CS315: Database Systems
Query Optimization

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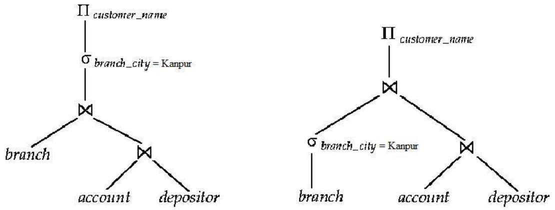
2nd semester, 2022-23

Tue 10:30-11:45, Thu 12:00-13:15

Evaluation Plan

- Equivalent expressions provide alternate ways of executing a query



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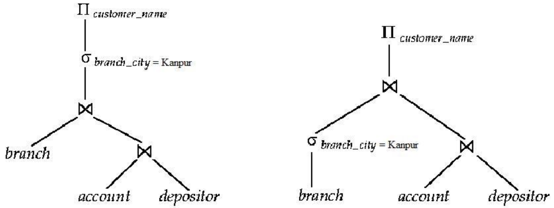
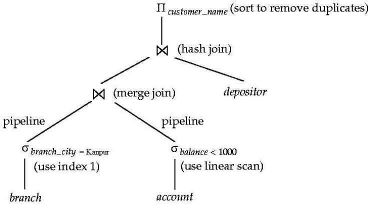


- An evaluation plan also specifies the algorithms



- Cost-based query optimization

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- Equivalent expressions are systematically generated by repeatedly applying equivalence rules and replacing one form by another
- Evaluation plans must account for all algorithms used
  - Merge join may be costlier than hash join, but since it provides a sorted output, a higher level aggregation will be faster

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12. $\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = \Pi_{L_1}(E_1) \bowtie_\theta \Pi_{L_2}(E_2)$
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18. $\Pi_L(E_1 \cup E_2) = \Pi_L(E_1) \cup \Pi_L(E_2)$

Example Schema

- branch (bname, bcity, assets)
- customer (cname, cstreet, ccity)
- account (ano, bname, bal)
- loan (lno, bname, amt)
- depositor (cname, ano)
- borrower (cname, lno)

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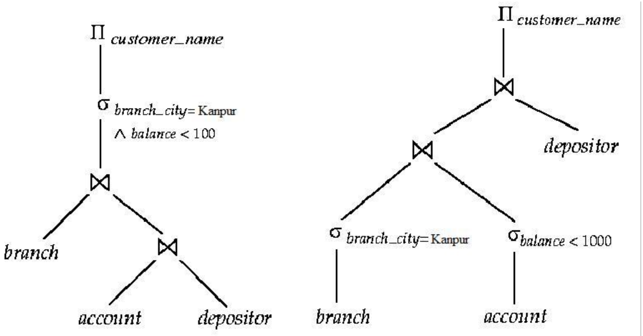
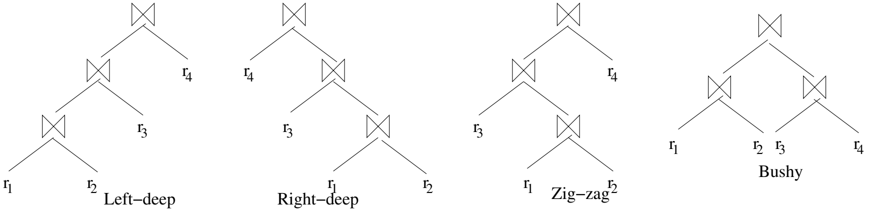


(a) Initial expression tree (b) Tree after multiple transformations

Join Trees

- Left-deep join tree
  - Right side of each join is a single relation, and not an intermediate result of join of two or more relations
- Similarly, right-deep join trees can be defined
- A tree where at least one child of an internal node is a single relation is called a zig-zag tree
- A general tree is also called a bushy tree



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  - Bushy trees: $(n-1)^{\text{th}}$ Catalan number $\frac{(2n-2)!}{n!(n-1)!}$
    - $C(1) = 1$; $C(n) = \sum_{i=1}^{n-1} C(i).C(n-i)$

Algorithm for Join Order

- Consider set S as the join of n relations
- S can be represented as $S' \bowtie (S - S')$ for any non-empty proper subset $S' \subset S$
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- Store solutions of subsets
- Interesting sort order: Useful order of records
  - Example: Merge join produces tuples in sorted order which makes later merge joins faster
  - Example: Sorted order makes later grouping and aggregation faster
- Algorithm should find the best subset for *each* interesting sort order

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- Perform joins earlier than cross products
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- Perform only left-deep (or right-deep) joins
  - Number of trees is less
  - Easier to incorporate other operations
  - Hybrid hash joins can be used

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- Perform joins earlier than cross products
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- Perform only left-deep (or right-deep) joins
  - Number of trees is less
  - Easier to incorporate other operations
  - Hybrid hash joins can be used
- Perform semantic optimizations
  - Find all employees earning more than their manager
  - May use domain knowledge to return empty result directly

Statistics

- For each relation r
  - Number of tuples n_r
  - Number of blocks b_r
  - Blocking factor, i.e., number of tuples that fit in a block $f_r = \lfloor n_r / b_r \rfloor$
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- If $R \cap S$ is a foreign key in S referencing R, then it is exactly n_s
- Every tuple of r can join with $n_s/v_s(A)$ when joining attribute is A, thereby prducing $n_r.n_s/v_s(A)$ joined tuples
- Reversing r and s, estimate becomes $n_s.n_r/v_r(A)$
- Lower size is the better estimate
- Histograms can improve the estimates

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