Gems from the world of data structures and algorithms

Point line query ...

Difficulty level: *high*

A line in a plane can be seen as dividing the plane into two regions. For any non-vertical line L, we can define these two regions as *upper* half-plane and *lower* half-plane. See Figure $1(i)$ for these concepts. Now we describe the problem.

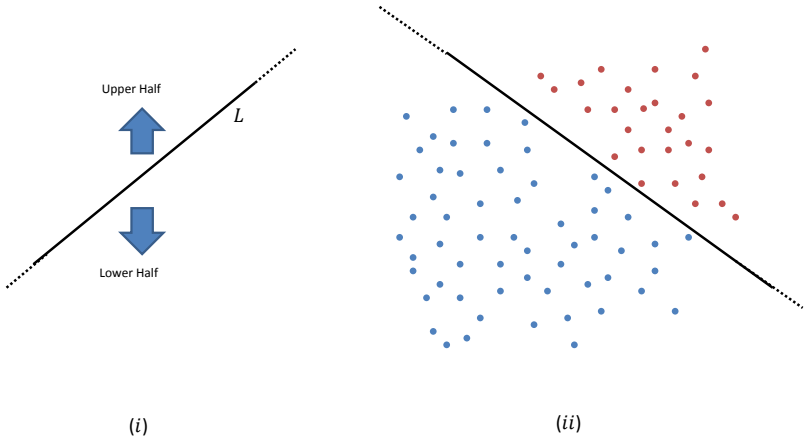

Figure 1: (i) The upper and lower half planes defined by a line. (ii) We need to report all red points to answer the query.

There is a set P of n points in a plane. You need to build an $O(n)$ size data structure such that given any non-vertical line L, all points of P that lie in the *upper* half-plane defined by L can be reported efficiently. Refer to Figure 1 (ii) for a better understanding. Your aim should be to achieve query time $O(k \log n)$, where k is the output size, i.e., the number of points lying in the corresponding upper half-plane of L. Please note that it is even possible to achieve $O(k + \log n)$ which is surely better than $O(k \log n)$. However, this involves more sophisticated tools from data structures.

Hints:

- This problem will test your patience and perseverance.
- Study Convex hull from some source.
- Get inspiration from Figure 2 shown on the following page.

Figure 2: The ideas and inspiration can be derived from any thing around us. All one needs is an open and aware mind :)