ESC201T : Introduction to Electronics

Lecture 15: Frequency Response

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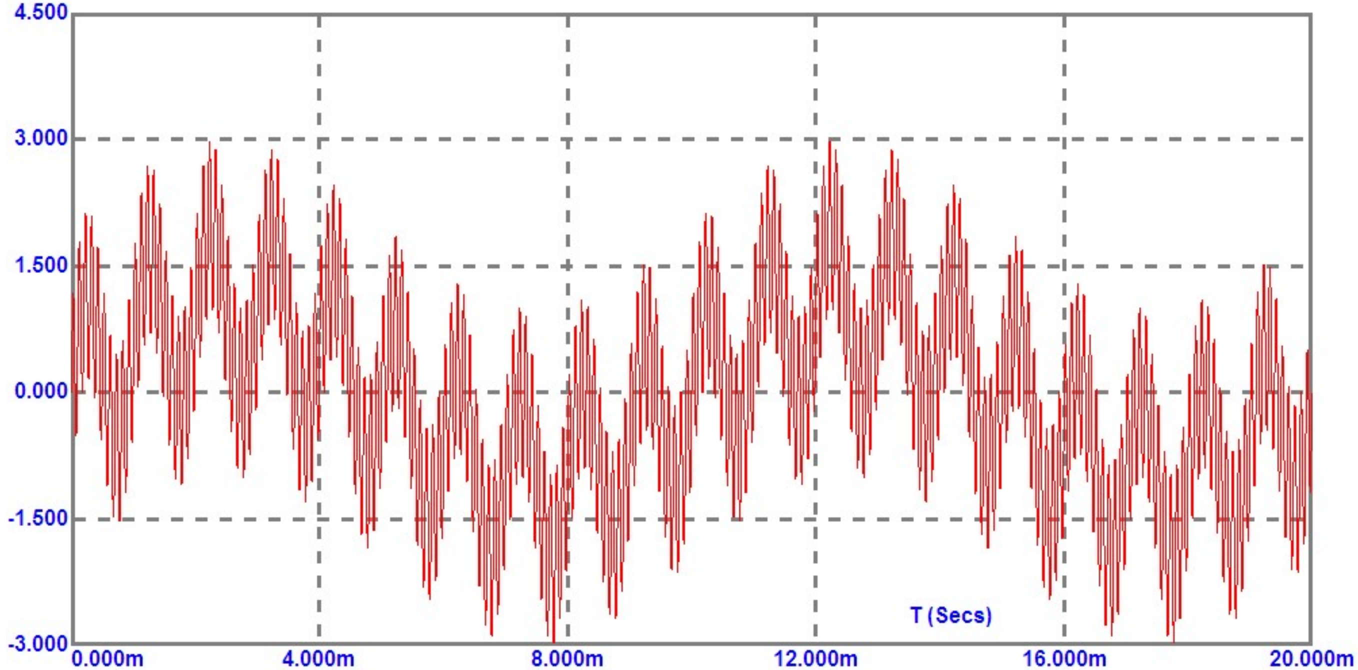
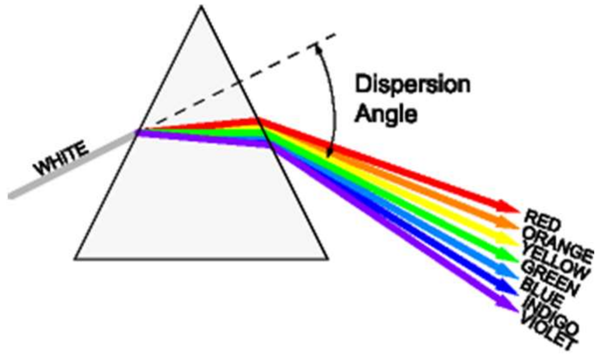
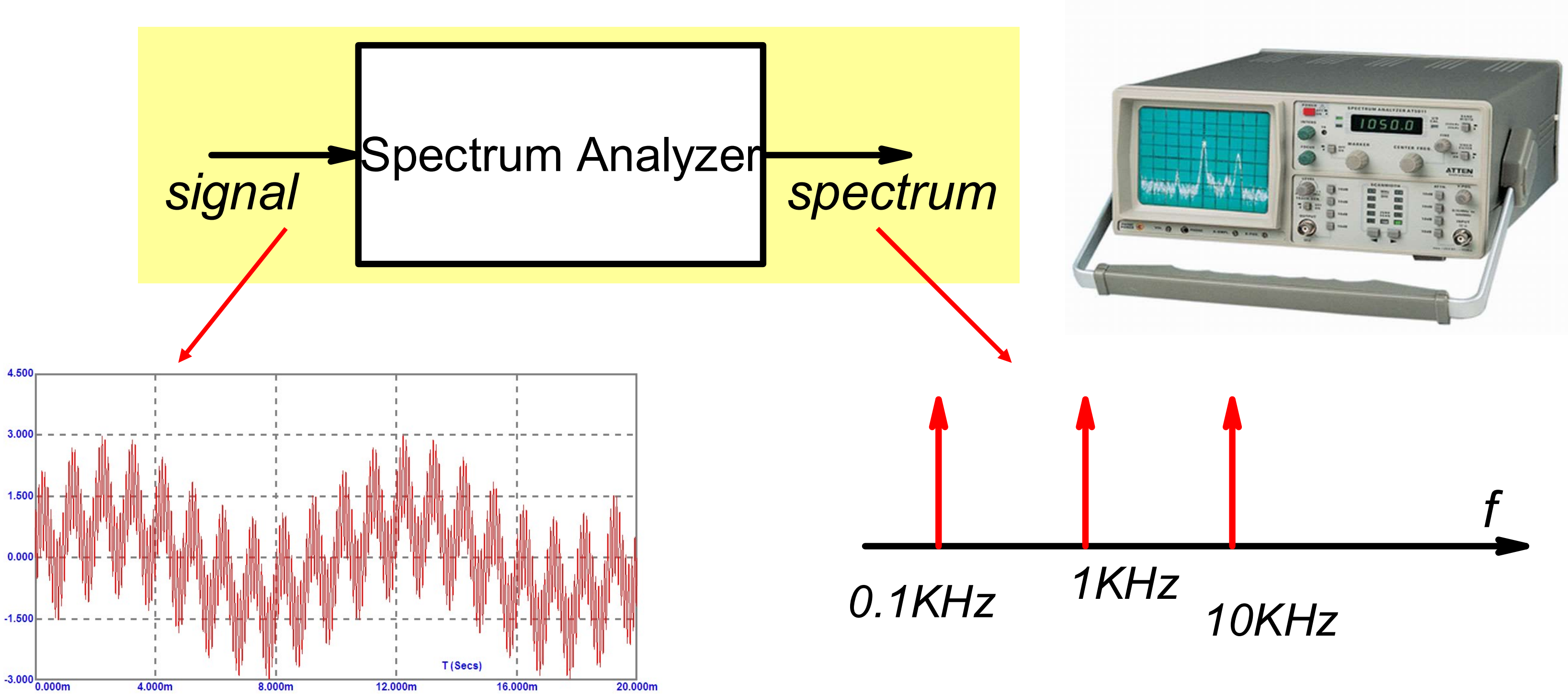
Time domain vs. Frequency domain analysis

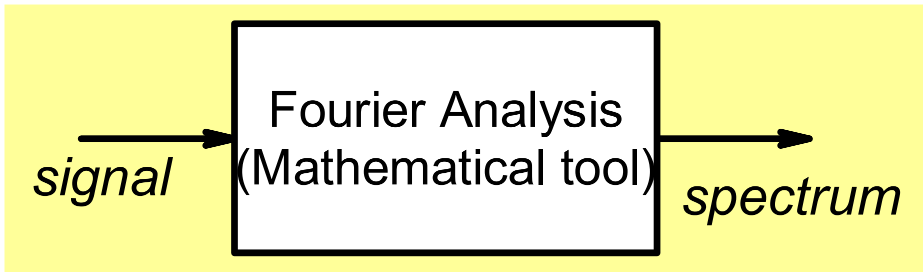
Signal





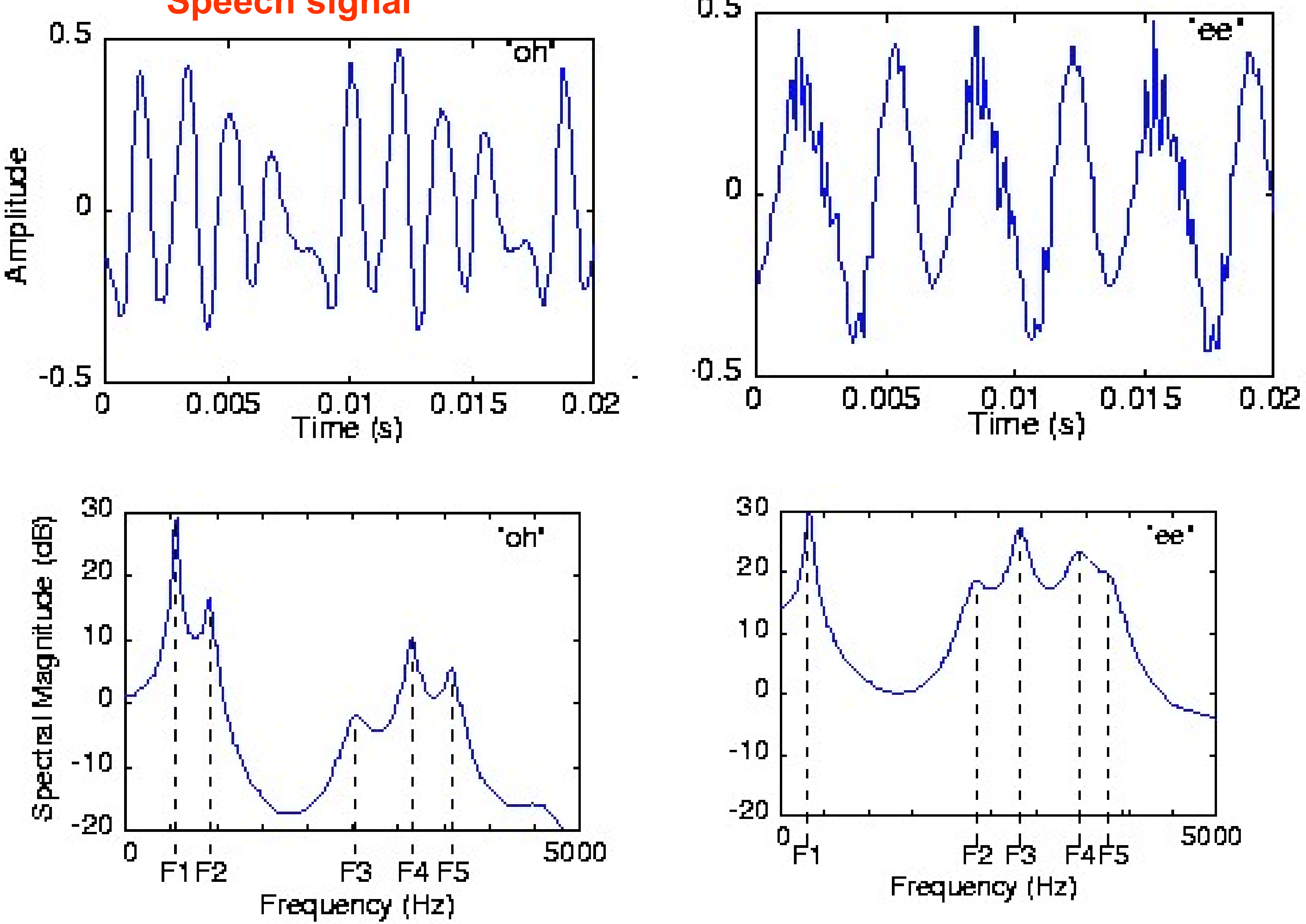


signal
Spectrum Analyzer
spectrum
4.500
3.000
1.500
0.000
-1.500
-3.000
0.000m
4.000m
8.000m
12.000m
16.000m
20.000m
T (Secs)
f
0.1KHz
1KHz
10KHz



