ESC201T : Introduction to Electronics

HW6: Solution

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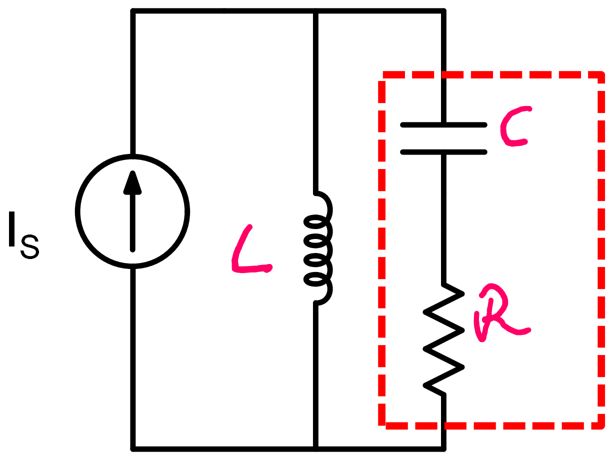
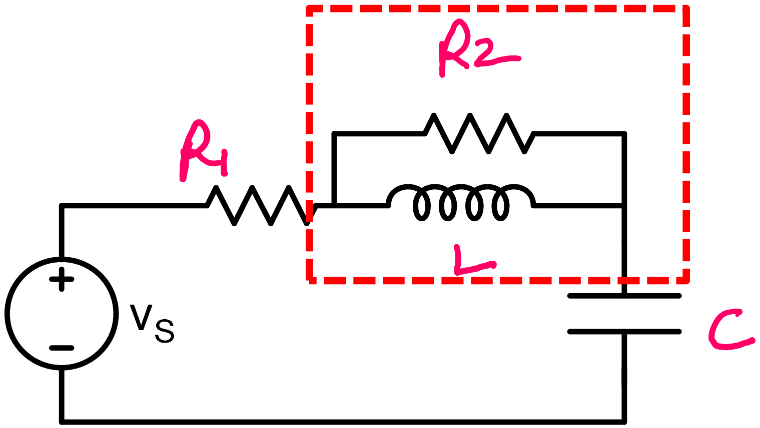
Q.1 Determine unity power factor frequencies for the circuit shown below





$$Z = R + \frac{1}{j\omega C} = \frac{1 + j\omega CR}{j\omega C}$$

$$Y = \frac{j\omega C}{1 + j\omega CR} = \frac{\omega^2 C^2 R}{1 + \omega^2 C^2 R^2} + \frac{j\omega C}{1 + \omega^2 C^2 R^2}$$

$$C_P = \frac{C}{1 + \omega^2 C^2 R^2}$$

$$\omega_O L = \frac{1}{\omega_O C_P} \Rightarrow zero\ power\ factor\ frequency$$

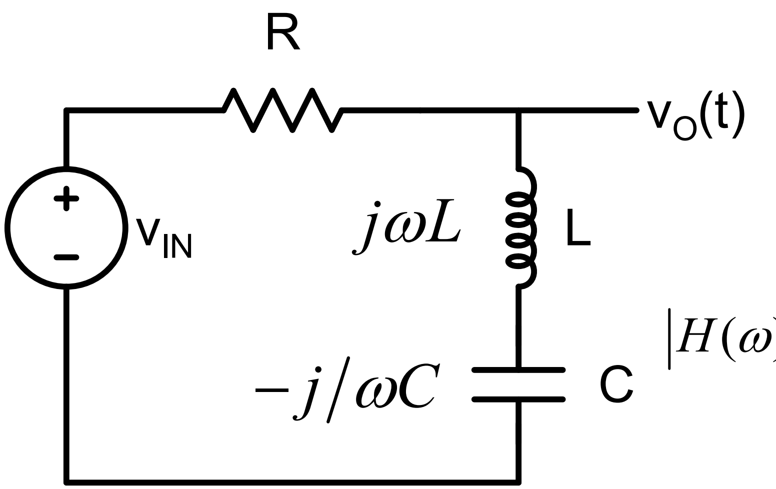
$$Y = \frac{1}{R_2} + \frac{1}{j\omega L} = \frac{R_2 + j\omega L}{j\omega L R_2}$$

$$Z = \frac{j\omega L R_2}{R_2 + j\omega L} = R_S + j\omega L_S$$

$$L_S = L \times \frac{R_2^2}{R_2^2 + \omega^2 L^2}$$

$$\omega_O L_S = \frac{1}{\omega_O C}$$

Q.2 Design a bandstop (or a notch filter) to remove a 50Hz noise from a 60Hz signal using a series RLC circuit. Assume L = 1H. The attenuation of 60Hz signal should be less than 3dB



$$|H(\omega)| = \left|\frac{V_O(\omega)}{V_{IN}(\omega)}\right| = \left|\frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}\right| = \frac{\frac{\omega^2}{\omega_O^2} - 1}{\sqrt{\omega^2 C^2 R^2 + (\frac{\omega^2}{\omega_O^2} - 1)^2}}$$

