ESC201T : Introduction to Electronics

HW3: Solution

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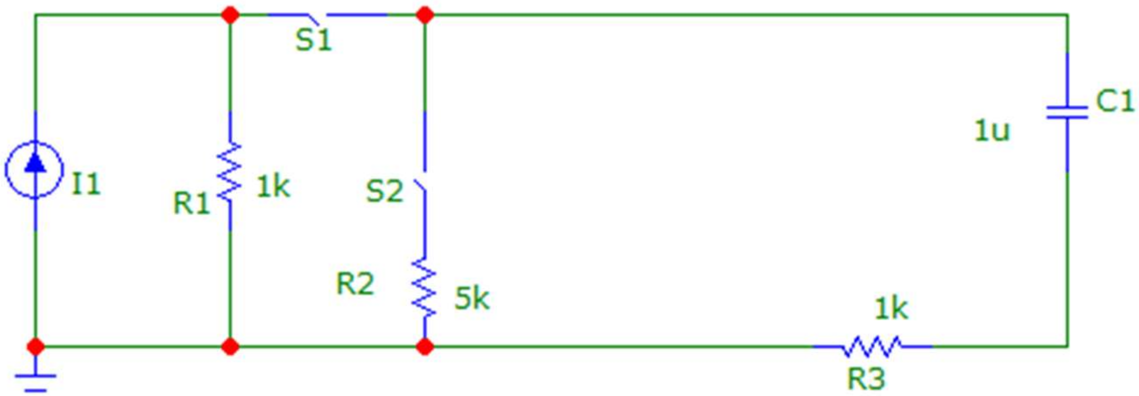
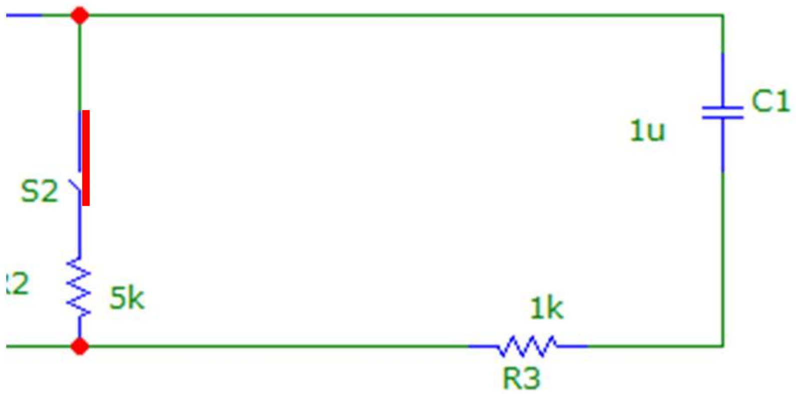
Q.1 Find the voltage across the capacitor as a function of time if the switch S2 is opened and S1 is closed at t = 0 . Assume that the current source has a value of 4mA and that before t = 0, both switches had been in their respective states for a long time so that steady state conditions can be assumed to prevail in the circuit prior to t = 0.



