ESC 201AT: Introduction to Electronics

Lecture 5: Toolbox For Circuit Analysis-2

Nodal and Mesh Analysis

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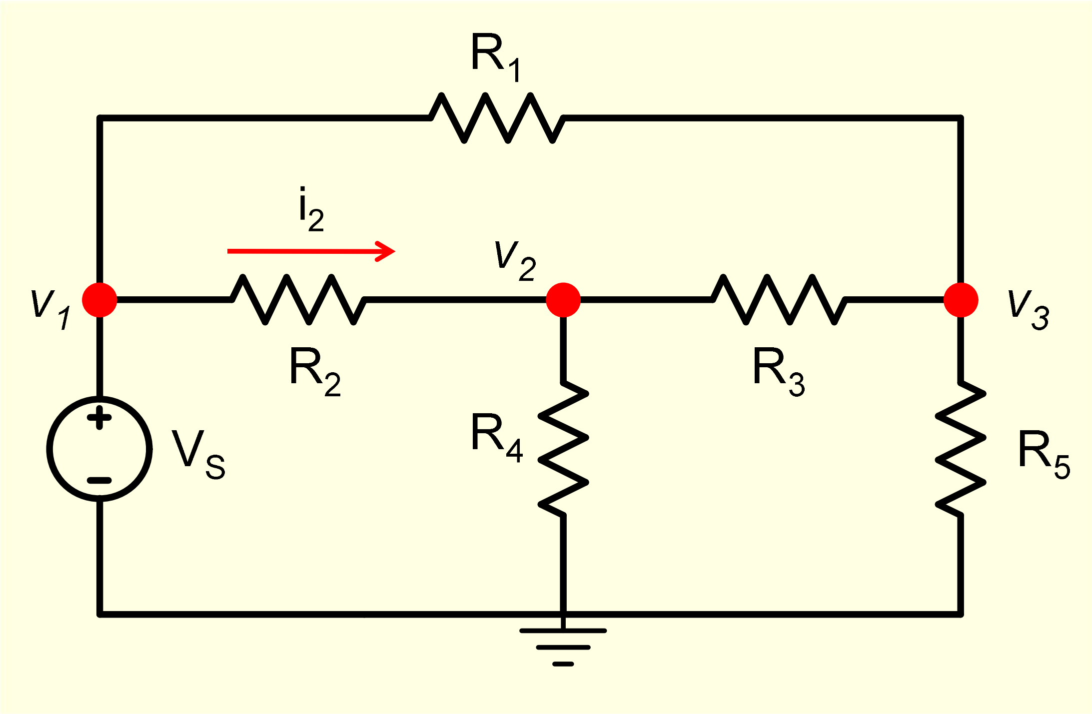
Circuit Analysis

Goal is to find voltages, currents and power in the circuit

If we know voltage and current, then power can be easily determined

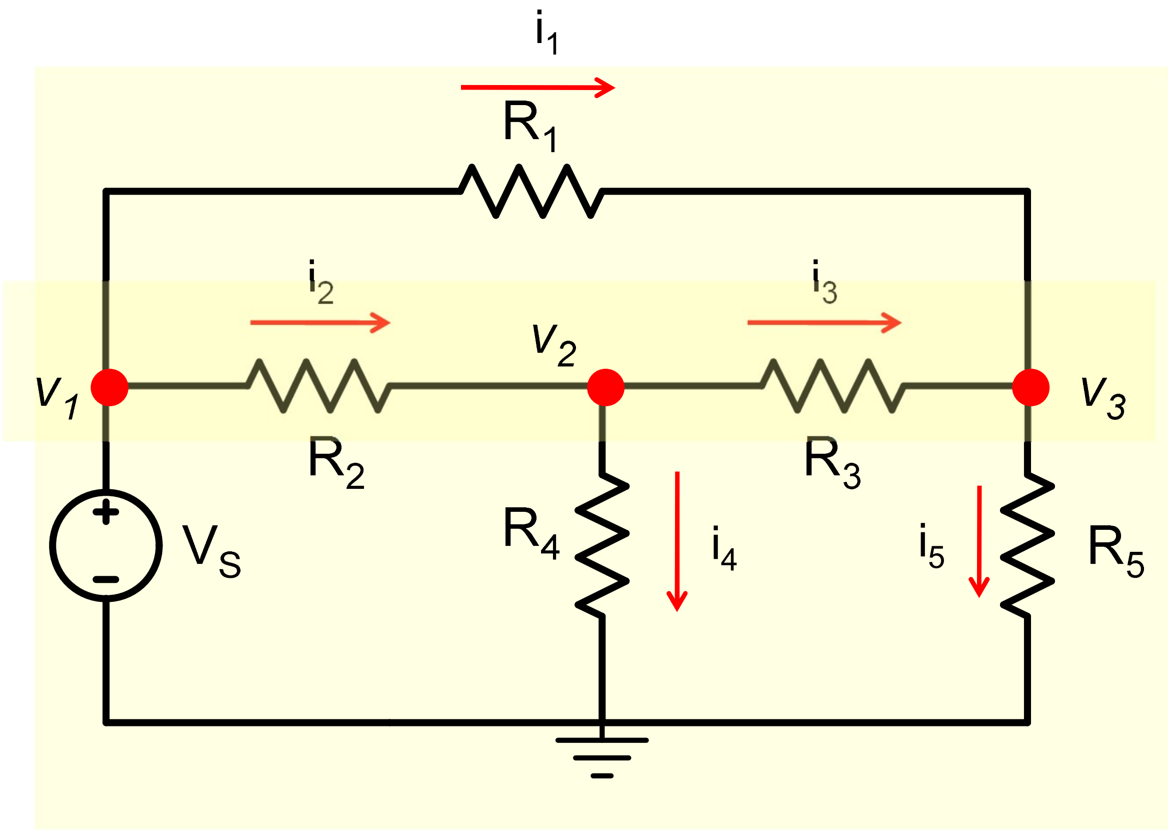
If we determine the voltages, then we can determine the currents using the models of circuit elements

Or if we determine the currents, then we can determine the voltages using the models of circuit elements



If we determine the voltages v_1, v_2,… then we can determine the currents as well

$$v_1 - v_2 = i_2 \times R_2$$



If we determine all the currents, then we can determine the voltages as well

$$v_1 - v_2 = i_2 \times R_2$$

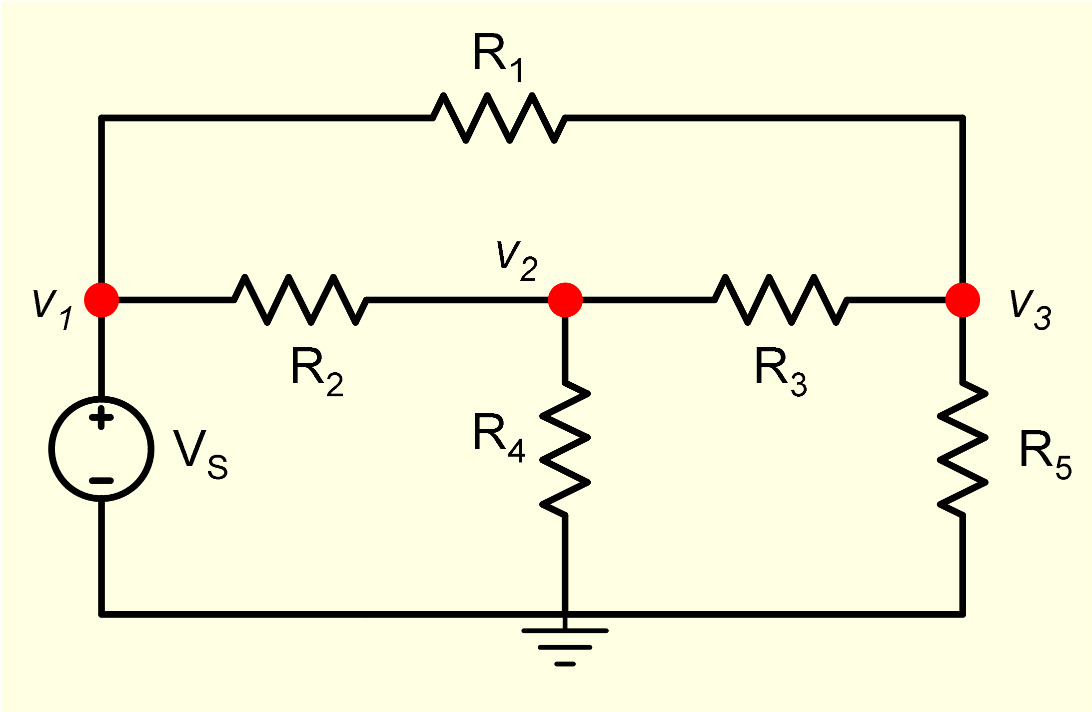
General Circuit Analysis Method: Nodal Analysis

In nodal analysis, the variables used to describe the circuit will be “Node Voltages”

– Nodal voltage are the voltages of each node with respect to a pre-selected reference node

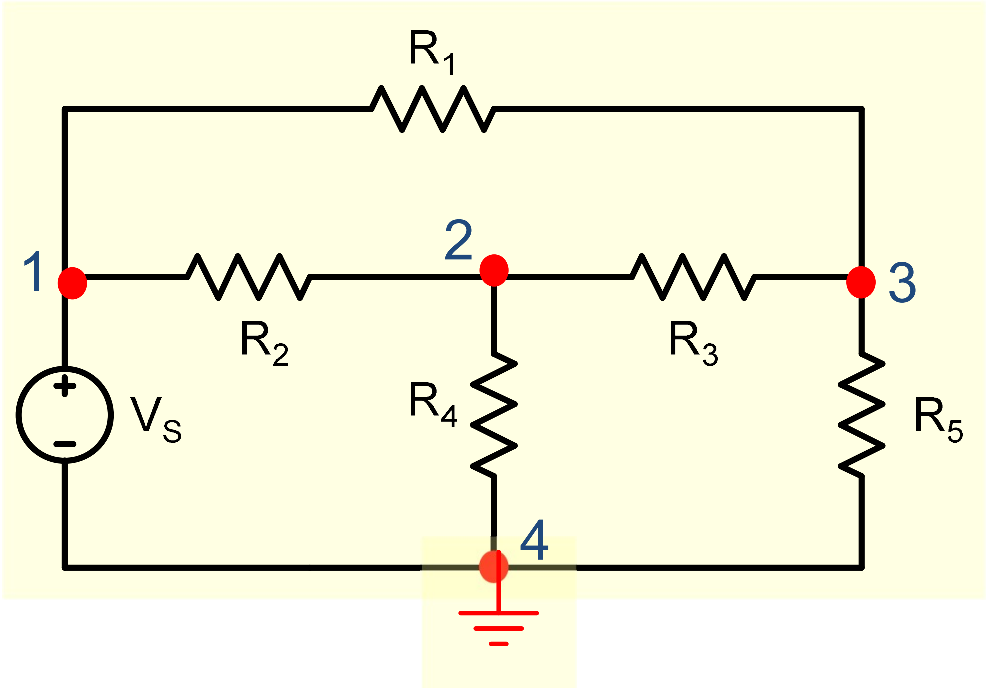
– Usually the reference node has many branches connected to it

– The reference node is also called ground



Nodal Analysis will give values of node voltages v_1, v_2 and v_3 with respect to the reference node

1. Identify and number the nodes. Choose one as a reference node
2. Writing KCL Equations in Terms of the Node Voltages



KCL at node-2 : $\frac{v_1 - v_2}{R_2} + \frac{0 - v_2}{R_4} + \frac{v_3 - v_2}{R_3} = 0$

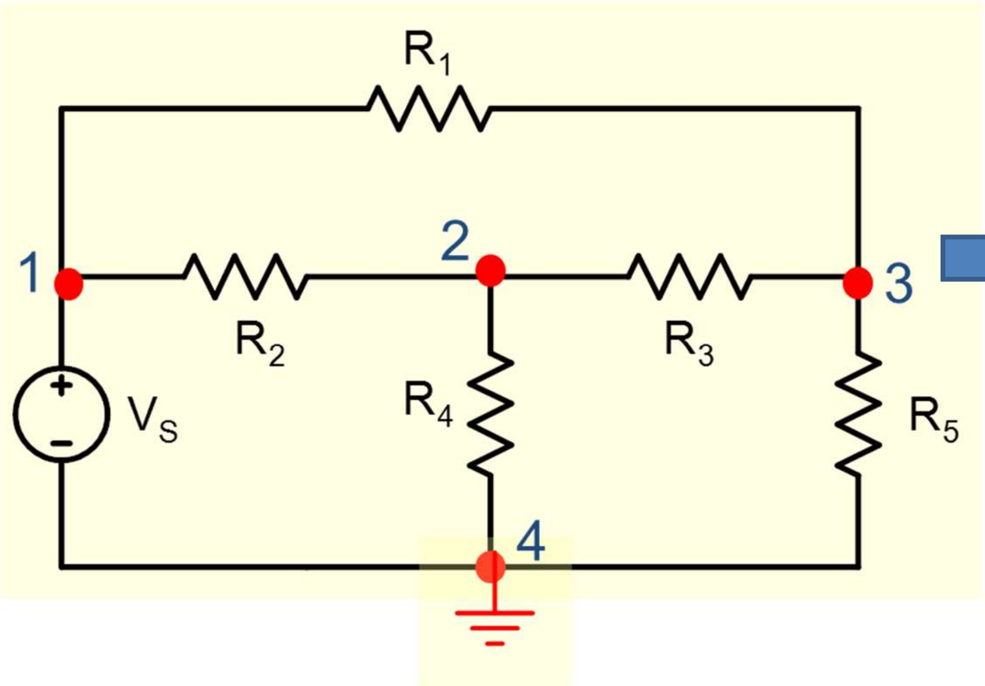
KCL at node-3 : $\frac{v_2 - v_3}{R_3} + \frac{0 - v_3}{R_5} + \frac{v_1 - v_3}{R_1} = 0$

$$v_1 = v_S$$

Solution of these equations will give the values of node voltages

Circuit Analysis

- ❑ Systematic method for Transformation of circuit into equations
- ❑ Solution of equations



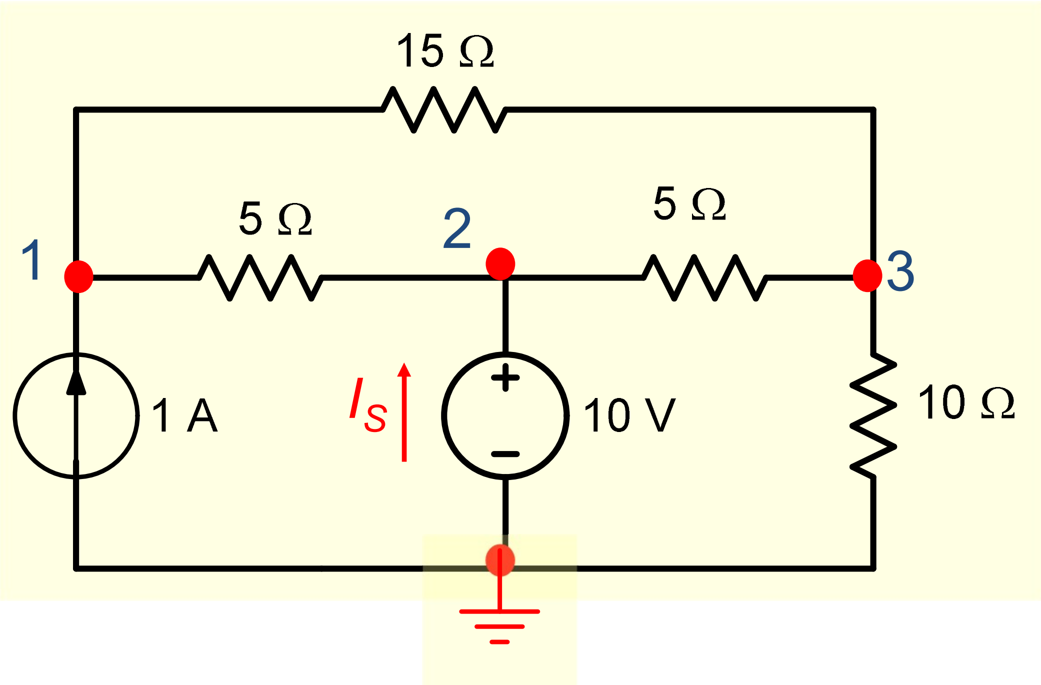
$$\text{KCL at node-2 : } \frac{v_1 - v_2}{R_2} + \frac{0 - v_2}{R_4} + \frac{v_3 - v_2}{R_3} = 0$$

$$\text{KCL at node-3 : } \frac{v_2 - v_3}{R_3} + \frac{0 - v_3}{R_5} + \frac{v_1 - v_3}{R_1} = 0$$

$$v_1 = v_S$$

The whole process is routine and can be automated by a computer program called circuit simulator.

Example



$$\text{KCL at node-1}: 1 + \frac{v_2 - v_1}{5} + \frac{v_3 - v_1}{15} = 0$$

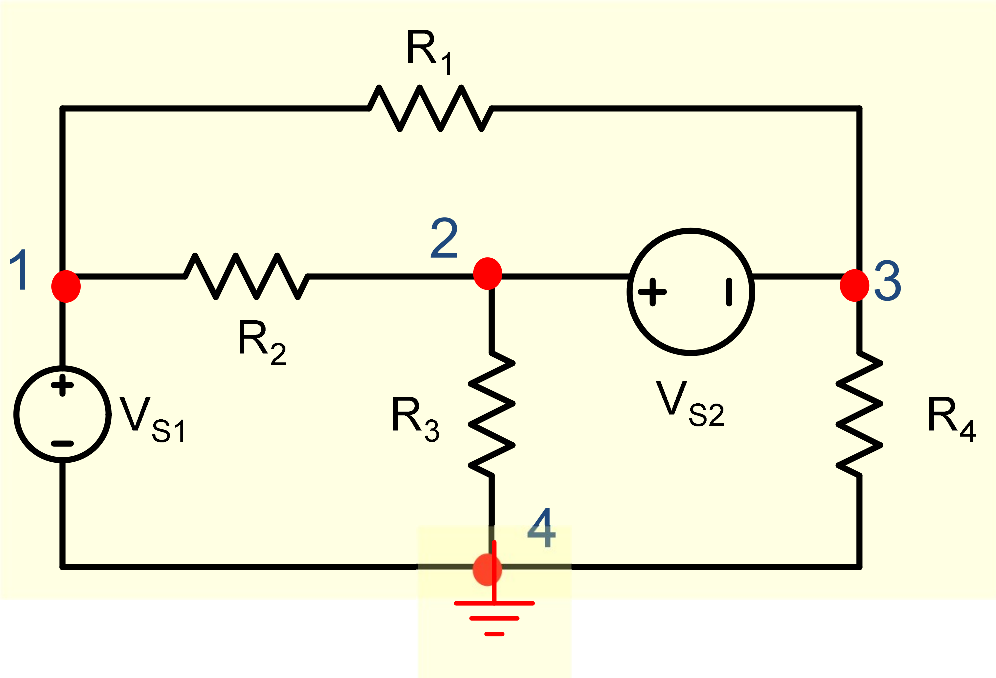
$$\text{KCL at node-3}: \frac{0 - v_3}{10} + \frac{v_2 - v_3}{5} + \frac{v_1 - v_3}{15} = 0$$

$$v_2 = 10\ V$$

$$v_3 = 7.857\ V \qquad v_1 = 13.2\ V$$

$$I_S = \frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{5} = -0.21\ mA$$

Circuit with "Floating" Voltage Sources

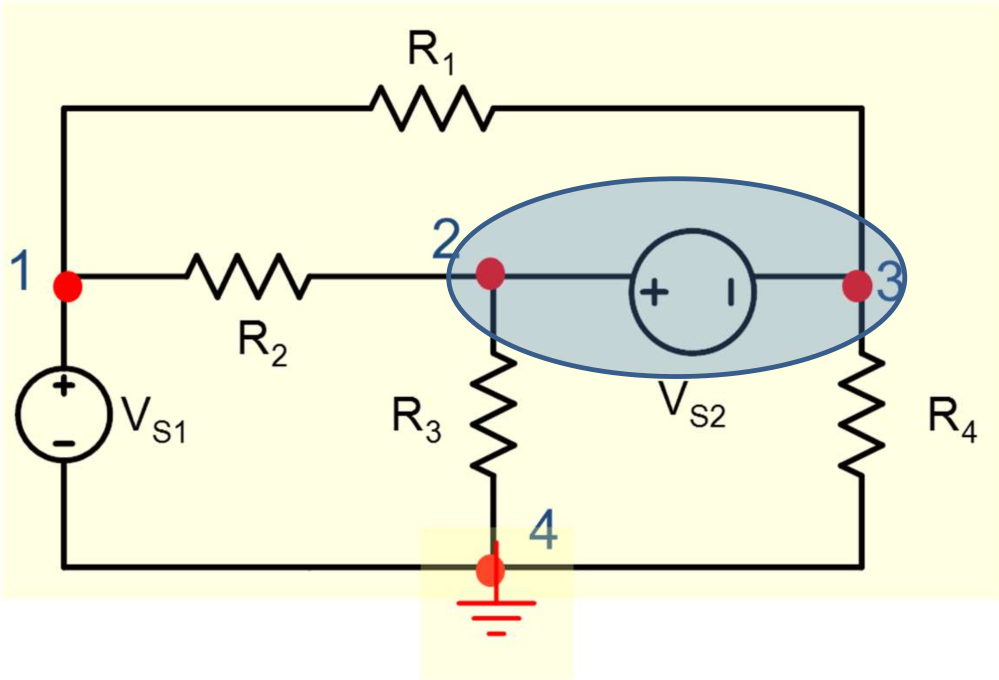


$$\text{KCL at node-2}: \frac{v_1 - v_2}{R_2} + \frac{0 - v_2}{R_3} + ? = 0$$

$$\text{KCL at node-3}: \frac{v_1 - v_3}{R_1} + \frac{0 - v_3}{R_4} + ? = 0$$

$$v_2 - v_3 = V_{S2}$$

$$v_1 = V_{S1}$$



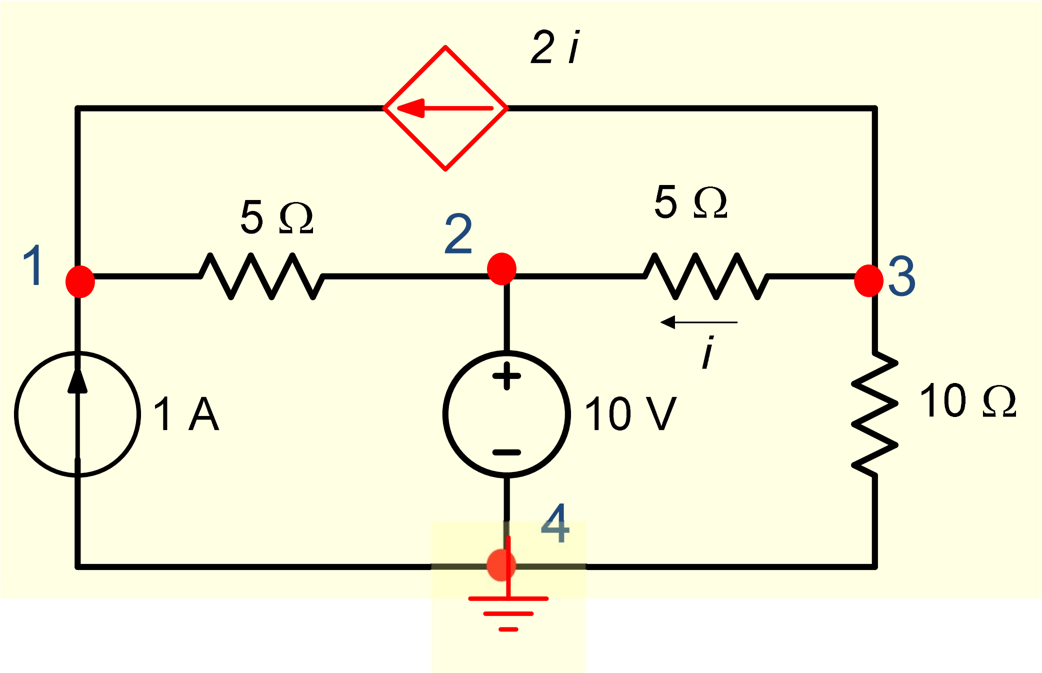
Node 2 and node 3 are merged together into a super node. KCL is applied to this super node

Sum of currents entering a super node is zero

KCL at super-node-(2+3) : $\frac{v_1-v_2}{R_2}+\frac{0-v_2}{R_3}+\frac{0-v_3}{R_4}+\frac{v_1-v_3}{R_1}=0$

$v_2-v_3=V_{S2}$ $\qquad v_1=V_{S1}$

Node-Voltage Analysis with a Dependent Source

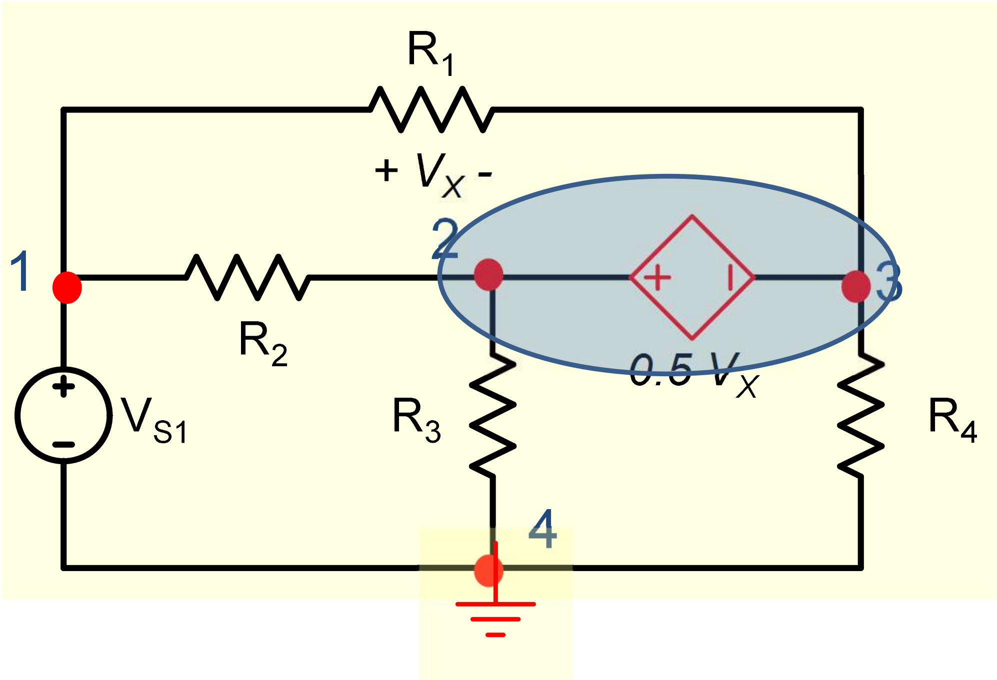


KCL at node-1 : $1 + \frac{v_2 - v_1}{5} + 2 \times i = 0$

KCL at node-3 : $\frac{0 - v_3}{10} + \frac{v_2 - v_3}{5} - 2 \times i = 0$

$v_2 = 10\ V$ $\quad$ $i = \frac{v_3 - v_2}{5}$

Solve the resulting system of equations in terms of node voltages



KCL at super-node-(2+3) : $\frac{v_1 - v_2}{R_2} + \frac{0 - v_2}{R_3} + \frac{0 - v_3}{R_4} + \frac{v_1 - v_3}{R_1} = 0$

$v_2 - v_3 = 0.5 v_X \qquad v_1 - v_3 = v_X$

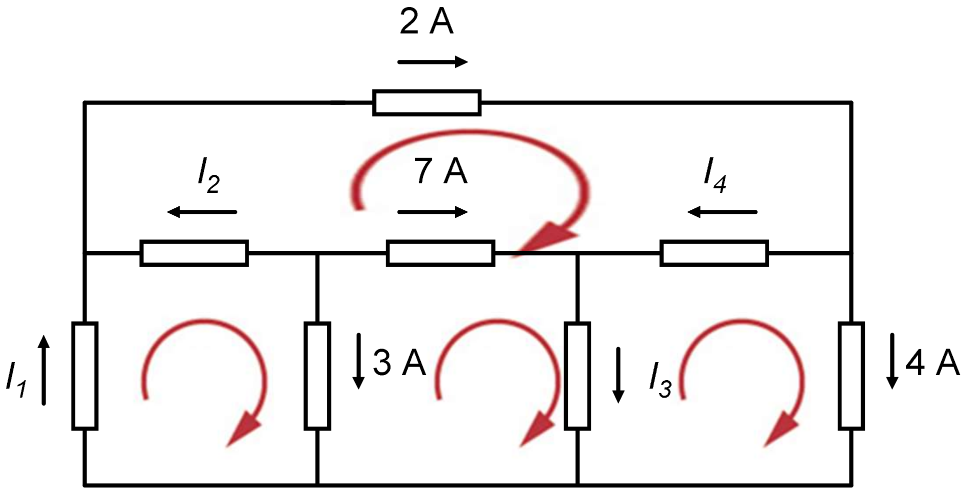
Mesh Analysis

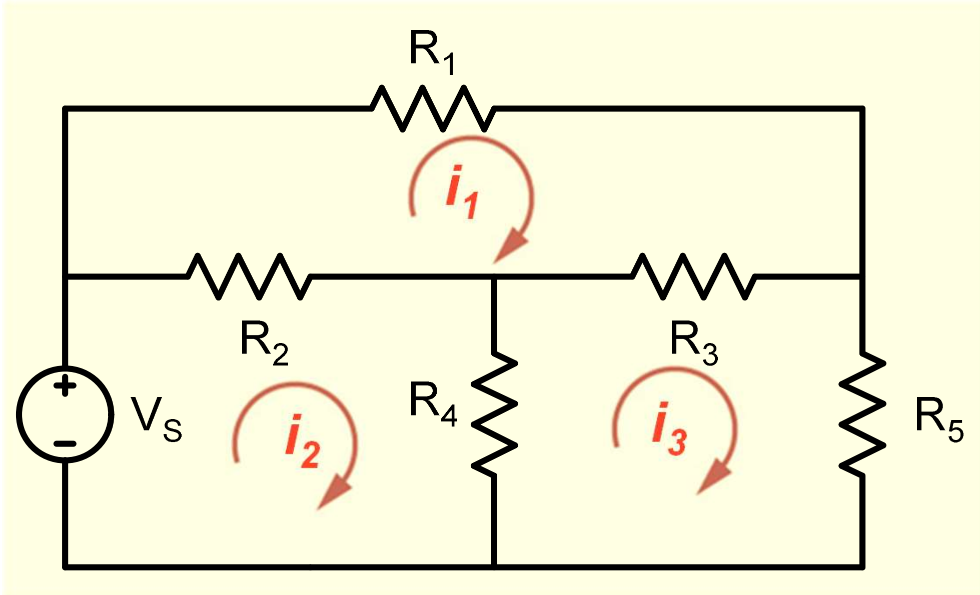
1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

2. Mesh analysis applies KVL to find unknown currents.

3. A mesh is a loop which does not contain any other loops within it.

Mesh Currents



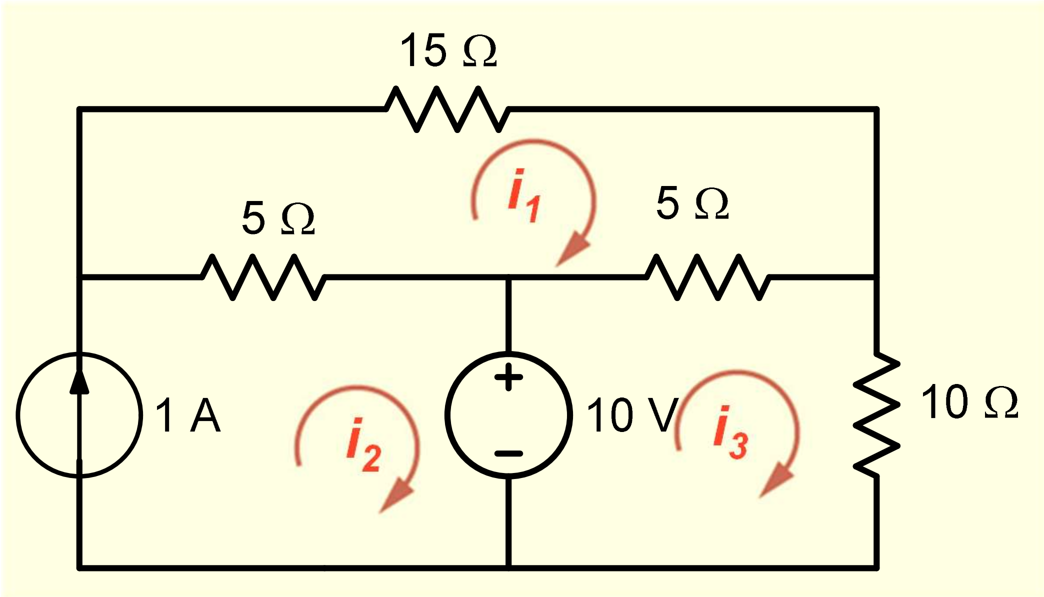


Mesh1: $i_1 \times R_1 + (i_1 - i_3) \times R_3 + (i_1 - i_2) \times R_2 = 0$

Mesh2: $-V_S + (i_2 - i_1) \times R_2 + (i_2 - i_3) \times R_4 = 0$

Mesh3: $i_3 \times R_5 + (i_3 - i_2) \times R_4 + (i_3 - i_1) \times R_3 = 0$

Solve the three equations to get mesh currents from which all branch currents can be determined



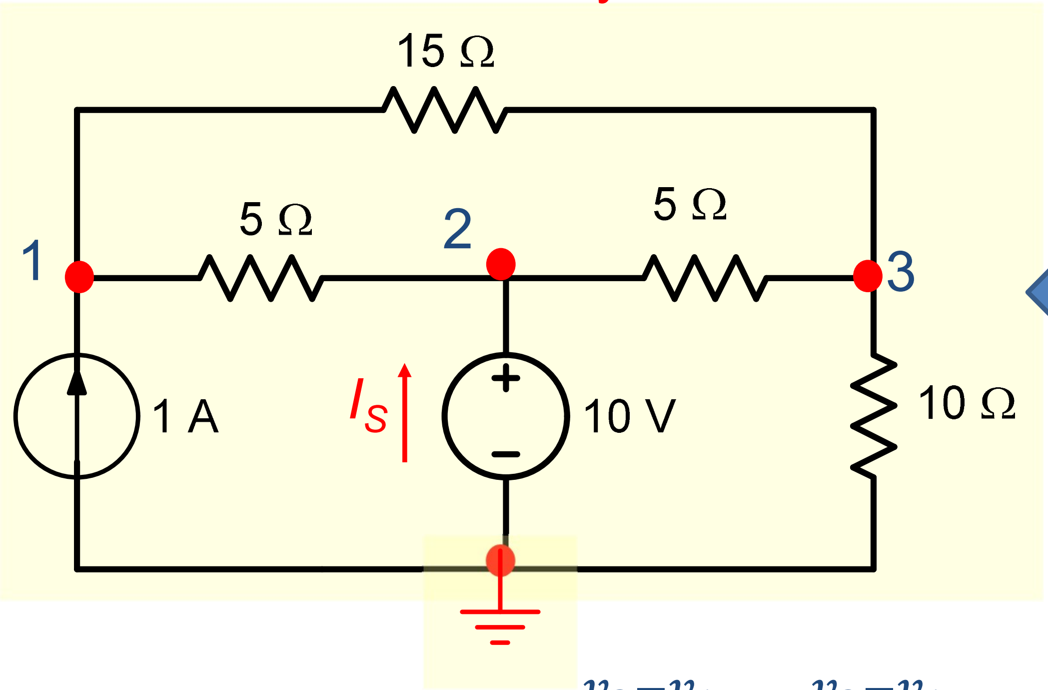
Mesh1: $i_1 \times 15 + (i_1 - i_3) \times 5 + (i_1 - i_2) \times 5 = 0$

Mesh2: $i_2 = 1$

Mesh3: $i_3 \times 10 - 10 + (i_3 - i_1) \times 5 = 0$

$$\Rightarrow i_1 = 0.356\ A\ ;\ i_3 = 0.78A$$

Nodal Analysis



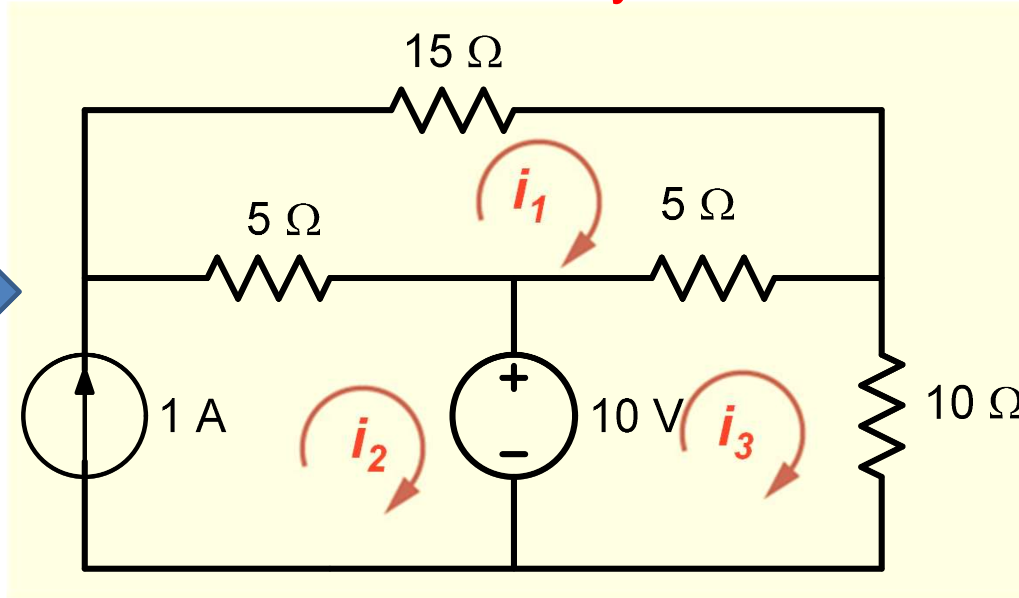
$$\text{KCL at node-1}: 1 + \frac{v_2 - v_1}{5} + \frac{v_3 - v_1}{15} = 0$$

$$\text{KCL at node-3}: \frac{0 - v_3}{10} + \frac{v_2 - v_3}{5} + \frac{v_1 - v_3}{15} = 0$$

$$v_1 = 13.2\,V \quad v_2 = 10\,V \quad v_3 = 7.857\,V$$

$$I_S = -0.21\,mA$$

Mesh Analysis



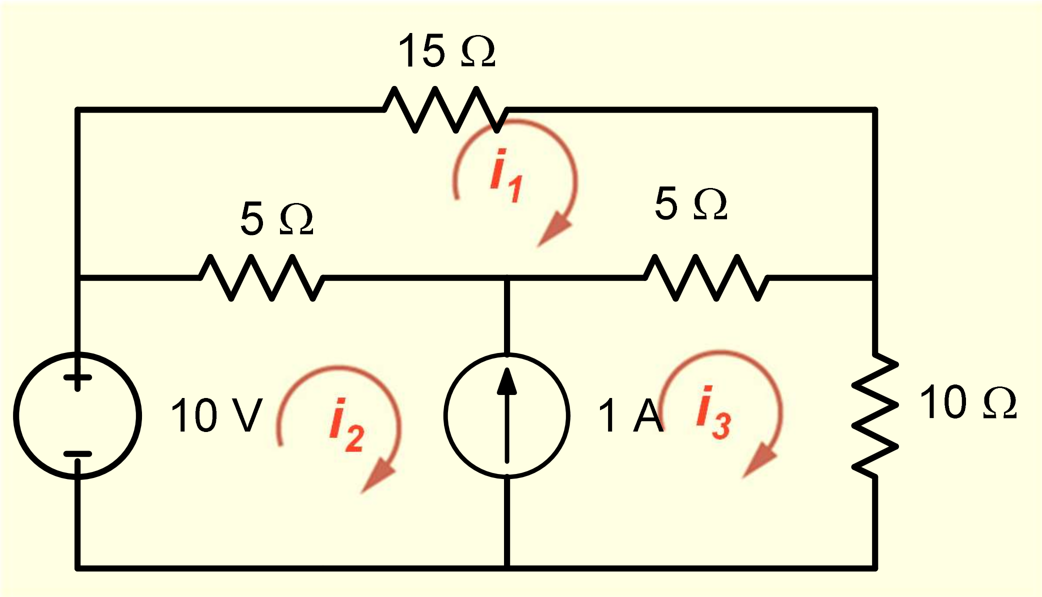
$$\text{Mesh1}: i_1 \times 15 + (i_1 - i_3) \times 5 + (i_1 - i_2) \times 5 = 0$$

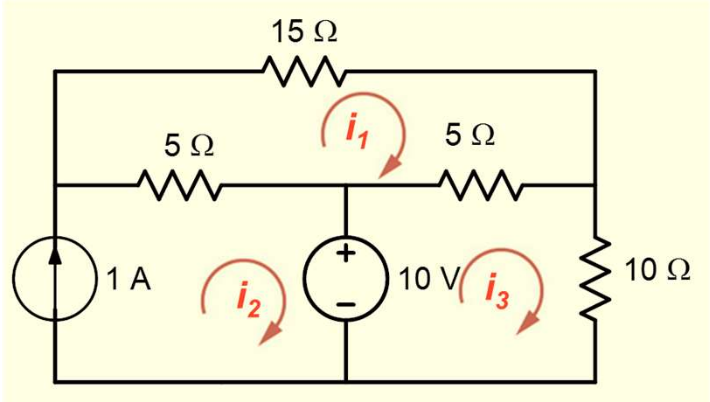
$$\text{Mesh2}: \; i_2 = 1$$

$$\text{Mesh3}: i_3 \times 10 - 10 + (i_3 - i_1) \times 5 = 0$$

$$\Rightarrow i_1 = 0.356A\,; i_3 = 0.7857A$$

Current source common to 2 mesh



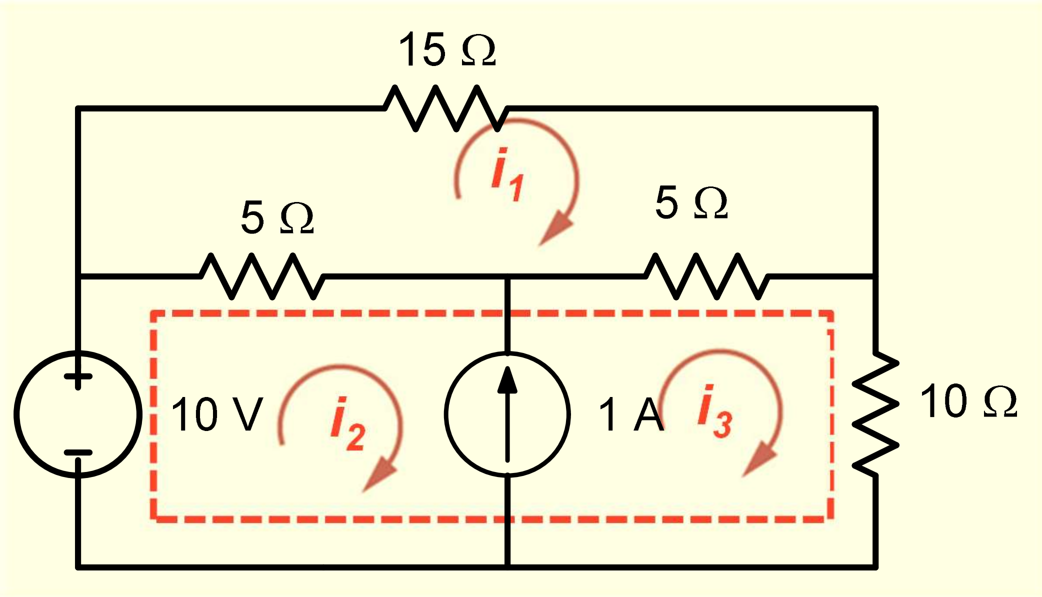


$$\text{Mesh1: } i_1 \times 15 + (i_1 - i_3) \times 5 + (i_1 - i_2) \times 5 = 0$$

