## RMS Value of a Periodic Signal

- The average or mean value $\tilde{x}_{ave}$ of a periodic signal $\tilde{x}(t)$ with a fundamental period $T$ is given by

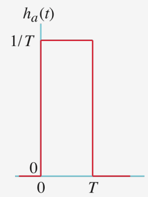
$$\tilde{x}_{ave} = \frac{1}{T}\int_0^T x(\tau)d\tau$$

- The signal $\tilde{x}(t)$ is processed by an analog system with an impulse response $h_a(t)$ as indicated in the next slide




## RMS Value of a Periodic Signal

$$h_a(t) = \begin{cases} 1/T, & \text{for } 0 < t < T, \\ 0, & \text{elsewhere} \end{cases}$$







## RMS Value of a Periodic Signal

- Its output $y(t)$ is given by

$$y(t) = \frac{1}{T}\int_0^t \tilde{x}(\tau)h_a(t-\tau)d\tau$$

- At $t = T$, the output reduces to

$$y(T) = \frac{1}{T}\int_0^T \tilde{x}(\tau)h_a(t-\tau)d\tau = \frac{1}{T}\int_0^T \tilde{x}(\tau)d\tau$$

which is the average of $\tilde{x}(t)$ over one period




## RMS Value of a Periodic Signal

- Thus, the RMS value of $\tilde{x}(t)$ can be evaluated by first passing the signal through a squarer and then processing its output by causal analog system with an impulse response

$$h_a(t) = \begin{cases} 1/T, & \text{for } 0 < t < T, \\ 0, & \text{elsewhere} \end{cases}$$




## RMS Value of a Periodic Signal

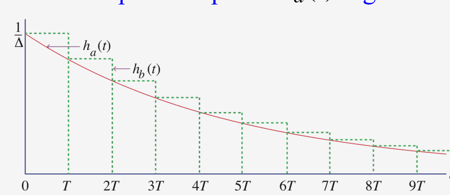
- In practice, it is not possible to design a causal analog system with an impulse response $h_a(t)$ given in the previous slide
- We can measure approximately the average value using a causal analog system with an impulse response given by

$$h_a(t) = \begin{cases} \frac{1}{\Delta}e^{-t/\Delta}, & \text{for } t > 0 \\ 0, & \text{elsewhere} \end{cases}$$




## RMS Value of a Periodic Signal

- Plot of impulse response $h_a(t)$ is given below



## RMS Value of a Periodic Signal

- We approximate $h_a(t)$ by an impulse response that is a sum of of narrow pulses of width $T$ and height $h_b(t)$ as shown by the dashed line in the previous figure
- Thus, the output $y(t)$ is now being computed using

$$y(t) \cong \frac{1}{T}\int_0^t \tilde{x}(\tau) h_b(t-\tau)d\tau$$

7



## RMS Value of a Periodic Signal

- For large values of $t$ we have

$$y(t) = \frac{1}{\Delta}\int_0^T x(\tau)d\tau + \frac{1}{\Delta}\int_T^{2T} x(\tau)e^{-T/\Delta}d\tau$$
$$+\frac{1}{\Delta}\int_{2T}^{3T} x(\tau)e^{-2T/\Delta}d\tau + \cdots$$

- From the expression for $\tilde{x}_{ave}$ given in Slide No. 1 we observe

8



## RMS Value of a Periodic Signal

$$\int_{(n-1)T}^{nT} x(\tau)d\tau = T \cdot \tilde{x}_{ave}$$

- Hence, the expression for $y(t)$ given in the previous slide reduces to

$$y(t) = \frac{T}{\Delta}\left[1 + e^{-T/\Delta} + e^{-2T/\Delta} + e^{-3T/\Delta} + \cdots\right]$$
$$= \frac{T}{\Delta(1-e^{-T/\Delta})} \cdot \tilde{x}_{ave}$$

9



## RMS Value of a Periodic Signal

- If $T/\Delta << 1$, then $e^{-T/\Delta} \cong 1 - \frac{T}{\Delta}$, and the expression for $y(t)$ given in Slide No. 9 reduces to

$$y(t) \cong \frac{T}{\Delta(1-1+\frac{T}{\Delta})} \cdot \tilde{x}_{ave} = \tilde{x}_{ave}$$

