CS315: Database Systems
Indexing

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2nd semester, 2022-23

Tue 10:30-11:45, Thu 12:00-13:15

Basics

- Indexing is used to speed up search
- A search key is used
- An index file consists of records or index entries which has two fields
    1. Search key: Attribute that is used for searching
    2. Pointer to the entire object or tuple
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- Two basic types of indices
  1. Ordered index: search keys are organized according to some order
  2. Hash index: search keys are organized according to a hash function

Static Hashing

- A hash function maps a key to a bucket
- A bucket is a unit of storage
- It is typically a disk block
- A key may need to be searched sequentially inside a bucket
- Results in hash file organization
- Example: mod n where n is the number of buckets

Hash Function

- Two important qualities of an ideal hash function
- Uniform: Total number of keys from the domain is spread uniformly over all the buckets
- Random: Number of keys in each bucket is same irrespective of the actual distribution of keys

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- Changing size of a database is a problem
- Periodic re-hashing is the only solution
- Dynamic hashing: h changes dynamically but deterministically

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- Family of functions $g(k) = \{h_1(k), \ldots, h_i(k), \ldots\}$
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- Example: bit representation

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- Multilevel index: primary index does not fit in memory
  - Outer index: Sparse primary index
  - Inner index: Dense primary index file

B-Tree

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- A B-tree of order Θ has the following properties:
  1. Leaf nodes are in same level, i.e., the tree is balanced
  2. Root has at least 1 key
  3. Other internal nodes have between Θ and 2Θ keys
  4. An internal node with k keys have $k + 1$ children
  5. Child pointers in leaf nodes are null
- Branching factor is between $\Theta + 1$ and $2\Theta + 1$
- Pointer to the object corresponding to a key is stored alongside

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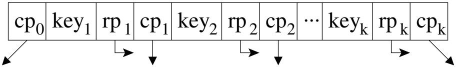
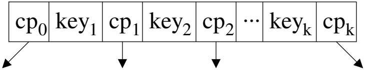


B-tree node



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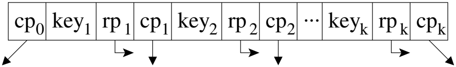

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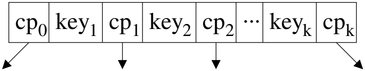

B+-tree node

- More keys can fit in a B+-tree
- Height may be less

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  - Quadtree, KD-tree, K-d-B-tree: Extension of BST
  - Data-partitioning
  - R-tree: Extension of B+-tree
  - Uses minimum bounding rectangles (MBRs)

Bitmap Index

- Attribute domain consists of a small number of distinct values
- A bitmap or a bit vector is an array of bits
- Each distinct value has an array of the size of the number of tuples
  - If the *i*-th bit is 1, tuple *i* has that value

Gender	Grade
Male	C
Female	A
Female	C
Male	D
Male	A

- Two sets of bit vectors
  - Male = (10011), Female = (01100)
  - A = (01001), B = (00000), C = (10100), D = (00010)

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- O/S allows efficient bitmap operations when they are packed in word sizes

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- drop index *i* deletes the index

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