ESC201T : Introduction to Electronics

Lecture Notes 18: Two Port Network

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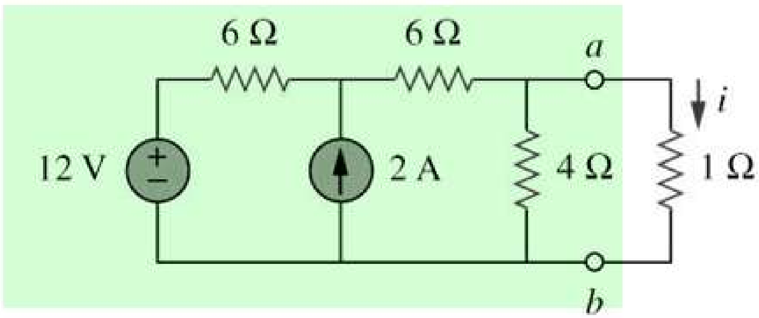
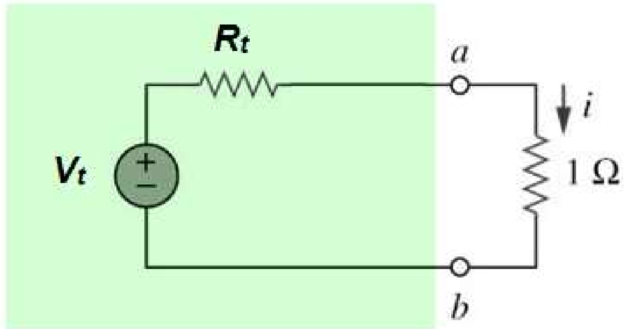
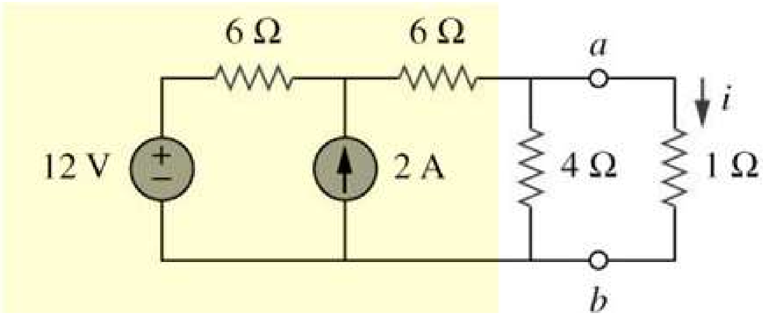
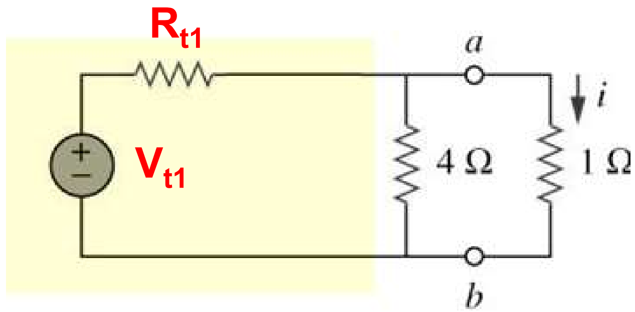
Techniques for Coping With Complexity: ABSTRACTION

An abstract representation is a simplified representation that has appropriate level of detail for the problem being addressed.



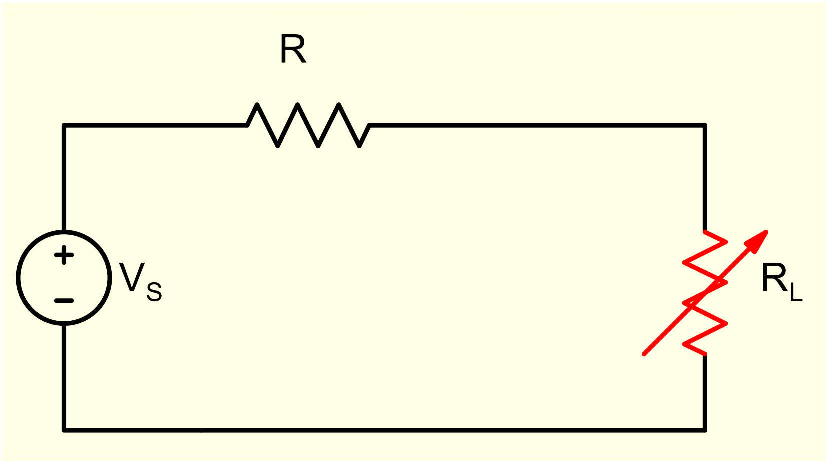






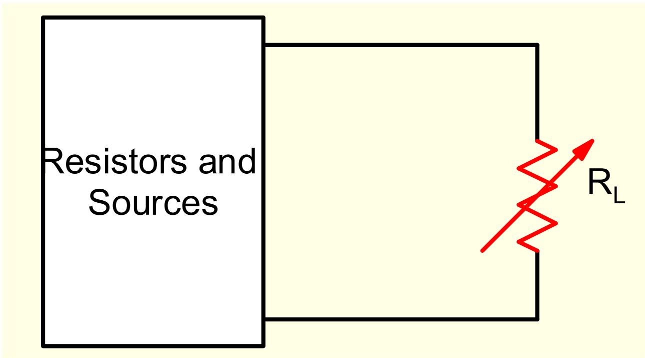
Circuit Transformation allows us to apply results derived for simple circuits to more complicated circuits as well.

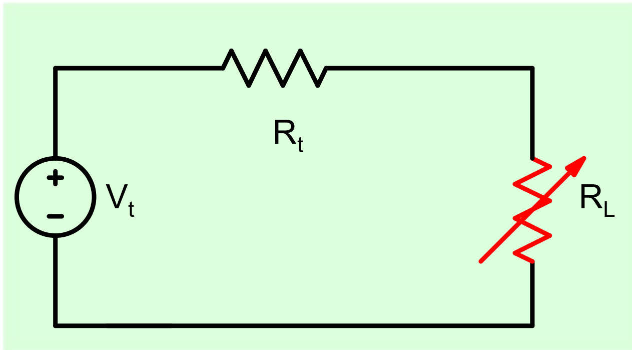
Maximum Power Transfer for dc circuits



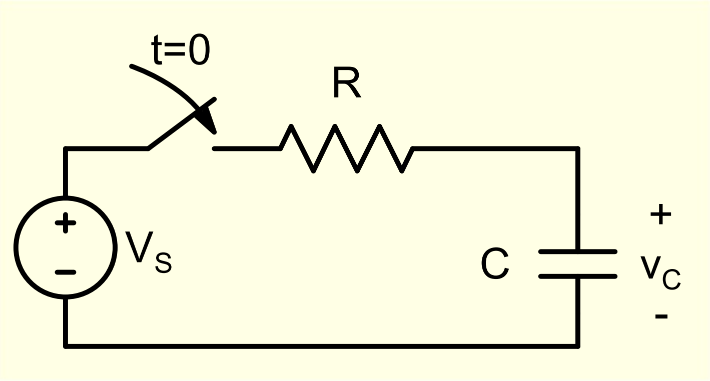
$$R_L = R$$

$$P_{L\max} = \frac{V_S^2}{4R_L}$$

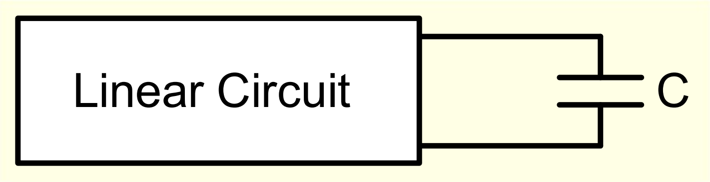


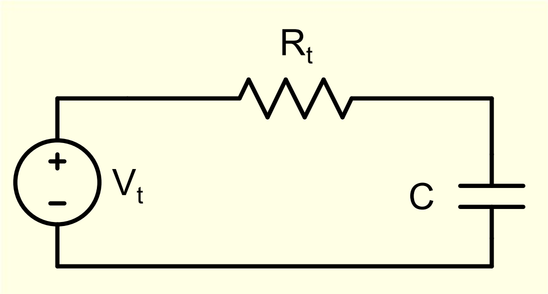


Maximum power is delivered to the load when $R_L = R_t$



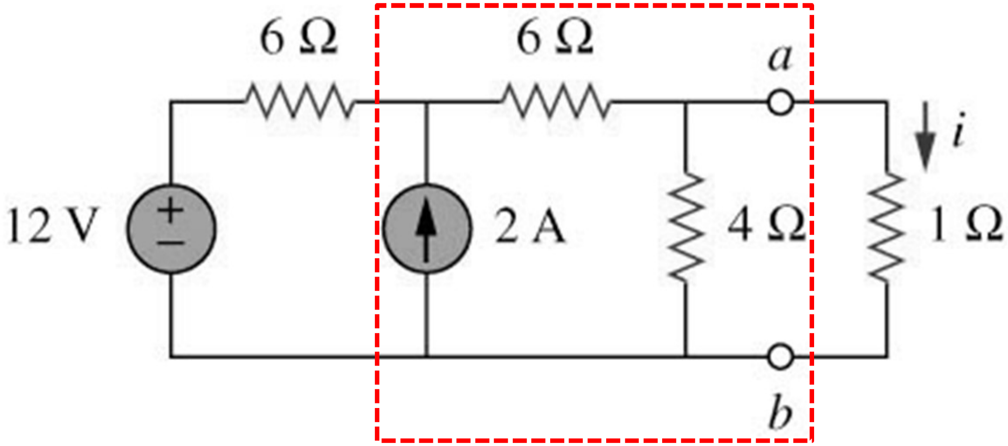
$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\ e^{-\frac{t}{RC}}$$





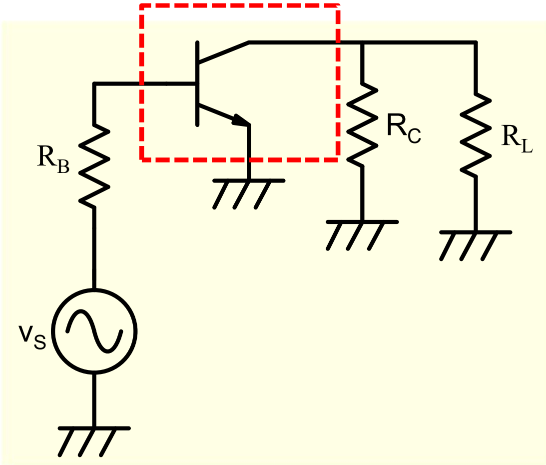
$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$

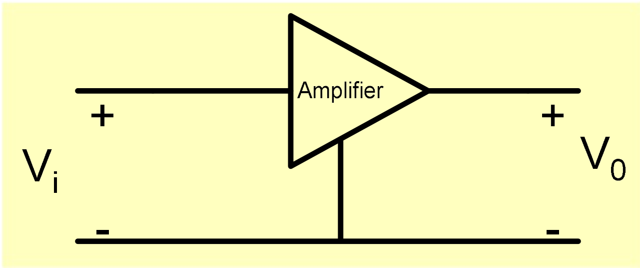
Thevenin's or Norton's Theorem are not Sufficient.

