ESC201T : Introduction to Electronics

Lecture 34: Minimization (Kmap)

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Design Flow

System Description

↓

Truth Table

↓

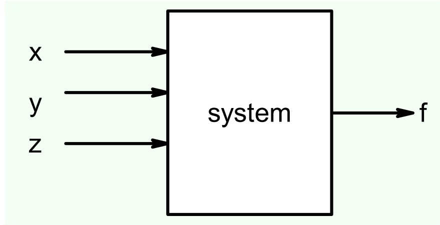
Boolean Expression

↓

Minimized Boolean Expression

↓

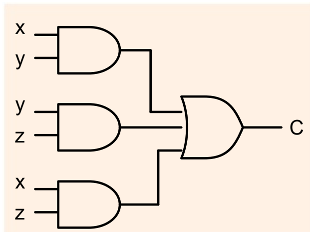
Gate Netlist



x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$\mathrm{f} = \overline{x}.\overline{y}.z + \overline{x}.y.z + x.\overline{y}.z + x.y.z$$

$$\Rightarrow \mathrm{f} = \overline{x}\,.\overline{z} + x.z$$



K-map representation of truth table

x	y	min term
0	0	$\bar{x} . \bar{y}$ m0
0	1	$\bar{x} . y$ m1
1	0	$x . \bar{y}$ m2
1	1	$x . y$ m3

x \ y	0	1
0	m_0	m_1
1	m_2	m_3

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

$\Rightarrow$

x \ y	0	1
0	0	1
1	1	0

$$f_2 = \sum(0,2,3)$$

x \ y	0	1
0	1	0
1	1	1

x \ y	0	1
0	1	0
1	0	1

$$f = \overline{x}.\overline{y} + x.y$$

3-variable K-map representation

x	y	z	min terms	
0	0	0	$\overline{x}\,.\,\overline{y}\,.\,\overline{z}$	m0
0	0	1	$\overline{x}\,.\,\overline{y}\,.\,z$	m1
0	1	0	$\overline{x}\,.\,y\,.\,\overline{z}$	m2
0	1	1	$\overline{x}\,.\,y\,.\,z$	m3
1	0	0	$x\,.\,\overline{y}\,.\,\overline{z}$	m4
1	0	1	$x\,.\,\overline{y}\,.\,z$	m5
1	1	0	$x\,.\,y\,.\,\overline{z}$	m6
1	1	1	$x\,.\,y\,.\,z$	m7

x \ yz	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$$\mathrm{f} = \overline{x}.\overline{y}.\overline{z} + \overline{x}.y.z + x.\overline{y}.z + x.y.z$$

4-variable K-map representation

w	x	y	z	min terms
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
⋮	⋮	⋮	⋮	⋮
1	1	1	0	m_{14}
1	1	1	1	m_{15}

wx \ yz	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$
\begin{aligned}
f = {} & \overline{w}.\overline{x}.\overline{y}.\overline{z} + \overline{w}.\overline{x}.y.z + \overline{w}.x.\overline{y}.z + \overline{w}.x.y.z \\
& + w.x.\overline{y}.\overline{z} + w.x.y.\overline{z} + w.\overline{x}.\overline{y}.\overline{z}
\end{aligned}
$$

Minimization using Kmap

$$f_2 = \sum(2,3)$$

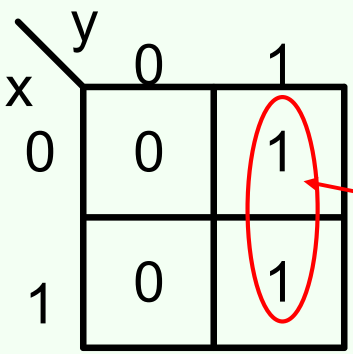
$$f = x.\overline{y} + x.y$$

$$f = x.(\overline{y} + y)$$

$$f = x$$

x \ y	0	1
0	0	0
1	1	1

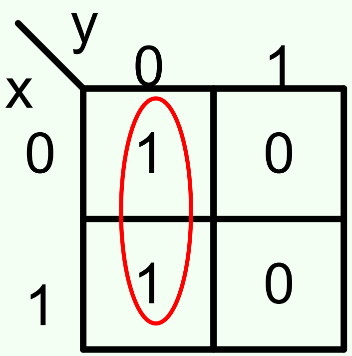
Combine terms which differ in only one bit position. As a result, whatever is common remains.



