, to July 31, 2022. For direct portfolio Sharpe ratio optimisation, we deploy deep learning models. An LSTM (long and short-term memory) model is developed and improved to direct portfolio Sharpe ratio. By changing model parameters, the method we offer surpasses the need to predict projected returns and enables us to optimise portfolio weights directly. The results show that our model performs the best overall over the evaluation period, including the economic uncertainty throughout the first three months of 2020. We compare our method to a wide variety of algorithms.

Keywords: Return, Portfolio Optimisation, Volatility, Minimum Variance Portfolio, Stock Price Prediction, Deep Learning, Optimum Risk Portfolio, Sharpe Ratio, LSTM, Prediction Accuracy

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CHAPTER 1

Introduction

In section 1.1 of this chapter, an overview of the background for the Portfolio Optimisation project is provided. The study work flow is then introduced step by step in section 1.2 as a result of the motives that have been provided. The motivation is discussed in section 1.3. The Research Workflow are then described in section 1.4.

1.1 Context

There are a number of theories for predicting stock prices and optimising portfolios in the research studies. Utilising artificial intelligence-based systems, machine learning-based models, and algorithms has likely become one of the most popular ways to anticipate stock values recently. [13][9]. It is also very popular to include text mining into social media platforms in order to increase the learning-based models' capacity for prediction accuracy [2]. Numerous researchers have constructed multi-objective optimisation strategies for reliable portfolio construction employing meta-heuristics and constraint-imposed heuristics.

1.2 Objectives

In this work, we explore the use of deep learning models for portfolio optimisation. [6]. We avoid this forecasting phase in favour of getting asset allocations immediately, in contrast to traditional techniques where anticipated returns are first forecasted (usually by econometric models) [8]. Since the prediction stages seek to reduce a prediction loss that is not equal to the portfolio's total return, the return prediction technique has not always been proven to maximise the performance of a portfolio [18]. In contrast, by explicitly maximising the Sharpe ratio, our technique maximises profit per unit of risk.[15]. Our method initially combines multiple features from diverse assets to produce a single observation in order to maximise the Sharpe ratio. The most important data is then extracted and produced as portfolio weights using a neural network.

1.3 Motivation

The first proposal of our end-to-end training system was made in [11]. However, there is not much discussion about how to maximise portfolios in these books since they primarily concentrate on improving the performance of a particular asset. Furthermore, although our data-set is up-to-date, their testing period ranges from 1970 to 1994. We also examine how our technique performs in the present COVID-19 problem.

1.4 Project work flow

According to the research objectives, the report will describe the work flow as below:

1. Selections of the Sectors and Stocks
2. Data Gathering
3. Volatility and Return on Investment
4. Stock Return Covariance and Correlation Matrices
5. Portfolio Return and Risk Estimation
6. Designing Minimum-Risk Portfolios
7. Designing Optimum-Risk Portfolios
8. Back Testing the Strategy

CHAPTER 2

Literature Review

2.1 Background

Allocating a certain amount of funds to a group of stocks or capital assets in a way that optimises both the investment's return and risk is known as portfolio design. The issue falls under the category of NP-hard computations, for which no algorithm with polynomial time complexity can find a solution. Since accurately predicting future stock price values is likewise a very difficult challenge, the complexity of the problem grows even further if the optimisation techniques need a robust evaluation of the risk and return associated with each component stock in the portfolio. The efficient market hypothesis proponents argue that it is difficult to anticipate a stock's future pricing with any degree of accuracy. On the other hand, many academics and financial experts have shown how they may use sophisticated models and clever algorithms to anticipate future stock values accurately. Researchers have proposed several solutions to the issue since Markowitz's foundational idea on portfolio optimisation using the minimum-variance technique [8]. Several other methods are also put forth in the literature to predict future stock prices. These methods include linear as well as non-linear regression, simple and weighted exponential smoothing, auto-regressive integrated moving averages (ARIMA), Machine Learning, vector auto-regression (VAR), and deep learning.

2.2 Key related research

Cardinality constraint and purchase limit-based techniques are put forward by several writers to address the flaws of the actual mean-variance optimisation strategy suggested by Markowitz, which has significant practical drawback [16]. These literature has a variety of recommendations for optimising a portfolio using techniques including genetic algorithms, swarm intelligence, fuzzy logic, and principal component analysis [3]. The technique that is most often used for reducing risks in a portfolio is generalised autoregressive conditional heteroscedasticity (GARCH) [14].

2.3 Research gaps

Since many strategies make decisions based on a predetermined environment with point risk-return estimation methods derived from historical data, a significant amount of effort has also been made to modelling and handling the uncertainty associated in market return, which is crucial step in the development of portfolio optimisation models[4].

Almost all Portfolio optimisation techniques optimise the Sharpe ratio using complete historical data, which reduces the model's efficiency as stock market conditions are constantly changing and becoming unpredictable.

2.4 Analysis

The study proposed an algorithmic method for creating optimised portfolios for certain industries. These portfolios may be used by an institution, an investment manager, or an individual to make wise stock investing choices that serve as a comprehensive reference for stock market investors in India. The report also suggests an Deep Learning model for stock asset allocation. The model's excellent degree of accuracy suggests that it may be used to real-world portfolio development with little mistake.

2.5 Conclusion

To address the institutional, investment manager, or individual challenge of investment portfolio optimisation, a great variety of material is easily accessible. To make it easier to decide which assets will amplify the portfolio and how to track their development over time to offer a more significant return to the organisation, investment manager, or individual, they all propose analytical approaches of portfolio optimisation.

In the present study, optimal portfolios have been created for 9 significant industries listed on the NSE using the minimum-variance, Maximum Diversification and Risk Parity Models. Three portfolios using each Model are constructed using the historical stock prices from 2000 to 2022. Then Risk and Return computed by the our model are compared with each one of them.

CHAPTER 3

Methodology

Here, we provide our conceptual framework and describe how gradient ascent may be used to maximise the Sharpe ratio. We go into depth on the operation of each part of our system and the many kinds of neural networks employed.

3.1 Objective Function

The anticipated return per unit of risk (For simplicity, excluding the risk-free rate) is the definition of the Sharpe ratio, which is used to evaluate the return of a portfolio with respect to risk of the portfolio:

$$S = \frac{E(P(w))}{Std(P(w))}$$

where $E(P(w))$ and $Std(P(w))$ are the estimated portfolio returns' mean and standard deviation. Specifically, we may maximise the objective function given below for a trading period of $t = 1, \cdots, T$:

$$L_T = \frac{E(P(w))}{\sqrt{E(P(w)^2) - (E(P(w))^2}}$$

$$E(P(w)) = \frac{1}{T}\sum_{t=1}^{T} P(w_t)$$

$P(w_t)$ is the realised portfolio return across n assets at time t and is represented as follows:

$$P(w_t) = \sum_{t=1}^{T} w_{i,t-1} \cdot r_{i,t}$$

where $r_{i,t} = (p_{i,t}/p_{i,t-1} - 1)$ represents the return of asset i. We set $\sum_i^n w_{i,t} = 1$ where $w_{i,t} \in [0, 1]$ to indicate the allocation ratio n (position) of asset i. In our strategy, we model $w_{i,t}$ for a long-only portfolio using a neural network f with parameters θ :

$$w_{i,t} = f(\theta \mid x_t)$$

If x_t stands for the most recent market data, and we avoid the traditional forecasting phase by connecting the inputs with positions to maximise Sharpe across the trading period T, namely L_T. We employ softmax outputs to satisfy the limitations imposed by a long-only portfolio, which call for weights to be positive and to add to one:

$$w_{i,t} = \frac{exp(\bar{w}_{i,t})}{\sum_j^n exp(\bar{w}_{j,t})}, \text{where } \bar{w}_{i,t} \text{ are the raw weights}$$

Unconstrained optimisation techniques could be employed to optimise such a framework. To maximise the Sharpe ratio, we apply gradient ascent explicitly. With an excellent derivation provided in [11] and [10], it is simple to calculate the gradient of L_T with reference to parameters θ. Once we have the value of $\partial L_T / \partial\theta$, we can repeatedly calculate this value from training data and change the parameters using gradient ascent:

$$\theta_{new} := \theta_{old} + \alpha \frac{\partial L_T}{\partial \theta}$$

where α denotes the learning rate, and the procedure may be repeated several times until the Sharpe ratio converges or the validation performance is optimised.

3.2 Mechanism/Algorithm

1. **Selections of the Sectors and Stocks** First, the present analysis will focus on nine crucial NSE sectors. Some industries include *pharmaceuticals, infrastructure, real estate, media, public sector banks, private sector banks, and large-cap, mid-cap, and small-cap companies*. The leading stocks in each sector and their relative contributions to the calculation of the overall sectoral index are included in monthly reports published by the NSE. The contributions are given percentage weights. D determined using the NSE report released on June 31, 2022 [12]. Following these actions, 28 stocks are picked for the nine specified industries.

2. **Data Gathering** The download function provided by the Python language's yfinance module is used to retrieve the historical prices of its 28 equities from the Yahoo Finance website. The stock prices are utilised for portfolio creation for 22 years, beginning on January 1, 2000, and ending in August 2022. Six elements of the web-extracted raw data are open, high, low, close, volume, and adjusted close. The other factors are neglected since the present study is based on univariate analysis, which only considers the stock's closing price for the period from

January 1, 2000, to August 31, 2022.

3. Volatility and Return on Investment Each stock's return and log return values are calculated daily based on the relative historical values of that stock. The change in the closing values over consecutive days is expressed by the daily return and the log return for a stock, and their logarithms are calculated in percentage terms. Using Python's pct change library function, the daily return and log returns are calculated. The values of each stock's daily and annual volatility are then calculated. A stock's daily volatility is the standard deviation of its daily returns. The volatility numbers are computed using the Python function std. Given that there are 254 working days in a year, the daily volatility of a stock is multiplied by the square root of 254 to get its annual volatility. According to an investor's perspective, a stock's annual volatility is a sign of how risky it is.

4. Stock Return Covariance and Correlation Matrices After obtaining each stock's return and volatility values, the stocks' covariance and correlation matrices are created based on their return values in the training dataset. These matrices provide vital info for the creation of a portfolio by assisting in comprehending the patterns of relationship among the stocks in a specific sector. Python methods called cov and corr are used to calculate the covariance and correlation matrices. The main optimisation goals of a portfolio design work are minimising risk and optimisation. The algorithm tries to distribute funds across stocks with little or no correlation in a diversified portfolio that minimises risk. It is feasible to identify these stocks by examining their covariance or correlation matrices.