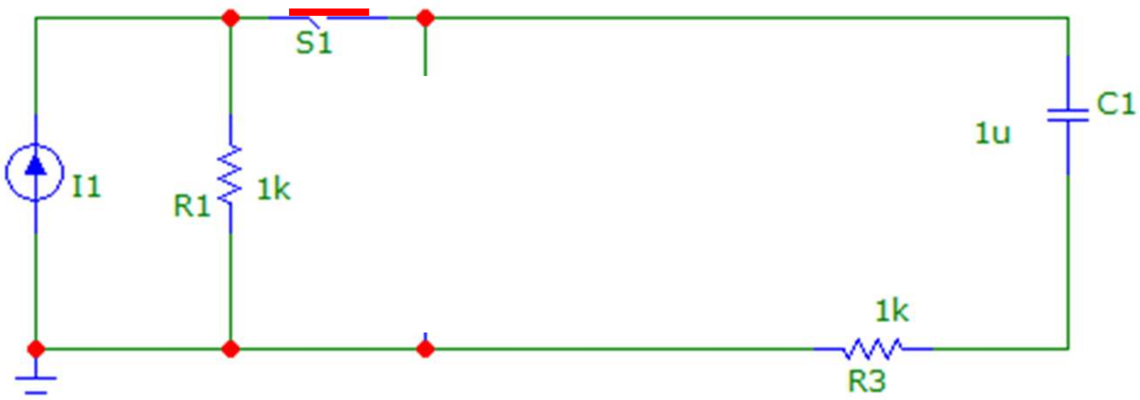
Before t= 0

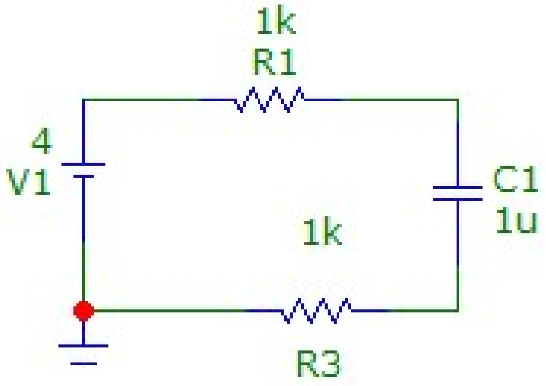
$$V_{C1}(t>0) = V_C(\infty) + \left(V_C(0^+) - V_C(\infty)\right) \times \exp(-\frac{t}{\tau})$$



$$\Rightarrow V_{C1} = 0 \Rightarrow V_{C1}(0^+) = 0$$

Circuit for t > 0



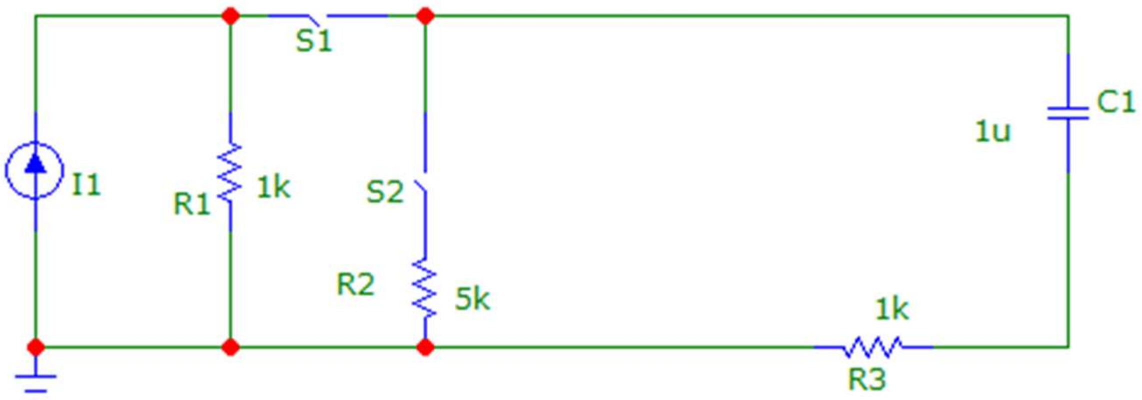


$$\tau = (1k + 6k) \times C_1 = 2ms$$

$$V_C(\infty) = 4\ V$$

$$V_{C1}(t > 0) = 4 \times (1 - \exp\left(-\frac{t}{\tau}\right))$$

Q.2 Suppose at t = 10ms , the switches go back to their original position. Determine the voltage across the capacitor at t = 20ms



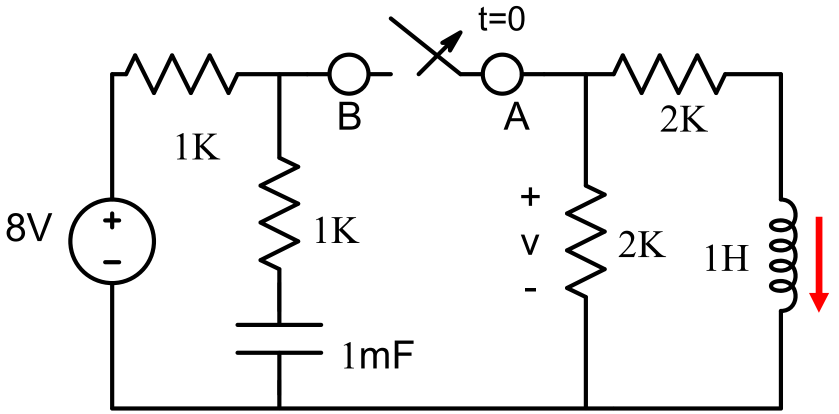
T = 10ms is 5 time constants so V_{C1} = 4V prior to switch changing positions

$$V_{C1}(t > 10ms) = V_C(\infty) + \left(V_C(10^+) - V_C(\infty)\right) \times \exp(-\frac{t - 10ms}{\tau_1})$$

