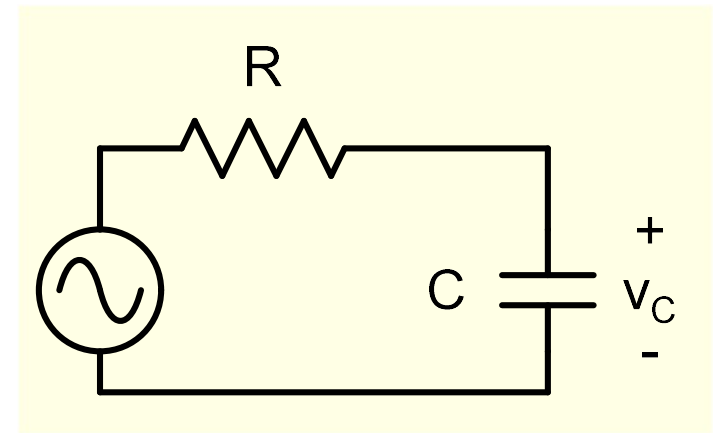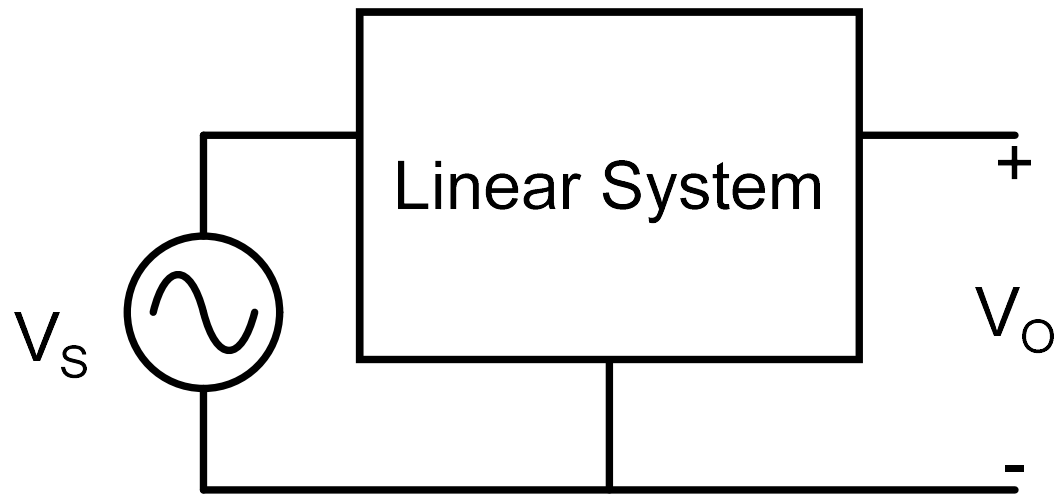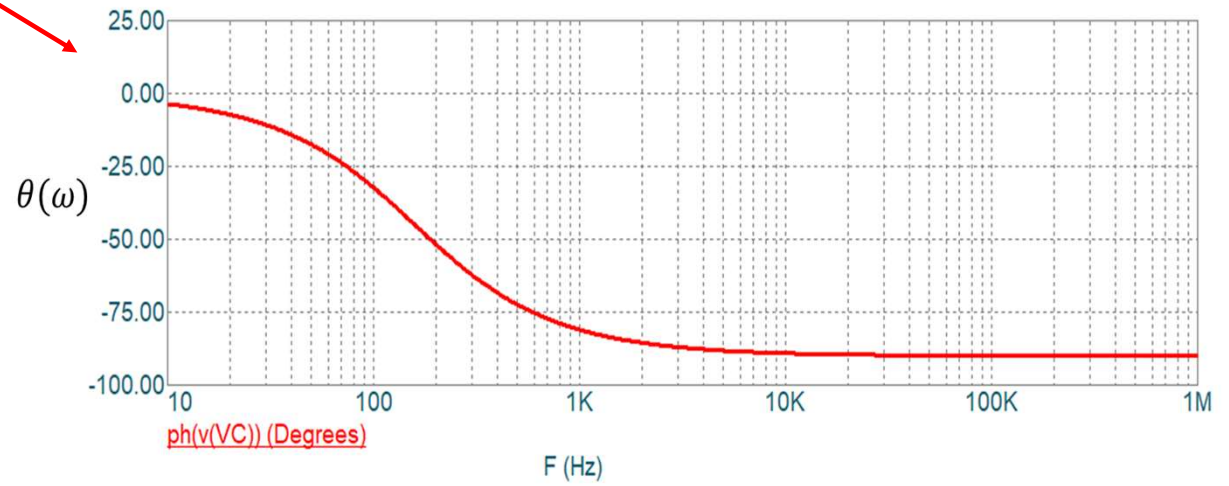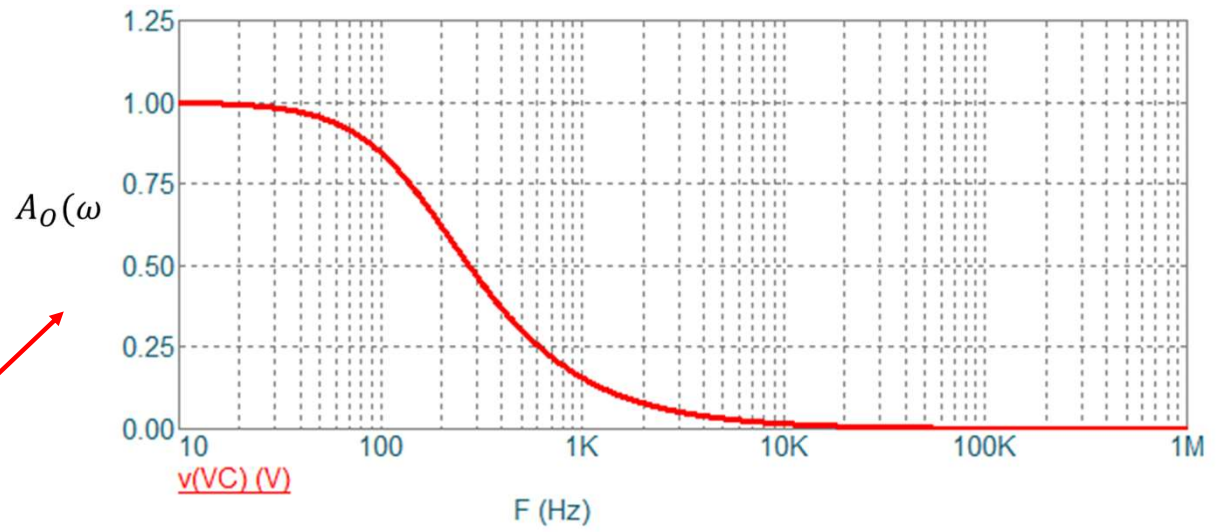# ESC201T : Introduction to Electronics