5. Portfolio Return and Risk Estimation The first set of portfolios is built at this stage using each sector's covariance and correlation matrices. The portfolios are first created by giving each of the 28 stocks in a particular sector the same weight. The equal-weighted portfolio's annual return and volatility numbers are calculated for each of the nine sectors. The anticipated return of a portfolio made up of n capital assets (i.e., stocks) is indicated as Exp(Ret) in (Eq 1), from which the return of a portfolio based on its historical return values is calculated.

$$Exp(Ret) = w_1 E(Ret_{c_1}) + w_2 E(Ret_{c_2}) + ...w_n E(Ret_{c_n}) \qquad \textit{(Eq 1)}$$

The annual return and volatility of the equal-weight portfolio of each sector are calculated using the stock's annual return and volatility metrics. The mean annual returns for each stock that makes up a portfolio are calculated using the Python function which is resembled by argument 'Y'. On the other hand, the daily volatility values are multiplied by a factor of the square root of 254 to obtain the annual volatility values for the equal-weight portfolios. The equal-weight portfolios serve as benchmarks for assessing the performance of other portfolios and provide one a baseline level of return and risk associated with the sectors over the training records. The return and risk projections made using equal-weighted portfolios, however, are very inaccurate predictors of future returns and dangers. Therefore, more accurate projections of the potential return and hazards are required. The creation of the portfolios with the lowest and highest levels of risk is necessary as a result.

6. Designing Minimum-Risk Portfolios At this stage, minimum-risk portfolios are created for the nine sectors to enhance the equal-weight portfolios. The lowest values for their variances are seen in portfolios with the lowest risk. A particular portfolio's variance, $Variance(P)$, depends on the variances of the stocks that make up the portfolio as well as the covariances between each pair, as shown by (Eq 2).

$$Variance(P) = \sum_{i=1}^{n} w_i s_i^2 + 2 * \sum_{i,j} w_i * w_j * Covar(i, j) \qquad (Eq2)$$

In (Eq 2), the weight given to stock i is defined by w_i, the covariance between stocks is obtained by $Covar(i, j)$, and the volatility of the stock is determined by its standard deviation which is represented by s_i. Each portfolio has 28 stocks, hence 784 terms are needed to calculate each portfolio's variance. While the remaining 756 items represent the covariances between each pair, only 28 of the terms are included in the weighted sum of the variances of the individual stocks. The portfolio with the lowest risk is the one that can identify the set of $w_i's$ that results in the portfolio's volatility being at its lowest value.

In order to determine which portfolio has the lowest risk for each sector, the *efficient frontier (EF)* employing many portfolios is first presented. The returns and volatility are shown along the y-axis and the x-axis, respectively, in the two-dimensional plot of a set of portfolios in the EF. The EF frontier is composed of the portfolio in the form of points that provide the highest level of return of a given volatility or the lowest volatility for a given level of return. The point at the furthest left position on the EF represents the portfolio with the least volatility and, as a consequence, the lowest risk. The efficient frontier is drawn by 5000 iterations of a Python programme looping through a portfolio of equities, randomly allocating weights to each item. Every one of the 5,000 points that the algorithm produces corresponds to a portfolio with a certain return and risk value. The EF points are those that provide the lowest volatility for a given return value or the highest return value for a given volatility.

The portfolio with the lowest risk is represented by the leftmost location on the EF out of all the produced points.

7. Designing Optimum-Risk Portfolios The investors in the stock market rarely follow the strategy of risk minimisation as proposed by the minimum- risk portfolio due to their low returns. Most often, the investors are ready to undertake higher risks if the associated returns are even higher. For optimising the risk and return in a portfolio, and thereby designing an optimum-risk portfolio, a metric called Sharpe Ratio is used, which is given by (3). For computing the optimum risk portfolio, the metric Sharpe Ratio of a portfolio is used. The Sharpe Ratio of a portfolio is given by (Eq 3).

$$Sharpe\ Ratio = \frac{Ret_{curr} - Ret_{risk\ free}}{Ris_{curr}} \qquad (Eq\ 3)$$

Ret_{curr}, $Ret_{risk\ free}$, and Ris_{curr} are used in (Eq. 3) to refer to the current portfolio return, the risk-free portfolio return, and the current portfolio risk (as measured by the portfolio's yearly standard deviation). The portfolio with a risk value of 4.24% is assumed to be risk-free. The optimal-risk portfolio maximises the Sharpe Ratio's value. When compared to the minimum-risk portfolio, the optimum-risk portfolio generates a very high return with a hardly noticeable increase in risk.

8. Back Testing the Strategy A minimal risk portfolio and an optimum risk portfolio are developed for the sectors using the training information from January 1, 2000, to December 31, 2021. On January 1, 2021, a fictional investor is established who invests a sum of one million Indian rupees (INR) in each sector in accordance with the advice of the best risk-weighted portfolio structure for that sector. Please take note that the figure of INR 1 million is just meant to serve as an example. The quantity or the currency will have no impact on the analysis. A regression model is created utilising the LSTM deep learning architecture to calculate the future stock price values and, in turn, estimate the future value of the portfolio. The stock prices for June 1, 2021, with a prediction horizon of one week, are anticipated on May 31, 2021, using the LSTM model. The expected rate of return for each portfolio is calculated using the predicted stock weights. Finally, the real rate of return is calculated on June 1, 2021, when the stock values are known. To assess the profitability of the portfolios and the LSTM model's predictive power, the anticipated and actual rates of return for each portfolio are compared.

3.3 Model Architecture

In Figure 2, we show the layout of our network. The input layer, neural layer, and output layer are the three key components of our model. In this architecture, cross-sectional characteristics are taken from input assets using neural networks to extract them. It has been said that features derived from deep learning models perform more effectively than conventional hand-crafted features [18]. Following the extraction of the features, the model generates portfolio weights, from which we acquire realised returns in order to maximise the Sharpe ratio. Each part of our technique is described in full below.

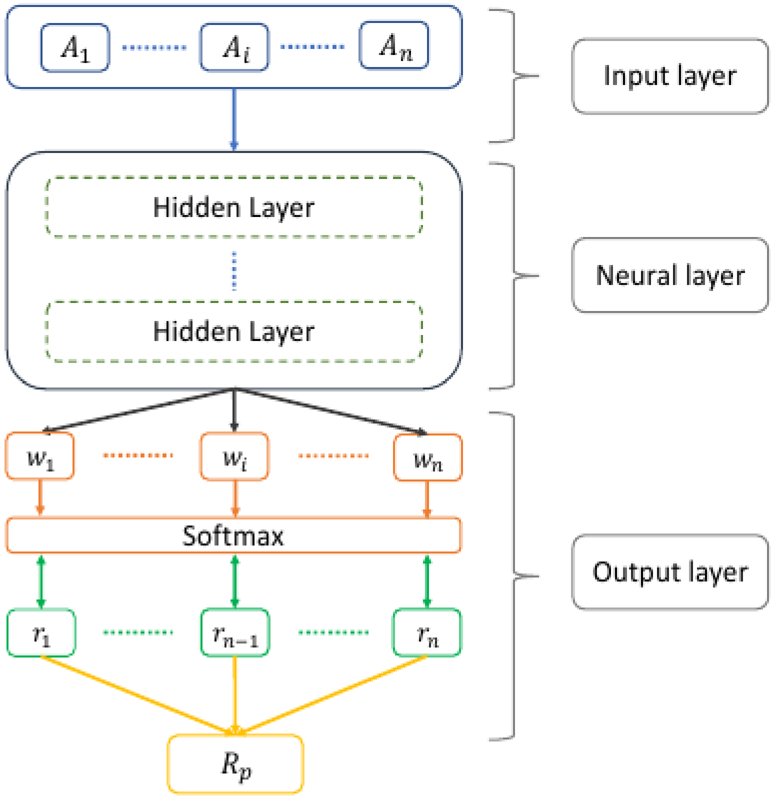

Figure 3.1: Model architecture schematic

Initial layer Each asset is designated as A_i, and a total of n assets make up the portfolio. Concatenating data from all assets creates a single input. The historical prices and returns of one asset, for instance, might be the input features with a dimension of $(k, 2)$, where k denotes the lookback window. The input dimension would be $(k, 2 \times n)$ if features were stacked across all assets. We may then feed this input into the network and anticipate the extraction of non-linear characteristics.

The neural layer A network may be created by stacking a number of hidden layers together; however, in fact, this step involves a lot of trials since there are several methods to combine hidden layers and the performance often relies on the architecture's design. Long Short-Term Memory (LSTM), Convolutional Neural Networks (CNN), and Fully Connected Neural Networks (FCN) have all been put to the test [5]. According to several studies [17], [7] , LSTMs provide the greatest overall performance for modelling daily financial data.

We see that the extreme overfitting issue with FCN is a concern. There are too many parameters as a consequence of it giving parameters to each input characteristic. The LSTM uses a cell structure with gate mechanisms to summarise and filter data from its extensive history, resulting in fewer trainable parameters and higher generalisation outcomes. As a result, oversmooth solutions are produced when CNNs with significant smoothing (typical of large convolutional filters) are used. We see that the convolution procedures and the parameter sharing architecture cause CNNs to overfilter the inputs. We do, however, point out that CNNs seem to be top-notch contenders for modelling high-frequency financial data, such as limit order books.

Outer layer We use the *softmax* activation function for the output layer to create a long-only portfolio, which by default applies restrictions to maintain portfolio weights in the positive and summing to one. We may multiply these portfolio weights by the related assets' returns $(r_1, r_2, \cdots, r_n)$ to get the realised portfolio returns since the number of output nodes $(w_1, w_2, \cdots, w_n)$ equals the number of assets in our portfolio (R_p). After obtaining the realised returns, we may compute the Sharpe ratio's gradients with respect to the model's parameters and apply gradient ascent to update the parameters.

CHAPTER 4

Experiments and results

This section discusses the various experiments pertaining to the proposed hypothesis and their findings.