Assuming V_{IN} = 1V and noting that Q = $1/\omega_O CR$

$$|V_O(\omega)| = \frac{Q \times (\frac{\omega}{\omega_O} - \frac{\omega_O}{\omega})}{\sqrt{1 + Q^2 (\frac{\omega}{\omega_O} - \frac{\omega_O}{\omega})^2}}$$

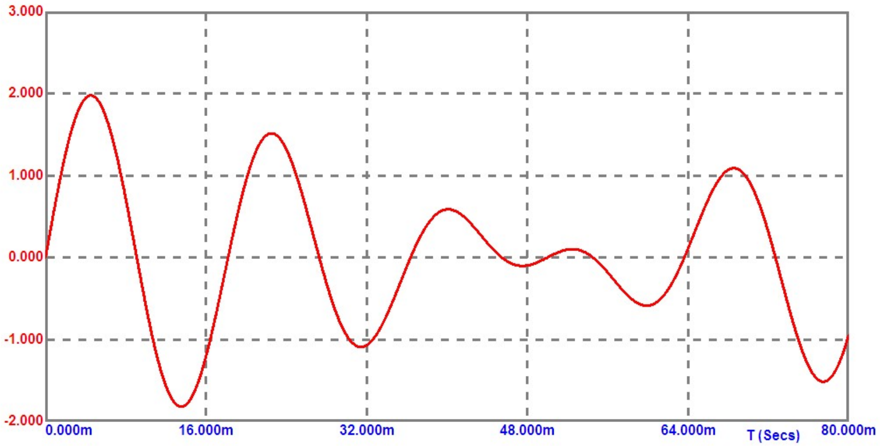
For $\omega = \omega_O$, $V_O = 0$ so the signal is rejected

$$\omega_O = 2 \times \pi \times 50 = 314.16 rad/s = \frac{1}{\sqrt{LC}} \Rightarrow C = 10.13 \mu F$$

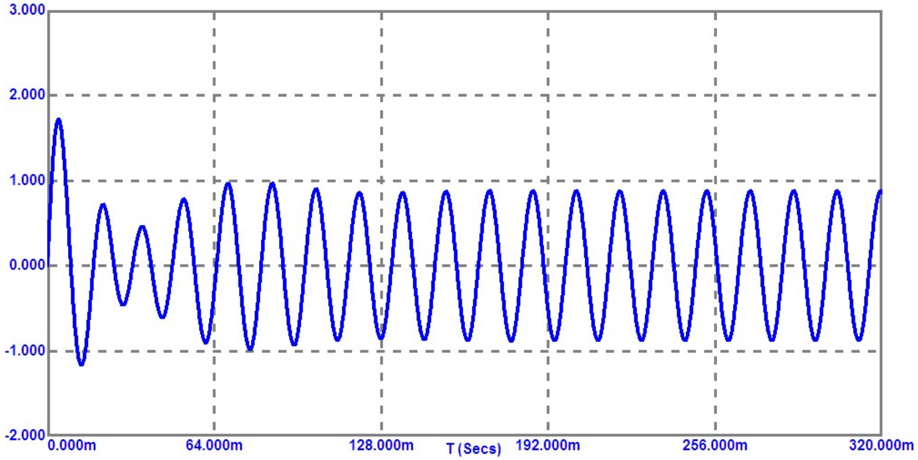
Less than 3dB attenuation for 60Hz signal implies

$$|V_O(\omega_1)| = \frac{Q \times (\frac{\omega_1}{\omega_O} - \frac{\omega_O}{\omega_1})}{\sqrt{1 + Q^2 (\frac{\omega_1}{\omega_O} - \frac{\omega_O}{\omega_1})^2}} \geq \frac{1}{\sqrt{2}} \qquad \omega_1 = 2\pi \times 60 \qquad \Rightarrow Q \geq 2.7$$

Let Q = 5. $$Q = \frac{\omega_O L}{R} \Rightarrow R \cong 63\Omega$$

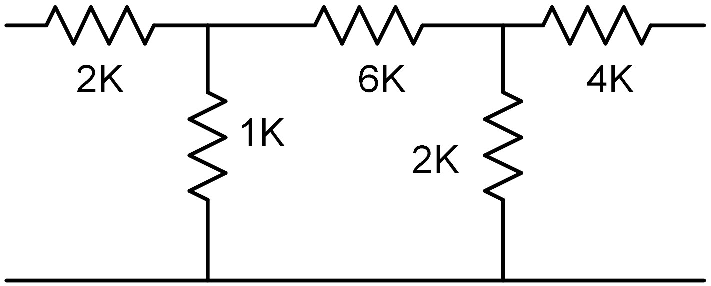


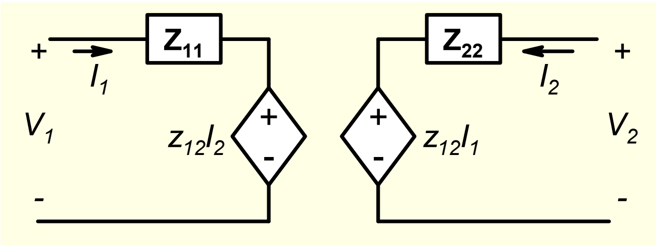
$$V_{in} = 1Sin(2\pi 50t) + 1Sin(2\pi 60t)$$