$$\tau = (1k + 5k) \times C_1 = 6ms \qquad V_C(\infty) = 0\ V$$

$$V_{C1}(t = 10ms) = 0 + 4 \times \exp\left(-\frac{20 - 10}{6}\right) = 0.756V$$

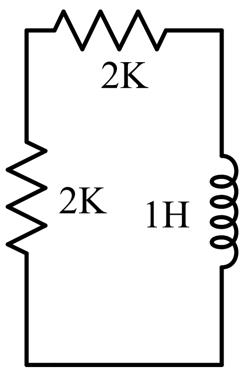
Q.3 For the circuit shown below, determine the voltage v across the 2K resistor as a function of time after the switch is opened at t= 0.



First find the inductor current

$$i_L(t) = i_L(\infty) + \{i_L(0^+) - i_L(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit after opening the switch (t > 0)



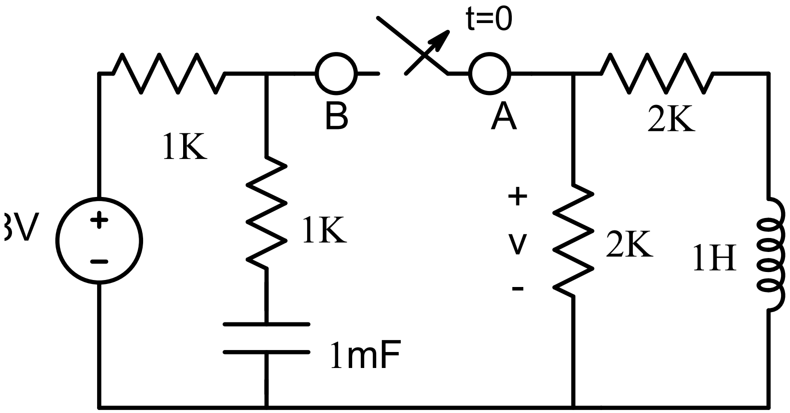
$$R_{eq} = 2K + 2K = 4K$$

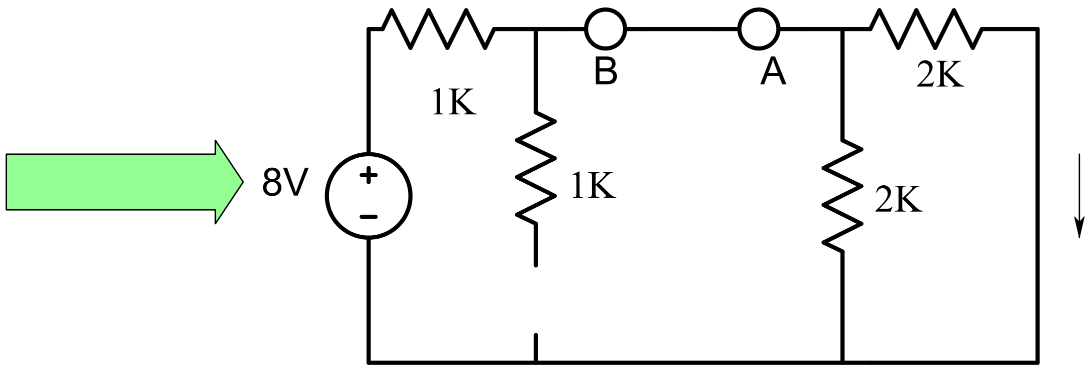
$$\tau = \frac{L}{R_{eq}} = 0.25ms$$

One can also see that : $i_L(\infty) = 0$

$$i_L(0^+) = i_L(0^-)$$

Circuit before opening the switch (t < 0) and assuming steady state condition:

