ESC201T : Introduction to Electronics

HW4: Solution

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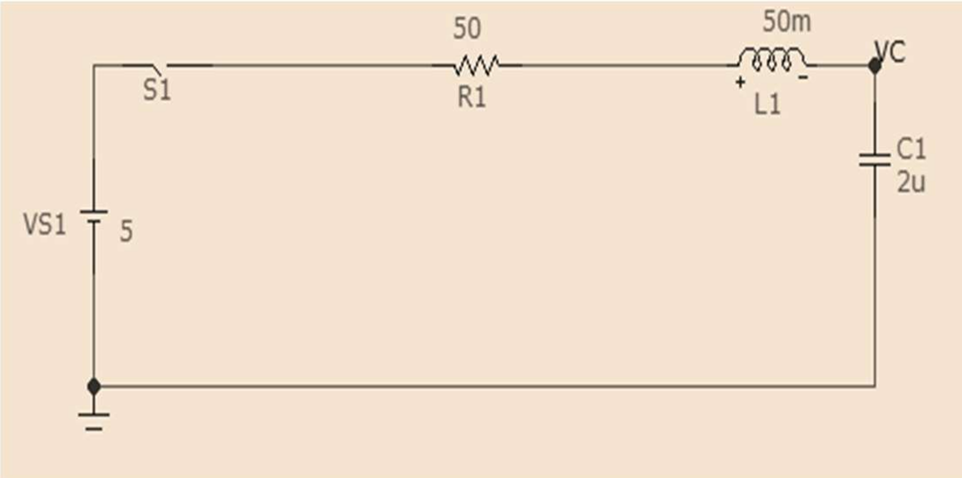
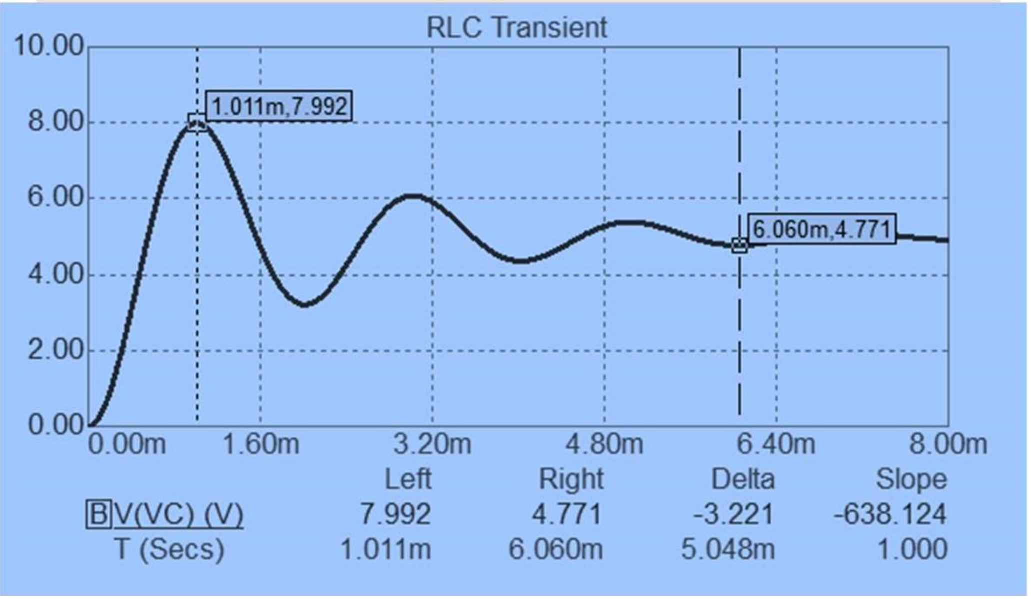
Q.1 In the circuit shown below, determine the frequency of oscillation that will be observed in the capacitor voltage after switch S1 is closed. Determine the time at which the output voltage Vo will settle to within 5% of its final value.





$$s = -\frac{\omega_o}{2Q} \pm j\omega_O\sqrt{1-\frac{1}{4Q^2}}$$

$$\omega_O = \frac{1}{\sqrt{L\times C}} = 3.16\text{K} \qquad Q = \frac{\omega_O L}{R} = 3.16$$

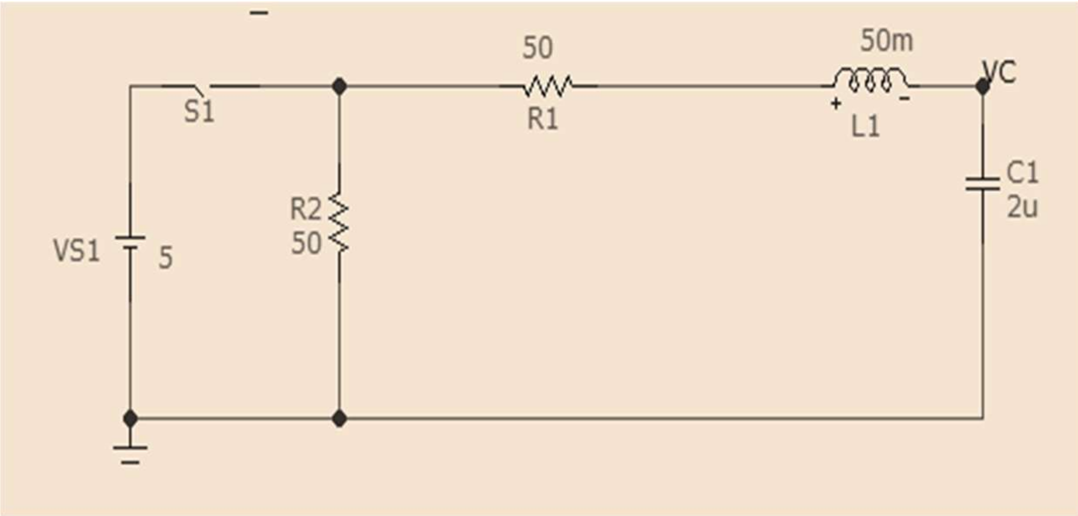
$$\omega_1 = \omega_O\sqrt{1-\frac{1}{4Q^2}} = 3.12\text{k}$$

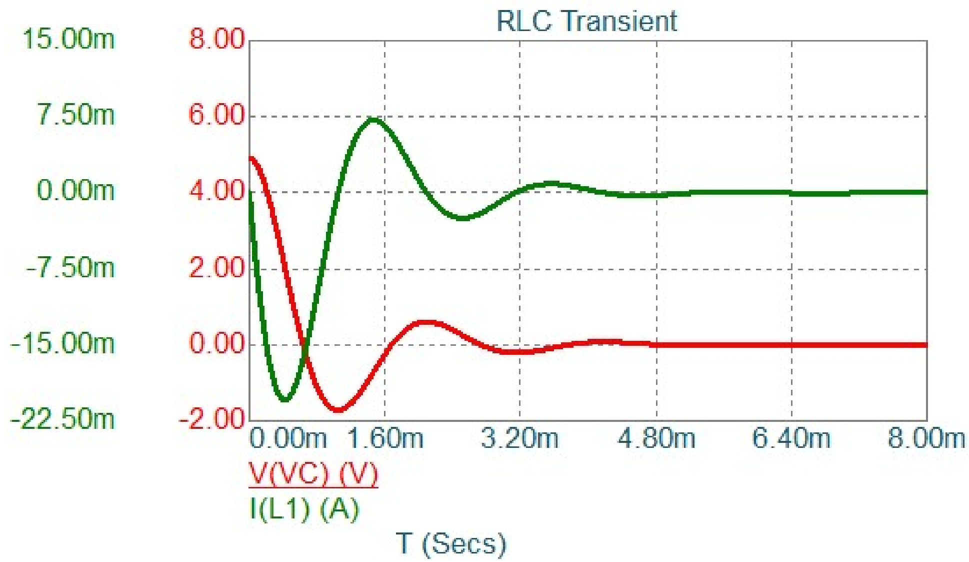
$$V_C(t) = 10 - e^{-\frac{R}{2L}t} \times (A \times Cos(\omega_1 t) + B \times Sin(\omega_1 t))$$

$$\tau = 2 \times \frac{L}{R} = 2ms$$

$$t_{sett} = 3 \times \tau = 6ms$$

Q.2 After the circuit shown above has reached steady state, the switch S1 is opened and an additional resistor is included to provide a current path as shown below. Sketch the qualitative variation of current and capacitor voltage. What is the maximum value of R2 for which oscillations in current would be observed?



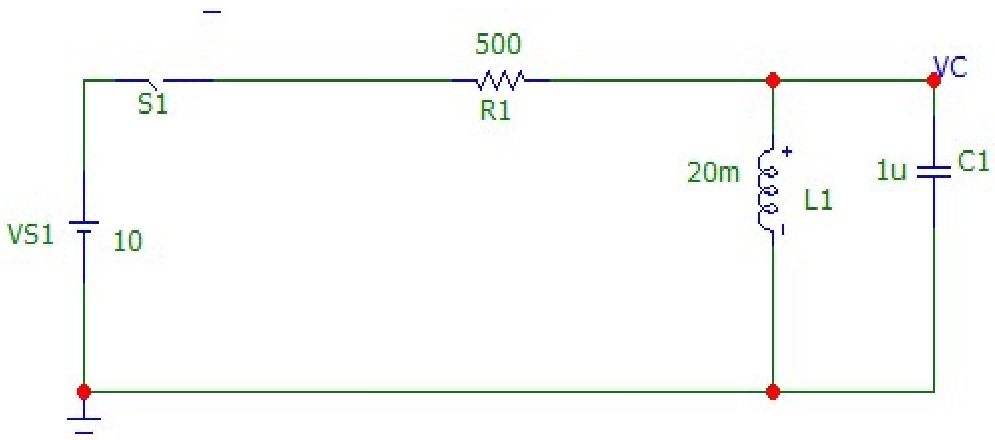


$$\omega_O = \frac{1}{\sqrt{L \times C}} = 3.16k$$

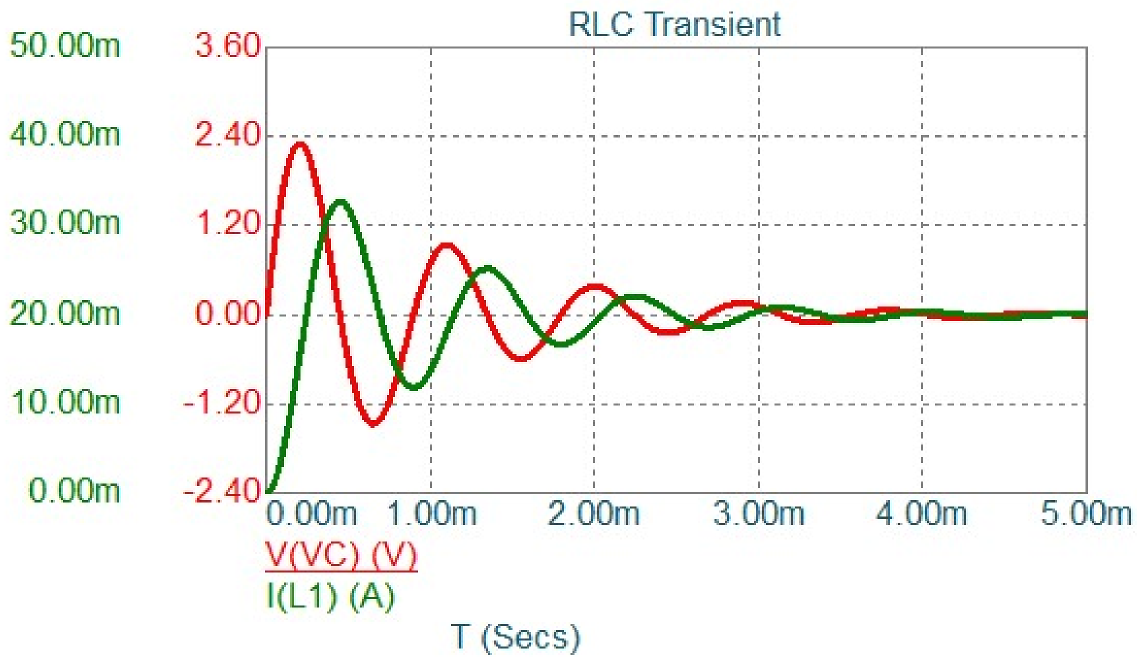
$$Q = \frac{\omega_O L}{R} \geq 0.5 \Rightarrow R \leq 316.2\Omega$$

$$R_2 \leq R - 50 = 266.2\Omega$$

Q.3 Sketch and explain the qualitative variation of current and capacitor voltage in the parallel LCR circuit shown below after switch S1 is closed

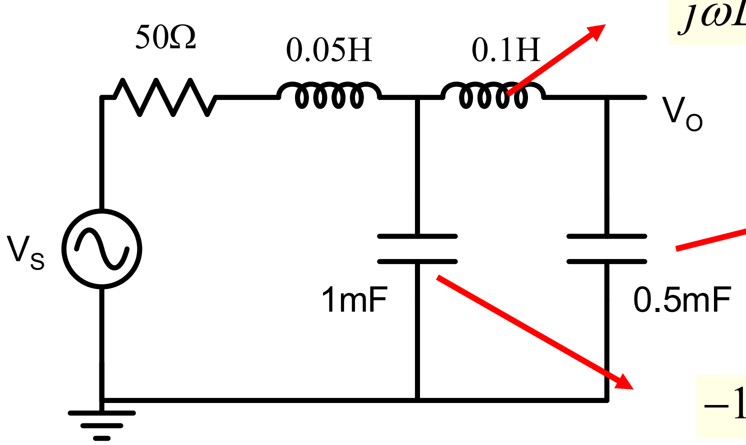


$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{R}{\omega_O L}$$



Q.4 Determine the output voltage as a function of time using the method of phasors for an input voltage of V_S = 5Cos(100t).

$$\omega = 100$$

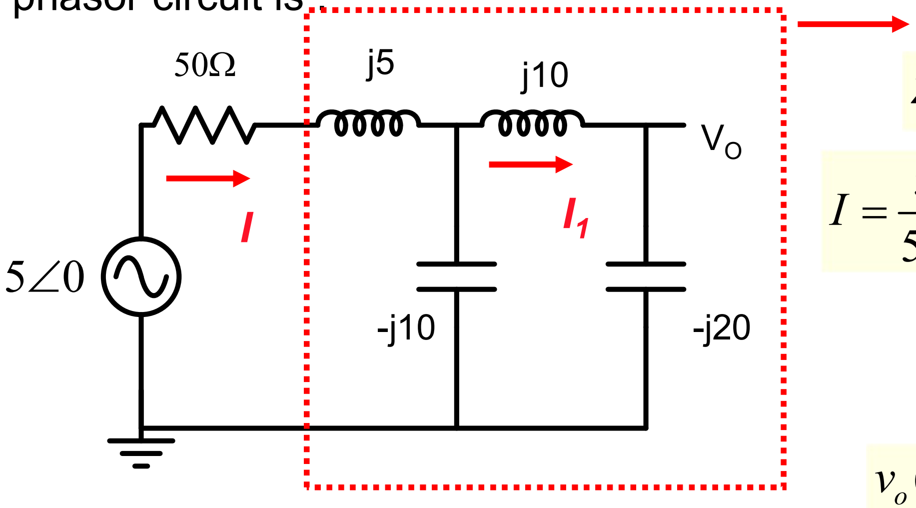


$$j\omega L = 10j$$

$$\frac{-j}{\omega \times 0.5 \times 10^{-3}} = -20j$$

$$-10j$$

The phasor circuit is :



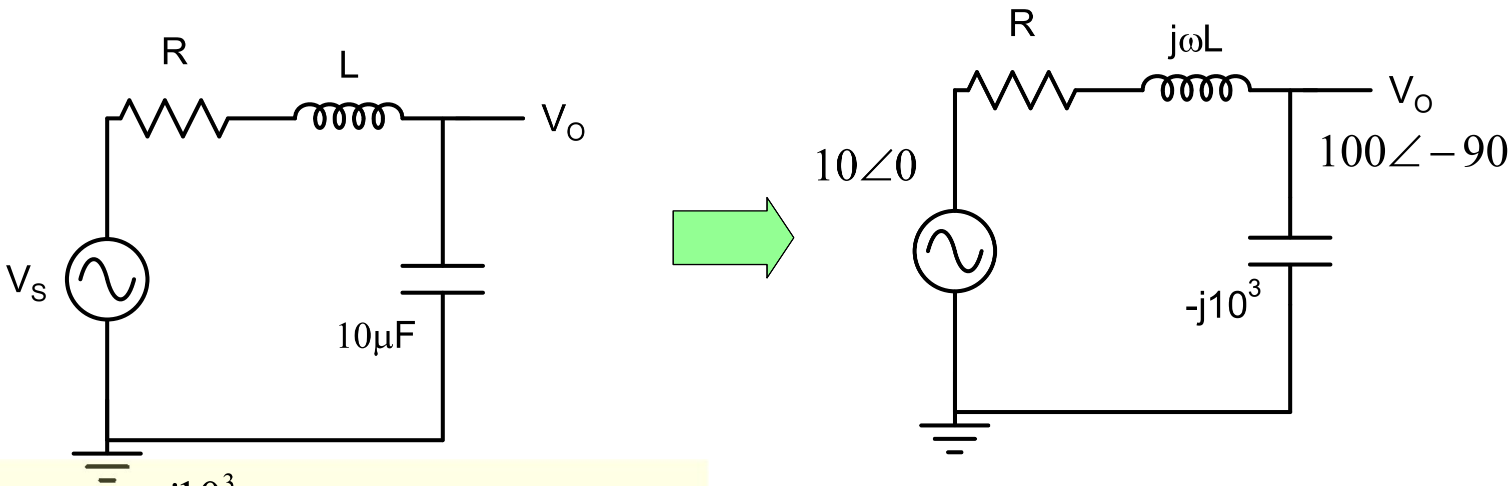
$$Z_{eq} = \{(-j20 + j10) \| -j10\} + j5 = 0$$

$$I = \frac{5}{50} = 0.1\angle 0 \qquad I_1 = \frac{0.1}{2}\angle 0$$

$$V_o = -j20 I_1 = 1\angle -90$$

$$v_o(t) = Cos(100t - 90)$$

Q.5 A student applied V_S = 10VCos(100t) as input to the circuit shown below and claimed that he measured V_O = **100VSin(100t)** across the capacitor. Is that possible? If so determine suitable values for inductor and resistor for such a condition to occur? Comment about the concept of voltage division in this case.



$$V_o = \frac{-j10^3}{R + j\omega L - j10^3} \times 10\angle 0 = 100\angle -90$$

$$\Rightarrow \frac{-j10^3}{R + j\omega L - j10^3} = -10j$$

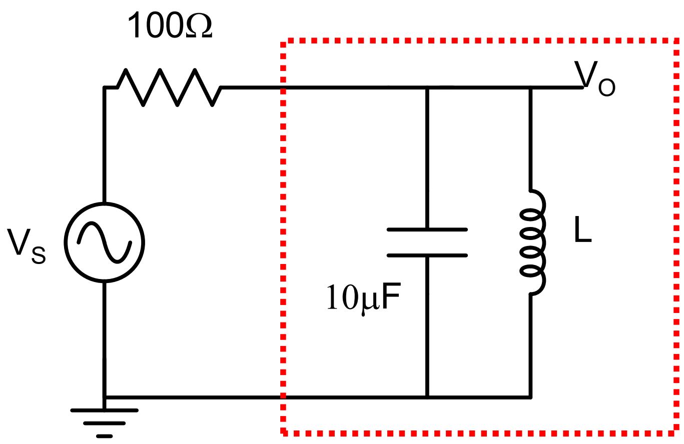
$$\frac{-j10^3}{R} = -10j \Rightarrow R = 100\Omega$$

$$\Rightarrow j\omega L - j10^3 = 0$$

$$\Rightarrow L = 10H$$

In a resistive series circuit one would expect voltage across any resistor to be less than the source voltage. However, due to negative impedance of capacitor, this does not hold for circuits containing capacitors and inductors. Hence voltage across a series element can be larger than source voltage !

Q.6 Determine suitable value for inductance such that output voltage V_O is maximum. For this value of inductance determine current in all the components (R,L and C). Assume that V_S = 10VCos(100t).



Let the equivalent impedance be Z_{eq}

$$Z_{eq} = \frac{j\omega L \times -j10^3}{j\omega L - j10^3}$$

$$V_o = \frac{Z_{eq}}{Z_{eq} + 100} V_S$$

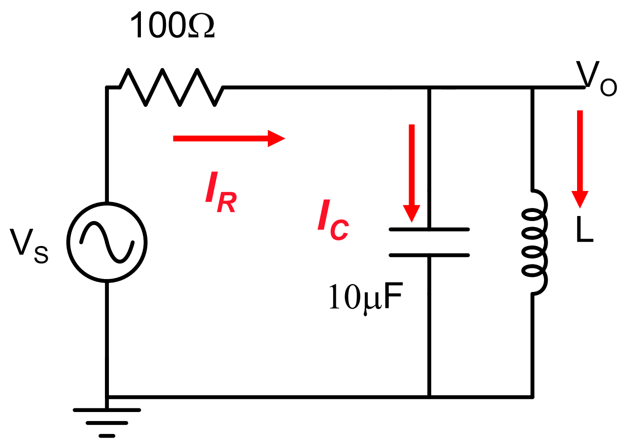
To obtain maximum output voltage, Z_{eq} should be as large as possible

One can note from the expression for Z_{eq} that one can obtain an infinite value if

$$j\omega L - j10^3 = 0 \Rightarrow L = 10H \qquad \Rightarrow V_o = V_S \qquad I_R = \frac{V_S}{100 + Z_{eq}} = 0$$

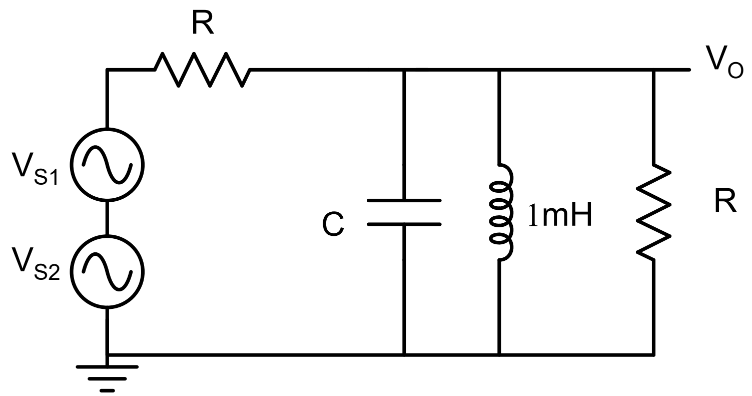
$$I_L = \frac{V_O}{j\omega L} = \frac{V_S}{j10^3} = 0.01\angle -90 \qquad I_C = \frac{V_O}{-j10^3} = \frac{V_S}{-j10^3} = 0.01\angle 90$$

$$i_L(t) = 0.01 \times Cos(100t - 90) = 0.01\,Sin(100t) \qquad i_C(t) = -0.01\,Sin(100t)$$



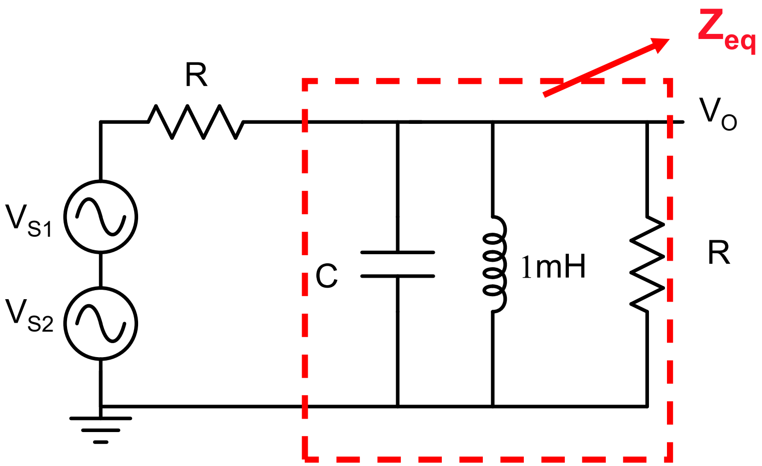
Note the surprising result that even though $I_R = 0$, I_L and I_C are non-zero.

Q.7 In the circuit shown below assume that the two input sinusoidal voltage sources represent two different radio channels. Determine suitable value of capacitor such that we can listen to channel-1 (represented by source V_{S1}) of frequency f_1 = 1000KHz. Determine also suitable value of R such that interference due to the second channel of frequency f_2 = 1100KHz is negligible (say power at the output due to V_{s2} is a factor of 1000 less than that due to V_{s1}) . V_o represents the output voltage. Use suitable approximations to simplify analysis.



We can use superposition theorem to determine the output voltage due to the two sources one at a time.

Since we would like to listen to channel-1 only, we would like to maximize output voltage due to V_{s1}:



$$V_{o1} = \frac{Z_{eq1}}{Z_{eq1}+R}V_{S1}$$

$$Z_{eq1} = R \,\Big\|\, j\omega_1 L \,\Big\|\, \frac{1}{j\omega_1 C}$$

We should maximize Z_{eq1}

$$Z_{eq1} = R \,\Bigg\|\, \frac{L/C}{j(\omega_1 L - \frac{1}{\omega_1 C})}$$

Make this term 0

$$\omega_1 L - \frac{1}{\omega_1 C} = 0 \Rightarrow C = \frac{1}{\omega_1^2 L} = 25.33\,pF$$

$$\Rightarrow V_{o1} = \frac{R}{R+R}V_{S1} = 0.5V_{S1}$$

We want power due to V_{S2} less by a factor of 1000 which means:

$$|V_{o2}| = \frac{|V_{o1}|}{\sqrt{1000}} = \frac{|V_{S1}|}{63.24}$$

$$\left| V_{S2} \times \frac{Z_{eq2}}{Z_{eq2}+R} \right| = \frac{|V_{S1}|}{63.24}$$