4.1 Description of Dataset

The goal of our approach is to establish a system that gives a strong reward-to-risk ratio. A diversified portfolio yields a better return per risk. Our dataset covers daily measurements between the years 2000 and 2022. Every quarter, we retrain our model and update the parameters using all the data we have up to that time. Our testing period spans the years 2000 to the end of June 2022, which includes the most current COVID-19-related crises.

4.2 Baseline Algorithms

We contrast our approach with a collection of standard algorithms. Many pension funds' reallocation techniques make up the first group of baseline models. These strategies allocate important assets according to a predefined allocation ratio, and they rebalance portfolios quarterly to keep these ratios constant. A portfolio may be chosen by investors depending on their preferred level of risk. Generally speaking, portfolios with a higher proportion of stocks would perform better but with more volatility.

Mean-variance optimisation (MV) [20], maximum diversification (MD) [32] and Risk Parity (RP) make up the second group of comparison models. To calculate the anticipated returns and covariance matrix, we employ moving averages with a 50-day rolling window. We choose weights for the portfolio that maximise the Sharpe ratio for MV on a daily basis. The diversity-weighted portfolio (DWP) from stochastic portfolio theory, as reported in [28], , links portfolio weights to asset market capitalisation, it is possible to consistently outperform the market index [10]. The final baseline method, known as Risk Parity Portfolio (RP), creates optimal portfolios by optimising the basic risk parity model [1].

4.2.1 Training Scheme

In this study, we model the portfolio weights using a single layer of 64-unit LSTM connectivity in order to optimise the Sharpe ratio. Instead of carefully adjusting the "appropriate" hyperparameters, we purposefully make our network simple to demonstrate the efficiency of our end-to-end training process. Each market index's closing prices and daily returns are included in our input, which is made up of observations over the previous 50 days. Keeping returns helps with the analysis of (Eq 7) and we can also use them as momentum characteristics in [26]. We are aware that returns may indeed be calculated from prices. We chose these frequently utilised features for our work since choosing features is not our main emphasis. Our network is trained using the Adam optimiser [17], and the mini-batch size is 64. For the purpose of optimising hyperparameters and preventing overfitting issues, we segregate 10% of every training data into a separate validation set. The validation set is used for any hyperparameter optimisation, leaving the test data for the final performance assessment and confirming the reliability of our findings. Our training method typically ends after 100 epochs.

4.3 Experiments

4.3.1 Correlation Heatmap

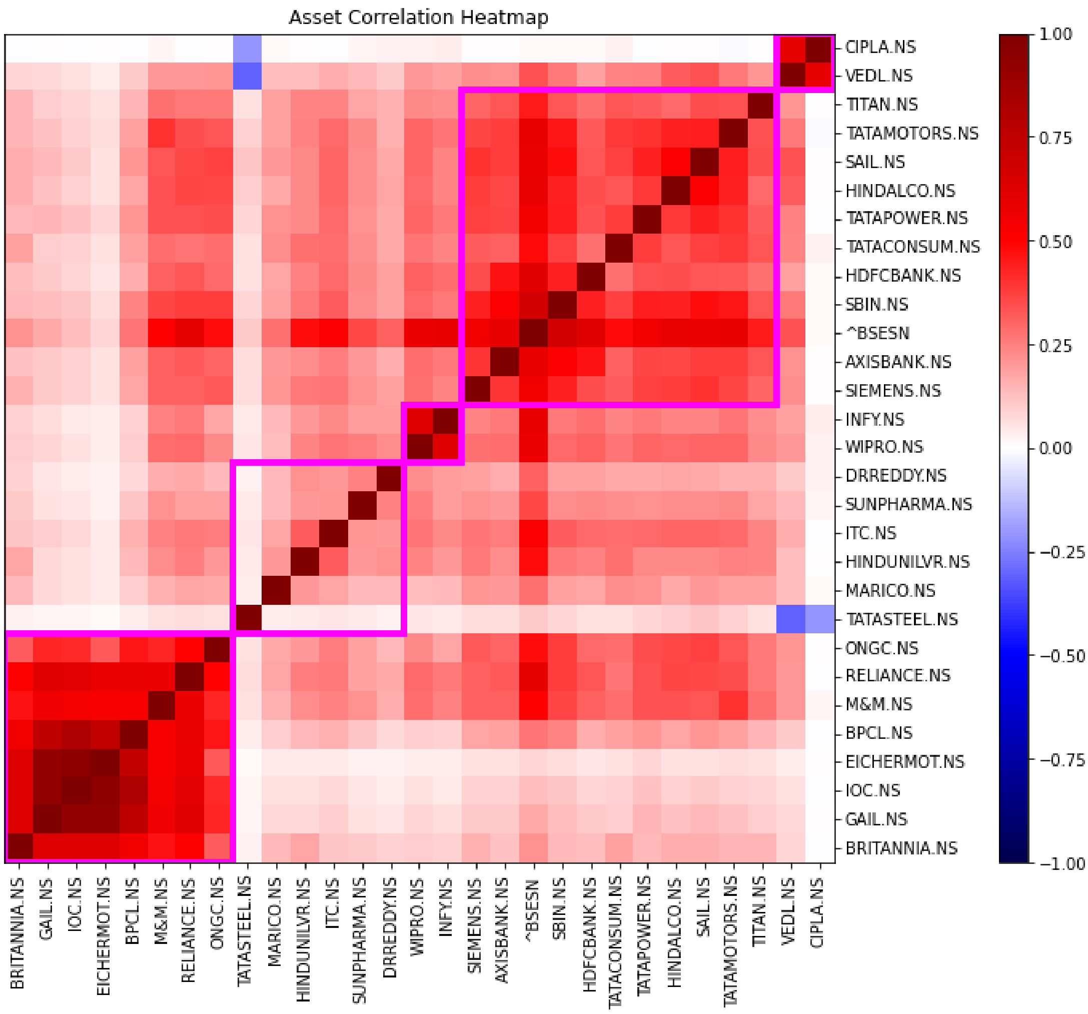

Figure 4.1: Correlation Heatmap

4.3.2 Experiments Parameters

4.3.2.1 Optimisation

Start date = 2000-01-01

End Date = 2022-07-31

Risk Free Rate = 0 (For Simplicity)

4.3.2.2 BackTesting

Start Money = 10,00,000

Commission = 0.5%

Silppage = 0.5%
Benchmark Index = BSE Sensex

4.3.3 Benchmark Experiment

4.3.3.1 Experiment Description

It's crucial to have a precise and clear method to measure your portfolio since a financial market's aspects are always changing. In order to do this, benchmark portfolios should provide investors with a realistic assessment of how their portfolio are performing relative to the important and specialised market categories.

We can evaluating our portfolio's total performance with respect to predetermined benchmarks, such as a market indices or a group of asset classes, by using a benchmark portfolio. These indexes are "passive" and unmanaged, in contrast to the actively managed investment portfolio that makes up your overall investment portfolio. As a result, benchmark portfolios may be used to assess the value a manager brings to your portfolio.

We are going to use BSE Sensex as Benchmark Portfolio

Theory The benchmark index of BSE is referred as Sensex. The 30 biggest and most popular companies on the BSE make up the Sensex, which serves as a bellwether for the Indian economy. It is market capitalisation-weighted and float-adjusted. Every year, between June and December, the Sensex is evaluated and Rebalanced. The Sensex is run by Standard & Poor's and is the earliest known stock index in India, having been founded in 1986. It is used by analysts and investors to track the economic cycles in India as well as the growth and decline of certain sectors.

