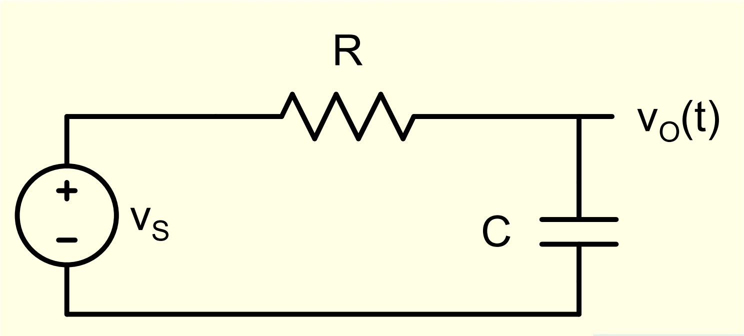
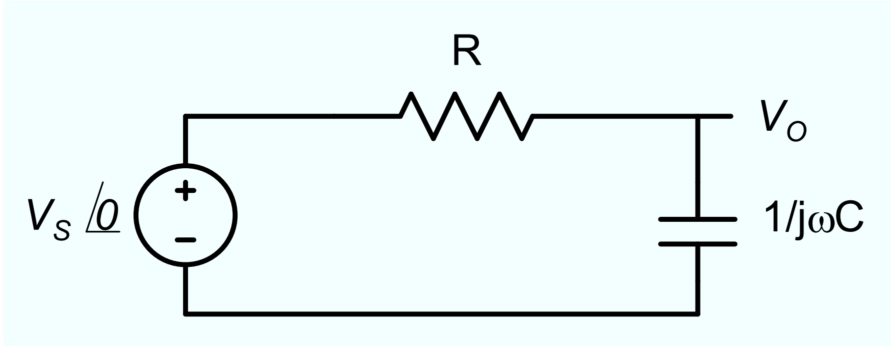
# ESC201T : Introduction to Electronics

## Lecture 16: Bode Plots and Filters

B. Mazhari
Dept. of EE, IIT Kanpur

# Frequency Response



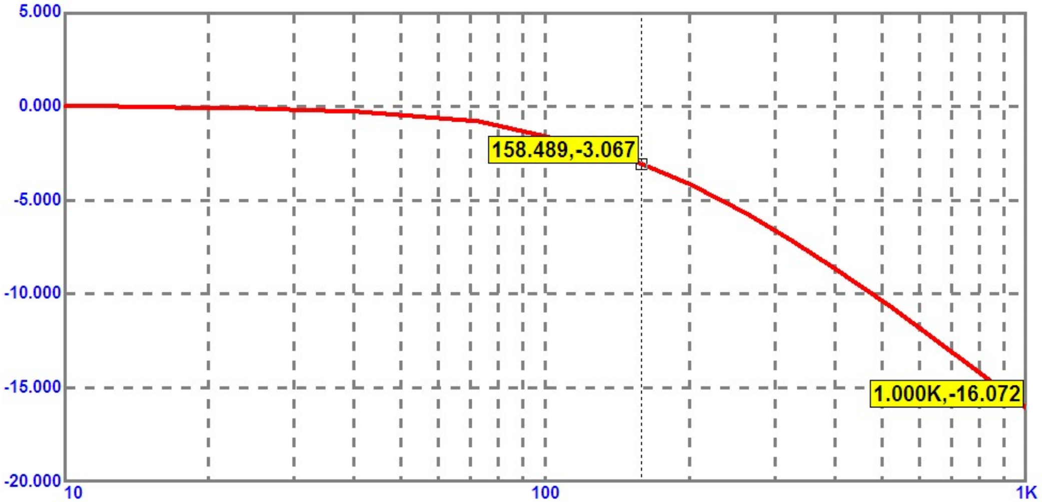

$$v_S\ (t) = v_{So} Cos(\omega t)$$

$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{1}{1 + j\omega CR}$$

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

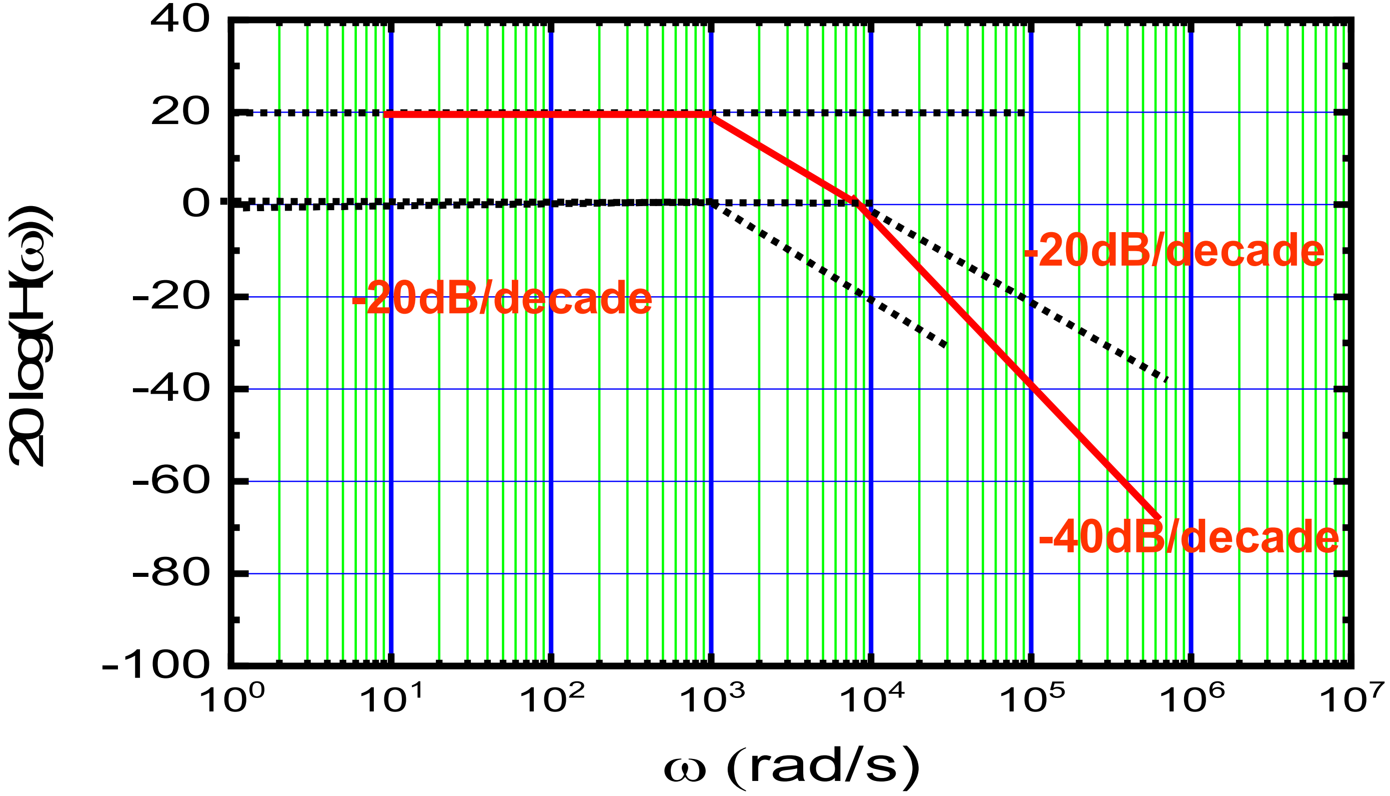
$$\phi(\omega) = -\tan^{-1}(\omega CR)$$

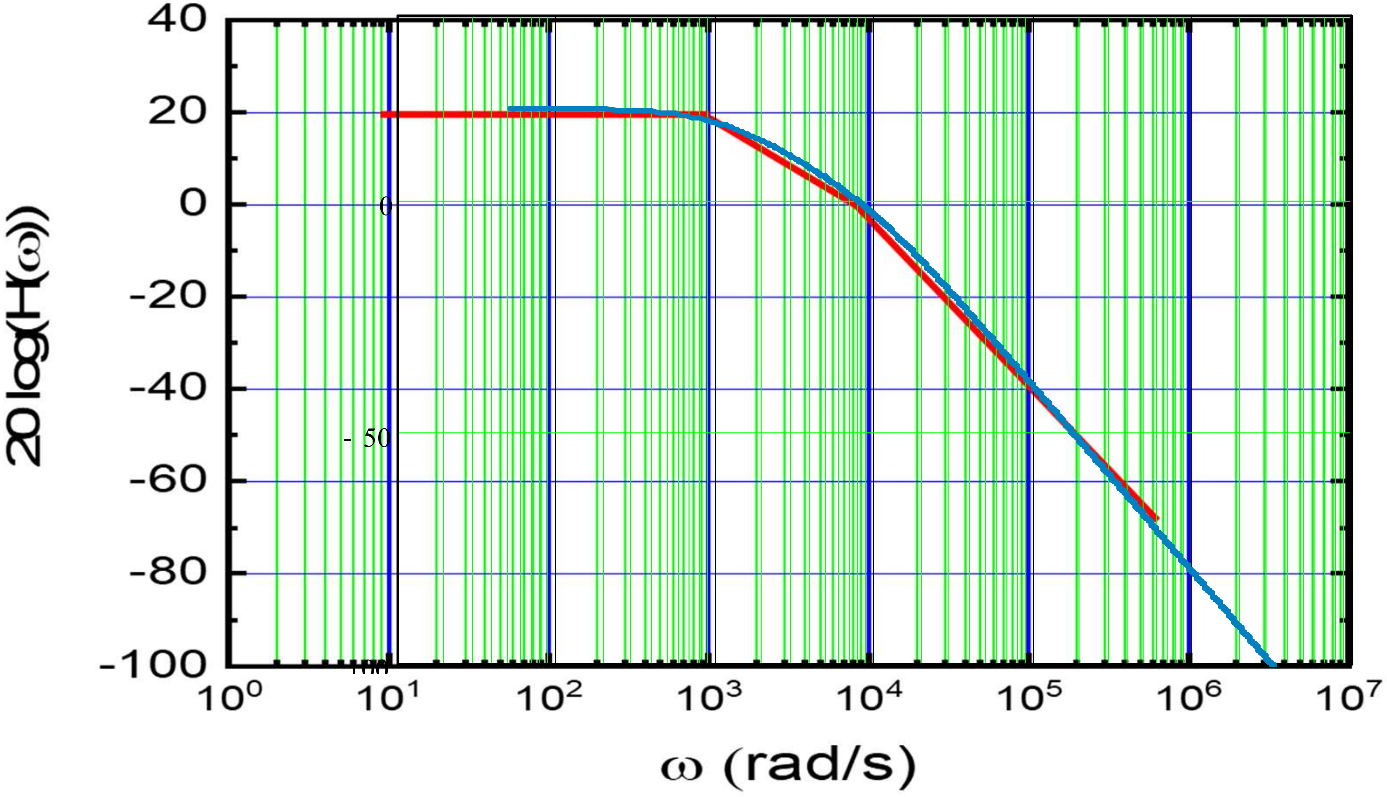


# Sketching of Transfer function

$$H(\omega) = \frac{10}{1 + j\dfrac{\omega}{10^3}} \times \frac{1}{1 + j\dfrac{\omega}{10^4}}$$

$$20\mathrm{Log}_{10}(|H(\omega)|) = 20 - 10Log_{10}(1 + (\frac{\omega}{10^3})^2) - 10Log_{10}(1 + (\frac{\omega}{10^4})^2)$$





