MTH 102A ASSIGNMENT-08 (ODE) **Mar 3 & 5, 2020.**

1. (a) Show that any homogeneous ODE $M(x,y)\,dx+N(x,y)\,dy=0$ can always be equivalently written in the form $y'=g(y/x)$, for some choice of g, in an interval not containing zero.

   (b) Show that the transformation $x=r\cos\theta$ and $y=r\sin\theta$ transforms a homogeneous ODE $M(x,y)\,dx+N(x,y)\,dy=0$ to a separable ODE in r and θ variable.

2. Solve the following first order ODEs using the appropriate method based on its type:

   (a) $2r(s^2+1)\,dr+(r^4+1)\,ds=0$.

   (b) $(x+4)(y^2+1)\,dx+y(x^2+3x+2)\,dy=0$.

   (c) $\left[x\tan\left(\frac{y}{x}\right)+y\right]\,dx-x\,dy=0$.

   (d) $(\sqrt{x+y}+\sqrt{x-y})\,dx+(\sqrt{x-y}-\sqrt{x+y})\,dy=0$ in a region such that $x>y>0$.

3. Show that the homogeneous ODE

$$(Ax^2+Bxy+Cy^2)\,dx+(Dx^2+Exy+Fy^2)\,dy=0$$

   is exact iff $B=2D$ and $E=2C$.

4. Determine whether the given ODE are exact and solve them when exact.

   (a) $\left(\frac{x}{y^2}+x\right)\,dx+\left(\frac{x^2}{y^3}+y\right)\,dy=0$.

   (b) $(y\sec^2 x+\sec x\tan x)\,dx+(\tan x+2y)\,dy=0$.

5. (a) Determine the constant A such that the ODE $(Ax^2y+2y^2)\,dx+(x^3+4xy)\,dy=0$ is exact. Solve the ODE if such an A exists.

   (b) Determine the most general function M such that the ODE $M(x,y)\,dx+(2ye^x+y^2e^{3x})\,dy=0$ is exact.

6. (a) Show that the ODE $(y^2+2xy)\,dx-x^2\,dy=0$ is not exact.

   (b) Determine the $n\in\mathbb{Z}$ such that y^n is an integrating factor of the above ODE.

   (c) Solve the resulting exact ODE after multiplying by the integrating factor obtained above.

   (d) Show that $y\equiv 0$ is a solution of the non-exact ODE but is not a solution of the exact ODE obtained after multiplying integrating factor.

7. Find the general solution of the following linear ODE using integrating factors:

   (a) $(x^2+x-2)y'+3(x+1)y=x-1$ in $(-2,\infty)$.

   (b) $y\,dx+(xy^2+x-y)\,dy=0$ in the upper half-plane of $\mathbb{R}^2$, i.e. $\{(x,y)\mid y>0\}$.