$$Z_{eq2} = R \,\Big\|\, j\omega_2 L \,\Big\|\, \frac{1}{j\omega_2 C}$$

Assuming $V_{S2} = V_{S1}$

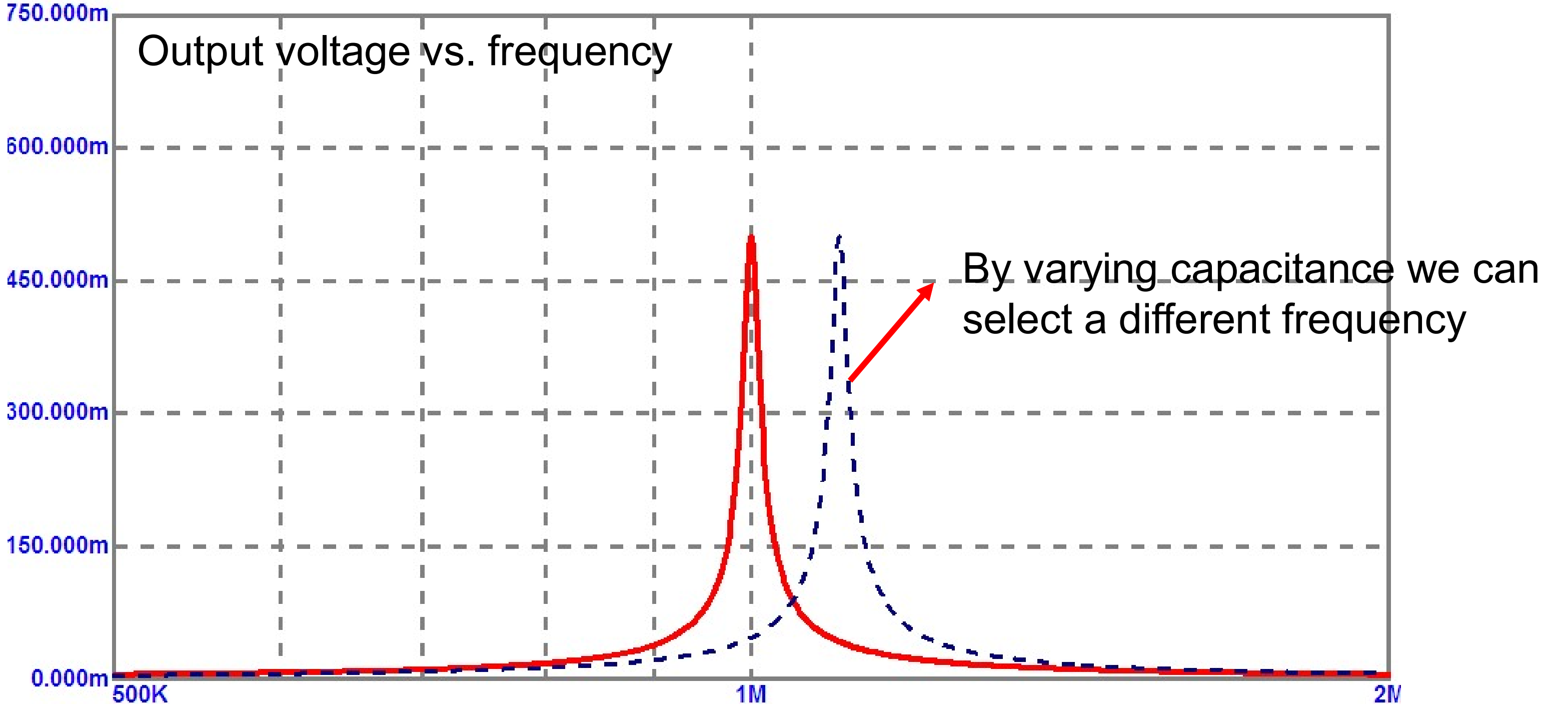
$$\left| V_{S2} \times \frac{Z_{eq2}}{Z_{eq2} + R} \right| = \frac{|V_{S1}|}{63.24}$$

$$\left| \frac{Z_{eq2}}{Z_{eq2} + R} \right| = \frac{1}{63.24}$$

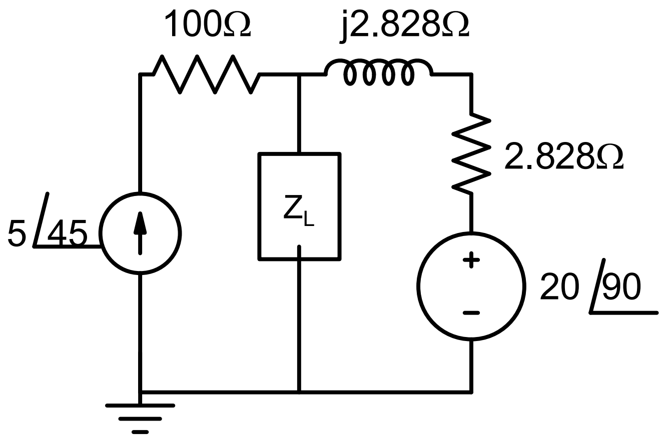
$$\Rightarrow R >> Z_{eq2} \,;\; R \cong 63.24 \left| Z_{eq2} \right|$$

$$Z_{eq2} = R \left\| j\omega_2 L \right\| \frac{1}{j\omega_2 C} \cong \frac{L/C}{j(\omega_2 L - \frac{1}{\omega_2 C})} = -j3.3\times10^4\,\Omega$$

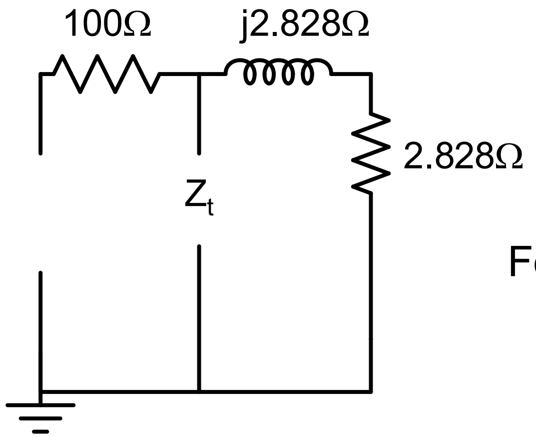
$$R \cong 63.24 \left| Z_{eq2} \right| = 2\times10^6$$



Q.8 Use Thevenin's theorem to determine Z_L such that maximum power is dissipated in the load impedance Z_L. Determine also the power dissipated in Z_L and power supplied by each source.