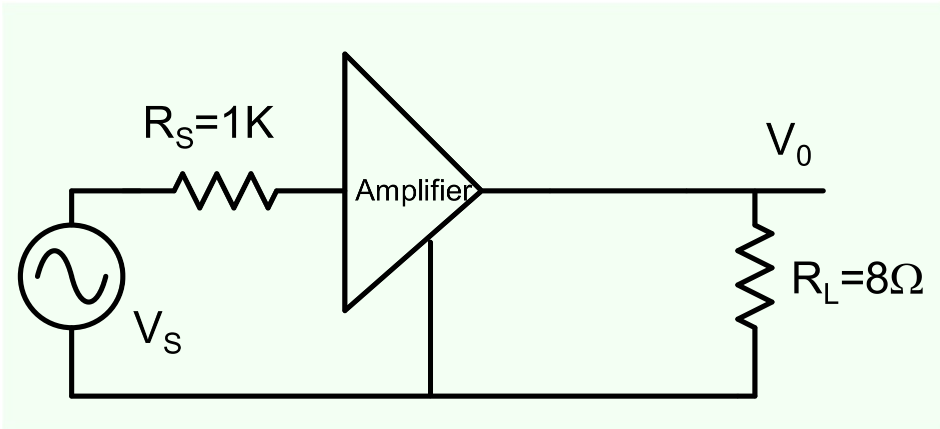


How do we build a simplified representation of only this portion of the circuit?

How do we analyze circuits containing new components?

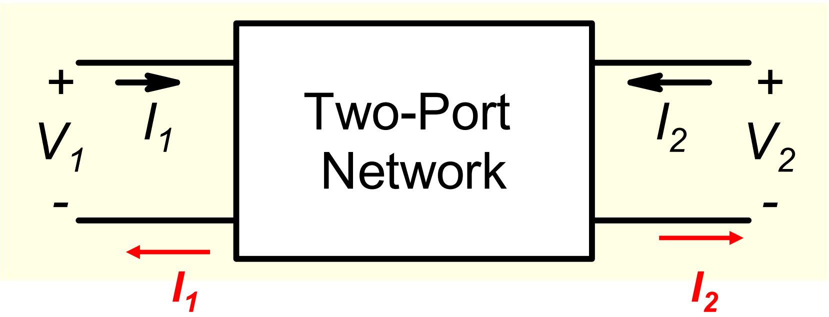






How do we specify amplifiers parameters?

Two-Port Networks



Port: A pair of terminals through which a signal can enter/leave the network

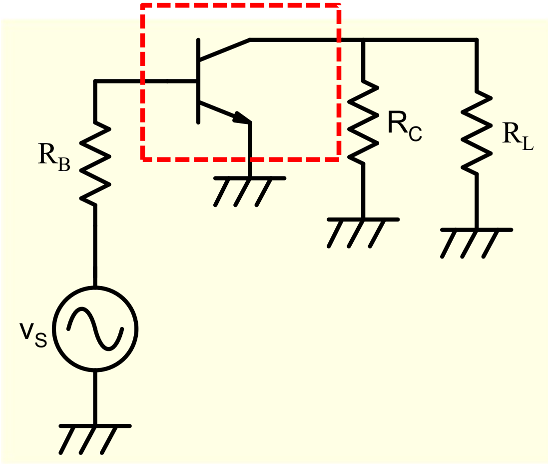
Constraints on analysis:

1. Linear elements only (R,L,C, dependent sources,..)

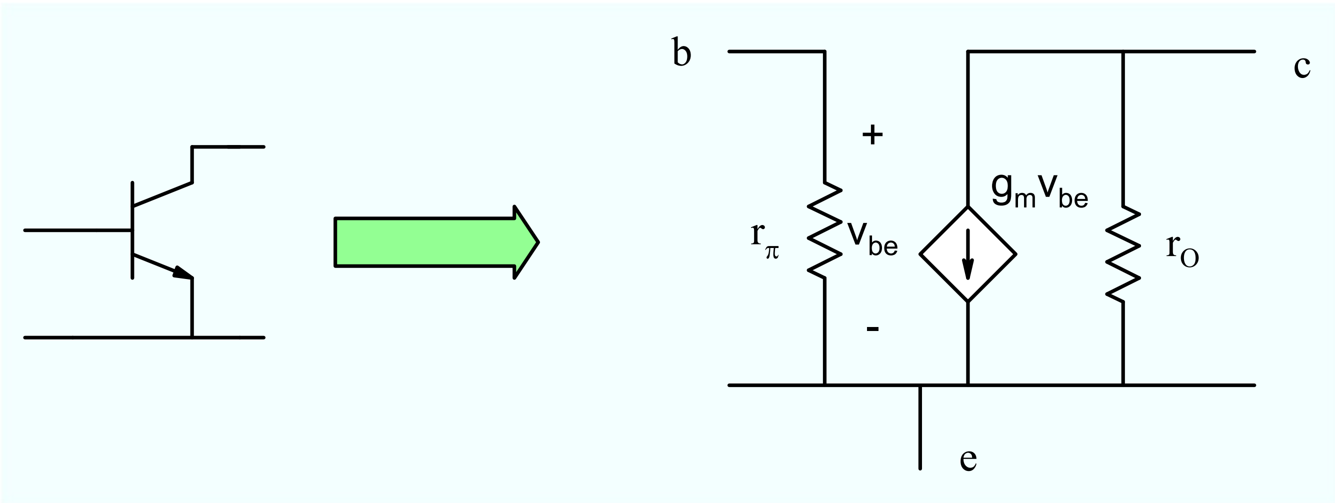
2. No independent sources or stored energy inside the network

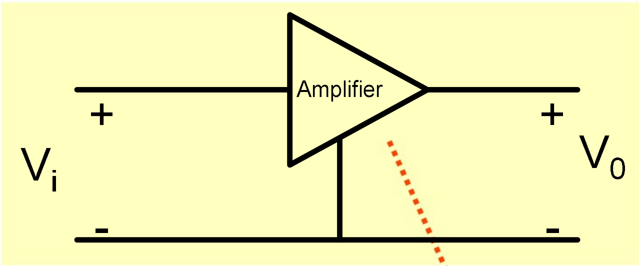
No matter how complicated is the circuit inside the two-port network, it can be represented by only four elements !

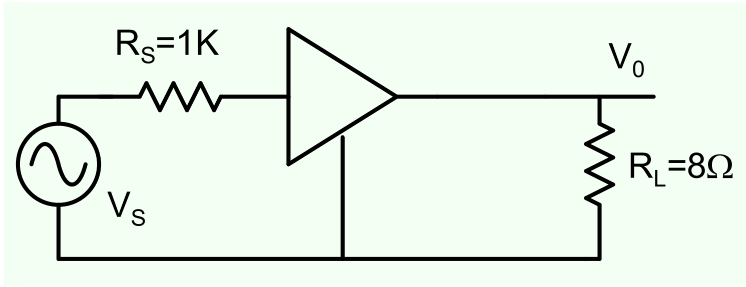
How do we analyze circuits containing new components?



Two port network allows the transistor to be represented in terms of familiar elements.

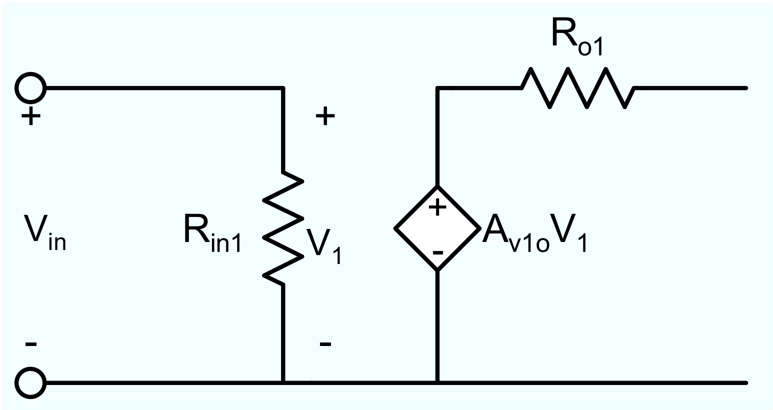


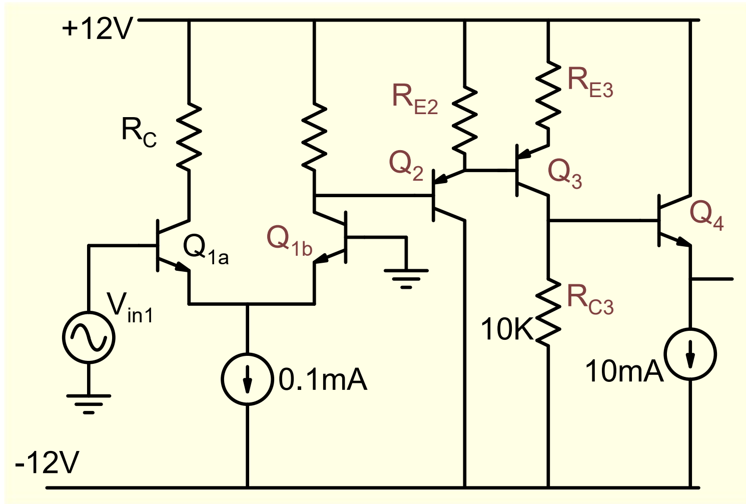




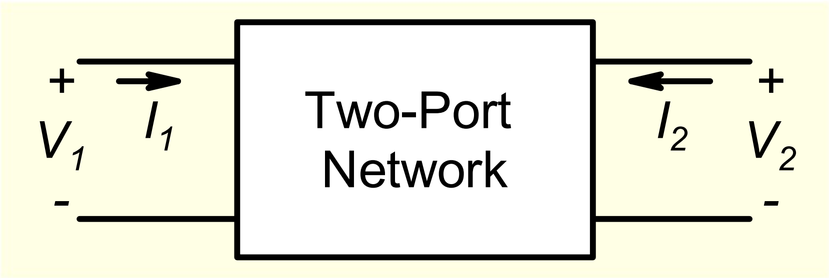
Two port network

How do we specify amplifiers parameters?





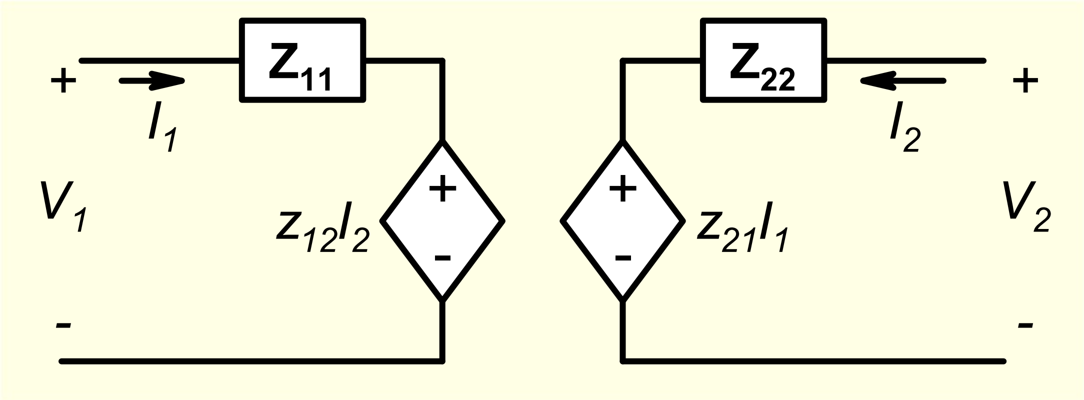
Z or Impedance Parameters



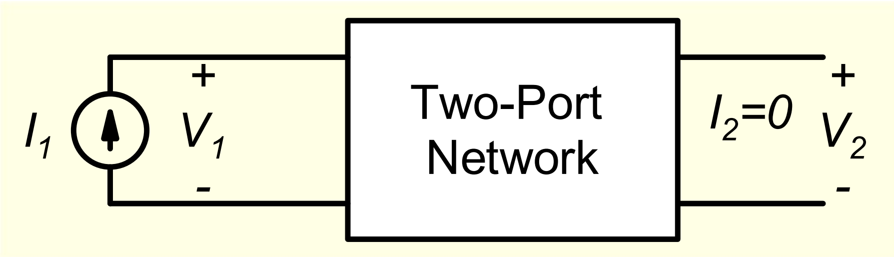
$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

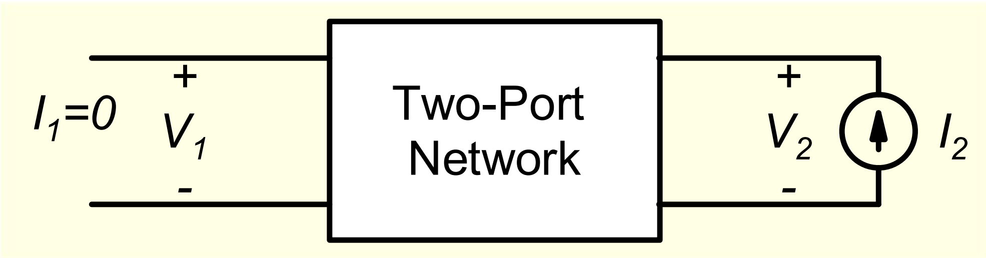
$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$



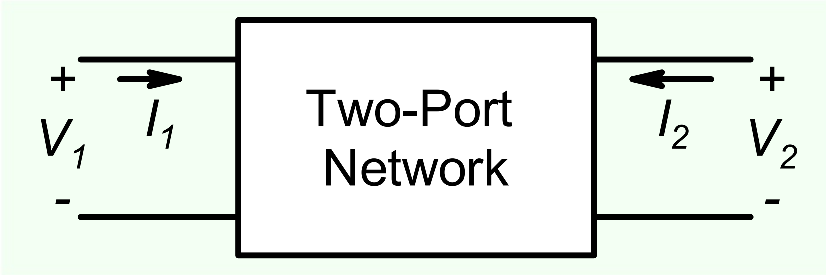
$$V_1 = z_{11} I_1 + z_{12} I_2 \qquad V_2 = z_{21} I_1 + z_{22} I_2$$



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$



$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$



$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

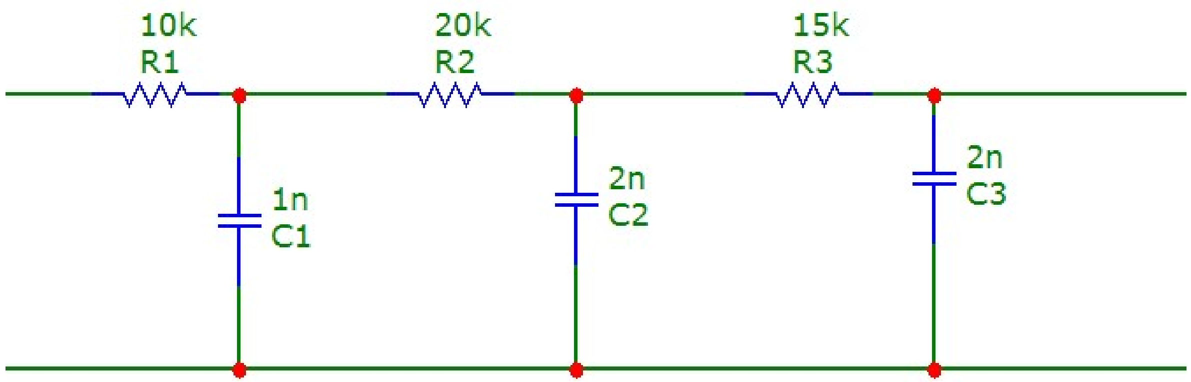
$\mathbf{z}_{11} =$ Open-circuit input impedance

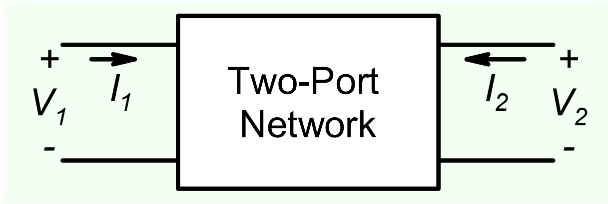
$\mathbf{z}_{12} =$ Open-circuit transfer impedance from port 1 to port 2

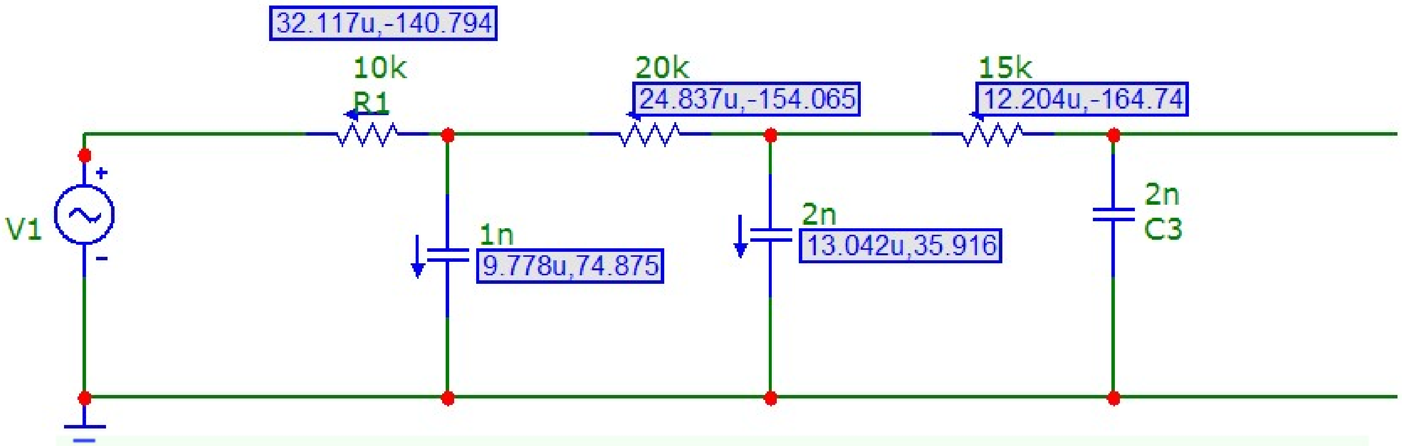
$\mathbf{z}_{21} =$ Open-circuit transfer impedance from port 2 to port 1