- Therefore, as $t \to \infty$, the output $y(t)$ of the analog system is precisely the average value $\tilde{x}_{ave}$

10



## RMS Value of a Periodic Signal

- The impulse response

$$h_a(t) = \begin{cases} \frac{1}{\Delta}e^{-t/\Delta}, & \text{for } t > 0 \\ 0, & \text{elsewhere} \end{cases}$$

can be realized by a causal analog lowpass filter with a transfer function

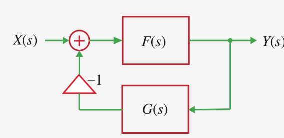
$$H_{LP}(s) = \frac{\frac{T}{\Delta}}{s + \frac{T}{\Delta}}$$

11



## Analog Feedback Systems

- The feedback connection of LTI analog systems shown below plays a major role in many control system design applications




12

## Analog Feedback Systems

- A few of these applications are described next
- The closed-loop frequency response of the feedback system is given by

$$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = \frac{F(j\Omega)}{1 + F(j\Omega)G(j\Omega)}$$

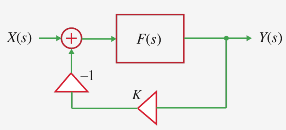



## Constant Gain Amplitude Scalar

- Most analog amplitude scalers have a frequency response $F(j\Omega)$ that is not constant but varies with frequency
- These devices can be made to have a constant gain, usually within a specified frequency range, using feedback as indicated in the next slide




## Constant Gain Amplitude Scalar




- Note: The system in the reverse path has a constant gain $K$




## Constant Gain Amplitude Scalar

- The closed-loop frequency response of this feedback system is thus

$$H(j\Omega) = \frac{F(j\Omega)}{1 + KF(j\Omega)}$$

- In the case where $|KF(j\Omega)| >> 1$, the closed-loop frequency response reduces to

$$H(j\Omega) \cong \frac{F(j\Omega)}{KF(j\Omega)} = \frac{1}{K}$$




## Constant Gain Amplitude Scalar

- In order to have a closed loop gain that is greater than 1, the amplitude scaler in the reverse path is an attenuator with a gain less than 1 which is easy to implement in practice



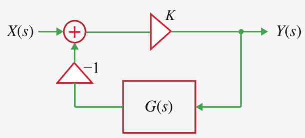

## Analog Inverse System Design

- The frequency response $G^{-1}(j\Omega)$ of a causal LTI analog system that is the inverse of a causal LTI analog system with a frequency response $G(j\Omega)$ is given by

$$G^{-1}(j\Omega) = \frac{1}{G(j\Omega)}$$

## Analog Inverse System Design

- An inverse system approximately satisfying the above relation can be designed using the feedback system shown below




19



## Analog Inverse System Design

- Note: The LTI analog system in the feed-forward path is an amplitude scaler with a constant gain $K$
- The closed-loop frequency response of this feedback system is given by

$$H(j\Omega) = \frac{K}{1 + KG(j\Omega)}$$

20



## Analog Inverse System Design

- For very large values of the constant $K$, the above equation reduces to

$$H(j\Omega) \cong \frac{K}{KG(j\Omega)} = \frac{1}{G(j\Omega)} = G^{-1}(j\Omega)$$

21



## Stabilization of Unstable System

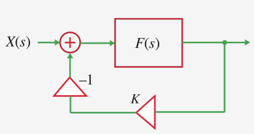
- An unstable causal LTI analog system with a frequency response $F(j\Omega)$ can be made BIBO stable by placing it in the feedforward path of a feedback system by appropriately choosing the causal LTI analog system $G(j\Omega)$ in the feedback path

22



## Stabilization of Unstable System

- For a first-order unstable analog system $F(j\Omega)$ , the compensator in the reverse path is simply an amplitude scaler with a gain $K$ as shown below