signal
Fourier Analysis
(Mathematical tool)
spectrum

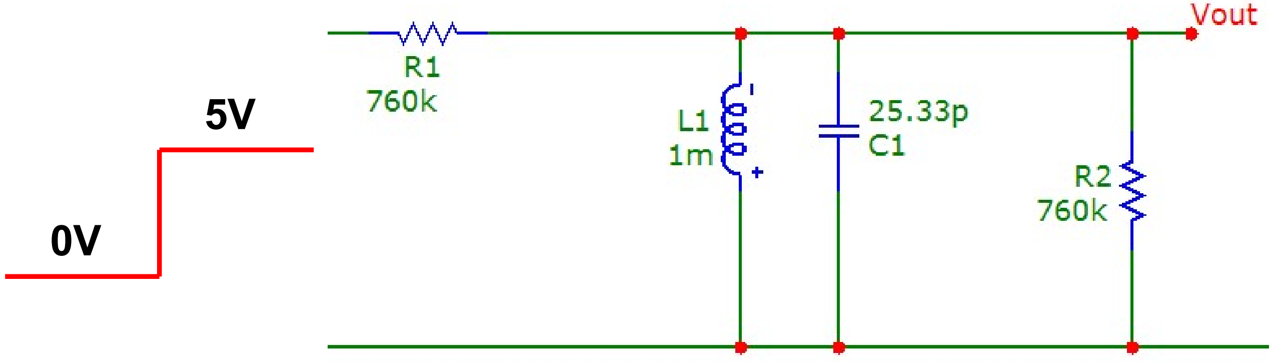
Speech signal

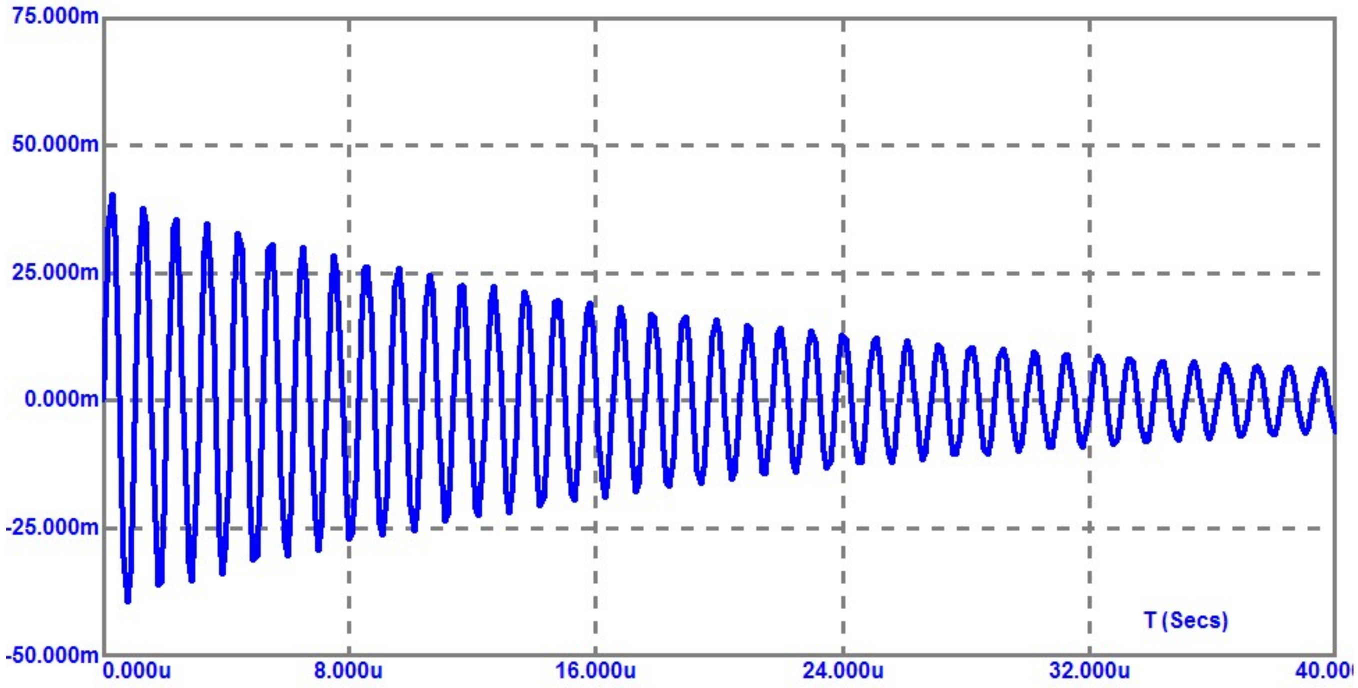


Don Johnson

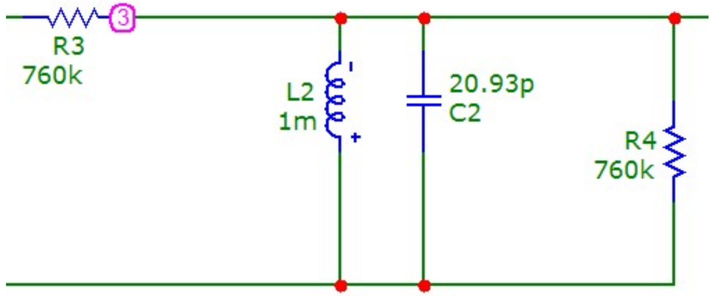
System

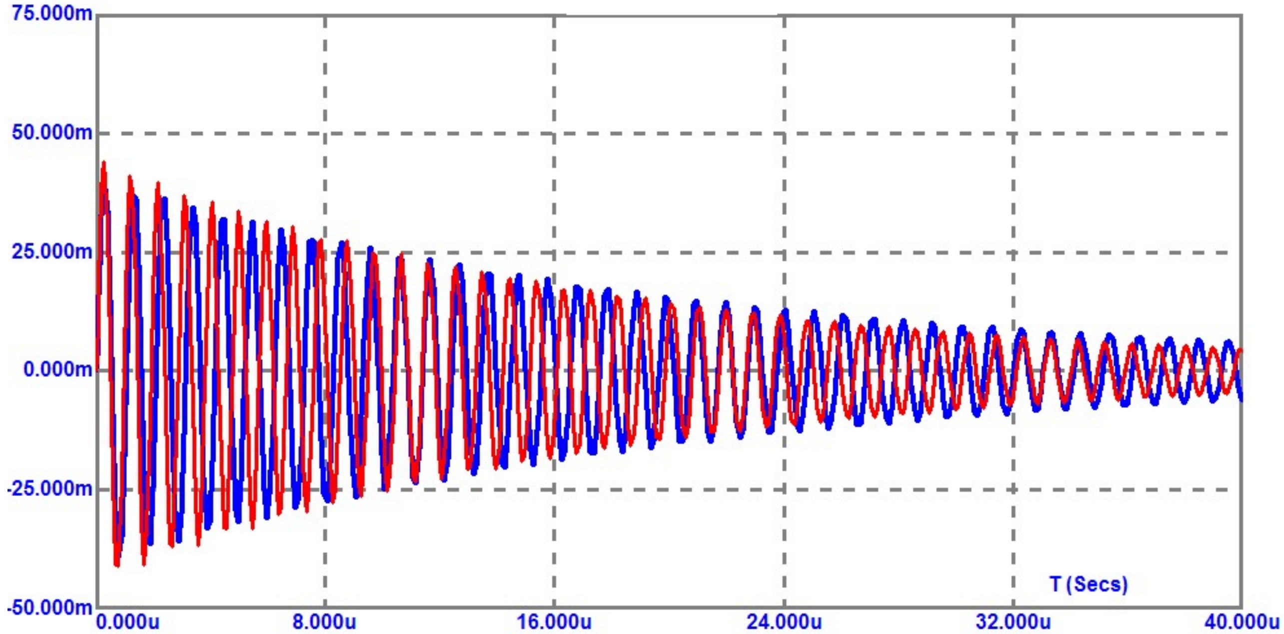
What does this circuit do ?





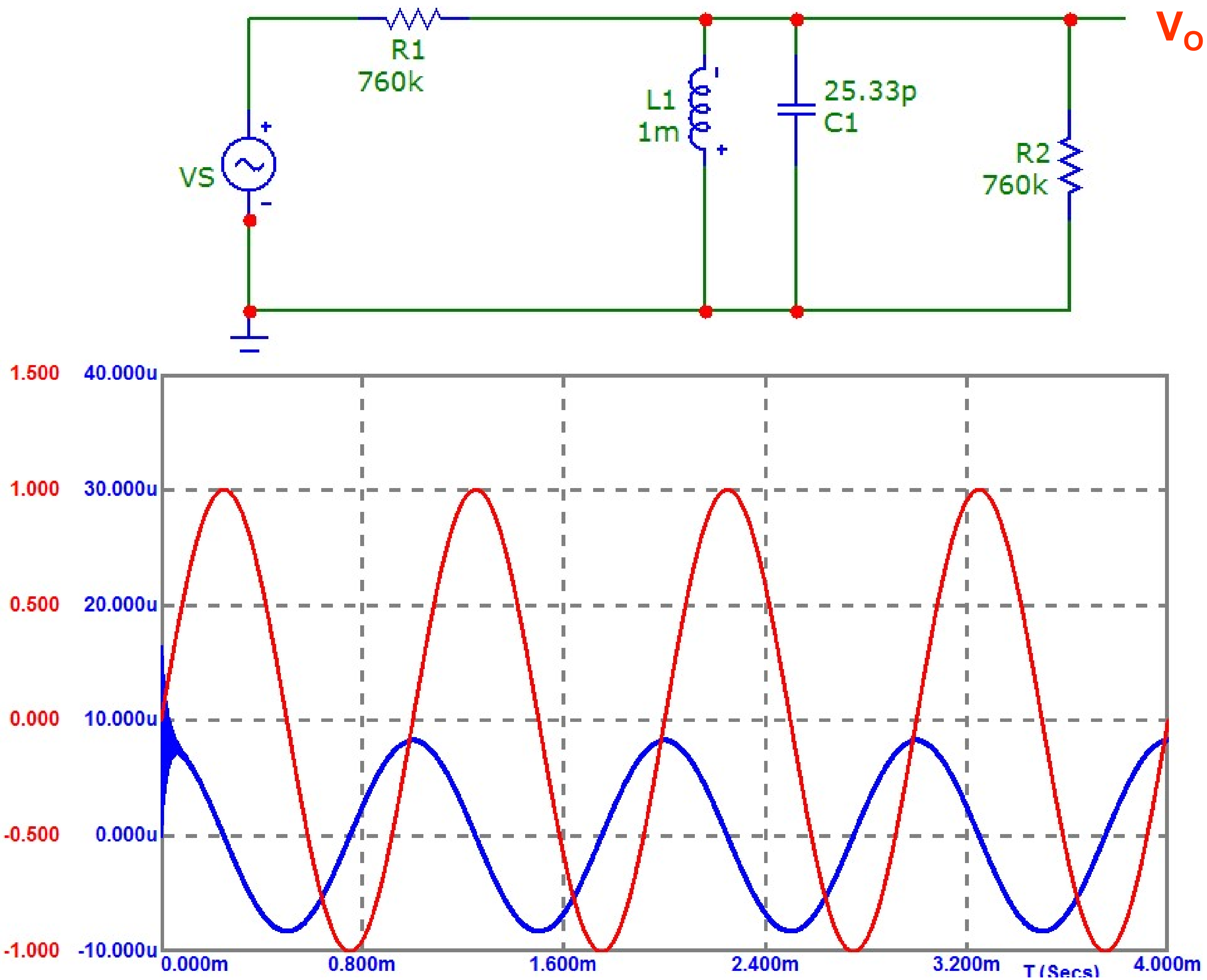
Suppose the capacitor is reduced to ~21pF.



