1. Identify all possible types of the following first order ODEs: (`Leisurely, not as part of tutorials, try computing the solution using the methods corresponding to each of the type.`)
   (a) $6x^2y\,dx - (x^3+1)\,dy = 0$.
   (b) $(y-1)\,dx + x(x+1)\,dy = 0$.
   (c) $e^{2x}y^2\,dx + (e^{2x}y - 2y)\,dy = 0$.
   (d) $(8x^3y - 12x^3)\,dx + (x^4+1)\,dy = 0$.
   (e) $(x^2+y^2)\,dx - 2xy\,dy = 0$.
   (f) $2(y^2+4)\,dx + (1-x^2)y\,dy = 0$.
   (g) $4xyy' = y^2 + 1$.
   (h) $y' = \frac{xy}{x^2+1}$.
   (i) $x^2y' + xy = \frac{y^3}{x}$.

2. In the lectures the integrating factor $\mu(x,y)$ for a linear ODE $y' + Q(x)y = R(x)$ is obtained under the assumption that $\mu(x,y) = \mu(x)$, i.e. μ is independent of y variable. Justify this assumption using the theorem on integrating factor for non-exact ODEs.

3. Solve the Bernoulli differential equations:
   (a) $y' - \frac{y}{x} = -\frac{y^2}{x}$ in $(0,\infty)$.
   (b) $\frac{dx}{dt} + \frac{t+1}{2t}x = \frac{t+1}{xt}$ in $(0,\infty)$.

4. Find the general solution of the following ODE:
   (a) $(5xy + 4y^2 + 1)\,dx + (x^2 + 2xy)\,dy = 0$ in $(0,\infty)$.
   (b) $(2xy^2 + y)\,dx + (2y^3 - x)\,dy = 0$.
   (c) $(5x + 2y + 1)\,dx + (2x + y + 1)\,dy = 0$.
   (d) $(3x - y + 1)\,dx - (6x - 2y - 3)\,dy = 0$.

5. Sketch isoclines with appropriate line elements and approximate solution curves for the ODE $y' = -\frac{x}{y}$.

6. Using Picard's method of successive approximations to compute the approximate solutions to the following IVP and compare with the closed-form solution:
   (a)
   $$\begin{cases} y' + xy &= x \\ y(0) &= 0. \end{cases}$$
   (b)
   $$\begin{cases} y' &= \frac{2}{3}\sqrt{y} \\ y(0) &= 0. \end{cases}$$

7. (a) Find the orthogonal trajectories of the family of ellipses centred at the origin, focus at $(a,0)$ and semi-major axis of length $2a$.
   (b) Find a family of oblique trajectories that intersect the family of curves $x + y = cx^2$ at angle θ such that $\tan\theta = 2$.