23



## Stabilization of Unstable System

- As here a portion of the output signal is being fed back to the input of the unstable system, the overall system is called a proportional feedback system
- Example: Consider a first-order LTI analog system with a real rational frequency response given by

$$F(j\Omega) = \frac{\alpha}{j\Omega + \lambda}, \ \lambda < 0$$

24

## Stabilization of Unstable System

- Note: The analog system is unstable as $\lambda < 0$
- The closed-loop frequency response of the feedback system is

$$H(j\Omega) = \frac{\dfrac{\alpha}{j\Omega+\lambda}}{1+\dfrac{K\alpha}{j\Omega+\lambda}} = \frac{\alpha}{j\Omega+(\lambda+K\alpha)}$$




## Stabilization of Unstable System

- It follows from the expression on the right-hand side of the above equation that the closed-loop system will be BIBO stable if

$$\lambda + K\alpha > 0$$

that is, if $K > -\lambda/\alpha$

- For example, if $\lambda = -3$ and $\alpha = 2$, then $K > 1.5$




## Analog Communication Systems

- Several applications of basic analog systems in the implementation of analog communication systems are described next




## Continuous-Time Amplitude Modulation

- For the transmission of a low-frequency analog signal over a channel, it is necessary to transform the analog signal to a high-frequency analog signal by means of a modulation operation
- At the receiving end, the low-frequency analog signal is extracted by demodulation

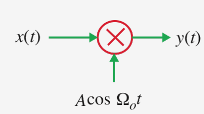



## Continuous-Time Amplitude Modulation

- There are four major types of modulation of analog signals
- We describe here the amplitude modulation scheme where the amplitude of a high-frequency carrier signal $A\cos(\Omega_o t)$ is varied by the low-frequency band-limited modulating signal $x(t)$ generating the modulated signal $y(t)$




## Continuous-Time Amplitude Modulation



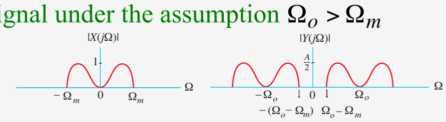

- The spectrum of the modulated signal $y(t)$ is given by

$$Y(j\Omega) = \frac{A}{2}X\big(j(\Omega-\Omega_o)\big) + \frac{A}{2}X\big(j(\Omega+\Omega_o)\big)$$

## Continuous-Time Amplitude Modulation

where $X(j\Omega)$ is the spectrum of $x(t)$

- Let $\Omega_m$ denote the highest frequency contained in $x(t)$
- Figure below shows the magnitude spectra of the modulating signal and the modulated signal under the assumption $\Omega_o > \Omega_m$







## Continuous-Time Amplitude Modulation

- The demodulation of the modulated signal $y(t)$ to recover the modulating signal $x(t)$ is implemented in two stages
- 1) The product of $y(t)$ with a sinusoidal signal of the same frequency as the carrier is formed:

$$r(t) = y(t)\cos(\Omega_o t) = Ax(t)\cos^2(\Omega_o t)$$

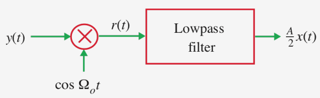
$$= \frac{A}{2}x(t) + \frac{A}{2}x(t)\cos(2\Omega_o t)$$




## Continuous-Time Amplitude Modulation

- A scaled replica of the modulating signal $x(t)$ is then recovered by passing it through a lowpass filter with a cutoff frequency $\Omega_c$ satisfying the relation as shown below

$$\Omega_m < \Omega_c < 2\Omega_o - \Omega_m$$







## Multiplexing and Demultiplexing

- For an efficient utilization of a wideband transmission channel, many narrow-bandwidth low-frequency analog signals are combined to form a composite wideband analog signal that is transmitted as a single signal




## Multiplexing and Demultiplexing

- The process of combining these signals is called multiplexing which is implemented to ensure that a replica of the original narrow-bandwidth low-frequency signals can be recovered at the receiving end
- The recovery process is called demultiplexing




## Multiplexing and Demultiplexing

- One widely used method of combining different voice signals in a telephone communication system is the frequency-division multiplexing (FDM) scheme
- Here, each voice signal, typically band-limited to a low-frequency band of width $2\Omega_m$, is frequency-translated into a higher frequency band using the amplitude modulation method

