MSO-203 B ASSIGNMENT 5
IIT, KANPUR

14th November , 2020

Multiple choice questions may have more than one correct answers. You have to submit all the questions.

1. Let $u : \mathbb{R}^2 \setminus (0,0) \to \mathbb{R}$ be a C^2 function satisfying

$$\triangle u = 0,\ (x,y) \in \mathbb{R}^2 \setminus (0,0).$$

Suppose u is of the form $u(x,y) = f((x^2+y^2)^{\frac{1}{2}})$, where $f : (0,\infty) \to \mathbb{R}$ is a non constant function, then

a) $\lim_{||(x,y)||\to 0} |u(x,y)| = \infty$
b) $\lim_{||(x,y)||\to 0} |u(x,y)| = 0$
c) $\lim_{||(x,y)||\to \infty} |u(x,y)| = \infty$
d) $\lim_{||(x,y)||\to \infty} |u(x,y)| = 0.$

2. Prove that Laplace equation is rotational invariant; that is, if $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ be the rotation matrix and we define the change of variable by $Y = AX,\ V(Y) = u(X)$, then show that $\triangle V = 0$, whenever $\triangle u = 0$. It is understood that $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ denotes arbitrary points.

3. (Neuman Boundary conditions on rectangles) Solve the following problem:

$$\begin{cases} \triangle u = 0 \quad \textbf{in } (0,a)\times(0,b), \\ u_x(a,y) = f(y), u_x(0,y) = 0, u_y(x,0) = 0 = u_y(x,b). \end{cases} \tag{1}$$

4. Show that the problem

$$\begin{cases} \triangle u = f \quad \textbf{in } \Omega, \\ u = g \quad \textbf{on } \partial\Omega. \end{cases} \tag{2}$$

can have at most one solution.

5. Deduce the expression of $\triangle u = 0$ in polar coordinates.