Digital Feedback System Examples

- Digital feedback systems are often used to model a variety of practical phenomenon
- Two such examples presented earlier are the algorithm for the computation of the square-root of a positive number and the modeling of the bank account paying compound interest

Amortization Model

- The digital implementation of the trapezoidal numerical integration is another example
- We provide next three additional examples

Amortization Model

- The paying of a debt by a fixed amount at equal intervals of time, such as monthly, to the lender is known as amortization

Amortization Model

- Part of the payment amount is used to reduce the remaining amount of principal and the remaining part is the interest on the loan
- The amortization model is a first-order difference equation given by

$$y[n] = (1+R)y[n-1] - x[n]$$

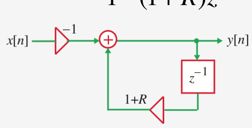
Amortization Model

- $y[n]$ is the amount of principal left after the payment has been made at the n-th period
- $x[n]$ is the amount being paid at the n-th period
- R is the rate of compounded interest at each period of payment

Amortization Model

- The transfer function $H(z)$ of the LTI digital system modeling the amortization process is given by

$$H(z) = -\frac{1}{1-(1+R)z^{-1}}$$



Amortization Model

- The amount paid at each period is a fixed number P, that is $x[n] = P$
- Its value can be determined knowing the total number N payments to be made to pay off the debt
- Let D denote the initial amount of loan, that is $y[0] = D$

Amortization Model

- It can be shown that

$$P = \frac{R(1+R)^N D}{(1+R)^N - 1}$$

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National Income Model

- Typically, the national income is computed at fixed intervals of time and thus can be represented as a discrete-time sequence
- The national income $y[n]$ is a sum of consumer expenditures $c[n]$, induced private investment $p[n]$, and government expenditure $x[n]$: $y[n] = c[n] + p]n] + x[n]$ where n denotes the time instant when the various sequences are summed

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National Income Model

- The model for the national income has been developed based on the following three properties:
- Property 1 - The consumer expenditure time instant n is proportional to the national income at time instant $n-1$, that is

$$c[n] = \alpha y[n-1]$$

where $\alpha > 0$

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National Income Model

- Property 2 - The induced private investment at time instant n is proportional to the increase in consumer expenditure from time instant $n-1$ to time instant n, that is

$$p[n] = \beta(c[n] - c[n-1])$$

where $\beta > 0$

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National Income Model

- Property 3 - The government expenditure remains constant for all values of the time instant n
- Substituting the expressions for $c[n]$ and $p[n]$ in $y[n] = c[n] + p]n] + x[n]$ we get after some algebra

$$y[n] - \alpha(1+\beta)y[n-1] + \alpha\beta y[n] = x[n]$$

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National Income Model

- The transfer function $H(z)$ of the LTI digital system modeling the national income is therefore given by

$$H(z) = \frac{1}{1 - \alpha(1+\beta)z^{-1} + \alpha\beta z^{-2}}$$

- A block-diagram representation of the national income model is shown in the next slide

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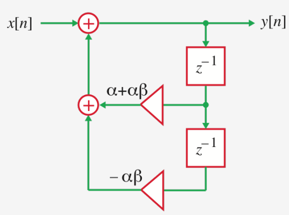
National Income Model



Inventory Model

- The inventory is the amount of consumer goods a manufacturer holds for sale during a fixed period of time
- At the beginning of a period, the manufacturer needs to check the inventory
- This will determine the number of units of goods to produce for direct sales during that period

Inventory Model

- This will ensure that a desired number of units of goods are left for inventory at the end of the period
- Let $y[n]$ represent the total income given by the total units of goods during the n-th period
- Let $x[n]$ denote the units of goods made during the same period

Inventory Model

- The inventory model is a second-order difference equation given by

  $$y[n]=(\alpha+\beta)y[n-1]-\alpha y[n-2]+x[n]$$

  where $\alpha y[n-1]$ represents the number of goods to be produced for direct sales during the n-th period and β determines the fraction of total income $\beta y[n]$ during the same period

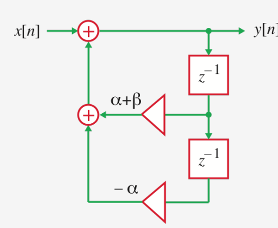
Inventory Model

- The transfer function $H(z)$ of the LTI digital system modeling the inventory is thus given by

  $$H(z)=\frac{1}{1-(\alpha+\beta)z^{-1}+\alpha z^{-2}}$$

- A block-diagram representation of the inventory model is shown in the next slide

