MSO-203 B ASSIGNMENT 4

IIT, KANPUR

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Multiple choice questions may have more than one correct answers. Submit problem number 1, 2, 3, 4. Maintain the order, in your submission, in which the questions are given.

1. Let $u : \mathbb{R} \to \mathbb{R}$ be the solution of the problem

$$\begin{cases} u_{tt} - u_{xx} = 0 & \textbf{in} \mathbb{R} \times [0, \infty), \\ u(x, 0) = f \\ u_t(x, 0) = g. \end{cases} \tag{1}$$

   Suppose $f(x) = g(x) = 0$, if $x \in \ \mathbb{R} \setminus [0, 1]$. Then

   a). $u(x, t) = 0, \ \forall (x, t) \in (-\infty, 0) \times (0, \infty)$.

   b). $u(x, t) = 0, \ \forall (x, t) \in (1, \infty) \times (0, \infty)$.

   c). $u(x, t) = 0, \ \{(x, t) \mid x + t < 0\}$.

   d). $u(x, t) = 0, \ \{(x, t) \mid x + t < 1\}$.

   e). None of the above.

2. Classify the following linear second order PDE as Elliptic, hyperbolic or Parabolic and deduce their cannonical form:

   a) $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$.

   b) $u_{xx} + 4u_{xy} + 4u_{yy} = 0$.

   c) (Tricomi Equation) $u_{xx} + xu_{yy} = 0$, in the region $x > 0$.

3. Solve the following wave equation by the method of separation of variables:

$$\begin{cases} u_{tt} - 9u_{xx} = 0 & \textbf{in } (0, \pi) \times (0, \infty), \\ u(x, 0) = 2 \\ u_t(x, 0) = 1. \end{cases} \tag{2}$$

4. Solve the following wave equation

$$\begin{cases} u_{tt} - 4u_{xx} = 0 & \textbf{in } \mathbb{R} \times (0, \infty), \\ u(x, 0) = \sin(x) \\ u_t(x, 0) = 1. \end{cases} \tag{3}$$

5. A advanced problem, not a part of the syllabus, but can be done with the information provided in the lectures with some knowledge differentiation inder integration.

   (Equipartition of Energy) Let $u \in C(\mathbb{R} \times [0, \infty))$ solves the initial value problem for the wave equation in one dimension:

$$
\begin{cases} u_{tt} - u_{xx} = 0 \quad \textbf{in } \times [0, \infty), \\ u(x, 0) = f \\ u_t(x, 0) = g. \end{cases} \tag{4}
$$

Suppose g, h are two given continuous functions that are identically 0 outside the set $(-1, 1)$. The kinetic energy and the potential energy of the solution are given by the following quantity

$$
K(t) := \frac{1}{2} \int_{-\infty}^{\infty} u_t^2(x, t) dx, \quad P(t) := \frac{1}{2} \int_{-\infty}^{\infty} u_x^2(x, t) dx.
$$

Prove that $K(t) + P(t)$ is a constant function of t (Conservation of energy).