4.3.3.2 Results

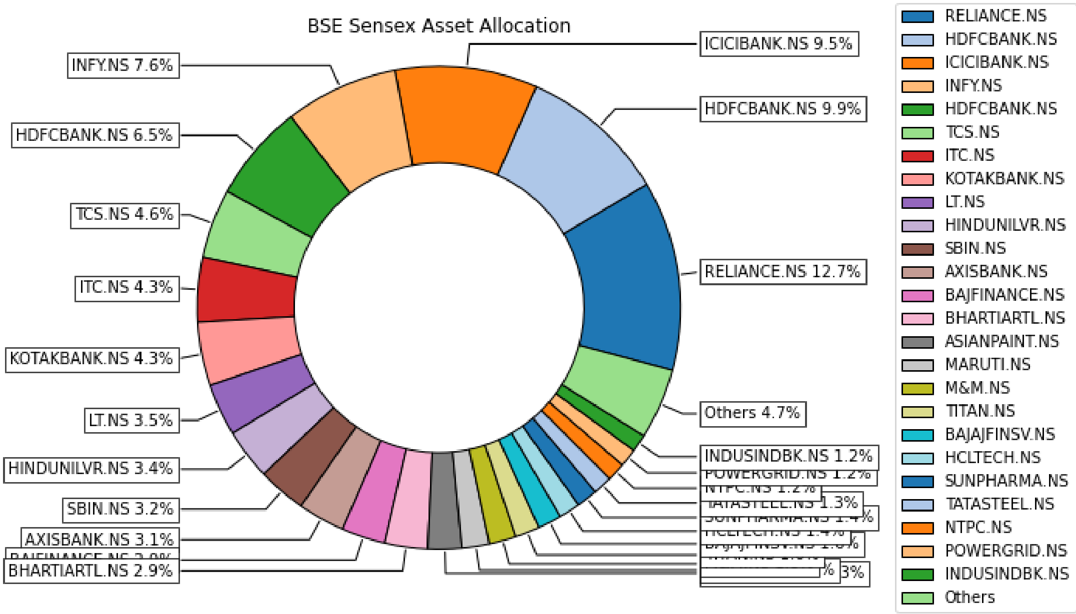

Figure 4.2: Allocation of assets of Sensex

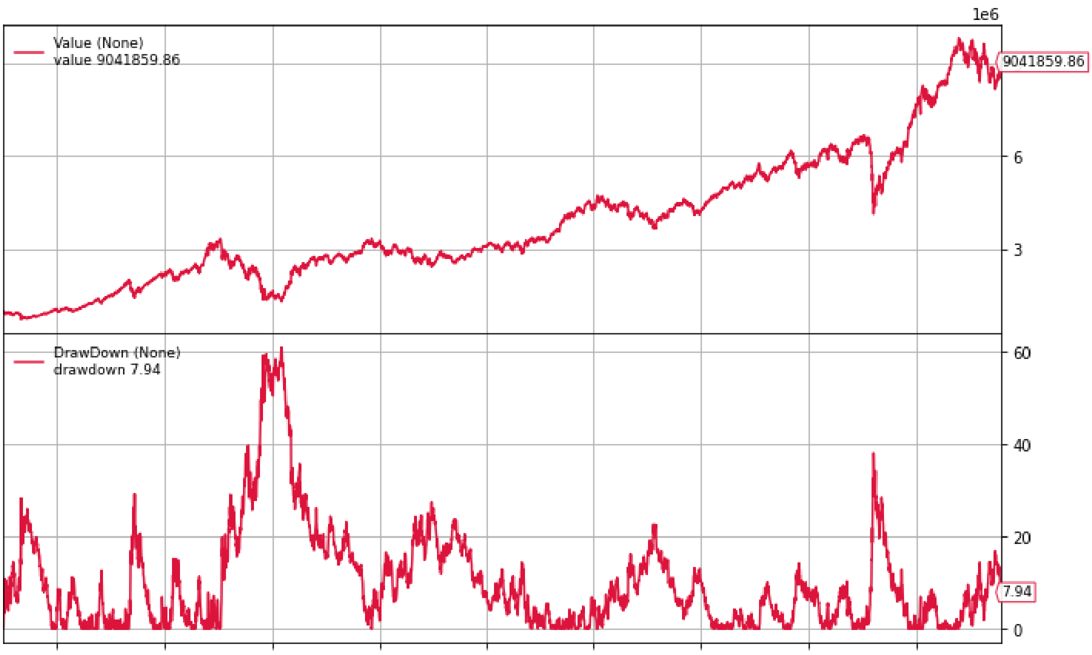

Figure 4.3: Backtest and Drawdown of Benchmark

Max Drawdown:	60.88%
CAGR:	10.54%
Sharpe:	0.514
Final Portfolio Value:	9041859.86

Table 4.1: Outcomes of the Benchmark Experiment

4.3.3.3 Conclusion

Since the inception of BSE Sensex in 2000 (base year), we have seen the worst downfall in 2008 of 60.88% and a significant decline in 2020. Sensex has given a CAGR of 10.54% since its inception, with a Sharpe ratio of 0.514. A lump sum investment of Rs 10 Lack in Sensex in 2000 would be Rs 90 Lack 41 thousand in Present Date

4.3.4 Mean Variance Optimisation

4.3.4.1 Experiment Description

The objective of creating a portfolio of assets using modern portfolio theory (MPT), also known as mean-variance optimisation, is to maximise anticipated return for a certain degree of risk. It formalises and broadens the concept of diversity in investment, which holds that having a variety of financial assets reduces risk compared to holding a single kind. Its main conclusion is that an investment's return and risk should not be evaluated on its own, but rather in the context of the risk and return of the whole portfolio. It substitutes asset price variation for risk.

Theory

$$\max_{w} \qquad \frac{R(w) - r_f}{\phi(w)}$$

Where:

$R(w)$ is the return function:

w: is the vector of weights of the portfolio.

$\phi(w)$: Standard Deviation of the portfolio.

r_f: is the risk free rate.