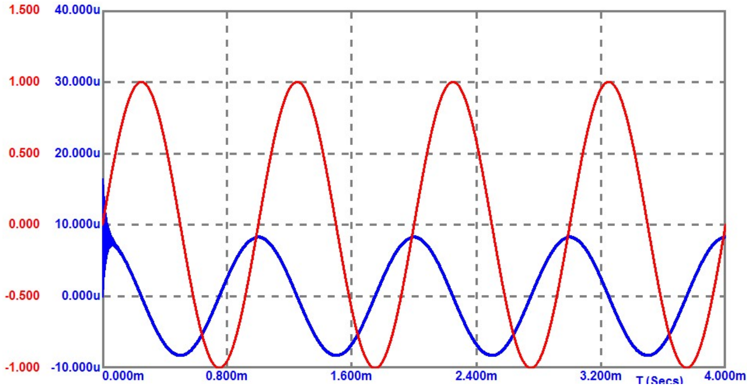


It is hard to find out what impact the change in capacitor has on circuit behavior

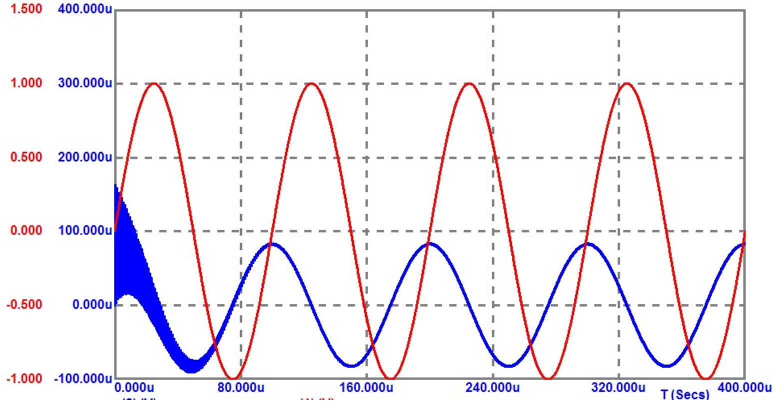
Frequency domain analysis



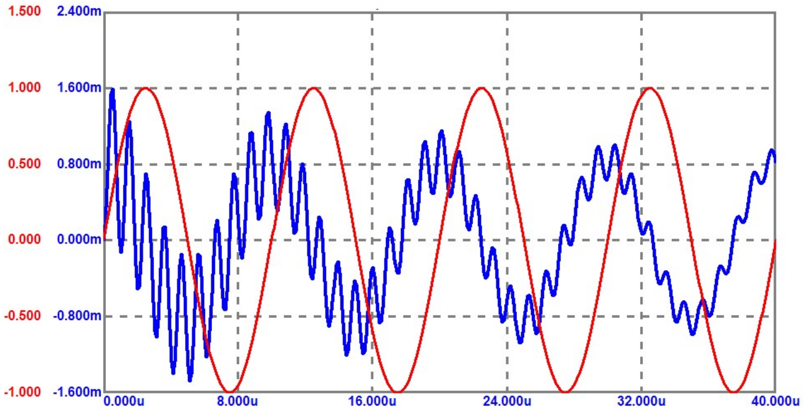
Measure response at many different frequencies for a constant input amplitude