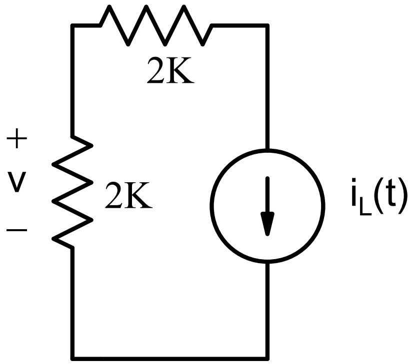




$$i_L(0^+) = i_L(0^-) = \frac{8}{(2K \| 2K) + 1K} \times 0.5 = 2mA$$

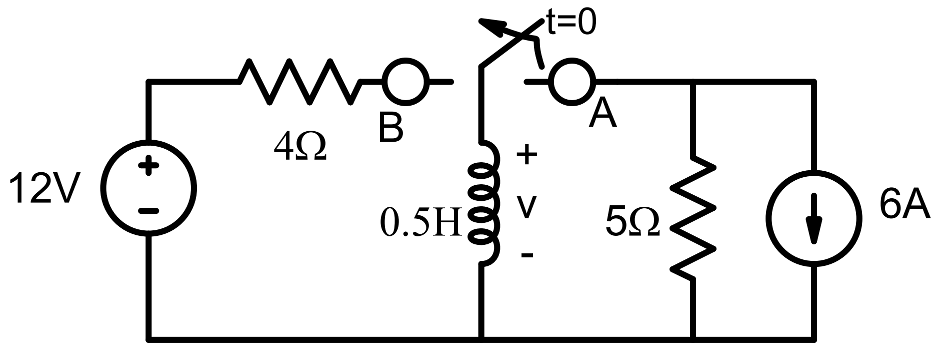
$$\Rightarrow i_L(t) = 2 \times e^{-4000t} mA$$

Voltage across the 2K resistor:



$$v(t) = -2 \times 10^3 \times i_L(t) = -4 \times e^{-4000t} V$$

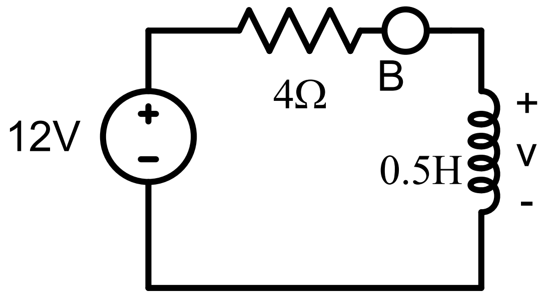
Q.4 Determine the current and voltage across the inductor as a function of time after the switch is connected to node B.



$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit after switch is connected to node B (t > 0)

$$\tau = \frac{L}{R_{eq}} = \frac{0.5}{4} = 0.125$$



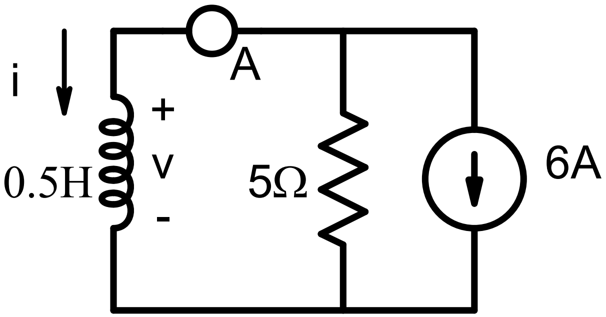
$$i(\infty) = \frac{12}{4} = 3\,A$$

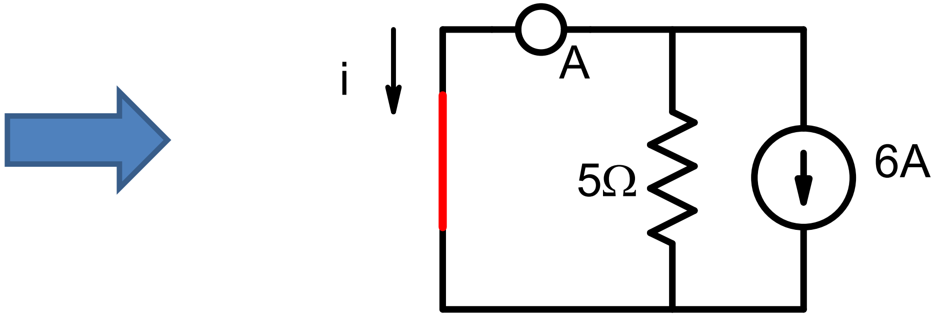
Next find current $i(0^+)$

Inductor Current cannot change instantly

$$i(0^-) = i(0^+)$$

Circuit before switch is connected to node B (t < 0)



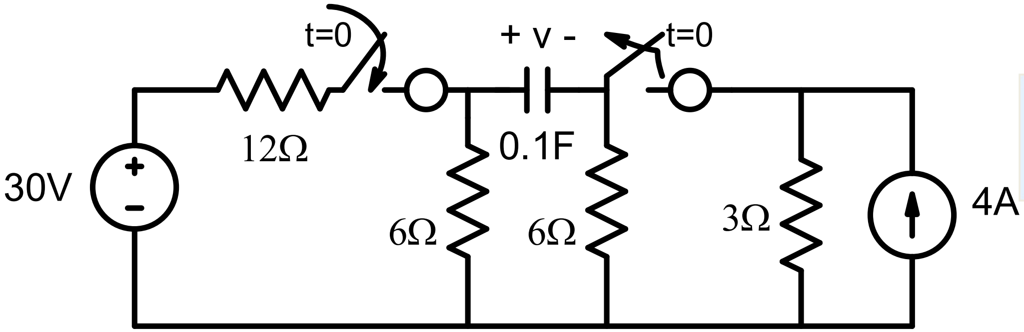


$$i(0^-) = i(0^+) = -6\,A$$

$$i(t) = 3 - 9 \times e^{-8t}$$

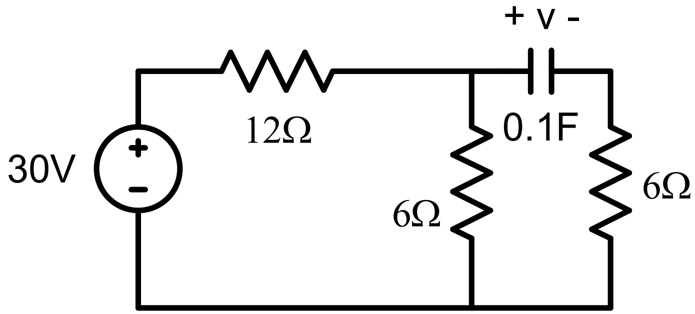
$$v = L\frac{di}{dt} = 36 \times e^{-8t}$$

Q.5 Determine the capacitor voltage as a function of time

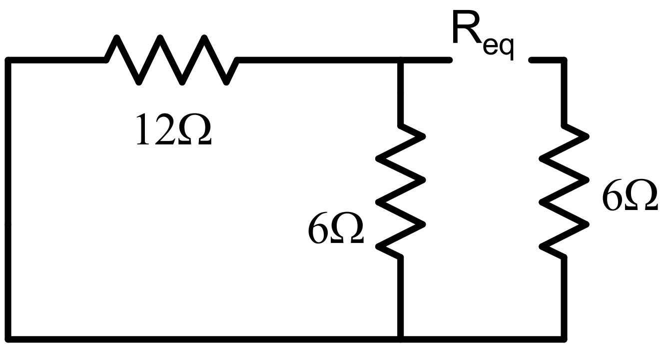


$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\}e^{-\frac{t}{\tau}}$$

Circuit after the switches are thrown (t > 0)



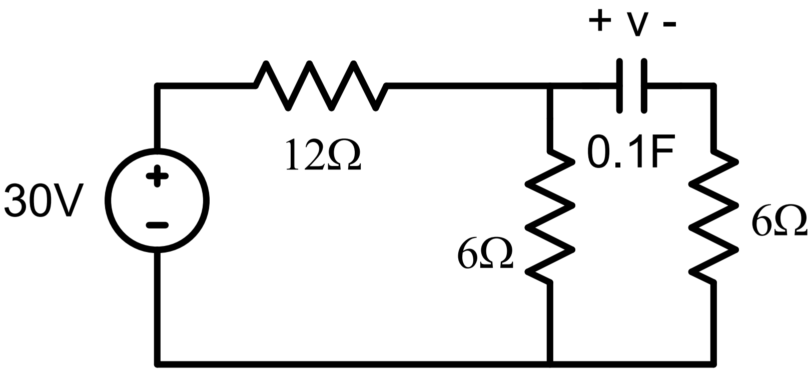
Circuit for finding equivalent resistance seen by the capacitance

