Polar form of z

Let $z = re^{i\theta}$.

$|z| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta} = r$

θ is called the *argument of z*. Denoted as $\arg(z)$.

It is a multi-valued function from $\mathbb{C}^* \to \mathbb{R}$.

The *principal value* of $\arg(z)$ is the unique value of $\arg(z)$ satisfying $-\pi < \arg(z) \leq \pi$. It is denoted as $\mathrm{Arg}(z)$.

Properties of $\arg(z)$

- $re^{i\theta} = r'e^{i\theta'} \iff r = r'$ and $\theta = \theta' + 2n\pi$.
- $\arg(z_1z_2) = \arg(z_1) + \arg(z_2)(mod\ 2\pi)$.
- $\arg(\frac{z_1}{z_2}) = \arg(z_1) - \arg(z_2)(mod\ 2\pi)$

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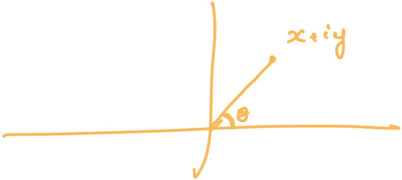
"error" factor of $2\pi n$.

If $x + iy = re^{i\theta}$ ~~and $-\pi/2 < \theta < \pi/2$~~ then $\tan\theta = (y/x)$.

$r = \boxed{|x+iy|}$

Recover r, θ from x, y ??

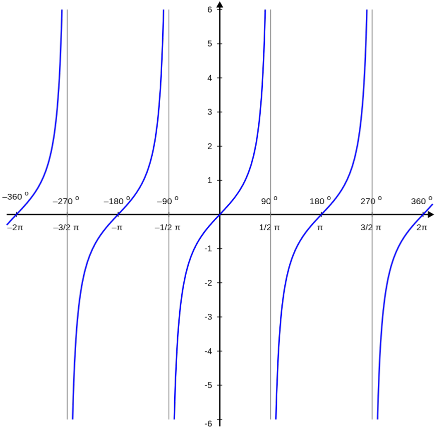


$(0,0)$ (x, y)

$\tan\theta = y/x$

$\theta = \tan^{-1}(y/x)$??

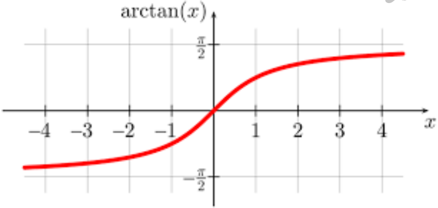
If $x + iy = re^{i\theta}$ ~~and~~ $\cancel{-\pi/2 < \theta < \pi/2}$ then $\tan\theta = (y/x)$.



RECALL

If $x + iy = re^{i\theta}$ and $-\pi/2 < \theta < \pi/2$ then $\tan\theta = (y/x)$.

$\tan^{-1} : \mathbb{R} \to (-\pi/2, \pi/2)$



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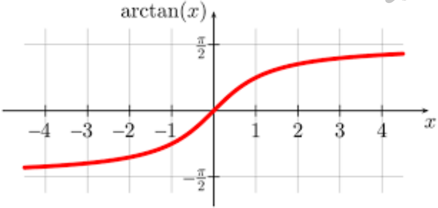

$\theta = \tan^{-1}(y/x)$

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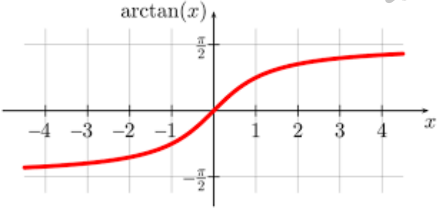

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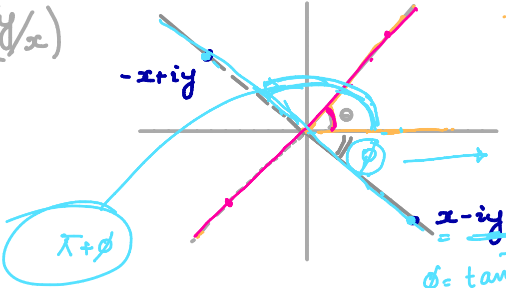
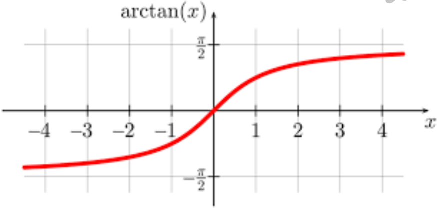


$(-\pi/2, 0)$

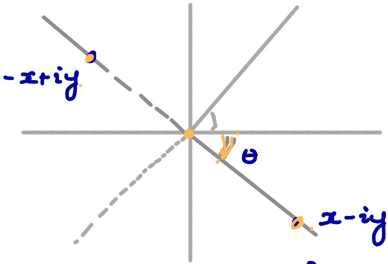
$\phi = \tan^{-1}(-y/x)$

If $x + iy = re^{i\theta}$ and $-\pi/2 < \theta < \pi/2$ then $\tan\theta = (y/x)$.

$\tan^{-1} : \mathbb{R} \to (-\pi/2, \pi/2)$



$\theta = \tan^{-1}(y/x)$



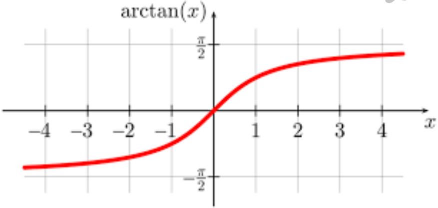
$\mathrm{Arg}(-x+iy) \in (\pi/2, \pi)$

$\mathrm{Arg}(x-iy) \in (-\pi/2, 0)$

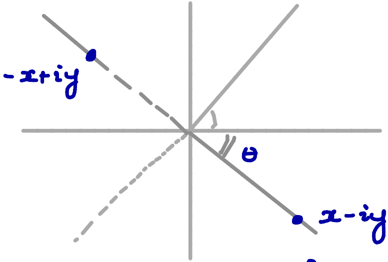
slope = $\tan\theta$.

If $x + iy = re^{i\theta}$ and $-\pi/2 < \theta < \pi/2$ then $\tan\theta = (y/x)$.

$\tan^{-1} : \mathbb{R} \to (-\pi/2, \pi/2)$



$\theta = \tan^{-1}(y/x)$



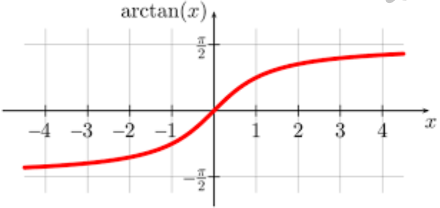
$\mathrm{Arg}(-x+iy) \in (\pi/2, \pi)$

$\mathrm{Arg}(x-iy) \in (-\pi/2, 0)$
$= \tan^{-1}(-y/x)$

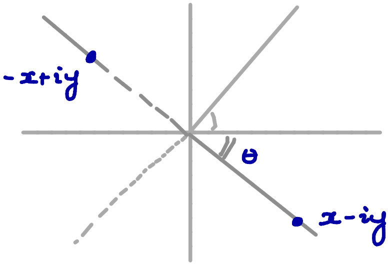
slope $= \tan\theta$.

If $x + iy = re^{i\theta}$ and $-\pi/2 < \theta < \pi/2$ then $\tan\theta = (y/x)$.

$\tan^{-1} : \mathbb{R} \to (-\pi/2, \pi/2)$



$\theta = \tan^{-1}(y/x)$



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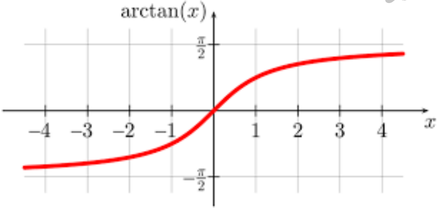