## Multiplexing and Demultiplexing

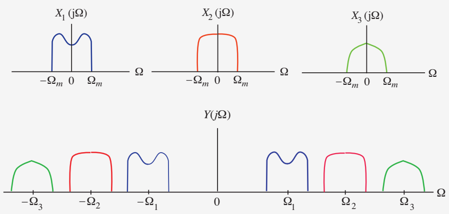
- The carrier frequency of adjacent amplitude-modulated signals is separated by $\Omega_o$ with $\Omega_o > 2\Omega_m$ to ensure that there is no overlap in the spectra of the individual modulated signals after they are added to form a baseband composite signal
- This signal is then modulated onto the main carrier developing the FDM signal and transmitted

37



## Multiplexing and Demultiplexing

- Figure below illustrates the frequency-division multiplexing scheme




38



## Multiplexing and Demultiplexing

- At the receiving end, the composite baseband analog signal is first extracted from the FDM signal by demodulation
- Next each individual frequency-translated signal is demultiplexed by passing the composite signal through a bandpass analog filter

39



## Multiplexing and Demultiplexing

- The center frequency of the bandpass filter is identical value to that of the corresponding carrier frequency and a bandwidth slightly greater than $2\Omega_m$
- The output of the bandpass analog filter is then demodulated using the method shown in Slide No. 32

40



## Quadrature Amplitude Modulation

- The DSB amplitude modulation is half as efficient as SSB amplitude modulation with regard to the utilization of the spectrum
- The quadrature amplitude modulation (QAM) method uses DSB modulation to modulate two different analog signals so that they both occupy the same bandwidth

41

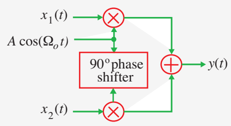


## Quadrature Amplitude Modulation

- Thus, QAM takes up only as much bandwidth as the SSB modulation method
- Let $x_1(t)$ and $x_2(t)$ be two band-limited low-frequency analog signals with a bandwidth of $\Omega_m$
- The QAM modulator modulates the two analog signals and combines them as indicated in the next slide

42

## Quadrature Amplitude Modulation




- The output $y(t)$ of the modulator is given by

$$y(t) = Ax_1(t)\cos(\Omega_o t) + Ax_2(t)\sin(\Omega_o t)$$




## Quadrature Amplitude Modulation

- The CTFT $Y(j\Omega)$ of the composite analog signal $y(t)$ is given by

$$Y(j\Omega) = \frac{A}{2}\left[X_1\left(j(\Omega-\Omega_o)\right) + X_1\left(j(\Omega+\Omega_o)\right)\right]$$
$$+\frac{A}{2}\left[X_2\left(j(\Omega-\Omega_o)\right) - X_2\left(j(\Omega+\Omega_o)\right)\right]$$

and occupies the same bandwidth as the modulated signal obtained by a DSB modulation




## Quadrature Amplitude Modulation

- To recover the original modulating analog signals, the composite analog signal is multiplied by both the in-phase and the quadrature components of the carrier separately, resulting in two signals:

$$r_1(t) = y(t)\cos(\Omega_o t)$$
$$r_2(t) = y(t)\sin(\Omega_o t)$$




## Quadrature Amplitude Modulation

- Substituting the expression for $y(t)$ we get

$$r_1(t) = \frac{A}{2}x_1(t) + \frac{A}{2}x_1(t)\cos(2\Omega_o t)$$
$$+\frac{A}{2}x_1(t)\sin(2\Omega_o t)$$
$$r_2(t) = \frac{A}{2}x_2(t) + \frac{A}{2}x_2(t)\cos(2\Omega_o t)$$
$$-\frac{A}{2}x_2(t)\sin(2\Omega_o t)$$




## Quadrature Amplitude Modulation

- Lowpass filtering of $r_1(t)$ and $r_2(t)$ by filters with a cutoff at $\Omega_m$ yields the two modulating signals as indicated below: