ESC201T : Introduction to Electronics

HW10: Solution

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ESC201T: HW 10 Solution

Q.1 Convert the following numbers into the number system indicated:

a. $(1010.011)_2$ to decimal

b. $(FA)_{16}$ to decimal

c. $(101110101101)_2$ into hexadecimal

d. $(FA)_{16}$ to binary

1 0 1 0 . 0 1 1

2^3 2^2 2^1 2^0 2^{-1} 2^{-2} 2^{-3}

$$(1010.011)_2 = 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0 +$$

$$= 0\times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-2} = 10.375$$

$$b.\ (F\,A)_{16} = 15\times 16^1 + 10\times 16^0 = 250$$

15 10

$$c.\ (101110101101)_2 = (1011)(1010)(1101)$$

$$= (B\,A\,D)_{Hex}$$

$$d.\ (FA)_{16} = (1111)(1010) = 11111010$$

Q.2 Convert the decimal number 27.25 into a binary number

27	remainder
13	1
6	1
3	0
1	1
0	1

27 = 1 1 0 1 1

0 .	25	
		x2
0 .	5	
		x2
1 .	0	

0.25 = $(.01)_2$

$$(27.25)_{10} = (11011.01)_2$$

Q.3 What is the largest decimal number that you can represent using 8bits? How many bits are required to represent decimal numbers less than or equal to 10^6?

Largest binary number is $(11111111)_2 = 255$

Let the number of bits be n. With these n bits the largest binary number is

$$2^{n-1} + 2^{n-2} + ... + 2 + 1 = 2^n - 1$$

$$2^n - 1 \geq 10^6$$

$$n \geq log_2(10^6 + 1)$$

N = 20 bits

Q.4 Determine the number system in which the following arithmetic operations have been carried out. Give justifications for your answer

a. 24+17=40

Let the number system be r

$$2r^1 + 4 + r^1 + 7 = 4r^1 + 0$$

$$\Rightarrow r = 11$$

b. 22 x 5 = 132

Let the number system be r

$$(2r^1 + 2) \times 5 = r^2 + 3r + 2$$

$$(r+1)(r-8) = 0 \Rightarrow r = 8$$

Q.5 Obtain 1's and 2's complement of the following binary numbers

a. 10000000

b. 10101010

c. 01110101

d. 10011100

Number	10000000	10101010	01110101	10011100
1's Complement	01111111	01010101	10001010	01100011
2's Complement	10000000	01010110	10001011	01100100

Q.6a What is the minimum number of bits required to represent -32 in 2's complement form?
b. 11011111 is a number in 2's complement. Is it positive or negative? What is its magnitude?

Binary representation for 32 is 100000. we need one more bit for sign to represent +32 in 2's complement.

$$(+32)_{10} = (0100000)_2$$

-32 is obtained by taking 2's complement of +32 $$(-32)_{10} = (1100000)_2$$

b. The number is negative because the sign bit is 1. Its magnitude can be obtained by taking its 2's complement

$$00100001 = (33)_{10}$$

Q.7 Carry out the following four operations using 8bit 2's complement representation:

±32±24 . Verify that operations have been properly carried out

$$(+32)_{10} = (0010\,0000)_2$$

$$(-32)_{10} = (1110\,0000)_2$$

$$(+24)_{10} = (0001\ \ 1000)_2$$

$$(-24)_{10} = (1110\ \ 1000)_2$$

+32+24:

```
  0 0 1 0 0 0 0 0
+ 0 0 0 1 1 0 0 0
  ---------------
  0 0 1 1 1 0 0 0
```

+56

-32+24:

```
  1 1 1 0 0 0 0 0
+ 0 0 0 1 1 0 0 0
  ---------------
  1 1 1 1 1 0 0 0
```

Negative number. Its magnitude is obtained by taking it's 2's complement which is 00001000 = 8

-32-24:

```
   1 1 1 0 0 0 0 0
+  1 1 1 0 1 0 0 0
  ----------------
 1 1 1 0 0 1 0 0 0
```

Negative number. Its magnitude is obtained by taking it's 2's complement which is 00111000 = 56

Q.8 Show that the Boolean expression $x+\overline{x}.y$ is equivalent to $x+y$ using basic postulates and theorems of Boolean algebra

postulates

P1: $x + 0 = x$

P2: $x + y = y + x$

P3: $x.(y+z) = x.y+x.z$

P4: $x + \overline{x} = 1$

$$x+\overline{x}.y = (x+\overline{x}).(x+y)$$
$$=1.(x+y) = x+y$$

Q.9 Reduce the following expressions to a minimum number of literals.

a. $f=(x+\overline{y}+\overline{z}).(\overline{y}+\overline{z})$

b. $f=(x+y).(\overline{y}+\overline{x})$

c. $f=ABCD+\overline{A}BD+AB\overline{C}D$

$$b.\ (x+y).(\overline{y}+\overline{x})$$
$$=x\overline{y}+x\overline{x}+\overline{y}\overline{y}+y\overline{x}$$
$$=x\overline{y}+y\overline{x}$$

$$a.\ (x+\overline{y}+\overline{z}).(\overline{y}+\overline{z})$$
$$=x\overline{y}+x\overline{z}+\overline{y}\overline{y}+\overline{y}\overline{z}+\overline{z}\overline{y}+\overline{z}\overline{z}$$
$$=x\overline{y}+x\overline{z}+(\overline{y}+\overline{y}\overline{z})+(\overline{z}\overline{y}+\overline{z})$$
$$=x\overline{y}+x\overline{z}+\overline{y}+\overline{z}$$
$$=\overline{y}+\overline{z}$$

$$c.\ ABCD+\overline{A}BD+AB\overline{C}D$$
$$=ABD+\overline{A}BD$$
$$=BD$$

Q.10 Obtain the truth table for the following function: (x.y+z)(y+x.z) and write it as sum of products (SOP) and product of sums (POS).

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

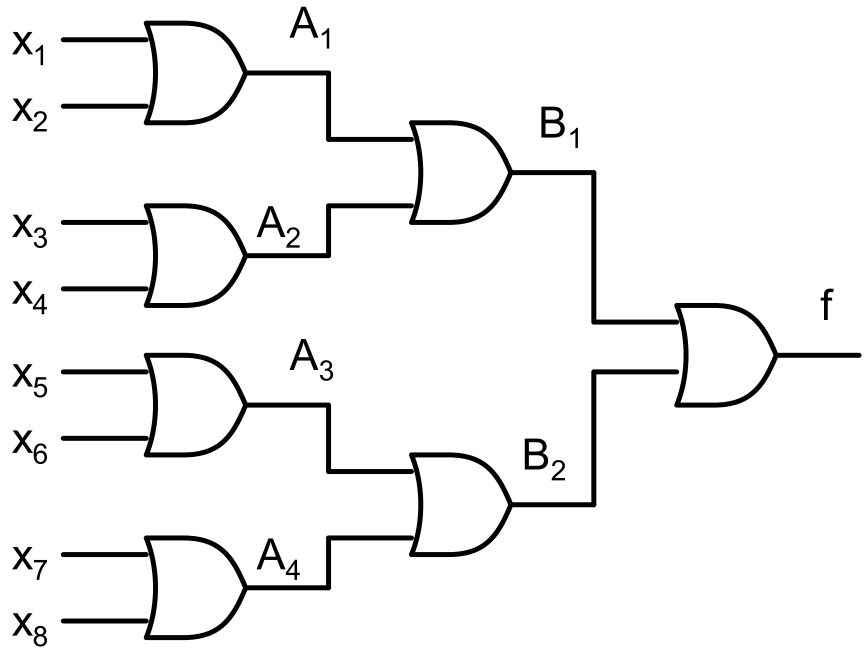
$$\text{SOP:}\ \ \mathrm{f} = \overline{x}.y.z + x.\overline{y}.z + x.y.\overline{z} + x.y.z$$

$$\text{PoS:}\ \ \mathrm{f} = (x + y + z).(x + y + \overline{z}).(x + \overline{y} + z).(\overline{x} + y + z)$$

Q.11 Implement an 8 input OR gate using only 2 input AND and 2 input OR gates

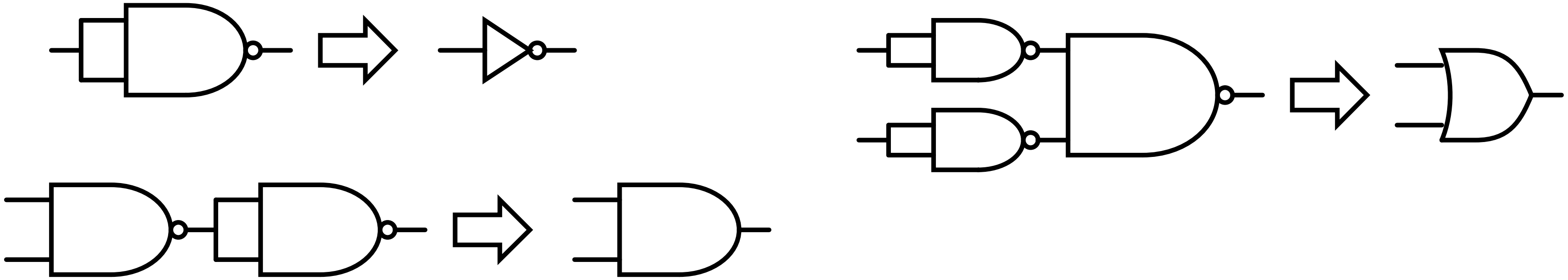
$$
\begin{aligned}
f &= x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \\
&= (x_1 + x_2) + (x_3 + x_4) + (x_5 + x_6) + (x_7 + x_8) \\
&= A_1 + A_2 + A_3 + A_4
\end{aligned}
$$

$$
\begin{aligned}
f &= (A_1 + A_2) + (A_3 + A_4) \\
&= B_1 + B_2
\end{aligned}
$$

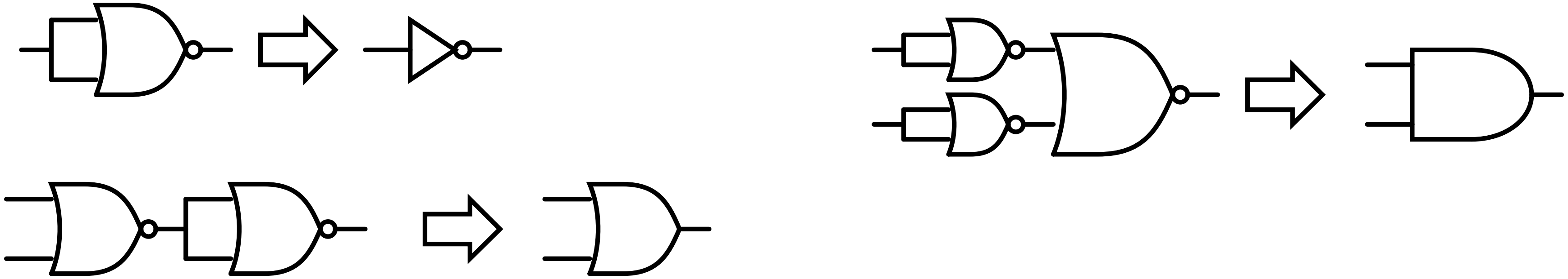


Q.12 Show that you can implement 2 input AND, 2 input OR and NOT gates using only 2 input NAND gates. Similarly show that you can implement 2 input AND, 2 input OR and NOT gates using only 2 input NOR gates