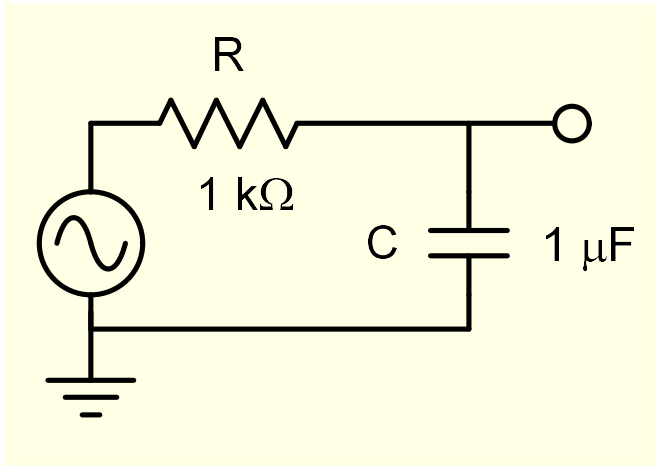## Lecture 14: Sinusoidal Steady State Analysis using Phasors
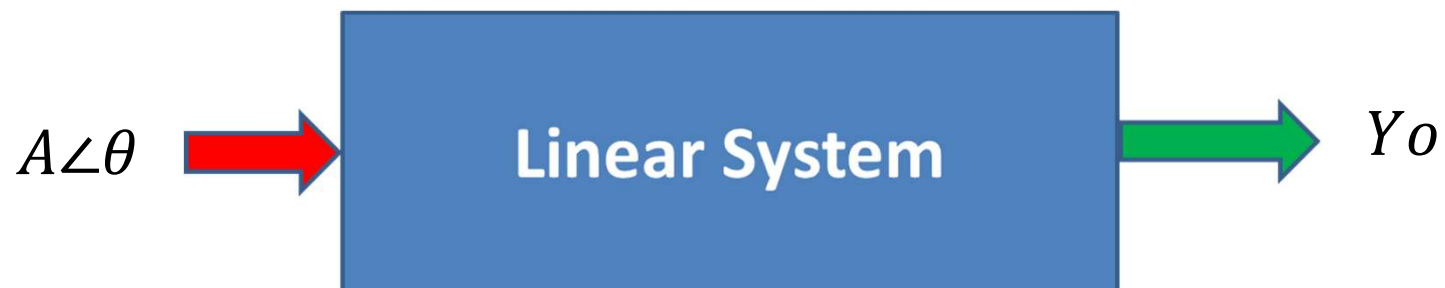
B. Mazhari
Dept. of EE, IIT Kanpur

$$V_S = 1 \times \cos(\omega t) \qquad V_O = A_O(\omega) \times \cos(\omega t + \theta(\omega))$$

$A_O(\omega)$ and $\theta(\omega)$ determine the complete characteristics of the system

R
1 kΩ
C
1 µF

$A_O(\omega$

v(VC) (V)

F (Hz)

$\theta(\omega)$

ph(v(VC)) (Degrees)

F (Hz)

