ESC201T : Introduction to Electronics

HW5: Solution

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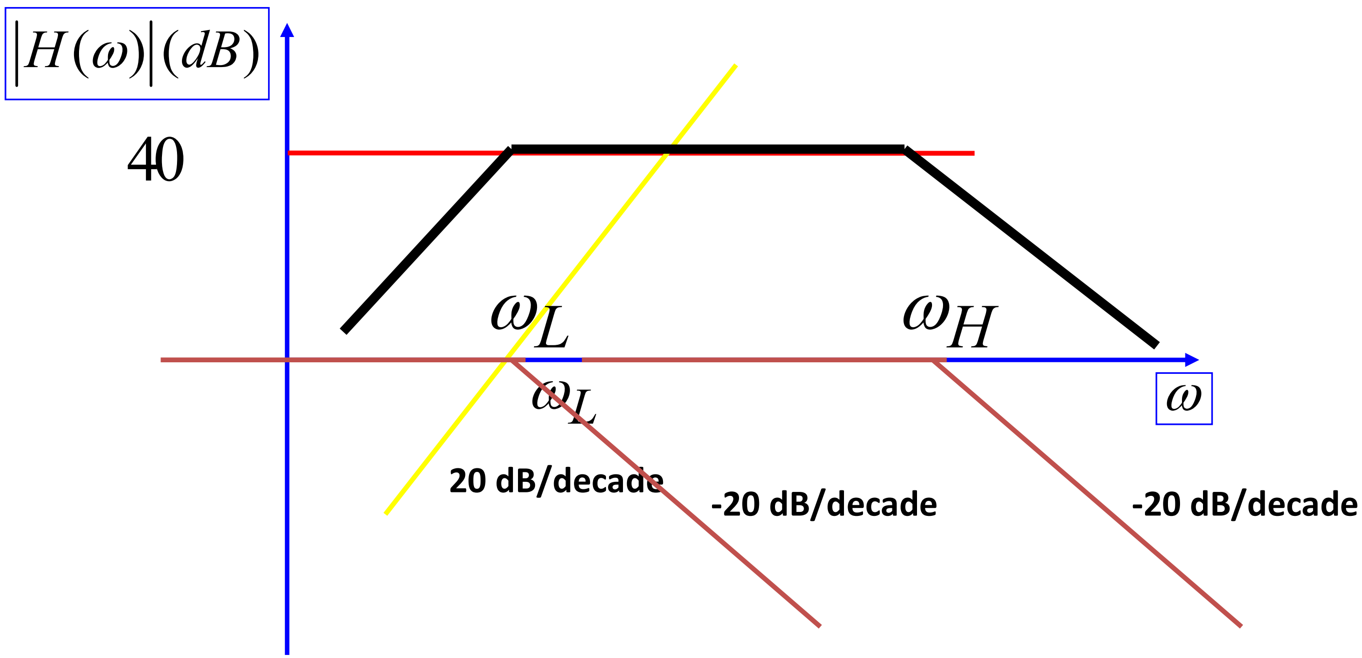
Q.1 An amplifier has a transfer function of the form $G(\omega) = \dfrac{V_O(\omega)}{V_{in}(\omega)} = \dfrac{100 \times j(\omega/\omega_L)}{\{1 + j(\omega/\omega_L)\}\{1 + j(\omega/\omega_H)\}}$. Sketch Bode plot of the transfer function and determine suitable values for corner frequencies such that amplifier can amplify audio frequencies in the range 20-20KHz equally well.

$\omega_H > \omega_L$

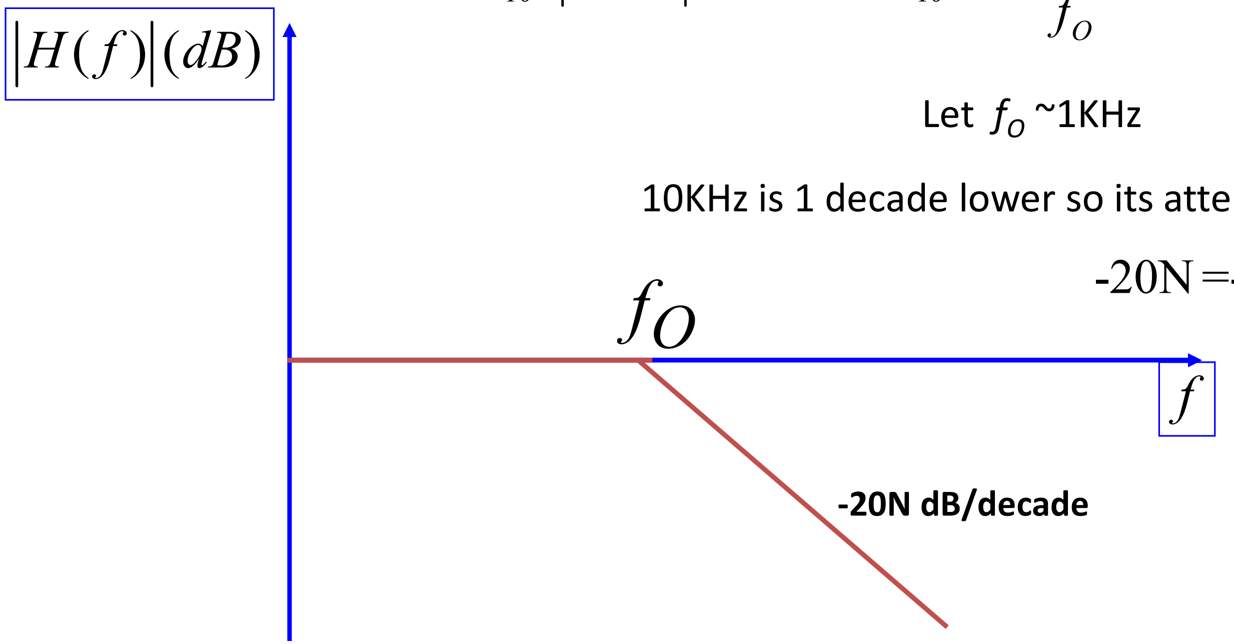
$$20\mathrm{Log}_{10}(|G(\omega)|) = 40 + 10Log_{10}(\frac{\omega}{\omega_L}) - 10Log_{10}(1 + (\frac{\omega}{\omega_L})^2) - 10Log_{10}(1 + (\frac{\omega}{\omega_H})^2)$$



$$\omega_L < 2\pi \times 20Hz; \omega_H > 2\pi \times 20KHz$$

Q.2 There is a low pass filter whose transfer function is of the form $|H(f)| = \dfrac{1}{\sqrt{1+(\frac{f}{f_O})^{2N}}}$. There is a signal with 1KHz and 10KHz sinusoids of equal magnitude. We would like to reject the 10KHz sinusoid. Determine suitable value for fo and N such that magnitude of 10KHz is -60dB lower as compared to 1KHz signal after passing through the filter.

$$20\mathrm{Log}_{10}(|H(f)|) = -10Log_{10}(1+(\frac{f}{f_O})^{2N})$$



Let f_O ~1KHz

10KHz is 1 decade lower so its attenuation will be

$$-20\mathrm{N} = -60 \Rightarrow \mathrm{N}=3$$

Q3 Draw the magnitude and phase Bode plots of the transfer function given below, for ω ranging between 0.01 rad/ sec to 100 rad/sec.