Implementation using NAND gates

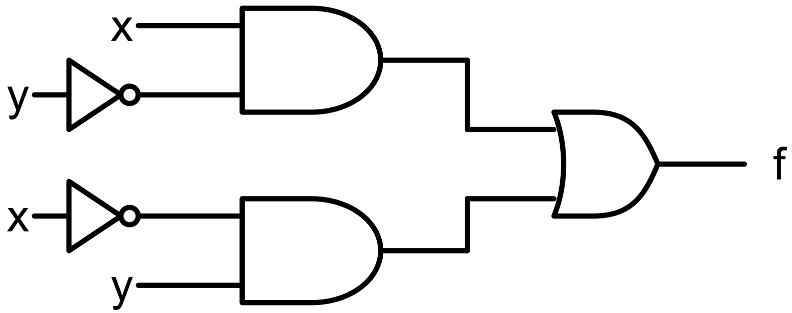


Implementation using NOR gates

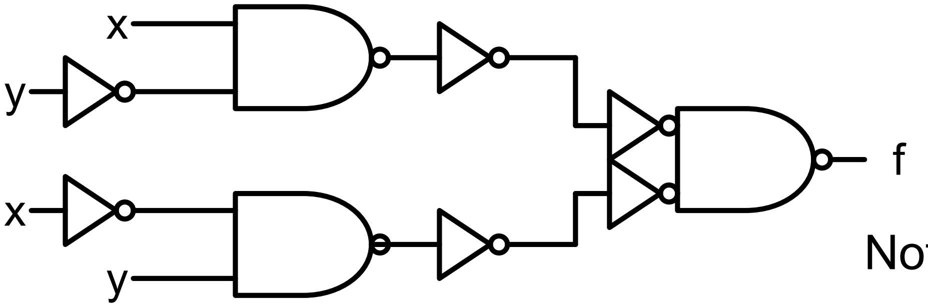


Q.13 Implement a 2-input exclusive OR gate with only 2 input NAND gates .

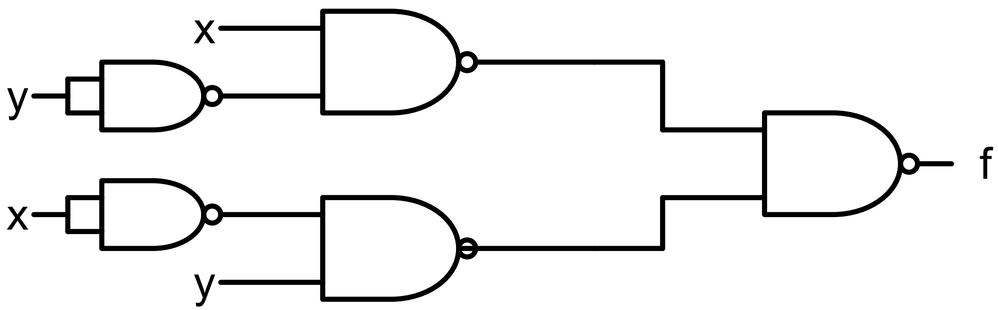
$$f = x.\overline{y} + \overline{x}.y$$



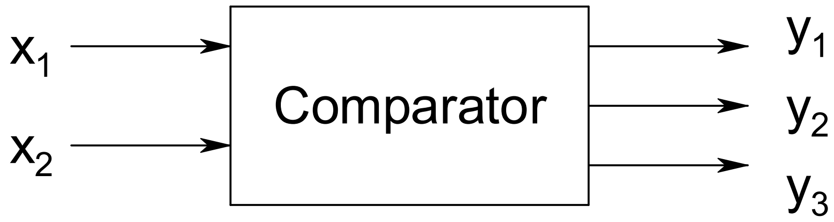
Using results from previous question, we can replace each gate by its NAND equivalent



Noting that 2 NOT gates in series cancel



Q.14 Figure below shows a block diagram of a comparator. From the description given , obtain first the truth table for outputs y1 , y2 and y3, then the Boolean expression and finally the gate netlist. Both inputs are 1-bit.



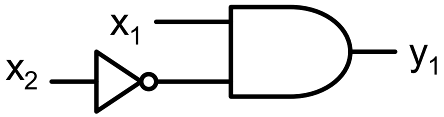
y_1 is 1 if and only if $x_1 > x_2$

y_2 is 1 if and only if $x_1 = x_2$

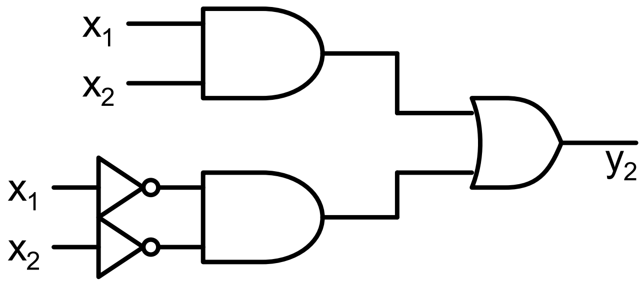
y_3 is 1 if and only if $x_1 < x_2$

x_1	x_2	y_1	y_2	y_3
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

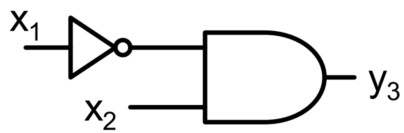
$$y_1 = x_1 . \overline{x_2}$$



$$y_2 = \overline{x_1} . \overline{x_2} + x_1 . x_2$$



$$y_3 = \overline{x_1} . x_2$$



Q.15 Simplify the following 4-variable functions into sum-of-products form using K-map.

