A Comparative Analysis of Deep Learning and Traditional Portfolio Optimization Models in Developed Financial Markets

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Divyanshu
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विश्वजीवनामृतं ज्ञानम्

ABV INDIAN INSTITUTE OF INFORMATION TECHNOLOGY AND MANAGEMENT

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CANDIDATES DECLARATION

I hereby certify that the work, which is being presented in the report, entitled **A Comparative Analysis of Deep Learning and Traditional Portfolio Optimization Models in Developed Financial Markets** , in partial fulfilment of the requirement for the award of the Degree of **Masters of Business Administration** and submitted to the institution is an authentic record of my own work carried out during the period *June 2023* to *October 2023* under the supervision of **Dr. Vishal Vyas**. I have also cited the references about the text(s)/figure(s)/table(s) from where they have been taken.

Date: Signatures of the Candidates

This is to certify that the above statement made by the candidates is correct to the best of my knowledge.

Date: Signatures of the Research Supervisor

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Abstract

Advancements in machine learning have opened up a wide range of new possibilities for using advanced computer algorithms, such as deep learning in portfolio risk management. However, very little evidence has been provided on the superior performance of deep learning models over traditional optimization models following the mean-variance framework in different financial market settings. This study uses two experiments with data from the Indian and U.S. securities markets to justify whether advanced machine learning models could outperform traditional portfolios' cumulative returns while optimizing the Sharpe ratio.

Keywords- Deep Learning, Optimization, Securities Market,

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CHAPTER 1

Introduction

In section 1.1 of this chapter, an overview of the background for the Portfolio Optimisation project is provided. The study work flow is then introduced step by step in section 1.2 as a result of the motives that have been provided. The motivation is discussed in section 1.3. The Research Workflow are then described in section 1.4.

1.1 Context

Portfolio optimization is a crucial element of asset management, aiming to maximize returns at a given risk level by selecting the optimal asset distribution within a portfolio. This concept was first pioneered by (Markowitz, 1952) leading to the development of Modern Portfolio Theory (MPT).

The primary advantage of constructing a diversified portfolio is the ability to reduce risk and create a smoother equity curve. Diversification allows for a higher return per unit of risk compared to investing solely in individual assets. This principle holds true as long as the assets in the portfolio are not highly correlated.

Indeed, while the benefits of diversification in portfolio allocation are indisputable, the process of selecting the "optimal" asset allocations is far from straightforward. This complexity arises from the dynamic nature of financial markets, which undergo significant changes over time. Assets that have exhibited strong negative correlations in the past may become positively correlated in the future, introducing an additional layer of risk and potentially compromising the portfolio's future performance.

Furthermore, the sheer breadth of available assets for portfolio construction poses a formidable challenge. For instance, in the context of the U.S. stock markets alone, there are more than 5000 individual stocks to consider. Additionally, a well-structured portfolio often extends beyond equities and may include bonds and commodities, significantly expanding the array of options available for allocation.

1.2 Objectives

To overcome the limitations associated with traditional mean-variance methods based on quadratic optimization, researchers have explored novel approaches in the field of portfolio optimization. These alternatives encompass both statistical and machine learning methods, offering promising avenues for improving portfolio management strategies.

Statistical approaches have been a focus of attention in this study. These methods include Autoregressive Conditional Heteroscedasticity (ARCH) introduced by (Engle and Granger, 1987) Autoregressive Integrated Moving Average (ARIMA), and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) proposed by (Bollerslev, 1986). These statistical techniques have traditionally been employed to model financial time series data and capture volatility dynamics.

In recent times, machine learning methods have gained significant traction for portfolio optimization, particularly in the context of forecasting. Approaches such as Neural Networks (NNs) (Bisoi et al., 2019) Support Vector Regression (SVR) (Chen, Liang, Hong and Gu, 2015) and ensemble learning (Zhou et al., 2019) have become popular choices. According to several comparative studies, machine learning exhibits greater capability in handling non-linear and non-stationary situations compared to statistical methods (Wang et al., 2020).

In addition to using machine learning models for return prediction in portfolio construction, this study introduces advanced methods that leverage Deep learning to directly optimize portfolios. This approach deviates from traditional techniques (McNeil et al., 2015) which typically involve forecasting projected returns, often using econometric models. Notably, these prediction stages do not guarantee optimal portfolio performance (Moody et al., 1998) as they seek to minimize a prediction loss that does not necessarily align with maximizing the portfolio's total gains.

In contrast, the proposed method focuses on optimizing the Sharpe ratio, which represents the return on investment per unit of risk. By directly optimizing this metric, it aims to enhance portfolio performance, offering a different and potentially more effective approach to portfolio optimization compared to traditional methods.

This innovative approach underscores the evolving landscape of portfolio optimization, as researchers increasingly explore machine learning and Deep learning techniques to address the challenges and complexities of modern financial markets.

1.3 Motivation

In recent years, there has been a growing utilization of machine learning techniques within the realm of finance, potentially influencing how hedge fund managers perceive the risk-reward ratio in financial markets (Wu et al., 2021). Notably, advanced machine learning algorithms have demonstrated remarkable achievements in various domains, including video games (Mnih et al., 2015) and board games (Silver et al., 2016).

Following the historic victory of the computer program AlphaGo over Lee Sedol, one of the most formidable human players in the game of Go, in 2016, professionals in the financial trading sector have developed a keen interest in deep learning methods, particularly Deep reinforcement learning (Meng and Khushi, 2019).

Machine learning techniques offer substantial potential in portfolio construction, with deep learning assuming an increasingly pivotal role within the industry (Bartram et al., 2021). This shift underscores the importance of leveraging advanced computational approaches to enhance decision-making processes and manage investment portfolios effectively.

Furthermore, in the context of wealth management, (Li et al., 2020) have highlighted the effective application of deep learning techniques, in optimizing asset allocation strategies. This indicates that machine learning, and deep learning in particular, have become integral tools for financial professionals seeking to navigate the complexities of modern wealth management and investment strategies.

1.4 Project work flow

The primary objective of this study is to conduct a comprehensive performance comparison of ten distinct portfolio construction methodologies, encompassing a wide array of approaches mentioned in the finance literature. These methodologies include mean-variance techniques (such as equally weighted portfolios, portfolios maximizing the Sharpe ratio, minimum variance portfolios, and portfolios aiming to maximize decorrelation), statistical approaches (comprising hierarchical risk parity, principal component analysis, and Holt's smoothing process), as well as deep learning models implemented through deep neural networks.