Linear System

$x_1(t) = A\cos(\omega t + \theta)$

$y_1(t)$

Linear System

$x(t) = Ae^{j\theta} \times e^{j\omega t}$

$Yo \times e^{j\omega t}$

$y_1(t) = Re(y(t))$

Linear System

$A\angle\theta$

Yo

Complex Impedances

$A\angle\theta$ → **Linear System** → Yo
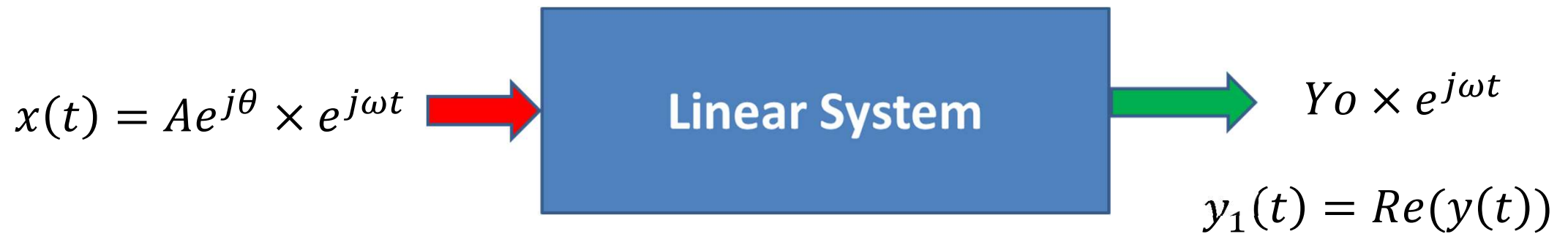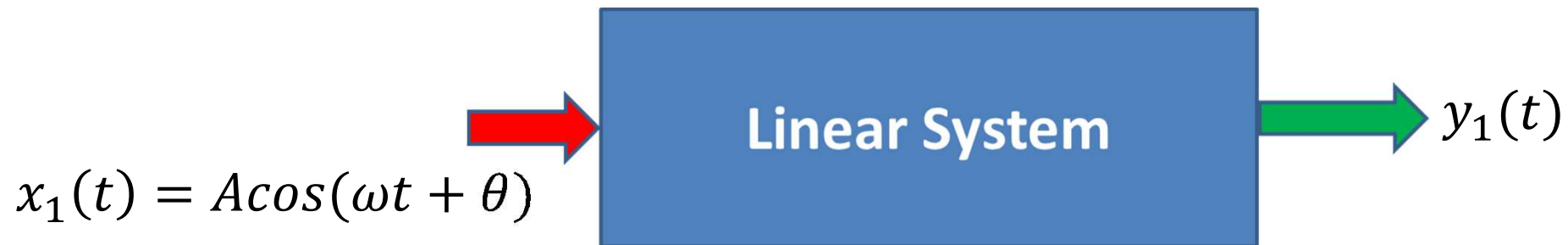
For the purpose of sinusoidal steady state analysis, circuit elements such as inductors and capacitors can be represented as Complex Impedances
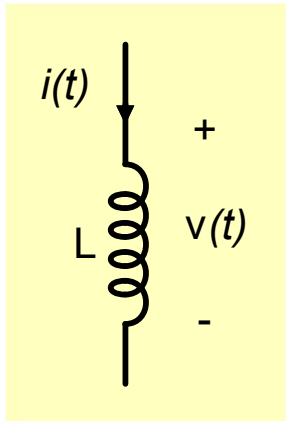
$$I_L = I_m \times e^{j(\omega t + \theta)} \rightarrow I_m \angle\theta$$

$$v_L = L \times \frac{dI_L}{dt} = j\omega L \times I_m \times e^{j(\omega t + \theta)}$$

$$\rightarrow V_L = j\omega L \times I_m \angle\theta$$
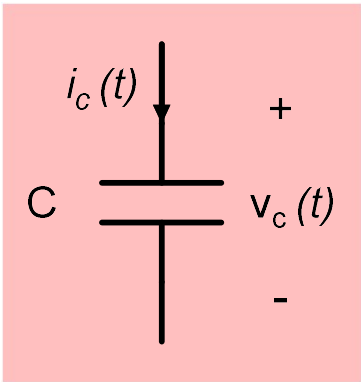
$$V_L = \omega L \angle 90° \times I_m \angle\theta$$