a. $\sum(1,5,6,7,14)$

b. $\sum(0,4,6,8)$

c. $\sum(0,1,4,6,8,9,14)$

d. $\sum(1,4,7,11,13,14)$

a)

x_1x_2 \ x_3x_4	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	0	0	0	1
10	0	0	0	0

$$F = \overline{x_1}\,\overline{x_3}x_4 + \overline{x_1}x_2x_3 + x_2x_3\overline{x_4}$$

b)

x_1x_2 \ x_3x_4	00	01	11	10
00	1	0	0	0
01	1	0	0	1
11	0	0	0	0
10	1	0	0	0

$$F = \overline{x_2}\,\overline{x_3}\,\overline{x_4} + \overline{x_1}x_2\overline{x_4}$$

c)

x_1x_2 \ x_3x_4	00	01	11	10
00	1	1	0	0
01	1	0	0	1
11	0	0	0	1
10	1	1	0	0

$$F = \overline{x_2}\,\overline{x_3} + \overline{x_1}\,\overline{x_3}\,\overline{x_4} + x_2x_3\overline{x_4}$$

d)

x_1x_2 \ x_3x_4	00	01	11	10
00	0	1	0	0
01	1	0	1	0
11	0	1	0	1
10	0	0	1	0

Cannot be minimized

Q.16 Simplify the following 4-variable functions into product-of-sums form using K-map

a. $\Pi\ (1,3,5,7,13,15)$

b. $\Pi\ (1,3,6,9,11,12,14)$

c. $\Pi\ (1,3,5,7,9,11,12,13,14,15,)$

d. $\Pi\ (0,1,3,4,5,7,12,13,15)$

a)

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$$F = (x_1 + \overline{x_4}).(\overline{x_2} + \overline{x_4})$$

b)

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00		0	0	
01				0
11	0			0
10		0	0	

$$F = (x_2 + \overline{x_4}).(\overline{x_2} + \overline{x_3} + x_4).(\overline{x_1} + \overline{x_2} + x_4)$$

c)

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00		0	0	
01		0	0	
11	0	0	0	0
10		0	0	

$$F = (\overline{x_1} + \overline{x_2}).\ \overline{x_4}$$

d)

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	0	0	
01	0	0	0	
11	0	0	0	
10				

$$F = (x_1 + x_3).(x_1 + \overline{x_4}).(\overline{x_2} + x_3).(\overline{x_2} + \overline{x_4})$$

Q.17 Simplify the following expressions into sum-of-products form using the don't care conditions (d) into account.

a.
$$F(A,B,C,D)=\sum(4,5,7,12,13,14)$$
$$d(A,B,C,D)=\sum(1,9,11,15)$$

b.
$$F(A,B,C,D)=\sum(1,2,12,13,14)$$
$$d(A,B,C,D)=\sum(8,9,10,11)$$

a)

AB \ CD	00	01	11	10
00	0	x	0	0
01	1	1	1	0
11	1	1	x	1
10	0	x	x	0

$$F=AB+B\overline{C}+BD$$

b)

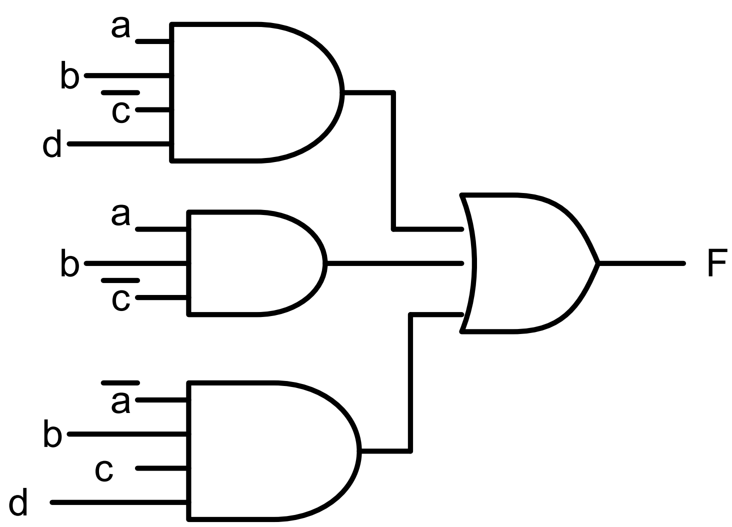
AB \ CD	00	01	11	10
00		1		1
01				
11	1	1		1
10	x	x	x	x

$$F=A\overline{C}+A\overline{D}+\overline{B}\,\overline{C}D+\overline{B}C\overline{D}$$

Q.18 For the Boolean expression given below, implement it using two levels of logic first as AND-OR and then as OR-AND.