Thevenin's impedance can be found from the equivalent circuit

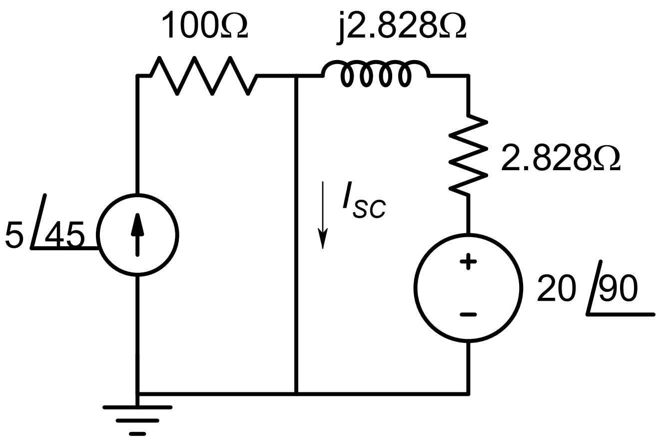


$$Z_t = 2.82 + j2.82 = 4\angle 45$$

For maximum power transfer, we should choose:

$$Z_L = \overline{Z_t} = 2.82 - j2.82 = 4\angle -45$$

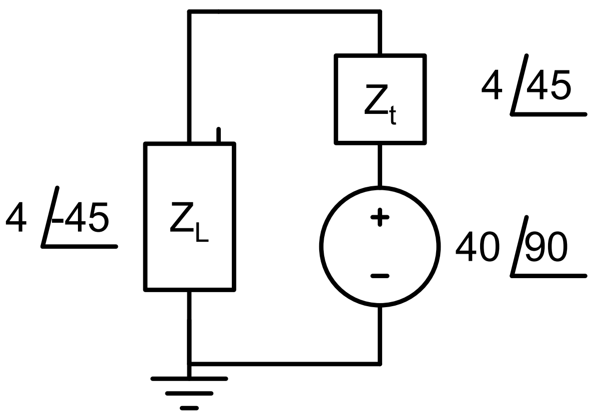
Thevenin's voltage can be found by first finding the short circuit current:



$$I_{SC} = 5\angle 45 + \frac{20\angle 90}{2.82 + j2.82} = 10\angle 45$$

$$V_t = I_{SC} \times Z_t = 10\angle 45 \times 4\angle 45 = 40\angle 90$$

Thevenin's equivalent circuit:

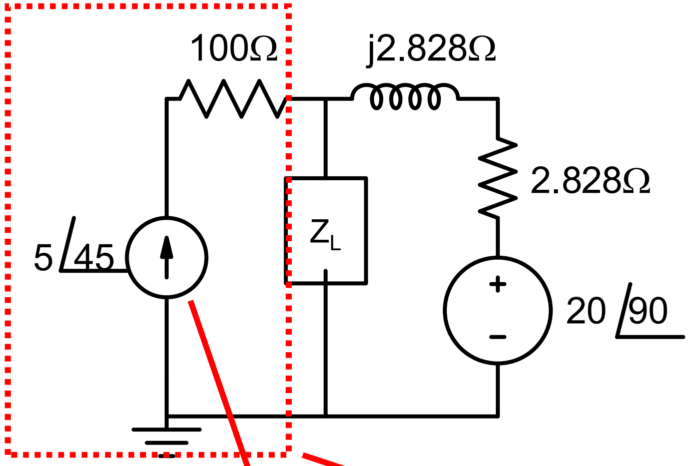


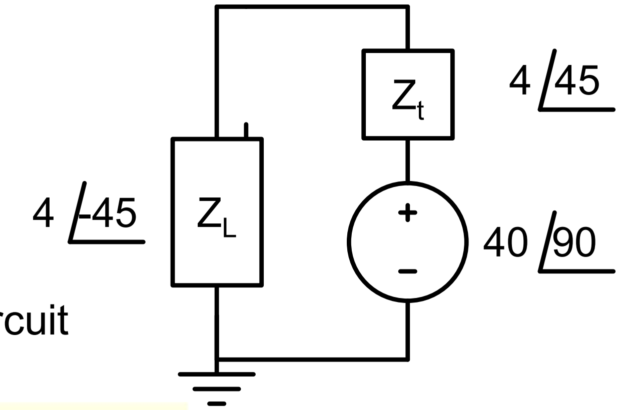
$$I_L = \frac{40\angle 90}{4\angle 45 + 4\angle -45} = j5\sqrt{2}$$