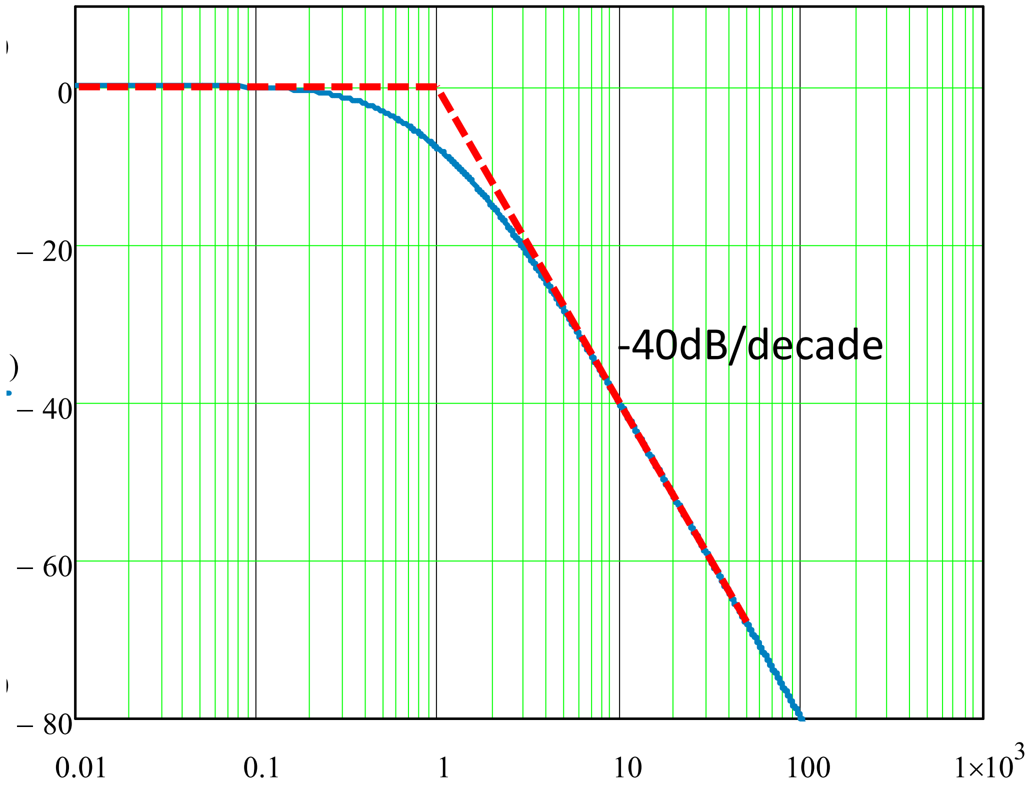
$$H(j\omega) = 2j\omega \,/\, [\,10\,(j\omega)^2 + 25\,(j\omega) + 10\,]$$

$$H(\omega) = \frac{2}{10} \times \frac{j\frac{\omega}{1}}{1 - \left(\frac{\omega}{1}\right)^2 + 2.5 \times j\left(\frac{\omega}{1}\right)}$$

$$H(\omega) = \frac{1}{1 - \left(\frac{\omega}{1}\right)^2 + 2.5 \times j\left(\frac{\omega}{1}\right)}$$

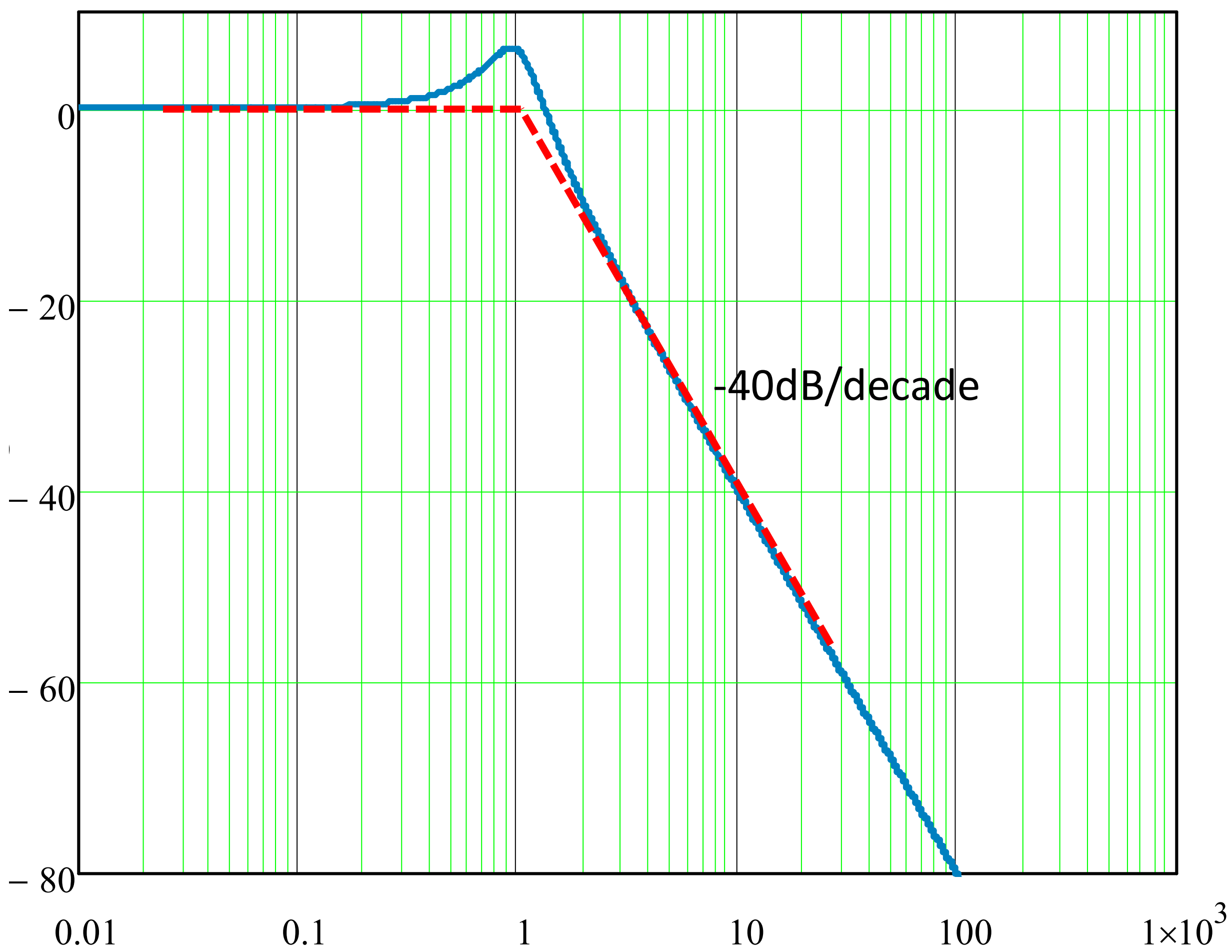
$$|H(1)| = -7.96dB$$

$$H(\omega) \approx \frac{-1}{\left(\frac{\omega}{1}\right)^2} for\ \omega \gg 1$$