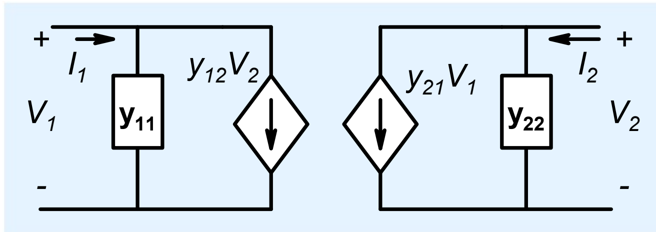


Output has only 60Hz component

Q.3 Determine Z, Y, H and G parameters for the circuit shown below







$$Z_{11} = \frac{V_1}{I_1}|I_2 = 0 \Rightarrow 2k + 1k\|8k = 2.89k\Omega$$

$$Z_{12} = \frac{V_1}{I_2}|I_1 = 0 \Rightarrow \frac{2k}{9k} \times 1k = 0.22\Omega$$

$$Z_{21} = \frac{V_2}{I_1}|I_2 = Z_{12} = 0.22k\Omega$$

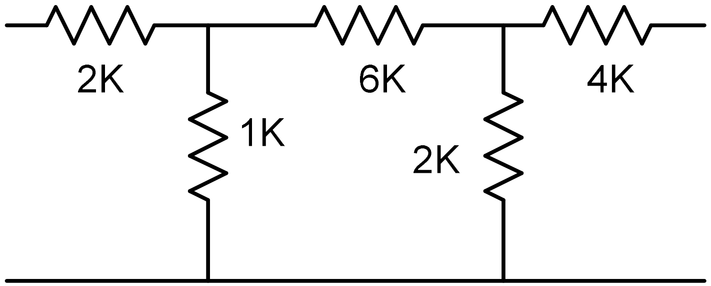
$$Z_{22} = \frac{V_2}{I_2}|I_1 = 0 |\Rightarrow 4k + 2k\|7k = 5.55\ k\Omega$$

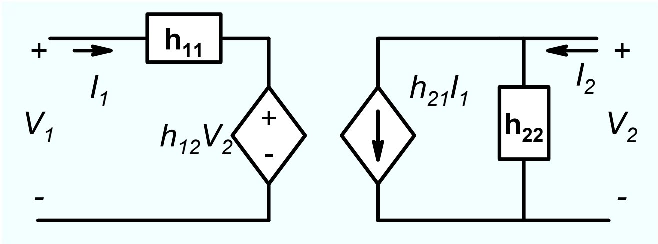
$$Y_{11} = \frac{I_1}{V_1}\ |V_2 = 0 \Rightarrow \frac{10^{-3}}{2 + 1\|(6 + 2\|4)}$$
$$= 3.4 \times 10^{-4}\Omega^{-1}$$

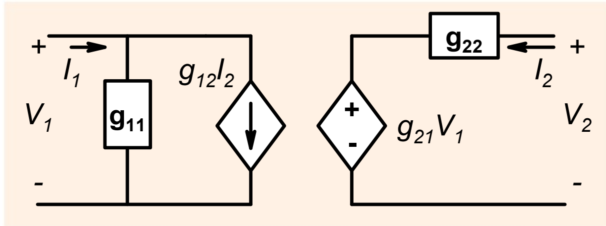
$$Y_{12} = \frac{I_1}{V_2}\ |V_1 = 0 \Rightarrow = 13.88 \times 10^{-6}\Omega^{-1}$$

$$Y_{21} = \frac{I_2}{V_1}\ |V_2 = 0 \Rightarrow = 13.88 \times 10^{-6}\Omega^{-1}$$

$$Y_{22} = \frac{I_2}{V_2}\ |V_1 = 0 \Rightarrow = 1.8 \times 10^{-4}\Omega^{-1}$$







$$h_{11} = \frac{V_1}{I_1} | V_2 = 0 \Rightarrow = 2.88 \times 10^3 \Omega$$

$$h_{12} = \frac{V_1}{V_2} | I_1 = 0 \Rightarrow = 4 \times 10^{-2}$$

$$h_{21} = -h_{12} = -4 \times 10^{-2}$$

$$h_{22} = \frac{I_2}{V_2} | I_1 = 0 \Rightarrow = 1.8 \times 10^{-4} \Omega^{-1}$$