4.3.4.2 Results

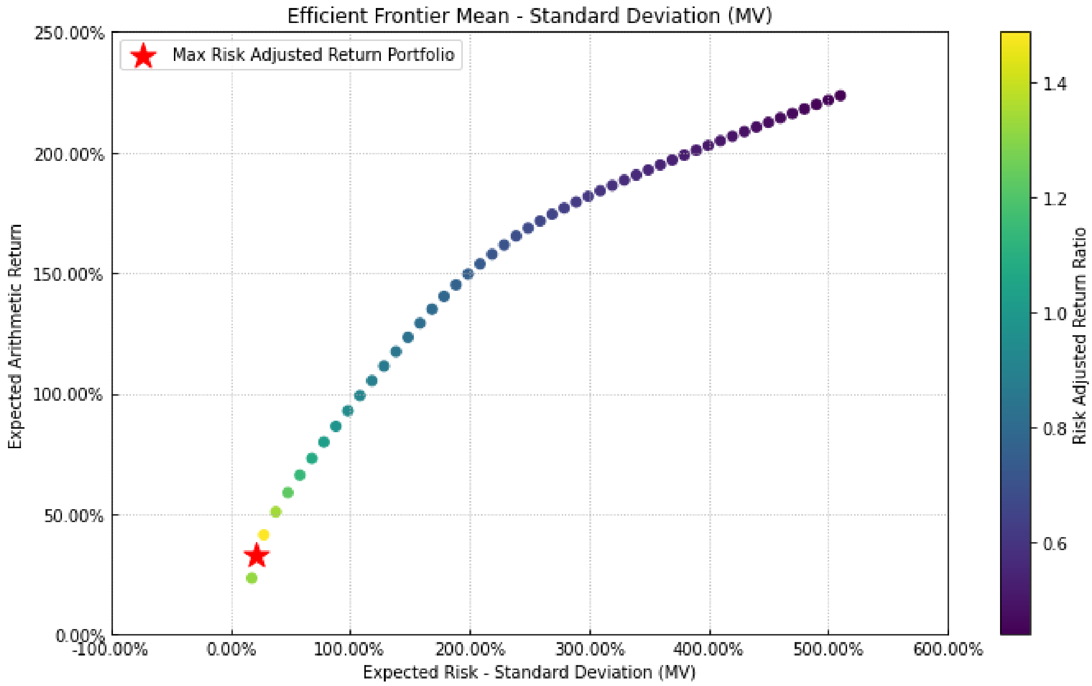

Figure 4.4: Efficient Frontier of Mean Variance Portfolio

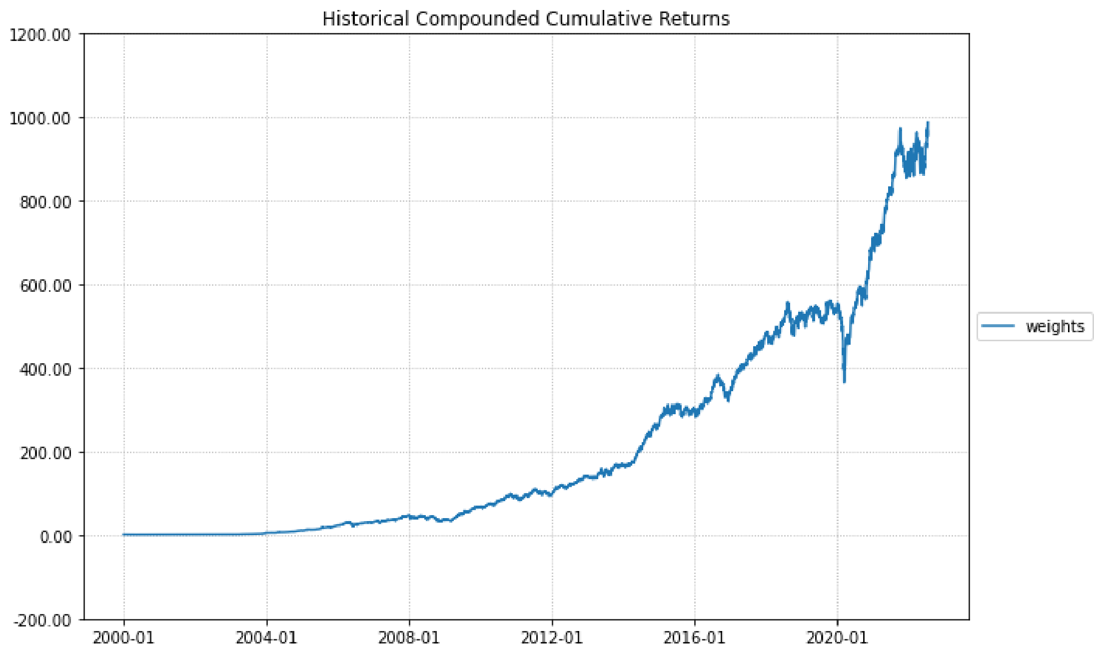

Figure 4.5: Historic Compounded Cumulative Returns using Mean Variance

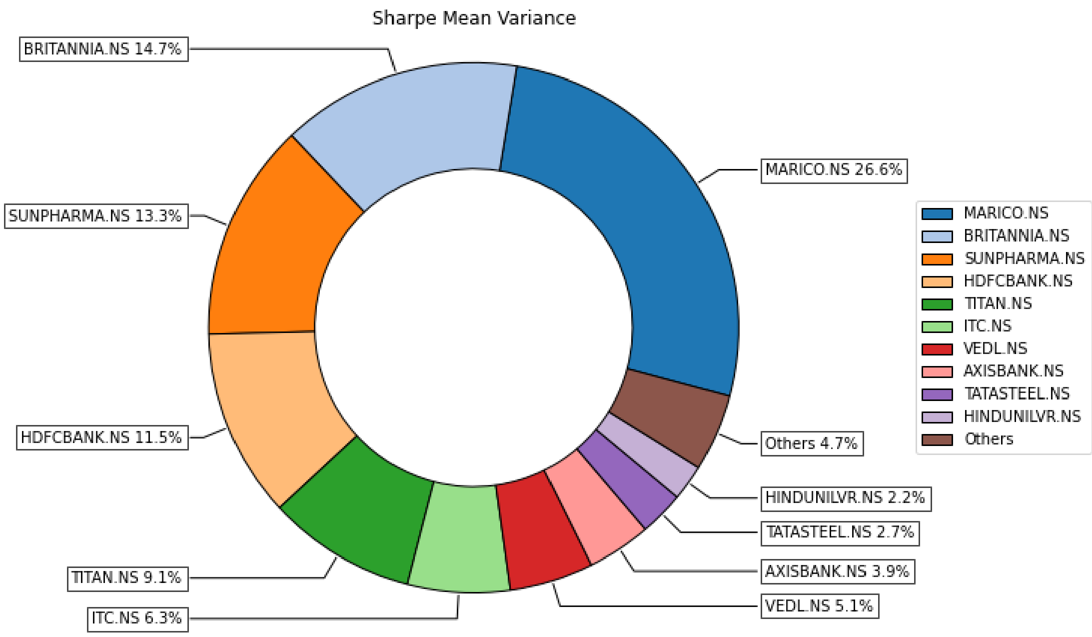

Figure 4.6: Allocation of assets using Mean Variance

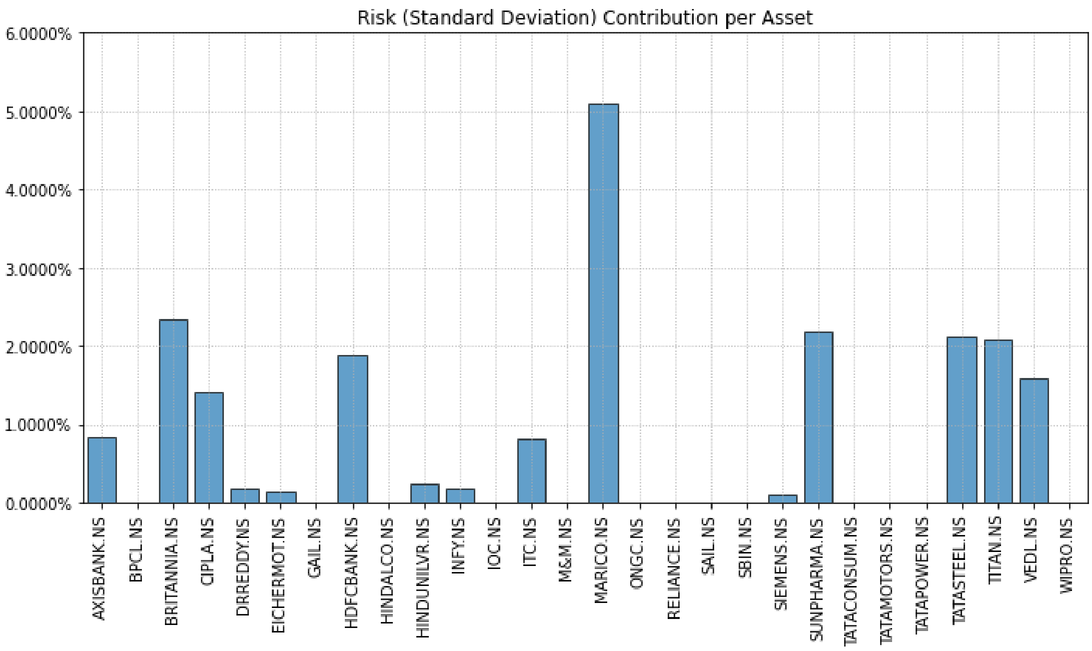

Figure 4.7: Risk Contribution Per Asset

4.3.4.3 Backtest and Drawdown of Mean Variance Oprimisation

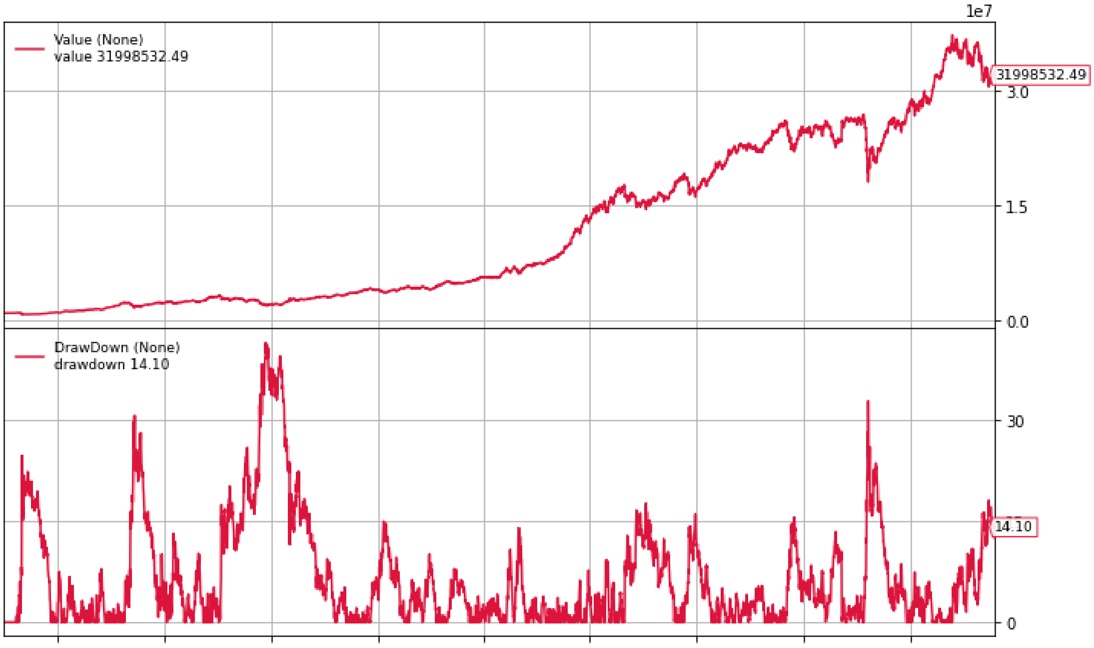

Figure 4.8: Backtest and Drawdown of Mean-Variance Optimisation

Max Drawdown:	41.59%
CAGR:	17.00%
Sharpe:	0.780
Final Portfolio Value:	31998532.49

Table 4.2: Outcomes of the Mean Variance Experiment

4.3.4.4 Conclusion

Since the Start of our study in 2000 (base year), We have seen the worst downfall in 2008 of 41.59% and a significant decline in 2020. Mean Variance optimisation has given a CAGR of 17.00% since its inception, with a Sharpe ratio of 0.780. A lump sum investment of Rs 10 Lack in portfolio using Mean Variance optimisation in 2000 would be Rs 3 Crore 19 Lack and 98 thousand in Present Date

4.3.5 Maximum Diversification Optimisation

4.3.5.1 Experiment Description

One of the most crucial factors to take into account when building an investing portfolio is spreading assets out to lower risk. One strategy is to diversify as much as possible. It is a portfolio strategy that seeks to build the most diversified portfolio achievable.

The strategy specifically aims to increase the diversification ratio. Hence, the greatest diversity method is a risk-based allocation strategy than ignores predicted returns. The strategy aims to create a portfolio with a wide range of investments.

Theory

$$\min_{w} \qquad \frac{P'(w)\cdot\Sigma}{\sqrt{P'VP}}$$

Where:

P: is the portfolio weights.

Σ: is the asset volatilities.

V: covariance matrix of these assets.

r_f: is the risk free rate.