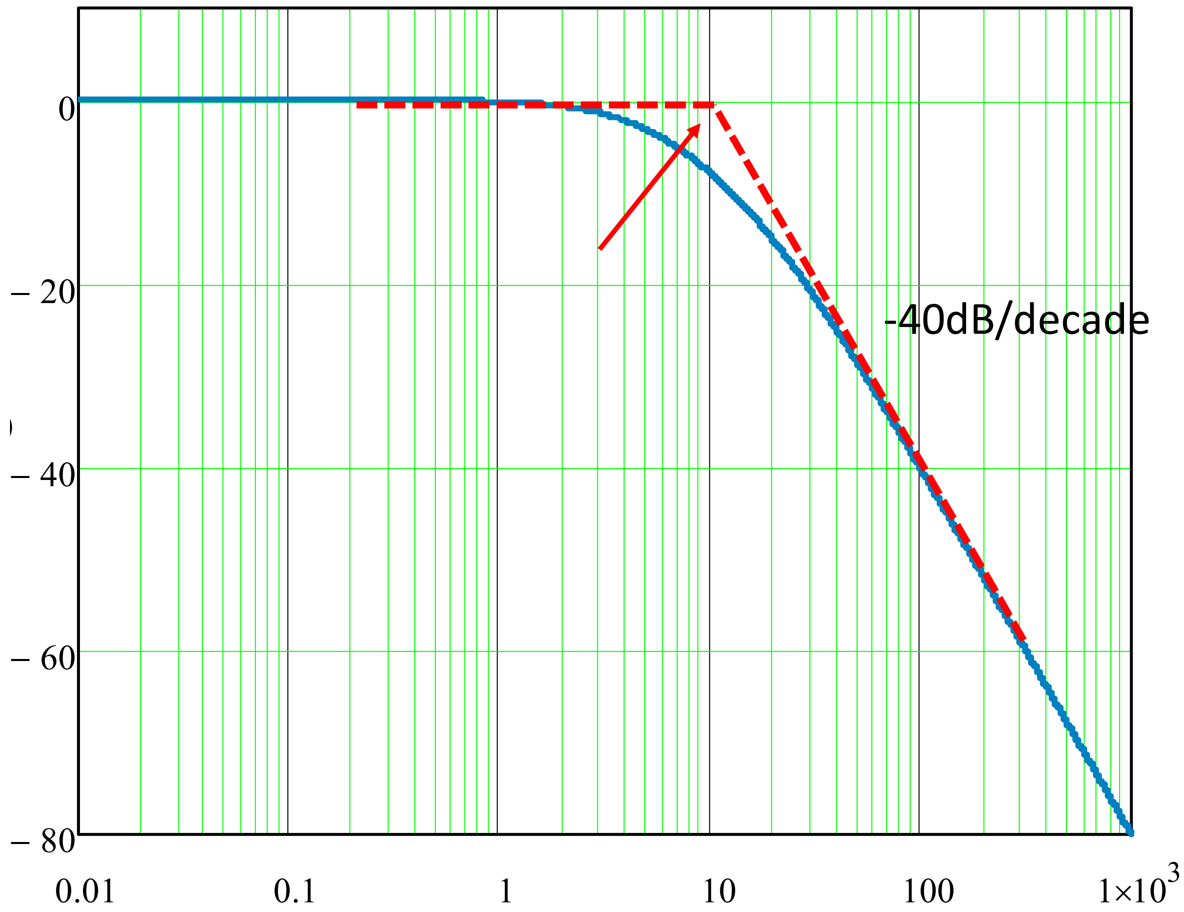


$$H(\omega) = \frac{1}{1 - \left(\frac{\omega}{1}\right)^2 + 0.5 \times j\left(\frac{\omega}{1}\right)}$$

$$|H(1)| = +6\ dB$$

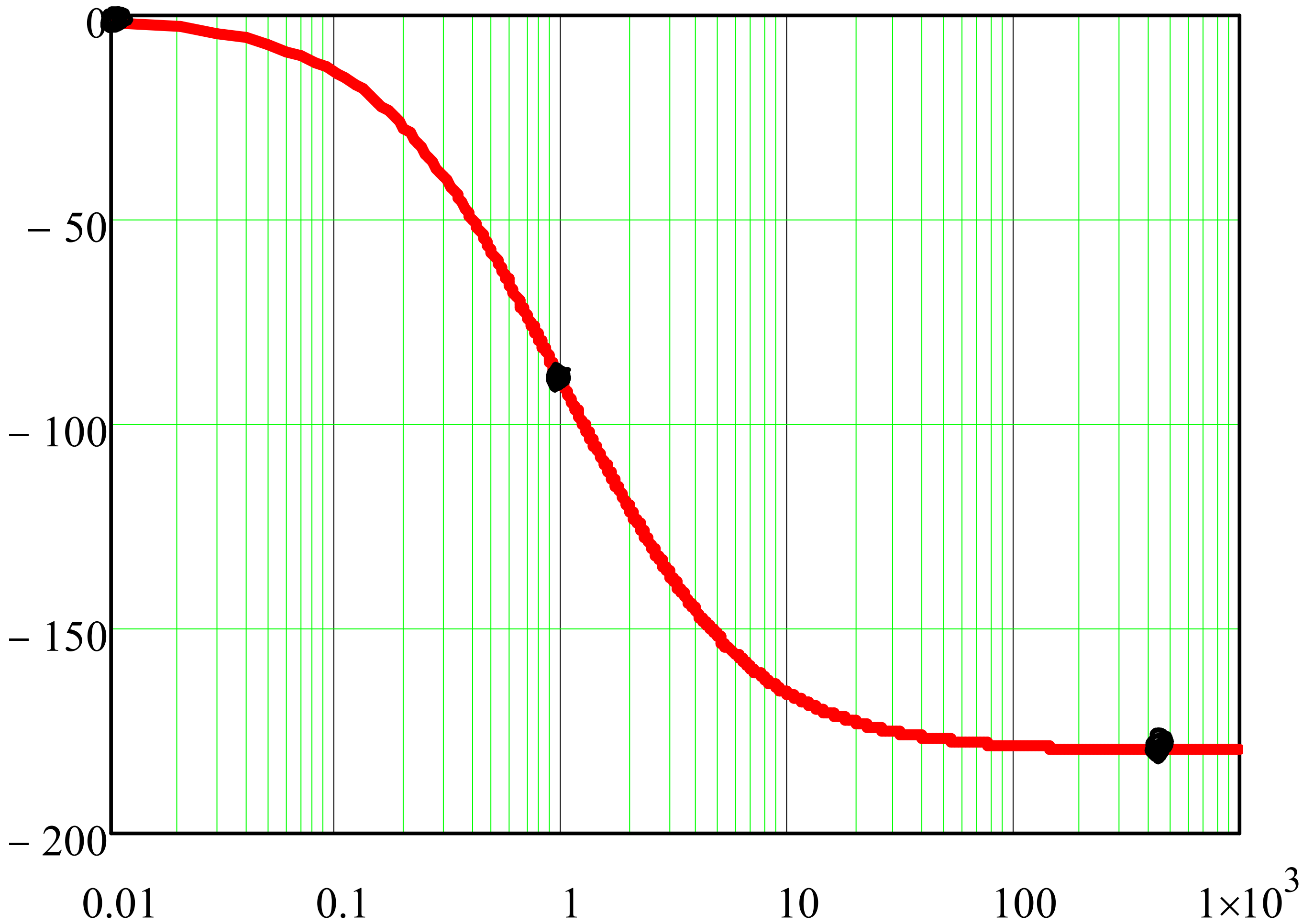


$$H(\omega) = \frac{1}{1 - \left(\frac{\omega}{10}\right)^2 + 2.5 \times j\left(\frac{\omega}{10}\right)}$$

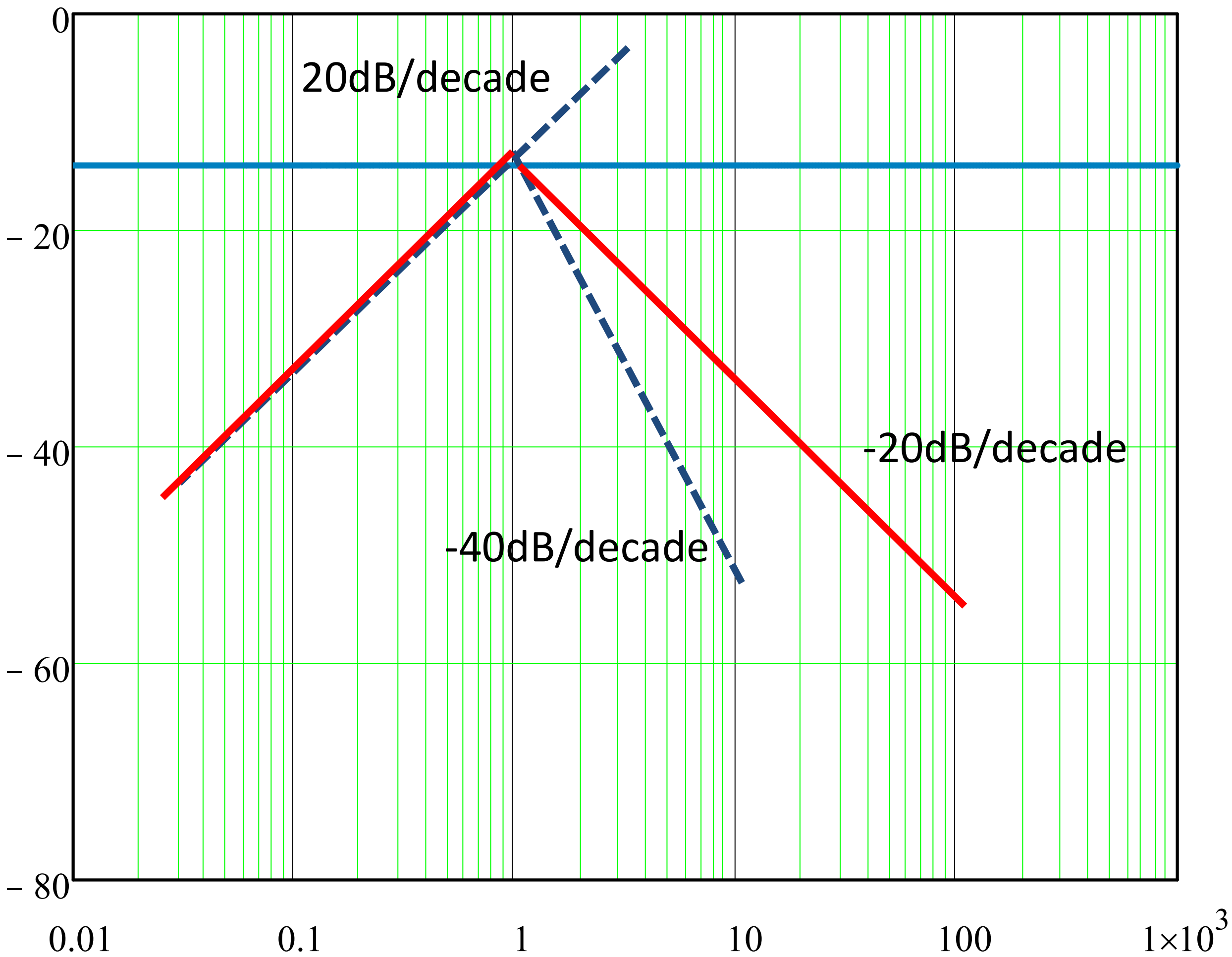


Phase

$$H(\omega) = \frac{1}{1 - \left(\frac{\omega}{1}\right)^2 + 2.5 \times j\left(\frac{\omega}{1}\right)}$$

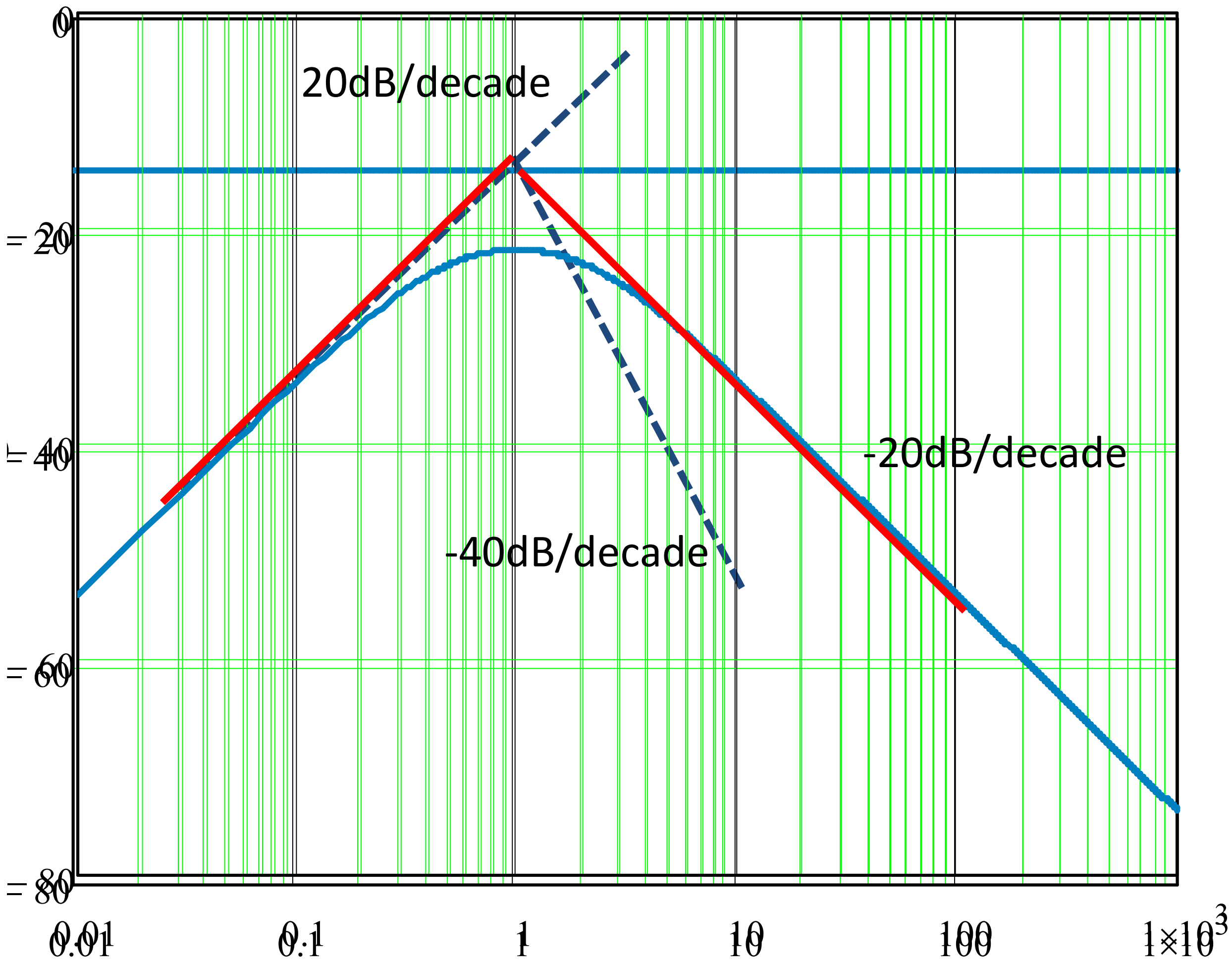


$$H(\omega) = \frac{2}{10} \times \frac{j\frac{\omega}{1}}{1 - \left(\frac{\omega}{1}\right)^2 + 2.5 \times j(\frac{\omega}{1})} \qquad 20\log(|H(\omega)| = -14 + 20\log\left(\frac{\omega}{1}\right) - 10\log(\left(\left(\frac{\omega}{1}\right)^2 - 1\right)^2 + 2.5^2 \times \left(\frac{\omega}{1}\right)^2)$$

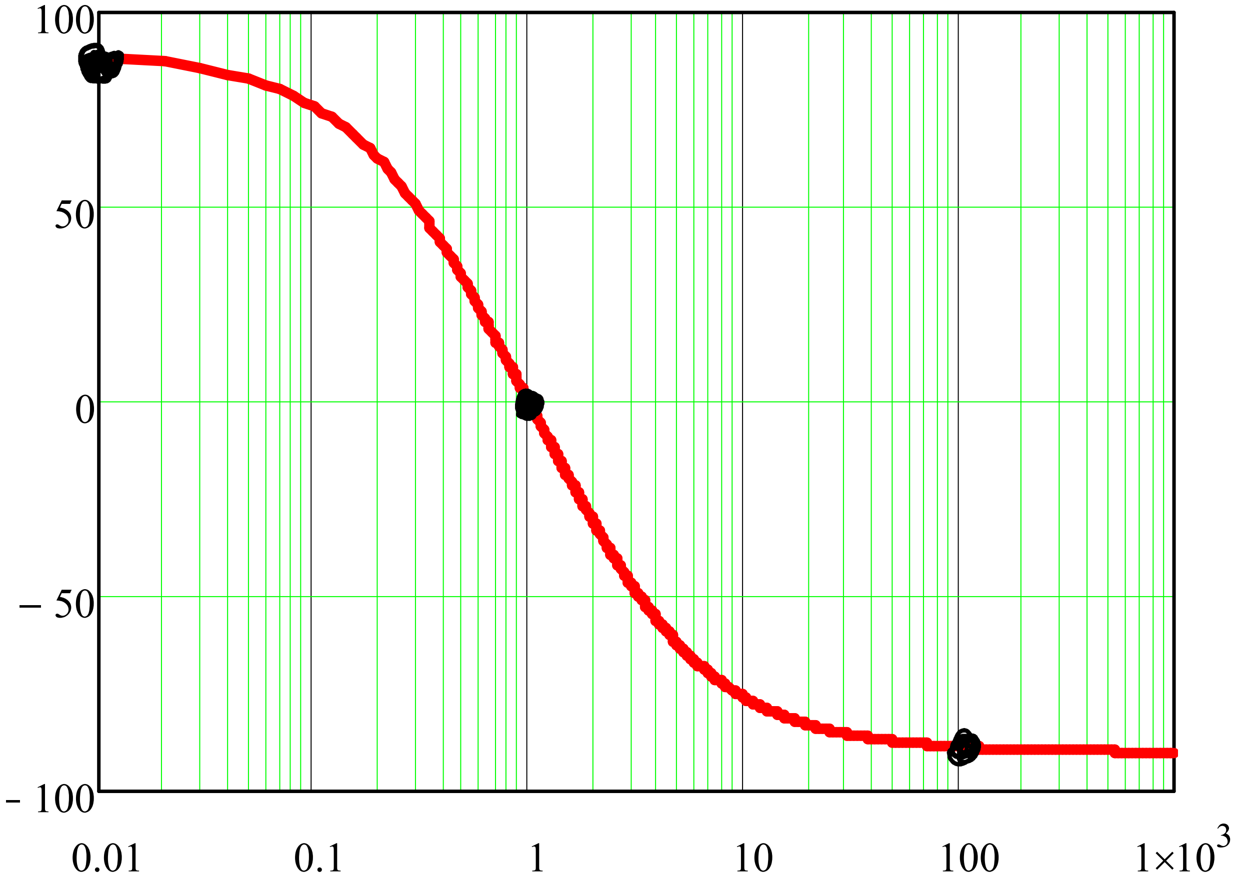


$$H(\omega)=\frac{2}{10}\times\frac{j\frac{\omega}{1}}{1-\left(\frac{\omega}{1}\right)^2+2.5\times j(\frac{\omega}{1})}$$

$$20\log(|H(\omega)|=-14+20\log\left(\frac{\omega}{1}\right)-10\log(\left(\left(\frac{\omega}{1}\right)^2-1\right)^2+2.5^2\times\left(\frac{\omega}{1}\right)^2)$$

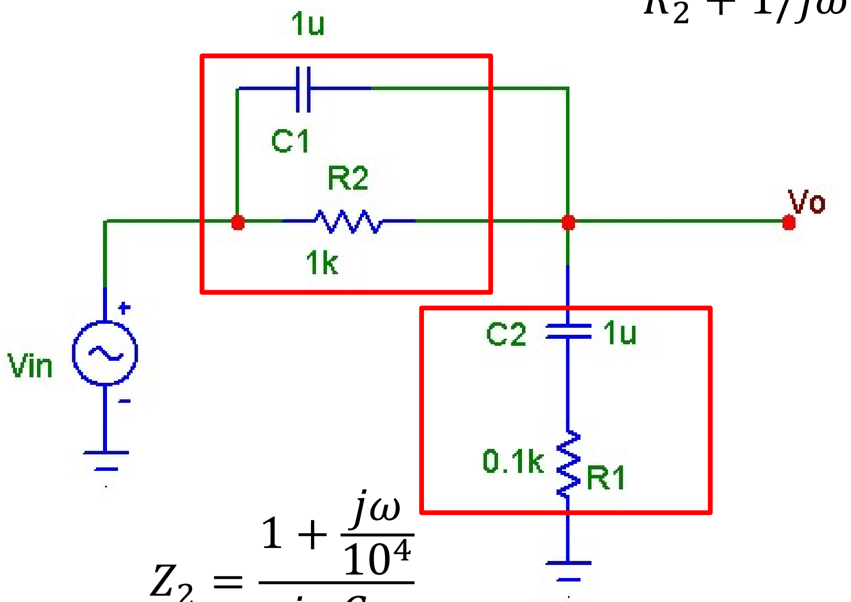


$$H(\omega) = \frac{2}{10} \times \frac{j\frac{\omega}{1}}{1 - \left(\frac{\omega}{1}\right)^2 + 2.5 \times j\left(\frac{\omega}{1}\right)}$$



Q.4 Derive the transfer function and sketch the Magnitude Bode plot of the circuit shown

$$Z_1 = R_2 \times \frac{\frac{1}{j\omega C}}{R_2 + 1/j\omega C} = \frac{10^3}{1 + \frac{j\omega}{10^3}}$$