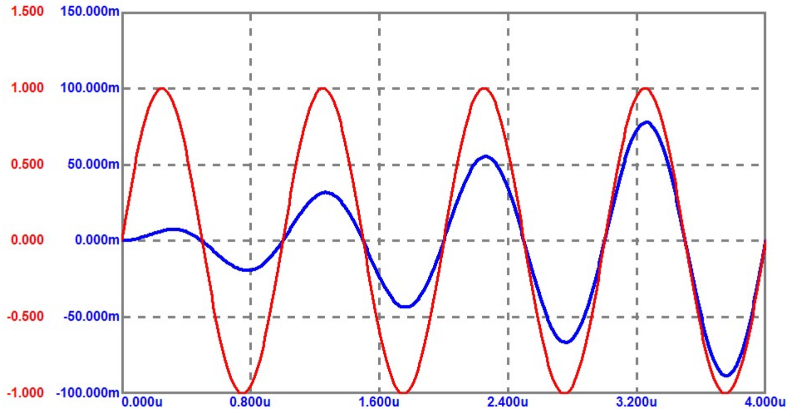
f=1KHz



f=10KHz

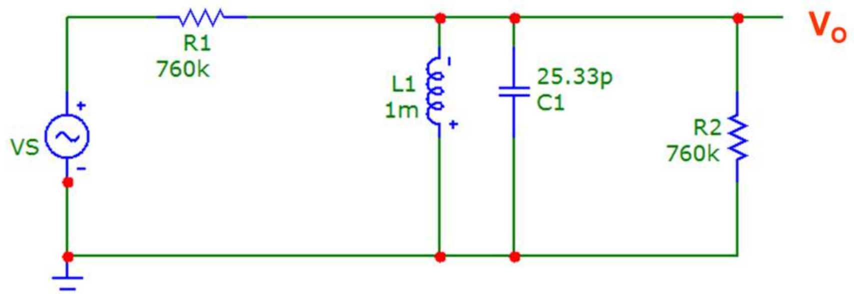


f=100KHz

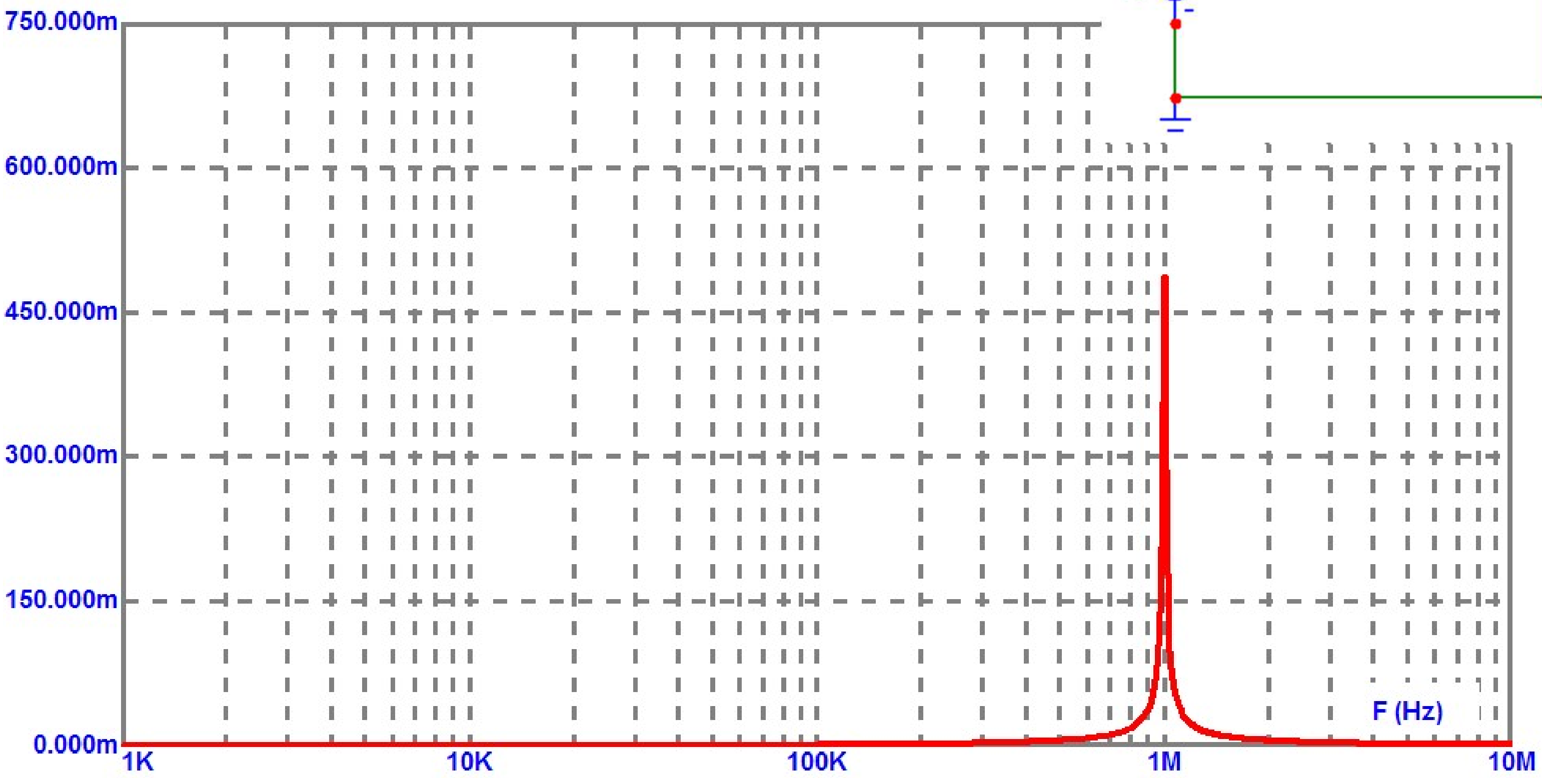


f=1000KHz

Plot the amplitude and phase as a function of frequency

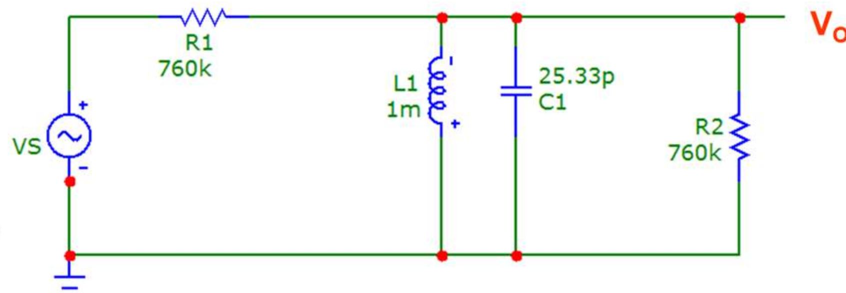


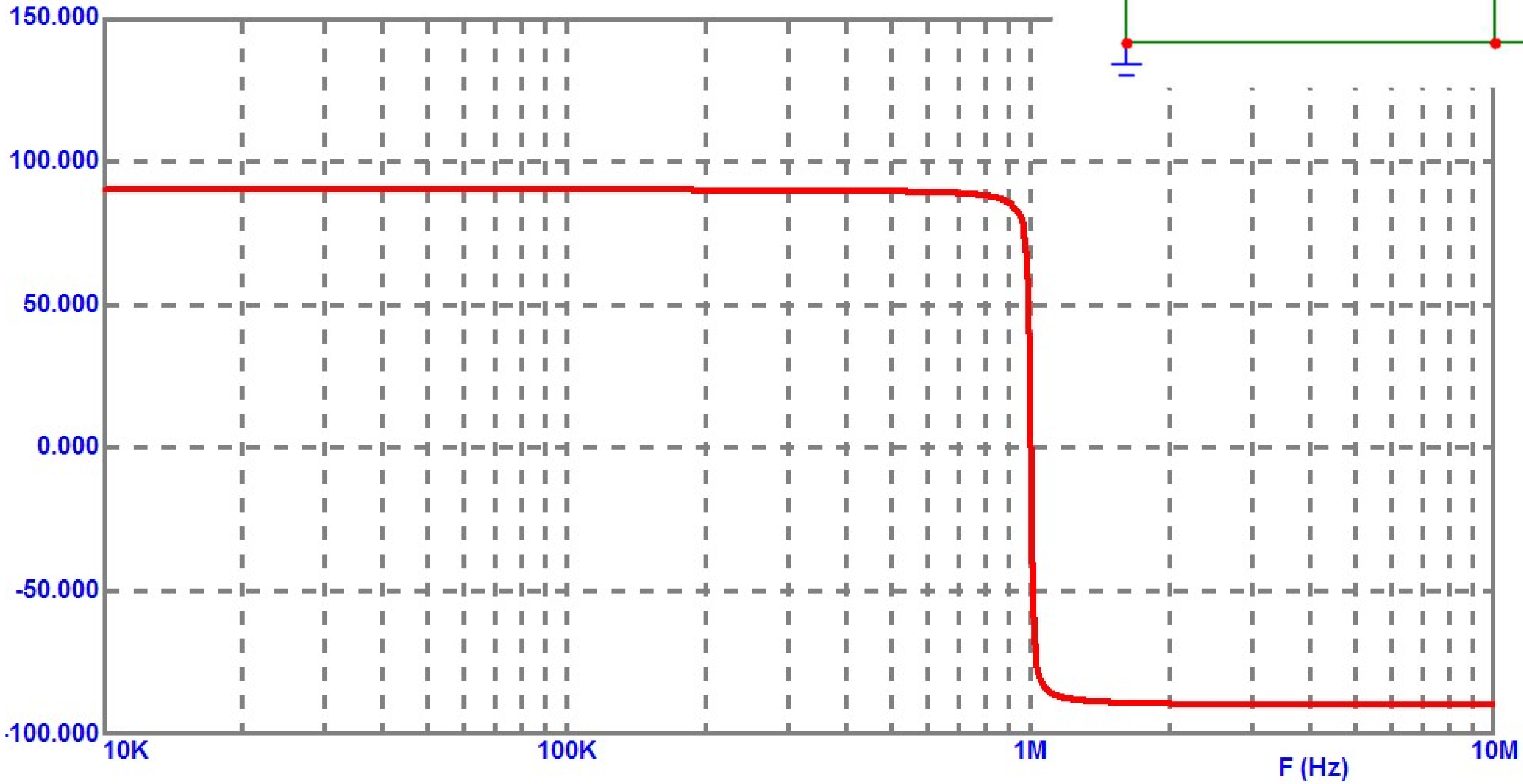
Amplitude as a function of frequency



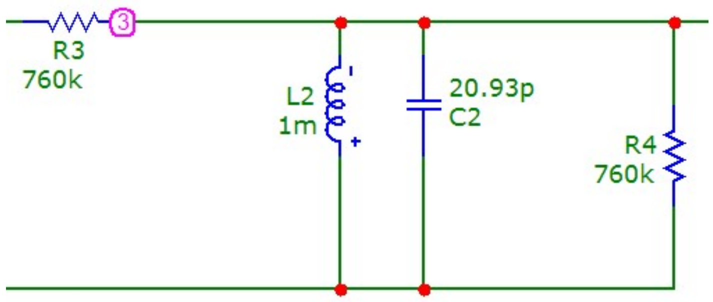
One can clearly see the frequency selective (often called a filter) nature of the circuit

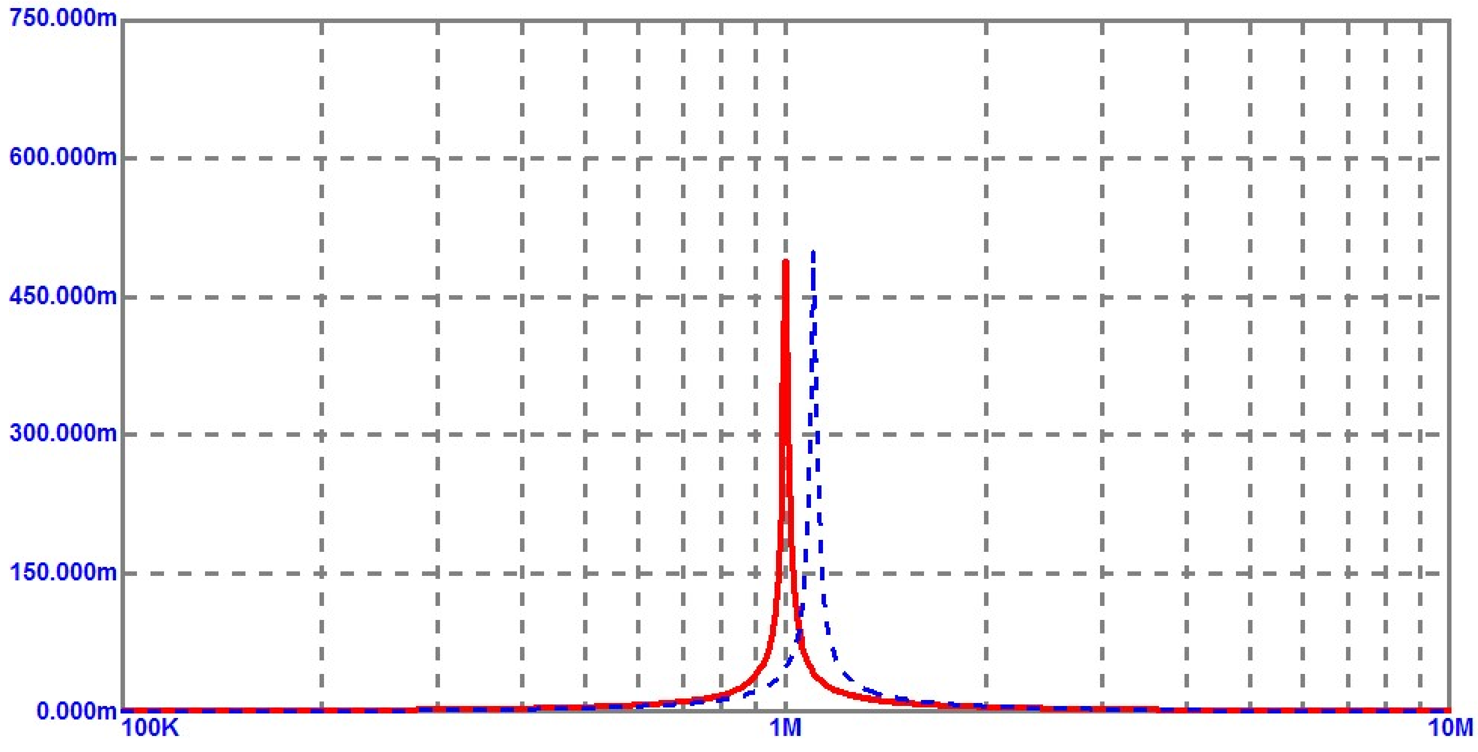
Phase as a function of frequency



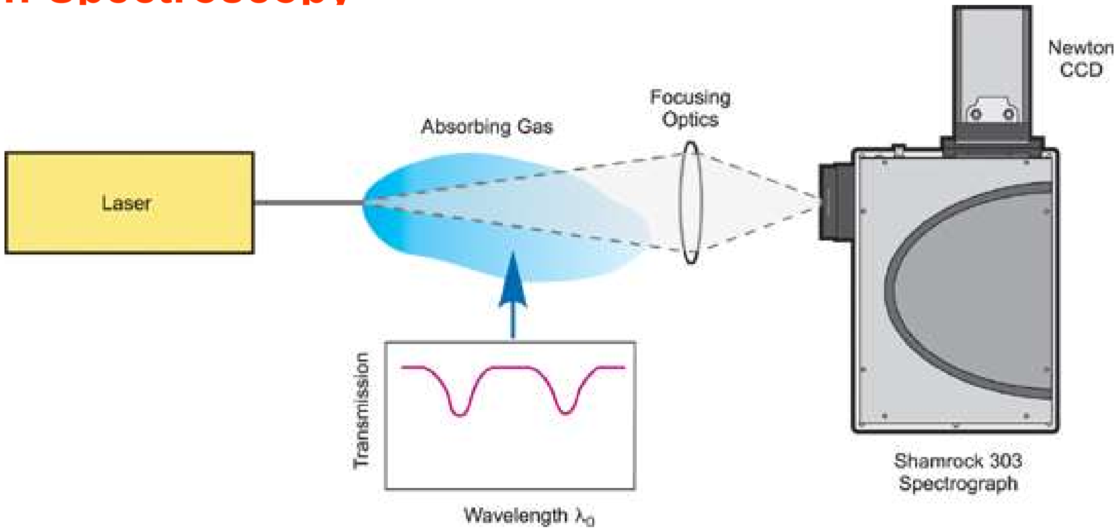


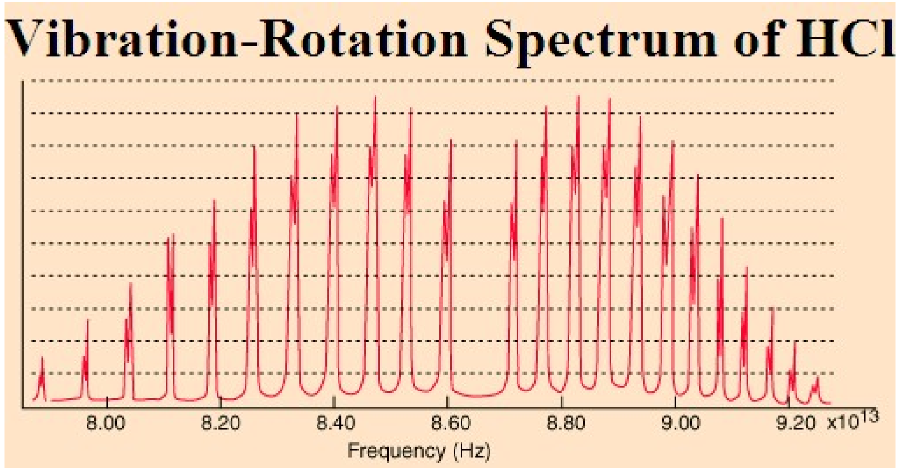
Suppose the capacitor is reduced to ~21pF.