$$\text{Mesh2: } -10 + (i_2 - i_1) \times 5 + ?$$

$$\text{Mesh3: } i_3 \times 10 + ? + (i_3 - i_1) \times 5 = 0$$

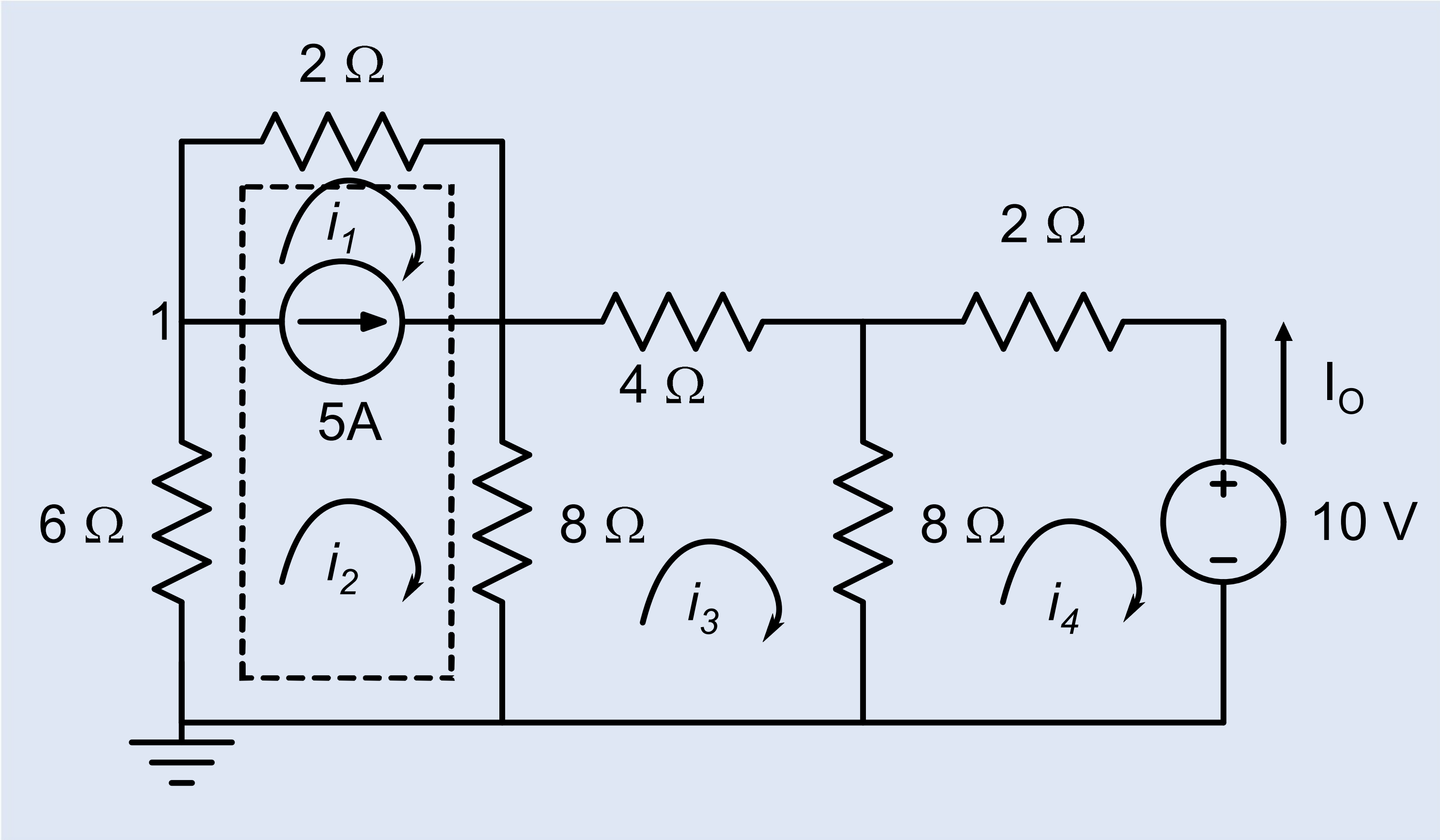
Super mesh

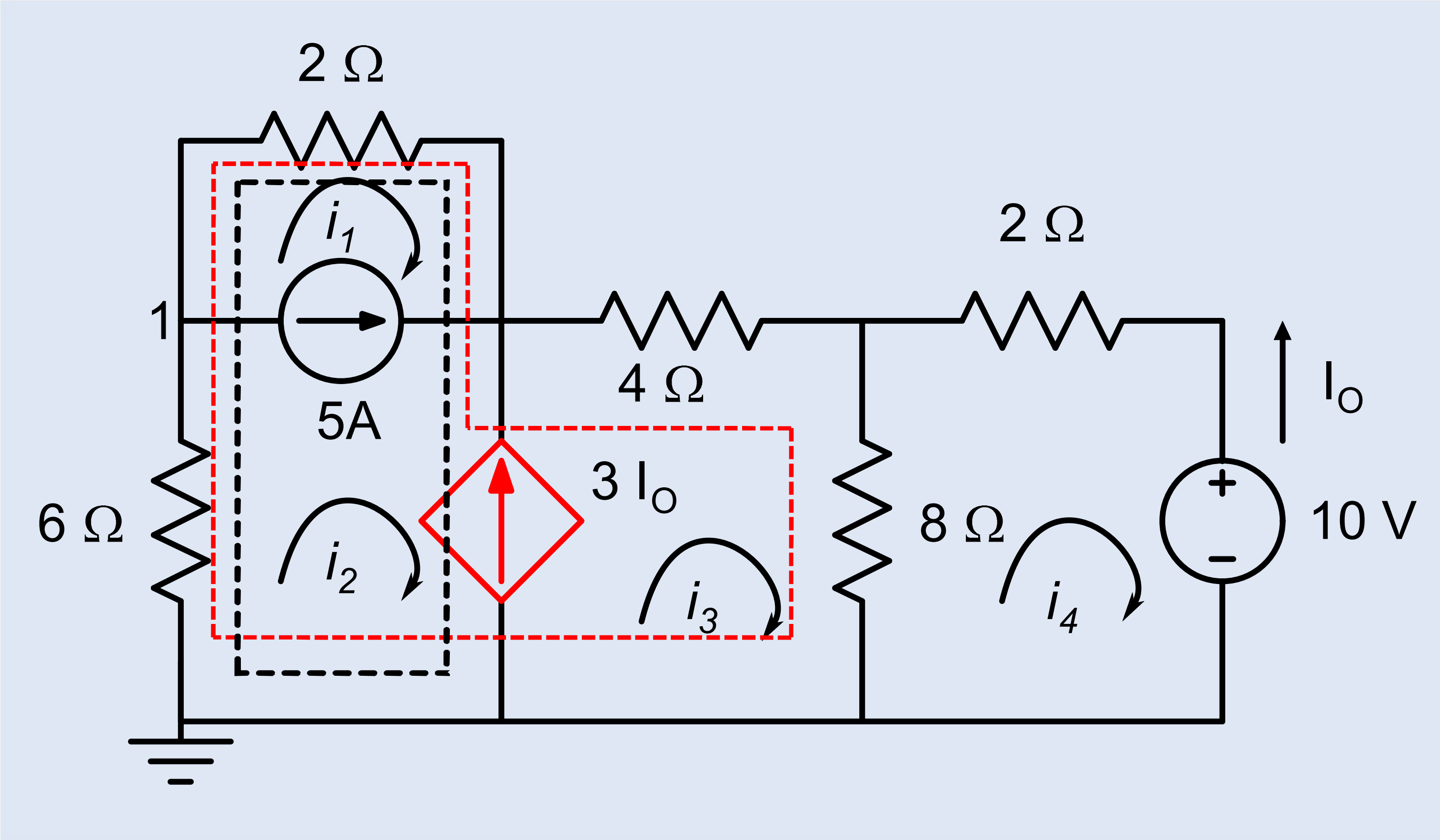


$$\text{Mesh1: } i_1 \times 15 + (i_1 - i_3) \times 5 + (i_1 - i_2) \times 5 = 0$$

$$\text{Super–Mesh1: } -10 + (i_2 - i_1) \times 5 + (i_3 - i_1) \times 5 + i_3 \times 10 = 0$$

$$i_3 - i_2 = 1$$


2 Ω
i1
1
5A
6 Ω
i2
8 Ω
4 Ω
i3
2 Ω
8 Ω
i4
IO
10 V

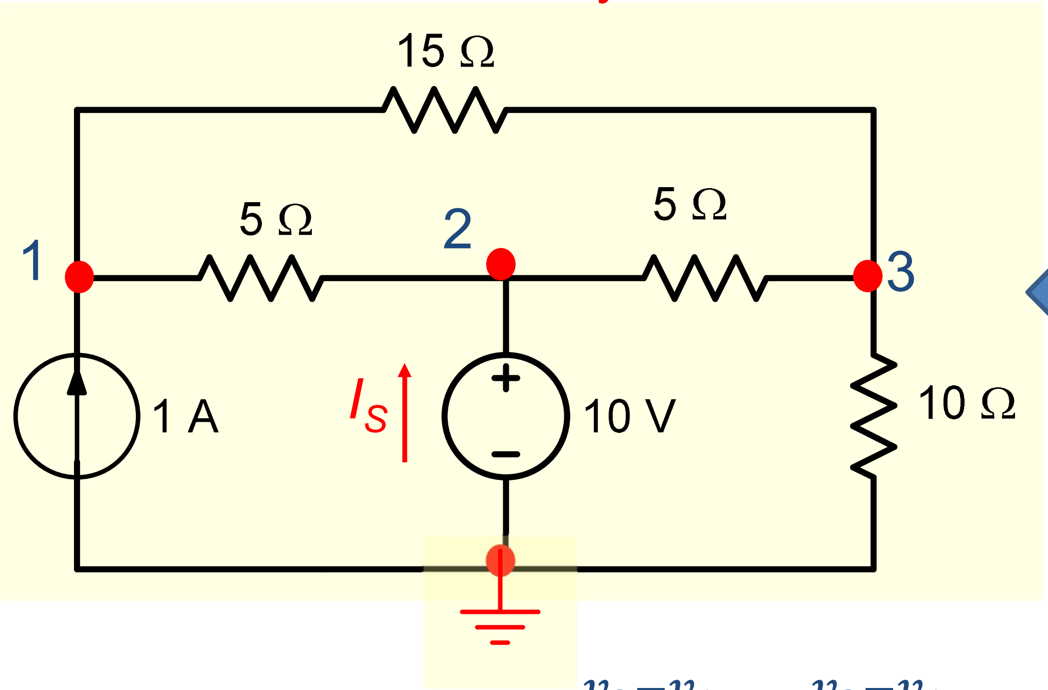


$$i_2 \times 6 + i_1 \times 2 + i_3 \times 4 + (i_3 - i_4) \times 8 = 0$$

$$(i_4 - i_3) \times 8 + i_4 \times 2 - 10 = 0$$

$$i_4 = -\mathrm{i}_O \qquad i_2 - i_1 = 5 \qquad i_3 - i_2 = 3i_O$$

Nodal Analysis



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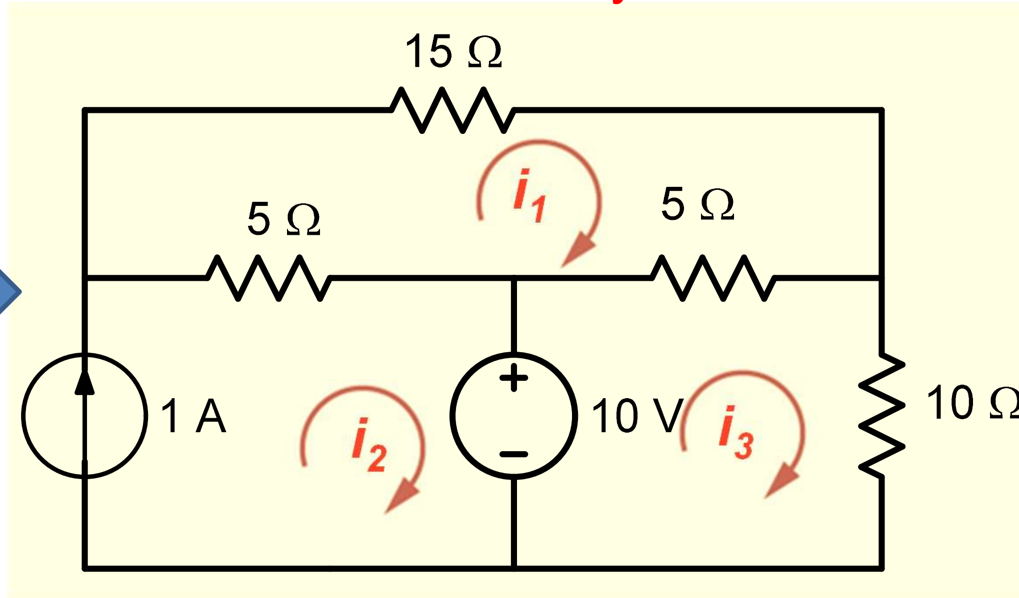
$$\text{KCL at node-3}: \frac{0 - v_3}{10} + \frac{v_2 - v_3}{5} + \frac{v_1 - v_3}{15} = 0$$

$$v_1 = 13.2\ V \quad v_2 = 10\ V \quad v_3 = 7.857\ V$$

$$I_S = -0.21\ mA$$

Mesh Analysis



$$\text{Mesh1}: i_1 \times 15 + (i_1 - i_3) \times 5 + (i_1 - i_2) \times 5 = 0$$