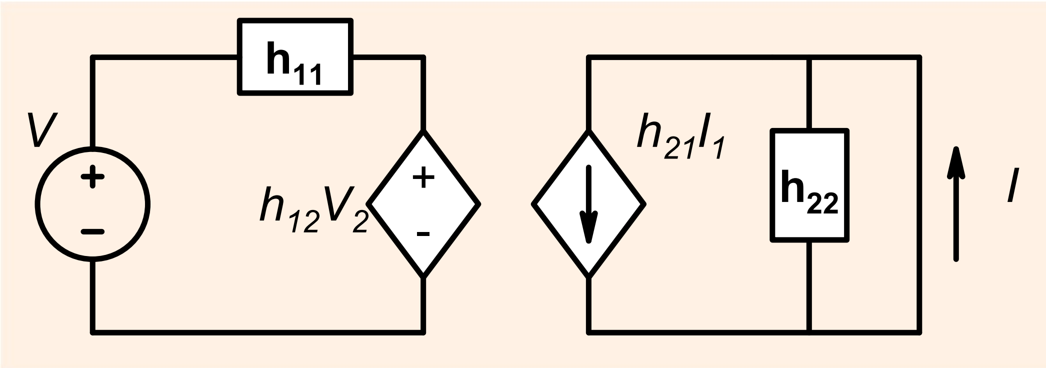
$$g_{11} = \frac{I_1}{V_1} \; | I_2 = 0 \Rightarrow = 3.46 \times 10^{-4} \Omega^{-1}$$

$$g_{12} = \frac{I_1}{I_2} \; | V_1 = 0 \Rightarrow = -0.077$$

$$g_{21} = -g_{12} = 0.077$$

$$g_{22} = \frac{V_2}{I_2} | V_1 = 0 \Rightarrow = 5.53k\, \Omega$$

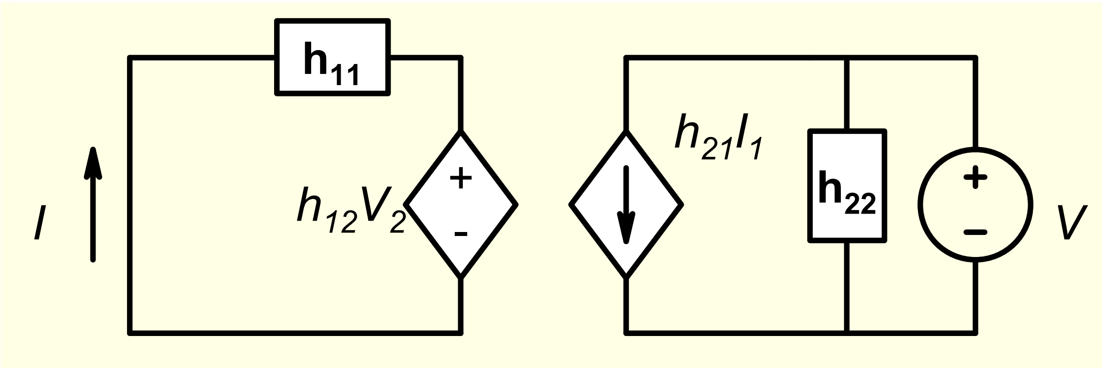
Q.4 Stating from the definition of a reciprocal network, show that h_{21} =- h_{12}