$\mathbf{z}_{22} =$ Open-circuit output impedance

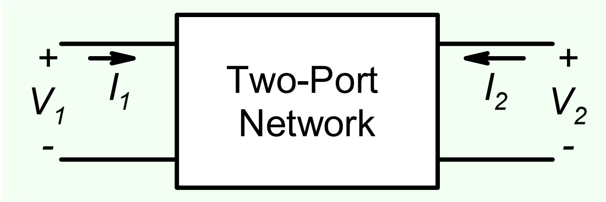
Example: Z representation

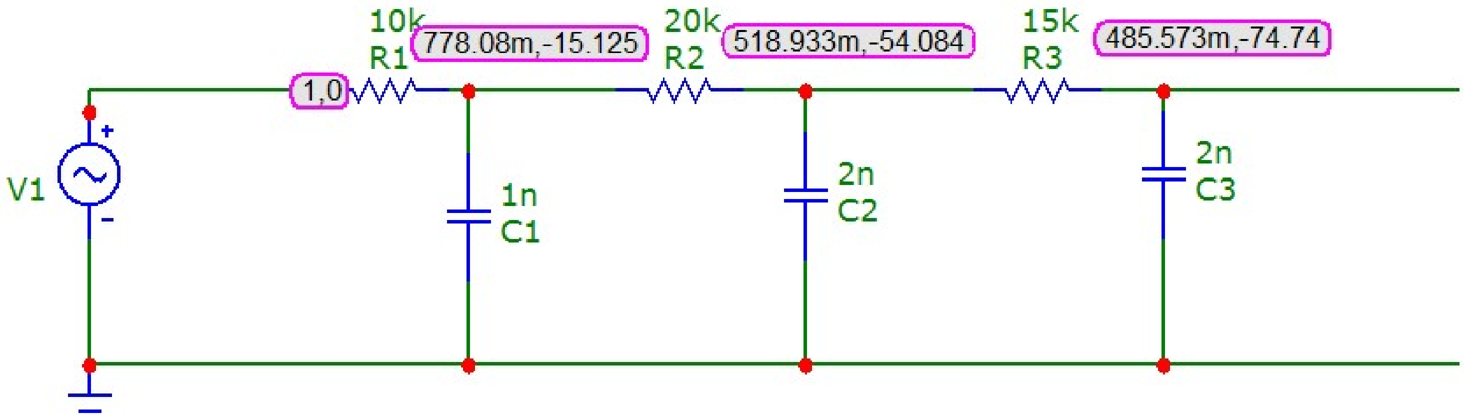




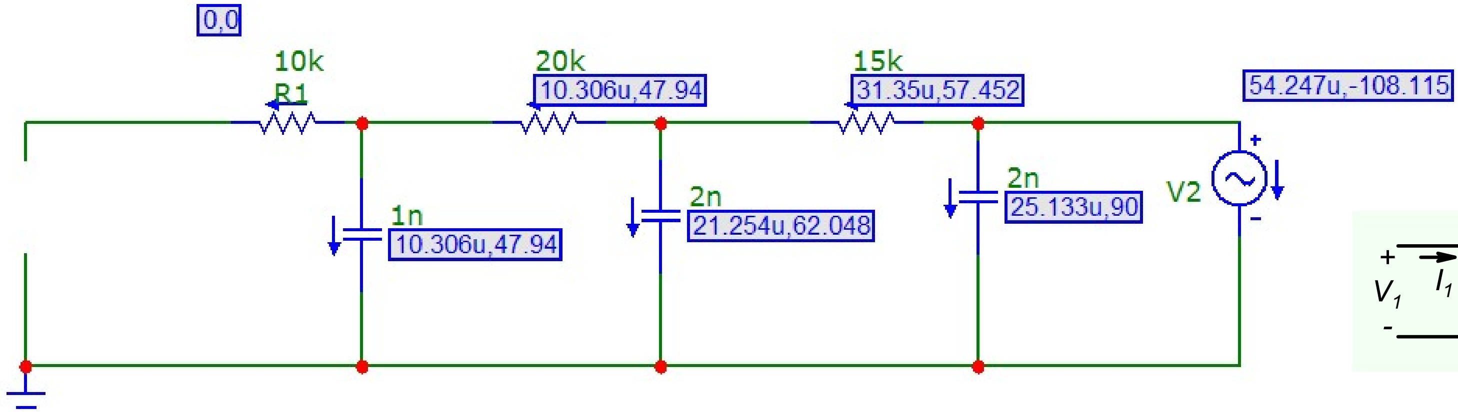


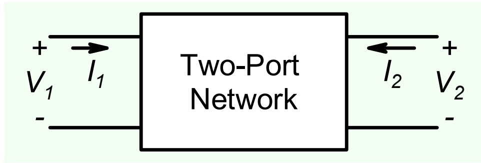
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{1\angle 0}{32.117\times 10^{-6}\angle -140.79+180} = 3.1\times 10^{4}\angle -39.2$$



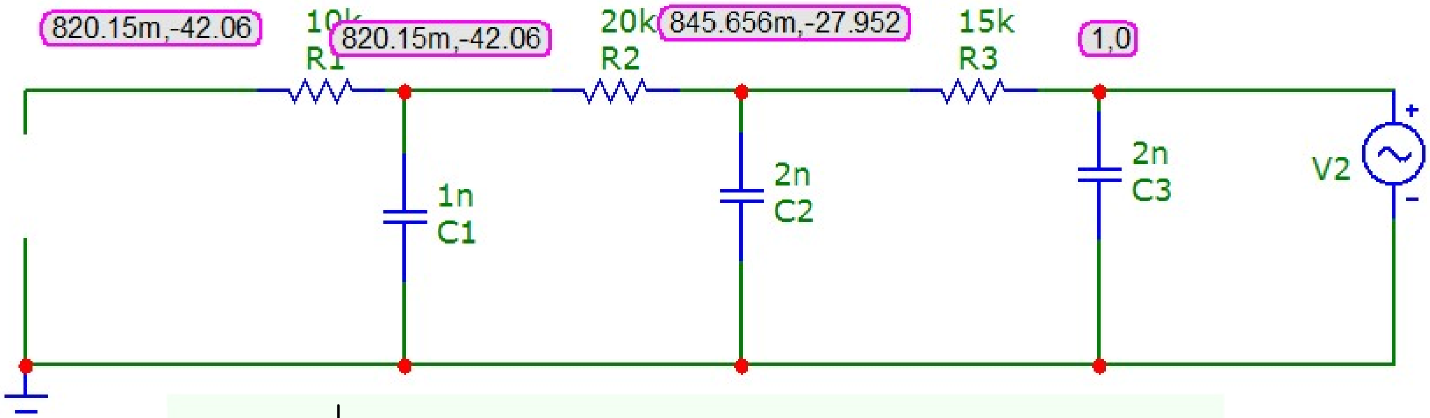


$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{0.485\angle -74.74}{32.117\times 10^{-6}\angle 39.21} = 1.5\times 10^{4}\angle -114$$

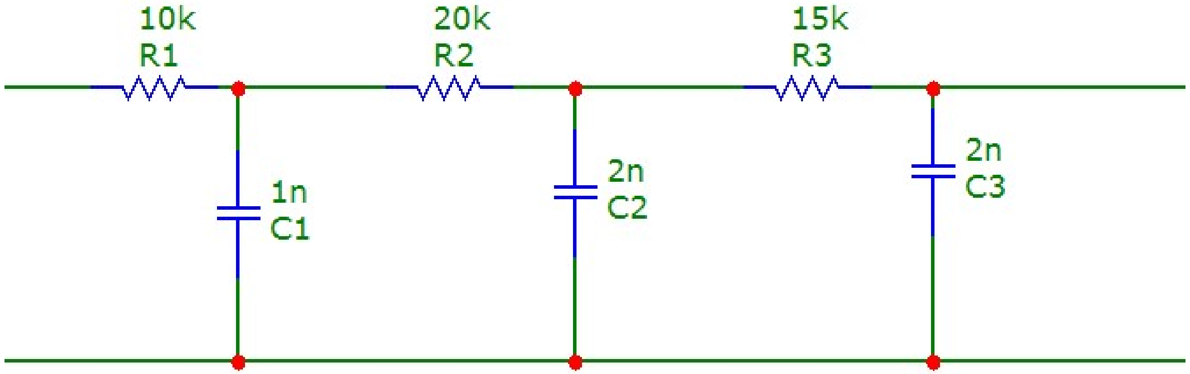


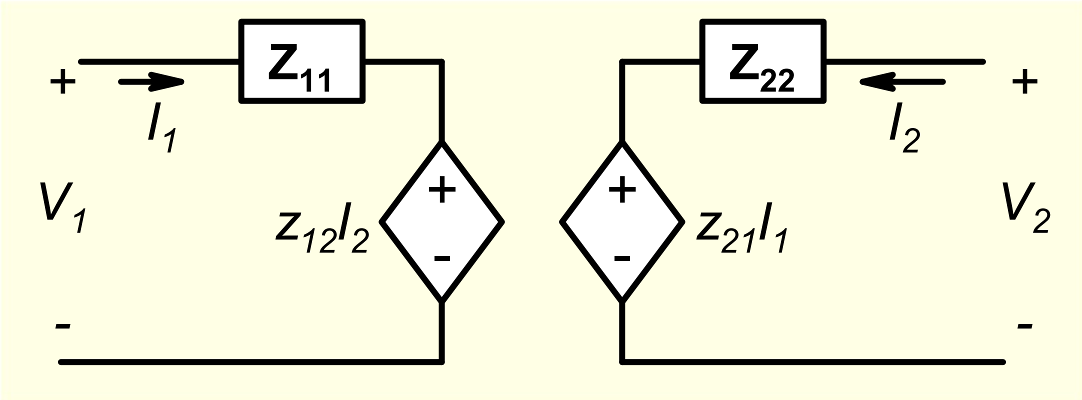


$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1\angle 0}{54.24\times 10^{-6}\angle -108.1+180} = 1.8\times 10^4\angle -71.9$$



$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0.82\angle -42}{54.24\times 10^{-6}\angle 71.9} = 1.5\times 10^4\angle -114$$





$$z_{11} = 3.1 \times 10^4 \angle -39.2$$

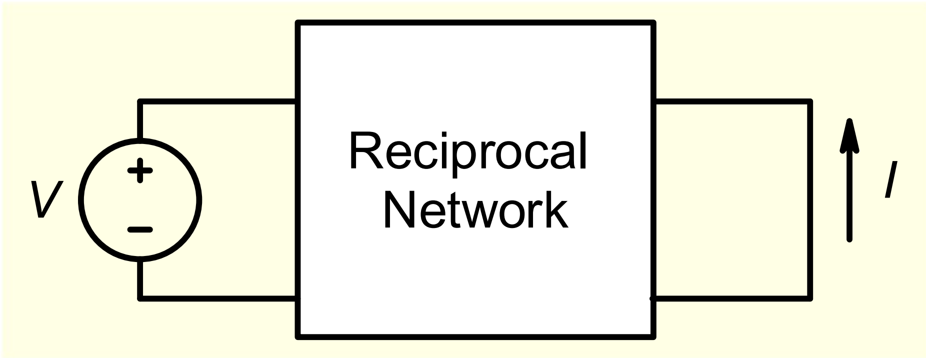
$$z_{22} = 1.8 \times 10^4 \angle -71.9$$

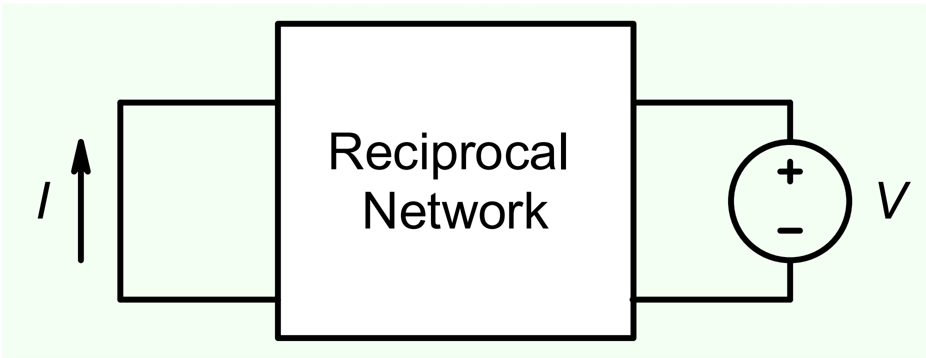
$$z_{12} = 1.5 \times 10^4 \angle -114$$