Financial asset prices exhibit a strong connection to their volatility patterns over time. Naturally, the predictability of stable stocks tends to surpass that of stocks with relatively higher levels of price noise. Therefore, to provide a more comprehensive assessment of the performance of various portfolio construction methods, the study incorporates multiple timeframes and asset types in its experimental design. This multifaceted approach allows for a more holistic understanding of how these methods perform under varying conditions.

The research employs two distinct experimental designs, each representing different timeframes and asset types. This approach enables a nuanced evaluation of the portfolio construction methods' effectiveness across diverse contexts, taking into account the varying characteristics of assets over different time horizons.

Additionally, to enhance the comparability of the methods under investigation, the study adopts a standardized input framework. Specifically, historical closing prices of financial assets serve as the sole input for all models. This standardized input approach ensures a level playing field, allowing for a fair comparison of the ability of these methods to process and leverage non-linear relationships. Notably, deep learning, as well as reinforcement learning, have demonstrated the capability to harness such non-linear relationships, potentially leading to superior results when compared to more linear methods commonly employed in portfolio construction.

In summary, this study adopts a rigorous and systematic approach to evaluate the performance of various portfolio construction methods, encompassing mean-variance, statistical, and deep learning techniques. By considering different timeframes, asset types, and utilizing a consistent input framework, the research aims to provide valuable insights into the effectiveness of these methods in optimizing portfolio construction within the realm of finance.

CHAPTER 2

Literature Review

2.1 Types of Portfolio

Gunjan and Bhattacharyya (2023) does categorization of portfolios based on the interplay of risk and return allows investors to tailor their investment strategies to align with their financial objectives and risk tolerance. This paper delineates five distinct portfolio categories, each catering to specific investment goals and preferences. Additionally, a behavioral perspective is introduced, expanding the classification into four types based on investor behavior and market dynamics.

2.1.1 Portfolio Categories

2.1.1.1 Aggressive Portfolio

- **Objective:** Aims for higher returns.
- **Approach:** Willing to undertake higher risks, often favoring companies in initial growth stages.
- **Examples:** Companies with substantial growth potential.

2.1.1.2 Defensive Portfolio

- **Objective:** Minimizes risk.
- **Approach:** Prefers companies offering daily need products, ensuring stability even in adverse conditions.
- **Examples:** Defensive industries providing essential goods and services.

2.1.1.3 Income Portfolio

- **Objective:** Generates income from dividends or recurring benefits.
- **Approach:** Similar to a defensive portfolio but focuses on consistent returns.
- **Examples:** Real estate, FMCG, and stable industries providing regular dividends.

2.1.1.4 Speculative Portfolio

- **Objective:** Pursues extremely high-risk opportunities.
- **Approach:** Often termed as gambling, involves investments in IPOs, initial product research, and takeover targets.
- **Examples:** Investments with high uncertainty and potential for significant gains or losses.

2.1.1.5 Hybrid Portfolio

- **Objective:** Balances risk and return optimally.
- **Approach:** Utilizes a mix of different assets based on risk-return profiles.
- **Examples:** Diversified portfolios combining assets with varying degrees of risk.

2.1.2 Behavioral Portfolio Categories

2.1.2.1 Transaction Cost Portfolio

- **Objective:** Reduces transaction costs during asset transactions.
- **Approach:** Utilizes time penalization techniques to minimize transaction fees and charges.
- **Examples:** Strategies focused on efficient buying and selling to minimize costs.

2.1.2.2 Robust Portfolio

- **Objective:** Reduces transaction costs in a sparse and robust portfolio selection process.
- **Approach:** Mitigates estimation errors inherent in optimization processes.
- **Examples:** Strategies that enhance robustness in portfolio selection.

2.1.2.3 Regularized Portfolio

- **Objective:** Addresses estimation errors in changing market conditions.
- **Approach:** Utilizes regularization techniques to handle uncertainties arising from market dynamics.
- **Examples:** Adaptive strategies that account for changing environments.

2.1.2.4 Reinforcement Learning Portfolio

- **Objective:** Adapts to changing market conditions through continuous learning.
- **Approach:** Utilizes reinforcement learning techniques to dynamically adjust to evolving market dynamics.
- **Examples:** Strategies that incorporate machine learning to adapt to real-time market changes.

This comprehensive classification of portfolios based on risk-return dynamics and investor behavior provides a versatile framework for investors to tailor their investment strategies. By understanding the nuances of each portfolio type, investors can make informed decisions that align with their financial objectives and risk tolerance, promoting a more diversified and adaptive approach to portfolio management.

2.2 The Indian Stock Market

The Indian stock market has various distinguishing features compared to established financial markets like the New York Stock Exchange (NYSE). Despite being relatively small compared to the US market, India's stock market is expanding rapidly and presents substantial investment prospects. The market has experienced remarkable growth, with the number of listed companies increasing significantly from just a few initially to over 5,000 on the two leading stock exchanges, the Bombay Stock Exchange (BSE) and the National Stock Exchange of India (NSE), by 2020 (Naik et al., 2020).

To foster the development of the stock market, the Indian government has implemented reforms aimed at attracting foreign investors and enhancing transparency and accountability within the market. Experts anticipate that India's economy will continue to grow robustly in the forthcoming years, making it an appealing destination for investors seeking high returns. Presently, India's stock market is home to the world's fourth-largest equity market, following the United States, China, and Japan. The benchmark Sensex index, which monitors 30 prominent companies, has surged 10% over the previous three months, while the broader Nifty 50 index has jumped 11% during the same period (M et al., 2018).

Investors can gain exposure to the Indian stock market through the two primary stock exchanges: the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE). Both exchanges follow the same trading mechanism, trading hours, and settlement process. As of June 2023, the BSE had 5,657 listed firms, while the NSE had 2,137 as of March 31, 2023. Most significant Indian companies are listed on both exchanges (Yadav and Sangeeta, 2023).

Foreign institutional investors, including mutual funds, pension funds, endowments, sovereign wealth funds, insurance companies, banks, and asset management companies, can invest directly in Indian stocks. High-net-worth individuals with a minimum net worth of $50 million can register as sub-accounts of a Foreign Institutional Investor (FII). However, individual foreign investors cannot invest directly in the Indian stock market.

(Surana, 2023) analysed that Indian stock market's liquidity employed multiple liquidity measures and utilized a Vector Auto-Regressive (VAR) approach. It found that the market displays interesting trading patterns among liquidity dimensions and emphasizes the significance of depth and tightness. The research revealed that the Indian stock market demonstrates greater persistence in depth, breadth, and immediacy but exhibits lower transaction costs compared to developed markets.