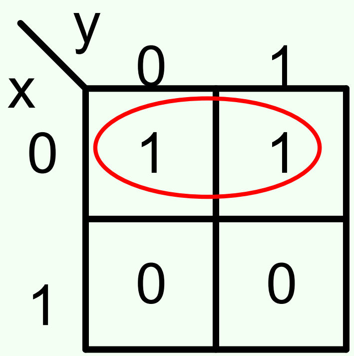
$$f = \bar{x}.y + x.y$$

$$f = (\bar{x}. + x).y$$

$$\Rightarrow f = y$$



$$\Rightarrow f = \bar{y}$$



$$\Rightarrow f = \bar{x}$$

Principle: $x + \overline{x} = 1$ and $x + x = x$

$$f_2 = \sum(0,2,3)$$

x \ y	0	1
0	0	1
1	1	1

$$f = x.\overline{y} + x.y + \overline{x}.y$$

$$f = x.(\overline{y} + y) + \overline{x}.y$$
$$= x + \overline{x}.y$$

$$f = x + \overline{x}.y + x.y$$
$$= x + (\overline{x} + x).y$$
$$= x + y$$

The idea is to cover all the 1’s with as few and as simple terms as possible

3-variable minimization

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$$y.z$$

$$x.z$$

$$\mathrm{f} = \overline{x}.\overline{y}.\overline{z} + \overline{x}.y.z + x.y.z + x.\overline{y}.z$$

$$\mathrm{f} = \overline{x}.\overline{y}.\overline{z} + y.z + x.z$$

3-variable minimization

x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

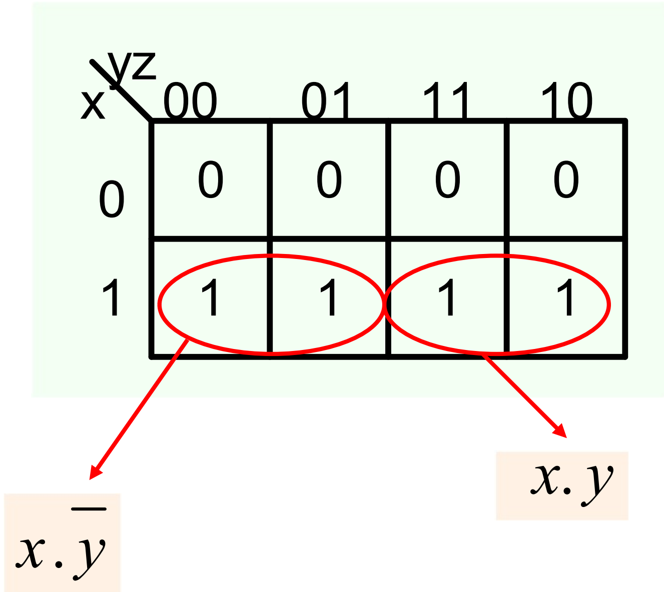
$$f = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.\bar{z} + x.y.z + x.\bar{y}.z$$

$\bar{x}.\bar{z}$

$x.z$

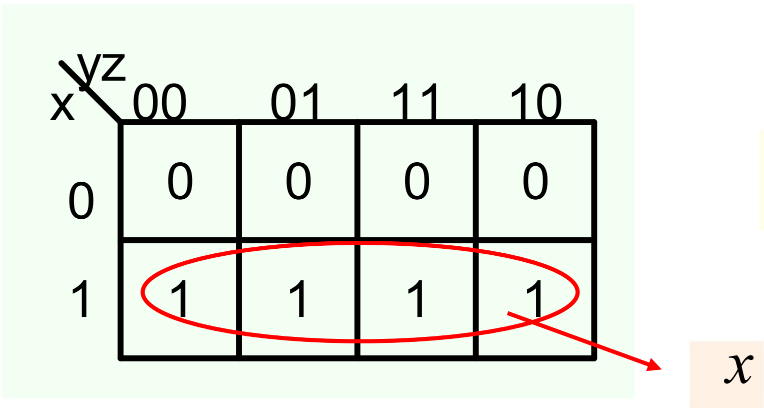
$$f = \bar{x}.\bar{z} + x.z$$

3-variable minimization



$$f = x.\overline{y}.\overline{z} + x.\overline{y}.z + x.y.z + x.y.\overline{z}$$

$$f = x.\overline{y} + x.y$$



$$f = x.(\overline{y} + y) = x$$

x \ yz	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$\bar{x}$

x \ yz	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$\bar{z}$

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

z

x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	1	1

$\bar{z}$

x

$$f = x + \bar{z}$$

Can we do this ?