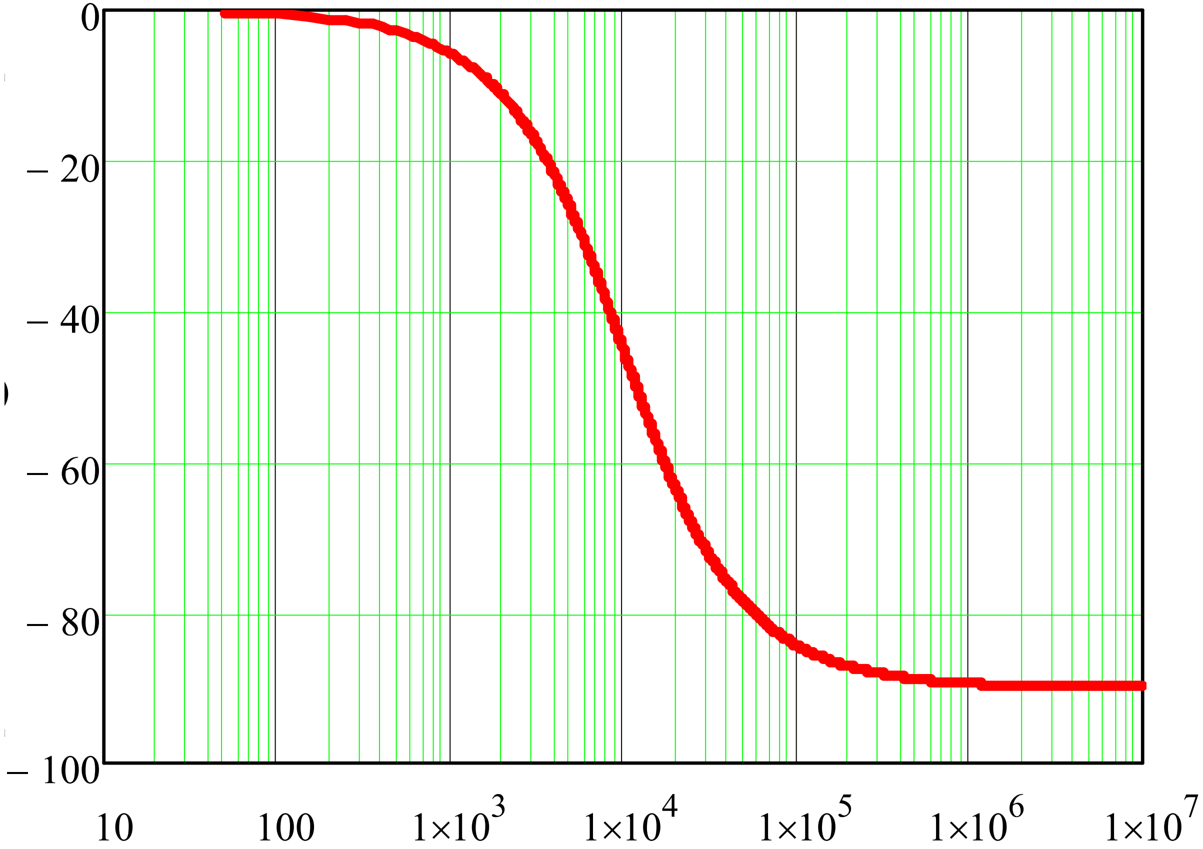
40
20
0
-20
-40
-60
-80
-100
0
- 50
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
$10^6$
$10^7$
ω (rad/s)
20 log(H(ω))

# Phase Plot

$$\varphi(\omega) = -\tan^{-1}(\omega/\omega_O)$$

$\omega \to 0, \phi \to 0 \quad \omega \to \infty, \phi \to -90°$

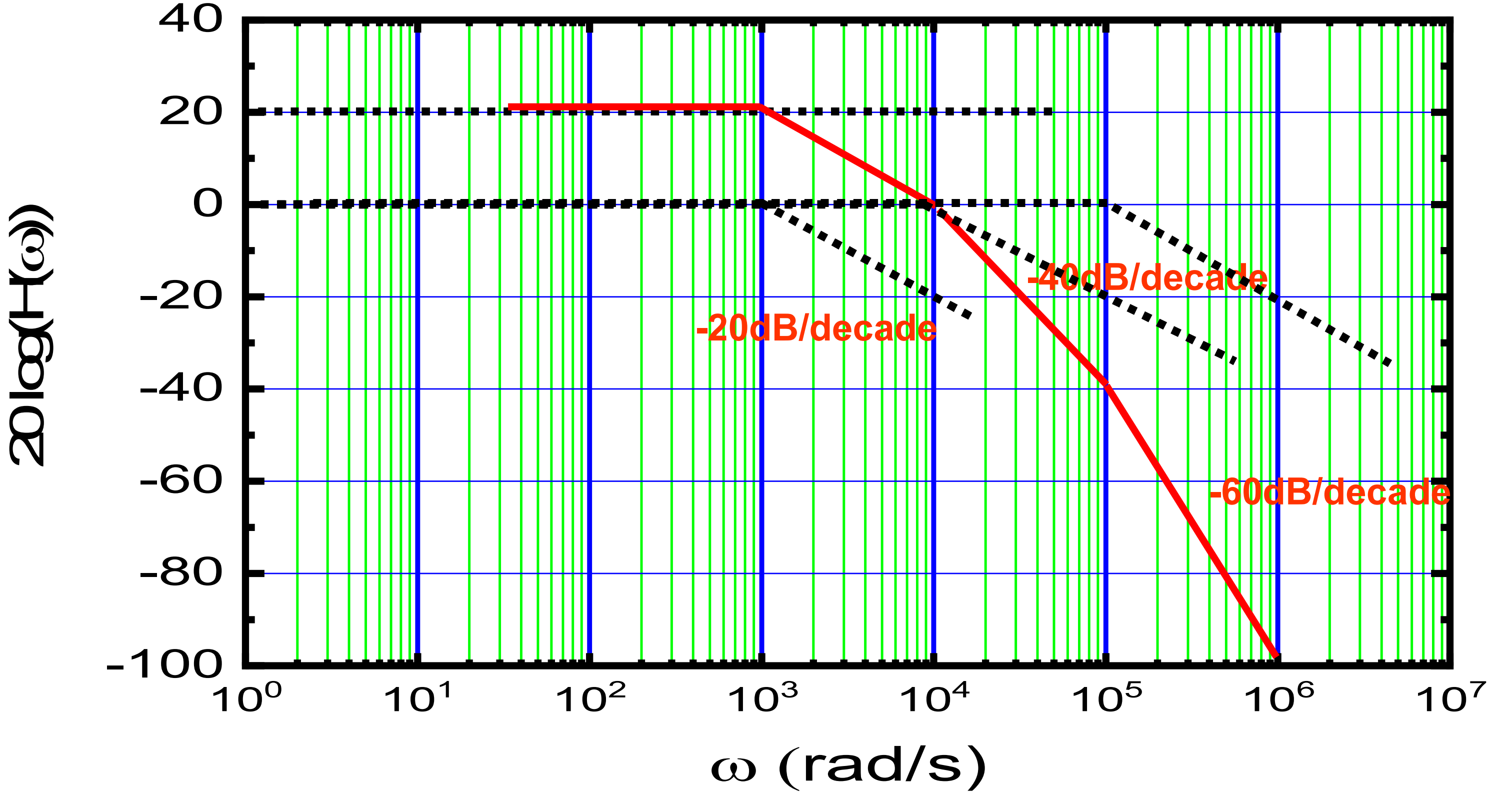
$\omega = \omega_O, \phi \to -45°$



## Sketching of Transfer function

$$H(\omega) = \frac{10}{1 + j\frac{\omega}{10^3}} \times \frac{1}{1 + j\frac{\omega}{10^4}} \times \frac{1}{1 + j\frac{\omega}{10^5}}$$

$$20\mathrm{Log}_{10}(|H(\omega)|) = 20 - 10Log_{10}(1 + (\frac{\omega}{10^3})^2) - 10Log_{10}(1 + (\frac{\omega}{10^4})^2) - 10Log_{10}(1 + (\frac{\omega}{10^5})^2)$$



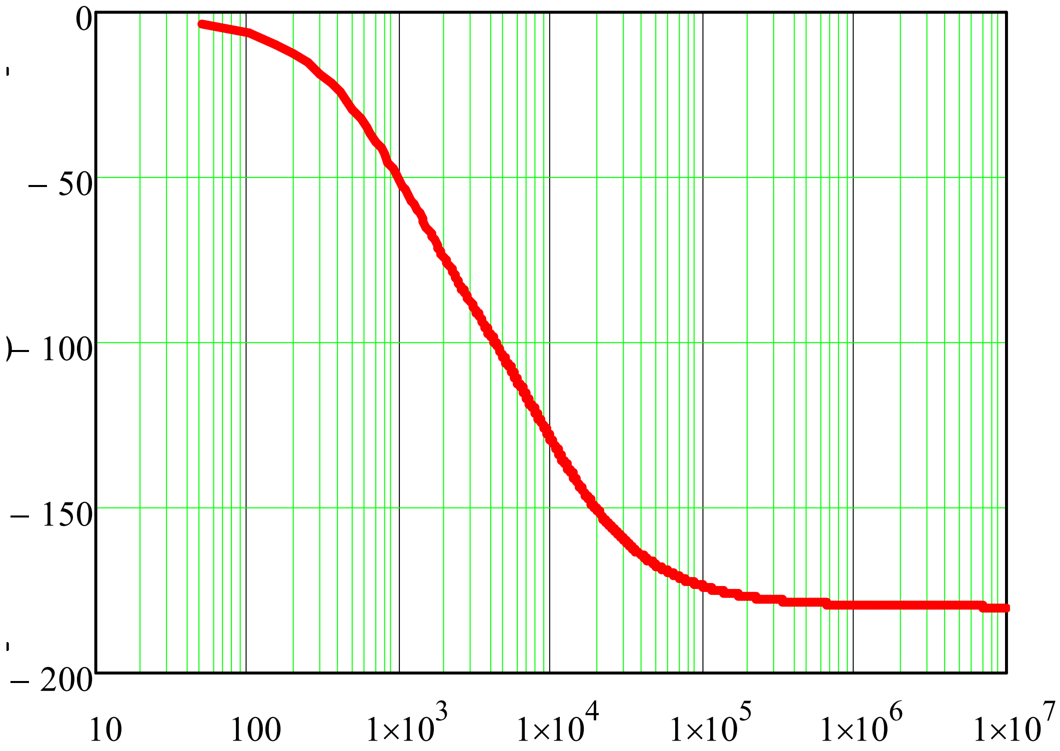
# Phase Plot

$$H(\omega) = \frac{10}{1 + j\dfrac{\omega}{10^3}} \times \frac{1}{1 + j\dfrac{\omega}{10^4}}$$