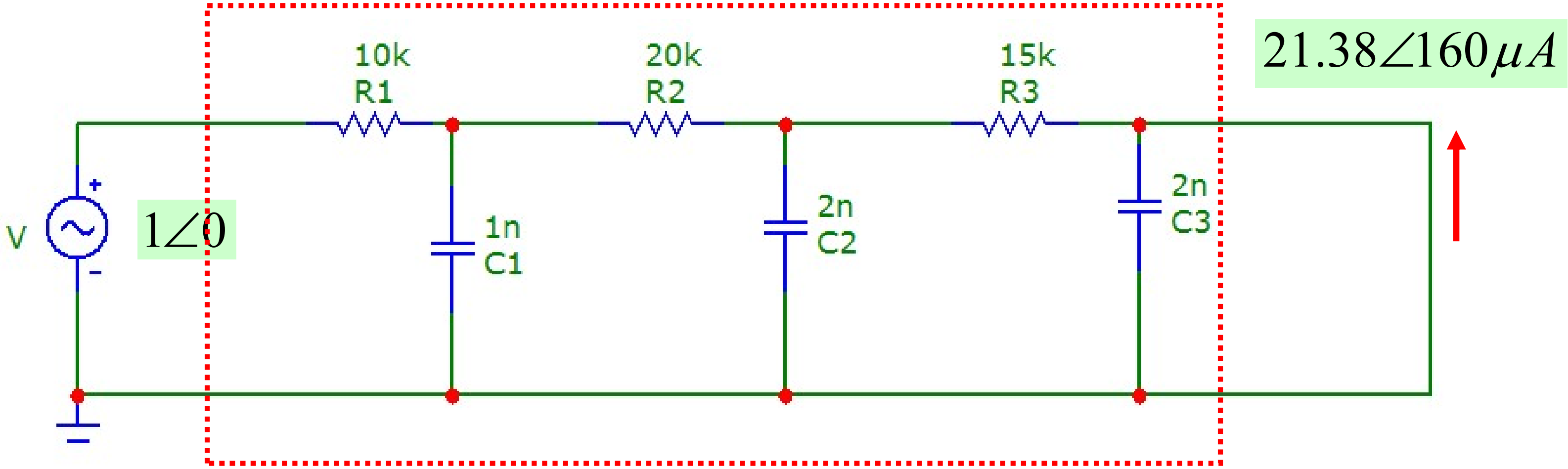
$$z_{21} = 1.5 \times 10^4 \angle -114$$

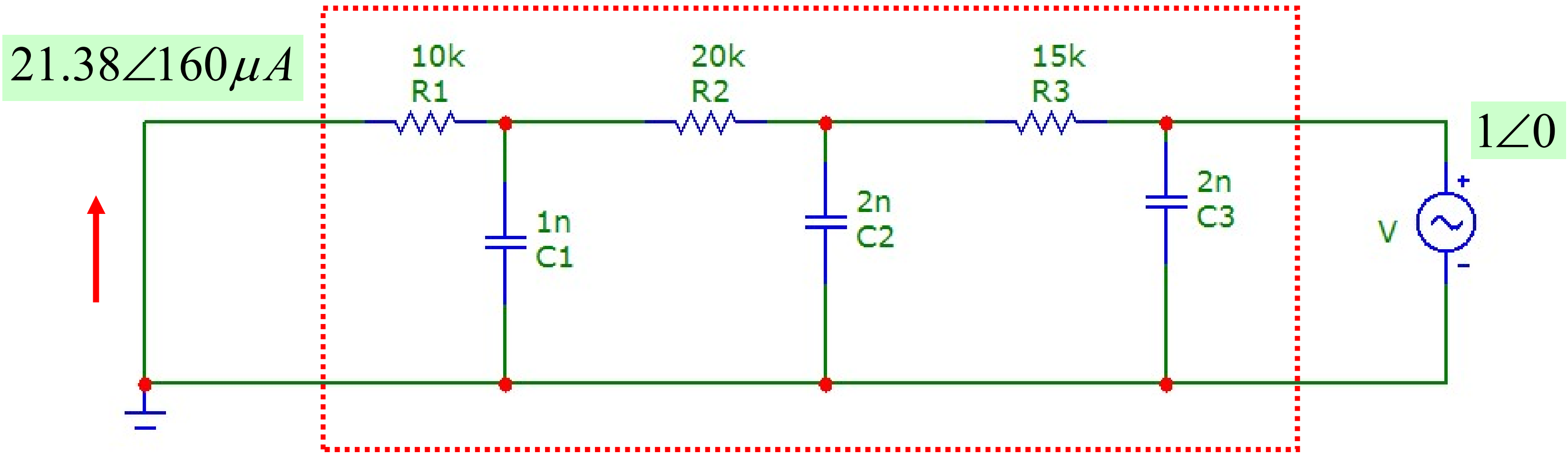
In general if there are no dependent sources in the network then: $z_{21} = z_{12}$

Reciprocal Networks



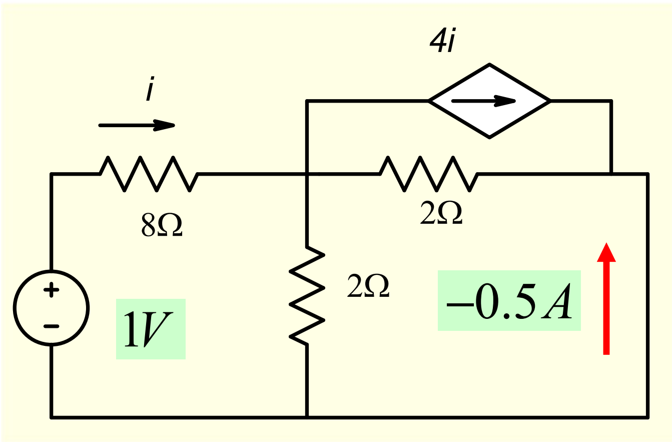


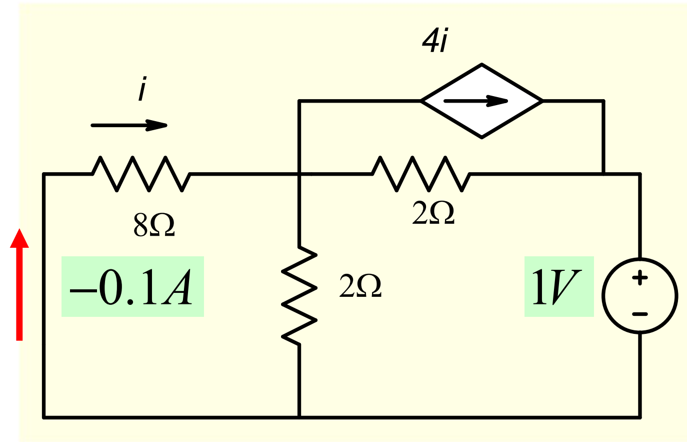

21.38∠160μA
10k
R1
20k
R2
15k
R3
V
1∠0
1n
C1
2n
C2
2n
C3


21.38∠160μA
10k
R1
20k
R2
15k
R3
1∠0
1n
C1
2n
C2
2n
C3
V

Reciprocal Networks

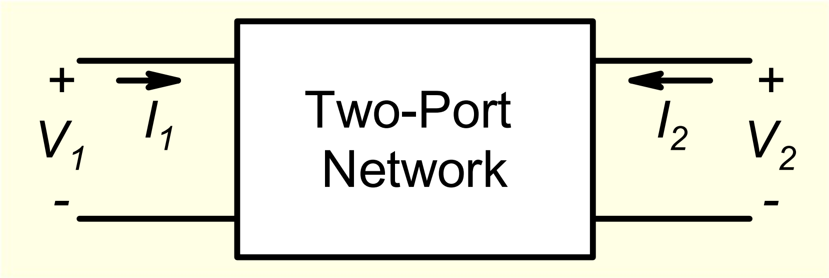
It can be shown that two port networks with no dependent sources are reciprocal

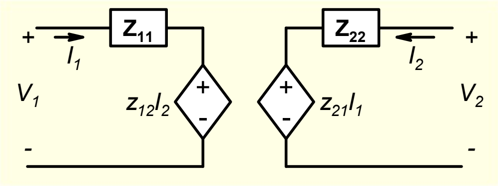




For reciprocal networks: $z_{12} = z_{21}$

Y or Admittance Parameters

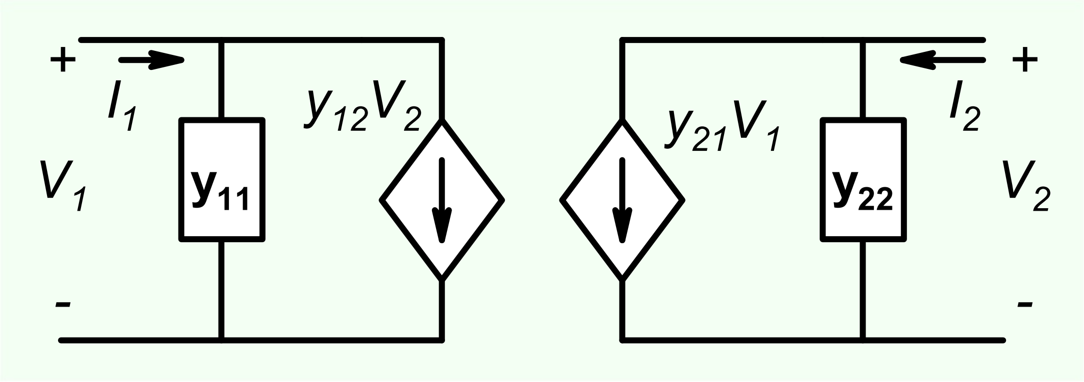


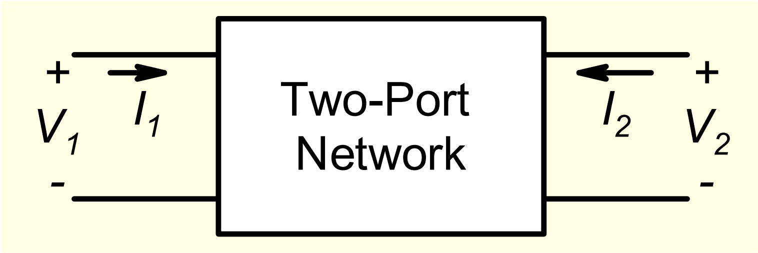


$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$





$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

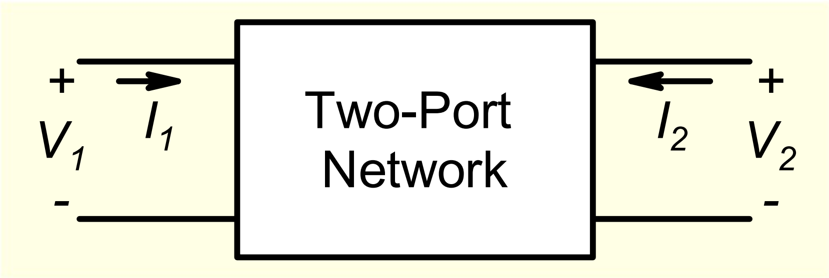
$\mathbf{y}_{11} =$ Short-circuit input admittance