$$R_L = 4Cos(-45) = \frac{4}{\sqrt{2}}$$

$$P_L = \frac{(5\sqrt{2})^2 R_L}{2} = 70.1W$$

Power due to current source





Using Thevenin's equivalent circuit

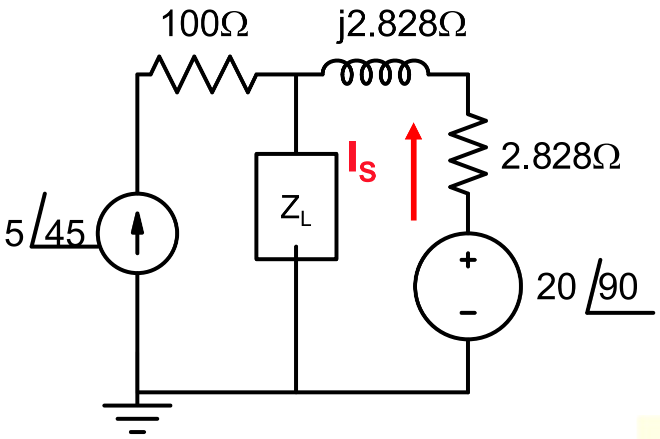
$$V_L = \frac{4\angle -45}{4\angle 45 + 4\angle -45} 40\angle 90 = 20\sqrt{2}\angle 45$$

$$P_X = \frac{20\sqrt{2}}{\sqrt{2}} \times \frac{-5}{\sqrt{2}} \times \cos(0) = -70.11W$$

$$P_I = -\frac{5^2}{2} \times 100 - 70.11 = 1.32KW$$

Note: power dissipated is positive and power supplied is negative

Power due to voltage source

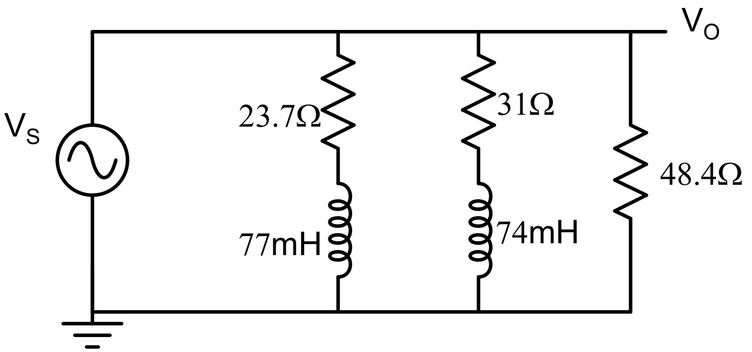


$$V_L = 20\sqrt{2}\angle 45$$

$$I_S = \frac{20\angle 90 - V_L}{2.82 + j2.82} = 5\angle 135$$

$$P_S = \frac{20}{\sqrt{2}} \times \frac{-5}{\sqrt{2}} \times \cos(-45) = -35.3W$$

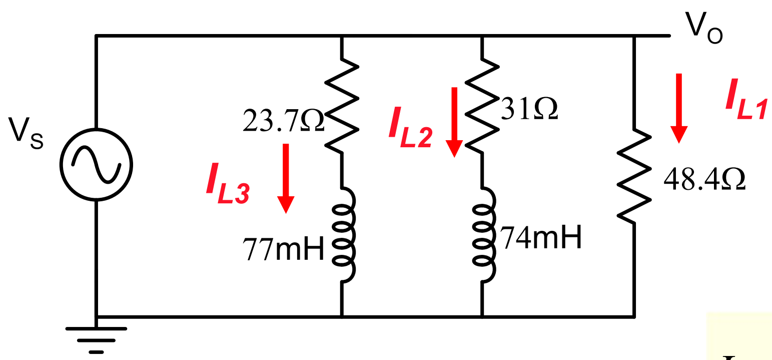
Q.9 For the circuit shown below, determine the power factor for the power supplied by the input. Determine a suitable value of capacitor which when connected in parallel to the source can raise the power factor to unity



$$\omega = 2 \times \pi \times 50 = 314 r/s$$

$$V_S = 311 Cos(\omega t)$$

Phasor Circuit



$$I_{L1} = 6.4A$$

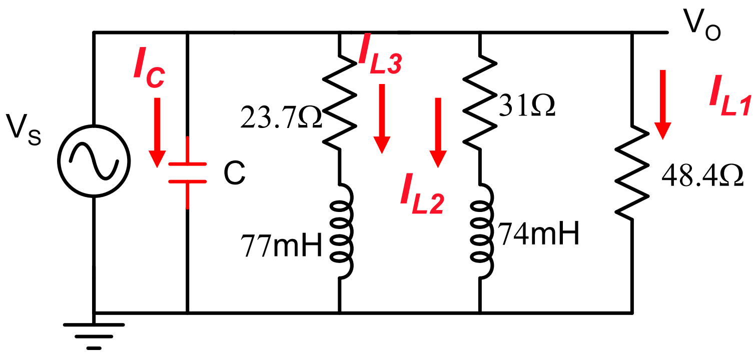
$$I_{L2} = \frac{V_S}{31 + j\omega \times 74 \times 10^{-3}} = 8\angle -37A$$

$$I_{L2} = \frac{V_S}{23.7 + j\omega \times 77 \times 10^{-3}} = 9.2\angle -45.6A$$

Total supply current: $$I_S = I_{L1} + I_{L2} + I_{L3} = 22.38\angle -30.55$$

Power Factor: $PF = Cos(30.55) = 0.86$

Power factor correction:



$$I_S = I_C + I_{L1} + I_{L2} + I_{L3} = 311\omega C\angle 90 + 22.38\angle -30.55$$

For PF = 1, the phase of current should be zero $\Rightarrow C = 0.116mF$

The new value of supply current is only 19.27A as compared to earlier value of 22.38A. Therefore losses in transmission line wires will be less here.

Suppose the second load is switched OFF. One can show that power factor drops to 0.94. As a result if one wishes to maintain PF = 1 then the capacitor value has to adaptively change with the load condition.