$\omega \to 0, \phi \to 0 \quad \omega \to \infty, \phi \to -90°$

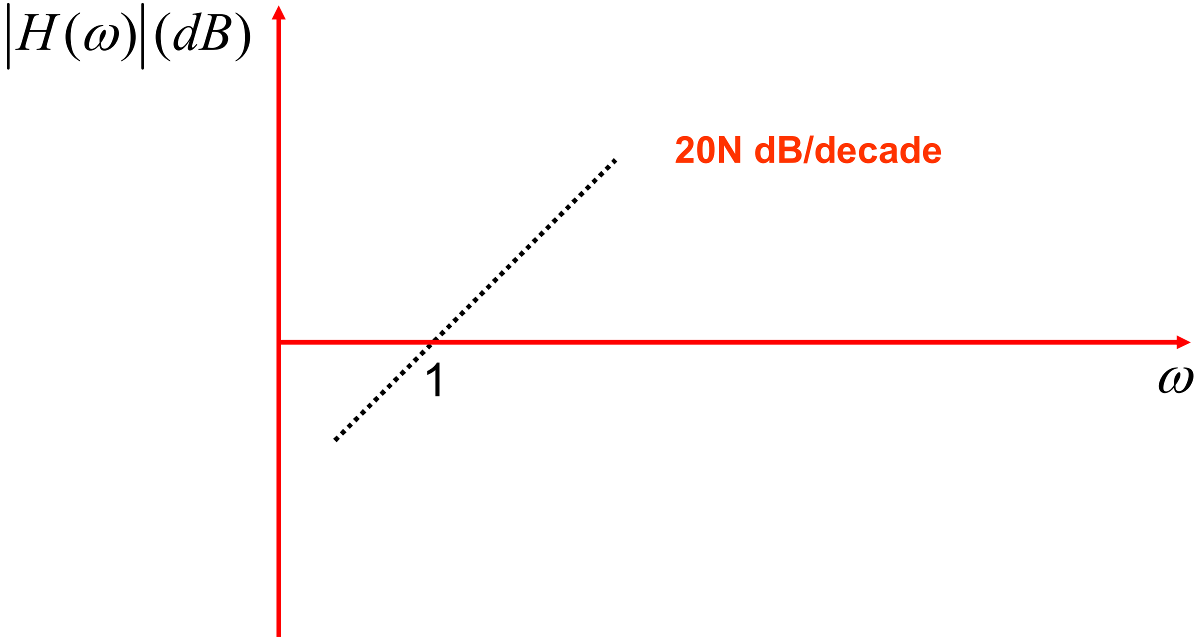
$\omega = \omega_O, \phi \to -45°$

$$\varphi(\omega) = -\tan^{-1}(\omega/10^3) - \tan^{-1}(\omega/10^4)$$

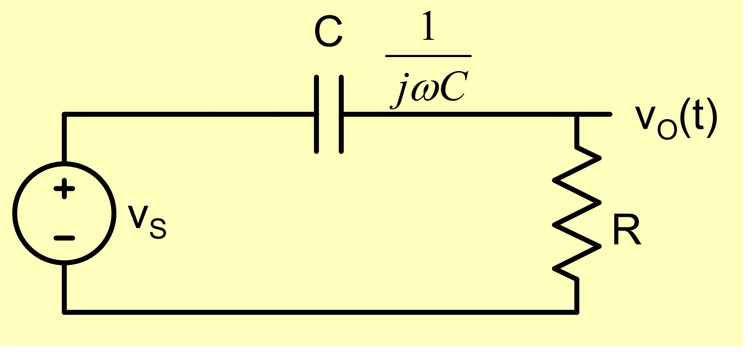


$$H(\omega) = (j\omega)^N$$

$$20\mathrm{Log}_{10}(|H(\omega)|) = 20N \times Log_{10}(\omega)$$



# Example




$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{j\omega CR}{1 + j\omega CR}$$

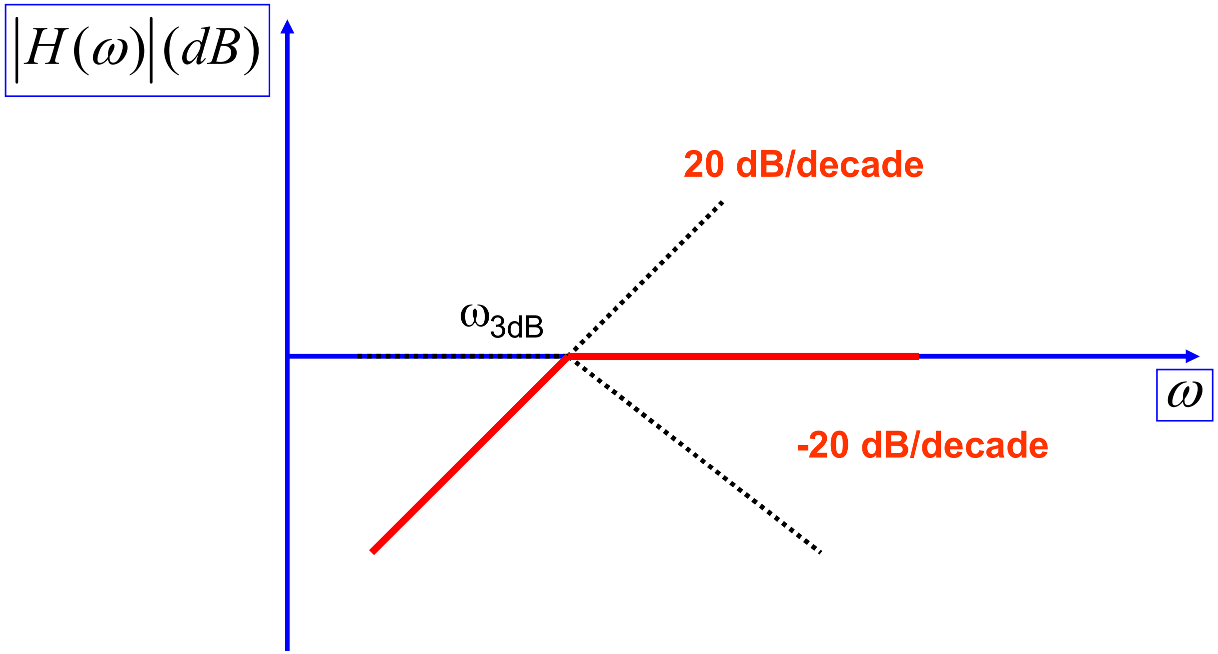
$$H(\omega) = \frac{j(\omega/\omega_{3dB})}{1 + j(\omega/\omega_{3dB})}$$

$$\omega_{3\mathrm{dB}} = \frac{1}{\mathrm{RC}} \quad ; \ \mathrm{f}_{3\mathrm{dB}} = \frac{1}{2\pi\mathrm{RC}}$$

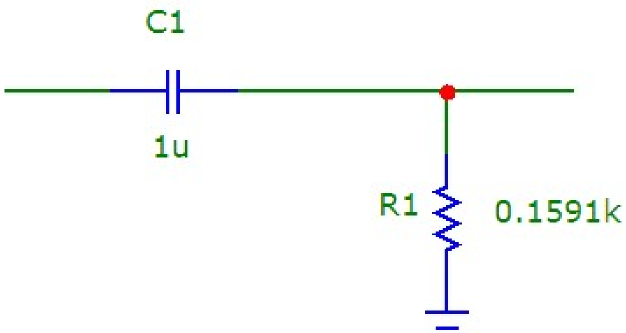
$$20\mathrm{Log}_{10}(|H(\omega)|) = 20log_{10}(\frac{\omega}{\omega_{3dB}}) - 10log_{10}(1 + (\frac{\omega}{\omega_{3dB}})^2)$$

$$\phi(\omega) = 90 - \tan^{-1}(\omega CR)$$

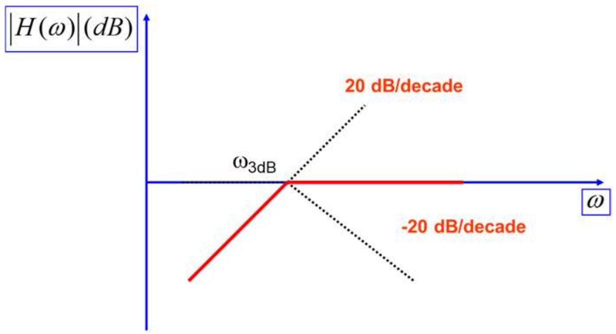
$$20\mathrm{Log}_{10}(|H(\omega)|) = 20log_{10}(\frac{\omega}{\omega_{3dB}}) - 10log_{10}(1+(\frac{\omega}{\omega_{3dB}})^2)$$



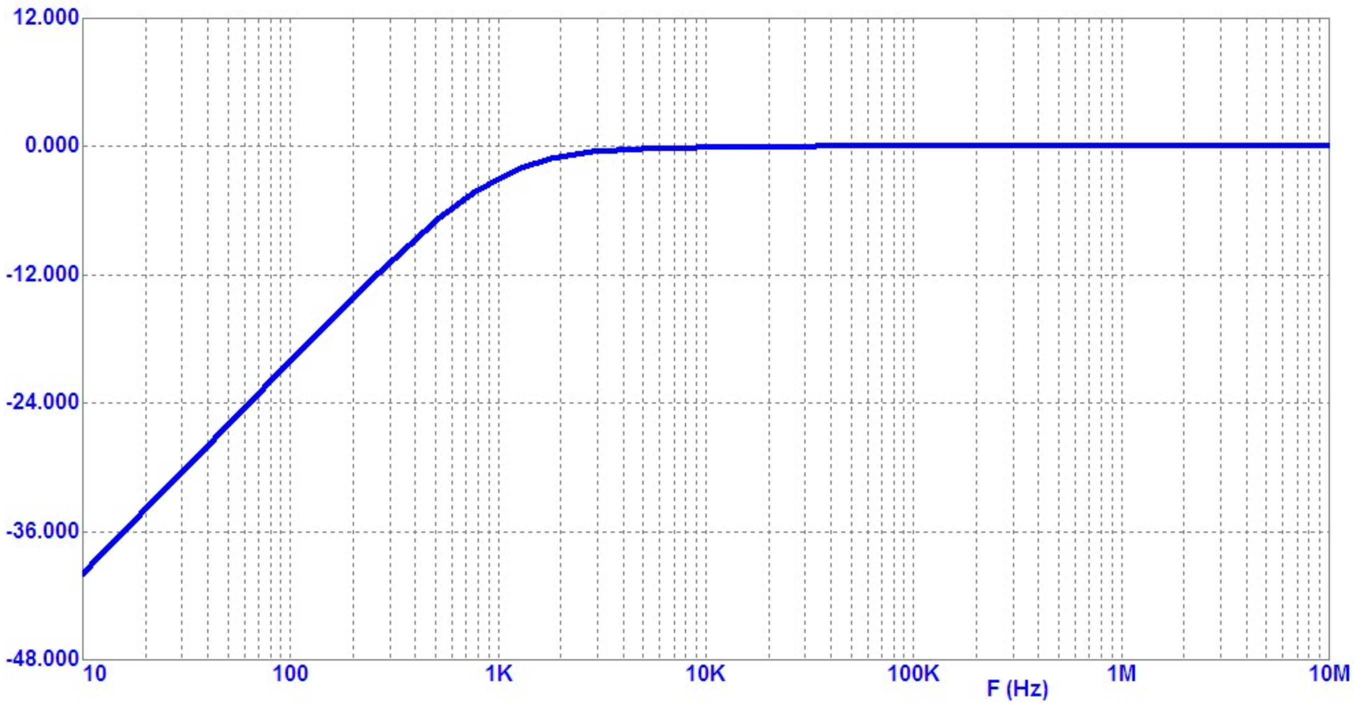

**High Pass Filter**




$$f_{3dB} = \frac{1}{2\pi RC} = 10^3\,Hz$$

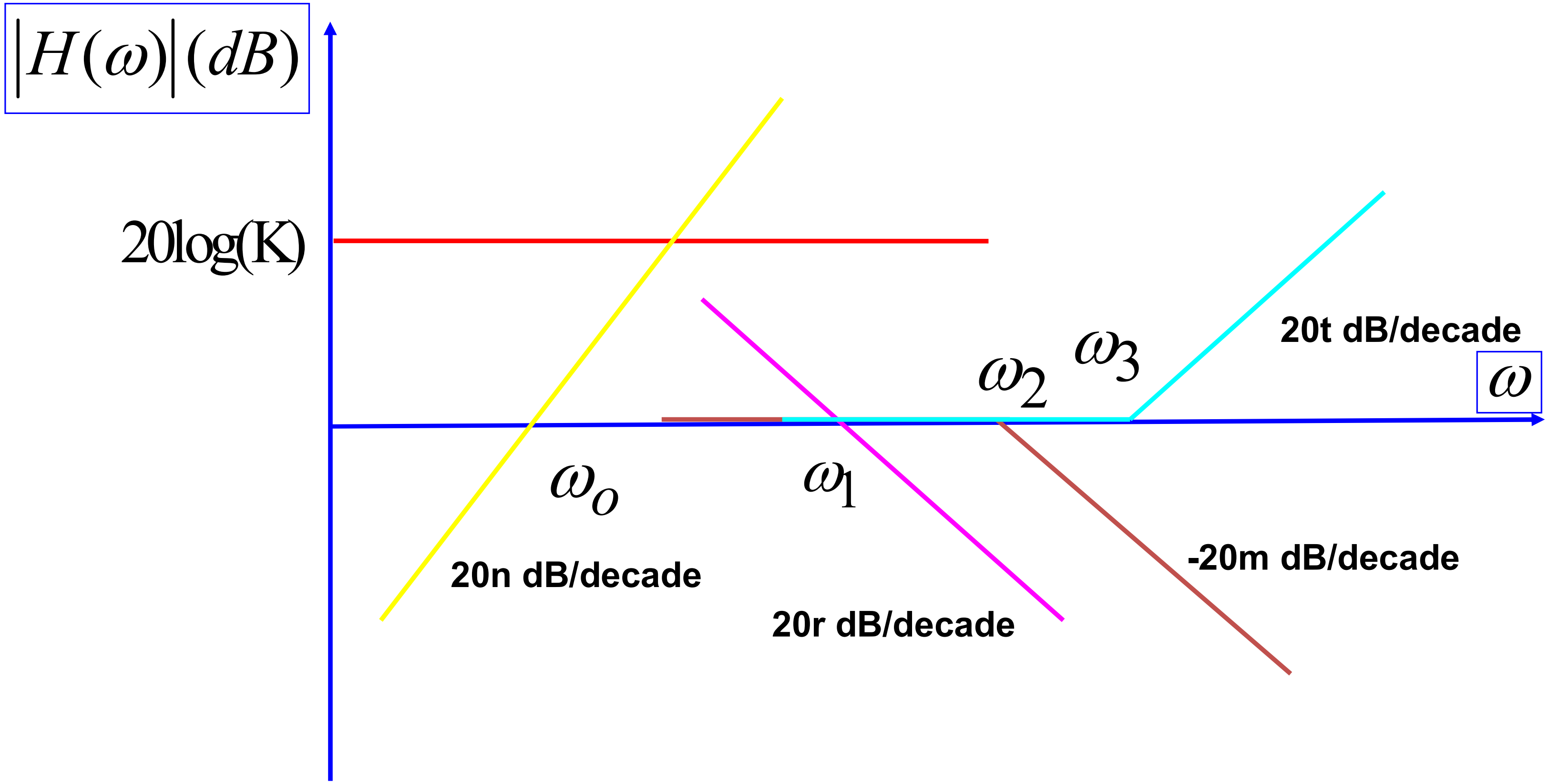



High Pass Filter



# Bode Plot segments

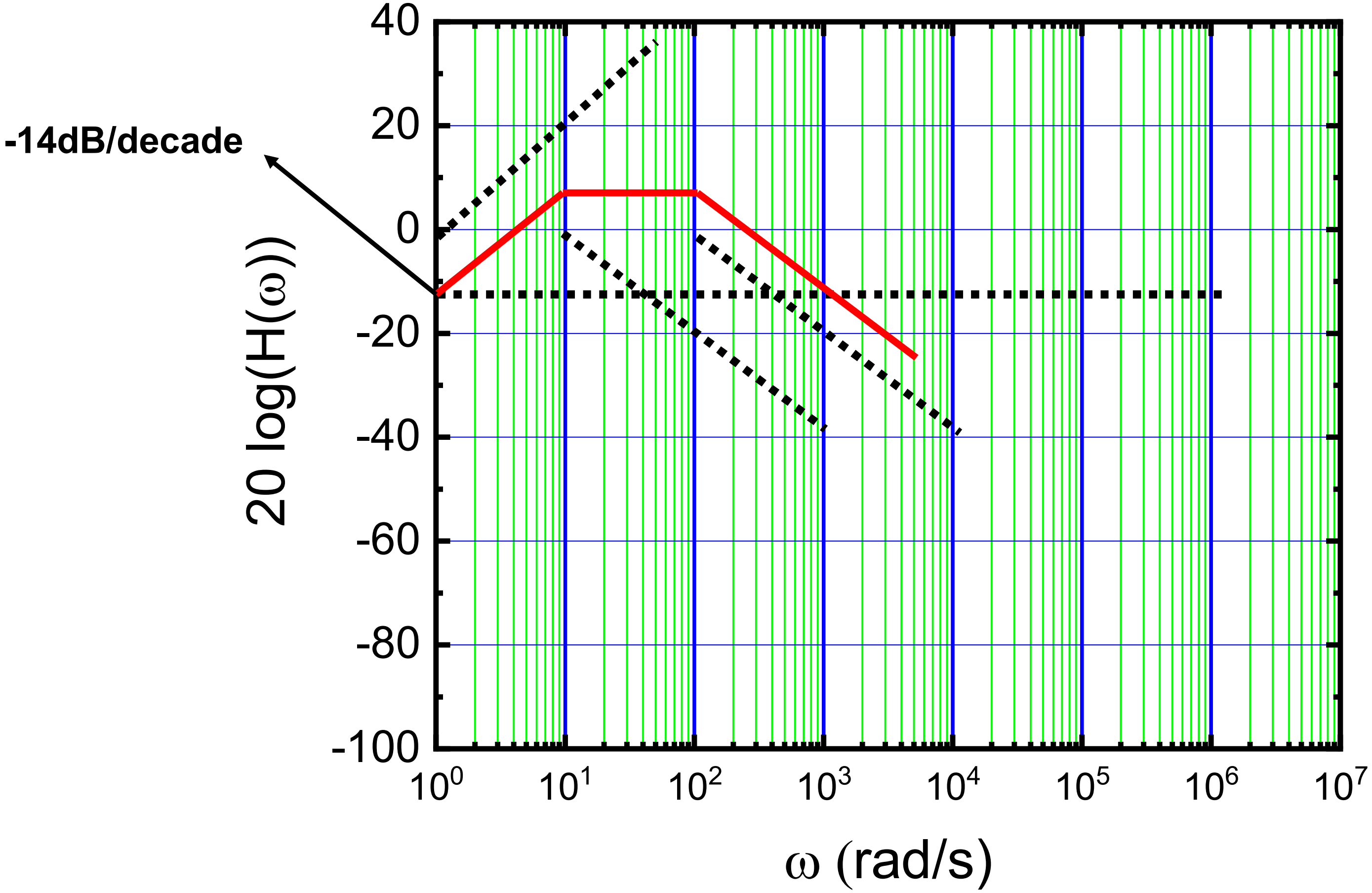
$$H(\omega) = K \times j(\omega/\omega_o)^n \times \frac{1}{j(\omega/\omega_1)^r}\frac{1}{\{1+j(\omega/\omega_2)\}^m} \times \{1+j(\omega/\omega_3)\}^t$$

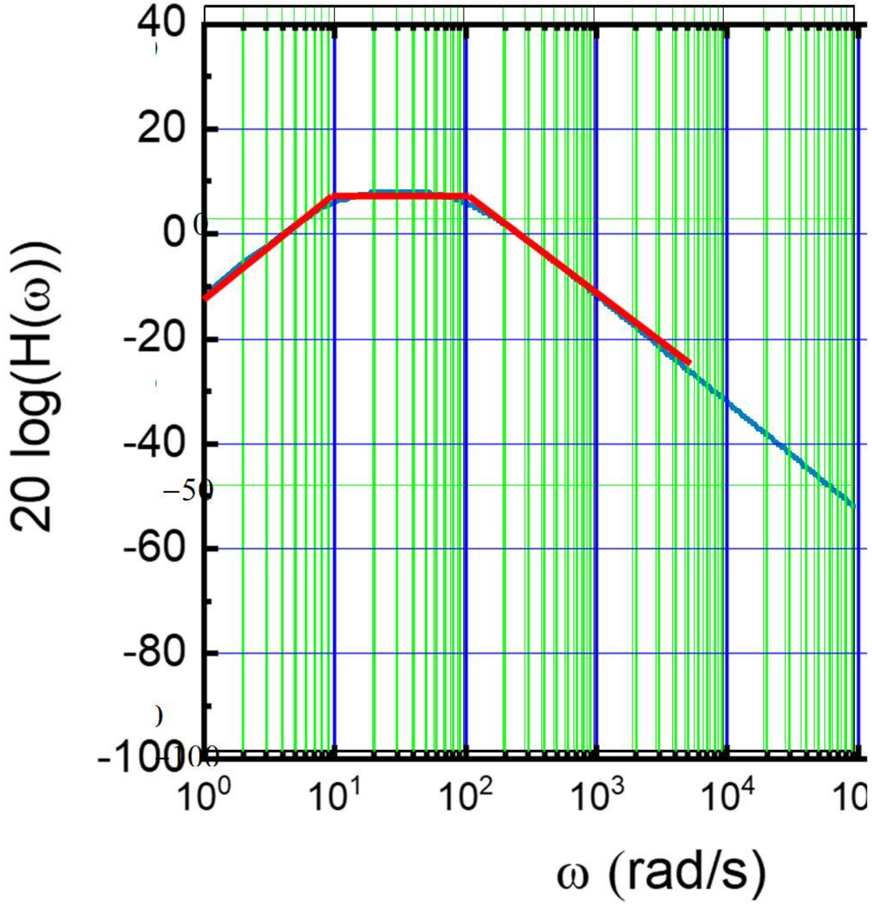


## Example:

$$H(\omega) = 200 \times j\omega \times \frac{1}{10 + j\omega} \times \frac{1}{100 + j\omega}$$