$\mathbf{y}_{12} =$ Short-circuit transfer admittance from port 2 to port 1

$\mathbf{y}_{21} =$ Short-circuit transfer admittance from port 1 to port 2

$\mathbf{y}_{22} =$ Short-circuit output admittance

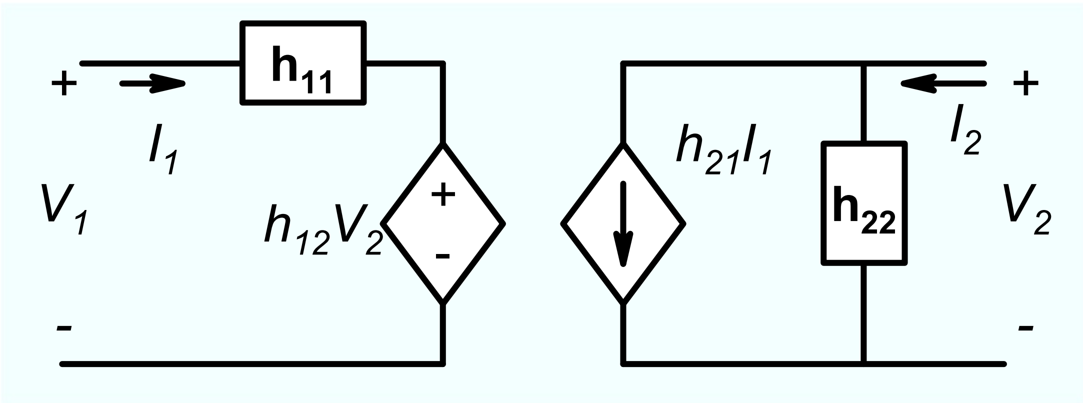
H or Hybrid Parameters



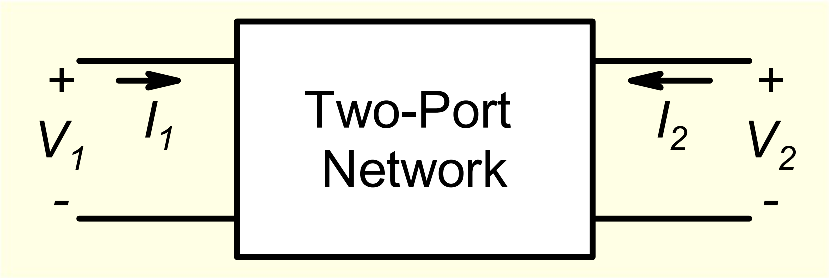
$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$



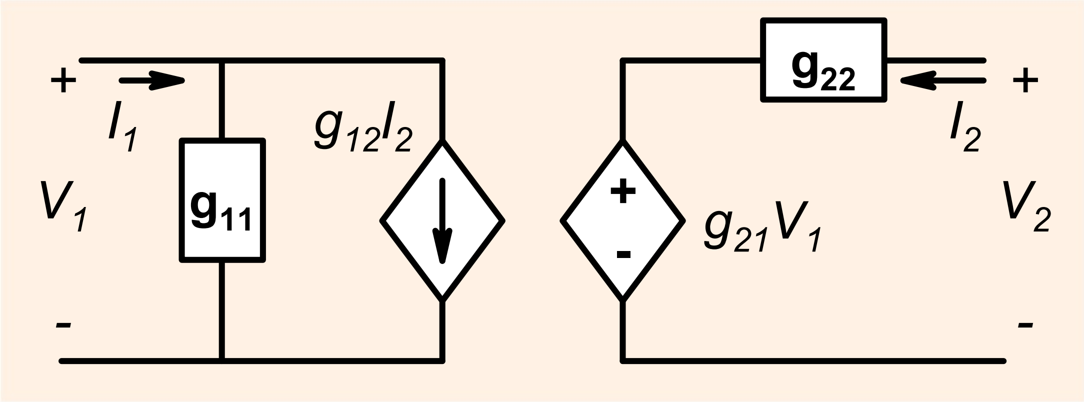
G or Inverse Hybrid Parameters



$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$



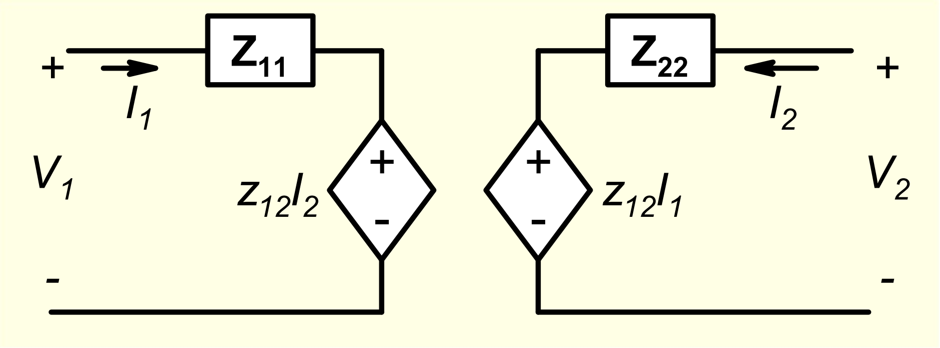
Z parameter representation for Reciprocal Networks

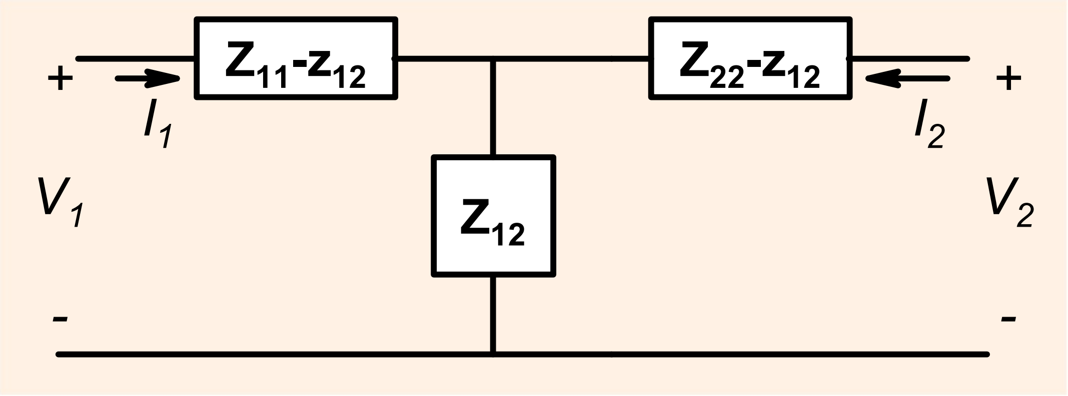
$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{12}I_1 + z_{22}I_2$$

$$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$$

$$V_2 = z_{12}(I_1 + I_2) + (z_{22} - z_{12})I_2$$





T-network

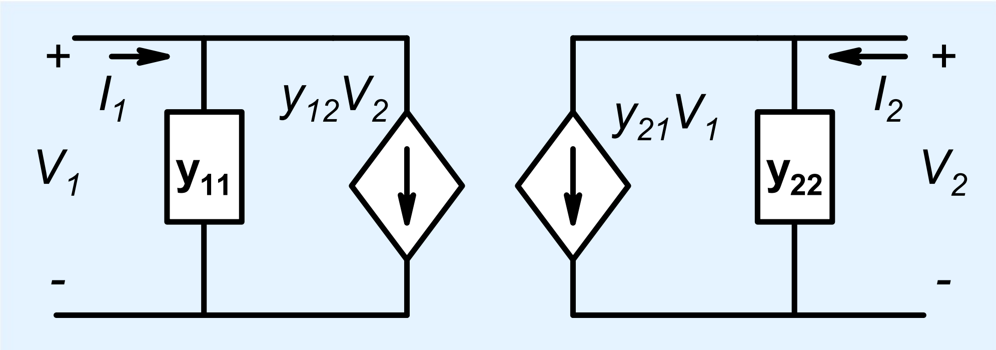
Y parameter Representation for Reciprocal networks

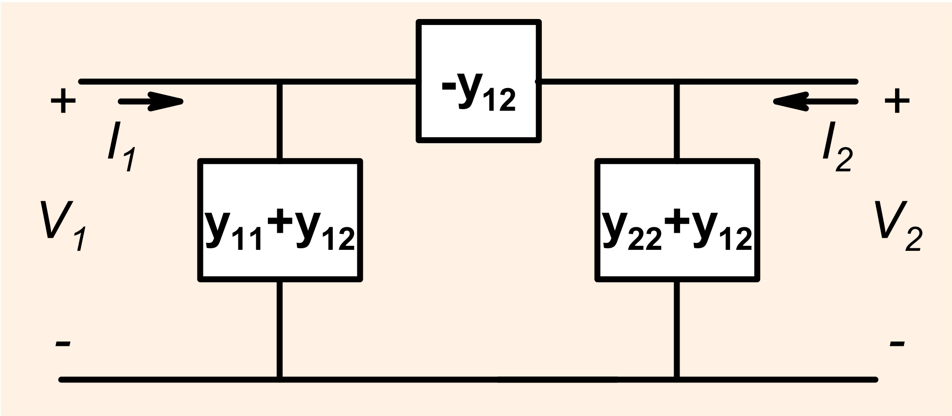
$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{12}V_1 + y_{22}V_2$$

$$I_1 = (y_{11} + y_{12})V_1 + y_{12}(V_2 - V_1)$$

$$I_2 = -y_{12}(V_2 - V_1) + (y_{22} + y_{12})V_2$$





π-network

Relationships between Parameters

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$I_1 = \frac{z_{22}}{\Delta_z} V_1 - \frac{z_{12}}{\Delta_z} V_2$$

$$I_2 = -\frac{z_{21}}{\Delta_z} V_1 + \frac{z_{11}}{\Delta_z} V_2$$

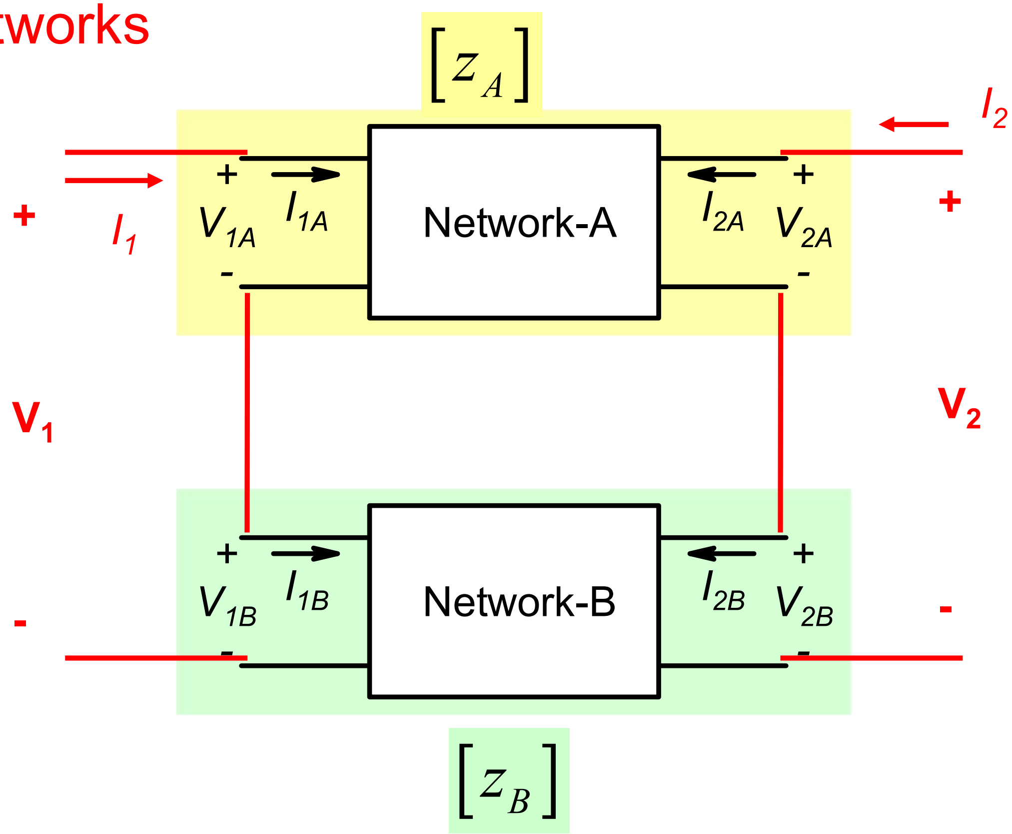
$$\Delta_z = z_{11} z_{22} - z_{12} z_{21}$$

