MSO-203 B ASSIGNMENT 1
IIT, KANPUR

18th October , 2020

1. Consider the following functions defined on the interval $[-\pi, \pi]$.

   a) $f(x) = |x|$
   b) $f(x) = |\sin(x)|$
   c) $f(x) = \sin|x|$
   d) $f(x) = x^2$.

   Which one of the above functions admits Fourier series expansion ? (It is understood that the functions are extended 2π periodically to whole of real line). Write down their Fourier series expansion.

2. Find the Fourier even half series of the function $f(x) = x$ on the interval $[0, L]$.

3. Consider the function defined by

$$g(x) := \sum_{n=1}^{\infty} \frac{4\cos((2n+1)x)}{\pi(2n+1)^2} + \sum_{m=1}^{\infty} \frac{2\sin((2m+1)x)}{(2m+1)}, \quad x \in \mathbb{R}.$$

   Then which of the following are correct:

   a) $g(\frac{\pi}{2}) = \frac{\pi}{2}$
   b) $g(0) = 0$
   c) $g(0) = \frac{\pi}{2}$
   d) $g(\frac{\pi}{2}) = -2$

   Hint: Work with the Fourier series of the periodical extension of the following function

$$f(x) = \begin{cases} x, & x \in [-\pi, 0) \\ \pi - x, & x \in (0, \pi]. \end{cases}$$

4. Pick the correct answers from the following:

   a) $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
   b) $\frac{\pi^2}{2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
   c) $0 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos(n\pi)$
   d) $0 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2}\cos(n\pi)$.

   Hint: Work with the Fourier transform of the periodical extension of the following function $f(x) = x^2$ *on the interval* $[-\pi, \pi]$

5. Find the half range series (both even and odd) for the following function:

$$f(x) = \begin{cases} 0, & x \in [0, \frac{\pi}{2}) \\ 1, & x \in [\frac{\pi}{2}, \pi]. \end{cases}$$

6. Apply Parsevals formula to the function $f(x) = x$ on $[-\pi, \pi]$ to find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.