$$I = h_{21} I_1$$

$$I_1 = \frac{V}{h_{11}}$$

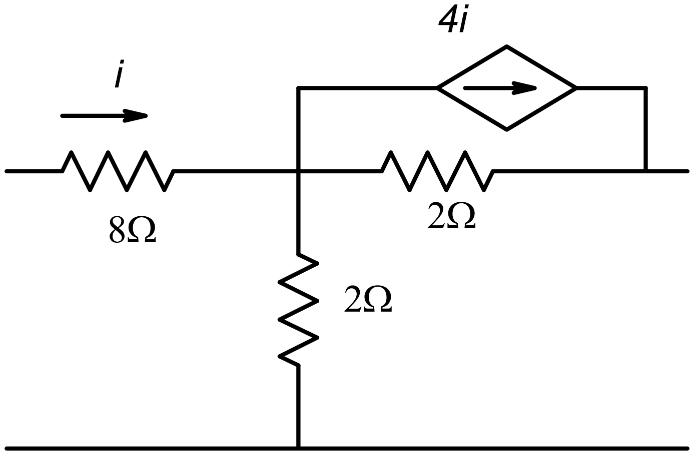
$$I = \frac{h_{21}}{h_{11}} V$$

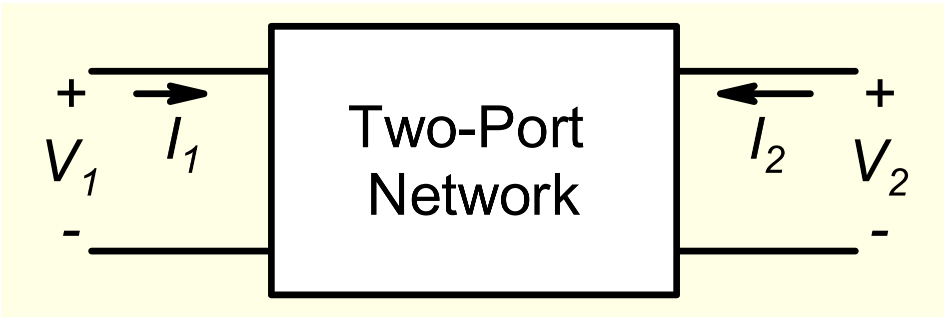


$$I = -\frac{h_{12} V}{h_{11}}$$

$$\Rightarrow h_{12} = -h_{21}$$

Q.5 Determine the Y parameters for the circuit shown below



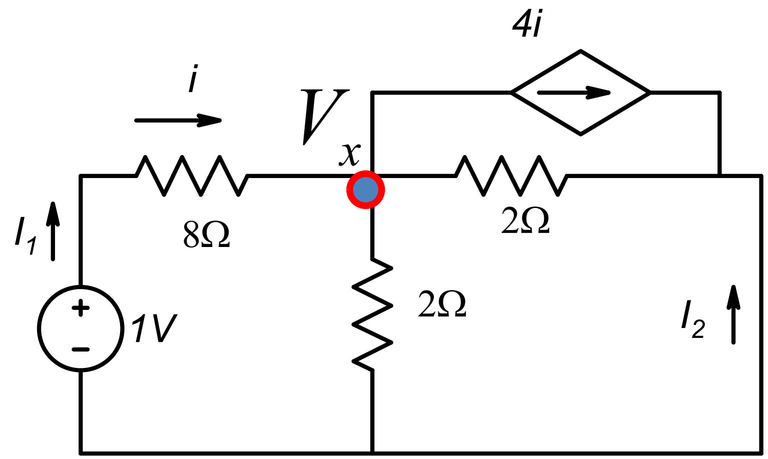


$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$



Nodal analysis gives: $$\frac{V_x - 1}{8} + \frac{V_x}{2} + \frac{V_x}{2} + 4i = 0 \qquad i = \frac{1 - V_x}{8}$$

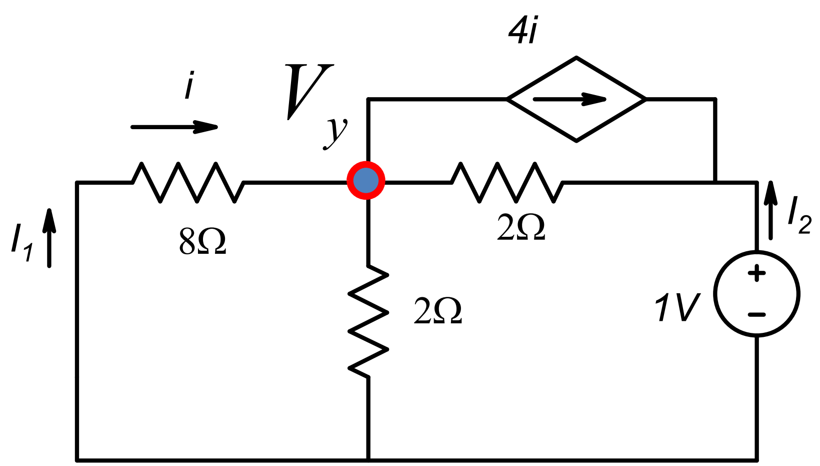
$$V_x = -\frac{3}{5}V \qquad i = 0.2A$$

$$i_2 = -4i - \frac{V_x}{2} = -0.5A$$

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = 0.2\Omega^{-1} \qquad y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = -0.5\Omega^{-1}$$