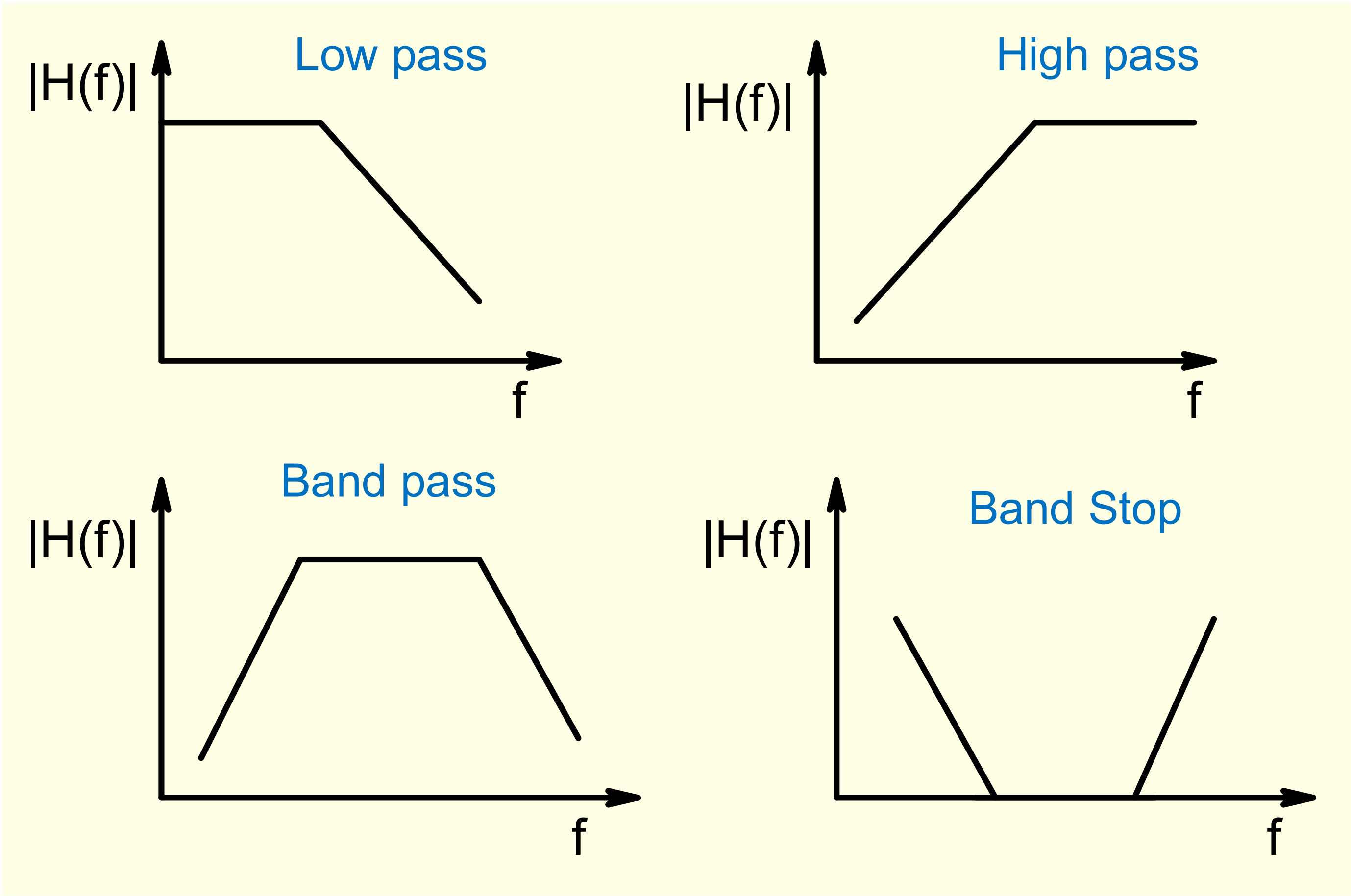
$$H(\omega) = 0.2 \times j\omega \times \frac{1}{1 + j\frac{\omega}{10}} \times \frac{1}{1 + j\frac{\omega}{100}}$$

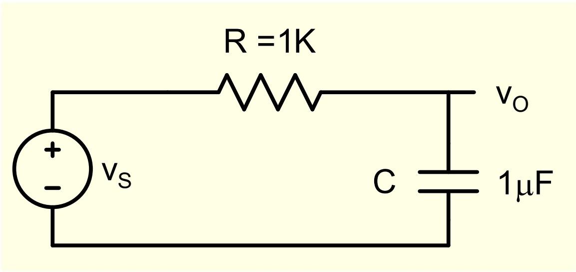


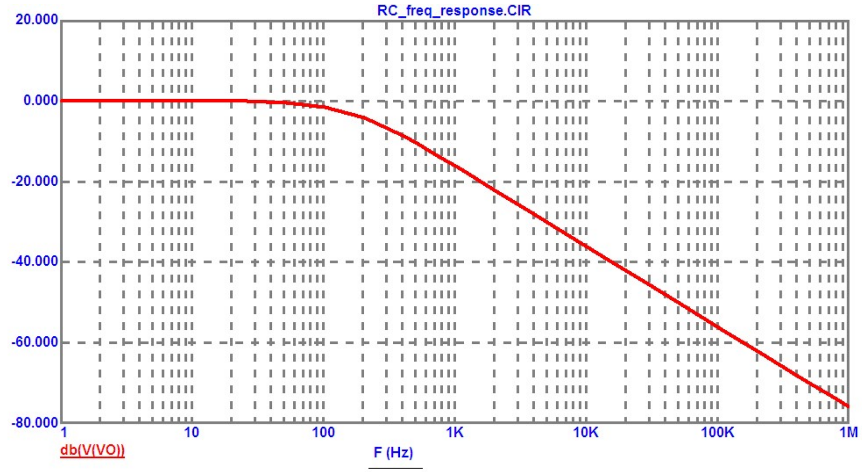


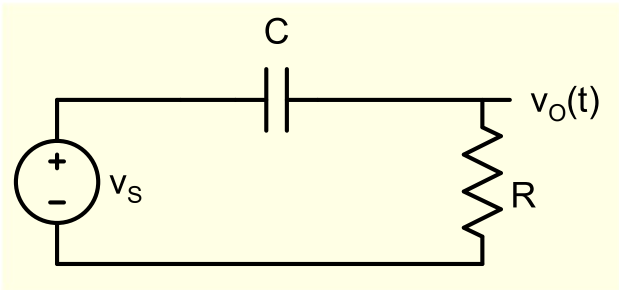
40
20
0
-20
-40
-60
-80
-100
20 log(H(ω))
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$
10
ω (rad/s)

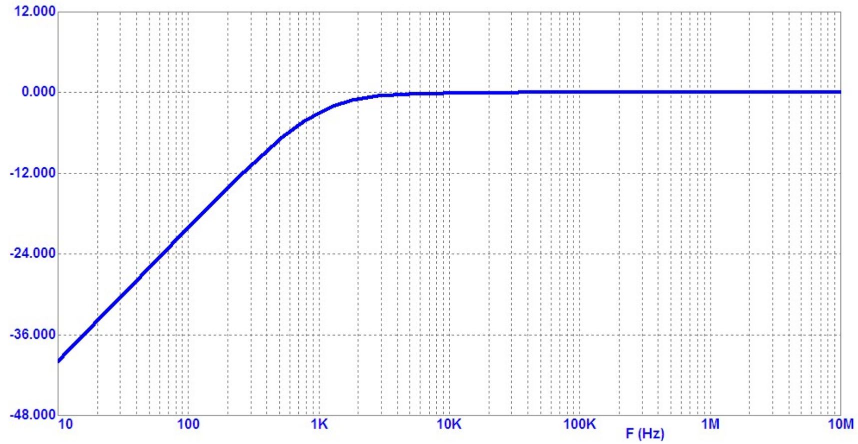
Filter -pass a band of frequency and reject the remaining



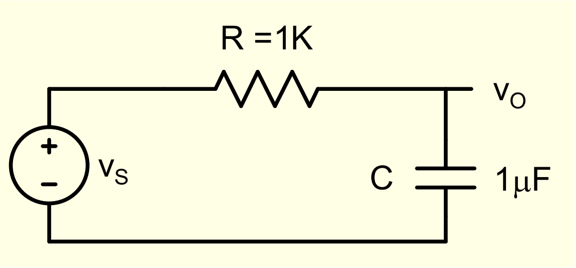

R =1K
vO
vS
C
1μF


RC_freq_response.CIR
20.000
0.000
-20.000
-40.000
-60.000
-80.000
1
10
100
1K
10K
100K
1M
db(V(VO))
F (Hz)

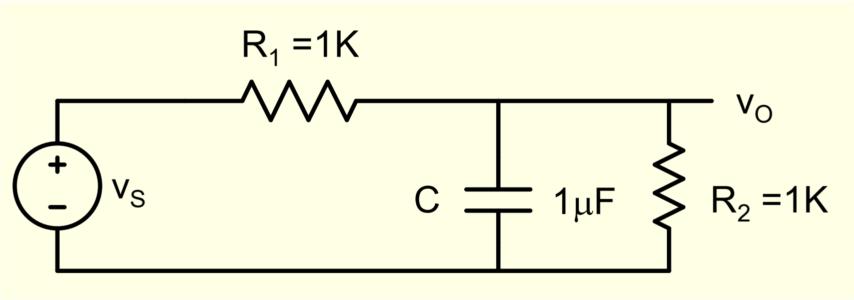

C
vO(t)
vS
R


12.000
0.000
-12.000
-24.000
-36.000
-48.000
10
100
1K
10K
100K
F (Hz)
1M
10M

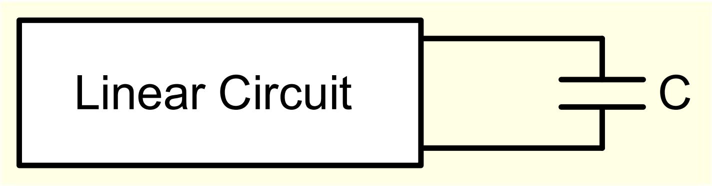
## 3dB Frequency of single capacitor filters




