MTH102 Topic-7 Problems

Section-1

1. Is the relation $v(x, y) = x^2 + y^2 + 25$ an implicit solution of the ODE $yy' + x = 0$ in $(-5, 5)$. (Introduction and Concept of Solutions)

2. Analyse the existence and uniqueness of solution for the IVP (Picard's Existence and Uniqueness Theorem)

$$\begin{cases} (x^2 - 2x)y' = 2(x-1)y \\ y(x_0) = y_0 \end{cases}$$

Section-2

1. Find the general solution of the ODE $y' = y^2 + 1 - x^2$. (First Order ODE)

2. Solve the following ODE: - $\frac{dx}{dt} + \frac{t+1}{2t}x = \frac{t+1}{xt}$ in $(0, \infty)$ (Bernoulli Equation)

Section-3

1. Let $I \subset R$ be an open interval and $Q : I \to R$ be a continuous function. (Picard's Theorem and mix of several Concepts)
   (a) Show that $y \equiv 0$ on I is a solution of the linear homogeneous ODE y'+Q(x)y = 0.
   (b) Show that if u is a solution of the ODE such that $u(x_0) = 0$ for some $x_0 \in I$ then $u \equiv 0$ on I.
   (c) If u and v are two solutions of the ODE such that $u(x_0) = v(x_0)$ for some $x_0 \in I$ then u(x) = v(x) for all $x \in I$.
   (d) Show that the set of all solutions of y' + Q(x)y = 0 in I form a vector space over R. Can a similar conclusion be made for any k-th order linear homogeneous ODE?
   (e) What is the dimension of the vector space of solutions of y' + Q(x)y = 0?