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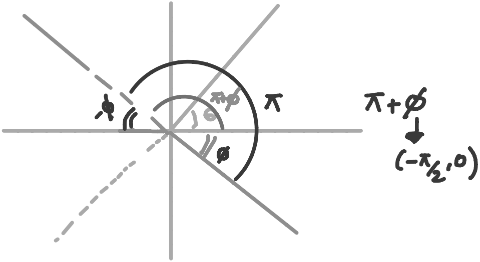
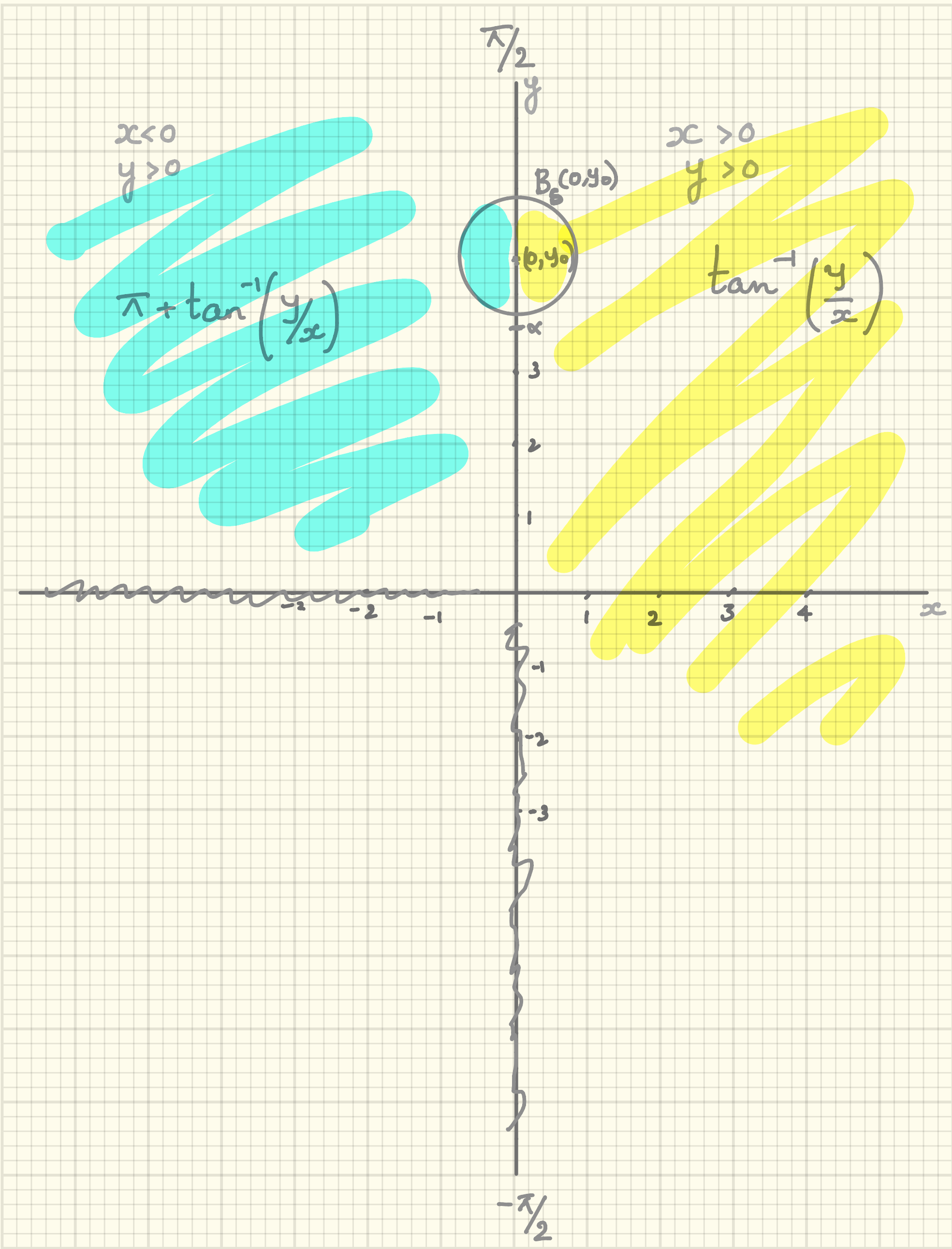


If $x + iy = re^{i\theta}$ and $-\pi/2 < \theta < \pi/2$ then $\theta = \tan^{-1}(y/x)$.
Since $\frac{y}{x} = \frac{-y}{-x}$, so,

