ESC 201T: Introduction to Electronics

Lecture 7: Toolbox For Circuit Analysis-4

Thevenin and Norton equivalent circuits
Source Transformation

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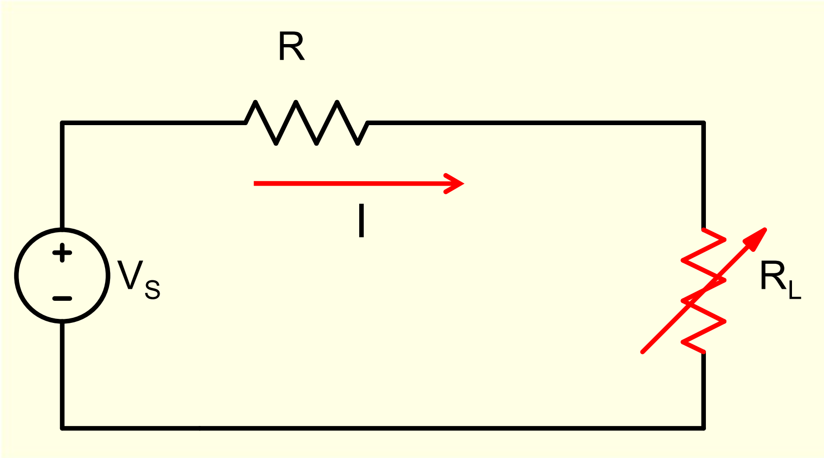
- Mesh and Nodal analysis are “brute-force” techniques that are not only time consuming and error prone to use but also yield **unstructured** expressions that are often unsuitable for gaining **insight** into operation of circuits and **modifying or designing** them.

- Need techniques that yield relatively simpler structured expressions that reveal the role of different components and that require less effort and are less error prone

Maximum Power Transfer for dc circuits



$$P_L = i^2 \times R_L$$

What value of R_L will give rise to maximum load power ?

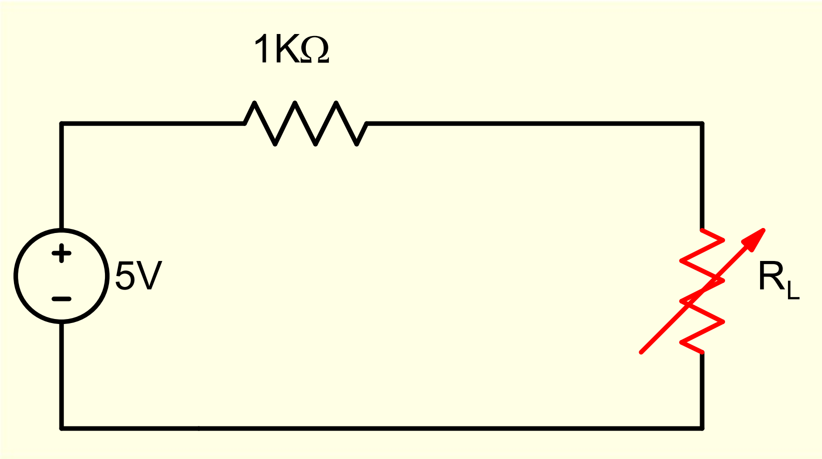
$$I = \frac{V_S}{R + R_L}$$

$$P_L = I^2 R_L = V_S^2 \times \frac{R_L}{(R + R_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0$$

$$R_L = R$$

$$P_{L\max} = \frac{V_S^2}{4 R_L}$$



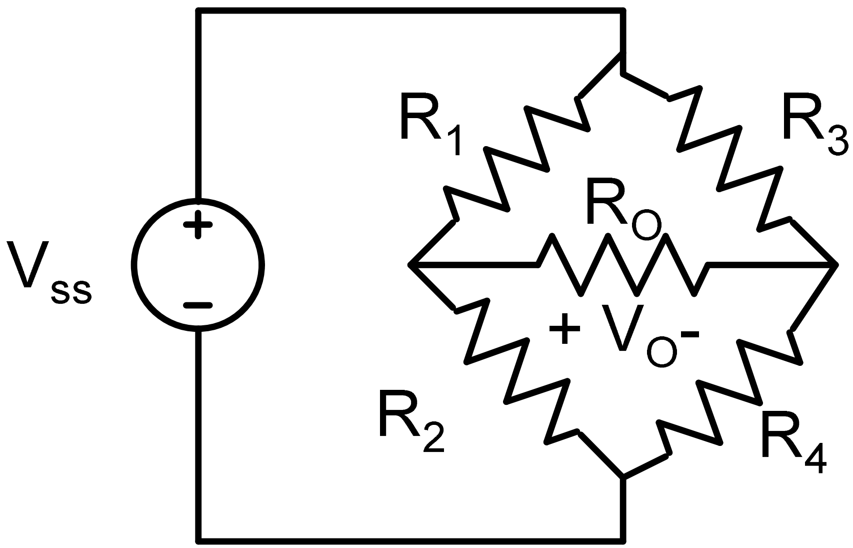
$$R_L = 1K \Rightarrow P_L = 6.25mW$$

$$R_L = 10K \Rightarrow P_L = 2mW$$

$$R_L = 0.2K \Rightarrow P_L = 3.47mW$$

Maximum power is delivered to the load when $\mathbf{R_L = R}$

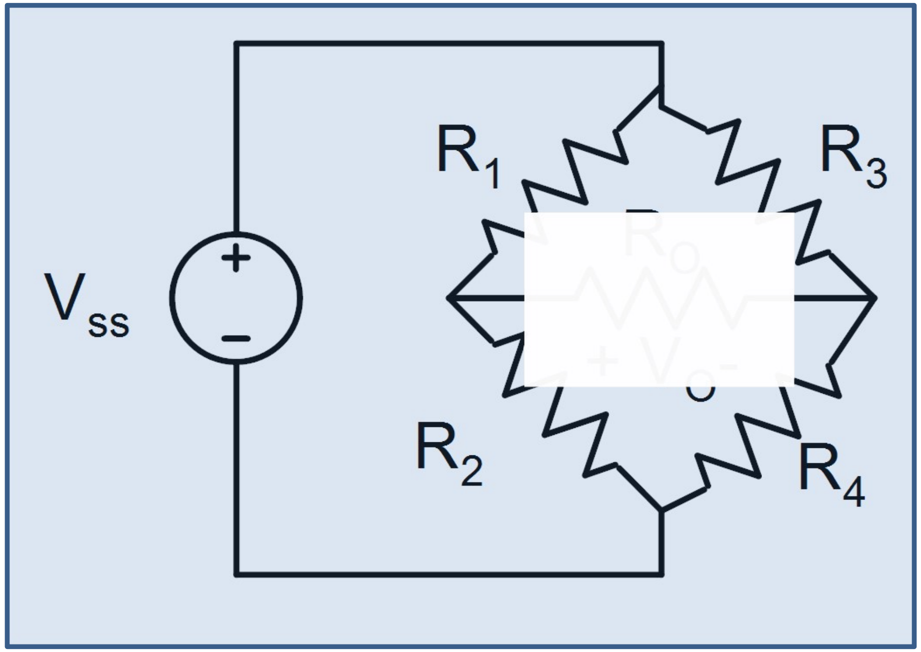
After deriving this useful result, how do we re-use it with other complicated circuits?



$$P_O = \frac{V_O^2}{R_O}$$

$$V_O = \frac{V_{SS} \times R_O \times (R_2 R_3 - R_1 R_4)}{R_2 R_4 R_0 + R_2 R_3 R_0 + R_1 R_4 R_0 + R_1 R_3 R_0 + R_2 R_3 R_4 + R_1 R_3 R_4 + R_1 R_2 R_4 + R_1 R_2 R_3}$$

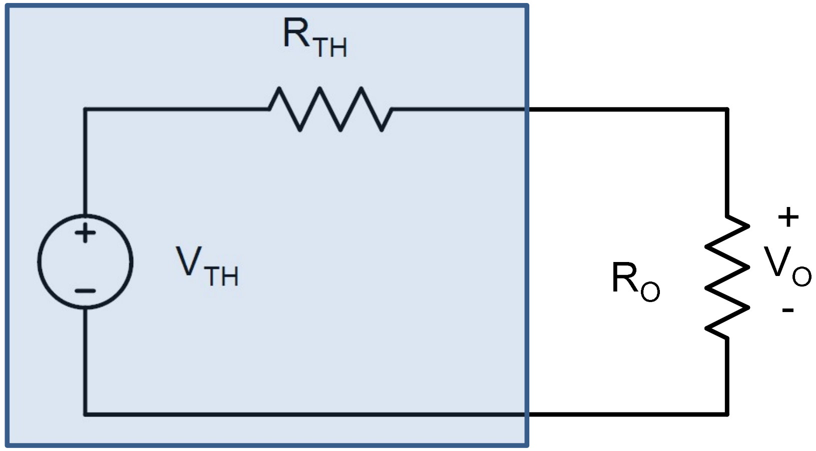
For what value of Ro is maximum power transferred to it?



$$V_O = V_{TH} \times \frac{R_O}{R_O + R_{TH}}$$

$$V_{TH} = V_{SS} \times \left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right)$$

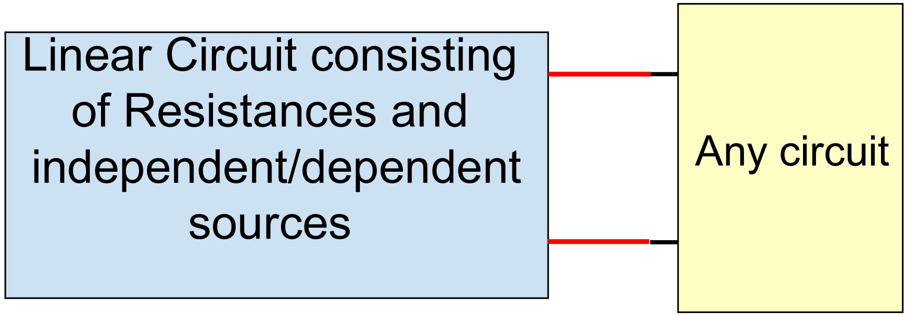
$$R_{TH} = (R_1 \parallel R_2) + (R_3 \parallel R_4)$$

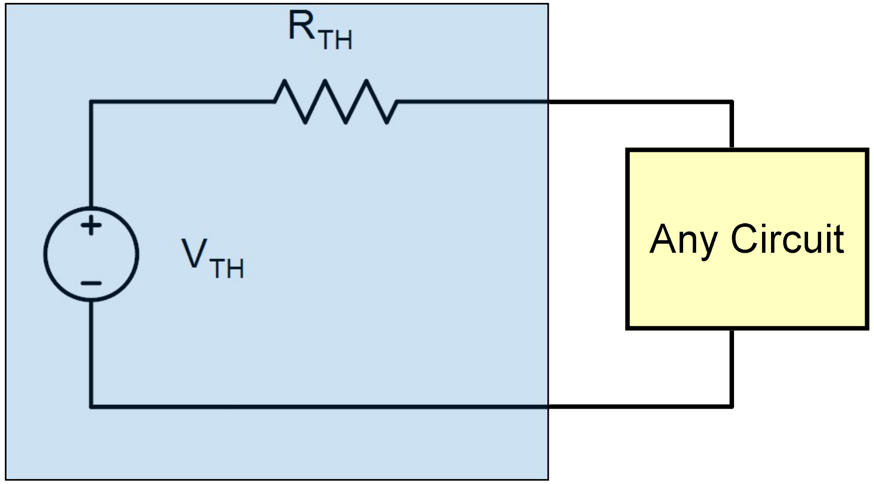


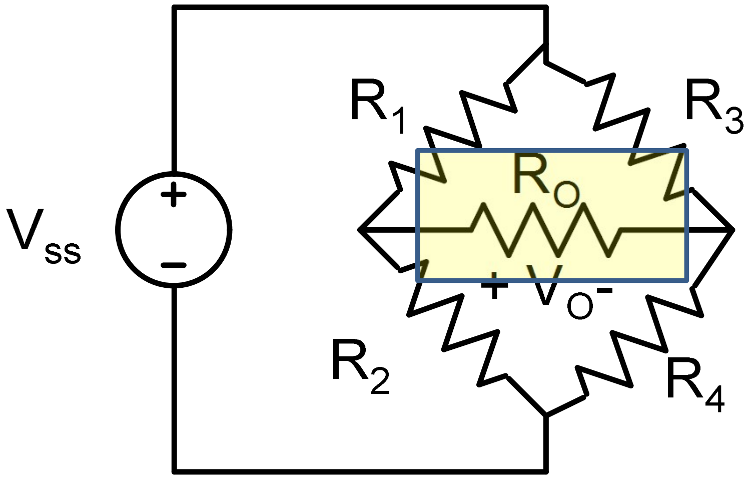
$$V_O = V_{SS} \times \left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right) \times \left\{\frac{R_O}{R_O+\{(R_1 \parallel R_2)+(R_3 \parallel R_4)\}}\right\}$$

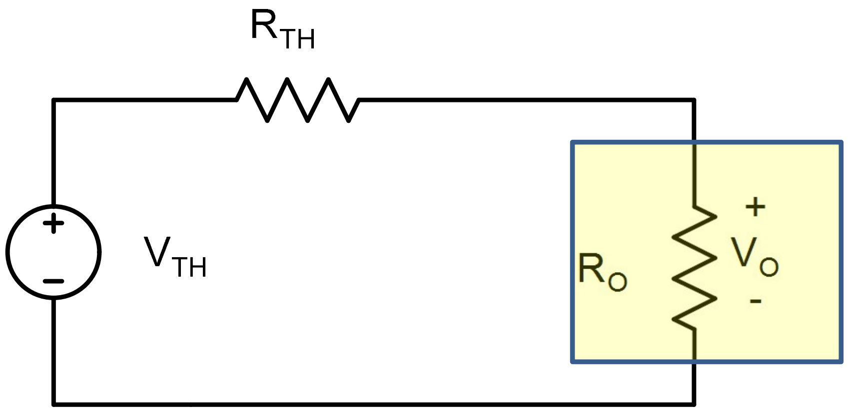
Maximum power will be transferred when R_O is chosen equal to R_{TH}