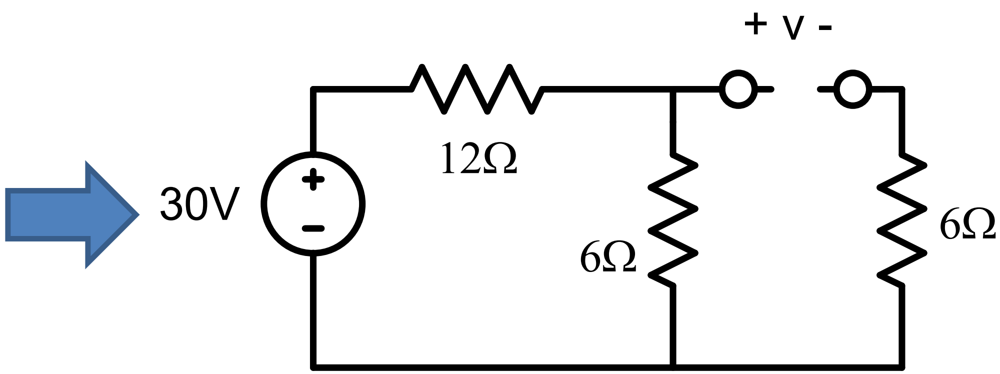


$$R_{eq} = (12 \| 6) + 6 = 10\Omega$$

$$\tau = C \times R_{eq} = 1.0s$$

We next find voltage long after closing the switch $v(t \to \infty) = 0$

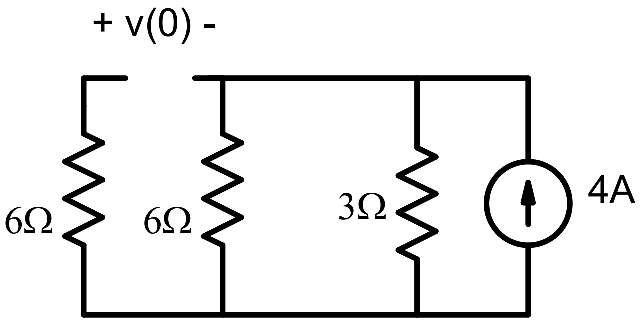




$$v(\infty) = \frac{6}{18} \times 30 = 10$$

To find v(0⁺) we note that capacitor voltage does not change instantly so it will remain the same as before the switches are thrown

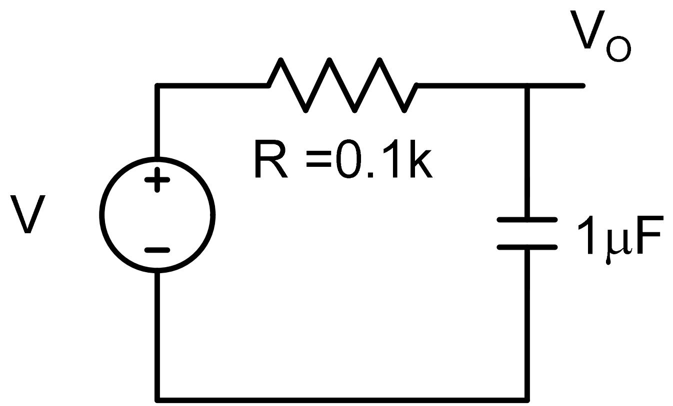
Circuit for t < 0 and assuming steady state condition:

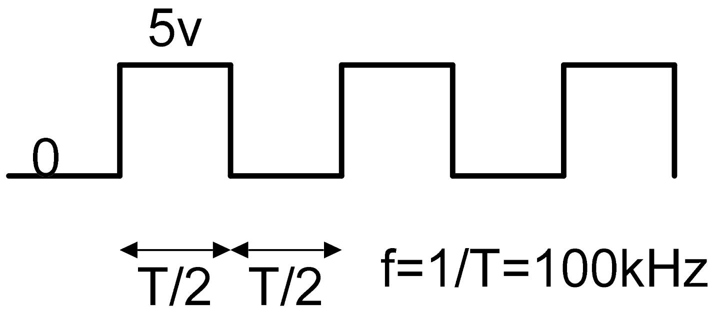


$$v(0^+) = -4 \times (6 \| 3) = -8V$$

$$v(t) = 10 - 18e^{-t}$$

Q.6 Sketch the output voltage for the circuit shown below if input is of the form shown on the right





$$RC = 10^2 \times 10^{-6} = 100\mu s \qquad \frac{T}{2} = 5\mu s$$

During the time interval : $0 \leq t \leq 0.5T$ The capacitor charges

$$v_o(T/2) = 5 \times (1 - e^{-\frac{T}{2RC}}) \cong 5 \times \frac{T}{2RC}$$

During the time interval: $0.5T \leq t \leq T$ The capacitor discharges

$$v_o(T) = 5 \times \frac{T}{2RC} e^{-\frac{T}{2RC}}$$

During the time interval : $T \le t \le 1.5T$ The capacitor charges again

$$v_o(t) = 5 + (5 \times \frac{T}{2RC} \times e^{-\frac{T}{2RC}} - 5) \times e^{-\frac{t-T}{RC}}$$

$$\Rightarrow\ v_o(1.5T) = 5 \times \frac{T}{2RC} + 5 \times \frac{T}{2RC} \times e^{-\frac{2T}{2RC}}$$