The Indian government has established rules allowing Foreign Institutional Investors (FIIs), Non-Resident Indians (NRIs), and Persons of Indian Origin (PIOs) to participate in the main and secondary capital markets via the Portfolio Investment Scheme (PIS). Through the stock exchanges in India, FIIs/NRIs are permitted to buy shares and debentures of Indian businesses. The corporation's paid-up capital determines the caps on overall FII and NRI/PIO investments. Public sector banks, including the State Bank of India, have a 20% threshold. The 24% FII investment ceiling can rise to the statutory or sectoral limit with authorization. Up to 24%, the 10% NRIs/PIOs ceiling may be raised. The reserve bank of India keeps track of these investments every day. (Reserve Bank of India, 2023)

2.3 Baseline method and optimization portfolio construction

Naïve diversification, also known as equal-weighted portfolio construction, is a simple approach for reducing a portfolio's idiosyncratic risk without sacrificing the expected rate of return. It involves allocating equal weights to assets in the portfolio. In contrast, portfolio optimization, pioneered by Harry Markowitz, aims to find the optimal allocation of weights that achieves an acceptable expected return with minimal volatility.

Modern Portfolio Theory (MPT), by (Markowitz, 1952), maximizes returns for a given risk level via mean-variance portfolio construction. The theory centers on the efficient frontier, guiding investors in optimizing expected returns for a chosen risk level.

Markowitz's efficient frontier is a core MPT concept. It's the part of the minimum-variance curve extending above and to the right of the global minimum variance portfolio. Rational, risk-averse investors prefer portfolios on this frontier for their superior risk-return trade-off. As risk escalates, the efficient frontier curve flattens, underscoring a key MPT principle: continually pursuing higher returns entails disproportionately more risk. Thus, effective diversification is crucial to balancing risk and return. In essence, MPT encourages investors to construct portfolios along the efficient frontier to maximize expected returns while recognizing the diminishing returns associated with escalating risk.

Market capitalization-weighted portfolios have garnered criticism for their inefficiency and long-term underperformance compared to equally weighted portfolios. This observation is supported by studies such as those by (Bolognesi et al., 2013) and (Malladi and Fabozzi, 2017). Additionally, research by (DeMiguel et al., 2009) not only reaffirmed the superior efficiency of equal-weighted portfolios over capitalization-weighted ones but also demonstrated that equal-weighting outperforms mean-variance-based portfolio strategies in out-of-sample testing.

Recent research, such as the study by (Taljaard and Maré, 2021) has highlighted a shift in the performance dynamics of equal-weighted portfolios. Specifically, it has been observed that an equal-weighted portfolio of stocks in the S&P 500 now significantly underperforms the market capitalization-weighted portfolio, especially in the short term. This contrasts with earlier findings.

Moreover, (Kritzman et al., 2010) argued that optimized portfolios, which are constructed through sophisticated optimization techniques, demonstrate superior out-of-sample performance when compared to equal-weighted portfolios. This suggests that optimization methods can provide better results than the traditional equal-weighting approach.

Given these evolving dynamics, it is prudent to consider a comprehensive comparison of various portfolio construction approaches, including both baseline methods like equal-weighting and more advanced optimization techniques. Such an assessment can provide a more nuanced understanding of the changing landscape of portfolio performance and help investors make informed decisions based on their specific objectives and investment horizons.

2.4 Machine Learning and optimization portfolio construction

The foundation of the Markowitz efficient frontier is constructed upon certain assumptions that have faced scrutiny when applied to real-world contexts (Ma et al., 2021). Specifically, it presupposes that all investors exhibit rational behavior and are uniformly risk-averse. Furthermore, the model assumes that every investor enjoys equal access to borrowing funds at a risk-free interest rate, despite this not aligning with the actual circumstances. Additionally, the traditional concept of the efficient frontier operates on the premise that asset returns conform to a normal distribution, whereas, in practice, asset returns frequently deviate significantly, often extending as far as three standard deviations from the mean.

Machine learning offers significant potential for the development of effective trading strategies, particularly in the realm of high-frequency trading, a feasibility that was previously limited (Arnott et al., 2019). The benefits of employing algorithmic machine learning in trading are extensive (Zhang et al., 2020), with a primary focus on enhancing alpha, or excess returns ((Sirignano and Cont, 2019) and (Zhang, Zohren and Roberts, 2019)). Much of the research concentrates on regression and classification pipelines, which involve forecasting excess returns or market movements over specific, predefined time horizons.

The utilization of machine learning techniques in finance has been on the rise, potentially influencing the way hedge fund managers assess risk-reward ratios within the financial market. Hedge funds, known for their adaptability, have been garnering increasing interest from investors (Wu et al., 2021). In a comprehensive research endeavor, (Wu et al., 2021) employed machine learning for hedge fund return prediction and selection, demonstrating that machine learning-based forecasting methods consistently outperformed the respective Hedge Fund Research indices across various scenarios. Among the four machine learning methods examined by (Wu et al., 2021), neural networks emerged as particularly noteworthy.

Concerning risk management, (Arroyo et al., 2019) have demonstrated the utility of machine learning in aiding venture capital investors in their decision-making processes. This technology assists in identifying investment prospects and evaluating associated risks. Additionally, (Jurczenko, 2020) has found that machine learning algorithms play a valuable role in enhancing stock risk forecasts, particularly when it comes to out-of-sample predictions of equity beta.

In a comprehensive research conducted by (Gu et al., 2020) it was found that machine learning tools surpass linear methods in terms of their predictive capabilities. The effectiveness of portfolios constructed using machine learning algorithms has been established, particularly for portfolios that have not undergone optimization, as highlighted by (Kaczmarek and Perez, 2021). Despite its widespread acceptance, the modern portfolio theory has faced criticism for its practical limitations ((Kolm et al., 2014) and (DeMiguel et al., 2009)). Consequently, a growing body of literature is dedicated to enhancing portfolio optimization techniques. This includes exploring alternatives such as replacing the statistical moments of asset returns with more reliable predictions (DeMiguel et al., 2009) or applying machine learning methods in place of traditional quadratic optimization, as proposed by (De Prado, 2016).

Reinforcement learning has found application in the domain of algorithmic trading, as highlighted by the work of Wen et al. (2021). Their research in options trading demonstrates the effectiveness of a reinforcement learning model, which outperforms the traditional buy-and-hold strategy in generating favorable returns. In the context of futures contracts, the study conducted by Zhang et al. (2020) establishes the superiority of trading strategies employing reinforcement learning over time series momentum strategies, yielding positive profits even in the presence of considerable transaction costs. Additionally, Kolm and Ritter (2019) explore the use of reinforcement learning to train a machine learning algorithm for options hedging in realistic scenarios, specifically identifying the minimal variance hedge based on a given transaction cost function.