$$y_{11} = \frac{z_{22}}{\Delta_z};\ y_{12} = -\frac{z_{12}}{\Delta_z};\ y_{22} = \frac{z_{11}}{\Delta_z};\ y_{21} = -\frac{z_{21}}{\Delta_z}$$

$$z_{11} = \frac{y_{22}}{\Delta_y};\ z_{12} = -\frac{y_{12}}{\Delta_y};\ z_{22} = \frac{y_{11}}{\Delta_y};\ z_{21} = -\frac{y_{21}}{\Delta_y}$$

$$\Delta_y = y_{11} y_{22} - y_{12} y_{21}$$

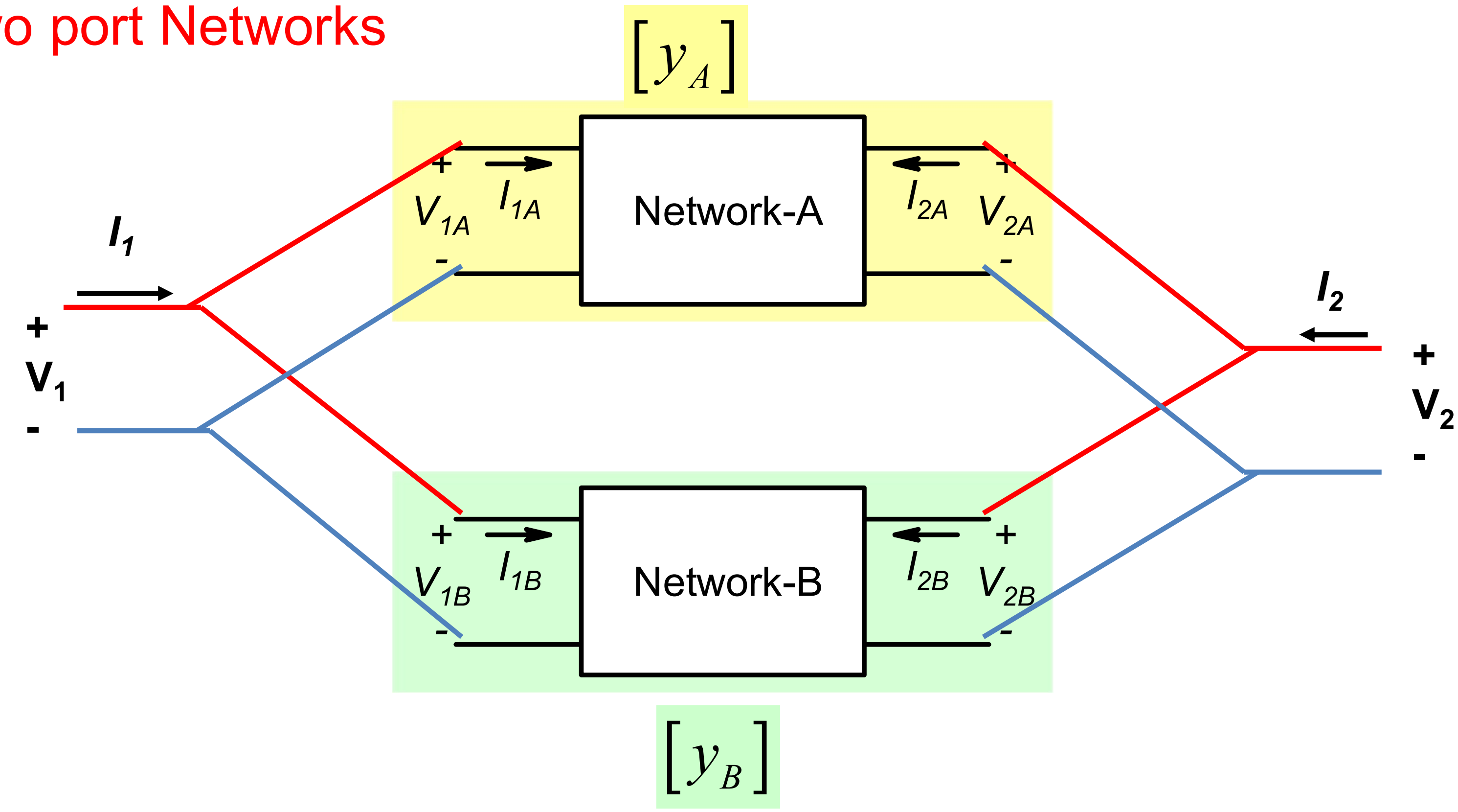
Series Two port Networks



$$[z] = [z_A] + [z_B]$$

$$z_{11} = z_{11a} + z_{11b}$$

Parallel Two port Networks

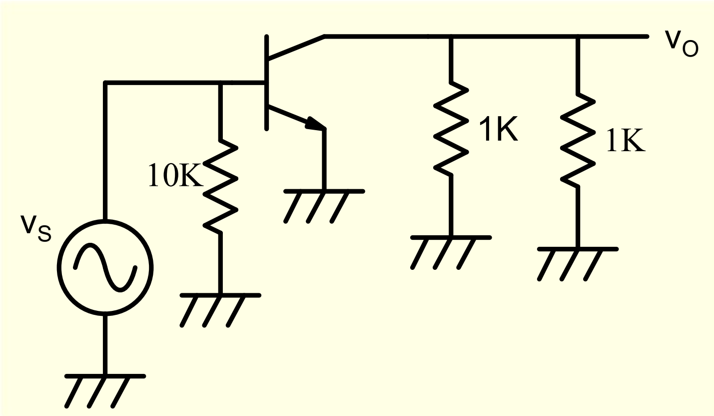


$$[y] = [y_A] + [y_B]$$

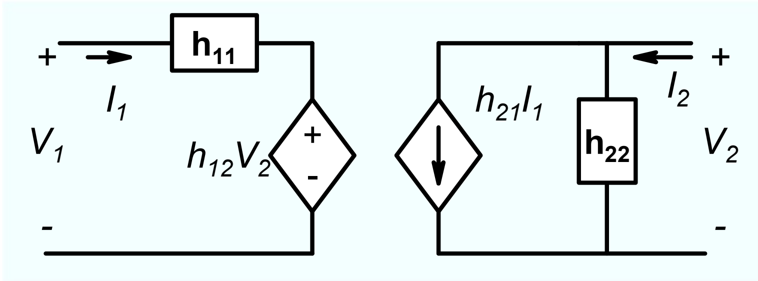
$$y_{11} = y_{11a} + y_{11b}$$

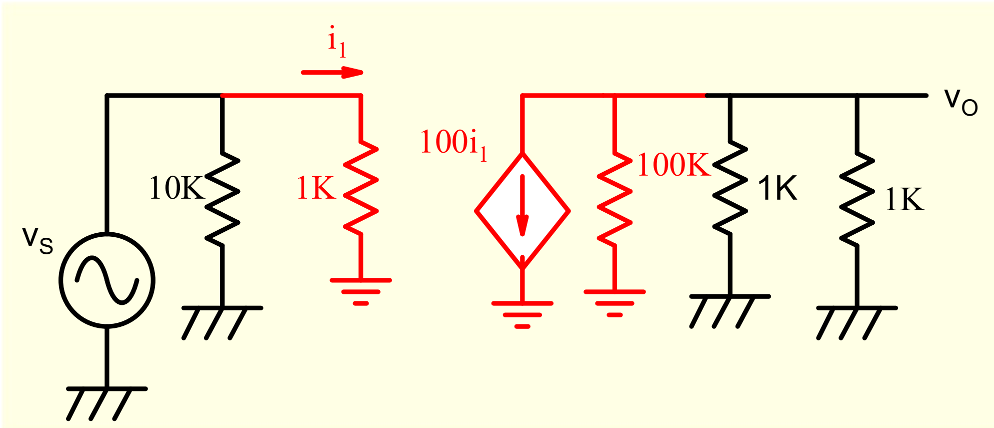
Example

BJT Transistor Circuit



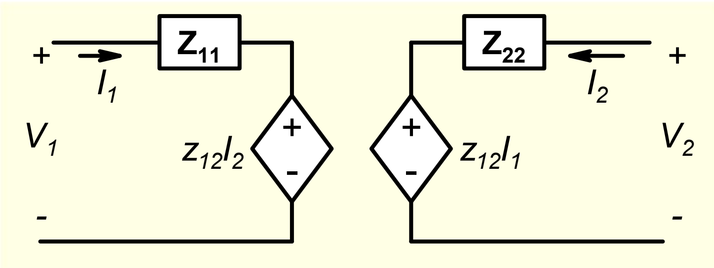
$$h_{11} = 1K\Omega;\ h_{12} \cong 0;\ h_{21} = 100;\ h_{22} = 10^{-5}\,S$$

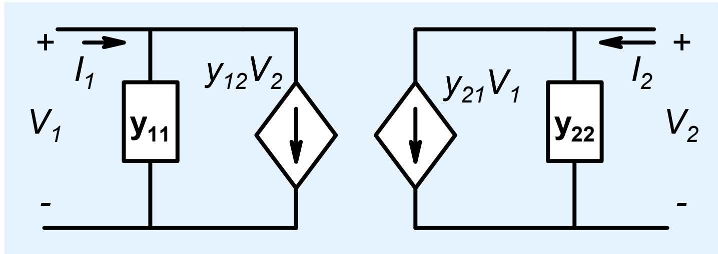


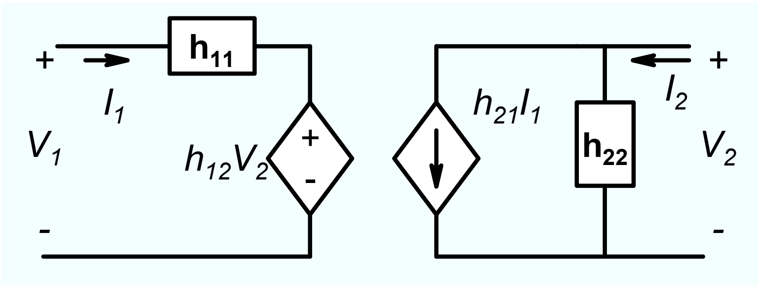


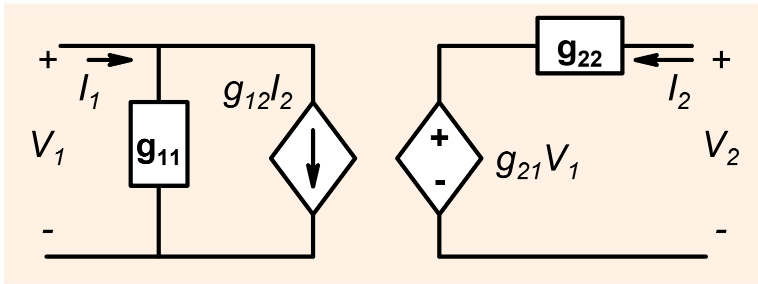
$$\frac{v_o}{v_s} \cong -50$$

Which parameters should one use?