Let x = T/2RC. After several cycles

$$v_o = 5x + 5x \times e^{-2x} + 5x \times e^{-4x} + \ldots\ldots = \frac{5x}{1 - e^{-2x}} \cong 2.5V$$

Alternative derivation

The charging of a capacitor is described by the expression

$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\} e^{-\frac{t}{\tau}}$$

If the time interval for charging is much smaller compared to time constant then the amount by which capacitor voltage charges is

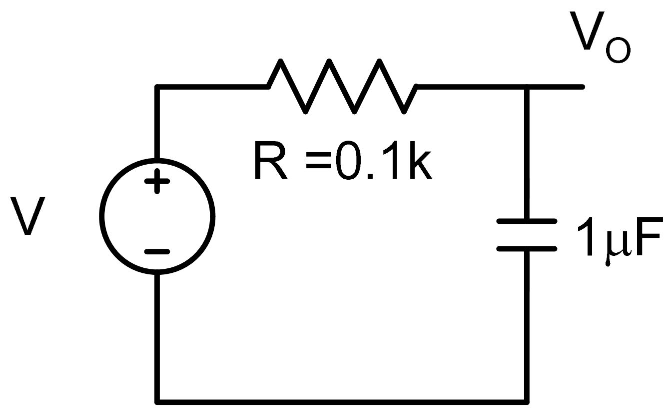
$$\Delta v_C \cong \{V(\infty) - V(0)\} \times \frac{\Delta t}{\tau}$$

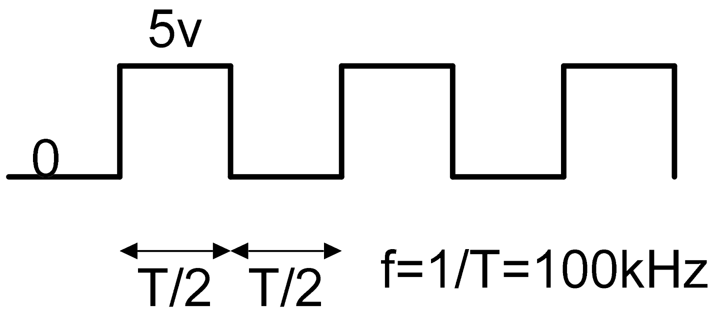
Note that, initially capacitor charges faster as V(0) is small but the rate decreases with time.

The discharging of a capacitor is described by the expression $v(t) = v(0) \times e^{-\frac{t}{\tau}}$

If the time interval for charging is much smaller compared to time constant then the amount by which capacitor voltage charges is

$$\Delta v_d \cong V(0) \times \frac{\Delta t}{\tau}$$





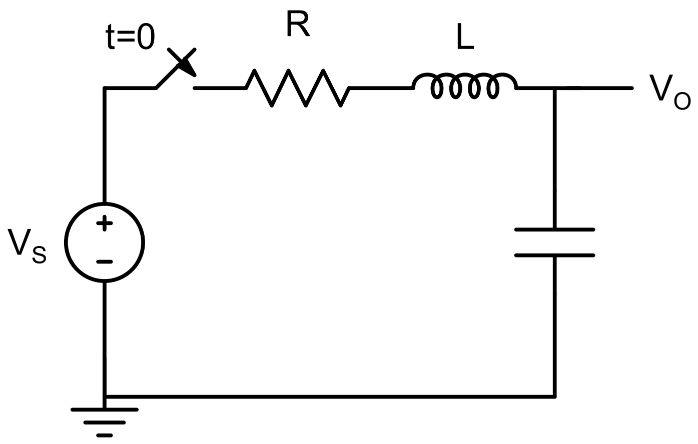
For the waveform shown above, initially the capacitor charges more than it discharges. As a result capacitor voltage rises. The voltage stabilizes when charge rate balances the discharge rate