In the realm of reinforcement learning, an agent interacts with its environment, learning an optimal course of action through trial and error, utilizing reward and penalty points for successful and erroneous actions, respectively. This approach is particularly relevant for problems involving sequential decision-making (Li, 2017). Unlike deep learning models, which rely on extensive historical datasets, reinforcement learning stands out due to its capacity for self-learning and adaptation based on the environment. In dynamically changing investment landscapes, where volatility is constant, the adaptability of reinforcement learning makes it a more attractive option compared to the reliance on historical datasets in deep learning. Successful integration of reinforcement learning in asset management and portfolio construction could potentially reshape the risk and return standards within the financial sector. However, empirical evidence supporting the application of reinforcement learning models in asset management remains limited as of now.

In the context of optimized strategies, (Kaczmarek and Perez, 2021) have demonstrated that when machine learning methods are employed for the preselection of stocks within portfolios, conventional portfolio optimization methods such as mean-variance and hierarchical risk parity exhibit an enhancement in the risk-adjusted returns of these portfolios. This improvement results in the outperformance of equal-weighted portfolios in out-of-sample analyses.

2.5 Deep Learning and optimization portfolio construction

Recently, there has been a growing interest in applying Deep learning techniques to portfolio allocation tasks within the field of finance. This surge in interest began following the seminal work of (Zhang, Zhong, Dong, Wang and Wang, 2019), leading to the emergence of a new branch of finance focused on the utilization of Deep learning methods in portfolio construction. Initially, these methods were applied to various financial domains, including cryptocurrencies, and as well as other asset classes.

Researchers have conducted comprehensive investigations to address the challenge of time series forecasting for stock returns using deep learning techniques ((Chiang et al., 2016), (Di Persio and Honchar, 2016), (Moghaddam et al., 2016)). Several studies propose that various Recurrent Neural Networks (RNN) exhibit superior performance compared to conventional financial time series models across diverse markets (Bao et al. (2017), Chen, Zhou and Dai (2015), 4Sermpinis et al. (2019)). The recurrent layer in RNN incorporates feedback loops, facilitating the retention of information in a "memory cell" over time. Nevertheless, its effectiveness diminishes when confronted with learning tasks that involve extended long-term temporal dependencies.

Long Short-Term Memory (LSTM) represents a distinctive category of Recurrent Neural Networks (RNN) that has demonstrated efficacy in text mining for forecasting stock returns Kraus and Feuerriegel (2017). LSTM incorporates specialized "memory cells" capable of preserving information over extended time intervals Hochreiter and Schmidhuber (1997). As a result, LSTM frequently exhibits superior performance in handling sequential data and making predictions in financial time series, surpassing the capabilities of traditional RNN models (Jiang et al. (2019), Nelson et al. (2017)). This advantage is particularly pronounced in the SRI context, where investors prioritize long-term returns over the short-term market's volatility.

Scholars have undertaken comparisons of various RNN architectures, including Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU) networks Samarawickrama and Fernando (2017). Some researchers have proposed that the Bi-directional LSTM (BiLSTM) could present a more favorable alternative for similar Optimization problems Chen et al. (2017). While LSTM and GRU, featuring a unidirectional flow of information, may suffice for many sequence prediction tasks, the BiLSTM model reviews the data in both forward and backward directions Schuster and Paliwal (1997). This dual-directional approach contributes to enhanced optimization accuracy, especially in the optimization of sequential data such as financial time series.

CHAPTER 3

Experimental design

3.1 Benchmark Portfolio

It's crucial to have a precise and clear method to measure your portfolio since a financial market's aspects are always changing. In order to do this, benchmark portfolios should provide investors with a realistic assessment of how their portfolio are performing relative to the important and specialised market categories.We can evaluating our portfolio's total performance with respect to predetermined benchmarks, such as a market indices or a group of asset classes, by using a benchmark portfolio. These indexes are "passive" and unmanaged, in contrast to the actively managed investment portfolio that makes up your overall investment portfolio. As a result, benchmark portfolios may be used to assess the value a manager brings to your portfolio.
We are going to use BSE Sensex as Benchmark Portfolio
Theory The benchmark index of BSE is referred as Sensex. The 30 biggest and most popular companies on the BSE make up the Sensex, which serves as a bellwether for the Indian economy. It is market capitalisation-weighted and loat-adjusted. Every year, between June and December, the Sensex is evaluated and Rebalanced. The Sensex is run by Standard & Poor's and is the earliest known stock index in India, having been founded in 1986. It is used by analysts and investors to track the economic cycles in India as well as the growth and decline of certain sectors Ma et al. (2021) .

3.2 Traditional Portfolio Optimization Methods

3.2.1 Equal-Weighted Portfolio Optimization

The equal-weighted portfolio optimization technique is a simple, yet effective strategy for allocating capital across different assets. Unlike traditional methods that rely on complex optimization algorithms and historical data, equal weighting assigns the same proportion of capital to each asset in the portfolio, regardless of its individual risk, return, or market capitalization DeMiguel et al. (2009).

The formula for calculating the weight of each asset in an equal-weighted portfolio is:

$$w_i = \frac{1}{N}$$

Where:

w_i is the weight of asset i.

N is the total number of assets in the portfolio.

3.2.2 Maximum Diversification Portfolio Optimization

One of the most crucial factors to take into account when building an investing portfolio is spreading assets out to lower risk. One strategy is to diversify as much as possible. It is a portfolio strategy that seeks to build the most diversified portfolio achievable. The strategy specifically aims to increase the diversification ratio. Hence, the greatest diversity method is a risk-based allocation strategy than ignores predicted returns. The strategy aims to create a portfolio with a wide range of investments (Theron and van Vuuren, 2018).

$$\min_{w} \qquad \frac{P'(w) \cdot \Sigma}{\sqrt{P'VP}}$$

Where:

P: is the portfolio weights.

Σ: is the asset volatilities.

V: covariance matrix of these assets.

r_f: is the risk free rate.

3.2.3 Risk Parity Portfolio Optimisation

An method to asset management known as risk parity optimisation places more emphasis on risk allocation rather than capital allocation, which is often referred to as volatility. According to the risk parity method, the risk parity portfolio may produce a better Sharpe ratio and can be more resilient to market downturns than the standard portfolio when asset allocations are modified to the same risk level. Risk parity is susceptible to substantial changes in correlation regimes, as was seen in Q1 2020, which caused risk-parity funds to perform much worse than average during the Covid-19 sell-off (Feng and Palomar, 2016).

$$\begin{aligned} \min_{w} \quad & \phi(w) \\ \text{s.t.} \quad & b\log(w) \geq c \\ & w \geq 0 \end{aligned}$$

Where:

w: weights of the portfolio.

b: risk contribution constraints.