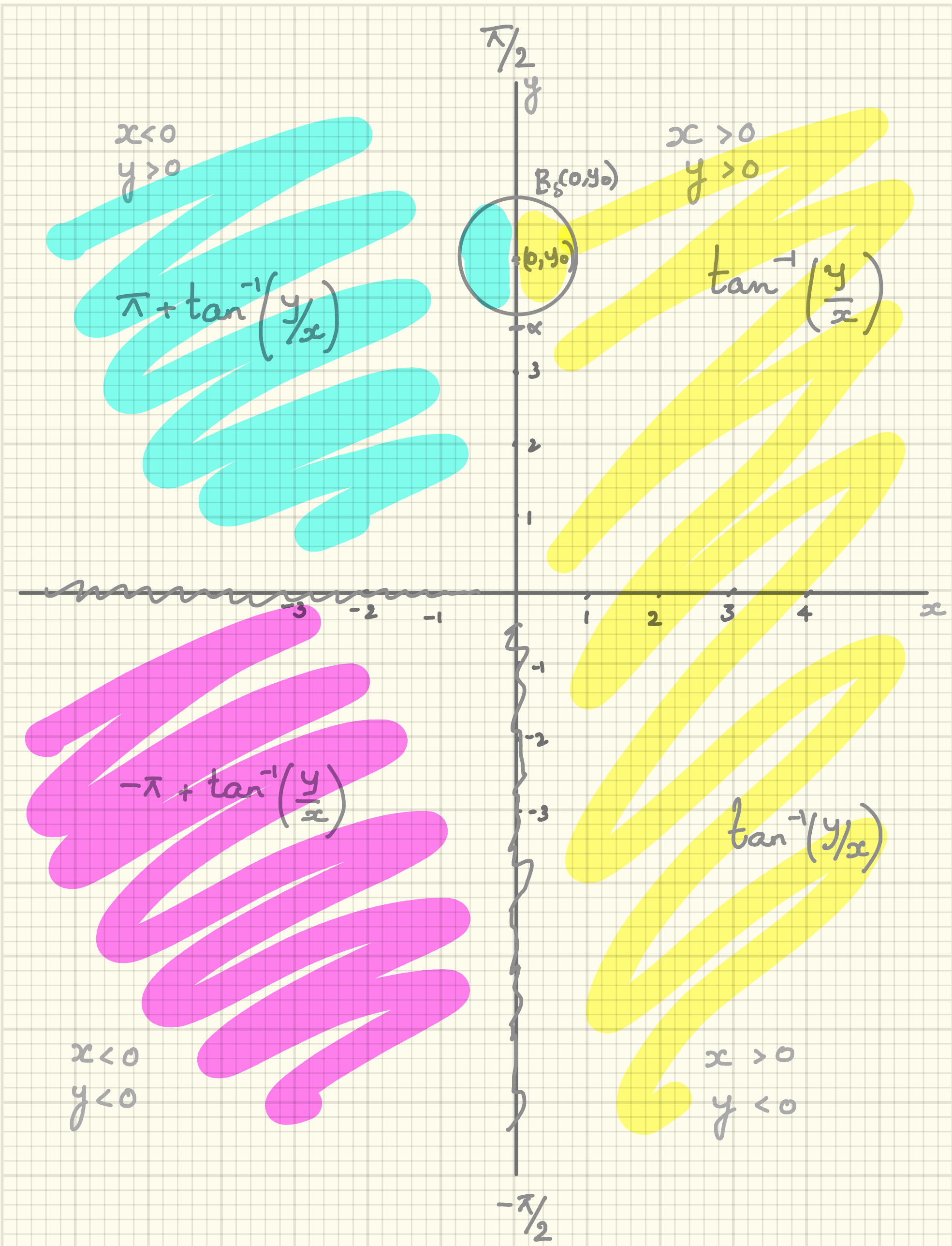
Convention

$$\operatorname{Arg}(z) = \begin{cases} \tan^{-1}(y/x), & \text{if } x > 0 \\ \pi + \tan^{-1}(y/x), & \text{if } x < 0, y \geq 0 \\ -\pi + \tan^{-1}(y/x), & \text{if } x < 0, y < 0 \\ -\dfrac{\pi}{2}, & \text{if } x = 0, y < 0 \\ \dfrac{\pi}{2}, & \text{if } x = 0, y > 0 \end{cases} \tag{1}$$