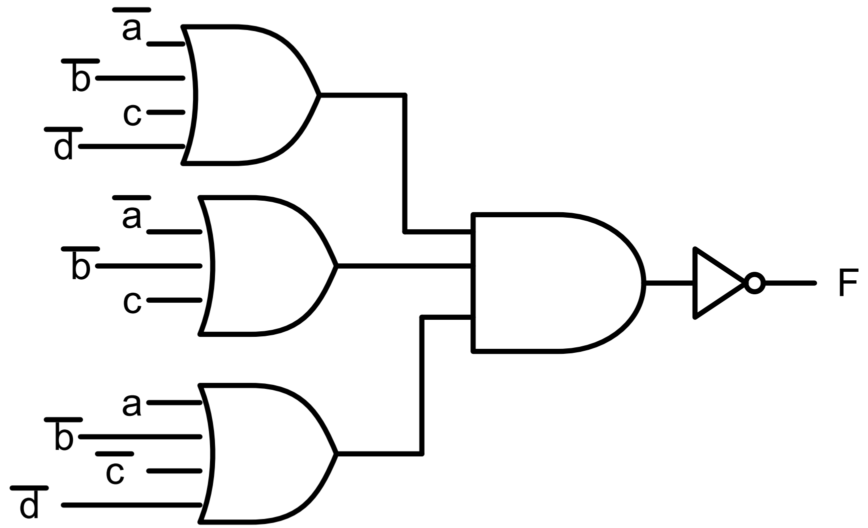
$$F(a,b,c,d)=(ab+cd)(\overline{a}b+\overline{c}d+a\overline{c})$$

$$F = ab\overline{c}d + ab\overline{c} + \overline{a}bcd$$



$$\overline{F} = (\overline{a}+\overline{b}+c+\overline{d}).(\overline{a}+\overline{b}+c).(a+\overline{b}+\overline{c}+\overline{d})$$

Implement $\overline{F}$ and then invert it.

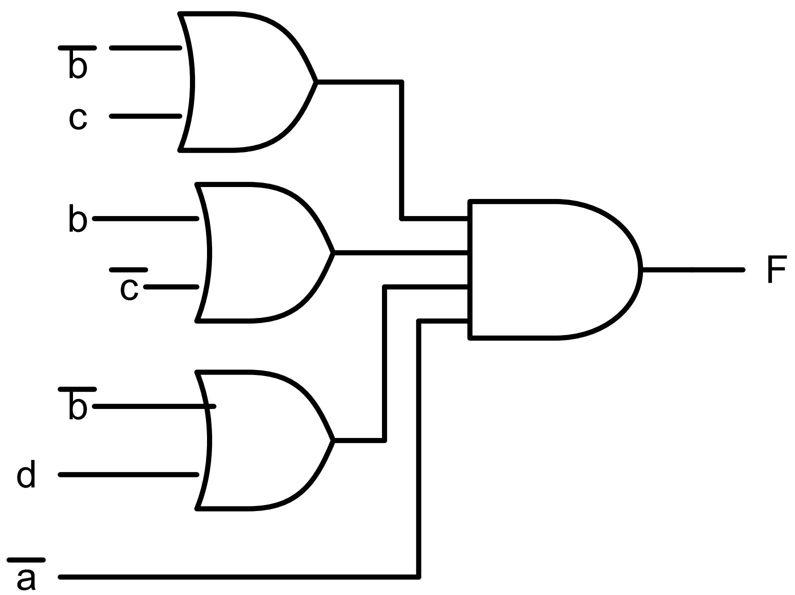


An alternative is to obtain PoS from K-map

$$F = ab\bar{c}d + ab\bar{c} + \bar{a}bcd$$

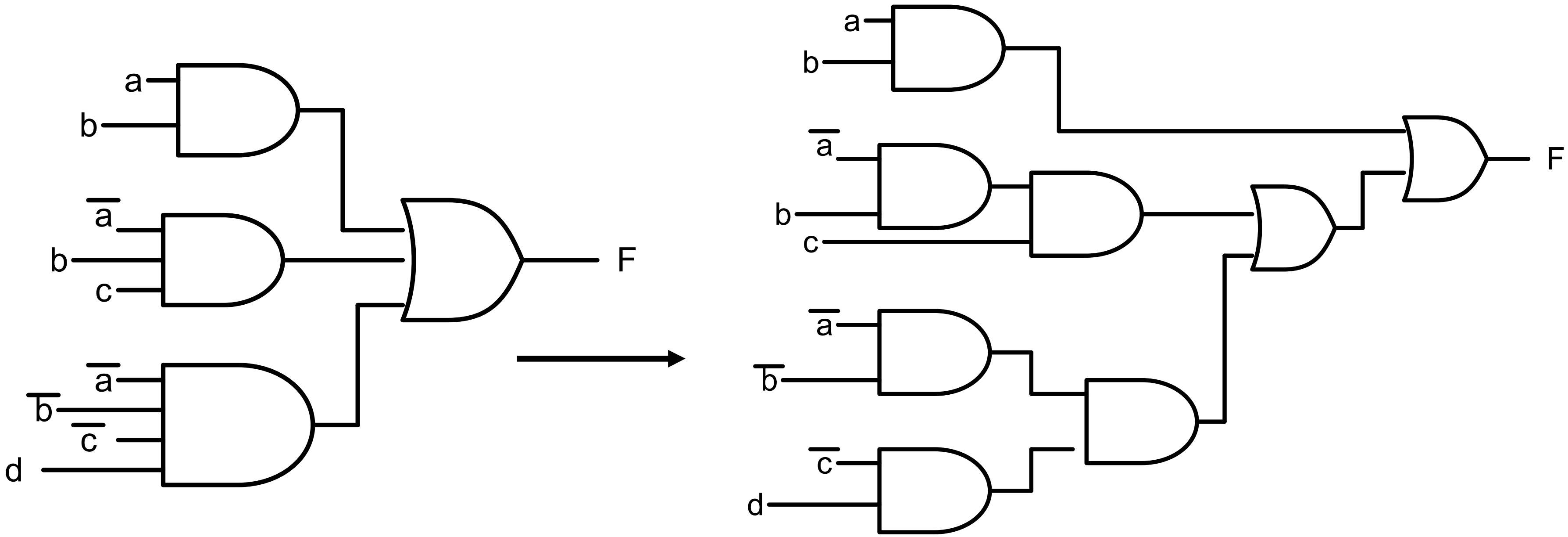
ab \ cd	00	01	11	10
00			0	0
01	0	0		0
11	0	0	0	0
10	0	0	0	0