$\phi(w)$: Standard Deviation of the portfolio

c: is an arbitrary constant.

3.2.4 Mean Variance Portfolio Optimisation

The objective of creating a portfolio of assets using modern portfolio theory (MPT), also known as mean-variance optimisation, is to maximise anticipated return for a certain degree of risk. It formalises and broadens the concept of diversity in investment, which holds that having a variety of financial assets reduces risk compared to holding a single kind. Its main conclusion is that an investment's return and risk should not be evaluated on its own, but rather in the context of the risk and return of the whole portfolio. It substitutes asset price variation for risk. (Markowitz, 1952)

$$\max_{w} \quad \frac{R(w) - r_f}{\phi(w)}$$

Where:

$R(w)$ is the return function:

w: is the vector of weights of the portfolio.

$\phi(w)$: Standard Deviation of the portfolio.

r_f: is the risk free rate.

3.2.5 Mean-Absolute Deviation Portfolio Optimisation

The mean-absolute deviation (MAD) model, initially proposed by (Konno and Yamazaki, 1991) and later refined by (Konno and Koshizuka, 2005), proves to be applicable in addressing extensive and highly diversified portfolio selection challenges. This approach finds utility in scenarios such as long-term asset liability management (ALM), as highlighted by the model developed by Clark et al. (2006), and in mortgage-backed security models, exemplified by the work of (Hayre, 2002). Particularly, these models are effective when investments are intended for prolonged periods and involve diversified portfolios.

An application of robust portfolio optimization utilizing MAD, as presented by Moon and Yao (2011), was implemented on diverse stock market data, specifically the NYSE dataset from 2003. Notably, this application demonstrated a reduction in computational complexity, yielding optimal results. MAD is observed to outperform mean-variance models based on (Markowitz, 1952) principles. However, it's important to note a limitation of this model: it penalizes both positive and negative deviations. The mathematical representation of this model is in accordance with the formulation by (Konno and Yamazaki, 1991).

$$\textit{minimize } w(x) = E \mid \sum_{j=1}^{n} R_j x_j - E \sum_{j=1}^{n} R_j x_j \mid$$

$$\text{subject to } \sum_{j=1}^{n} E \mid R_j \mid x_j \geq \rho M_0, \text{ and}$$

$$\sum_{j=1}^{n} x_j = M_0,$$

$0 \leq x_j \leq u_j, j = 1, 2, ..., n$ where R_j is the return of asset j, x_j is the amount invested in asset j, *rho* is a parameter representing the minimal rate of return required by an investor, M_0 is the total amount of fund and u_j is the maximum amount of money which can be invested in an asset j.

3.2.6 Minimax Portfolio Optimisation

The Minimax (MM) model, as introduced by Cai, X. and Teo, K.L. and Yang, X.Q. and Zhou, X.Y. (2004) and further discussed by Li et al. (2019), utilizes the minimum return as a risk metric. When dealing with multivariate and normally distributed asset returns, both Minimax Cai, X. and Teo, K.L. and Yang, X.Q. and Zhou, X.Y. (2004) and the mean-variance model by (Markowitz, 1952) yield equivalent results. Notably, Minimax exhibits advantages in scenarios where returns deviate from normal distribution. Its efficiency, attributed to linear programming, makes it a fast and adaptable choice for complex models and constraints. However, a drawback of Minimax lies in its sensitivity to outliers, rendering it unsuitable for situations with missing historical data. The mathematical representation of this model aligns with the formulation proposed by Cai, X. and Teo, K.L. and Yang, X.Q. and Zhou, X.Y. (2004).

$$
\begin{aligned}
& max\ M_p \\
& \text{subject to } \sum_{j=1}^{N} w_j y_j - M_p \geq 0, t = 1, 2, ..., T, \\
& \sum_{j=1}^{N} w_j \overline{y_j} \geq G, \\
& \sum_{j=1}^{N} w_j \leq W, \\
& w_j \geq 0, j = 1, 2, ..., N
\end{aligned}
$$

where, y_i is the return on one dollar invested in a security j in a time period t, $\bar{y}_i$ is the average return on security j, w_j is the portfolio allocation to security j, M_p is the minimum return on portfolio, G is the minimum level of return and W is the total allocation.

3.2.7 Lower partial moment Portfolio Optimisation

The Lower Partial Moment (LPM), initially introduced by Nawrocki (1992) and later discussed by Brogan, A.J. and Stidham Jr., S. (2008), employs a series of moments to assess downside risk within a portfolio. Continuous research has identified various types of moments that can be considered in the context of LPM. These multiple lower moments (N) are systematically examined as part of the LPM framework. The methodology is then utilized in calculating performance metrics such as the Sortino ratio, Omega ratio, and Kappa ratio, as described by Chen (2016). The formal definition of this model aligns with the formulation proposed by Nawrocki (1992).

$$LPM_\alpha(\tau, Ri) = \int_{-\infty}^{\tau} (\tau - R)^\alpha \, \partial F(R)$$

where, α is the degree of LPM, τ is the target return, R is the return, and $\partial F(R)$ is the cumulative distribution function of the asset return R.

3.3 Machine Learning Models

3.3.1 Random forest (RF)

The Random Forest (RF), a nonparametric and nonlinear model, was initially introduced by Ho (1995) and later expounded upon by Breiman in 2001. Renowned for its ability to mitigate overfitting issues due to its inherent convergence Breiman (2001), RF has become a popular choice for stock prediction tasks (Ballings et al. (2015); Booth et al. (2014); Qin et al. (2013)). The key parameters influencing the performance of

RF include the number of decision trees, the maximum depth of the trees (referred to as max-depth), the minimum number of samples required to split an internal node (min-samples-split), the minimum number of samples necessary to form a leaf node (min-samples-leaf), and the number of features considered when seeking the best split (max-features).

3.3.2 Support vector regression (SVR)

Support Vector Regression (SVR) is a well-established machine learning model that has found extensive applications in stock market prediction (Emir (2013); Lu et al. (2009); Matías and Reboredo (2012); Rasel et al. (2015)). Employing Vapnik's Structural Risk Minimization (SRM) principle, SVR effectively addresses various regression challenges. Rooted in statistical learning theory, SVR guides the regulation of generalization, aiming to strike the optimal balance between model complexity and empirical risk.