Absorption Spectroscopy



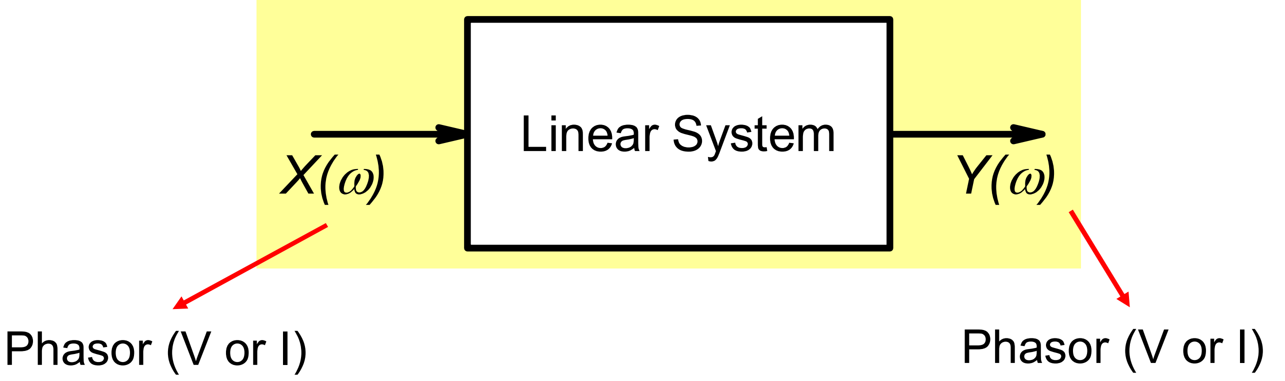


Bond length can be estimated from the spectra

Analysis of signals and systems in frequency domain often provides useful insight into their behavior.

Frequency domain analysis

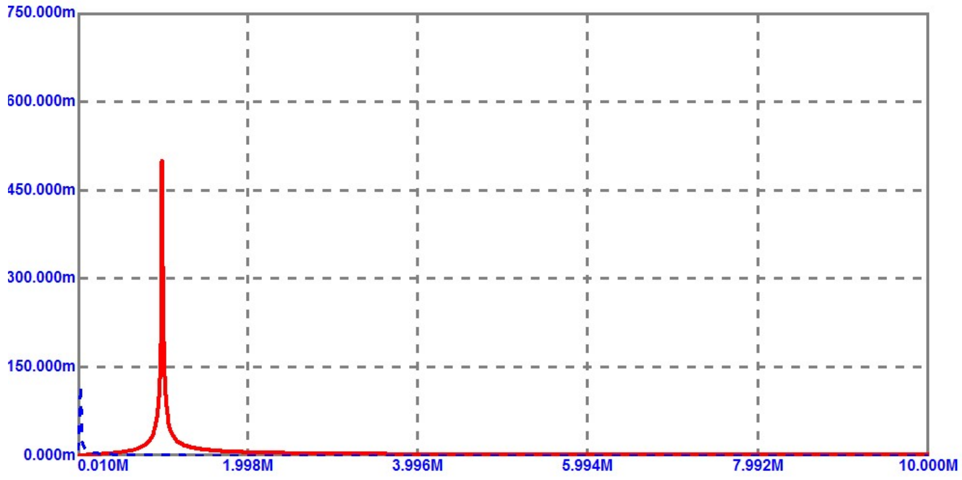
Transfer function is a useful tool for finding the frequency response of a system

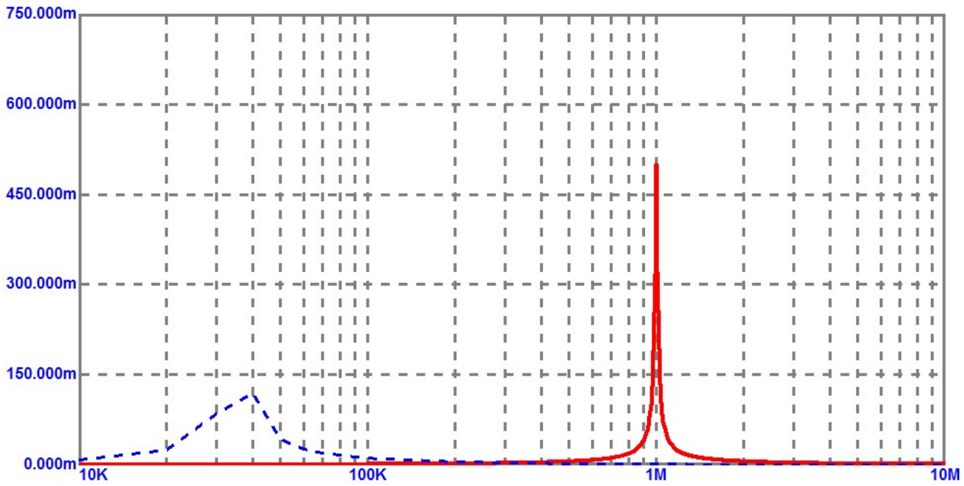


$$\text{Transfer Function:} \quad H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

Transfer function has a magnitude and a phase

Because of the wide dynamic range of frequency, plotting frequency on log axis is often more revealing !





Insight is a strong function of information representation

The magnitude of transfer function is often specified in **decibels**

$$G_{dB} = 10\log_{10}\left(\frac{P_2}{P_1}\right)$$

Because power is proportional to V^2 or I^2, voltage gain and current gain in decibels is specified as

$$G_{dB} = 20\log_{10}\left(\frac{V_2}{V_1}\right) \qquad G_{dB} = 20\log_{10}\left(\frac{I_2}{I_1}\right)$$

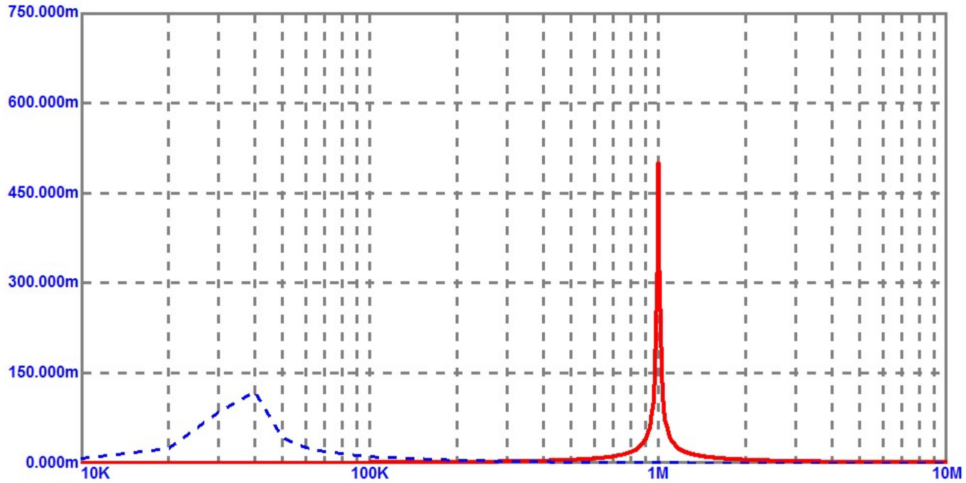
Decibel scale is more convenient for our perception of hearing

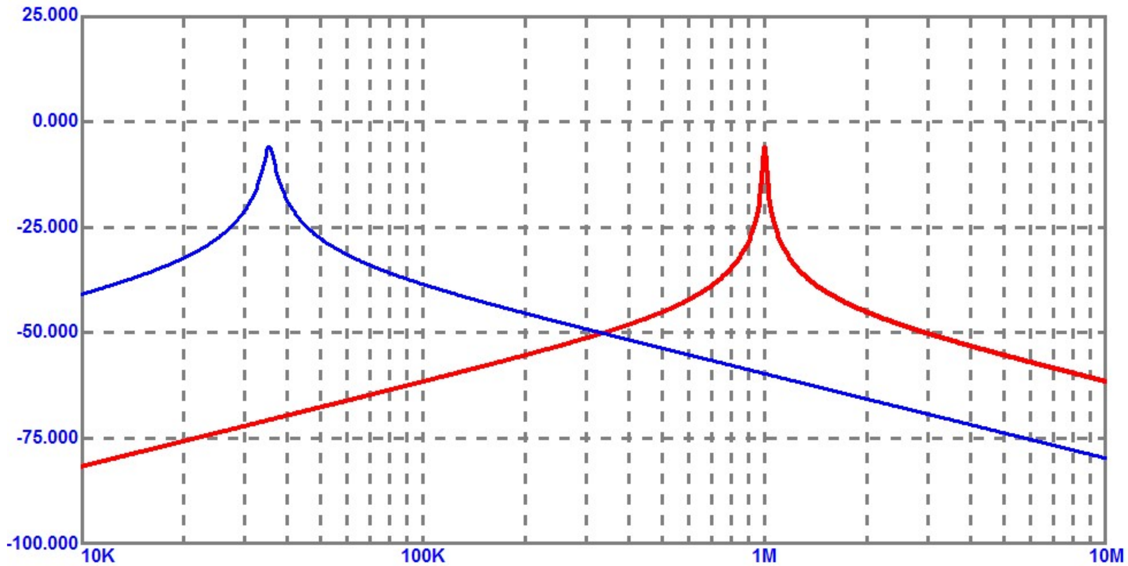
Change in Sound Pressure Level	Apparent Change in Loudness
3 dB	Just noticeable
5 dB	Clearly noticeable
10 dB	Twice or half as loud
20 dB	4 times or 1/4 as loud

Decibel Scale

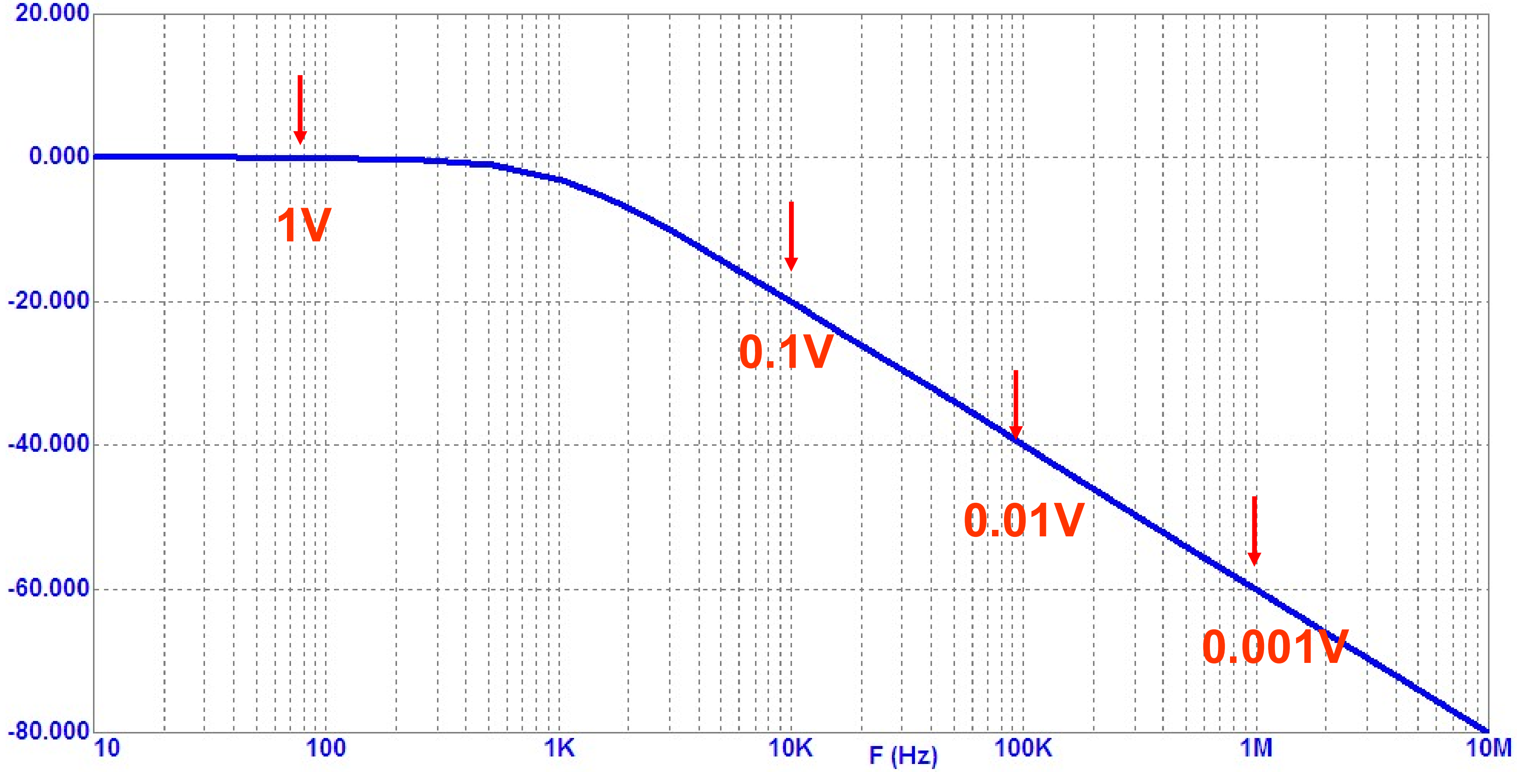
G	$20Log_{10}(G)$
1000	60
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	-3
0.5	-6
0.1	-20
0.01	-40

Decibel scale often reveals more information about behavior





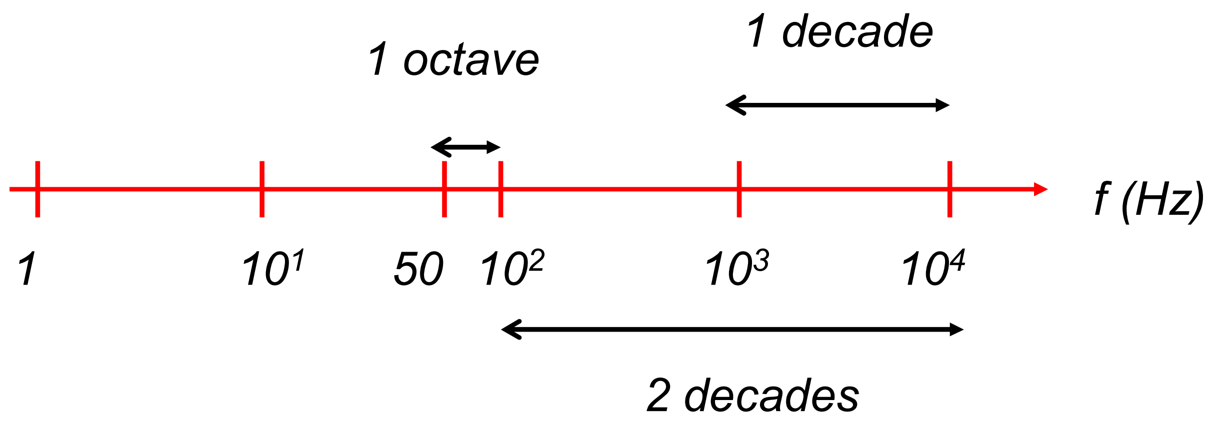
dB Scale



G	$20Log_{10}(G)$
1000	60
100	40
10	20
2	6
√2	3
1	0
1/√2	-3
0.5	-6
0.1	-20
0.01	-40

A plot of the decibel magnitude of transfer function versus frequency using a logarihmic scale for frequency is called a **Bode plot**

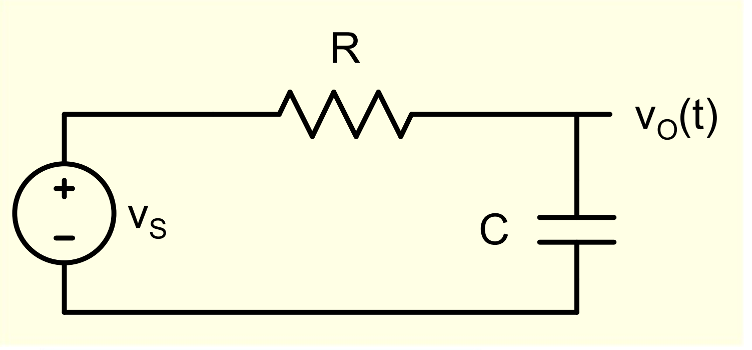
Logarithmic frequency scale



$$\text{No. of decades} = \log\left(\frac{f_2}{f_1}\right)$$

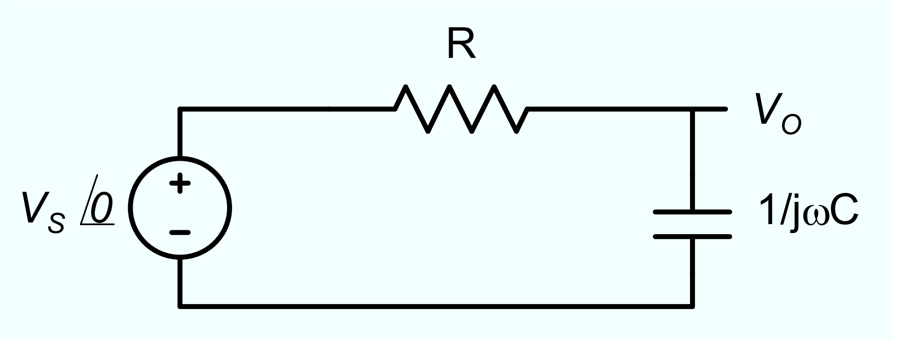
$$\text{No. of octaves} = \log_2\left(\frac{f_2}{f_1}\right) = \frac{\log\left(\frac{f_2}{f_1}\right)}{\log(2)}$$

How do we determine transfer function?



$$v_S\ (t) = v_{S_o} Cos(\omega t)$$

$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$



$$H(\omega) = \frac{1}{1 + j\omega CR}$$

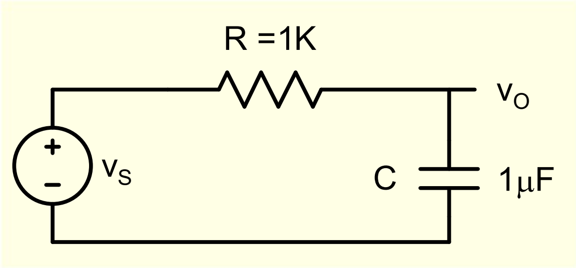
$$|H(\omega)| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

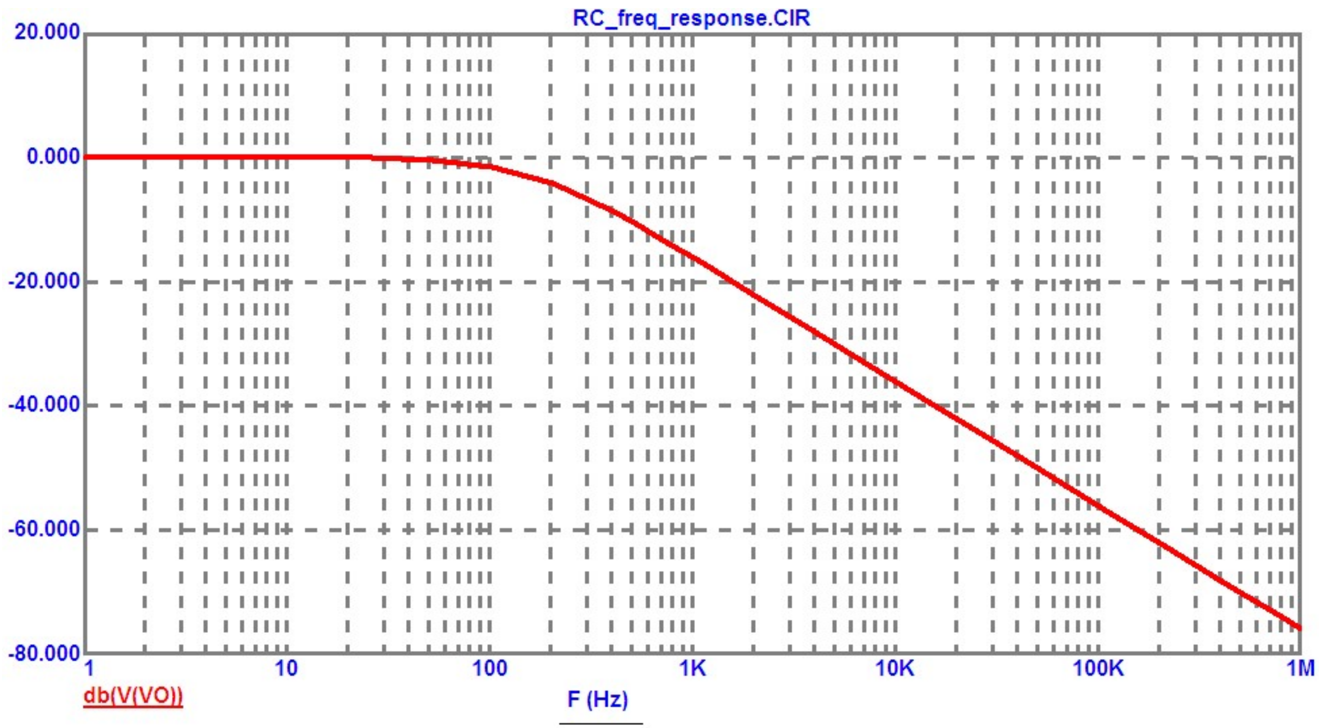
$$\phi(\omega) = -\tan^{-1}(\omega CR)$$

Magnitude plot

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$$

$$20\mathrm{Log}_{10}(|H(\omega)|) = -10 Log_{10}(1+\omega^2 C^2 R^2)$$

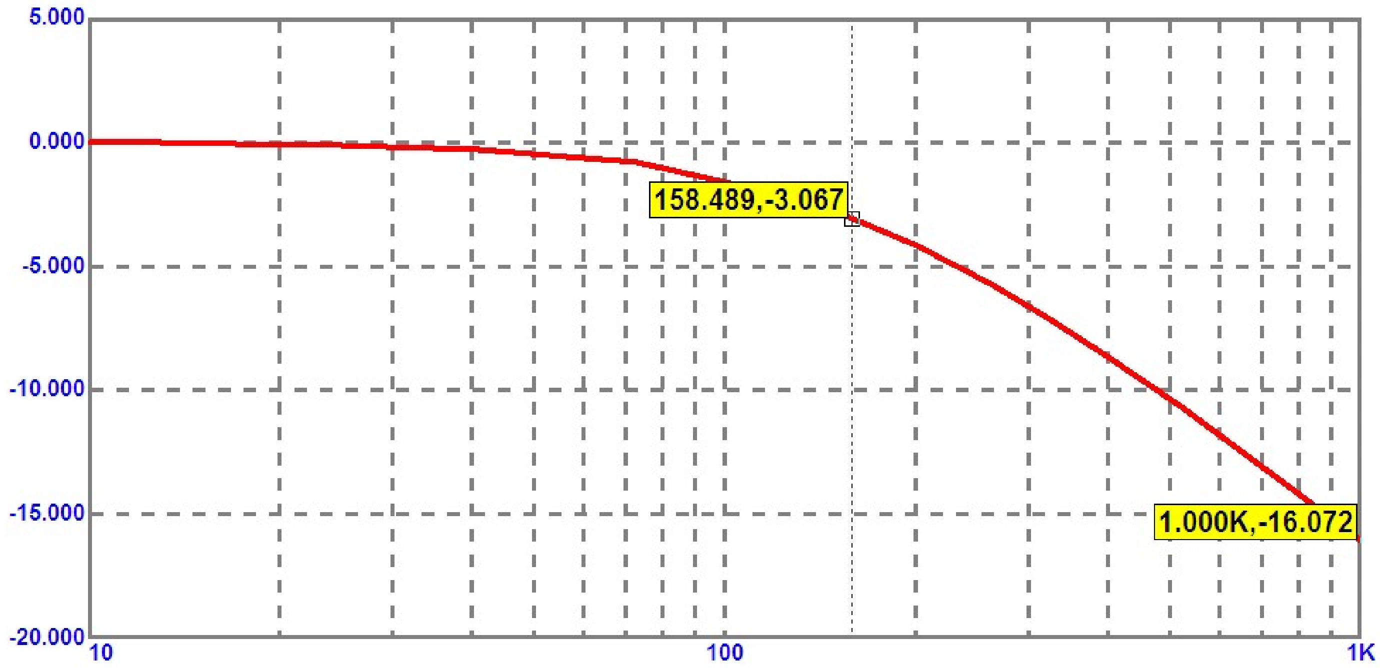




$$20\mathrm{Log}_{10}(|H(\omega)|) = -10Log_{10}(1+\omega^2C^2R^2)$$
$$= -10Log_{10}(1+\frac{\omega^2}{\omega_{3dB}^2})$$

$$\omega_{3\mathrm{dB}} = \frac{1}{\mathrm{RC}} \quad ; \; \mathrm{f}_{3\mathrm{dB}} = \frac{1}{2\pi \mathrm{RC}}$$

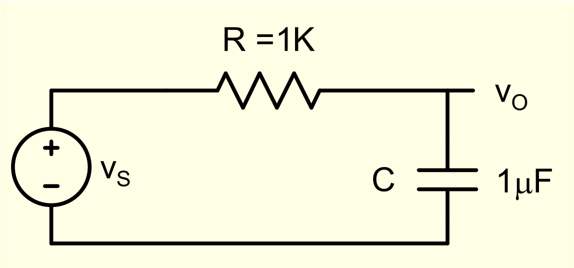
For $\omega = \omega_{3\mathrm{dB}} \;\; 20\mathrm{Log}_{10}(|H(\omega_{3dB})|) = -3dB$



$$H(\omega)=\frac{1}{1+j\omega CR}$$

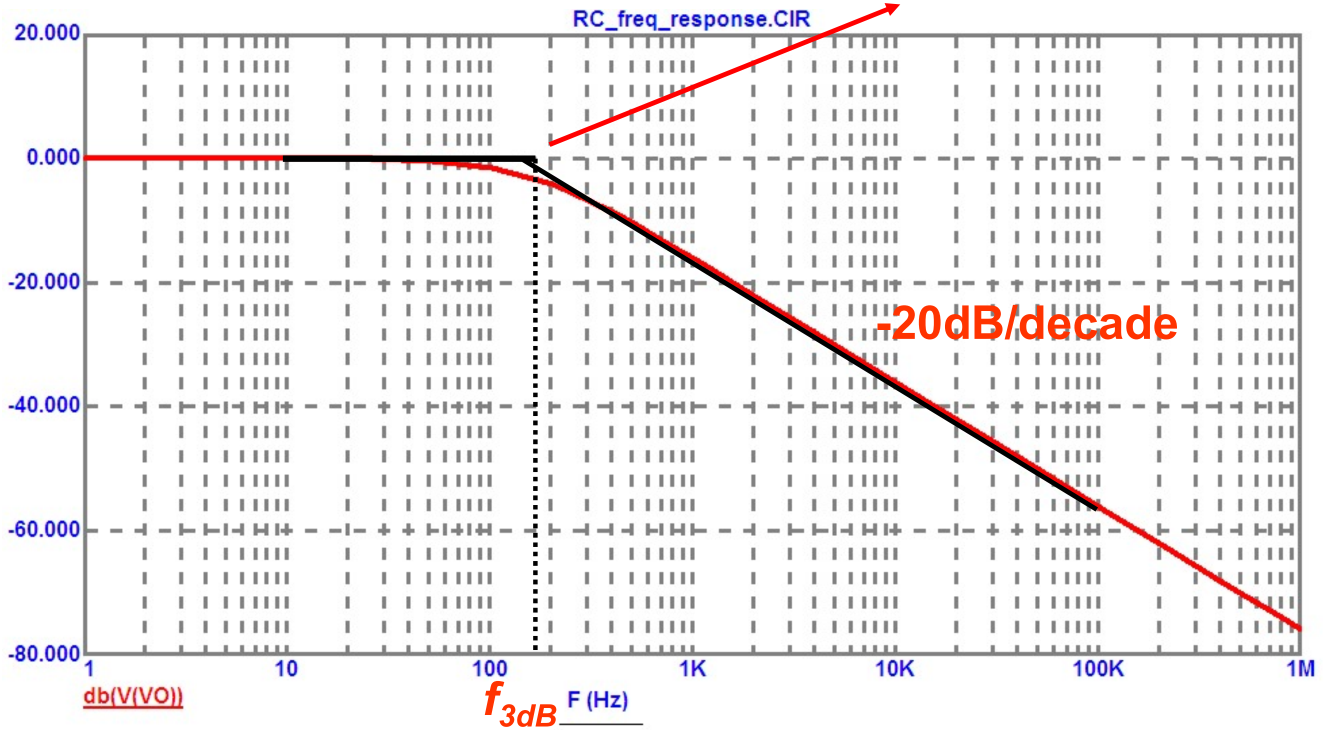
$$|H(\omega)|=\frac{1}{\sqrt{1+\omega^2C^2R^2}}$$

$$20\mathrm{Log}_{10}(|H(\omega)|)=-10Log_{10}(1+\frac{\omega^2}{\omega_{3dB}^2})$$



For $\omega >> \omega_{3\mathrm{dB}}$ $20\mathrm{Log}_{10}(|H(\omega)|)\cong -20Log_{10}(\frac{\omega}{\omega_{3dB}})$

Also called corner frequency or half power frequency



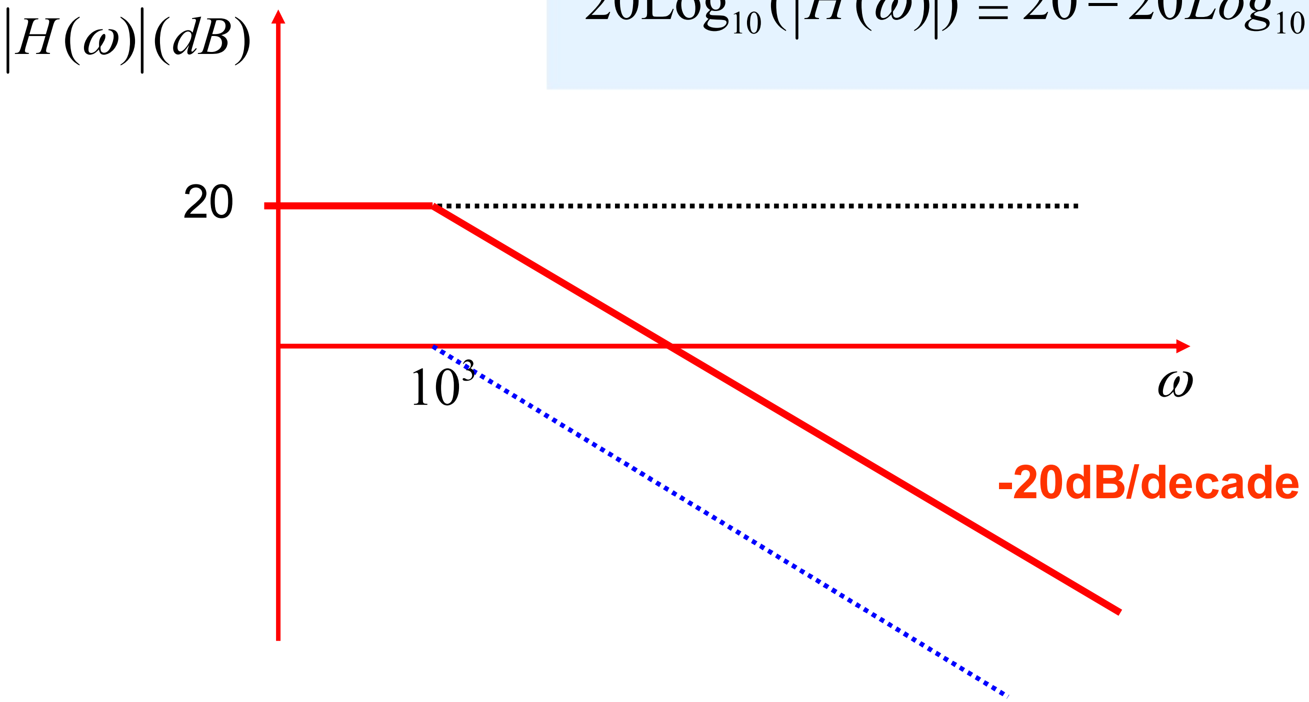
Example

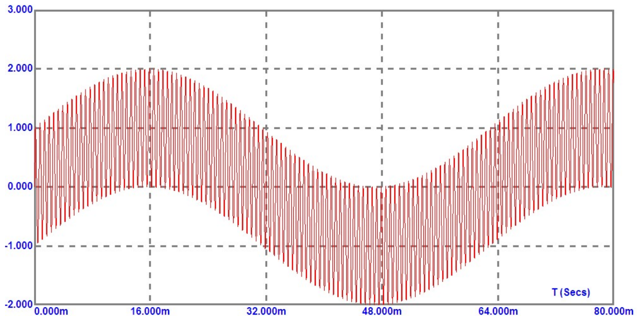
$$H(\omega)=\frac{10}{1+j\omega 10^{-3}}$$

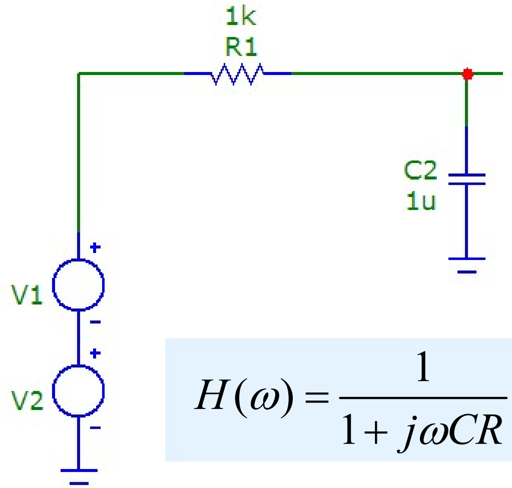
$$20\mathrm{Log}_{10}(|H(\omega)|)=20-10Log_{10}(1+\frac{\omega^2}{\omega_{3dB}^2})$$

$$\omega_{3dB}=10^3$$

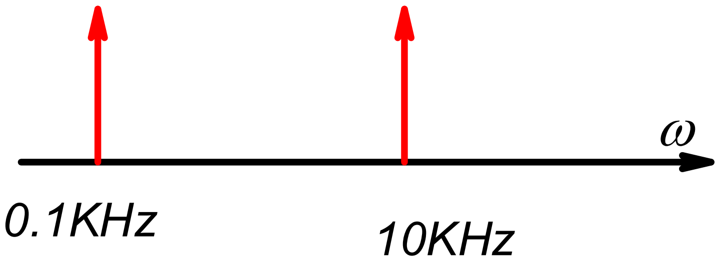
$$20\mathrm{Log}_{10}(|H(\omega)|)\cong 20-20Log_{10}(\frac{\omega}{\omega_{3dB}})$$