$$\omega_{3dB} = \frac{1}{RC} = 10^3\, rad/s$$

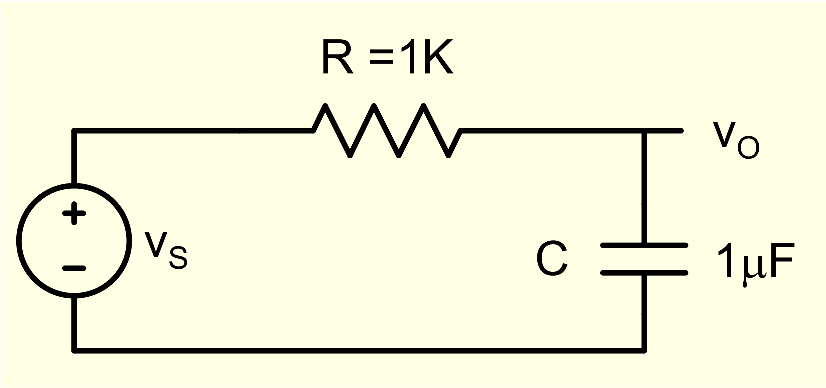


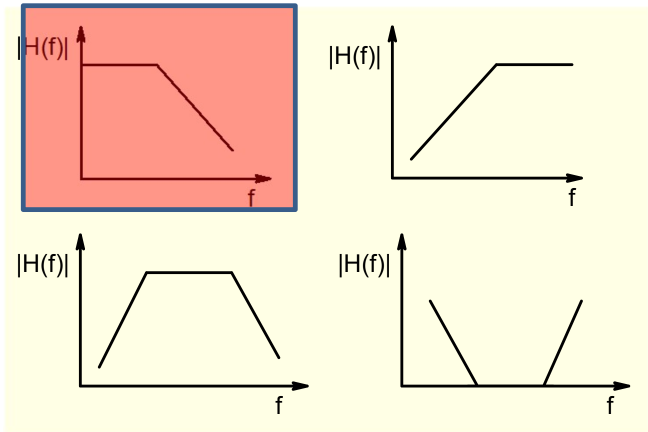

$$\omega_{3dB} = \frac{1}{R_1 \| R_2\, C}$$




$$\omega_{3dB} = \frac{1}{\tau} = \frac{1}{R_{eq} C}$$

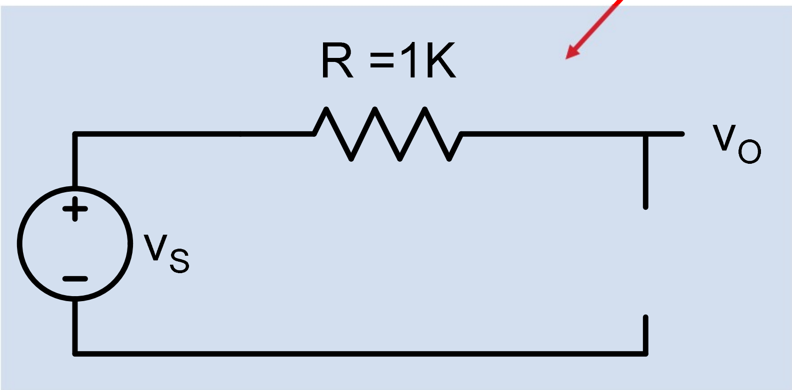
One can often tell the type of filter by looking at behavior at very low and very high frequencies and keeping in mind that capacitor offers very high impedance at low frequencies and very low impedance at high frequencies