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Note that each encirclement should represent a single product term. In this case it does not.

$$
\begin{aligned}
\mathrm{f} &= x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z \\
&= x.\bar{y} + x.z
\end{aligned}
$$

We do not get a single product term. In general we cannot make groups of 3 terms.

Can we use kmap with the following ordering of variables?

x \ yz	00	01	10	11
0	0	0	0	0
1	0	1	1	0

Can we combine these two terms into a single term ?

$$\mathrm{f} = x.\overline{y}.z + x.y.\overline{z}$$
$$= x.(\overline{y}.z + y.\overline{z})$$

Note that no simplification is possible. Kmap requires information to be represented

x \ yz	00	01	10	11
0	0	1	0	1
1	0	0	0	0

These two terms can be combined into a single term but it is not easy to show that on the diagram.

$$
\begin{aligned}
\mathrm{f} &= \overline{x}.\overline{y}.z + \overline{x}.y.z \\
&= \overline{x}.(\overline{y} + y).z = \overline{x}.z
\end{aligned}
$$

Kmap requires information to be represented in such a way that it is easy to apply the principle $x + x = 1$

4-variable minimization

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$\overline{w}.y.z$

$\overline{w}.x.z$

$w.\overline{y}.\overline{z}$

$$f = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + \overline{w}.\overline{x}.\overline{y}.\overline{z} + w.x.y.\overline{z}$$

But is this the simplest expression ?

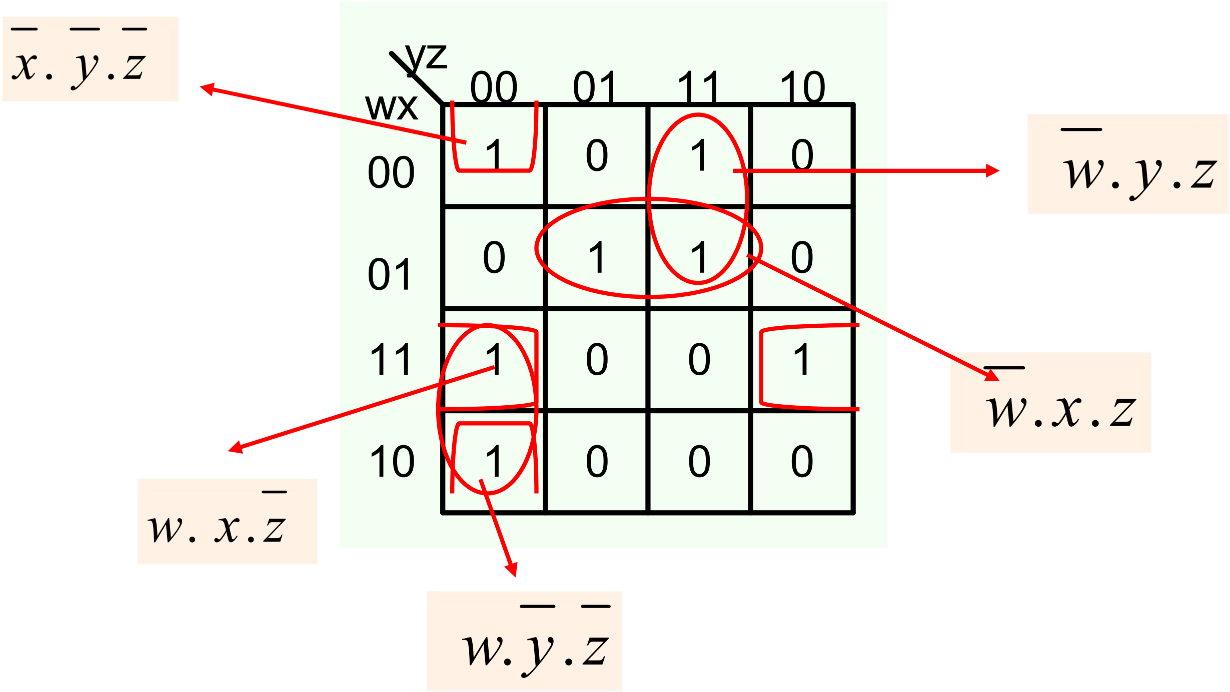
wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$w.\,x.\overline{y}.\overline{z}+w.\,x.y.\overline{z}=w.\,x.\overline{z}$$

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$w.\,\overline{x}.\overline{y}.\overline{z}+\overline{w}.\,\overline{x}.\overline{y}.\overline{z}=\overline{x}.\,\overline{y}.\overline{z}$$

4-variable minimization



$$f = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

Is this the best that we can do ?

Cover the 1's with minimum number of terms

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$\mathrm{f} = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$\mathrm{f} = \overline{w}.y.z + \overline{w}.x.z + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

4-variable minimization

wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

$$\mathrm{f} = \overline{w}.x.\overline{y} + w.\overline{x}.\overline{z} + \overline{w}.\overline{y}.\overline{z}$$

$$\mathrm{f} = \overline{w}.x.\overline{y} + w.\overline{x}.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

Groups of 4

wx \ yz	00	01	11	10
00	0	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	0	0

$\overline{w}.x$

$\overline{y}.z$

wx \ yz	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$x.z$

$w.z$

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$x.\bar{z}$

wx \ yz	00	01	11	10
00	0	1	1	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	0

$\bar{x}.z$

wx \ yz	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$\bar{x}.\bar{z}$

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	0	0	0
11	0	0	0	0
10	1	0	1	0

??

Groups of 8

wx \ yz	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

z

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

x

wx \ yz	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$\overline{z}$

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

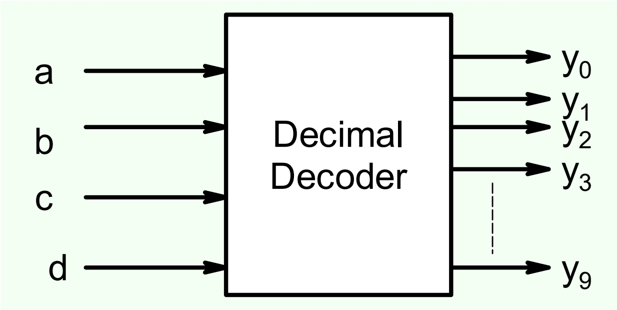
$\overline{x}$

Examples

wx \ yz	00	01	11	10
00	0	1	0	1
01	1	1	1	1
11	1	1	1	1
10	0	0	0	1

wx \ yz	00	01	11	10
00	0	1	0	1
01	1	1	0	1
11	1	1	1	1
10	0	0	0	1

Don't care terms



Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

$$y_3 = \overline{a}.\overline{b}.c.d$$

a	b	c	d	$y_0 y_1 y_2 y_3 y_4 y_5 y_6 y_7 y_8 y_9$
0	0	0	0	1 0 0 0 0 0 0 0 0 0
0	0	0	1	0 1 0 0 0 0 0 0 0 0
0	0	1	0	0 0 1 0 0 0 0 0 0 0
0	0	1	1	0 0 0 1 0 0 0 0 0 0
0	1	0	0	0 0 0 0 1 0 0 0 0 0
0	1	0	1	0 0 0 0 0 1 0 0 0 0
0	1	1	0	0 0 0 0 0 0 1 0 0 0
0	1	1	1	0 0 0 0 0 0 0 1 0 0
1	0	0	0	0 0 0 0 0 0 0 0 1 0
1	0	0	1	0 0 0 0 0 0 0 0 0 1
1	0	1	0	x x x x x x x x x x
1	0	1	1	x x x x x x x x x x
1	1	0	0	x x x x x x x x x x
1	1	0	1	x x x x x x x x x x
1	1	1	0	x x x x x x x x x x
1	1	1	1	x x x x x x x x x x

Don’t care terms can be chosen as 0 or 1. Depending on the problem, we can choose the don’t care term as 1 and use it to obtain a simpler Boolean expression