Thévenin Equivalent Circuit

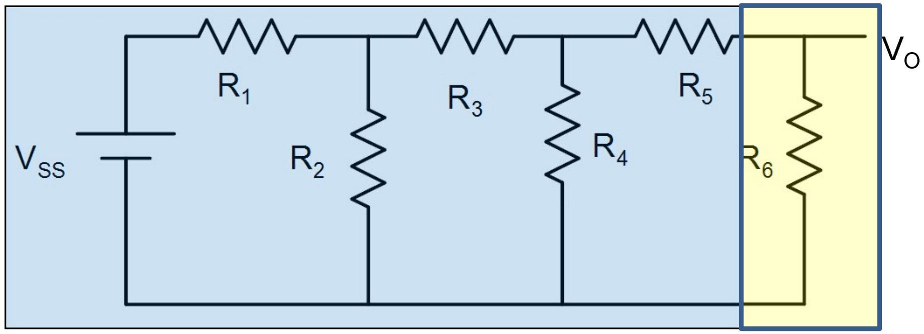


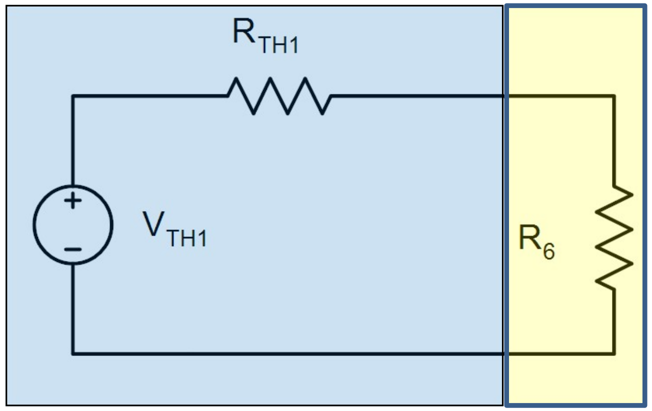


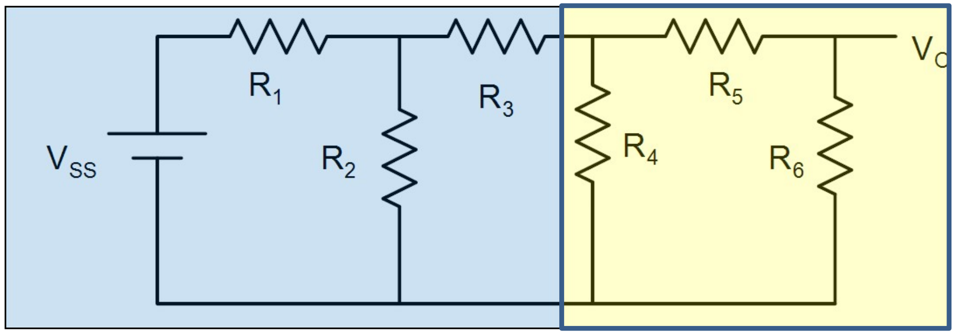


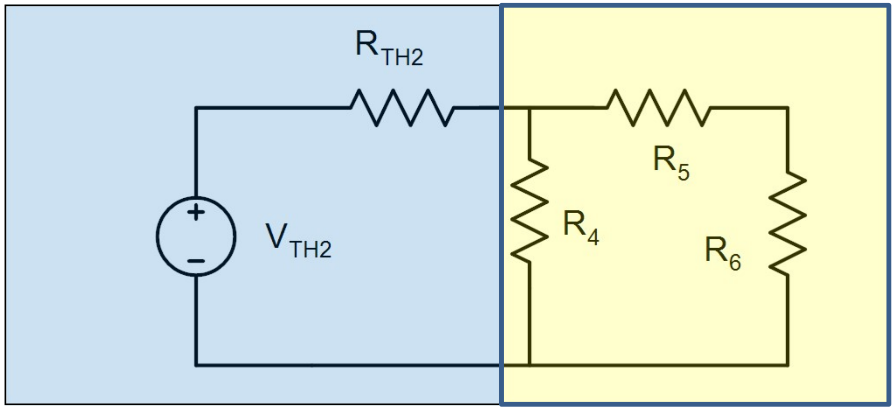


We can create Thevenin’s equivalent for any part of the circuit

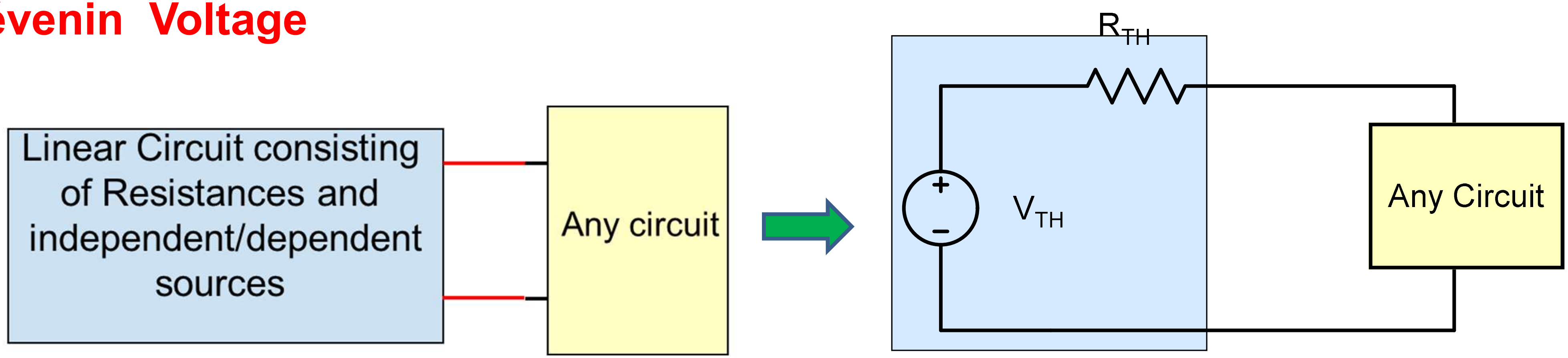




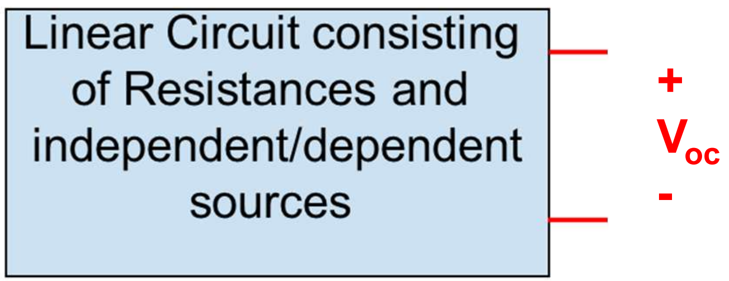


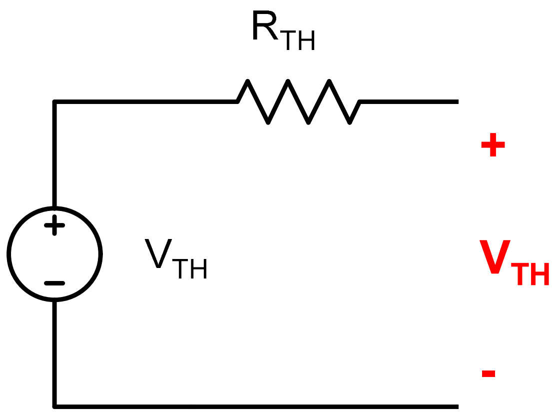


Thévenin Voltage



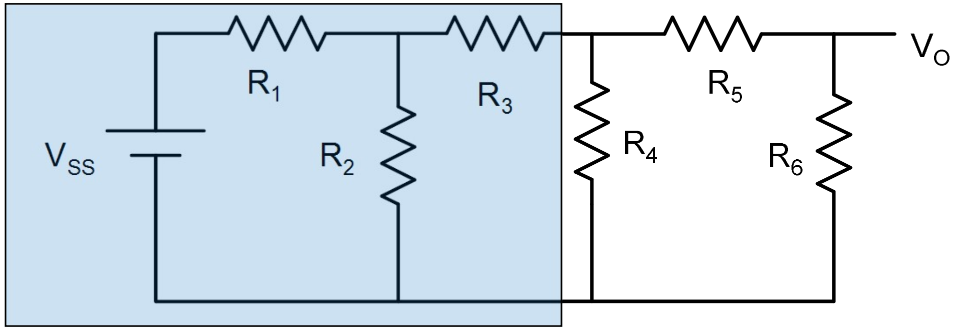
What is V_{TH} ?

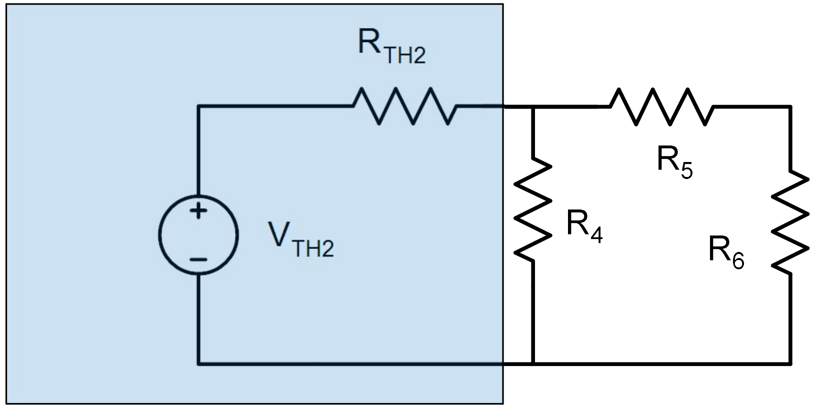


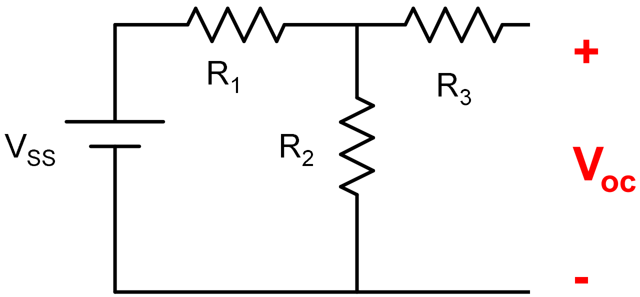


$$V_{TH} = V_{OC}$$

Example

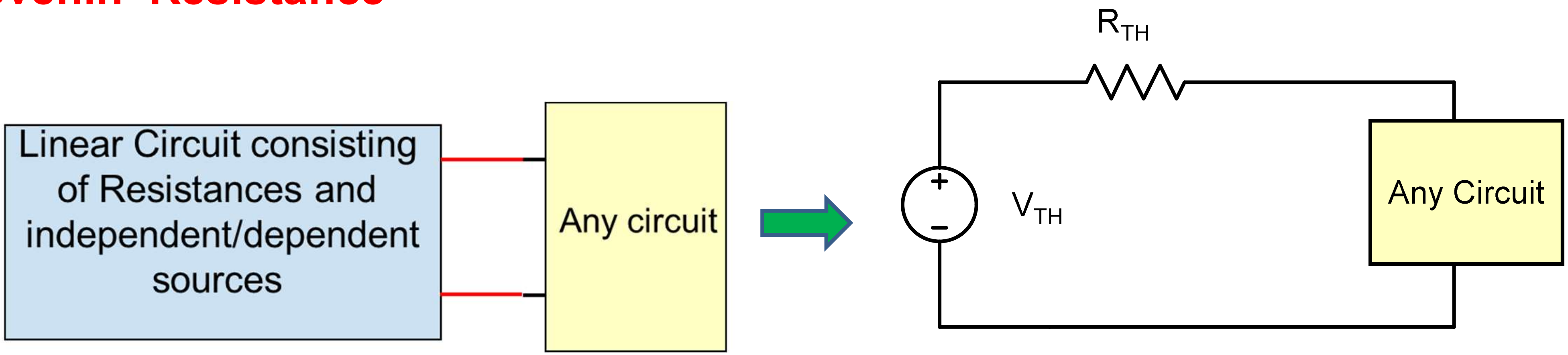




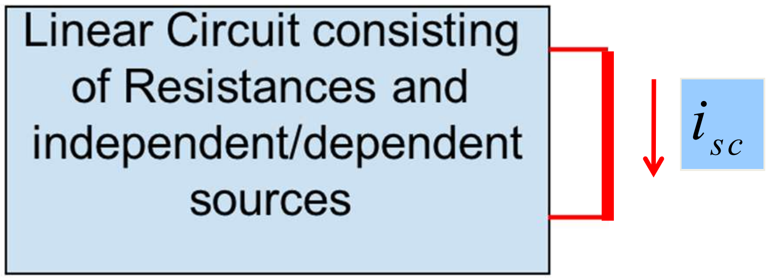


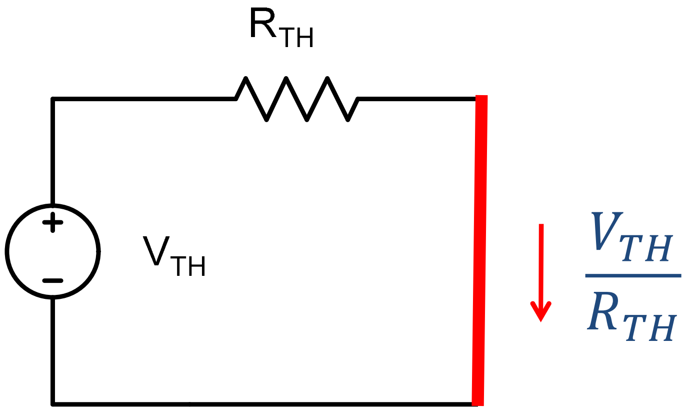
$$V_{TH2} = V_{OC} = V_{SS} \times \frac{R_2}{R_1 + R_2}$$

Thévenin Resistance



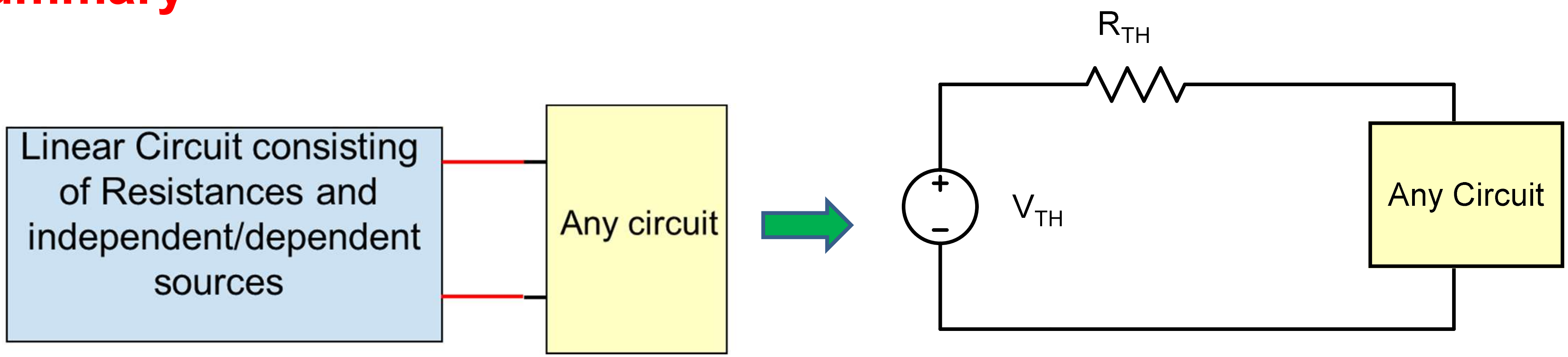
What is R_{TH} ?

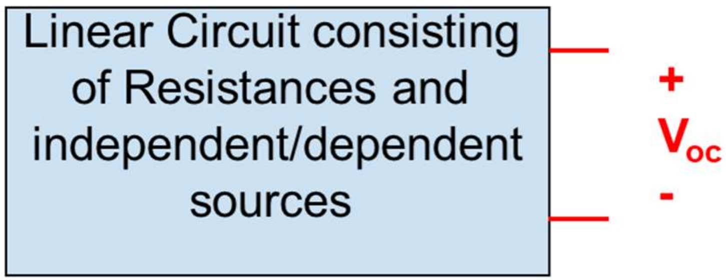


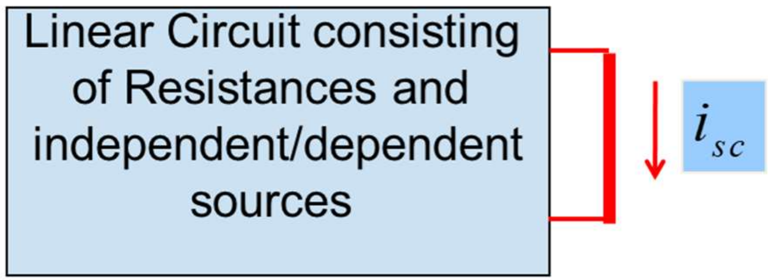


$$I_{SC} = \frac{V_{TH}}{R_{TH}} \Rightarrow R_{TH} = \frac{V_{OC}}{I_{SC}}$$

Summary



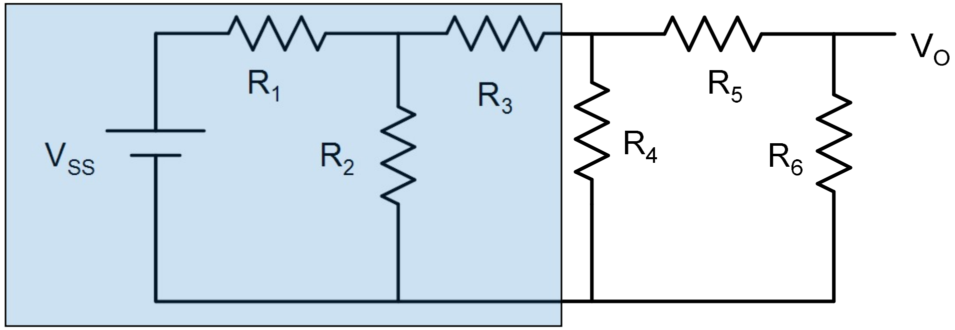


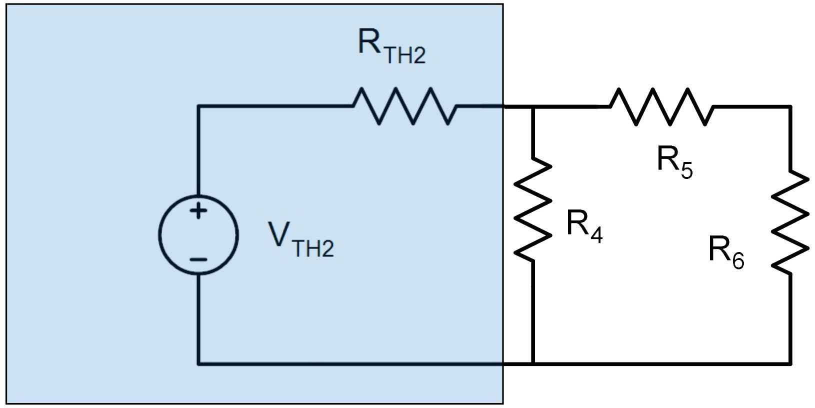


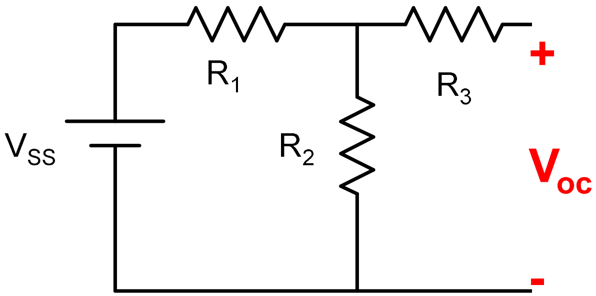
$$V_{TH} = V_{OC}$$

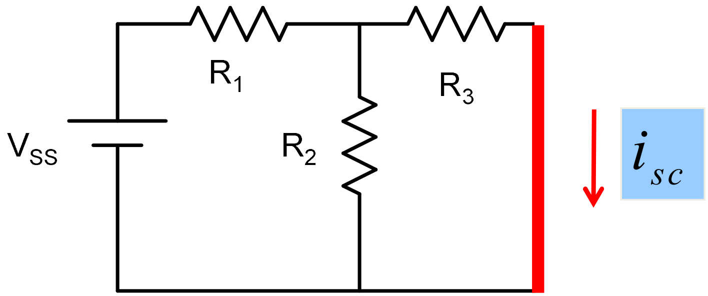
$$R_{TH} = \frac{V_{OC}}{I_{SC}}$$

Example







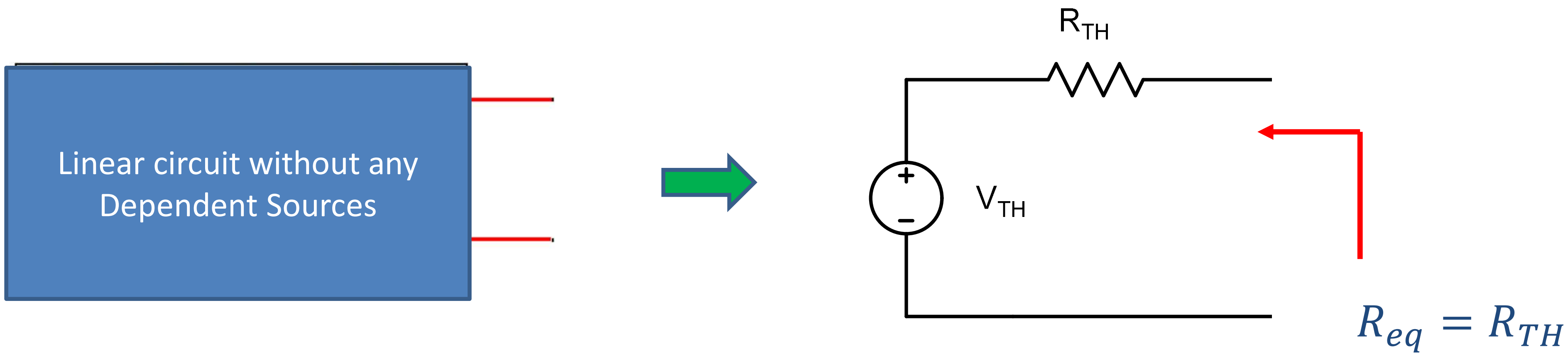


$$V_{TH2} = V_{OC} = V_{SS} \times \frac{R_2}{R_1 + R_2}$$

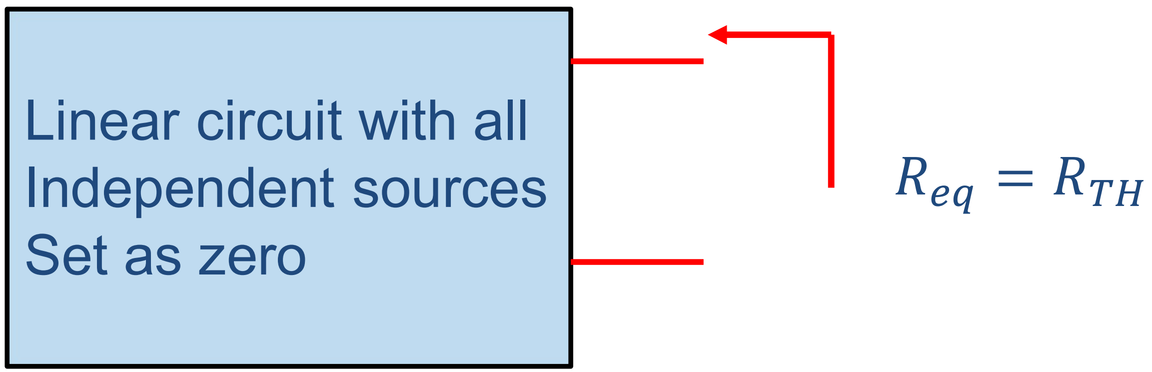
$$I_{SC} = \frac{V_{SS}}{R_1 + R_2 \| R_3} \times \frac{R_2}{R_2 + R_3}$$

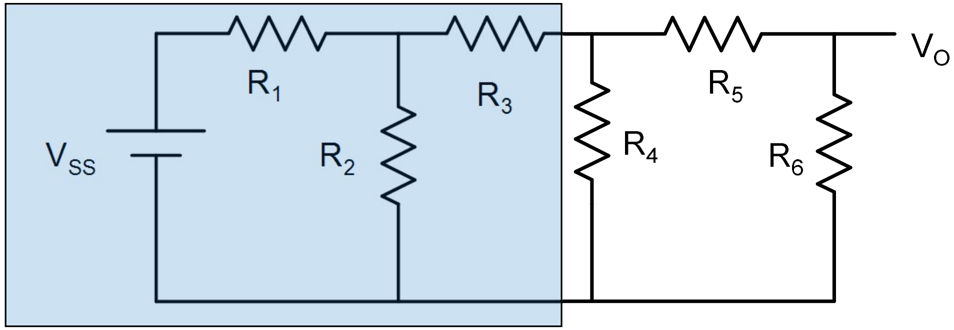
$$R_{TH2} = \frac{V_{OC}}{I_{SC}} = \frac{R_2 + R_3}{R_1 + R_2} \times (R_1 + R_2 \| R_3)$$

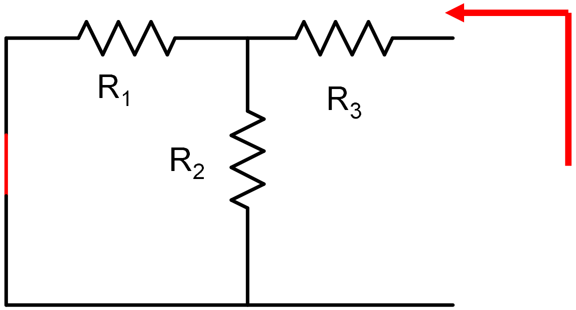
Thévenin Resistance: Alternative Method in the absence of dependent sources



If we make all independent sources zero, then $V_{TH} = 0$



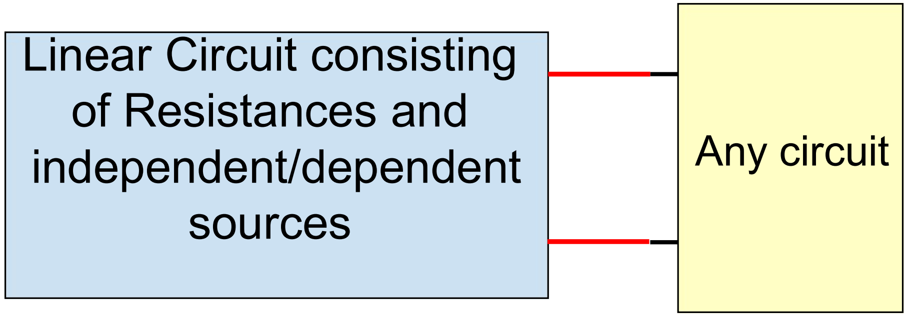


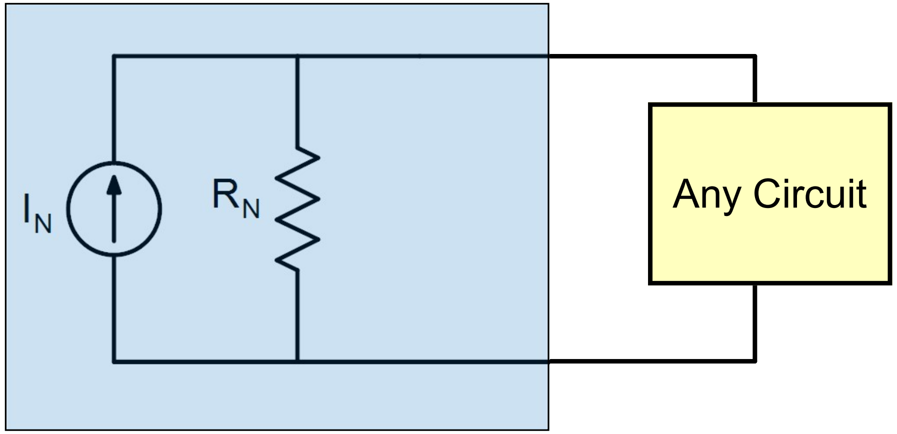


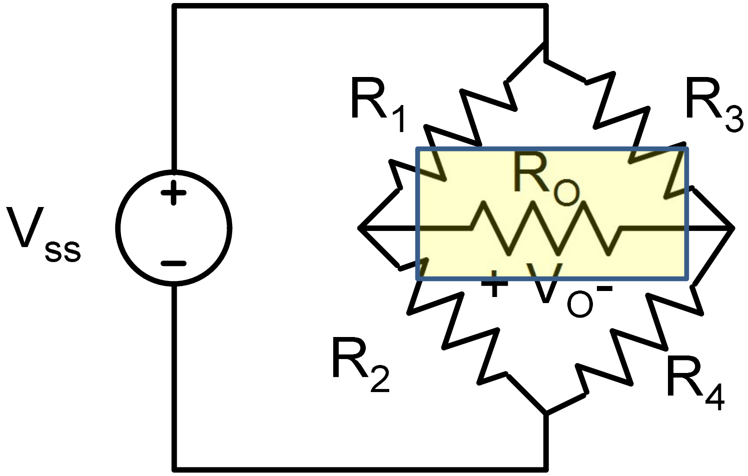
$$R_{eq} = R_{TH2} = \mathrm{R}_1 \| R_2 + R_3$$

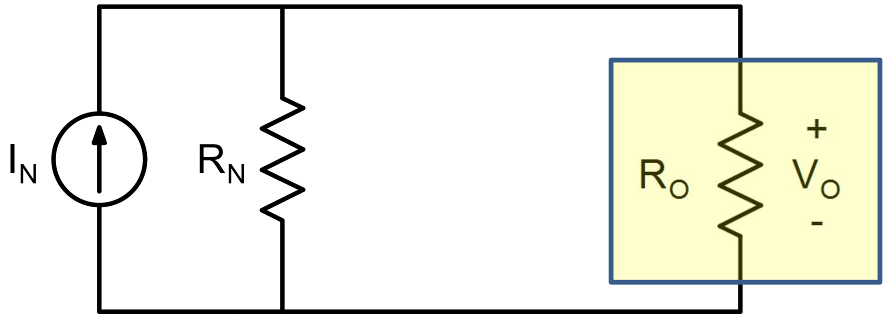
$$R_{TH2} = \frac{V_{OC}}{I_{SC}} = \frac{R_2 + R_3}{R_1 + R_2} \times (R_1 + R_2 \| R_3)$$