Current through the inductor lags the voltage by 90°

$$V_L = j\omega L \times I_L$$

$$V_L = Z_L \times I_L \qquad Z_L = j\omega L$$

This is like ohms law relationship between phasor voltage and current

Capacitor

$$i_c = C \frac{dv_c}{dt}$$

$$v_C(t) = V_M \times e^{j(\omega t + \;)}$$

$$V_C = V_M \angle \theta$$

$$i_C(t) = j\omega C V_M \times e^{j(\omega t + \theta)}$$

$$I_C = j\omega C V_M \angle \theta$$

$$I_C = j\omega C V_M \angle \theta + 90°$$

In a capacitor, current leads voltage by 90^0

$$I_C = j\omega C \times V_M \angle \theta$$

$$V_C = I_C \times Z_C$$

$$Z_C = \frac{1}{j\omega C} = -j \frac{1}{\omega C}$$

Resistor

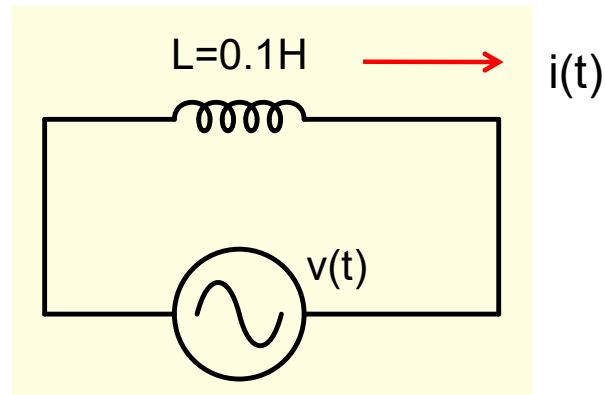$$v_R(t) = V_M \cos(\omega t + \theta)$$

$$i_R(t) = \frac{V_M}{R} \cos(\omega t + \theta)$$

$$V_R = V_M \angle \theta$$

$$I_R = \frac{V_M}{R} \angle \theta$$

$$I_R = \frac{V_R}{R}$$

Example-1

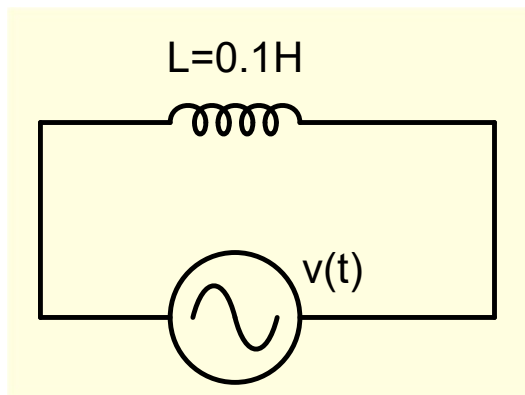$$v(t) = 2\cos(200t + 45) \qquad \omega = 200$$

$$V_L = 2\angle 45 \qquad V_L = I_L \times j\omega L \Rightarrow I_L = \frac{V_L}{j\omega L}$$
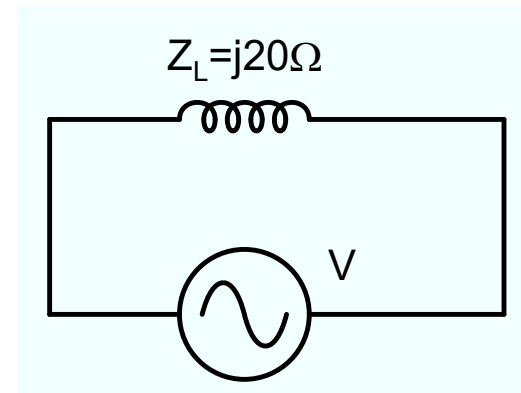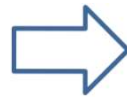
$$I_L = \frac{2\angle 45}{j20} = \frac{2\angle 45}{20\angle 90} = 0.1\angle -45$$

$$i(t) = 0.1\cos(200t - 45)$$

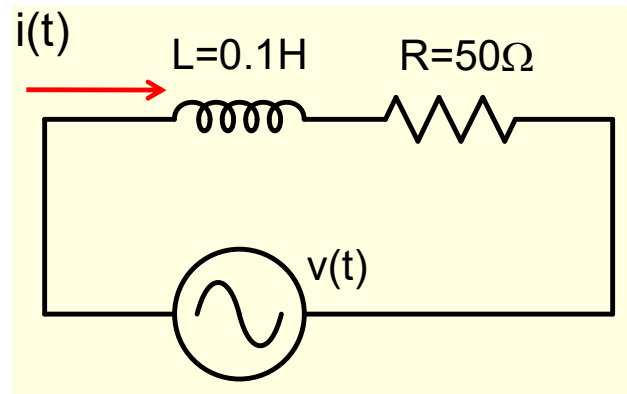L=0.1H

v(t)

$$Z_L = j\omega L$$

$Z_L = j20\Omega$

V

$$I_L = \frac{V_L}{j20}$$

Carry out analysis with phasors keeping in mind that we can always transform phasor to the sinusoidal voltage or current as the case maybe.
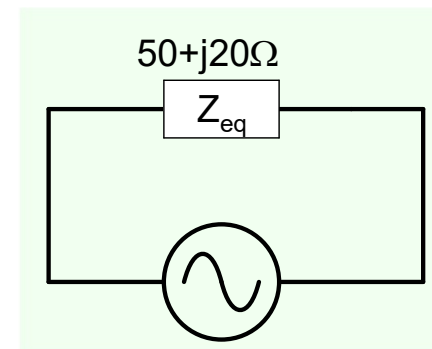
Example-2

