Series

- A series is a sum of well-defined terms of a sequence of real or complex numbers and variables

Classification of Series

- A finite series contains a finite number of terms
- An infinite series contains an infinite number of terms

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Series

- For brevity, a series is written using the Greek symbol $\sum$
- Example: $\sum_{n=0}^{\infty} b_n$ is an infinite series
- Example: $\sum_{n=0}^{N} b_n$ is a finite series
- The sum of all terms in a finite series is finite if each term in the series has finite value

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Series

- The sum of all terms in an infinite series may or may not be finite
- If the sum of all terms is finite, then the infinite series is said to be a convergent series, otherwise it is a divergent series

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Arithmetic Series

Types of Series

Arithmetic Series

- The terms $\{a_n\}$ of arithmetic series are from an arithmetic sequence
- The difference between two consecutive terms of an arithmetic series is constant

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Arithmetic Series

- An infinite arithmetic series is a divergent series
- A finite arithmetic series is a convergent series

Example –

- Determine the sum $S = \sum_{k=1}^{N} a_k$ where $a_k = a + (k-1)\delta$

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Arithmetic Seriess

- The first term of the series is $a_1 = a$
- The last term of the series is $a_N = a + (N-1)\delta$
- The sum S can be written as

$$S = \sum_{k=1}^{N}[a_1 + (k-1)\delta] = Na_1 + \sum_{k=1}^{N}(k-1)\delta$$

and also as

$$S = \sum_{k=1}^{N}[a_N - (k-1)\delta] = Na_N - \sum_{k=1}^{N}(k-1)\delta$$

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Arithmetic Series

- Adding these two expressions for S we get
  $$2S = N(a_1 + a_N) = Na + N[a + (N-1)\delta]$$
  which leads to $S = Na + \dfrac{N(N-1)\delta}{2}$
- It follows from the above
  $$\sum_{k=1}^{N} k = N + \frac{N(N-1)}{2} = \frac{N(N+1)}{2}$$

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Geometric Series

Geometric Series

- One commonly encountered convergent series is the infinite length geometric series given by
  $$\boxed{\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}, \ |\alpha| < 1,}$$
  where α is a complex number

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Geometric Series

- Another series that is sometimes encountered is the finite-length geometric series
  $$\boxed{\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1 \\ N, & \alpha = 1 \end{cases}}$$

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Geometric Series

- In one application we have $e^{j2\pi(k-\ell)/N}$, which when substituted in the previous expression leads to, for r an integer,
  $$\sum_{n=0}^{N-1} e^{j2\pi(k-\ell)n/N} = \frac{1-e^{j2\pi(k-\ell)n}}{1-e^{j2\pi(k-\ell)n/N}}$$
  $$= \begin{cases} 0, & \text{for } k \neq \ell \\ N, & \text{for } k = \ell + rN \end{cases}$$

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Modulo Operation

Definition

- Let the range of integer values of the argument be $0,1,2,\ldots,N-1$
- Let K be an arbitrary integer defining the argument
- If K is outside the above range of integer values, then it is replaced with
  $$R = K + \gamma N$$

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Modulo Operation

where γ is an integer chosen so that takes a value in the range $0,1,2,\ldots,N-1$

- The integer R obtained using the modulo operation is called the residue
- The modulo operation is denoted by the notation
  $$\boxed{\langle K \rangle_N = K \text{ modulo } N}$$
  where $\langle K \rangle_N$ is the residue

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Modulo Operation

Example: Let $N = 9$ and $K = 37$

We have then $R = 37 + \gamma 9$

- If we choose $\gamma = -4$, then we get

$$R = 37 - 4 \times 9 = 1$$

which is an integer in the range of integer values $0,1,2,\ldots,8$

- Hence, $\langle 37 \rangle_9 = 1$

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Modulo Computation Using MATLAB

- The function `mod` can be used to implement the modulo operation

Example: The MATLAB statement

```
r = mod(37,9)
```

yields

```
r
= 1
```

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Greatest Common Divisor

Definition

- The divisor or factor D of an integer N is an integer dividing N with a zero remainder
- The greatest common divisor (GCD) between two or more non-zero positive integers is the largest common integer factor of these integers