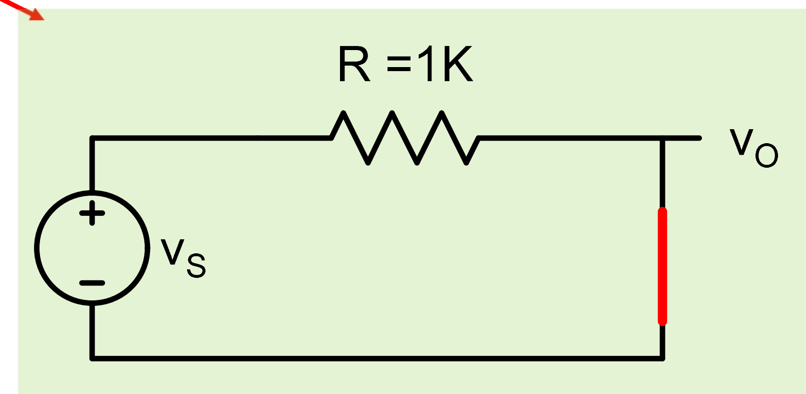




Low *f*

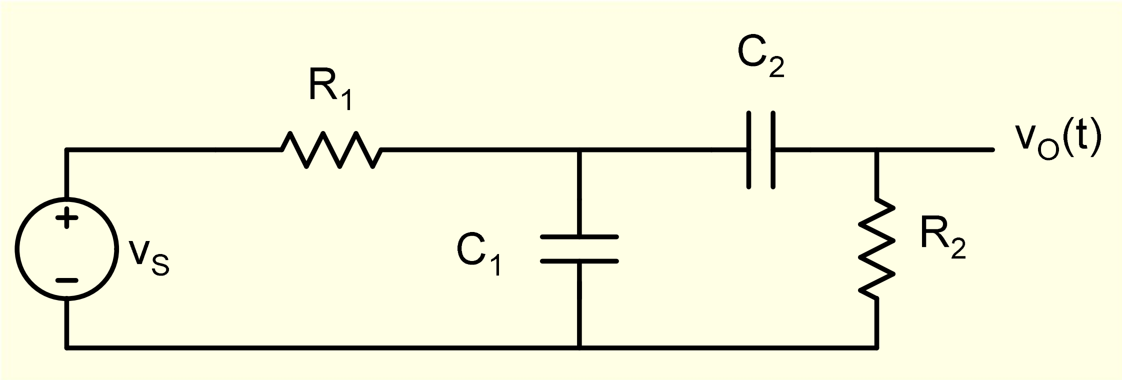
High *f*

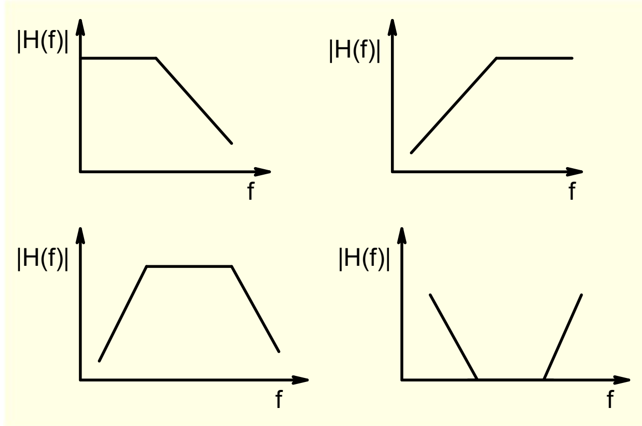



$Vo \sim V_S$



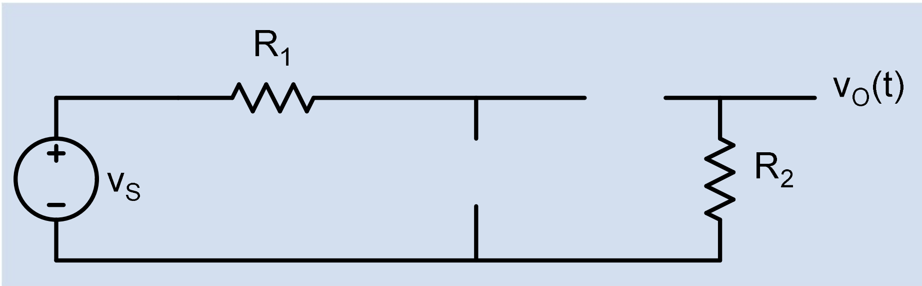

$Vo \sim 0$



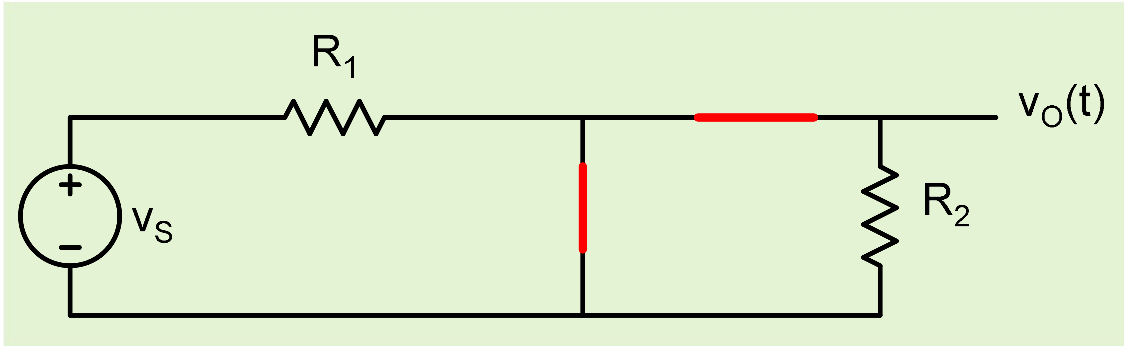




Low *f*

High *f*

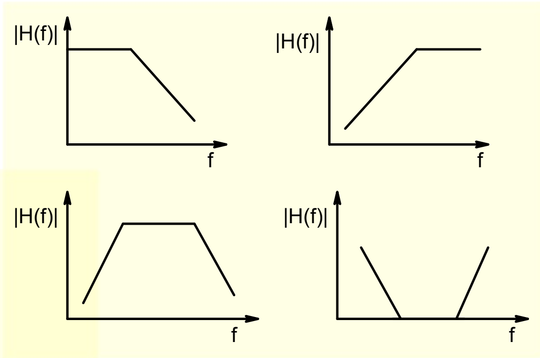


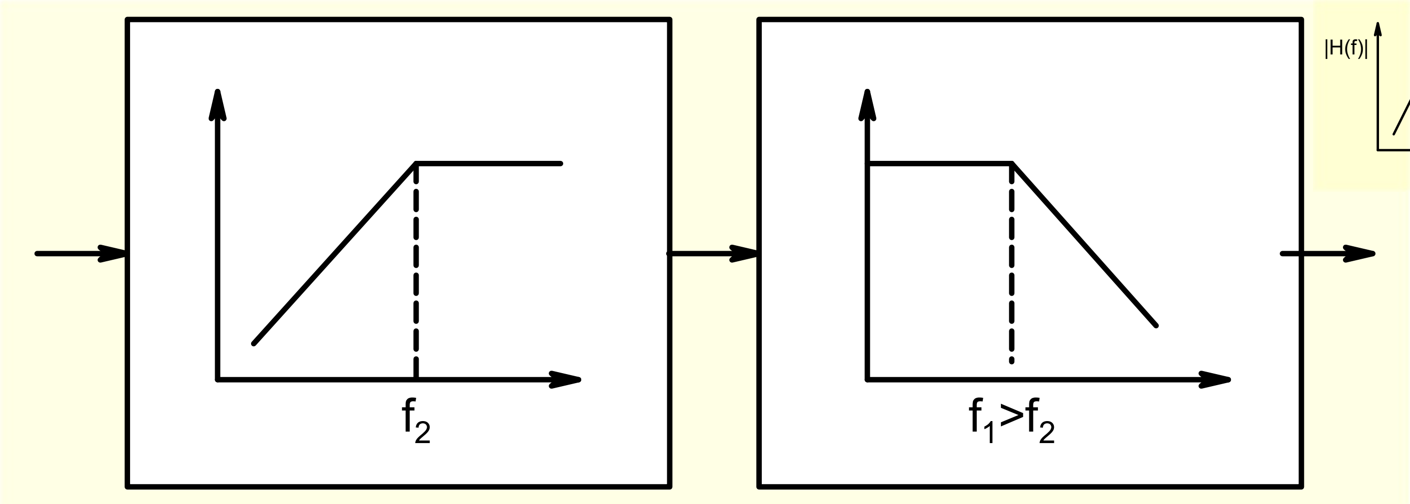

Vo ~0

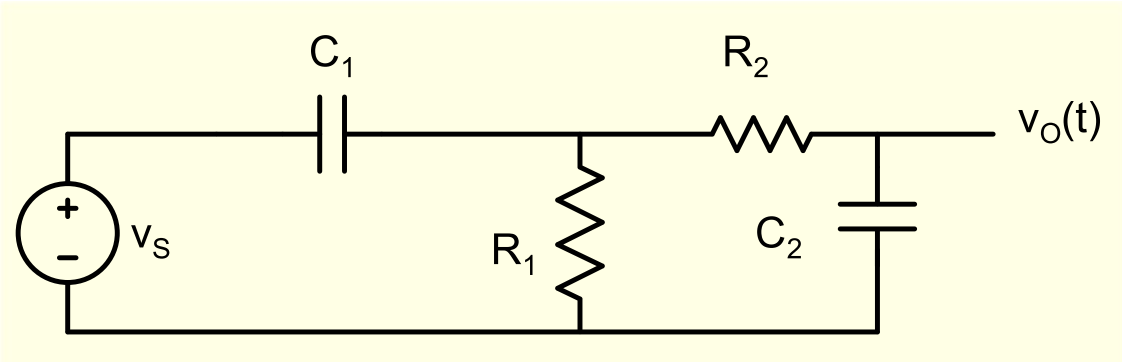



Vo ~0

# Bandpass Filter



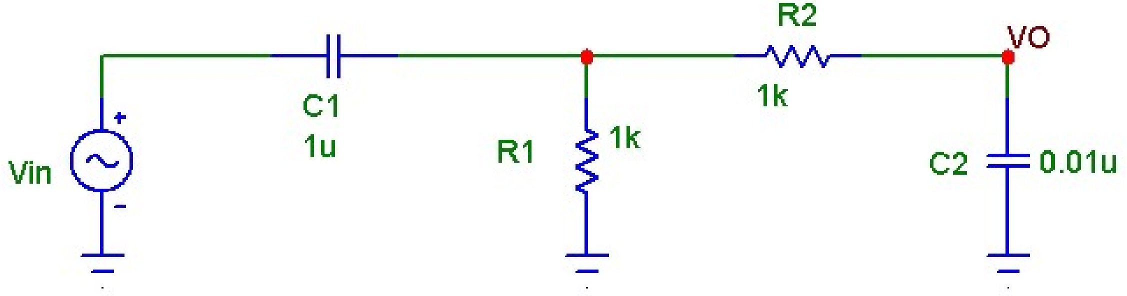



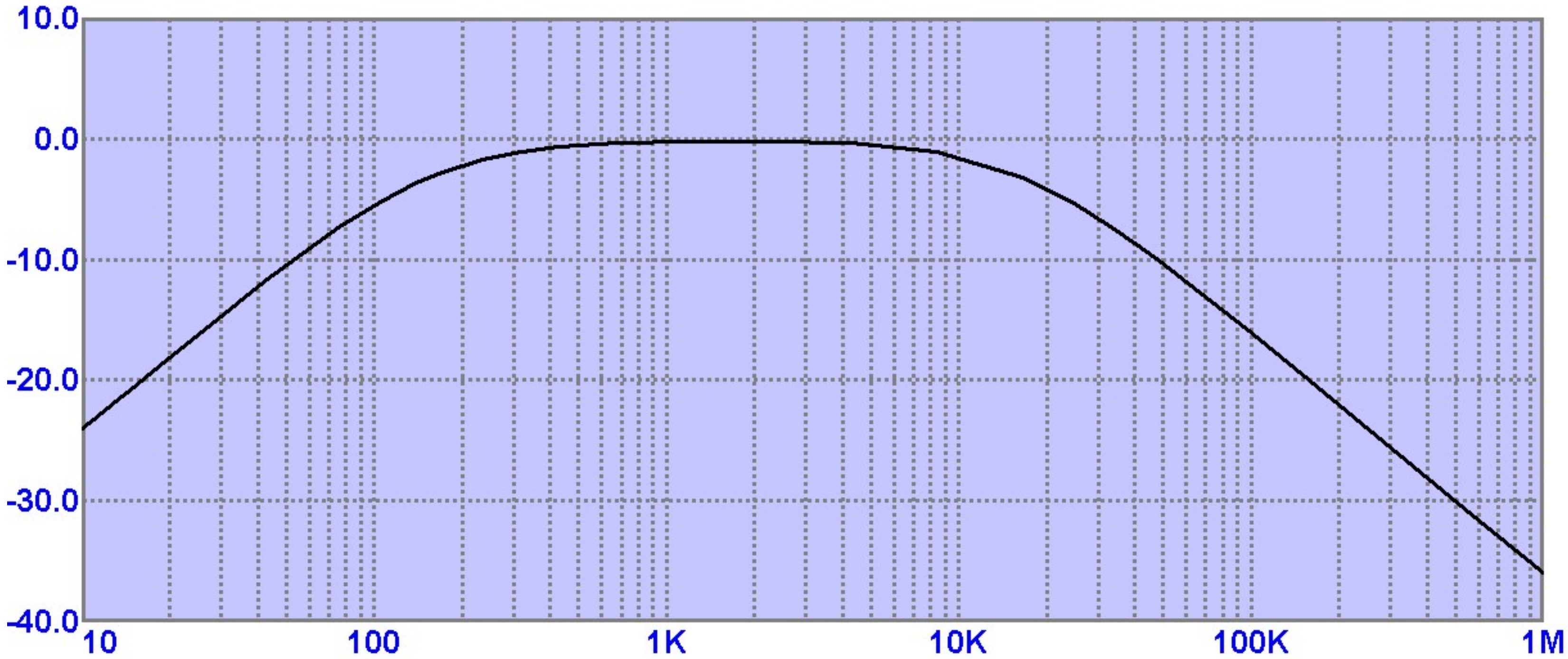




$$f_2 \cong \frac{1}{2\pi R_1 C_1}\ ;\ f_1 \cong \frac{1}{2\pi R_2 C_2}$$

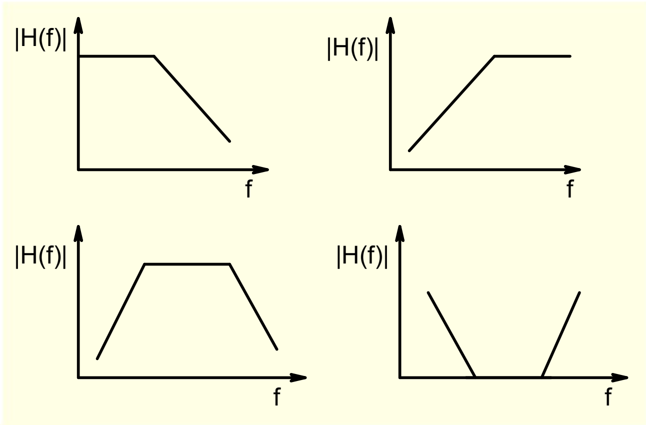
Example: Band Pass filter

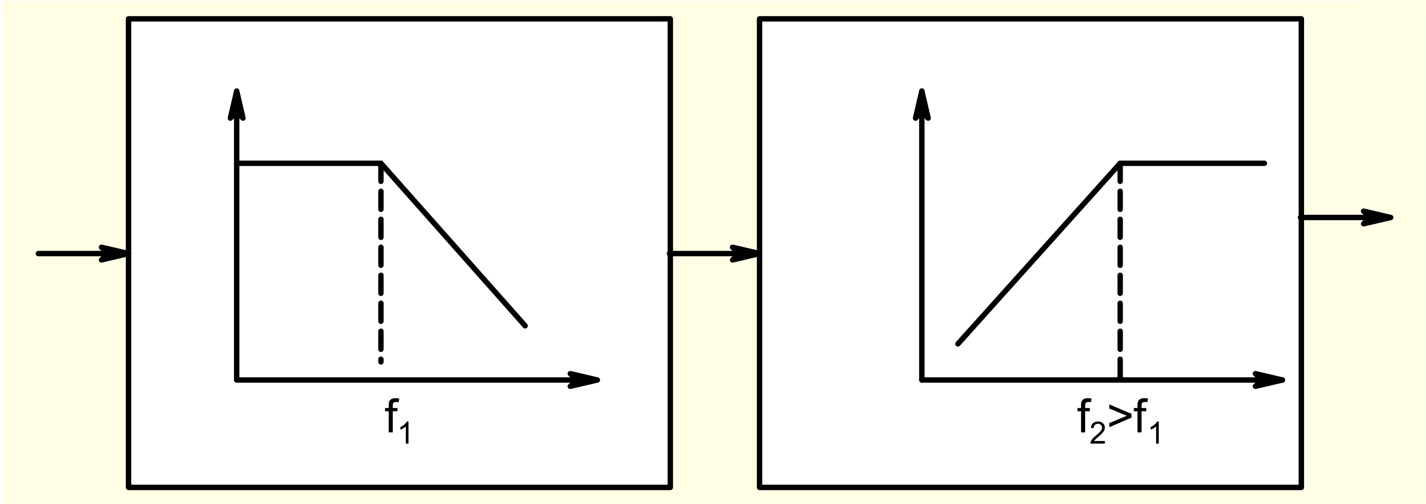
$$f_2 \cong \frac{1}{2\pi R_1 C_1} \;;\; f_1 \cong \frac{1}{2\pi R_2 C_2}$$