$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}$$

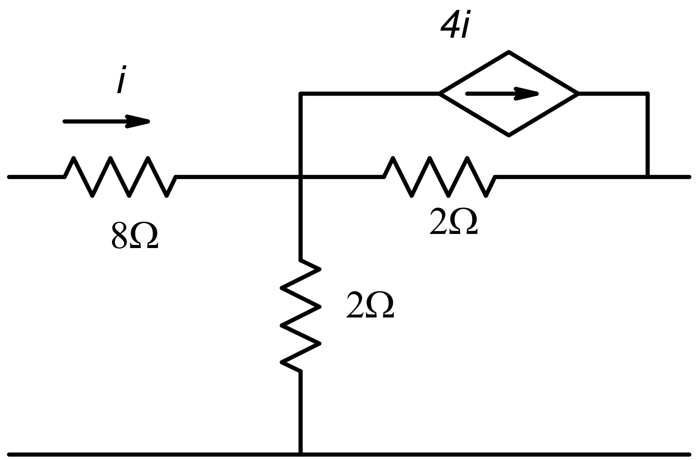


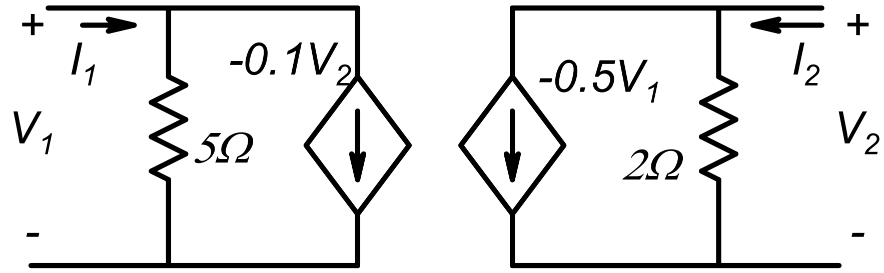
Nodal analysis gives: $$\frac{V_y - 1}{2} + \frac{V_y}{2} + \frac{V_y}{8} + 4i = 0 \qquad i = \frac{0 - V_y}{8}$$

$$V_y = \frac{4}{5}V \qquad i = -0.1A$$

$$i_2 = -4i + \frac{1 - V_y}{2} = 0.5A$$

$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = 0.5\Omega^{-1} \qquad y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = -0.1\Omega^{-1}$$