$$F = \bar{a}.(\bar{b}+c).(b+\bar{c}).(\bar{b}+d)$$

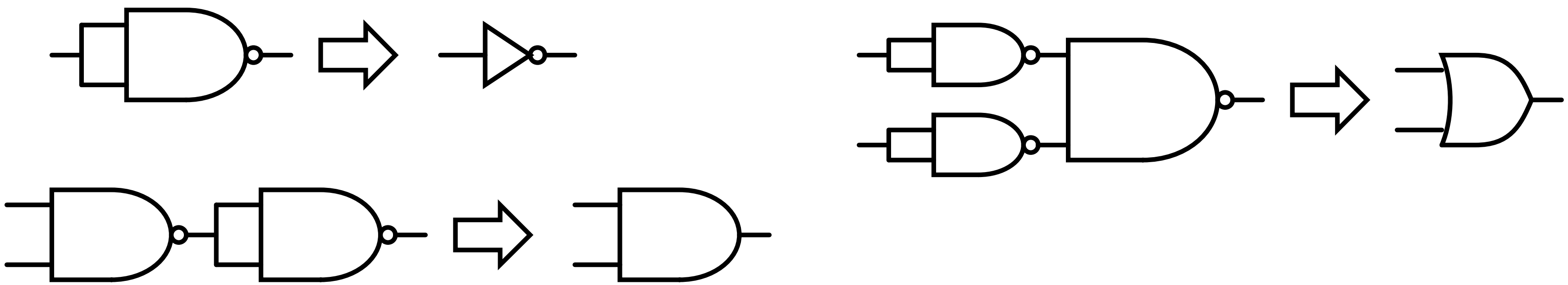


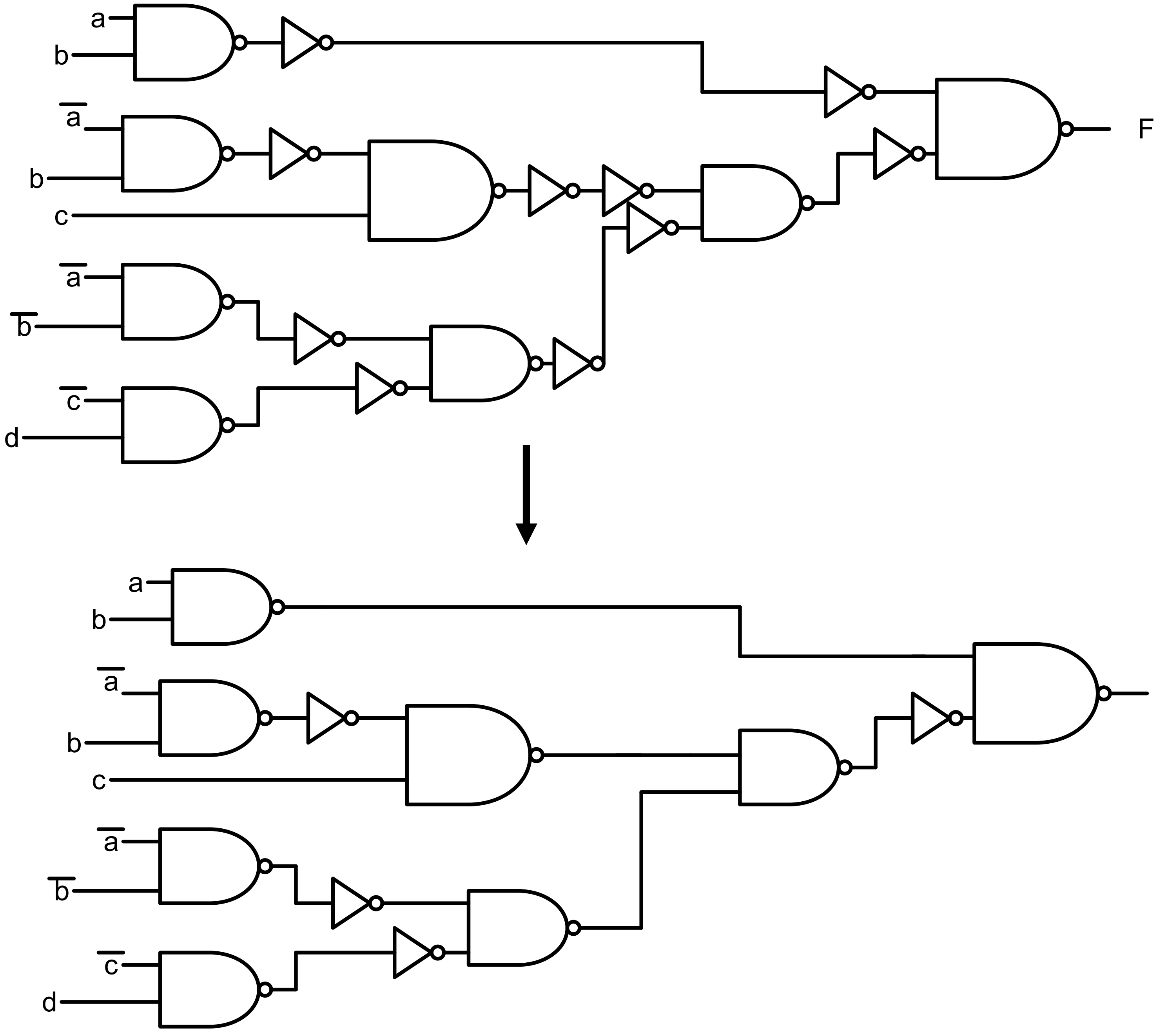
Q.19 Implement the following expression using only 2-input NAND gates and then repeat the problem with only 2 input NOR gates.