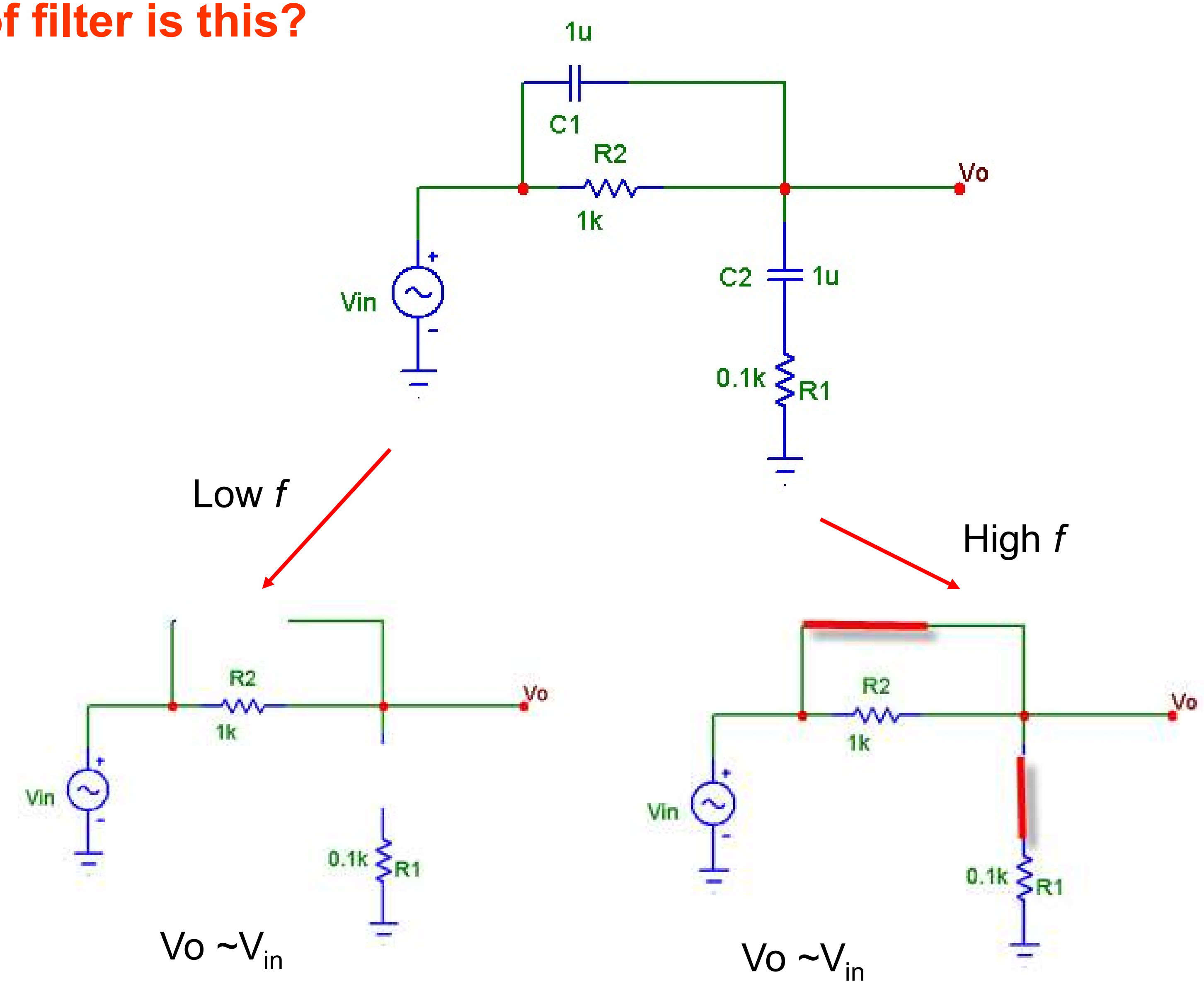
# Band-Stop Filter



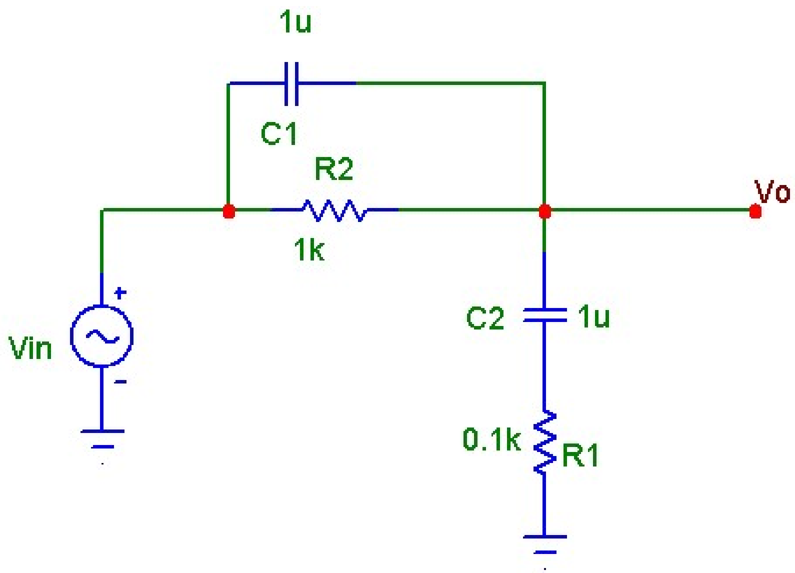



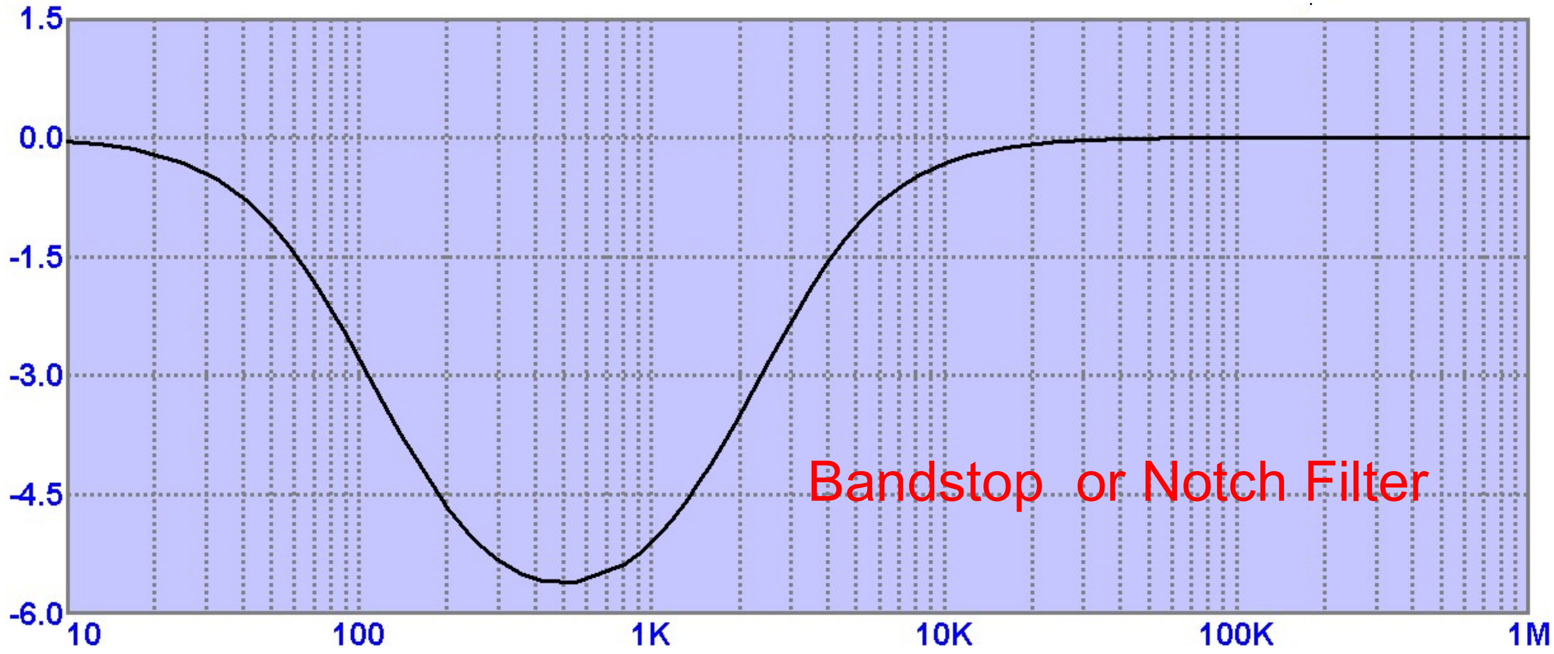

Will this work?

## What kind of filter is this?

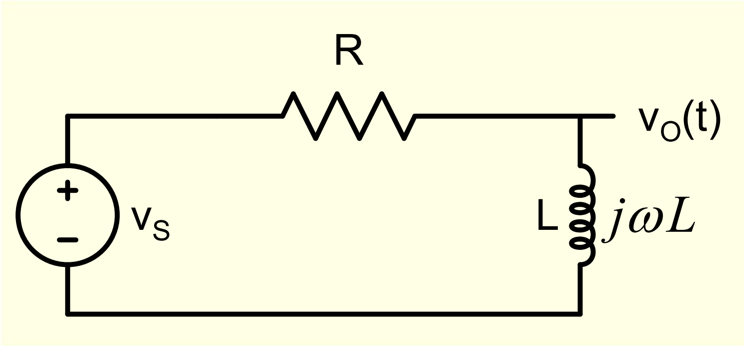


# What does this circuit do?



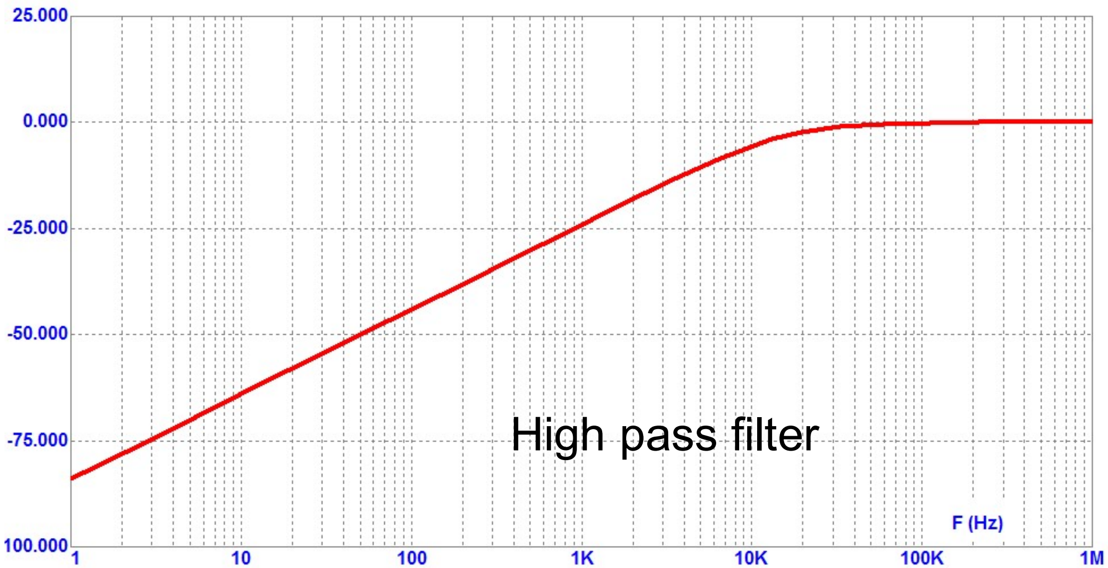
# R-L Circuits (Filters)



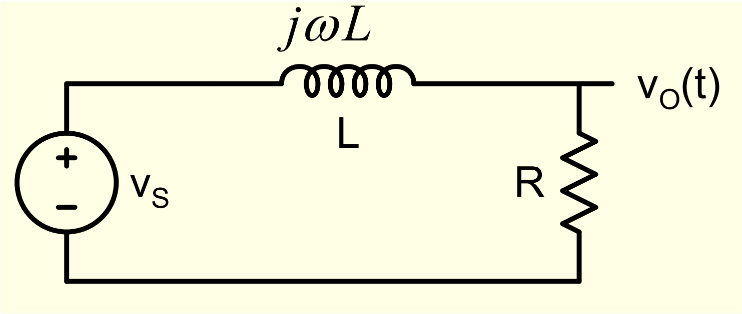

$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j(\omega/\omega_{3dB})}{1 + j(\omega/\omega_{3dB})}$$

$$\omega_{3dB} = \frac{R}{L}$$



## R-L Circuits




$$H(\omega) = \frac{V_O(\omega)}{V_S(\omega)}$$

$$H(\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j(\omega/\omega_{3dB})}$$

$$\omega_{3dB} = \frac{R}{L}$$