

1. Find the general solution of the following ODE using the given solution:
 - (a) e^{2x} is a solution of $(2x + 1)y'' - 4(x + 1)y' + 4y = 0$.
 - (b) $\frac{1}{x}$ is a solution of $x(1 - x)y'' + 2(1 - 2x)y' - 2y = 0$.
 - (c) x is a solution of $(1 - x^2)y'' - 2xy' + 2y = 0$.
2. Show that $\frac{\sin x}{\sqrt{x}}$ is a solution of $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$ in any $I \subset (0, \infty)$. Consequently, find its general solution.
3. Find the general solution of the following linear, homogeneous, constant coefficient ODE:
 - (a) $y'' - 4y' + 3y = 0$.
 - (b) $y'' + 2y'' + (\omega^2 + 1)y = 0$ where ω is a real number.
 - (c) $y^{(5)} - 2y^{(4)} + y^{(3)} = 0$.
 - (d) $y^{(6)} - 2y^{(3)} + y = 0$.
 - (e) $y^{(4)} + 2y^{(3)} + 6y'' + 2y' + 5y = 0$ given that $\sin x$ is a solution of the ODE.
4. Solve the following IVP:
 - (a) $y'' + 4y' + 4y = 0$ with $y(0) = 1$ and $y'(0) = -1$.
 - (b) $y'' - 2y' - 3y = 0$ with $y(0) = 1$ and $y'(0) = 3$.
5. Find the general solution of the following linear, inhomogeneous, constant coefficient ODE (using method of undetermined coefficients)
 - (a) $y^{(4)} + 2y^{(3)} - 3y'' = 18x^2 + 16xe^x + 4e^{3x} - 9$.
 - (b) $y'' + y = \sin x + (1 + x^2)e^x$.
 - (c) $y'' - y = e^{-x}(\sin x + \cos x)$.
 - (d) $y^{(3)} - 3y'' - y' + 3y = x^2e^x$.
6. Find the general solution of the following linear, inhomogeneous constant coefficient ODE (using method of variation of parameters):
 - (a) $y'' - 2y' + y = xe^x \ln x$ in $(0, \infty)$.
 - (b) $y'' + y = \cot^2 x$.
7. Find the general solution of the following ODE using both the method of undetermined coefficients and the variation of parameters:
 - (a) $y'' + 4y = 2\cos^2 x + 10e^x$.
 - (b) $y'' + y = x \sin x$.
8. Find the general solution of the following linear, inhomogeneous, variable coefficient ODE:
 - (a) $x^2y'' - x(x + 2)y' + (x + 2)y = x^3$ for $x > 0$ given that $y = x$ is a solution of the homogeneous part.

- (b) $x^2y'' + xy' + 4y = 2x \ln x$ in $(0, \infty)$.
- (c) $(x + 2)^2y'' - (x + 2)y' - 3y = 0$ in $(-2, \infty)$.
- (d) $x^2y'' + 2xy' - 12y = 0$.
- (e) $x^2y'' + 5xy' + 13y = 0$.
- (f) $x^2y'' - xy' + y = 0$.