$$v(t) = 2 \cos(200t + 45) \qquad \omega = 200$$

$$Z_L = j\omega L$$
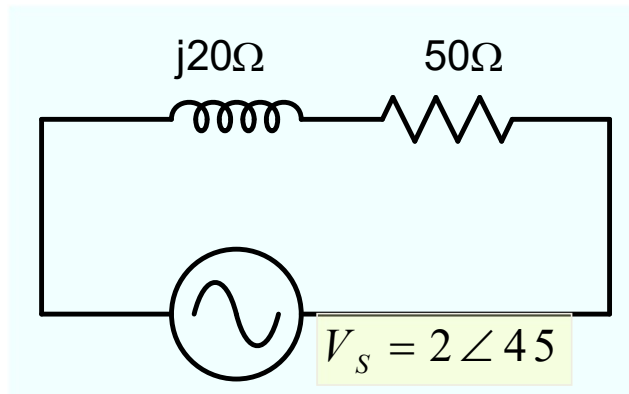

$$V_S = 2\angle 45$$
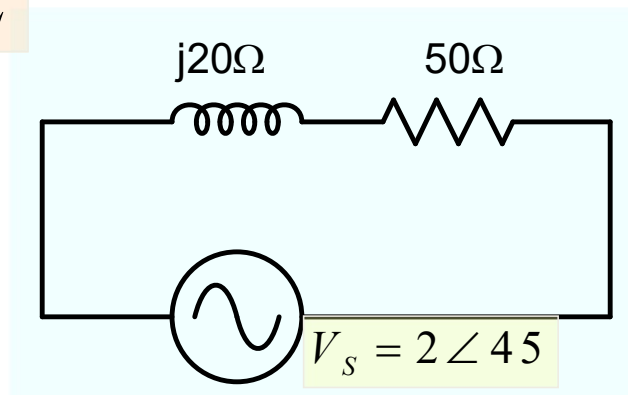
$$I = \frac{2\angle 45}{50 + j20} = \frac{2\angle 45}{53.85\angle 21.8} = 0.037\angle 23.2 \qquad i(t) = 0.037 \cos(200t + 23.2)$$

j20Ω   50Ω

$$V_S = 2\angle 45$$
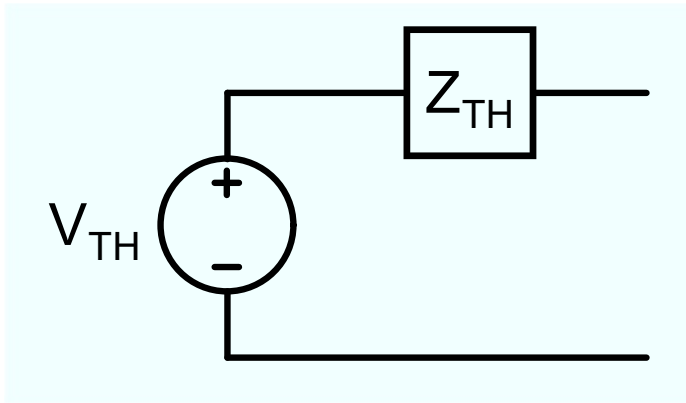
$$V_R = 2\angle 45 \times \frac{50}{50 + j20}$$

Concept of voltage or current division can be used as before

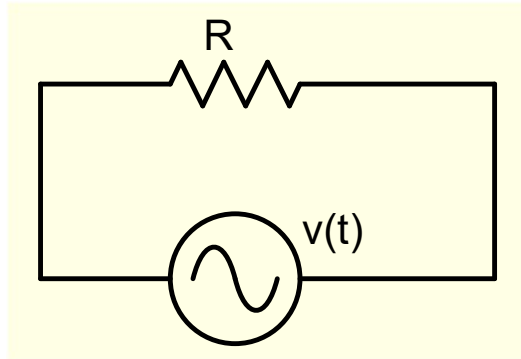# Thevenin and Norton equivalent circuit

Thevenin voltage is a phasor and Thevenin impedance is in general a complex impedance
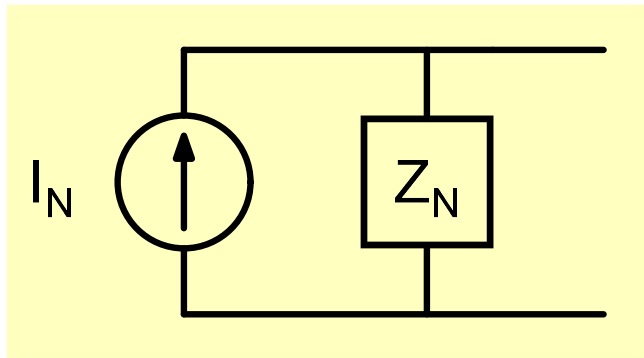
Similarly Norton current is a phasor and impedances are complex numbers.