$$F(a,b,c,d)=ab+\overline{a}bc+\overline{a}\,\overline{b}\,\overline{c}d$$



Use the following transformation





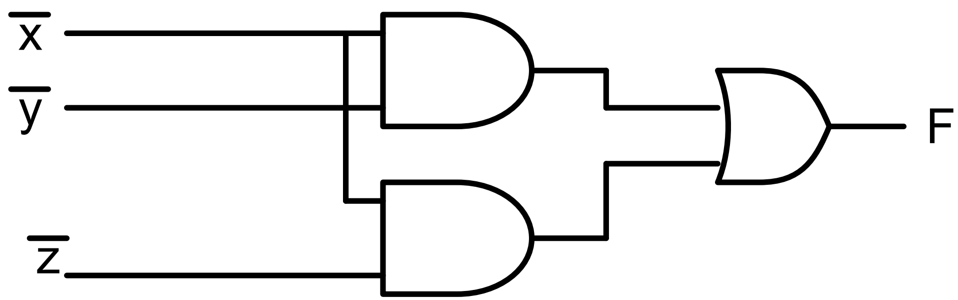
a
b
$\overline{a}$
b
c
$\overline{a}$
$\overline{b}$
$\overline{c}$
d
F
a
b
$\overline{a}$
b
c
$\overline{a}$
$\overline{b}$
$\overline{c}$
d
F

Q.20 Design a combinational circuit with 3 inputs and 1 output
(a) The output is 1 when the binary value of the inputs is less than 3. The output is 0 otherwise
(b) The output is 1 when the binary value of inputs is an odd number.

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

x \ yz	00	01	11	10
0	1	1	0	1
1	0	0	0	0

$$F = \overline{x}\,\overline{y} + \overline{x}\,\overline{z}$$



b.

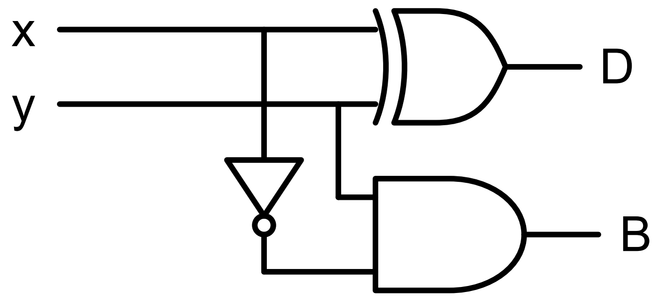
x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