Norton Equivalent Circuit

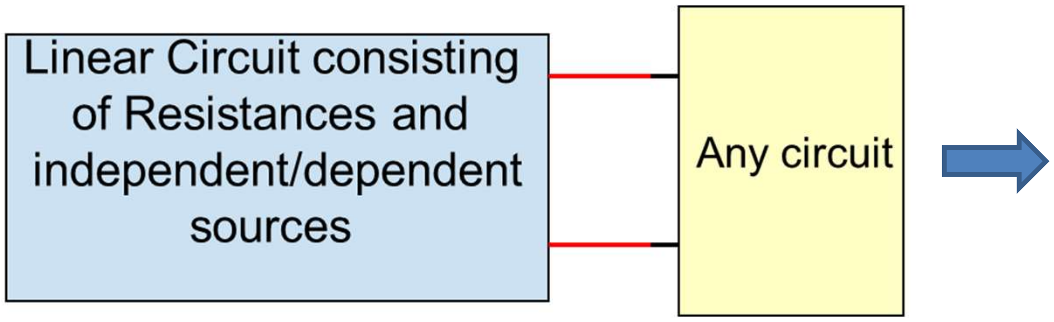


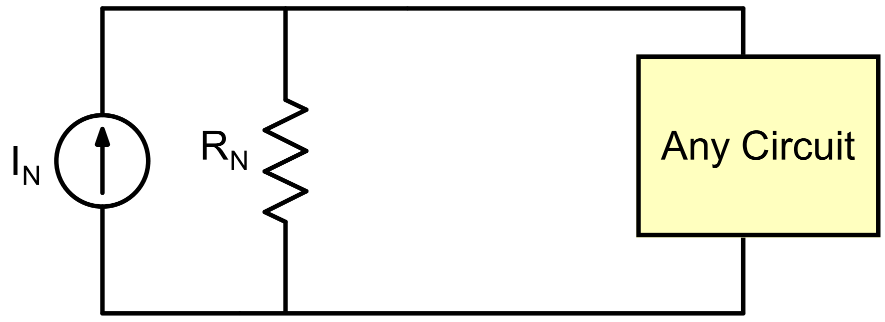


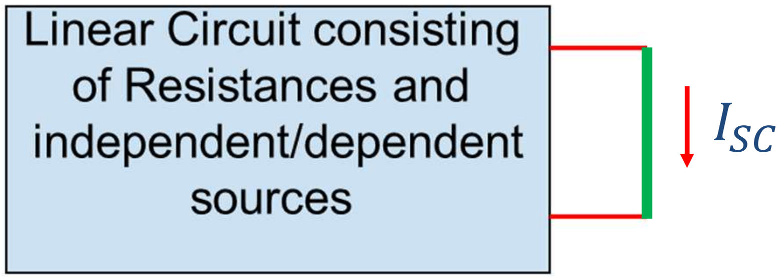


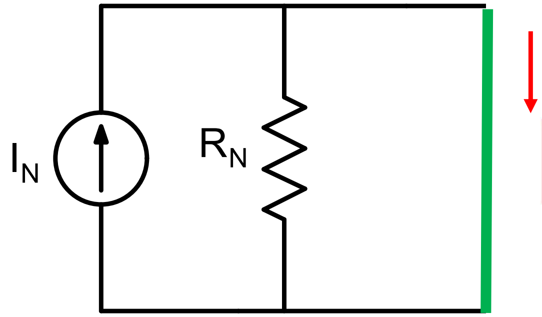


Norton's Current



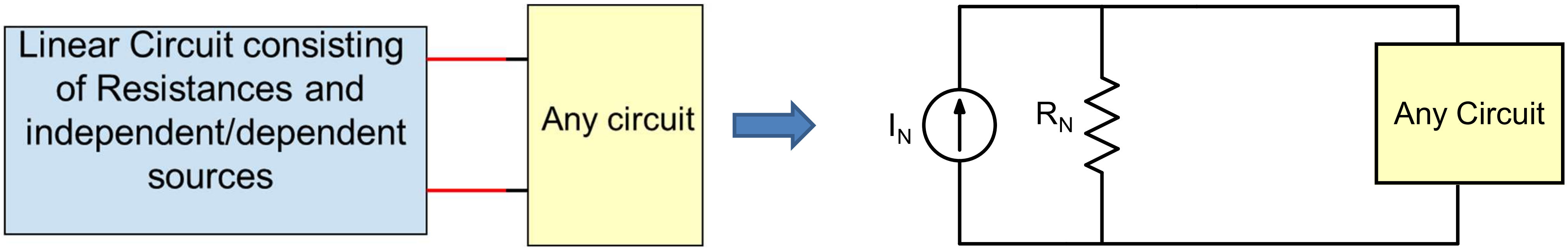


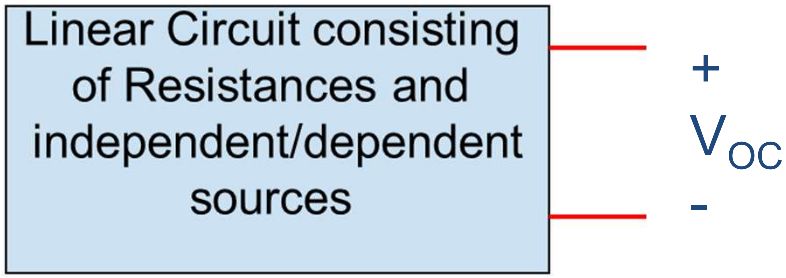


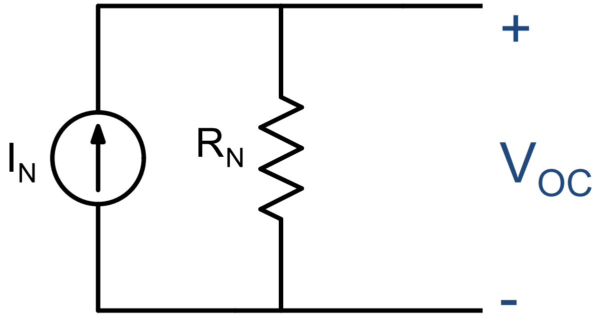


$$I_N = I_{SC}$$

Norton’s Resistance



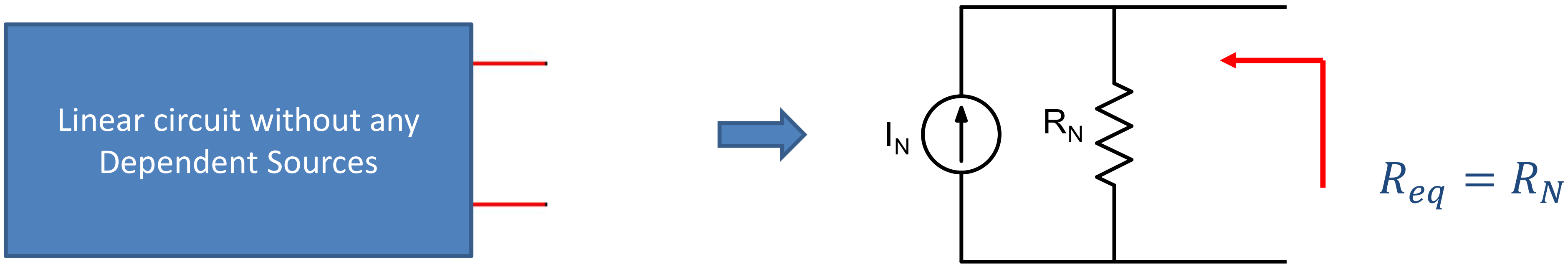




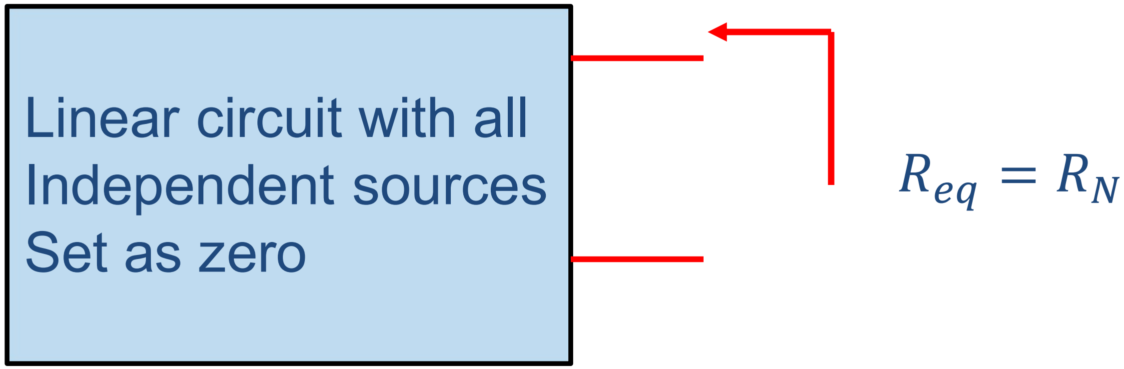
$$R_N = \frac{V_{OC}}{I_{SC}}$$

$$V_{OC} = I_N \times R_N$$

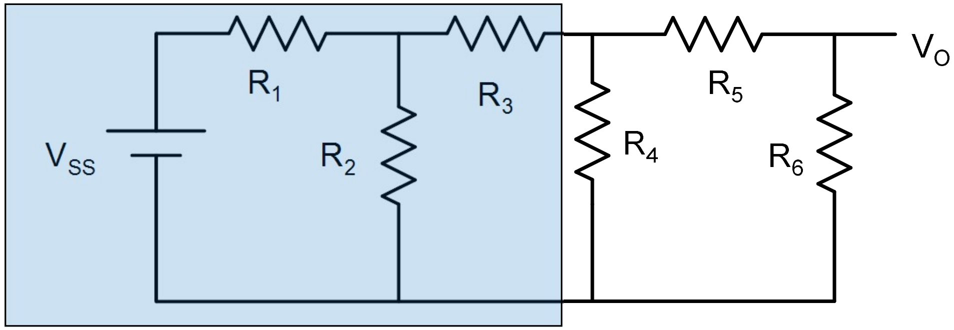
$$I_N = I_{SC}$$

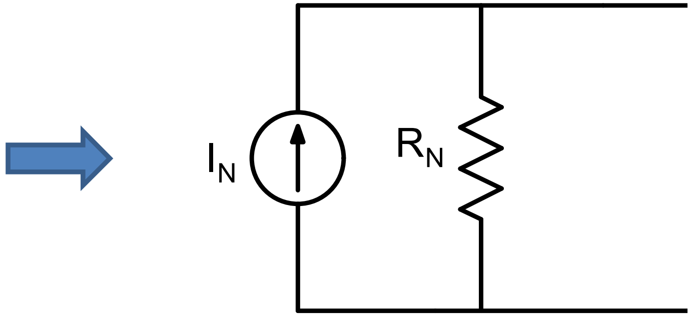


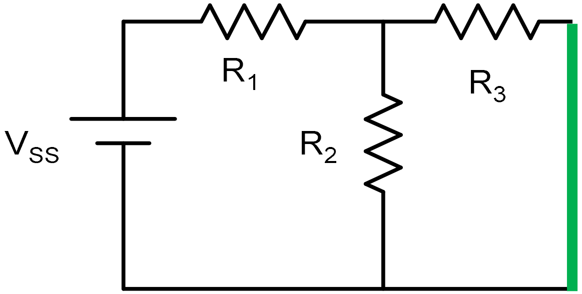
If we make all independent sources zero, then $I_N = 0$



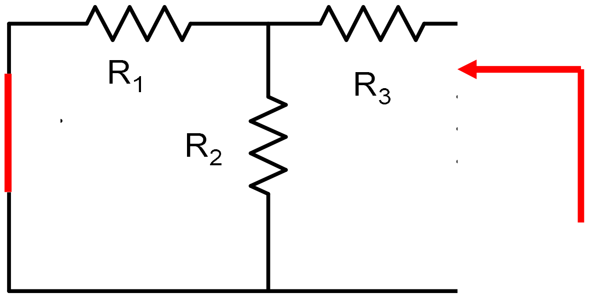
Example





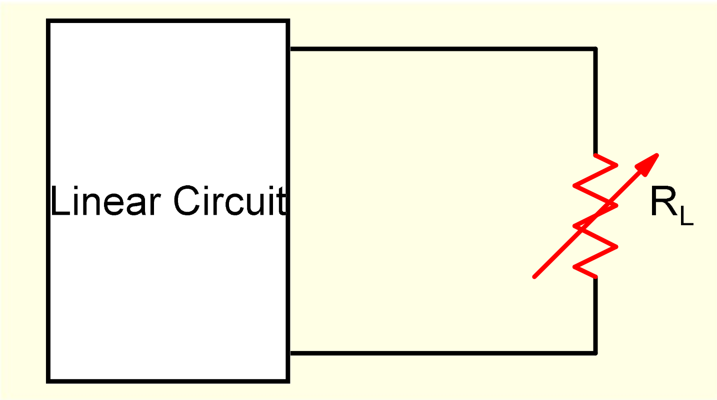


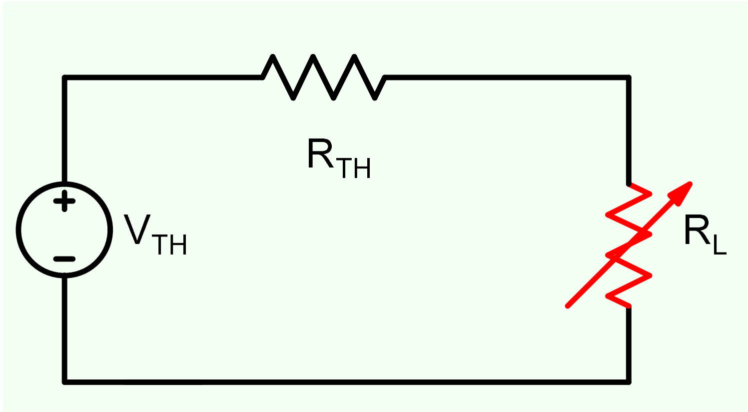
$$I_N = I_{SC} = \left(\frac{V_{SS}}{R_1 + R_2 \| R_3}\right) \times \left(\frac{R_2}{R_2 + R_3}\right)$$