Power dissipation with sinusoidal Voltage

$$p = \frac{v(t)^2}{R}$$

$$p_{avg} = \frac{1}{T} \int_0^T \frac{v(t)^2}{R} \, dt$$
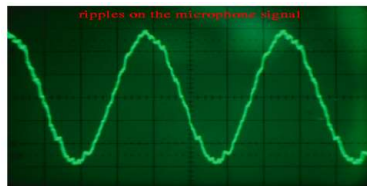
Average

X: $x_1, x_2, x_3, \ldots\ldots\ldots x_N$

$$x_{avg} = \frac{1}{N}\sum x_i$$

If X is continuous,, then its average over a time t_1

$$x_{avg} = \frac{1}{t_1}\int_0^{t_1} x(t)\, dt$$

For periodic signals

ripples on the microphone signal

$$x_{avg} = \frac{1}{T}\int_0^{T} x(t)\, dt$$

Average Power

$$p_{avg} = \frac{1}{T} \int_0^T \frac{v(t)^2}{R} \, dt$$

We would like to express it like the dc power dissipated in a resistor $\quad p = \frac{V^2}{R}$

$$p_{avg} = \frac{[\sqrt{\frac{1}{T} \int_0^T v(t)^2 \, dt}]^2}{R}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 \, dt} \qquad\qquad p_{avg} = \frac{V_{rms}^2}{R}$$

This is true for any periodic waveform

RMS Value of a Sinusoid

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 \, dt}$$

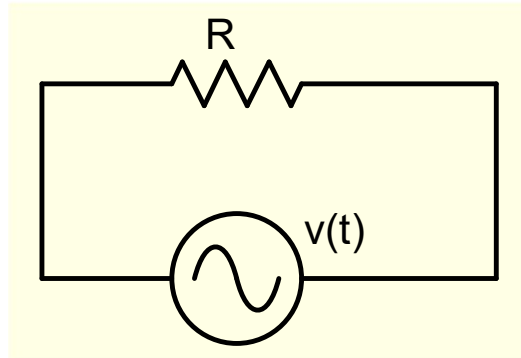$$v(t) = V_m \cos(\omega t + \theta)$$

$$\int_0^T \cos^2(\omega t + \theta) \, dt = \int_0^T \frac{1 - \cos(2\omega t + 2\theta)}{2} \, dt$$

$$= 0.5T - \frac{1}{4\omega} \sin(2\omega t + 2\theta) \Big|_0^T = 0.5T$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Power dissipation with sinusoidal Voltage

$$v(t) = V_m \cos(\omega t + \theta)$$

$$p_{avg} = \frac{V_{rms}^2}{R}$$

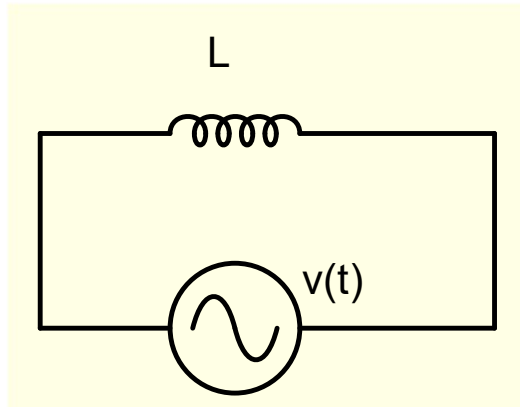$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$p_{avg} = \frac{V_m^2}{2R}$$

$$i(t) = I_m \cos(\omega t + \theta)$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

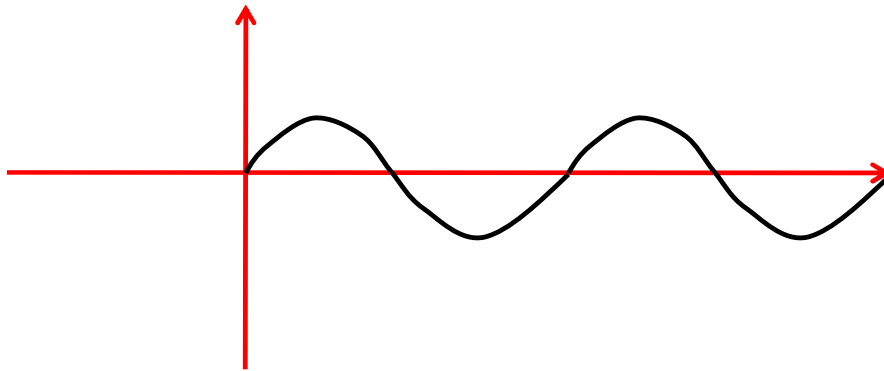$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i(t)^2 \, dt}$$

$$p_{avg} = 0.5 I_m^2 R$$

L

$$v(t) = V_m \cos(\omega t) \qquad i(t) = I_m \cos(\omega t - 90°) = I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t)\sin(\omega t)$$

$$= \frac{V_m I_m}{2}\sin(2\omega t) + \frac{V_m I_m}{2}\sin(0)$$

$$= \frac{V_m I_m}{2}\sin(2\omega t)$$

v(t)