3.3.3 Autoregressive integrated moving average (ARIMA) model

ARIMA is a classical statistical model, which is often used in stock prediction. The ARIMA model can be presented as follows:

$$(1 - \sum_{i=1}^{p} \phi_i L^i)(1 - L_i)^d \cdot r_t = \delta + (1 - \sum_{i=1}^{q} \theta_i L^i)\varepsilon_t$$

where p, d, q, L, ϕ_i, θ_i and ε_t represent the number of autoregressive terms, the number of difference times, the number of moving average terms, lag operator, autoregressive parameter, moving average parameter and error term respectively Yu et al. (2020). p,d,q need to be determined before using ARIMA model. Since ARIMA model needs some restrict hypotheses such as stationarity test, this paper set the values of p,d,q before each training process.

3.3.4 Mean-variance with forecasting (MVF) model

Markowitz (1952) is recognized as the precursor of modern finance theory, having introduced the mean-variance (MV) model that provided a mathematical approach to balancing the trade-off between maximizing expected return and minimizing risk. In accordance with the framework proposed by Yu et al. (2020), this study integrates the predictive outcomes related to returns to enhance the MV model for constructing the Mean-Variance-Forecast (MVF) model.

The MVF model essentially represents a multi-objective optimization problem. The subsequent equations articulate the formulation of the MVF model.

$$\min \sum_{i,j=1}^{n} x_i x_j \sigma_{ij}$$

$$\max \sum_{i=1}^{n} x_i \hat{r}_i$$

$$\max \sum_{i=1}^{n} x_i \bar{\varepsilon}_i$$

$$\text{Subject to} \quad \sum_{i=1}^{n} x_i = 1$$

$$0 \leq x_i \leq 1, \quad i = 1, 2, \ldots, n$$

where x_i denotes the proportion of asset i in the portfolio, n is the number of assets in the portfolio, σ_{ij} is the covariance of asset i and j, $\hat{r}_i$ denotes the predicted return of asset i, and $\bar{\varepsilon}_i$ represents the average predictive errors of asset i over the sample period. Ma et al. (2021) sets the sample period as 20 trading days to build the MVF model according to Yu et al. (2020). In other words, $\hat{r}_i$ is the predicted return of asset i at time t, and $\bar{\varepsilon}_i$ means the average predictive error of asset i over the past 20 trading days, i.e., time $t, t-1, \ldots, t-19$. The predictive error of asset i at time t equals to $\varepsilon_i = r_i - \hat{r}_i$, where r_i represents the actual return of asset i. Eqs. (4)-(5) mean maximization of the expected portfolio return and the sample period's abnormal return, respectively.

The equal-weighted method is often used to convert the above multiple objective portfolio optimization to a single objective model (Yu et al. (2020)). Thus, the MVF model becomes the following form:

$$\min \sum_{i,j=1}^{n} x_i x_j \sigma_{ij} - \sum_{i=1}^{n} x_i \hat{r}_i - \sum_{i=1}^{n} x_i \bar{\varepsilon}_i$$

$$\text{Subject to} \quad \sum_{i=1}^{n} x_i = 1$$

$$0 \leq x_i \leq 1, \quad i = 1, 2, \ldots, n$$

3.3.5 Omega with forecasting (OF) model

The Omega ratio was initially introduced by Keating and Shadwick (2002) and has since been widely utilized for portfolio construction due to its ability to overcome the limitations of the Sharpe ratio (Gilli et al. (2011); Kane et al. (2009); Kapsos et al. (2014)). The Omega ratio is defined as follows:

$$\omega = \frac{E(y_i) - \tau}{E\left[\tau - y_i\right]} + 1$$

where τ represents the threshold for dividing returns into expected (revenue) and unexpected (loss), determined by investors, and y_i signifies the random return of asset i. Since the Omega ratio requires the probability distribution of asset returns, the solution becomes biased and overly optimistic when this distribution is imprecise Kapsos et al. (2014). To address this issue, Kapsos et al. (2014) introduced the worst-case Omega ratio (WCOR) and modified the Omega model as follows:

$$\max \psi$$

$$\text{Subject to}$$

$$\delta\left(\sum_{j=1}^{n} x_j \bar{r}_j^i - \tau\right) - (1-\delta)\frac{1}{T^i}\sum_{t=1}^{T^i} \eta_t^i \geq \psi$$

$$\eta_t^i \geq -\sum_{j=1}^{n} x_j \bar{r}_j^i + \tau$$

$$\eta_t^i \geq 0$$

$$\sum_{j=1}^{n} x_j = 1$$

$$0 \leq x_i \leq 1$$

$$t = 1, 2, \ldots, T^i; \quad i = 1, 2, \ldots, l; \quad j = 1, 2, \ldots, n$$

where x_i represents the proportion of asset i in the portfolio, η_{it} is an auxiliary variable used to linearize this portfolio model, δ denotes the risk-return preference of this model, T_i signifies the sample period of the i-th distribution, $\bar{r}_{ij}$ represents the sample period's mean return of the i-th distribution, l is the number of distributions, and n is the number of assets in the portfolio. This paper sets δ, T_i, τ, and l as 0.5, 20, 0, and 1, respectively, according to Yu et al. (2020).

Subsequently, return predictive results can be combined with this model to build the Omega Forecast (OF) model, similar to the MVF model:

$$\max \psi \quad \max \sum_{i=1}^{n} x_i \hat{r}_i \max \sum_{i=1}^{n} x_i \bar{\varepsilon}_i$$

$$\text{Subject to}$$

$$0.5\left(\sum_{i=1}^{n} x_i \bar{r}_i\right) - 0.5\sum_{t=1}^{20} \eta_t \geq \psi$$

$$\eta_t \geq -\sum_{i=1}^{n} x_i \bar{r}_i$$

$$\eta_t \geq 0$$

$$\sum_{i=1}^{n} x_i = 1$$

$$0 \leq x_i \leq 1$$

$$t = 1, 2, \ldots, 20; \quad i = 1, 2, \ldots, n$$

Similarly, the multi-objective optimization model can be converted to a single objective model, following Yu et al. (2020):

$$\text{(26)} \quad \min -\psi - \sum_{i=1}^{n} x_i \hat{r}_i - \sum_{i=1}^{n} x_i \bar{\varepsilon}_i$$