4.3.5.2 Results

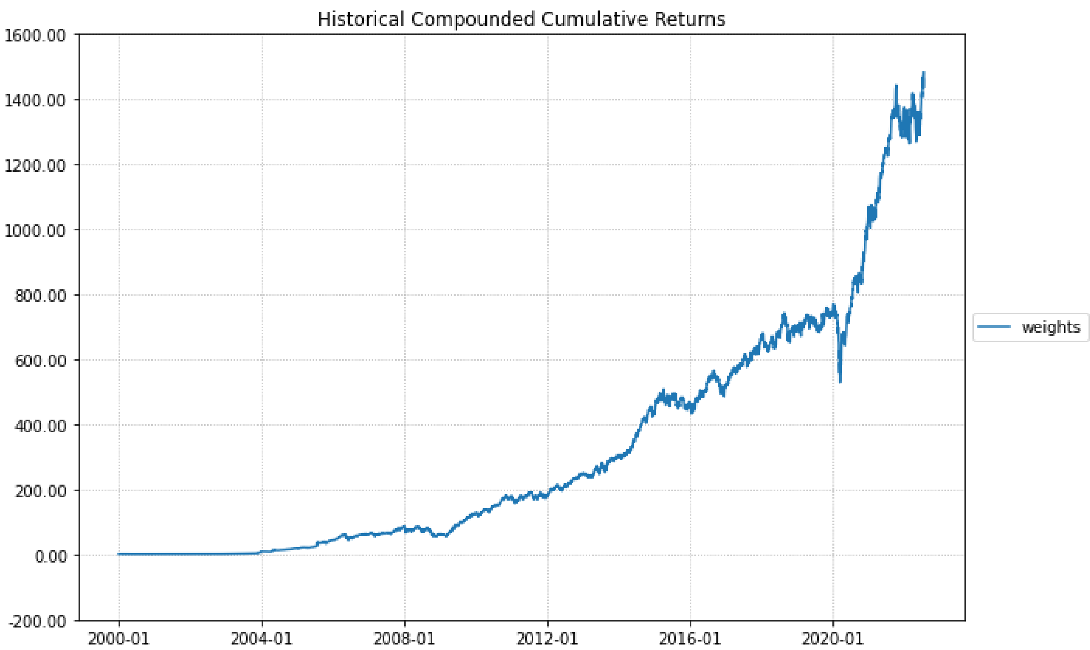

Figure 4.9: Historic Compounded Cumulative Returns Using Maximum Diversification

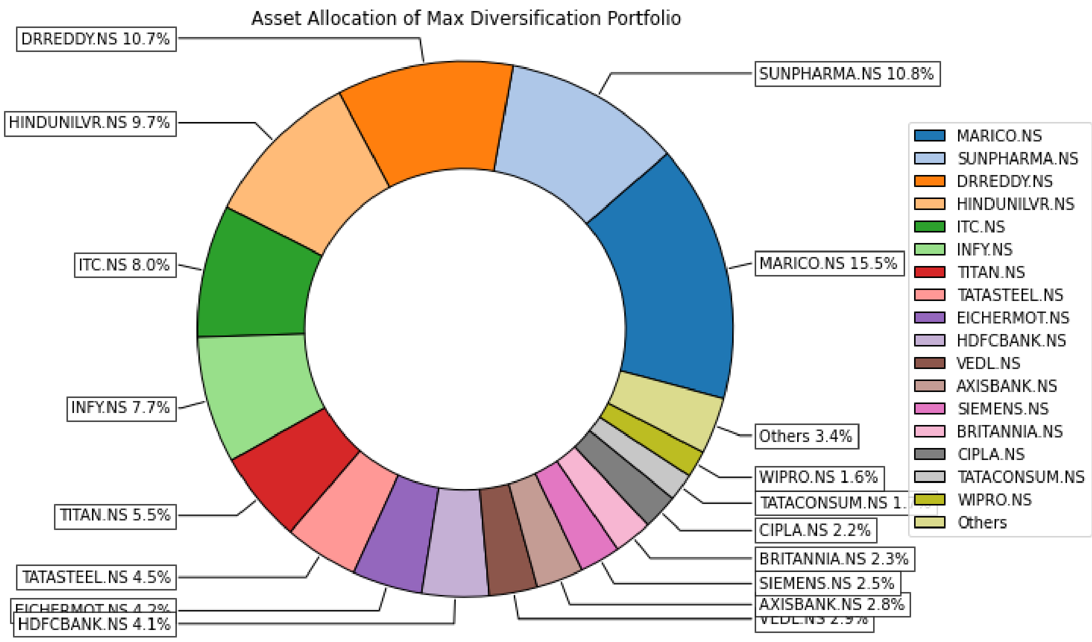

Figure 4.10: Allocation of assets using Maximum Diversification

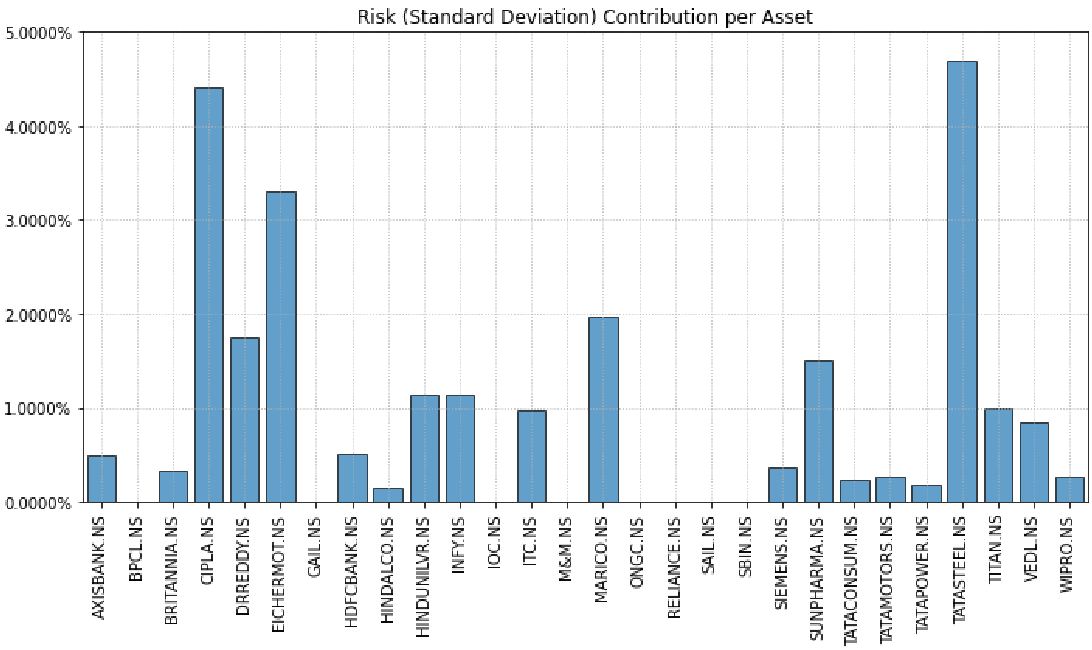

Figure 4.11: Risk Contribution Per Asset

4.3.5.3 Back Testing With Maximum Diversification Strategy on Assets

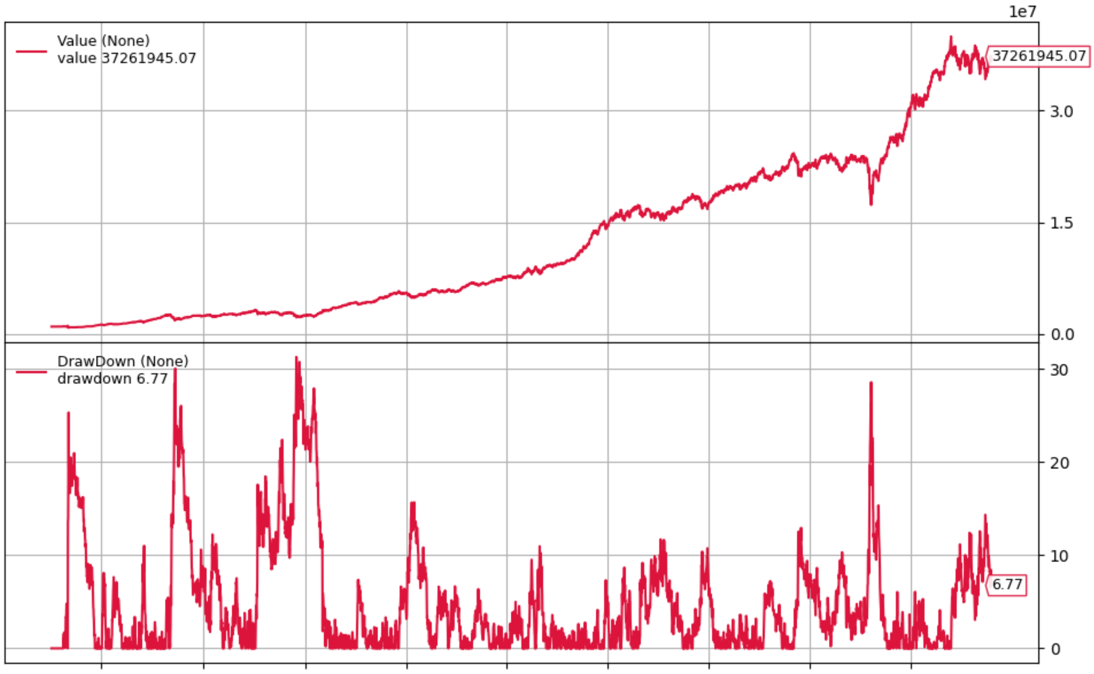

Figure 4.12: Backtest and Drawdown of Maximum Diversification Optimisation

Max Drawdown:	31.31%
CAGR:	17.85%
Sharpe:	0.911
Final Portfolio Value:	37261945.07

Table 4.3: Outcomes of the Maximum Diversification Experiment

4.3.5.4 Conclusion

Since the Start of our study in 2000 (base year), We have seen the worst downfall in 2008 of 31.31% and a significant decline in 2020. Maximum Diversification optimisation has given a CAGR of 17.85% since its inception, with a Sharpe ratio of 0.911. A lump sum investment of Rs 10 Lack in portfolio using Mean Variance optimisation in 2000 would be Rs 3 Crore 72 Lack and 61 thousand in Present Date

4.3.6 Risk Parity Portfolio Optimisation

4.3.6.1 Experiment Description

An method to asset management known as risk parity optimisation places more emphasis on risk allocation rather than capital allocation, which is often referred to as volatility. According to the risk parity method, the risk parity portfolio may produce a better

Sharpe ratio and can be more resilient to market downturns than the standard portfolio when asset allocations are modified to the same risk level. Risk parity is susceptible to substantial changes in correlation regimes, as was seen in Q1 2020, which caused risk-parity funds to perform much worse than average during the Covid-19 sell-off.

Theory

$$\begin{aligned} \min_{w} \quad & \phi(w) \\ \text{s.t.} \quad & b \log(w) \geq c \\ & w \geq 0 \end{aligned}$$

Where:

w: weights of the portfolio.

b: risk contribution constraints.

$\phi(w)$: Standard Deviation of the portfolio

c: is an arbitrary constant.