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Greatest Common Divisor

- GCD of two non-zero positive integers N_1 and N_2 is usually denoted as $\text{GCD}(N_1, N_2)$

Example: Determine GCD(36,42)

- The factors of 36 and 42 are determined by expressing each as a product of two integers:

$$36 = 36 \times 1 = 18 \times 2 = 12 \times 3 = 9 \times 4 = 6 \times 6$$
$$42 = 42 \times 1 = 21 \times 2 = 14 \times 3 = 7 \times 6$$

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Greatest Common Divisor

- Thus, the factors of 36 are

1, 2, 3, 4, 6, 9, 12, 18

and the factors of 42 are

1, 2, 3, 6, 7, 14, 21

- The common factors between 36 and 42 are

1, 2, 3, 6

- The largest common factor is 6
- Hence, GCD(36,42) = 6

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Greatest Common Divisor

- The factors of 42 are determined by expressing it as a product of two integers:

$$42 = 42 \times 1 = 21 \times 2 = 14 \times 3 = 7 \times 6$$

- Thus, the factors of 42 are

1, 2, 3, 6, 7, 14, 21

- The common factors between 36 and 42 are

1, 2, 3, 6

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GCD Computation Using MATLAB

- The MATLAB function to compute the GCD is `gcd`
- The MATLAB statement

```
D = gcd(36,42)
```

yields

```
D =
   6
```


Least Common Multiple

Definition

- The multiples of a non-negative integer, called natural number, are given by the product of this number by each member of the set of natural numbers

Example: The multiples of 4 are given by

$$4\times 1,\ 4\times 2,\ 4\times 3,\ 4\times 4$$

that is, 4, 8, 12, 16, …

Least Common Multiple

- The least common multiple (LCM) of two or more natural numbers is the smallest multiple that is common to these numbers

Example: Determine LCM(12,18)

- The multiples of the number 12 are

  12, 24, 36, 48, 60, 72, …

  and the multiples of the number 18 are

  18, 36, 54, 72, 90, 108, …

Least Common Multiple

- Two common multiples of 12 and 18 are thus 36 and 72
- Smallest common multiple is 36
- Hence, LCM(12,18) = 36

LCM Computation Using MATLAB

- MATLAB function to compute the LCM is `lcm`

LCM Computation Using MATLAB

Example: The MATLAB statement

```
M = lcm(12,18);
```

yields

```
M =
36
```


Relation Between GCD and LCM

- The LCM and the GCD of two natural numbers N_1 and N_1 are related through:

$$\mathrm{LCM}(N_1,N_2)=\frac{N_1N_2}{GCD(N_1,N_2)}$$

Decibel

- In some applications, the ratio of two physical quantities is described in terms of a logarithmic unit called decibel
- It is logarithm to base 10 of the ratio multiplied by 10
- If $\mathcal{P}_1$ and $\mathcal{P}_2$ denote the two power values, their ratio $\mathcal{R}_{dB}$ in decibel is given by

$$\mathcal{R}_{dB} = 10\log_{10}\left(\frac{\mathcal{P}_1}{\mathcal{P}_2}\right)\ dB$$

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Decibel

- Ratio of two signal amplitudes A_1 and A_2 in dB is computed using

$$\mathcal{R}_{dB} = 20\log_{10}\left(\frac{A_1}{A_2}\right)\ dB$$

- If the power ratio in dB is a positive number, then it is called as the gain
- If the power ratio in dB is a negative number, then it is called as the loss

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Decibel

- Ratio of the signal power to that of the background noise is called the signal-to-noise ratio, usually abbreviated as SNR or S/N
- The signal-to-noise ratio is usually specified in dB
- Logarithm to the base 10 of a positive number can be computed in MATLAB using the function `log10`

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Decibel

Example: Let the input power to an analog amplifier be 12 watts and the output power be 360 watts

- The gain $\mathcal{G}_{dB}$ in dB of the amplifier is then

$$\mathcal{G}_{dB} = 10\log_{10}\left(\frac{360}{12}\right) = 10\log_{10}(30)$$
$$= 14.7712\ \ dB$$

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