$$R_{eq} = R_N = (R_1 \| R_2) + R_3$$

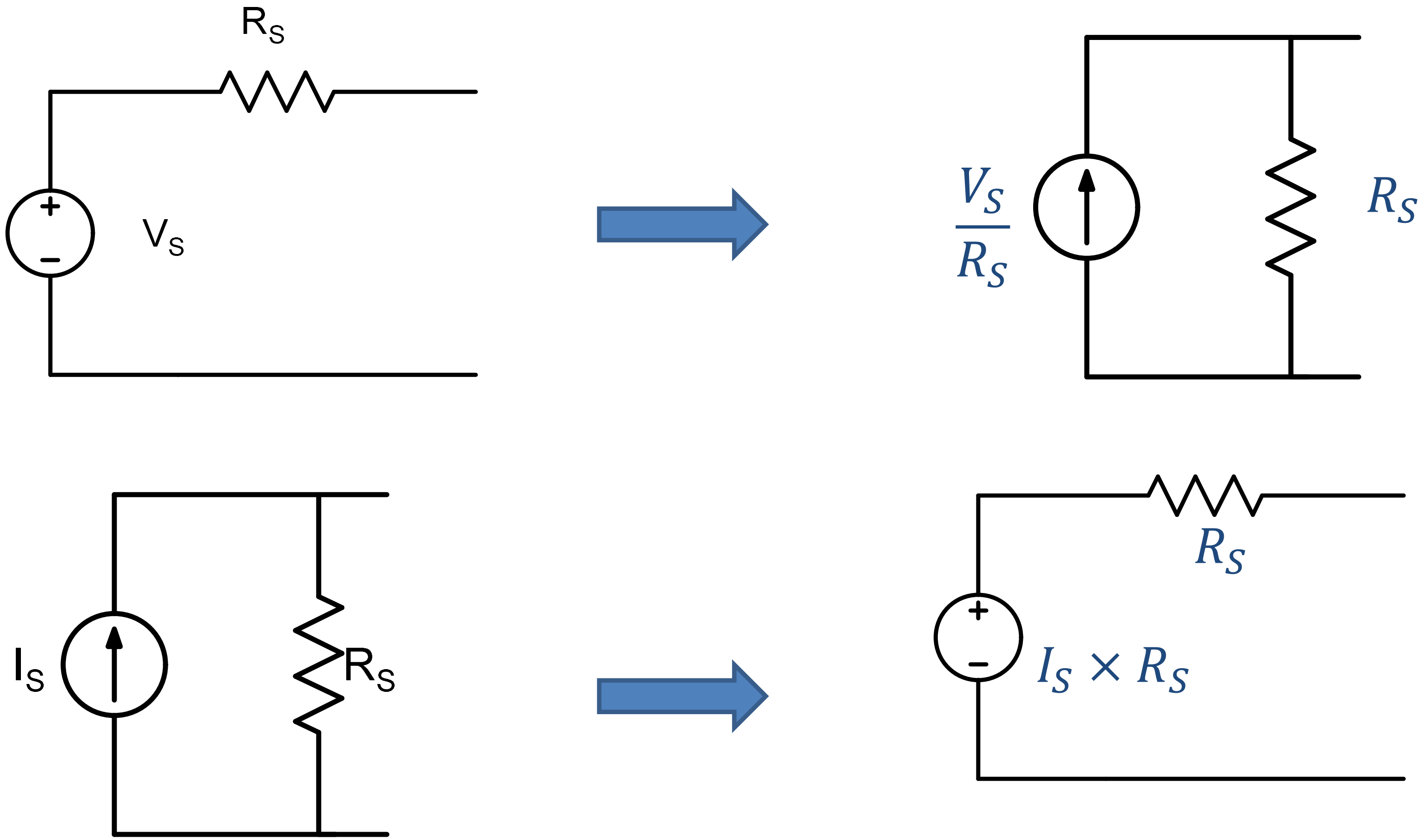
Maximum Power Transfer : General Case



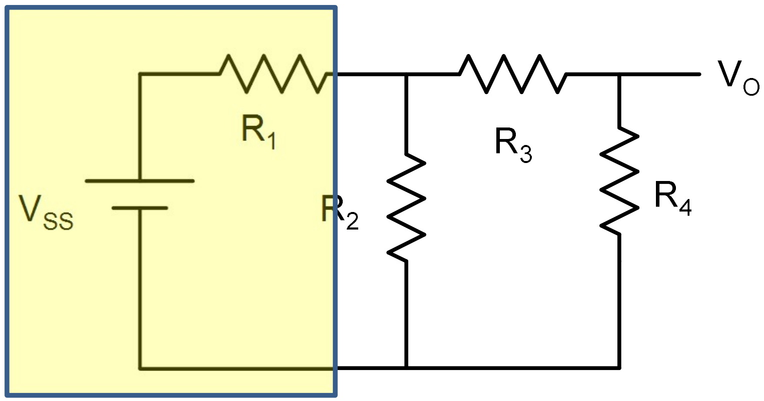


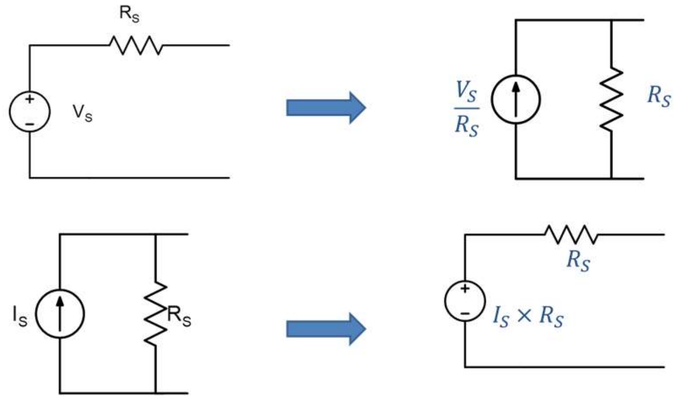
Maximum power is delivered to the load when $\mathbf{R_L = R_{TH}}$

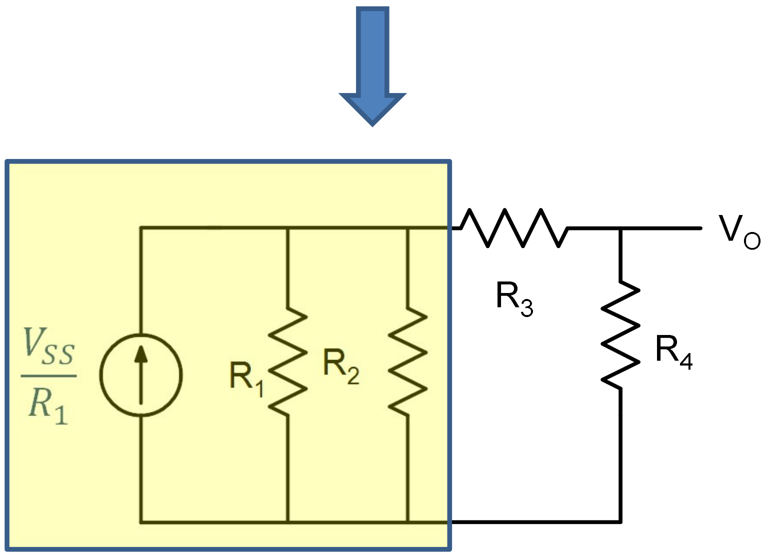
Source Transformation

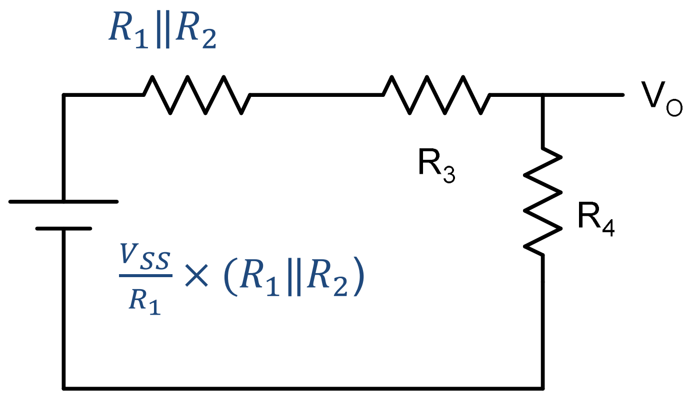


Example









$$\frac{V_{SS}}{R_1} \times R_1 \| R_2 \times \frac{R_4}{R_4 + R_3 + (R_1 \| R_2)}$$