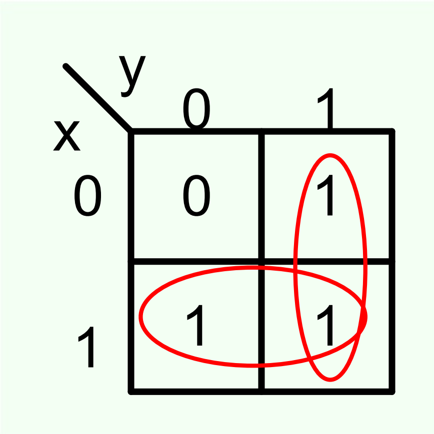
Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

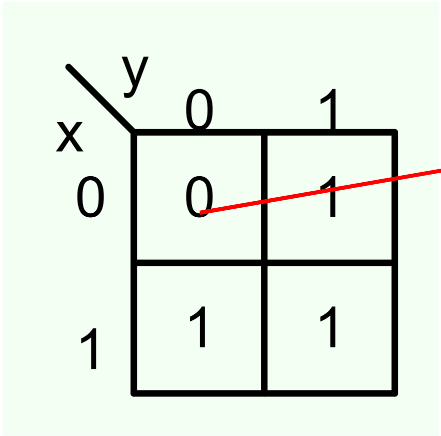
$$y_3 = \overline{b}.c.d$$

Don’t care terms should only be included in encirclements if it helps in obtaining a larger grouping or smaller number of groups.

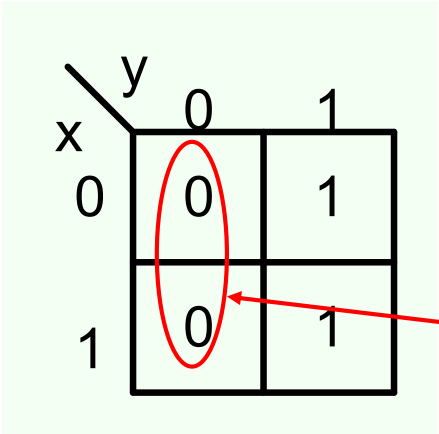
Minimization of Product of Sum Terms using Kmap



$$f = x + \bar{x}.y + x.y$$
$$= x + (\bar{x} + x).y$$
$$= x + y$$



$$f = x + y$$



$$f = y$$

x \ y	0	1
0	1	0
1	1	0

$$\Rightarrow \mathrm{f} = \overline{y}$$

x \ y	0	1
0	1	1
1	0	0

$$\Rightarrow \mathrm{f} = \overline{x}$$

x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$$\overline{x}. + z$$

$$x + \overline{z}$$

$$\mathrm{f} = (\overline{x}\ . + z).(x + \overline{z})$$

$$\Rightarrow \mathrm{f} = \overline{x}\ .\overline{z} + x.z$$

$x + y + z$

wx \ yz	00	01	11	10
00	0	1	0	1
01	1	1	1	1
11	1	0	1	1
10	0	0	0	0

$x + \overline{y} + \overline{z}$

$\overline{w} + y + \overline{z}$

$\overline{w} + x$

$$f = (x + y + z).(x + \overline{y} + \overline{z}).(\overline{w} + y + \overline{z}).(\overline{w} + x)$$

Example

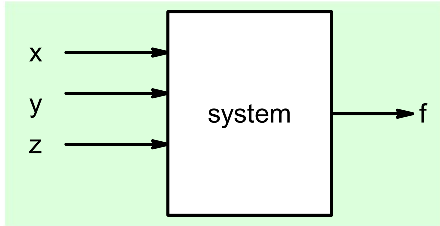
Obtain the minimized PoS by suitably using don’t care terms

wx \ yz	00	01	11	10
00	1	x	0	1
01	1	0	1	1
11	0	x	1	1
10	1	x	1	x

$$f = (x + w + \bar{z}).(\bar{x} + \bar{w} + y).(y + \bar{z})$$

Design Flow

System Description → **Truth Table** → **Boolean Expression** → **Minimized Boolean Expression** → **Gate Netlist**



x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$\mathrm{f} = \bar{x}.\bar{y}.z + \bar{x}.y.z + x.\bar{y}.z + x.y.z$$

$$\Rightarrow \mathrm{f} = \bar{x}\,.\bar{z} + x.z$$