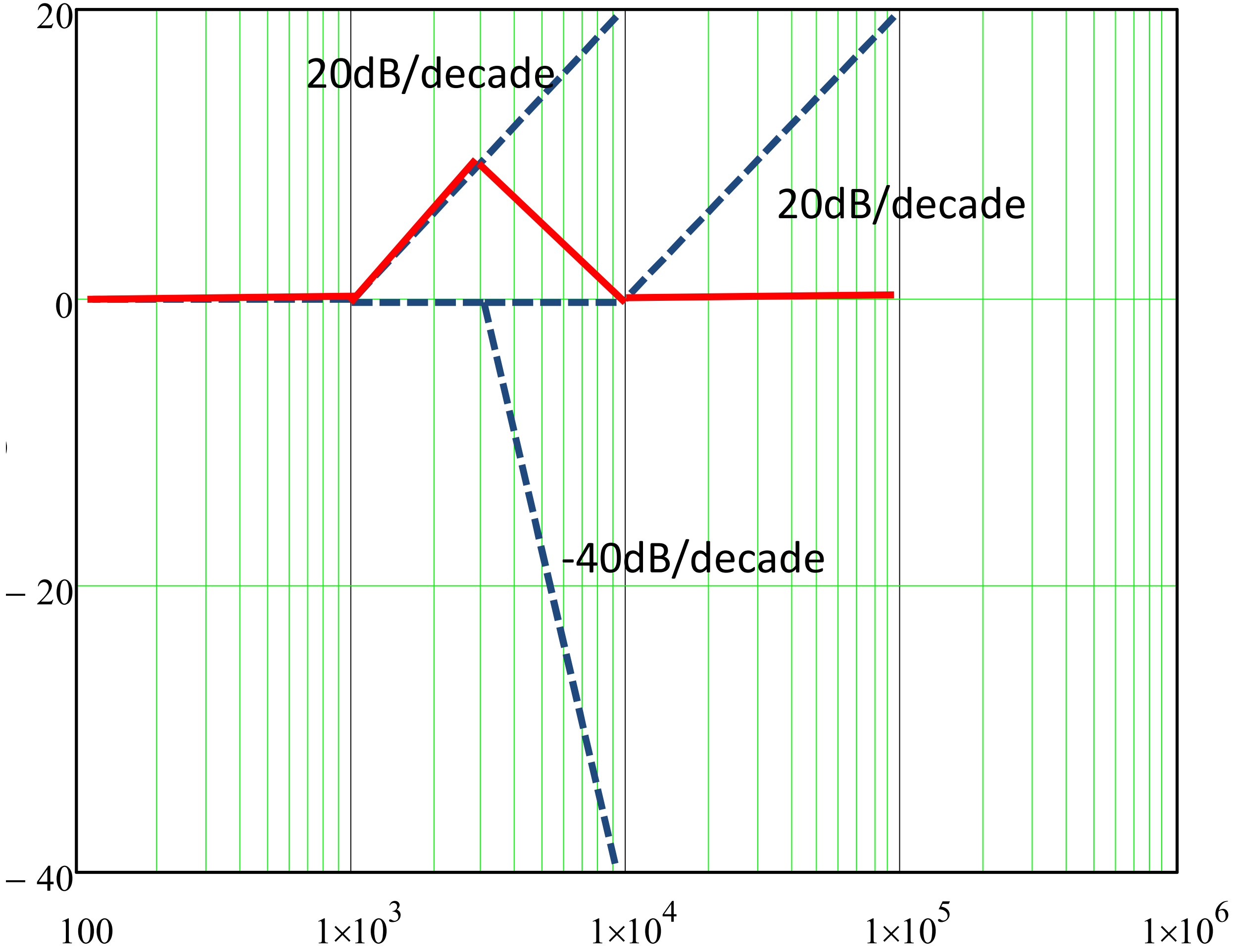


$$H(\omega) = \frac{Z_2}{Z_1 + Z_2}$$

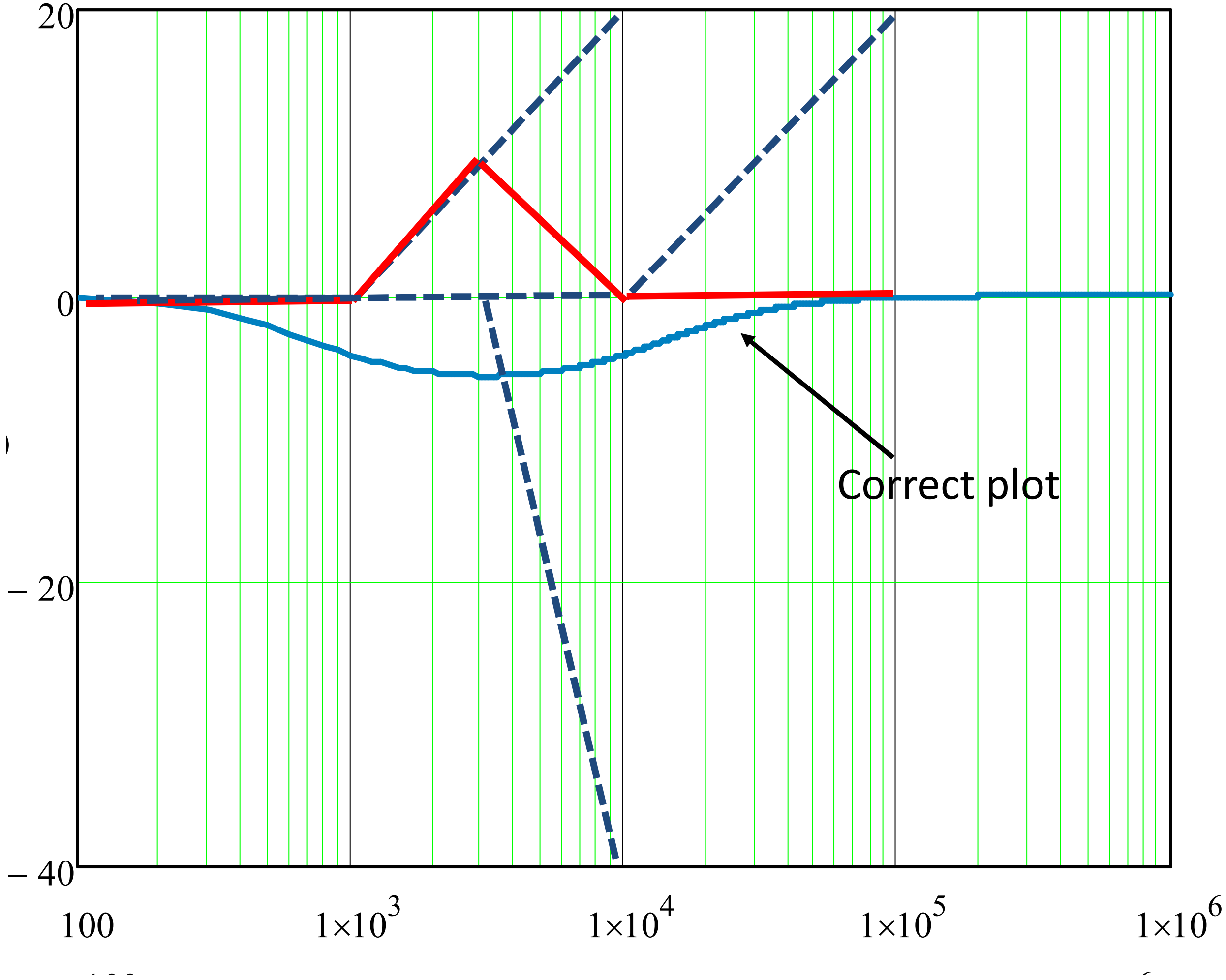
$$Z_2 = \frac{1 + \frac{j\omega}{10^4}}{j\omega C_2}$$

$$\mathrm{H}(\omega) = \frac{\left(1 + \frac{j\omega}{10^4}\right) \times \left(1 + \frac{j\omega}{10^3}\right)}{1 + 6.64j\,\frac{\omega}{\omega_O} + \left(\frac{\omega}{\omega_O}\right)^2}; \omega_O = 3.16 \times 10^3 rad/s$$

$$\mathrm{H}(\omega) = \frac{\left(1 + \frac{j\omega}{10^4}\right) \times \left(1 + \frac{j\omega}{10^3}\right)}{1 + 6.64j\frac{\omega}{\omega_O} + \left(j\frac{\omega}{\omega_O}\right)^2}; \omega_O = 3.16 \times 10^3 rad/s$$



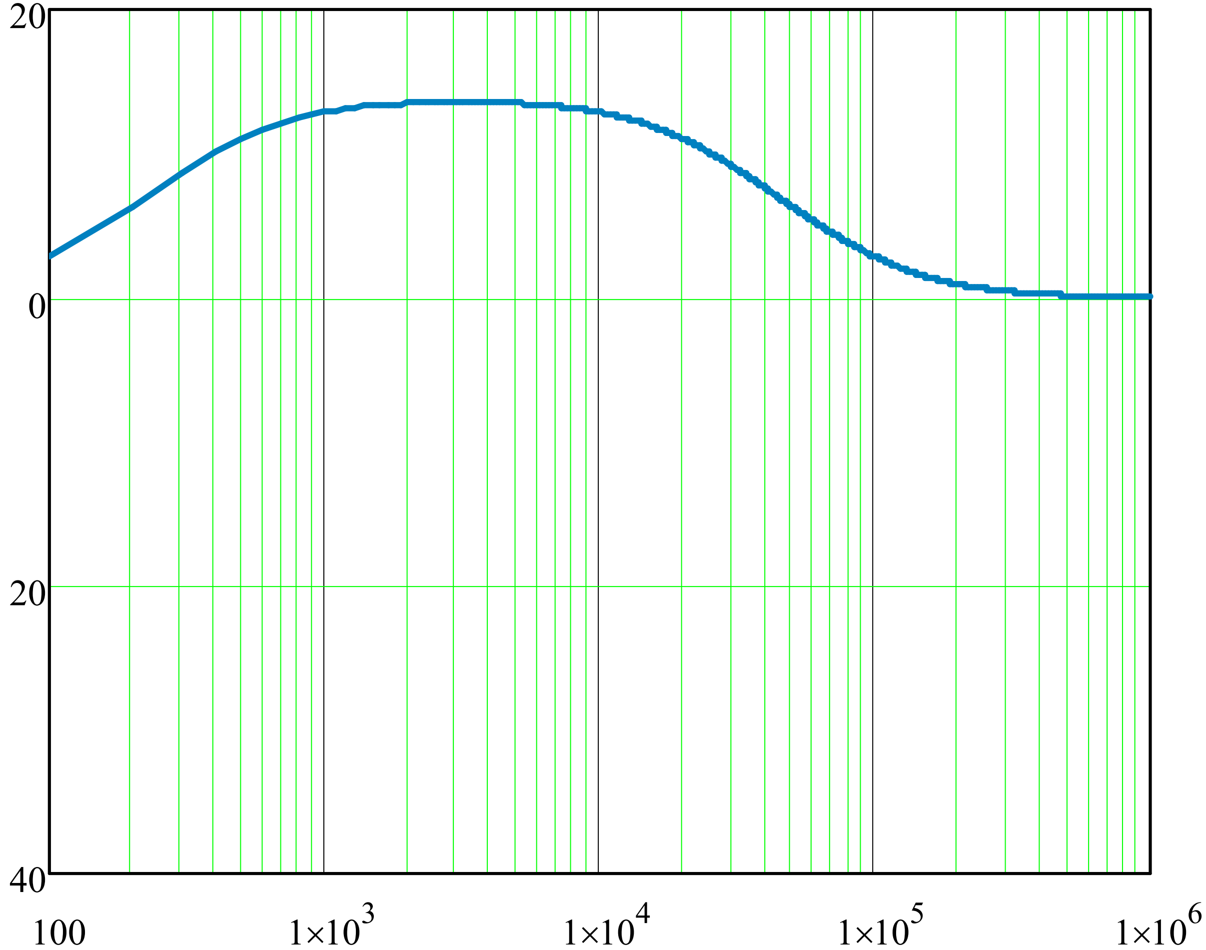
$$H(\omega) = \frac{\left(1 + \frac{j\omega}{10^4}\right) \times \left(1 + \frac{j\omega}{10^3}\right)}{1 + 6.64j\frac{\omega}{\omega_O} + \left(j\frac{\omega}{\omega_O}\right)^2}; \omega_O = 3.16 \times 10^3 rad/s$$



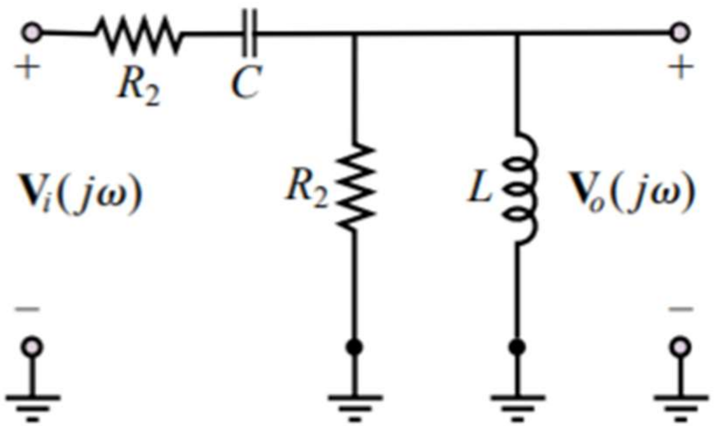
$$\mathrm{H}(\omega) = \frac{\left(1 + \frac{j\omega}{10^4}\right) \times \left(1 + \frac{j\omega}{10^3}\right)}{1 + 6.64j\frac{\omega}{\omega_O} + \left(j\frac{\omega}{\omega_O}\right)^2}; \omega_O = 3.16 \times 10^3 rad/s$$

$$\mathrm{H}(\omega) = \frac{\left(1 + \frac{j\omega}{10^5}\right) \times \left(1 + \frac{j\omega}{10^2}\right)}{1 + 6.64j\frac{\omega}{\omega_O} + \left(j\frac{\omega}{\omega_O}\right)^2}; \omega_O = 3.16 \times 10^3 rad/s$$

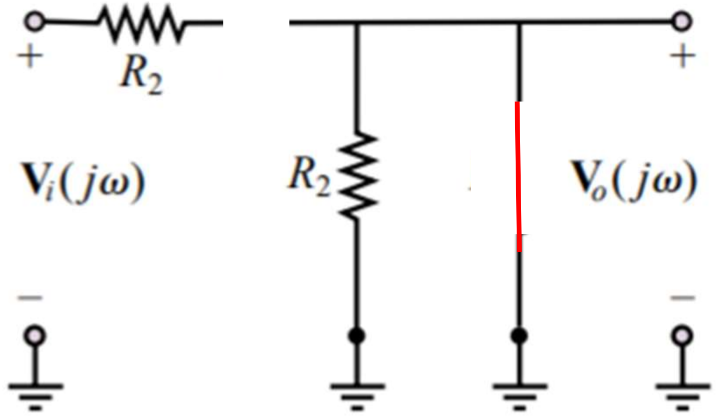
Corner frequencies are sufficiently far apart



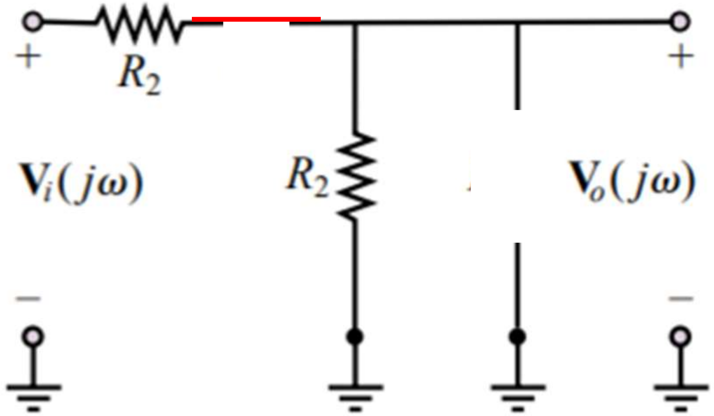
Q.5 Determine the nature of filter for the circuit shown below



Low frequency



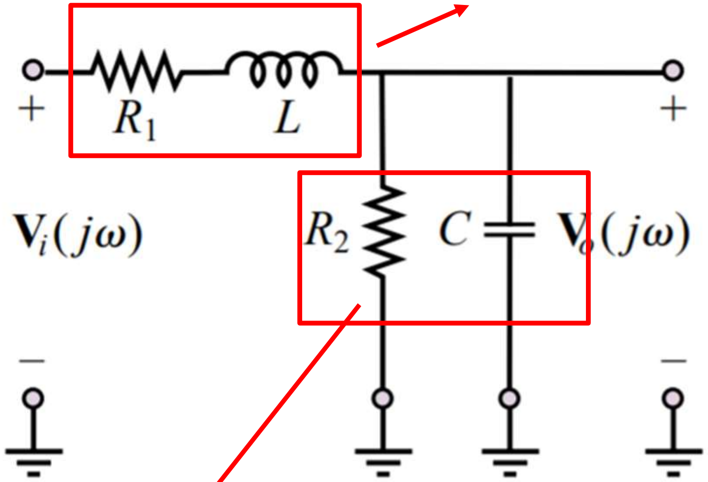
High frequency



$$\Rightarrow \textit{High Pass Filter}$$

Q.6 Determine the transfer function and sketch the Bode plot for the circuit shown below

$$Z_1 = R_1 + j\omega L$$



$L = 11$ mH $\quad C = 0.47$ nF

$R_1 = 2.2$ kΩ $\quad R_2 = 3.8$ kΩ

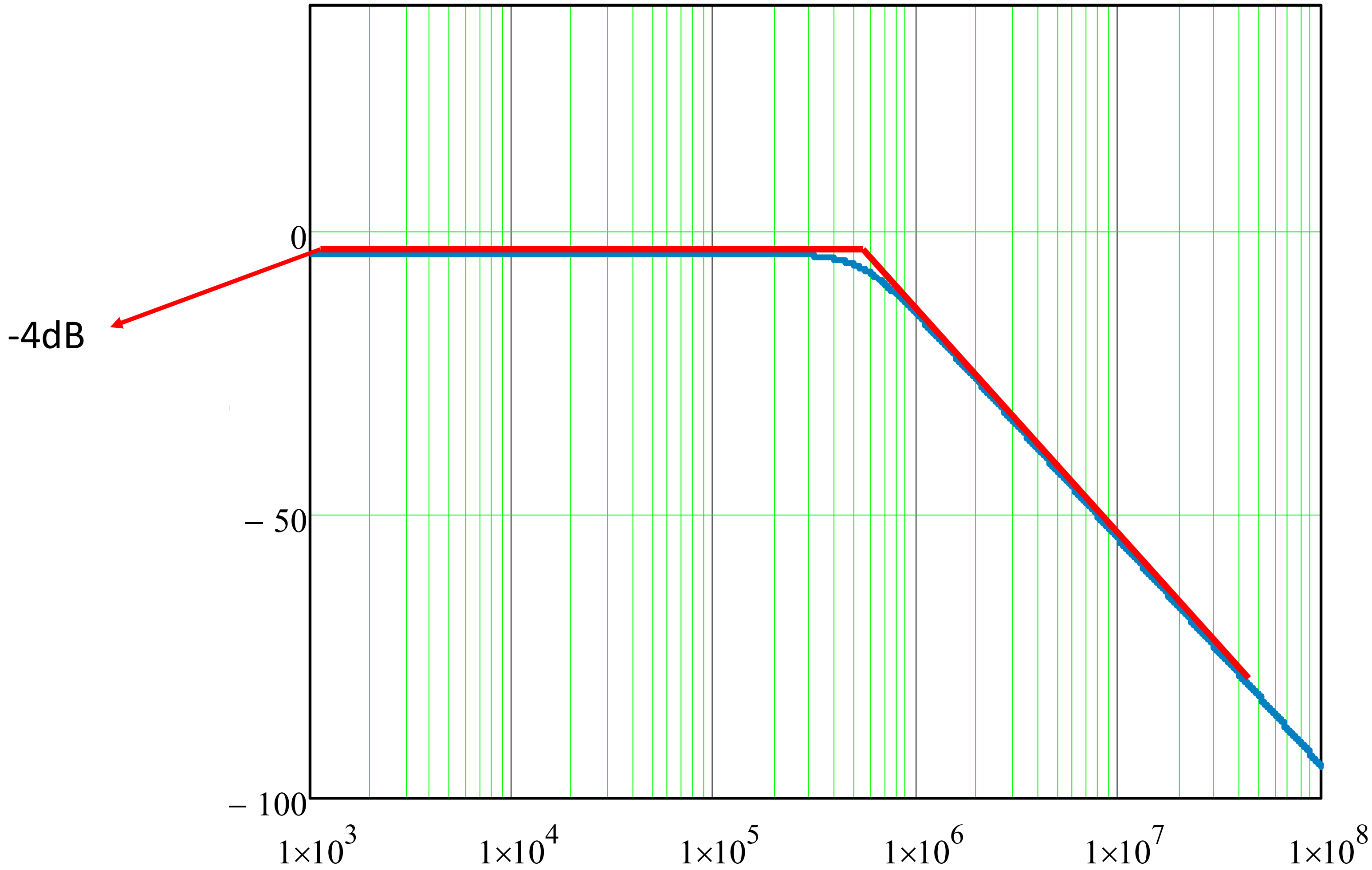
$$H(\omega) = \frac{Z_2}{Z_2 + Z_1} = \frac{\dfrac{R_2}{1 + j\omega R_2 C}}{R_1 + j\omega L + \dfrac{R_2}{1 + j\omega R_2 C}}$$

$$Z_2 = R_2 \times \frac{\dfrac{1}{j\omega C}}{R_2 + 1/j\omega C} = \frac{R_2}{1 + j\omega R_2 C}$$

$$H(\omega) = \frac{R_2}{R_1 + R_2} \times \frac{1}{1 + \dfrac{j\omega L + j\omega R_1 R_2 C}{R_1 + R_2} + \dfrac{(j\omega)^2 (R_2 CL)}{R_1 + R_2}}$$

$$H(\omega) = \frac{0.63}{1 + 1.375\dfrac{j\omega}{\omega_O} + \left(j\dfrac{\omega}{\omega_O}\right)^2}; \omega_O = 5.5 \times 10^5 rad/s$$

$$H(\omega) = \frac{0.63}{1 + 1.375\,\frac{j\omega}{\omega_O} + \left(j\,\frac{\omega}{\omega_O}\right)^2}\,;\omega_O = 5.5\times10^5 rad/s$$



$$H(\omega) = \frac{0.63}{1 + 1.375\frac{j\omega}{\omega_O} + \left(j\frac{\omega}{\omega_O}\right)^2}; \omega_O = 5.5 \times 10^5 rad/s$$