$$p_{avg} = 0$$

- Average power absorbed by
an inductor is zero

$$v(t) = V_m \cos(\omega t) \qquad i(t) = I_m \cos(\omega t + 90°) = -I_m \sin(\omega t)$$
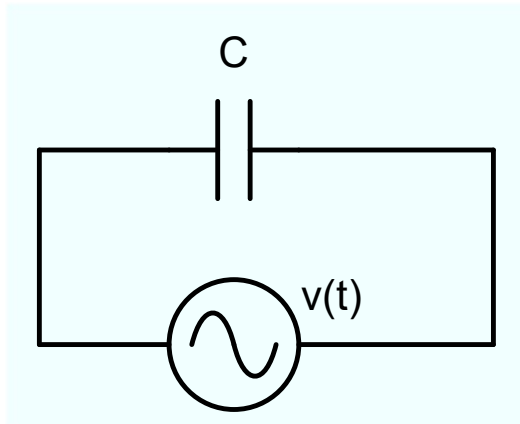
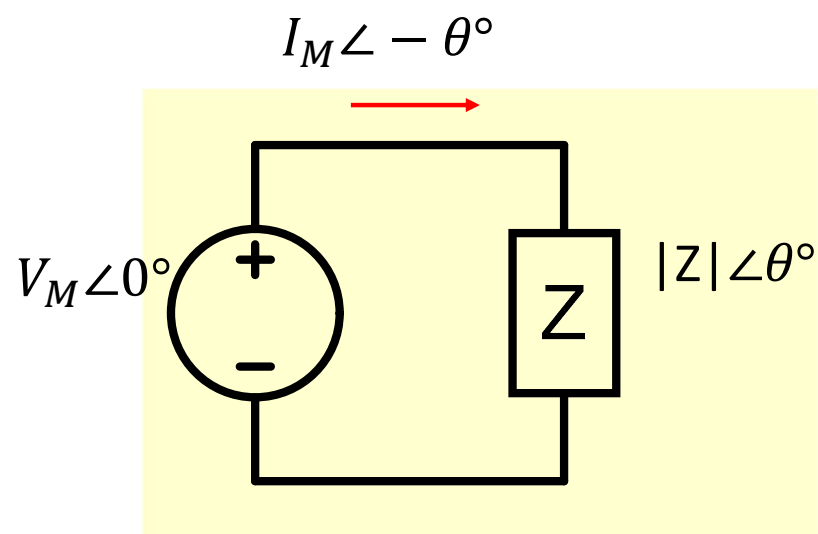$$p(t) = v(t)i(t) = -V_m I_m \cos(\omega t) \sin(\omega t)$$

$$= -\frac{V_m I_m}{2} \sin(2\omega t) - \frac{V_m I_m}{2} \sin(0)$$

$$= -\frac{V_m I_m}{2} \sin(2\omega t) \qquad p_{avg} = 0$$

- Average power absorbed by
  a capacitor is zero

General Rule

$$v(t) = V_m \, Cos(\omega t)$$

$$i(t) = I_m \, Cos(\omega t - \theta)$$

$I_M \angle -\theta°$

$V_M \angle 0°$ +   Z $|Z| \angle \theta°$ −

Average Power : $p = \dfrac{1}{T} \displaystyle\int_0^T v(t) \times i(t)\, dt$

$$P = V_{rms} I_{rms} \cos\theta$$
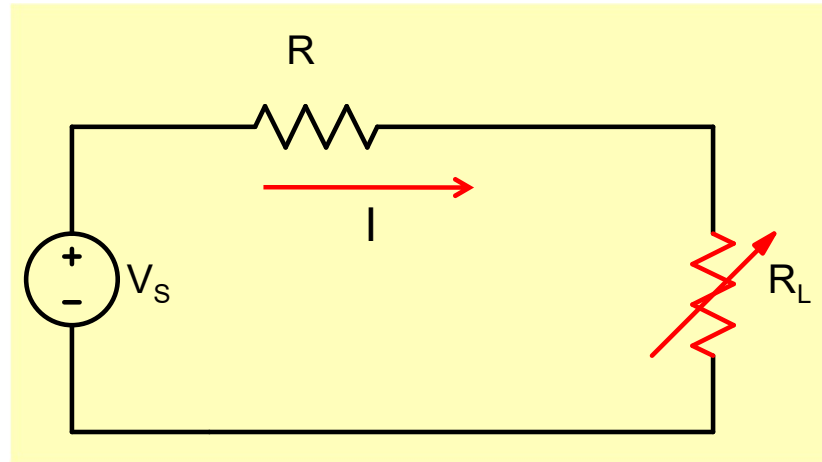
Power Factor $PF = \cos\theta$

For a resistor PF = 1, while for inductor and capacitor it is 0

$$j\omega L = \omega L \angle 90 \;;\; -j\frac{1}{\omega C} = \frac{1}{\omega C} \angle -90$$