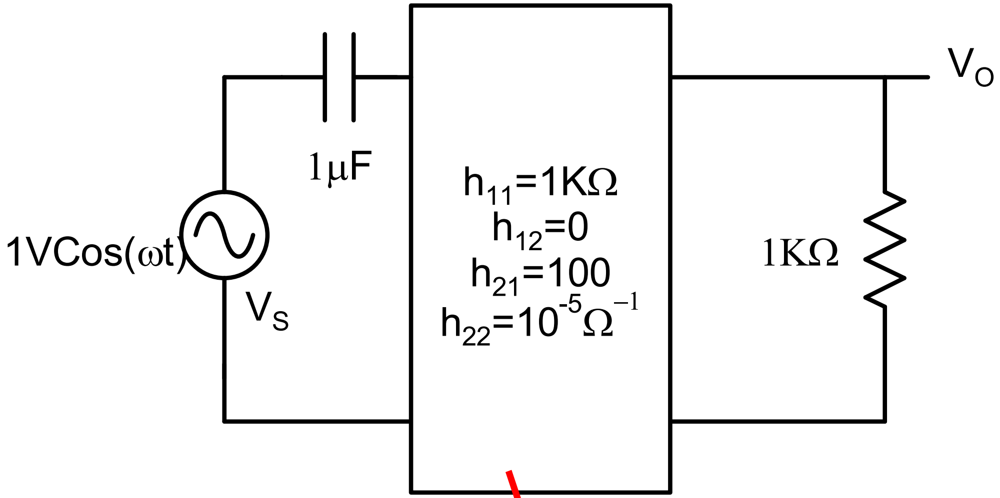
$$y_{11} = 0.2\Omega^{-1}$$

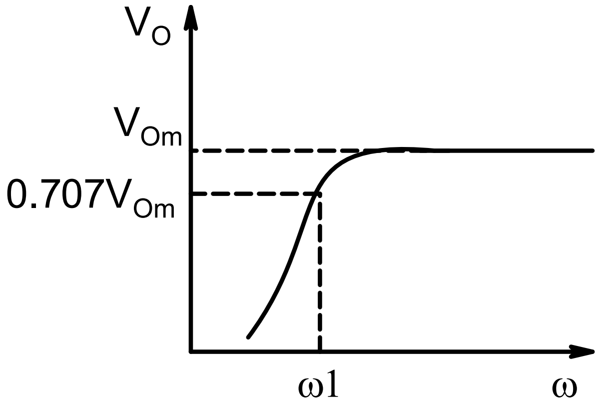
$$y_{22} = 0.5\Omega^{-1}$$

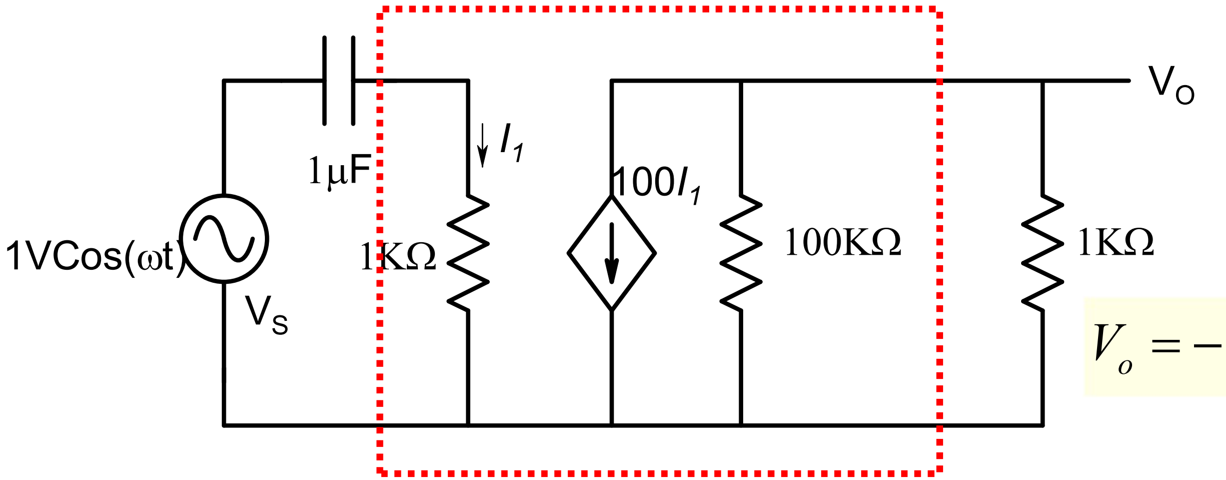
$$y_{21} = -0.5\Omega^{-1}$$

$$y_{12} = -0.1\Omega^{-1}$$

Q.6 For the circuit shown below on the left, the variation of magnitude of output voltage with frequency is shown on the right. Determine the voltage V_{om} and frequency $\omega 1$.







$$I_1 = \frac{1\angle 0}{10^3 - \frac{j}{\omega C}}$$

$$V_o = -100 I_1 \times 10^5 \| 10^3 = -9.9\times 10^4 I_1$$

$$V_o = -\frac{9.9\times 10^4}{10^3 - \dfrac{j}{\omega C}}$$

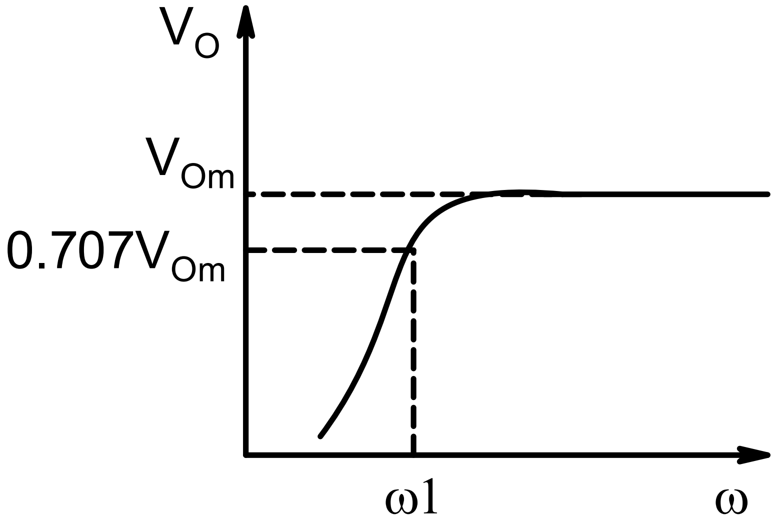
$$\left|V_o\right| = \frac{9.9\times 10^4\,\omega C}{\sqrt{10^6\,\omega^2 C^2 + 1}}$$

One notes that for ω = 0, output voltage is zero and increases as frequency increases.

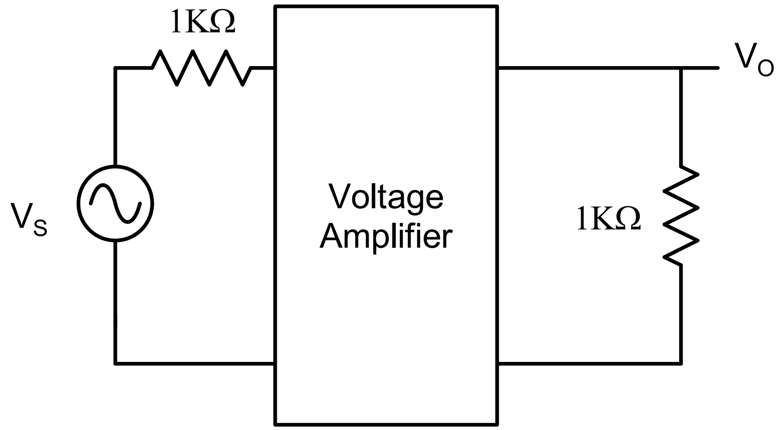
For very large ω, Vo can be approximated as