4.3.6.2 Results

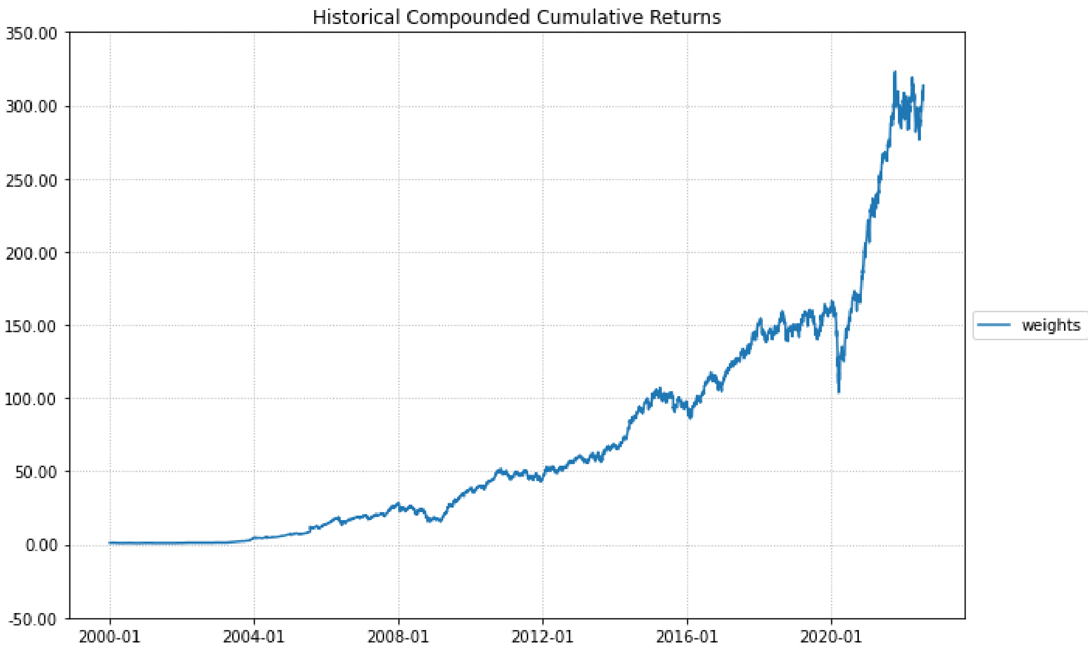

Figure 4.13: Historic Compounded Cumulative Returns Using Risk Parity

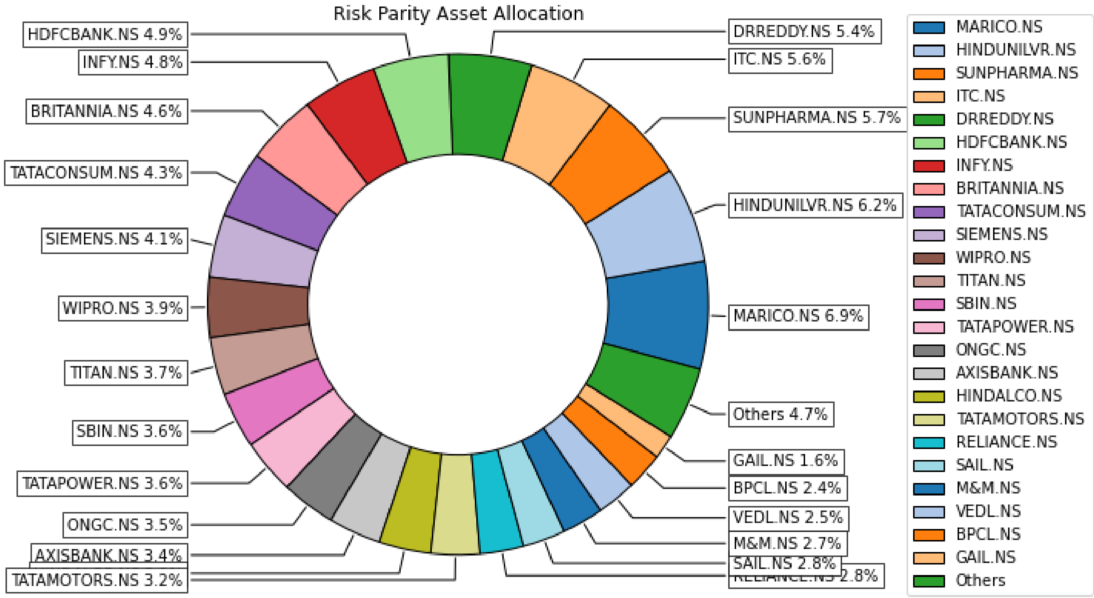

Figure 4.14: Allocation of assets using Risk Parity

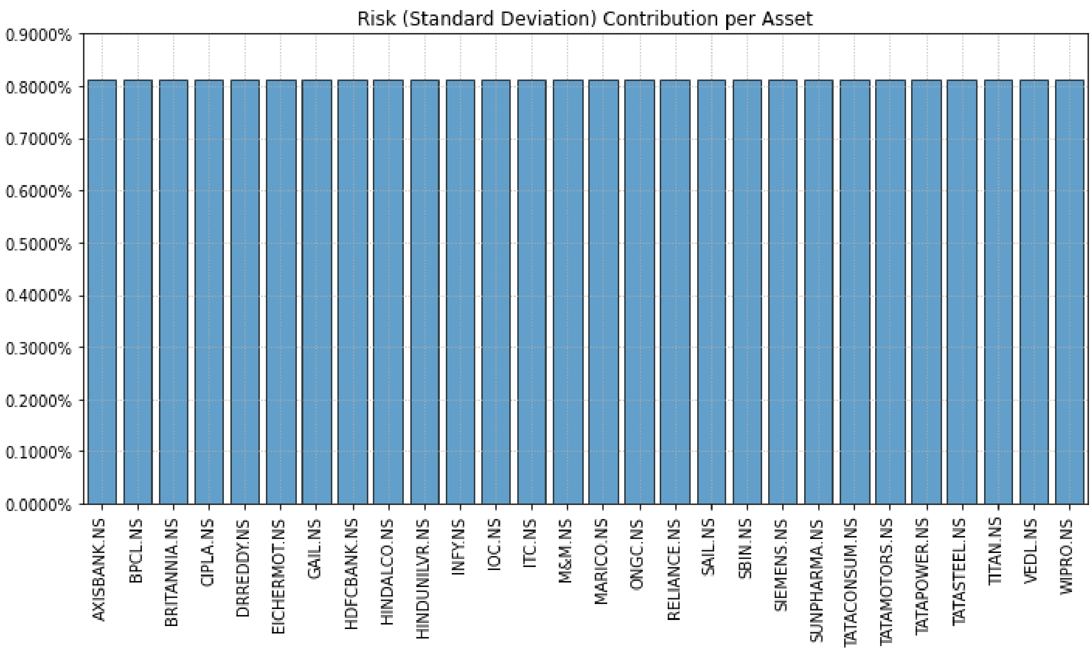

Figure 4.15: Risk Contribution Per Asset

4.3.6.3 Back Testing With Risk Parity Strategy on Assets

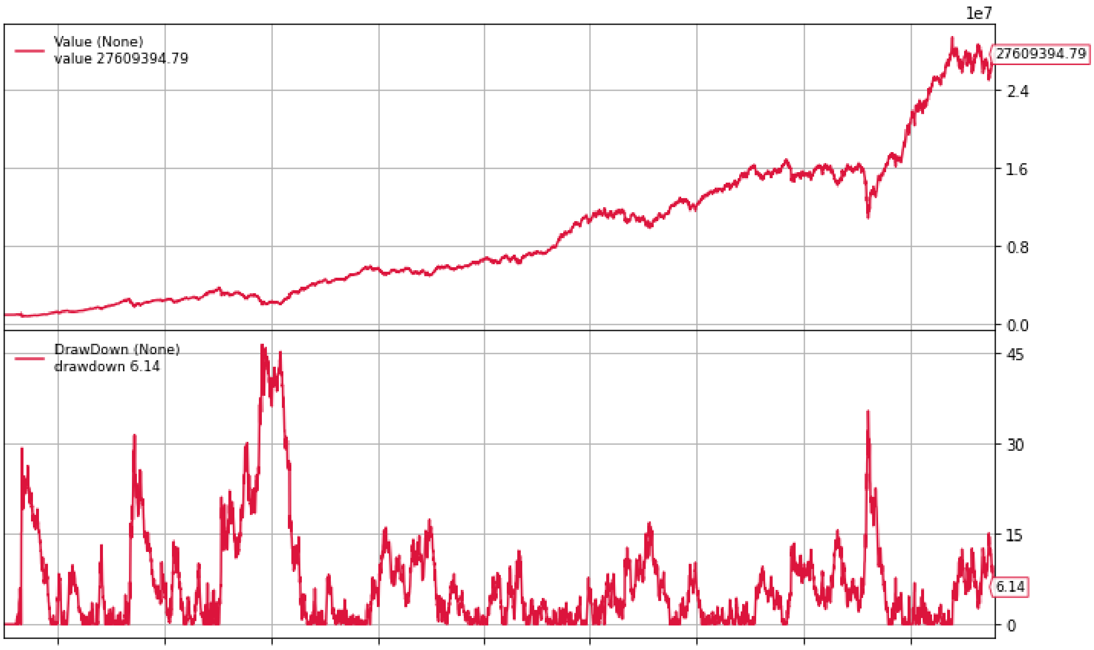

Figure 4.16: Backtest and Drawdown of Risk Parity Optimisation

Max Drawdown:	46.45%
CAGR:	16.27%
Sharpe:	0.697
Final Portfolio Value:	27609394.79

Table 4.4: Outcomes of the Risk Parity Experiment

4.3.6.4 Conclusion

Since the Start of our study in 2000 (base year), We have seen the worst downfall in 2008 of 46.45% and a significant decline in 2020. Risk Parity optimisation has given a CAGR of 16.27% since its inception, with a Sharpe ratio of 0.697. A lump sum investment of Rs 10 Lack in portfolio using Mean Variance optimisation in 2000 would be Rs 2 Crore 76 Lack and 9 thousand in Present Date

4.3.7 LSTM based Portfolio Optimisation

4.3.7.1 Experiment Description

This study considers using deep learning models to directly optimise a portfolio. We avoid this forecasting phase in favour of getting asset allocations immediately, in contrast to traditional techniques where anticipated returns are first forecasted (usually by econometric models). Since the prediction stages aim to minimise a prediction loss that is not the total reward from the portfolio, the return forecasting technique is not guaranteed to maximise the performance of a portfolio, as shown by many studies. In contrast, our strategy maximises return per unit of risk by explicitly optimising the Sharpe ratio. In order to maximise the Sharpe ratio, our approach first joins several characteristics from various assets to create a single observation. A neural network is then used to extract the most important information and output portfolio weights.

Theory

As Discussed in section 3.1

Figure 4.17 displays the model's schematic design. The asset's daily closing prices for the previous 125 days are used as the model's input. The 125-day input data contains two features: close values and percentage change, and it has a form of shape $(125, 2)$. The input layer accepts the data and sends it on to the 64-node initial LSTM layer. The shape produced by the LSTM layer is (125, 64). This indicates that each LSM layer node extracts 64 characteristics from each row throughout the input information. The middle layer is Flatten layer, which converts an n-dimensional array into a 1-dimensional array. The output of the dense layer, which consists of 28 nodes, provides the Asset Allocation's anticipated value. The output layer employs the softmax activation function, commonly referred to as softargmax or the normalised exponential function. An Early Stopping callback Function is implemented that enables us to give an any large number of learning epochs and discontinue learning whenever the observed parameter stops enhancing