Inventory Model



Examples of Digital Communication Systems

- Almost all communication systems are now being implemented using digital signal processing methods
- We describe example of two such systems

Amplitude Modulation

- In the discrete-time amplitude modulation scheme, a high-frequency carrier sequence $A\cos(\omega_o n)$ modulates a real low-frequency band-limited modulating sequence $x[n]$ with the carrier frequency ω_o
- The carrier frequency is smaller than half of the sampling frequency of $x[n]$

Amplitude Modulation

- The sequence $v[n]$ generated by the modulation is given by

$$v[n] = Ax[n]\cos(\omega_o n)$$

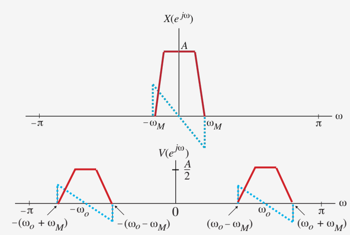
- The DTFT $V(e^{j\omega})$ of $v[n]$ is given by

$$V(e^{j\omega}) = \frac{A}{2}X(e^{j(\omega-\omega_o)}) + \frac{A}{2}X(e^{j(\omega+\omega_o)})$$

where $X(e^{j\omega})$ is the DTFT of $x[n]$

Amplitude Modulation

- Plots of $X(e^{j\omega})$ and $V(e^{j\omega})$ are shown below



Amplitude Modulation

- As can be seen from these plots, the DTFT $V(e^{j\omega})$ of the modulated signal $v[n]$ has two parts centered at $\pm\omega_o$ with a bandwidth $2\omega_M$ which is twice the bandwidth of $X(e^{j\omega})$
- The part of the $V(e^{j\omega})$ located in the frequency range $\omega_o \le (\omega_o + \omega_M)$ is known as the upper sideband

Amplitude Modulation

- The part of the $V(e^{j\omega})$ located in the frequency range $(\omega_o + \omega_M) \le \omega_o$ is known as the lower sideband
- Both sidebands have the same information content

Amplitude Modulation

- In the single sideband modulation scheme, only one of the sidebands is transmitted for an efficient utilization of the transmission medium
- A preferred method of the generation of the signal containing one of the two sidebands is based on the use of a Hilbert transformer

Amplitude Modulation

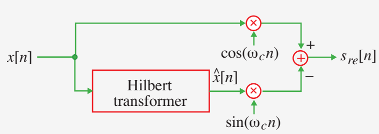
- It is implemented by modulating the analytic signal $v[n] = x[n] + j\hat{x}[n]$ and transmitting either the real part or the imaginary part of the modulated signal $s[n]$ given by

$$s[n] = (x[n] + j\hat{x}[n])(\cos(\omega_o n) + j\sin(\omega_o n))$$
$$= (x[n]\cos(\omega_o n) - \hat{x}[n]\sin(\omega_o n))$$
$$+ j(\hat{x}[n]\cos(\omega_o n) + x[n]\sin(\omega_o n))$$

Amplitude Modulation

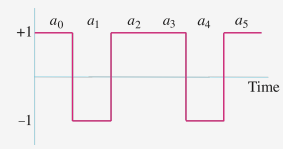
- Figure below shows the method for generating the real part of $s[n]$ given by

$$s_{re}[n] = x[n]\cos(\omega_o n) - \hat{x}[n]\sin(\omega_o n)$$



Discrete Multitone Communication System

- Binary data are normally transmitted serially as a pulse train, as indicated below

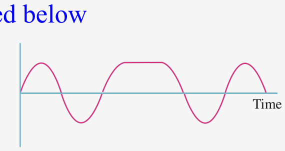


Discrete Multitone Communication System

- To faithfully extract the information transmitted, the receiver requires complex equalization procedures to compensate for channel imperfection and to make full use of the channel bandwidth

Discrete Multitone Communication System

- For example, the pulse train in Slide No. 28 arriving at the receiver may appear as indicated below



Discrete Multitone Communication System

- To alleviate the problems encountered with the transmission of data as a pulse train, frequency-division multiplexing with overlapping sub-channels has been proposed

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Discrete Multitone Communication System

- In such a system, each binary digit a_r, $0 \le r \le N-1$, modulates a subcarrier sinusoidal signal $\cos(2\pi rt/T)$, as indicated in the next slide, for the transmission of the binary data
- The modulated subcarriers are summed and transmitted as one composite analog signal