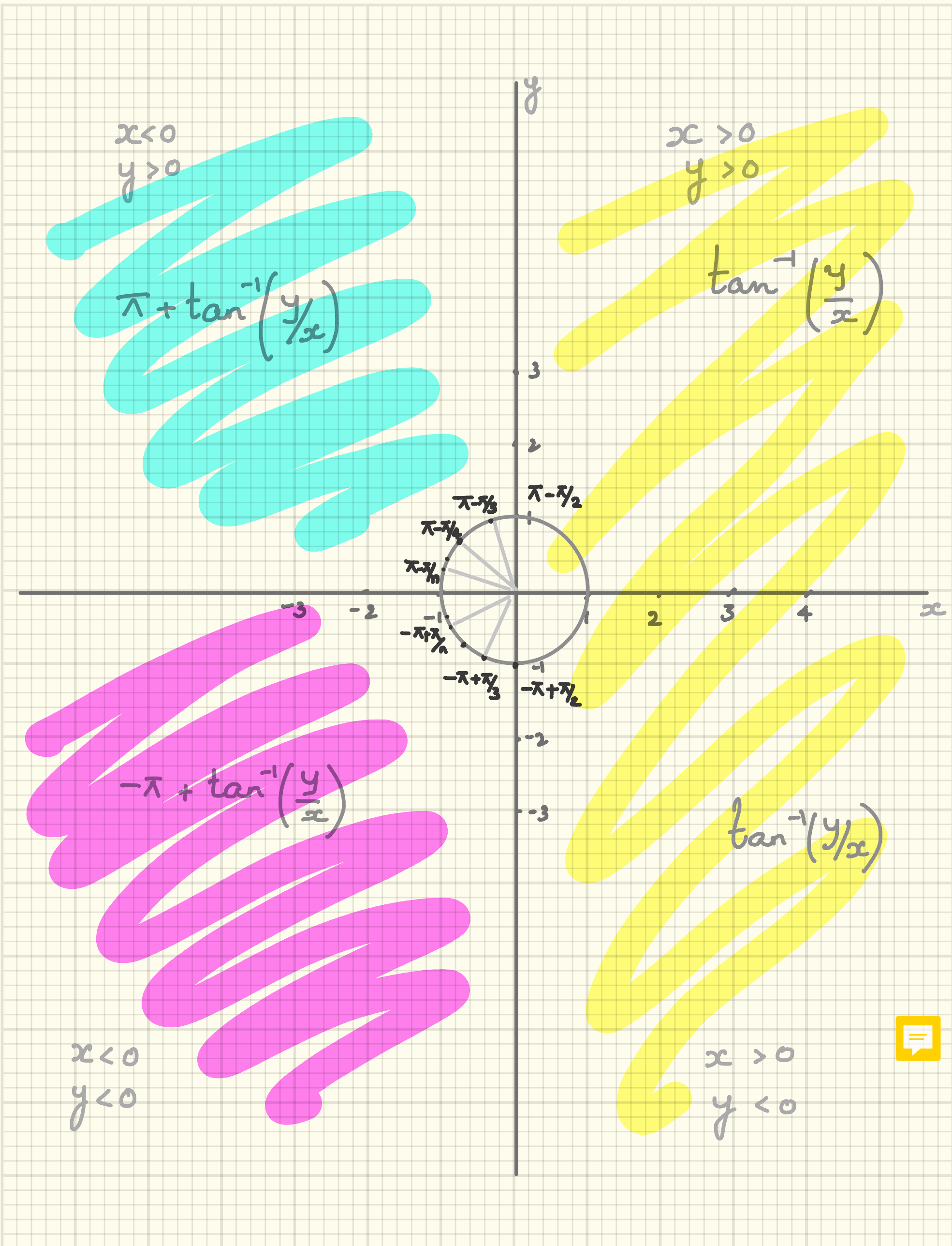
y
x
x<0
y>0
$\pi + \tan^{-1}(y/x)$
x >0
y >0
$\tan^{-1}\left(\frac{y}{x}\right)$
3
2
1
-3
-2
-1
1
2
3
4
-1
-2
-3
$-\pi + \tan^{-1}\left(\frac{y}{x}\right)$
$\tan^{-1}(y/x)$
x<0
y<0
x >0
y <0



π/2
y
x<0
y>0
x>0
y>0
$B_\delta(0,y_0)$
$(0,y_0)$
$\pi+\tan^{-1}(y/x)$
$\tan^{-1}\left(\frac{y}{x}\right)$
x
-π/2



$\pi/2$
y
$x<0$
$y>0$
$x>0$
$y>0$
$B_\delta(0,y_0)$
$(0,y_0)$
$\pi + \tan^{-1}(y/x)$
$\tan^{-1}\left(\frac{y}{x}\right)$
$-\pi + \tan^{-1}\left(\frac{y}{x}\right)$
$\tan^{-1}(y/x)$
$x<0$
$y<0$
$x>0$
$y<0$
x
$-\pi/2$



x<0
y>0
π + tan⁻¹(y/x)
x>0
y>0
tan⁻¹(y/x)
y
x
π − π/2
π − π/3
π − π/4
π − π/6
−π + π/6
−π + π/3
−π + π/2
−π + tan⁻¹(y/x)
tan⁻¹(y/x)
x<0
y<0
x>0
y<0