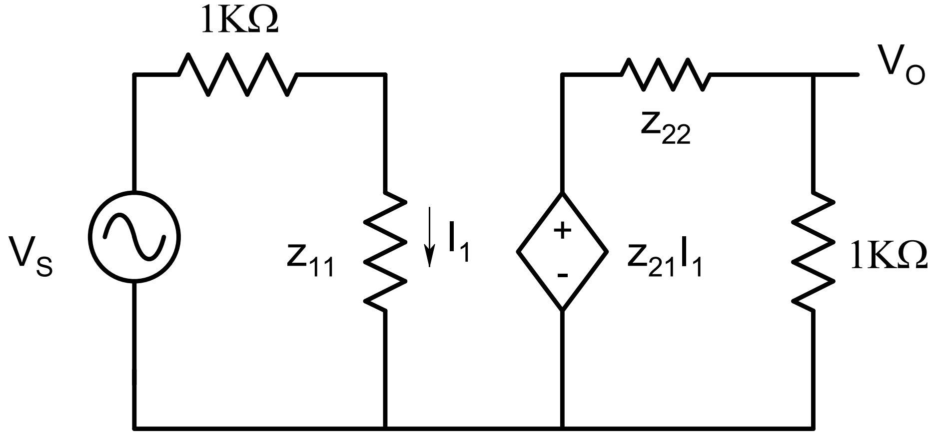
$$\left|V_{om}\right| \cong \frac{9.9\times 10^4\,\omega C}{10^3\,\omega C} \cong 99$$

$$\frac{99}{\sqrt{2}} = \frac{9.9\times 10^4\,\omega_1 C}{\sqrt{10^6\,\omega_1^{\,2} C^2 + 1}} \Rightarrow \omega_1 = 10^3\,rad/s$$



Q.7 The two port network shown below is a voltage amplifier. One can design the amplifier for different values of z parameters under the constraint $\frac{z_{21}}{z_{22}} = 100$ and z_{12} = 0. Determine suitable values for the z parameters such that voltage gain $\frac{V_O}{V_S}$ is maximized.





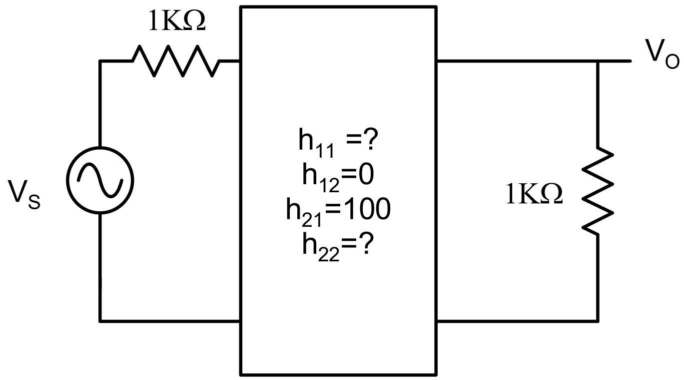
$$I_1 = \frac{V_S}{z_{11} + 1K} \qquad V_O = \frac{1K}{z_{22} + 1K} z_{21} I_1$$

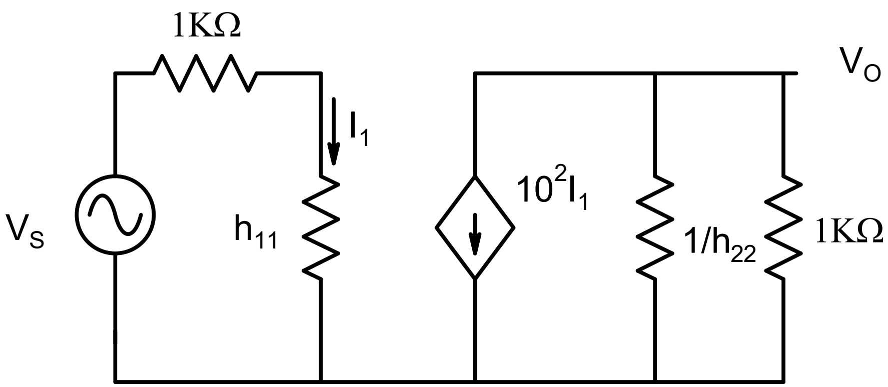
$$\frac{V_O}{V_S} = \frac{1K}{z_{22} + 1K} \times z_{21} \times \frac{1}{z_{11} + 1K}$$

$$\frac{V_O}{V_S} = \frac{z_{22}}{z_{22} + 1K} \times \frac{1K}{z_{11} + 1K} \times 10^2$$

$\Rightarrow z_{22} >> 1K$; $z_{11} << 1K$ Choose z_{22} = 10K, z_{11} = 0.1K $\rightarrow z_{21} = 10^6$

Q.8 Determine the values of h_{11} and h_{22} for the amplifier shown below such power delivered to the load is maximized





To maximize power we need to maximize current in the 1K load resistor

$$I_L = 10^2 \frac{V_S}{h_{11} + 1K} \times \frac{1}{1 + h_{22} \times 10^3}$$

$$\Rightarrow h_{11} << 1K \ ; \ h_{22} 10^3 << 1$$

Choose h_{11} = 0.1K, $h_{22} = 10^{-4}$