$$\text{Mesh2}: \ i_2 = 1$$

$$\text{Mesh3}: i_3 \times 10 - 10 + (i_3 - i_1) \times 5 = 0$$

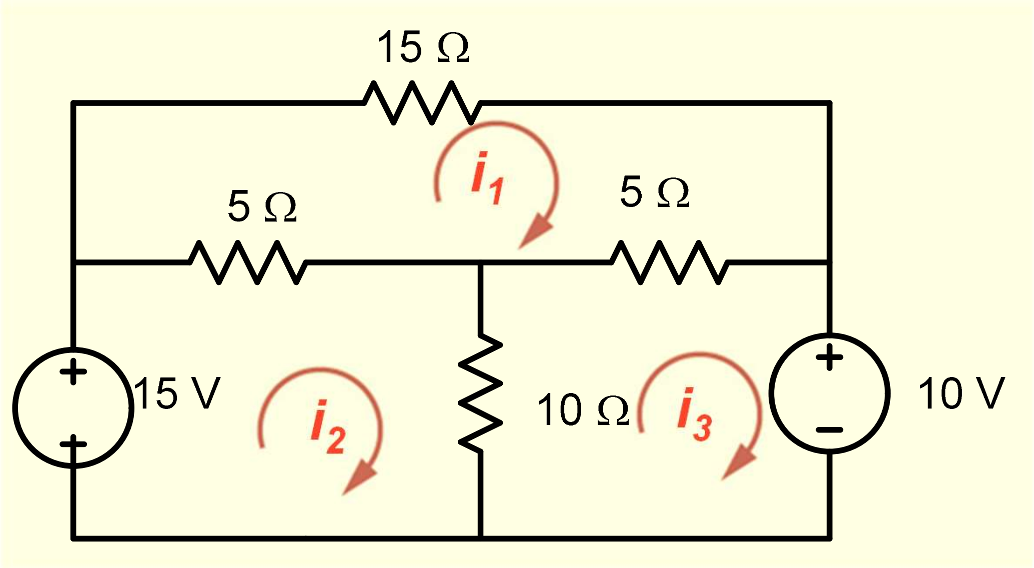
$$\Rightarrow i_1 = 0.356A\,; i_3 = 0.7857A$$

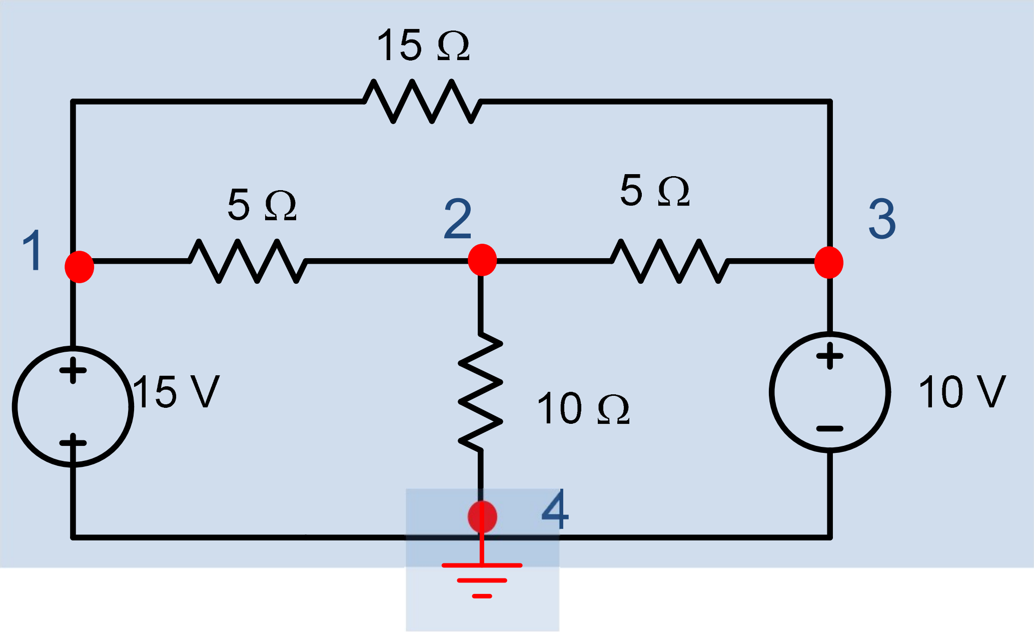
Nodal vs. Mesh Analysis

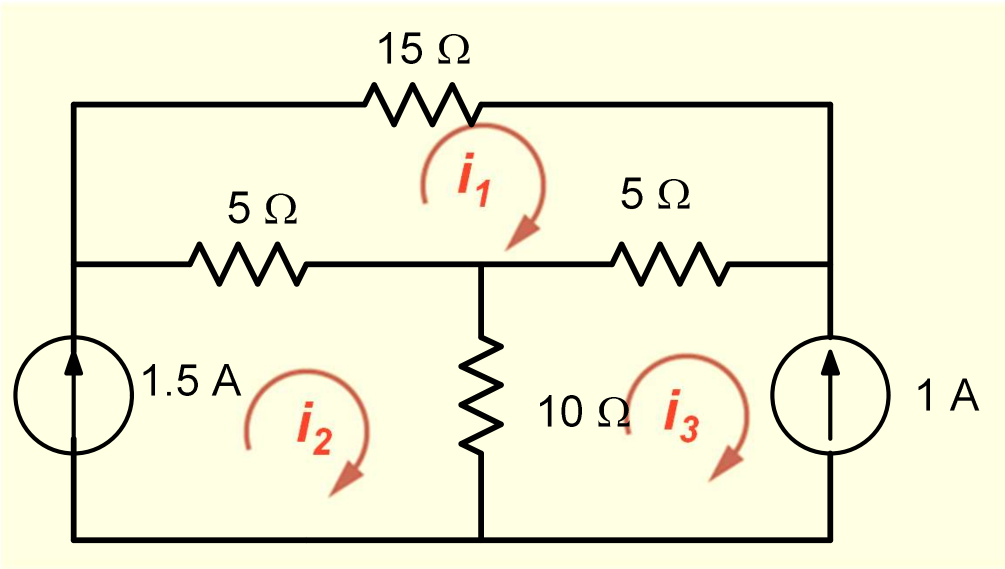
Select the method that results in the smaller number of equations. For example:

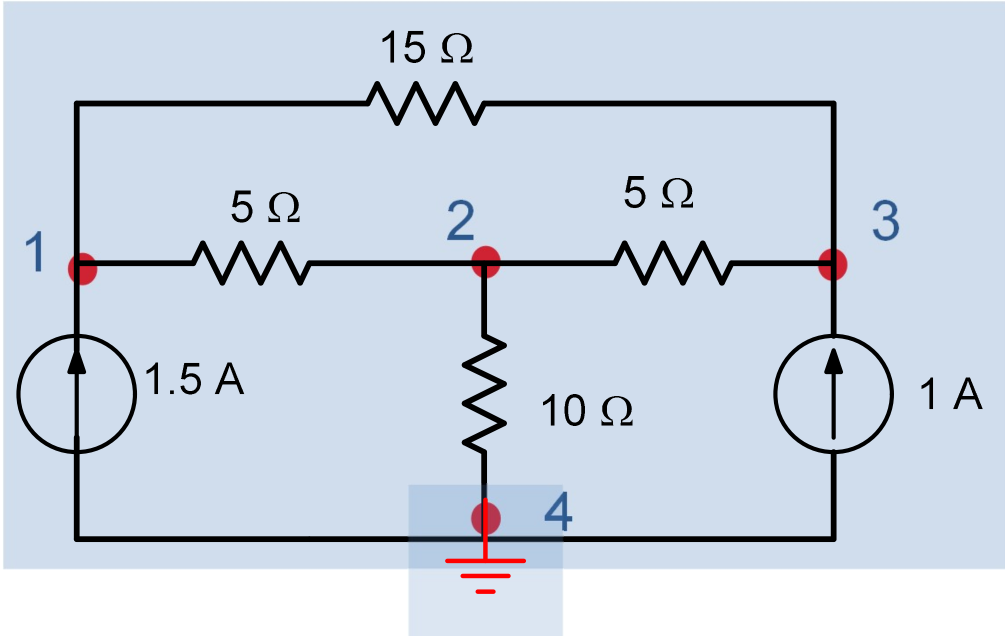
1. Choose nodal analysis for circuit with fewer unknown node voltages than unknown meshe currents.

2. If node voltages are required, it may be expedient to apply nodal analysis. If branch currents are required, it may be better to use mesh analysis.









Mesh analysis requires lesser number of equations to be solved here

Both methods are suitable for automated analysis