Multiplication

So matrix multiplication can be seen in this way.

$$(AB)[i,:] = a_{i1} B[1,:] + a_{i2} B[2,:] + \dots + a_{in} B[n,:] \rightarrow \text{Playing with rows of } B$$

$$(AB)[:,i] = A[:,1]\, b_{1i} + A[:,2]\, b_{2i} + \dots + A[:,n]\, b_{ni} \rightarrow \text{Playing with columns of } A.$$

$$AB = \begin{bmatrix} A[1,:] & B \\ A[2,:] & B \\ \vdots & \\ A_m[m,:] & B \end{bmatrix}$$

$$A[1,:]B = a_{11} B[1,:] + a_{12} B[2,:] + \dots a_{1n} [n,:]$$

$$B = \begin{bmatrix} B[:,1] & B[:,2] & \dots & B[:,n] \end{bmatrix}$$

$$AB = \begin{bmatrix} AB[:,1] & AB[:,2] & \dots & AB[:,n] \end{bmatrix}$$

$$AB[:,1] = A[:,1]\, B_{11} + A[:,2]\, B_{21} + \dots A[:,n]\, B_{n1}$$

RREF

Defn A matrix $A_{m \times n}$ is said to be in RREF (Row Reduced Echelon Form) if

Ladder like

1) All the zero rows of A are at the bottom
2) The first non-zero entry of a non-zero row is 1. This '1' is called the pivot entry.
3) Every other entry in the pivoted column is 0.
4) The pivot of $(i+1)^{th}$ row, if it exists, comes to the right of pivot entry of i^{th} row $1 \le i \le m-1$

Solving Linear Systems

$$\begin{aligned} x + y \quad - 2u + v &= 2 \\ z + u + 2v &= 3 \\ v + w &= 3 \\ v + 2w &= 5 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 3 \\ 3 \\ 5 \end{bmatrix}$$

compute RREF ([A b])

RREF([A b]) = RREF ([C d])

$$[C \ d] = \left[\begin{array}{cccccc|c} 1 & 1 & 0 & -2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

RREF(A) = C

rank (A) = Pivots in C = r

= 4 = r

rank ([A b]) = Pivots in [C d]

= 4 = r_0

$r = r_0$ (Hence the system is consistent).

Now solve $Cx = d$ and the x will also be the solution of $Ax = b$.

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$$\begin{bmatrix} \boxed{1} & 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & \boxed{1} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$x + y - 2u = 1$ $\qquad$ $z + u = 1$ $\qquad$ $v = 1$

$x = 1 + 2u - y$ $\qquad$ $z = 1 - u$ $\qquad$ $w = 2$

So the solution set is –

$$\begin{pmatrix} x \\ y \\ z \\ u \\ v \\ w \end{pmatrix} = \begin{bmatrix} 1 + 2u - y \\ y \\ 1 - u \\ u \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} + u \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving Homogeneous Linear System

Date ___/___/___ $\qquad$ Saathi

The solution $X = 0$ of $AX = 0$ is called a trivial solution.

A solution $X \neq 0$ of $AX = 0$ is called a non-trivial soln.

If $A_{m \times n} X_{n \times 1} = 0$ and $m < n \Rightarrow$ Non-Trivial Soln. (Infinite)

If $m \geq n$ (i) Also if $r = \min(m, n) \Rightarrow$ Trivial Soln.

(ii) If $r < n \Rightarrow$ Non Trivial solution.