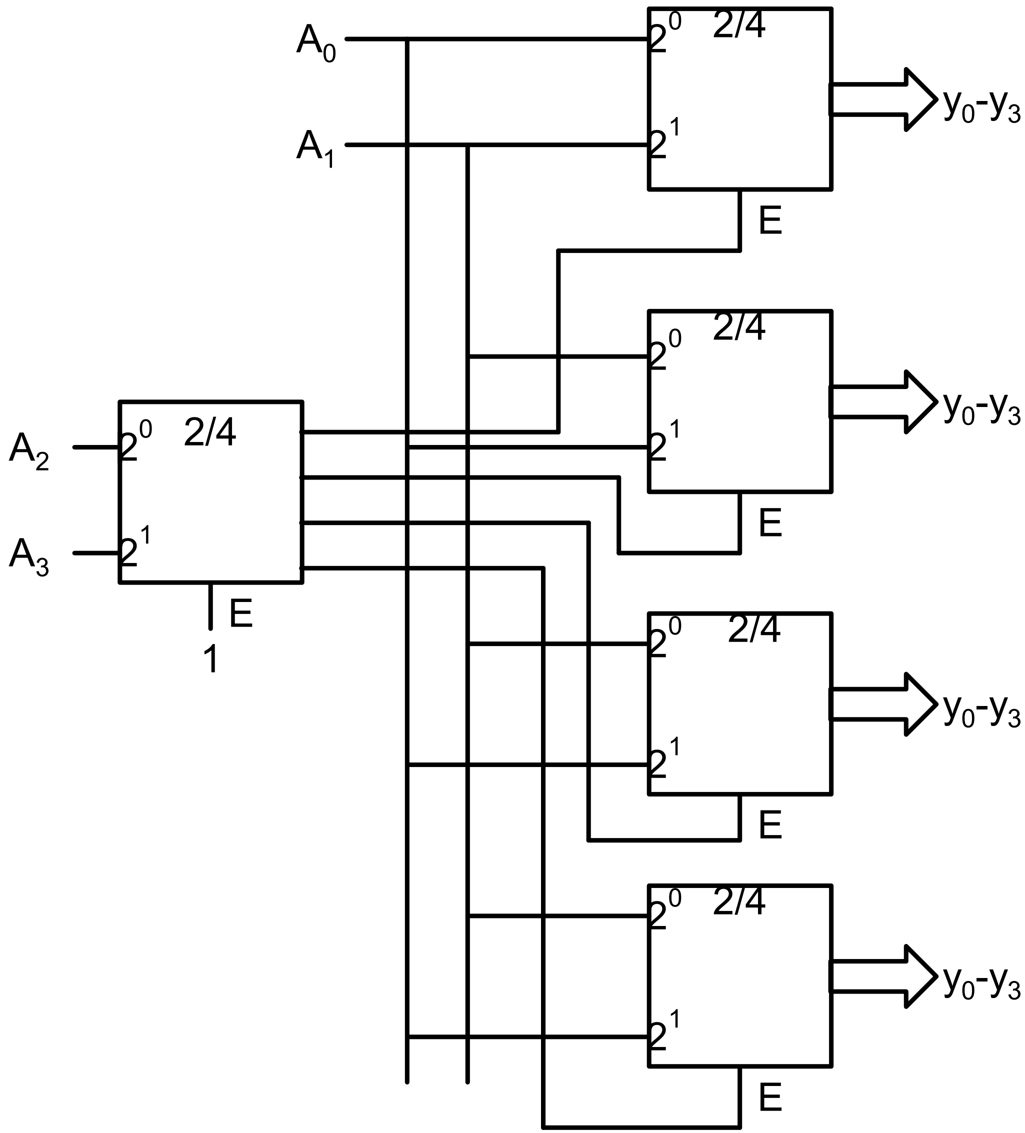
$$F = z$$

Q.21 Design a half subtractor circuit with inputs x and y and outputs Diff. and B_{out}. The circuit subtracts the bits x-y and places the result in Diff. and borrow in B_{out}

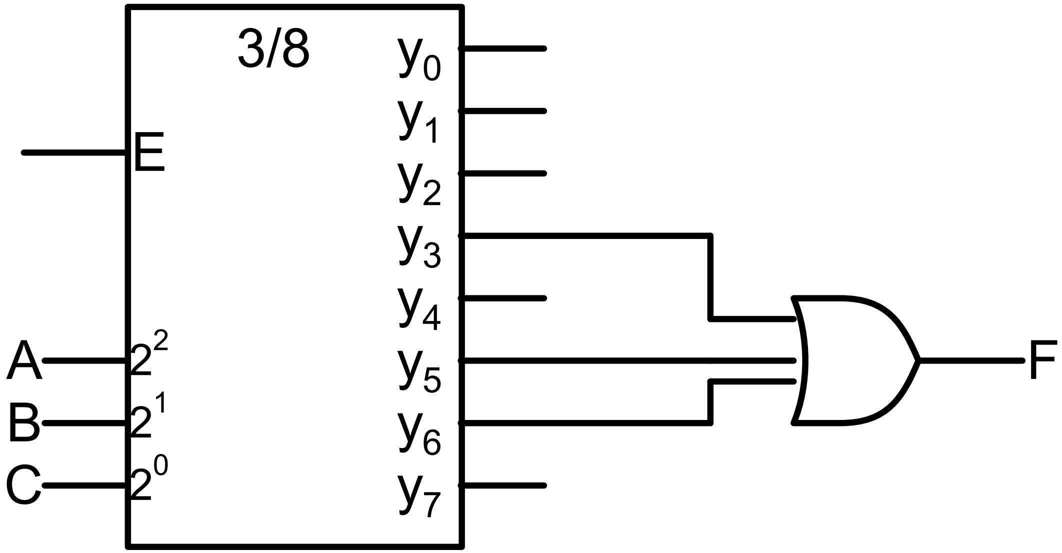
x	y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



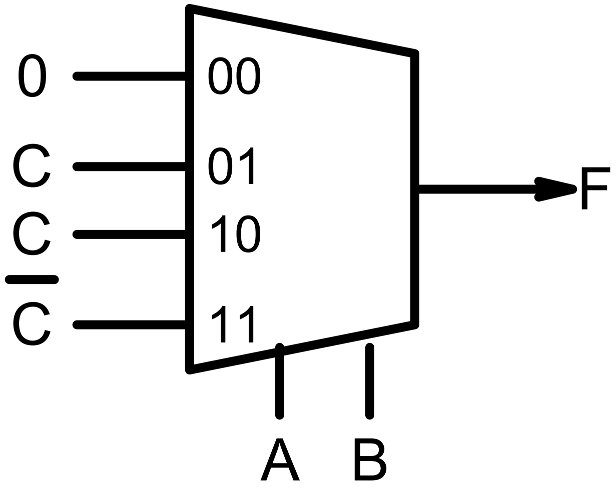
Q.22 Construct a 4-to-16 line decoder with five 2-to-4 line decoders with enable input



Q.23 Implement the following function using a decoder: $F(A,B,C)=\sum(3,5,6)$



Q.24 Implement the above function using a multiplexer



A	B	C	F	
0	0	0	0	*F=0*
0	0	1	0	
0	1	0	0	*F=C*
0	1	1	1	
1	0	0	0	*F=C*
1	0	1	1	
1	1	0	1	*F=$\overline{C}$*
1	1	1	0	