Subject to

$$0.5\left(\sum_{i=1}^{n} x_i \bar{r}_i\right) - 0.5 \sum_{t=1}^{20} \eta_t \geq \psi$$

$$\eta_t \geq -\sum_{i=1}^{n} x_i \bar{r}_i$$

$$\eta_t \geq 0$$

$$\sum_{i=1}^{n} x_i = 1$$

$$0 \leq x_i \leq 1$$

$$t = 1, 2, \ldots, 20; \quad i = 1, 2, \ldots, n$$

3.4 Deep Learning Models

3.4.1 Deep multilayer perceptron (DMLP)

DMLP, a classic Artificial Neural Network (ANN), distinguishes itself from the multilayer perceptron (MLP) by incorporating a greater number of hidden layers. While MLP is theoretically capable of approximating arbitrary functions in terms of mapping abilities Principe et al. (1999), empirical evidence suggests that DMLP often outperforms MLP with few hidden layers Orimoloye et al. (2020); Singh and Srivastava (2017). The DMLP model comprises three components: the input layer, hidden layer, and output layer. In this study, stochastic gradient descent is employed to train DMLP, and the early stopping technique is implemented to mitigate overfitting issues during the training process. The primary hyperparameters of DMLP include hidden nodes, hidden layers, optimizer, learning rate, activation function, loss function, batch size, and patience. Following the recommendation by Orimoloye et al. (2020), the rectified linear unit (relu) function is selected as the activation function.

3.4.2 Long short term memory (LSTM) neural network

The Long Short-Term Memory (LSTM) neural network, a type of recurrent neural network, was introduced to address the limitations of conventional recurrent neural networks by preserving long-term information Graves and Schmidhuber (2005). This attribute is primarily attributed to the presence of memory cells in the hidden layer. The typical architecture of an LSTM neural network includes an input layer, hidden layer, and output layer. In this study, training of the LSTM neural network is conducted using stochastic gradient descent, and early stopping techniques are employed to mitigate overfitting. The considered hyperparameters for the LSTM neural network encompass hidden nodes, hidden layers, learning rate, batch size, patience, dropout rate, recurrent dropout rate, activation function, optimizer, and loss function. As per the recommendation by (Orimoloye et al., 2020) the rectified linear unit (relu) function is chosen as the activation function.

3.4.3 Convolutional neural network (CNN)

The Convolutional Neural Network (CNN), an innovative type of Artificial Neural Network (ANN), was introduced by LeCun et al. (1995). CNNs are commonly utilized in computer vision and image processing, consistently demonstrating impressive performance (Ji et al. (2012); Long et al. (2015)). In recent years, researchers have explored the application of CNNs in stock price prediction, yielding promising results (Hoseinzade and Haratizadeh (2019); Sezer and Sezer and Ozbayoglu (2018)). The typical structure of a CNN involves multiple consecutive convolutional layers and pooling layers, followed by fully connected layers. Given that stock returns are time series data, this study employs a one-dimensional (1D) CNN for stock return prediction. Similar to the LSTM neural network, the stochastic gradient descent method is utilized for training the CNN, and early stopping is employed to address overfitting concerns. The hyperparameters considered for the CNN in this study encompass filter numbers, convolutional layers, maxpooling layers, fully connected layers, fully connected layer nodes, learning rate, patience, batch size, activation function, optimizer, and loss function.

CHAPTER 4

Methodology

4.1 Mechanism/Algorithm

1. Selections of the Sectors and Stocks First, the present analysis will focus on nine crucial NSE sectors. Some industries include *pharmaceuticals, infrastructure, real estate, media, public sector banks, private sector banks, and large-cap, mid-cap, and small-cap companies*. The leading stocks in each sector and their relative contributions to the calculation of the overall sectoral index are included in monthly reports published by the NSE. The contributions are given percentage weights. D determined using the NSE report released on Nov 31, 2023 NSE (Nov 2023). Following these actions, 28 stocks are picked for the nine specified industries.

2. Data Gathering The download function provided by the Python language's yfinance module is used to retrieve the historical prices of its 28 equities from the Yahoo Finance website. The stock prices are utilised for portfolio creation for 23 years, beginning on January 1, 2000, and ending in Nov 2023. Six elements of the web-extracted raw data are open, high, low, close, volume, and adjusted close. The other factors are neglected since the present study is based on univariate analysis, which only considers the stock's closing price for the period from January 1, 2000, to Nov, 30 2023.

3. Volatility and Return on Investment Each stock's return and log return values are calculated daily based on the relative historical values of that stock. The change in the closing values over consecutive days is expressed by the daily return and the log return for a stock, and their logarithms are calculated in percentage terms. Using Python's pct change library function, the daily return and log returns are calculated. The values of each stock's daily and annual volatility are then calculated. A stock's daily volatility is the standard deviation of its daily returns. The volatility numbers are computed using the Python function std. Given that there are 254 working days in a year, the daily volatility of a stock is multiplied by the square root of 254 to get its annual volatility. According to an investor's perspective, a stock's annual volatility is a sign of how risky it is.

4. Stock Return Covariance and Correlation Matrices After obtaining each stock's return and volatility values, the stocks' covariance and correlation matrices are created based on their return values in the training dataset. These matrices provide vital info for the creation of a portfolio by assisting in comprehending the patterns of relationship among the stocks in a specific sector. Python methods called cov and corr are used to calculate the covariance and correlation matrices. The main optimisation goals of a portfolio design work are minimising risk and optimisation. The algorithm tries to distribute funds across stocks with little or no correlation in a diversified portfolio that minimises risk. It is feasible to identify these stocks by examining their covariance or correlation matrices.

5. Portfolio Return and Risk Estimation The first set of portfolios is built at this stage using each sector's covariance and correlation matrices. The portfolios are first created by giving each of the 28 stocks in a particular sector the same weight. The equal-weighted portfolio's annual return and volatility numbers are calculated for each of the nine sectors. The anticipated return of a portfolio made up of n capital assets (i.e., stocks) is indicated as Exp(Ret) in (Eq 1), from which the return of a portfolio based on its historical return values is calculated.

$$Exp(Ret) = w_1 E(Ret_{c_1}) + w_2 E(Ret_{c_2}) + ...w_n E(Ret_{c_n}) \qquad (Eq\ 1)$$

The annual return and volatility of the equal-weight portfolio of each sector are calculated using the stock's annual return and volatility metrics. The mean annual returns for each stock that makes up a portfolio are calculated using the Python function which is resembled by argument 'Y'. On the other hand, the daily volatility values are multiplied by a factor of the square root of 254 to obtain the annual volatility values for the equal-weight portfolios. The equal-weight portfolios serve as benchmarks for assessing the performance of other portfolios and provide one a baseline level of return and risk associated with the sectors over the training records. The return and risk projections made using equal-weighted portfolios, however, are very inaccurate predictors of future returns and dangers. Therefore, more accurate projections of the potential return and hazards are required. The creation of the portfolios with the lowest and highest levels of risk is necessary as a result.