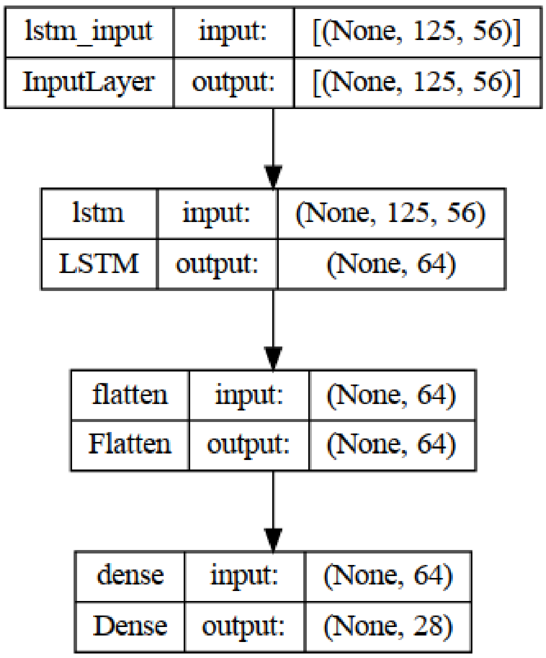

Figure 4.17: LSTM model's Schematic diagram

4.3.7.2 Results

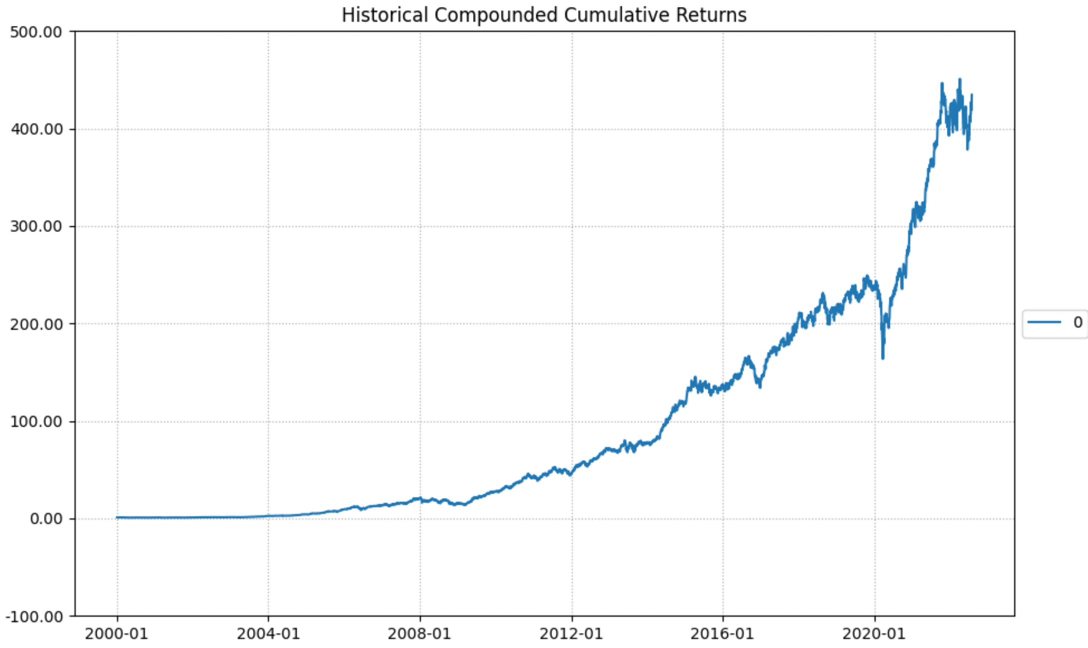

Figure 4.18: Historic Compounded Cumulative Returns Using LSTM

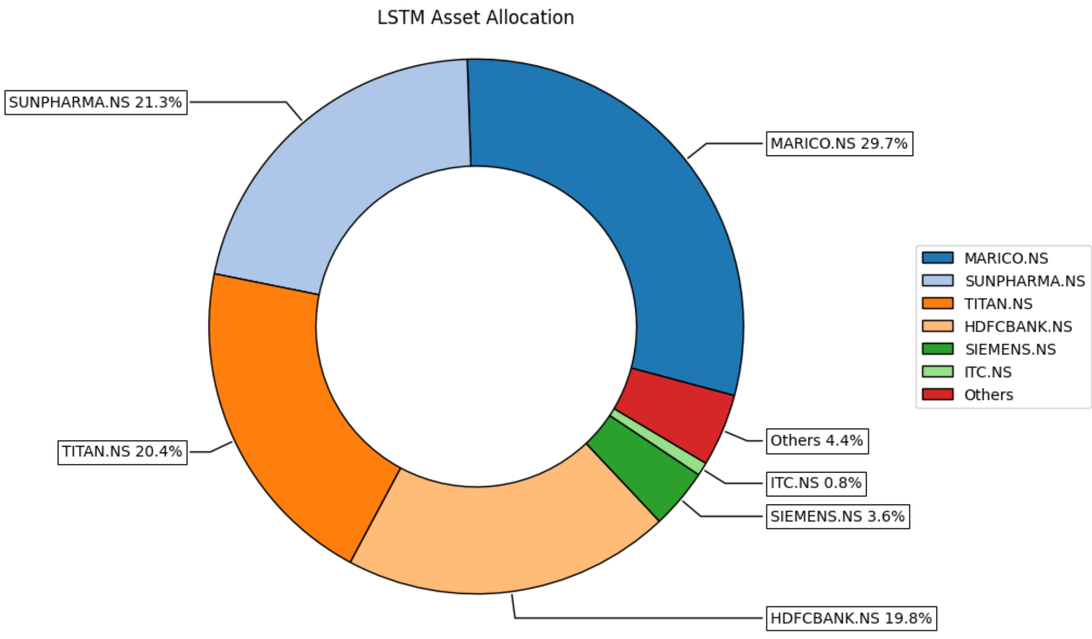

Figure 4.19: Allocation of assets using LSTM

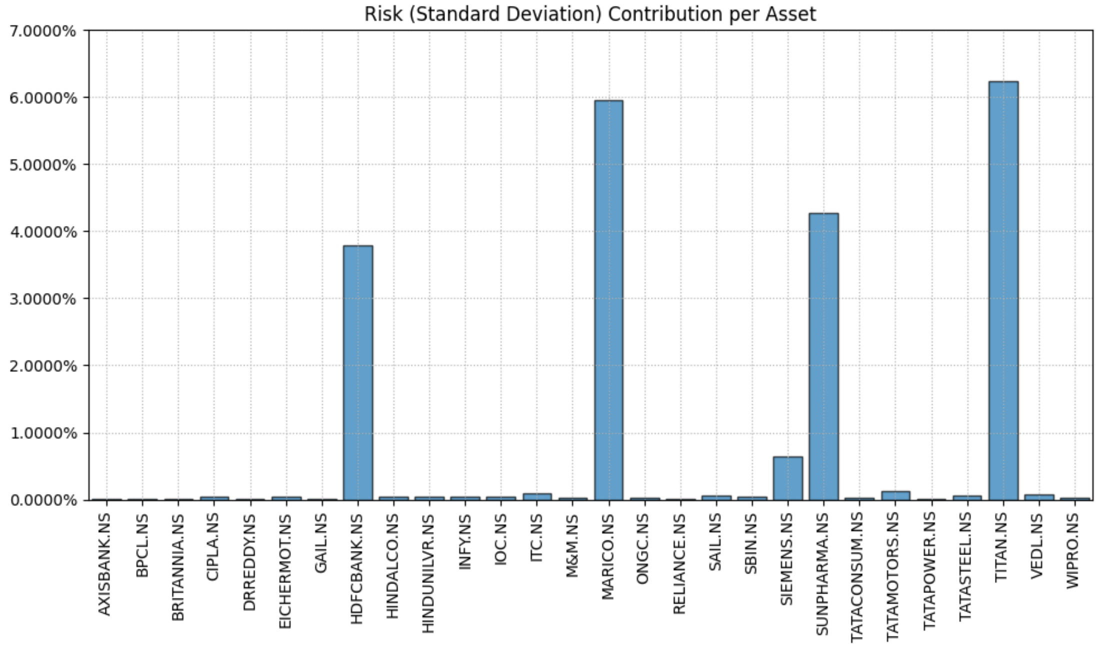

Figure 4.20: Risk Contribution Per Asset

4.3.7.3 Back Testing With LSTM Strategy on Assets

Figure 4.21: Backtest of LSTM Optimisation

Max Drawdown:	55.34%
CAGR:	20.07%
Sharpe:	0.966
Final Portfolio Value:	60135137.08

Table 4.5: Outcomes of the LSTM Experiment

4.3.7.4 Conclusion

Since the Start of our study in 2000 (base year), We have seen the worst downfall in 2008 of 55.34% and a significant decline in 2020. LSTM optimisation has given a CAGR of 20.07% since its inception, with a Sharpe ratio of 0.966. A lump sum investment of Rs 10 Lack in portfolio using Mean Variance optimisation in 2000 would be Rs 6 Crore 1 Lack and 35 thousand in Present Date

4.4 Results and Conclusion

	Benchmark	MV	MD	RP	LSTM
Max Drawdown:	60.88%	41.59%	31.31%	46.45%	55.34%
CAGR:	10.54%	17.00%	17.85%	16.27%	20.07%
Sharpe:	0.514	0.780	0.911	0.697	0.966
Final Portfolio Value:	90.41 Lack	3.2 Crore	3.73 Crore	2.76 Crore	6.01 Crore

Table 4.6: Comparison of Outcomes of different Experiments

As in Table 4.6, We can conclude that Our LSTM model has outperformed all other optimisation methods with a CAGR of 20.07% while maintaining a Highest Sharpe ratio of 0.966. Meanwhile, the Sharpe ratio is of Maximum Diversification Optimisation with a value of 0.911 and above average return of 17.85%. The industry Standard Mean-Variance Optimisation technique has given a return of 17% with a Sharpe ratio of 0.780. Other than that, Risk parity optimisation has shown 16.27% CAGR and a reasonable Sharpe ratio of 0.697. The Benchmark has performed inferior to our experiments with a return of 10.54% CAGR and a Sharpe Ratio of 0.514

CHAPTER 5

Discussions and conclusion

In this chapter, the work is concluded and future plan is presented and the limitation of the work and possible future extensions are described respectively.

5.1 Final Conclusion

In this study, we use deep learning techniques to directly optimise the Sharpe ratio of a portfolio. By updating model parameters via gradient ascent, this pipeline avoids the limitations of conventional forecasting and enables us to better manage portfolio weights. BSE Sensex has been utilised as the benchmark portfolio in this study. We contrast our approach with other well-known techniques, such as risk parity optimisation, maximum diversification, classical mean-variance optimisation reallocation techniques. Our testing period, which runs from January 2000 to July 2022, encompasses the current COVID-19 situation. The outcomes demonstrate that our model offers the greatest performance, and a thorough analysis of our model's performance in the actual scenario demonstrates the logic and viability of our approach.

5.2 Contribution

This study contributes to the field since there is no publication in the literary works that extensively and collectively covers the following topics: practical limitations, probabilistic and analytical approaches, different forms of performance evaluation, evaluation of strategy using back testing , and use of programming language and Deep Learning.

5.3 Limitations

There are two major limitations in this study that could be addressed in future research.

- Current Study Parameters can only include one Benchmark
- The study focused on only stock portfolios while there are many different types of assets whose value is appreciated over time.
- The stock data availability limits this study at the start of the benchmark period as stocks get listed at a different point in time

5.4 Future scope

The next phase of this endeavour will include:

- Our goal is to examine portfolio performance using various objective functions. We may optimise the Sortino ratio or any other metric of the efficiency of the portfolio because to the flexible framework of our methodology.
- We will Concentrate to build a portfolio utilising ETFs and market indexes rather than using individual assets. This significantly lowers the range of available assets, and these indexes have shown strong correlations.
- Instead of just one, Our Technique can also include more market indices and sector-specific indices for Benchmarking Process

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