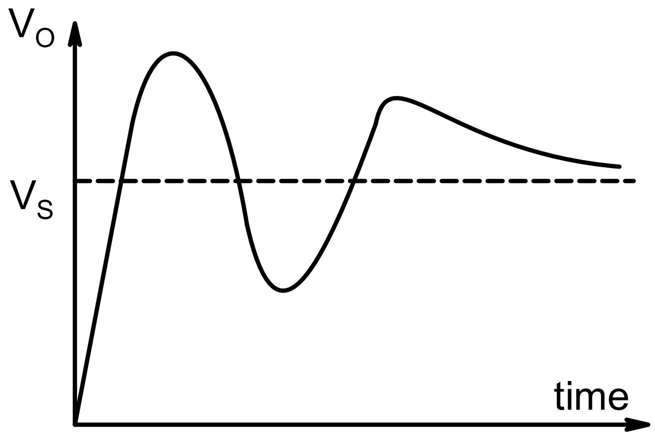
$$\Delta v_C = \Delta v_d$$

$$\{V(\infty) - V(0)\} \times \frac{\Delta t}{\tau} = V(0) \times \frac{\Delta t}{\tau}$$

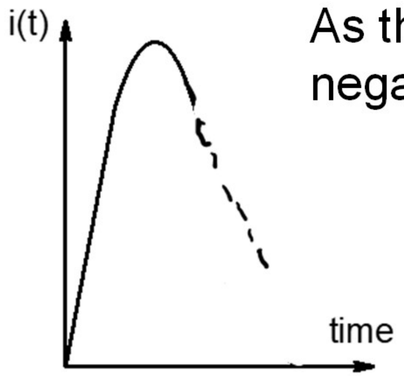
$$V(0) = 0.5V(\infty)$$

Q.7 In the circuit shown below, the initial values of inductor current and capacitor voltage are zero. At time t = 0, the switch was closed and the voltage measured across the capacitor is shown on the right. Using minimal number of equations, explain qualitatively how capacitor voltage can overshoot the supply voltage.





As the switch is closed, the current starts increasing from an initial value of zero (because inductor current cannot change instantly). This current charges the capacitor. Eventually the capacitor is fully charged to V_S and current falls to zero. The current in the circuit is thus expected to have the form shown below:



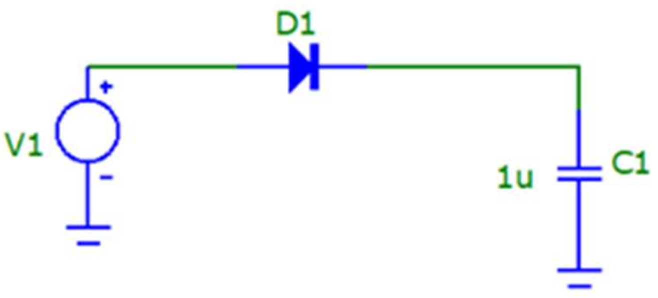
As the current decreases , the voltage across the inductor becomes negative .

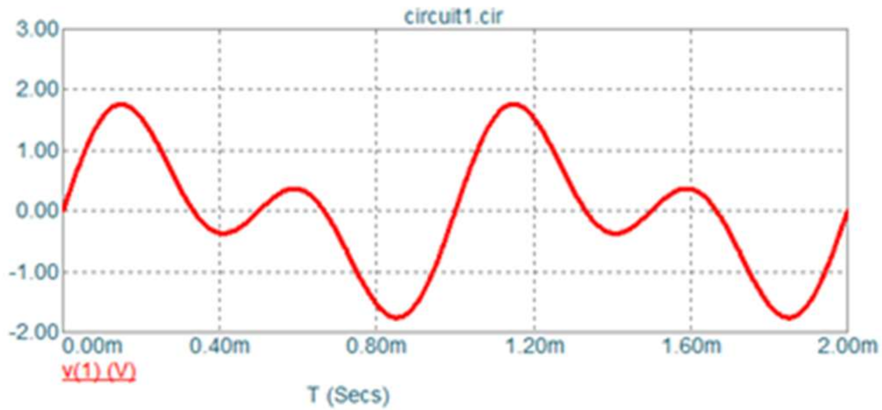
$$v_L = L\frac{di}{dt}$$

$$v_O = V_S - v_R - v_L$$

One can see that because of negative v_L, under appropriate conditions, v_O can become larger than V_S momentarily.

Q.8 In the circuit shown below, the diode is ideal in the sense that it does not conduct at all in reverse direction and in forward direction, voltage drop across it is zero. Sketch the output voltage across the capacitor for the input voltage V1 shown on the right.





If at anytime the output voltage falls below input voltage, the diode will turn on and capacitor will quickly charge but if input falls below output then diode will be OFF and capacitor will retain its charge and voltage. Thus the capacitor voltage will be equal to the peak input voltage as shown below