MSO-203 B ASSIGNMENT 6

IIT, KANPUR

20th, November, 2020

This assignment will not be graded, do it for yourself.

1. Using the method of separation of variables solve the following heat equations

$$\begin{cases} u_t - 9u_{xx} = 0 \quad \textbf{in } (0,\ell) \times (0,\infty), \\ u(0,t) = 0, u(\ell,t) = 0, \ t > 0, \\ u(x,0) = \sin(\frac{\pi}{\ell}t), \ x \in (0,\ell). \end{cases} \tag{1}$$

2. Consider the following problem:

$$\begin{cases} u_t - u_{xx} = 0 \quad \textbf{in } (0,\pi) \times (0,T), \\ u(0,t) = 0, u(\ell,t) = 0, \ 0 < t < T, \\ u(x,0) = \sin^2 x, \ x \in (0,\pi), \end{cases} \tag{2}$$

Using maximum principal show that $0 \le u(x,t) \le e^{-t}\sin x$ on $(0,\pi) \times (0,T)$.

3. Consider the following problem

$$y^{''}(t) + (\sin(t) + 5)y(t) = 0 \quad \text{in } \mathbb{R}.$$

Let y_1 denotes a non-trivial solution (not identically zero function) of the above problem. Then show y_1 has infinitely many zeros, that is, y_1 vanishes at infinitely many points on $\mathbb{R}$.

4. Solve the following heat equations:

$$\begin{cases} u_t - 9u_{xx} = 0 \quad \textbf{in } (0,\ell) \times (0,\infty), \\ u(0,t) = 10, u(\ell,t) = 100, \ t > 0, \\ u(x,0) = \sin(\frac{\pi}{\ell}t), \ x \in (0,\ell). \end{cases} \tag{3}$$

5. Consider a function $v : (0,\infty) \to \mathbb{R}$ with the property $v(r), v'(r)$ tends to 0 as $r \to \infty$. Considering a function

$$u(x,t) = t^{-\frac{1}{2}}v(\frac{x}{\sqrt{t}})$$

as the solution of the heat equation

$$u_t = u_{xx}, \ x \in \mathbb{R}, t > 0,$$

find v.