Current in phase with voltage gives rise to power dissipation

Maximum Power Transfer for dc circuits

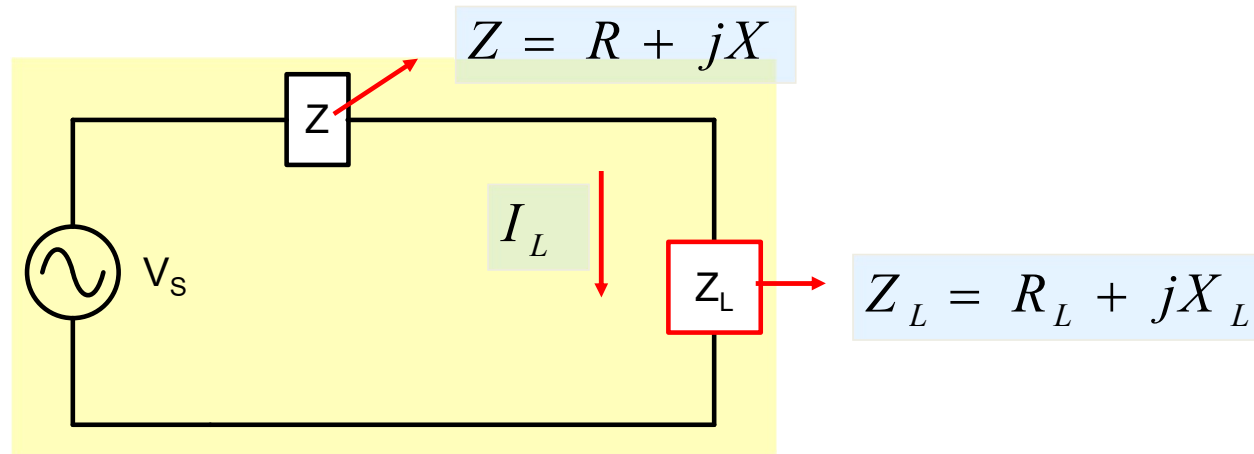What value of R_L will give rise to maximum load power ?

$$I = \frac{V_S}{R + R_L}$$

$$P_L = I^2 R_L = V_S^2 \times \frac{R_L}{(R + R_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0$$

$$R_L = R$$

$$P_{L\,max} = \frac{V_S^2}{4 R_L}$$

Maximum Power Transfer for sinusoidal input

$$Z = R + jX$$

$$Z_L = R_L + jX_L$$

$$I_L = \frac{V_S}{R + R_L + j(X + X_L)}$$

$$P_L = \frac{\frac{V_S^2}{2}}{(R + R_L)^2 + (X + X_L)^2} R_L$$

For maximum load power : $X_L = -X$
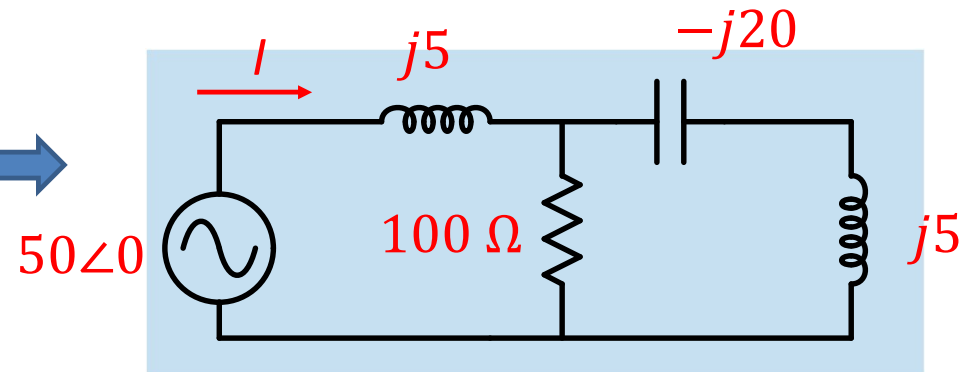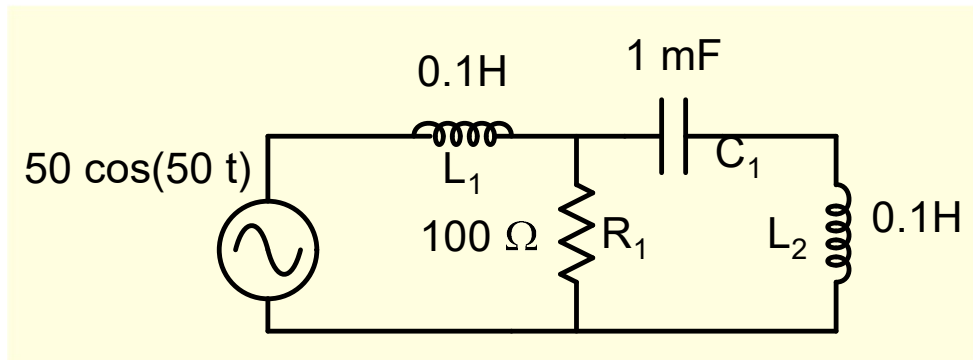
$$P_L = 0.5 \times \frac{V_S^2}{(R + R_L)^2} R_L$$

Choose $R_L = R$ to maximize load power

$$Z_L = \overline{Z}$$

Maximum power is transferred to the load when load is complex conjugate of source impedance

Example-3

Determine all you can about the given circuit

$$\omega = 50$$

$$Z = j5 + 100\|(-j15) = 2.2 - j9.67 \qquad I = \frac{50}{Z} = 1.12 + j4.96 = 5\angle 77.2$$

$$P_S = \frac{50}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \cos(77.2) = 27.97W$$

Voltage across R1 ?

$$V_{R1} = I \times (100\|(-j15)) = 74.8\angle -4.3$$

$$\frac{V_{R1}^2}{2R_1} = 27.97W$$