6. Designing Minimum-Risk Portfolios At this stage, minimum-risk portfolios are created for the nine sectors to enhance the equal-weight portfolios. The lowest values for their variances are seen in portfolios with the lowest risk. A particular portfolio's variance, $Variance(P)$, depends on the variances of the stocks that make up the portfolio as well as the covariances between each pair, as shown by (Eq 2).

$$Variance(P) = \sum_{i=1}^{n} w_i s_i^2 + 2 * \sum_{i,j} w_i * w_j * Covar(i, j) \qquad (Eq2)$$

In (Eq 2), the weight given to stock i is defined by w_i, the covariance between stocks is obtained by $Covar(i, j)$, and the volatility of the stock is determined by its standard deviation which is represented by s_i. Each portfolio has 28 stocks, hence 784 terms are needed to calculate each portfolio's variance. While the remaining 756 items represent the covariances between each pair, only 28 of the terms are included in the weighted sum of the variances of the individual stocks. The portfolio with the lowest risk is the one that can identify the set of $w_i's$ that results in the portfolio's volatility being at its lowest value.

In order to determine which portfolio has the lowest risk for each sector, the *efficient frontier (EF)* employing many portfolios is first presented. The returns and volatility are shown along the y-axis and the x-axis, respectively, in the two-dimensional plot of a set of portfolios in the EF. The EF frontier is composed of the portfolio in the form of points that provide the highest level of return of a given volatility or the lowest volatility for a given level of return. The point at the furthest left position on the EF represents the portfolio with the least volatility and, as a consequence, the lowest risk. The efficient frontier is drawn by 5000 iterations of a Python programme looping through a portfolio of equities, randomly allocating weights to each item. Every one of the 5,000 points that the algorithm produces corresponds to a portfolio with a certain return and risk value. The EF points are those that provide the lowest volatility for a given return value or the highest return value for a given volatility.

The portfolio with the lowest risk is represented by the leftmost location on the EF out of all the produced points.

7. Designing Optimum-Risk Portfolios The investors in the stock market rarely follow the strategy of risk minimisation as proposed by the minimum- risk portfolio due to their low returns. Most often, the investors are ready to undertake higher risks if the associated returns are even higher. For optimising the risk and return in a portfolio, and thereby designing an optimum-risk portfolio. When compared to the minimum-risk portfolio, the optimum-risk portfolio generates a very high return with a hardly noticeable increase in risk.

8. Back Testing the Strategy A minimal risk portfolio and an optimum risk portfolio are developed for the sectors using the training information from January 1, 2000, to Nov 30, 2023. On Nov 30, 2023, a fictional investor is established who invests a sum of one million Indian rupees (INR) in each sector in accordance with the advice of the best risk-weighted portfolio structure for that sector. Please take note that the figure of INR 1 million is just meant to serve as an example. The quantity or the currency will have no impact on the analysis. A model is created utilising the Differen architectures to calculate the future stock price values and, in turn, estimate the future value of the portfolio. The expected rate of return for each portfolio is calculated using the predicted stock weights. Finally, the real rate of return is calculated on Nov 30, 2023, when the stock values are known. To assess the profitability of the portfolios and the model's predictive power, the anticipated and actual rates of return for each portfolio are compared.

4.1.1 Model Architecture

Figure 4.1 illustrates our model's structure. It consists of three primary components: an input layer, a processing layer utilizing a neural network, and an output layer. This architecture leverages neural networks to extract cross-sectional features from input data, aligning with research suggesting superior performance of deep learning features compared to traditional hand-crafted ones [18]. Following feature extraction, the model generates portfolio weights used to calculate returns and maximize the Sharpe ratio.

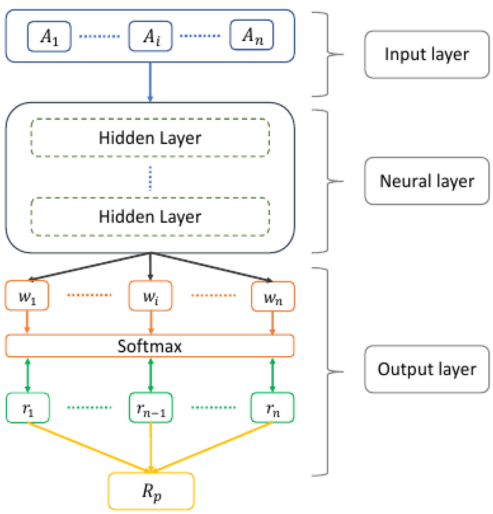

Figure 4.1: Model architecture schematic

4.1.2 Detailed Breakdown

4.1.2.1 Input Layer

Each asset (represented as A_i) is part of a portfolio containing n assets. We combine data from all assets into a single input. This input could be historical prices and returns for one asset, with a dimension of $(k, 2)$ where k represents the lookback window. Stacking features across all assets would result in a dimension of $(k, 2 \times n)$. This input is then fed into the network for extracting non-linear characteristics.

4.1.2.2 Processing Layer (Neural Network)

This layer can be built by stacking multiple hidden layers. However, designing this layer requires experimentation as various hidden layer combinations exist, and performance often hinges on the architecture's design. We explored Long Short-Term Memory (LSTM), Convolutional Neural Networks (CNN), and Fully Connected Neural Networks (FCN) [5]. Studies suggest LSTMs achieve the best overall performance for daily financial data modeling [17, 7].

- **Fully Connected Neural Networks (FCNs):** A major concern with FCNs is their tendency to overfit due to the large number of parameters assigned to each input feature.
- **Long Short-Term Memory (LSTMs):** LSTMs utilize a cell structure with gate mechanisms to summarize and filter data from extended histories. This leads to fewer trainable parameters and improved generalization capabilities.
- **Convolutional Neural Networks (CNNs):** While CNNs excel at modeling high-frequency financial data (like limit order books), their smoothing properties (often associated with large convolutional filters) can generate overly smooth solutions. Additionally, the convolution process and parameter sharing architecture of CNNs can lead to overfiltering of inputs.

4.1.2.3 Output Layer

To create a long-only portfolio, we employ the softmax activation function in the output layer. This function inherently enforces constraints, ensuring portfolio weights are positive and sum to one. Since the number of output nodes $(w_1, w_2, ..., w_n)$ corresponds to the number of assets (n) in the portfolio, these weights can be multiplied by the respective asset returns $(r_1, r_2, ..., r_n)$ to obtain the realized portfolio return (R_p). With the realized returns, we can calculate the gradients of the Sharpe ratio concerning the model's parameters and utilize gradient ascent for parameter updates.

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