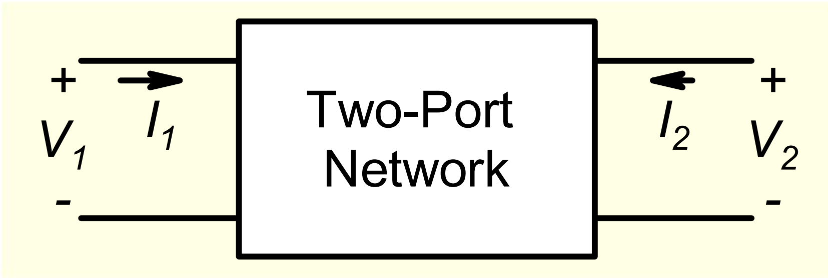




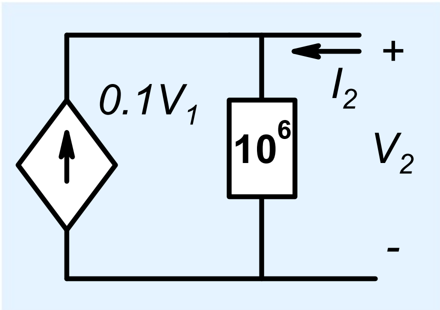


Inherent **nature of the system** and **nature of problem** being addressed often determine the choice

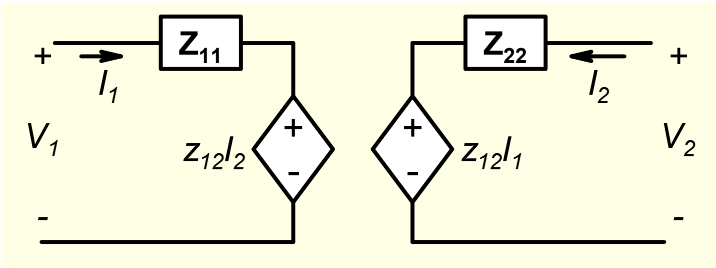
Nature of the system



Suppose there is a system which when viewed from port-2 looks very much like a ideal dependent current source

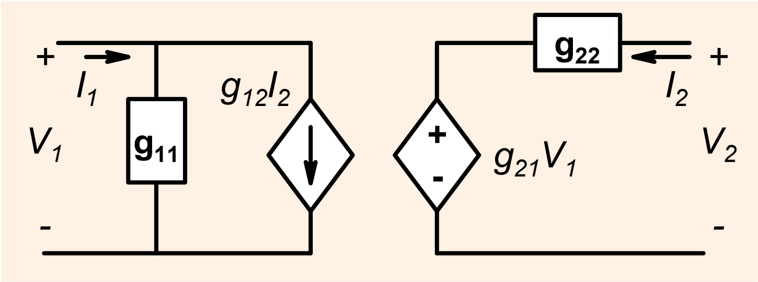


Modeling this system with either z or g parameters can be problematic



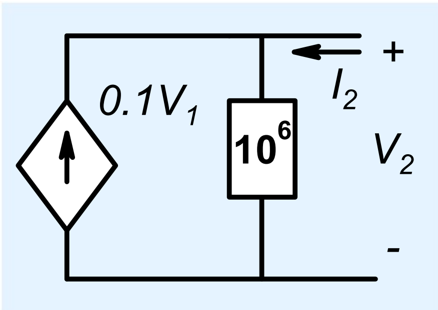
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

If we apply V_1 = 1V, $V_2 = 10^5$ V which is obviously not possible !

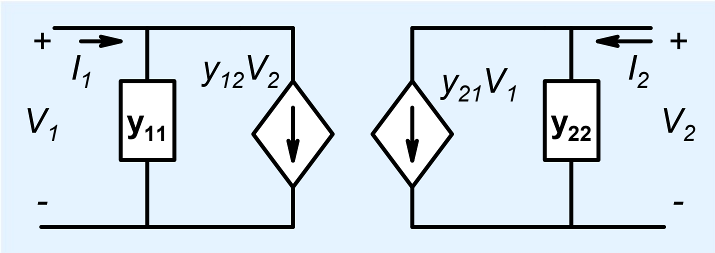


$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$

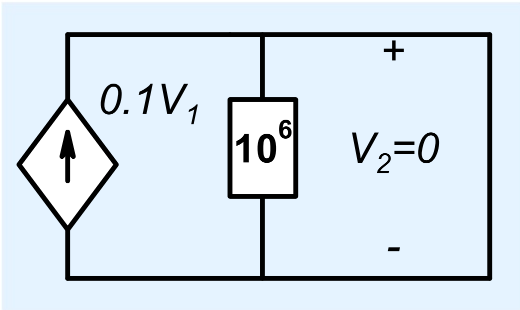


Same problem occurs here as well. If we make I_2 zero then V_2 tends to make very large !

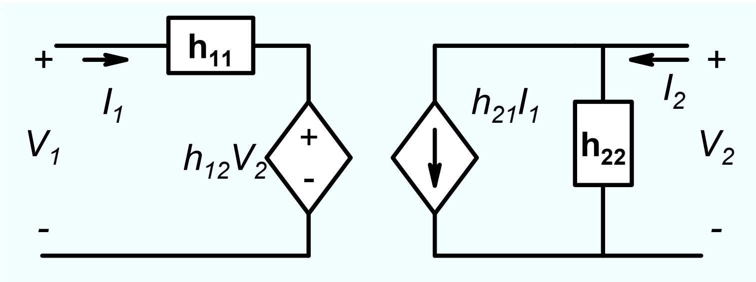


$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$



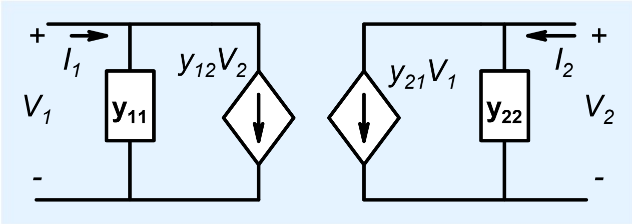
Determination of Y parameters requires V_2 and not I_2 to be made zero and thus causes no problems. Same is true for h parameters.

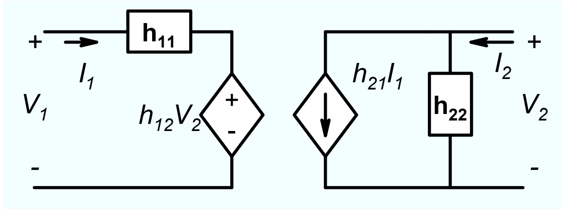


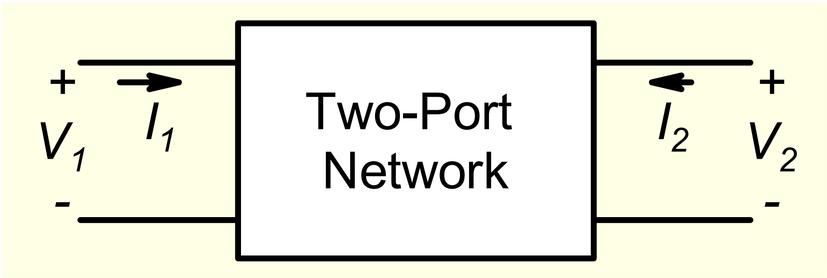
$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

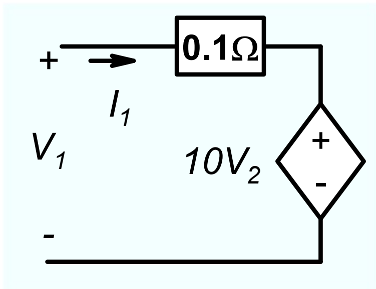
Our choice reduces to :



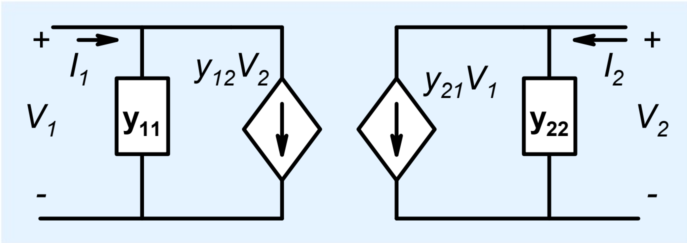




Suppose our system which when viewed from port-1 looks very much like a ideal dependent voltage source



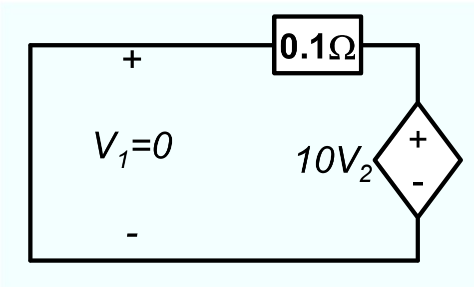
Can we directly determine Y parameters?



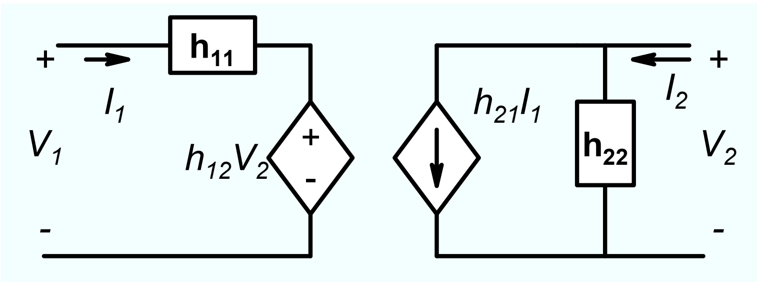
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}, \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

To determine y_{12} or y_{22}, we have to short the input voltage



If V_2 = 1V, current flowing would be 100A which would again cause problems



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Summary : 2-port parameters