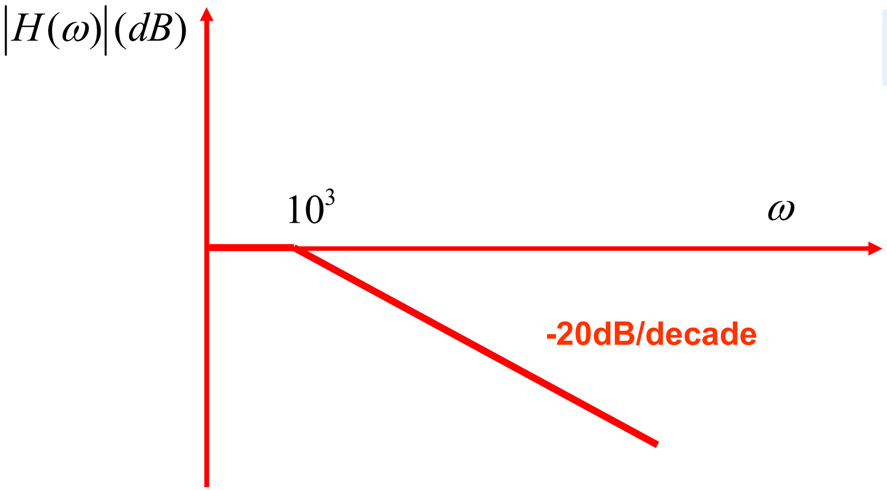




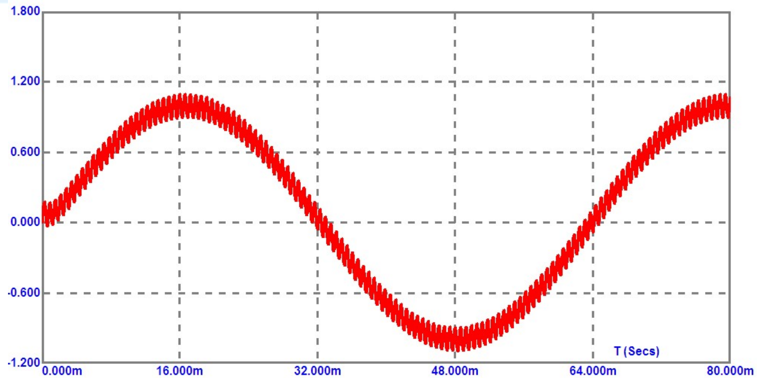
$$H(\omega)=\frac{1}{1+j\omega CR}$$

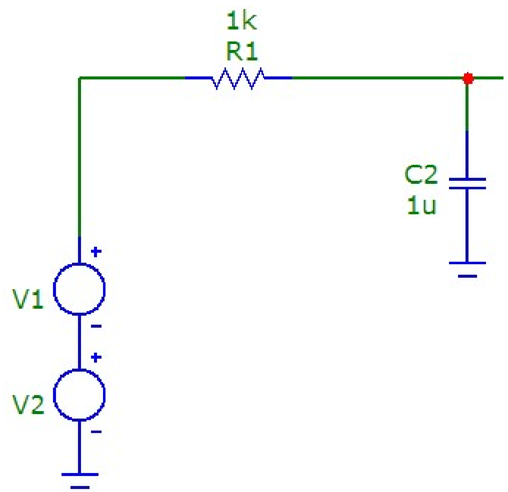


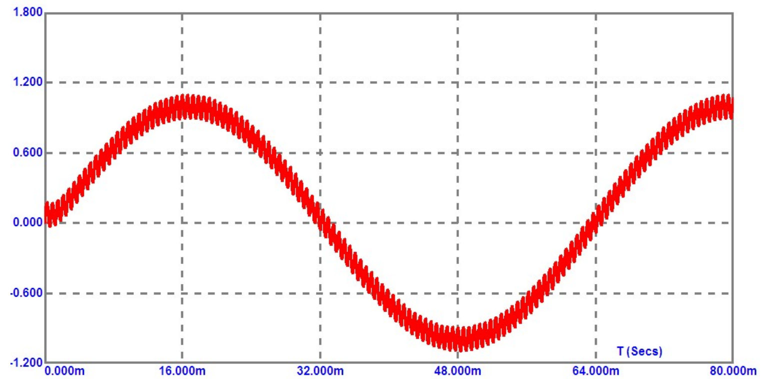
$$H(\omega)=\frac{1}{1+j\omega 10^{-3}}=\frac{1}{1+j\dfrac{\omega}{10^{3}}}$$

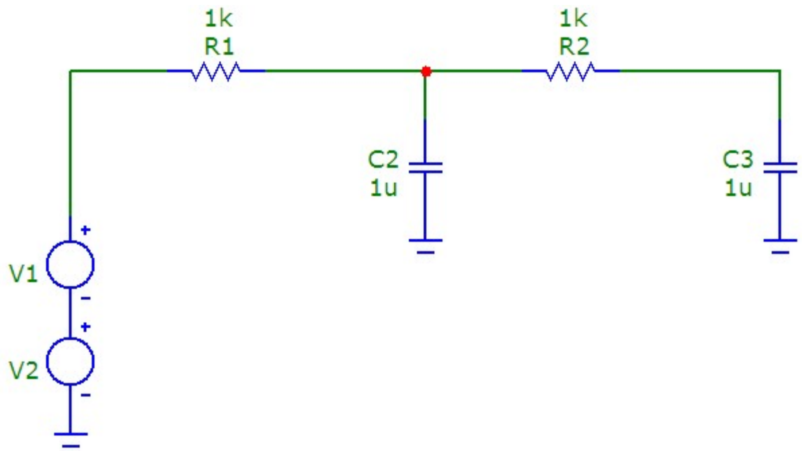


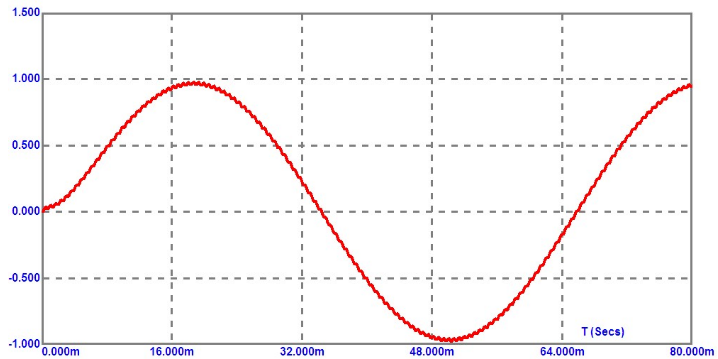
$$V_O(t)=1Sin(100t)+0.1Sin(10^{3}t)$$

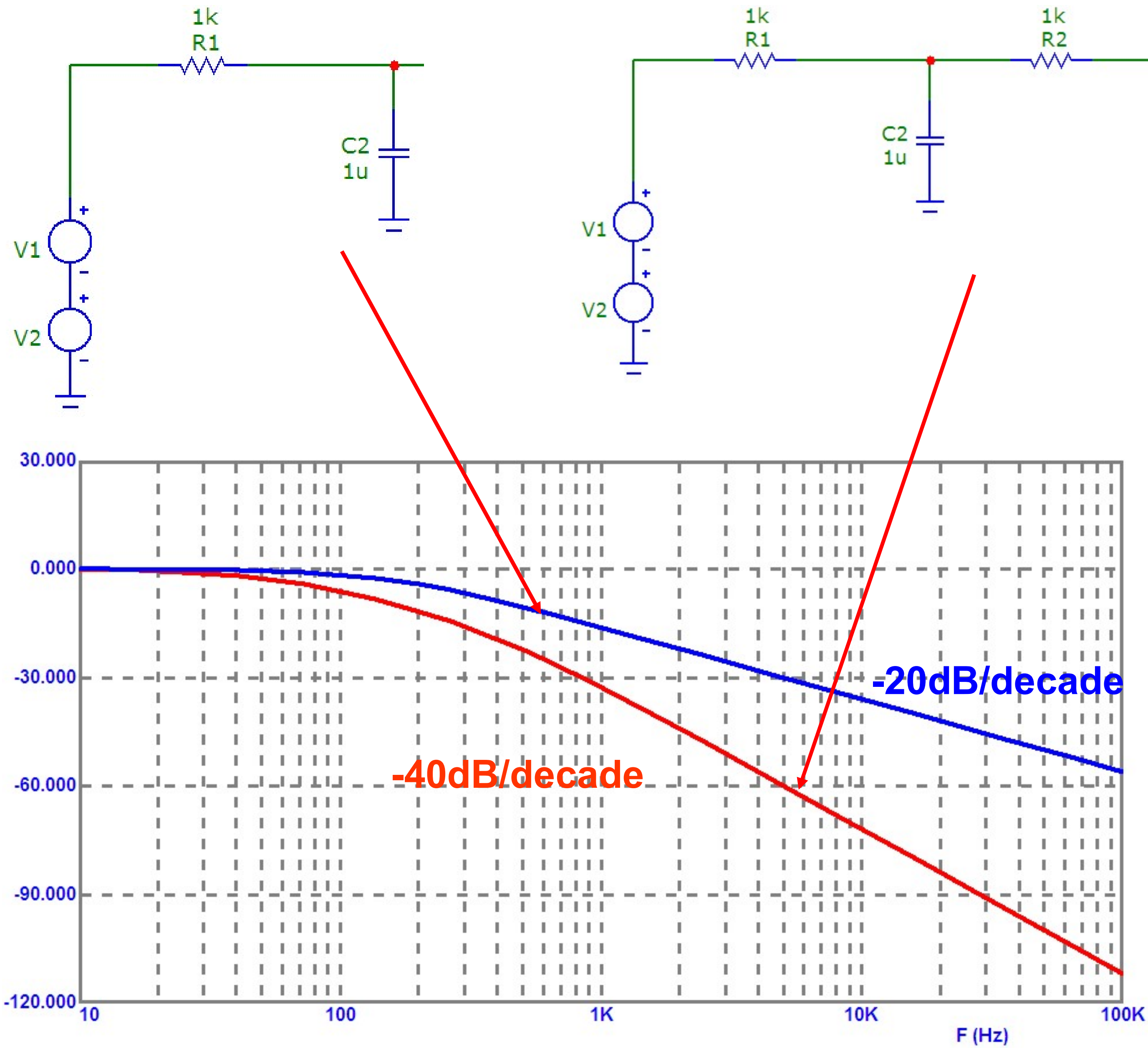



1k
R1
C2
1u
V1
V2


1.800
1.200
0.600
0.000
-0.600
-1.200
T (Secs)
0.000m
16.000m
32.000m
48.000m
64.000m
80.000m


1k
R1
1k
R2
C2
1u
C3
1u
V1
V2


1.500
1.000
0.500
0.000
-0.500
-1.000
T (Secs)
0.000m
16.000m
32.000m
48.000m
64.000m
80.000m



1k
R1
C2
1u
V1
V2
1k
R1
1k
R2
C2
1u
C3
1u
V1
V2
30.000
0.000
-30.000
-60.000
-90.000
-120.000
10
100
1K
10K
100K
F (Hz)
-20dB/decade
-40dB/decade

Adding more RC stages, makes the characteristics sharper