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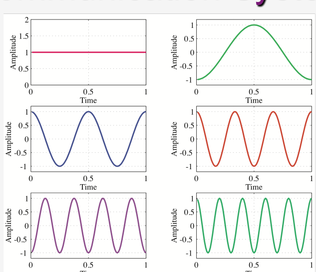
Discrete Multitone Communication System



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Discrete Multitone Communication System

- At the receiver, the analog signal is passed through a bank of coherent demodulators whose outputs are tested to determine the digits transmitted
- This is the basic idea behind the multicarrier modulation/demodulation scheme for digital data transmission

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Discrete Multitone Communication System

- A widely used form of the multicarrier modulation is the discrete multitone transmission (DMT) scheme
- Here, the modulation and demodulation processes are implemented via the discrete Fourier transform (DFT)

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Discrete Multitone Communication System

- This approach leads to an all-digital system, eliminating the arrays of sinusoidal generators and the coherent demodulators
- We outline here the basic idea behind the DMT scheme

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Discrete Multitone Communication System

- Let $\{a_k[n]\}$ and $\{b_k[n]\}$, $0 \le k \le M-1$, be two $M-1$ real-valued data sequences operating at a sampling rate of F_T that are to be transmitted
- Define a new set of complex-valued sequences $\{\alpha_k[n]\}$ of length $N = 2M$ as indicated in the next slide

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Discrete Multitone Communication System

$$\alpha_k[n] = \begin{cases} a_0[n], & k = 0 \\ a_k[n] + jb_k[n], & 1 \le k \le \frac{N}{2} - 1 \\ b_0[n], & k = \frac{N}{2} \\ a_k[n] - jb_k[n], & \frac{N}{2} + 1 \le k \le N-1 \end{cases}$$

- We apply an inverse DFT to $\{\alpha_k[n]\}$

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Discrete Multitone Communication System

- It transforms $\{\alpha_k[n]\}$ to a new set of N sequences given by

$$u_\ell[n] = \frac{1}{N} \sum_{k=0}^{N-1} \alpha_k[n] W_N^{-\ell k}, \quad 0 \le \ell \le N-1$$

where $W_N = e^{-j2\pi/N}$

- Note: $\{u_\ell[n]\}$ is a real-valued sequence

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Discrete Multitone Communication System

- Each of these N signals is then up-sampled by a factor of N and time-interleaved
- This generates a composite signal $\{x[n]\}$ operating at a rate of NF_T that is assumed to be equal to $2F_c$

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Discrete Multitone Communication System

- The composite signal is converted into an analog signal $x_a(t)$ by passing it through a D/A converter, followed by an analog reconstruction filter
- The analog signal $x_a(t)$ is then transmitted over the channel

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Discrete Multitone Communication System

- At the receiver, the received analog signal $y_a(t)$ is passed through an analog anti-aliasing filter and then converted into a digital signal $\{y[n]\}$ by an S/H circuit, followed by an A/D converter operating at a rate of $NF_T = 2F_c$

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Discrete Multitone Communication System

- The received digital signal is then de-interleaved by a delay chain containing $N-1$ unit delays
- Their outputs are next down-sampled by a factor of N, generating the set of signals $\{v_\ell[n]\}$

Discrete Multitone Communication System

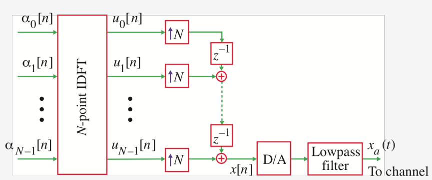
- Applying the DFT to these N signals, we finally arrive at N signals $\{\beta_k[n]\}$ given by

$$\beta_k[n] = \sum_{\ell=0}^{N-1} v_\ell[n] W_N^{\ell k}, \quad 0 \le k \le N-1$$

- The figures in the next two slides shows schematically the overall DMT scheme

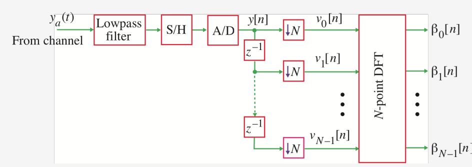
Discrete Multitone Communication System

- Transmitter



Discrete Multitone Communication System

- Receiver



Discrete Multitone Communication System

- If we assume the frequency response of the channel to have a flat passband and assume the analog reconstruction and anti-aliasing filters to be ideal lowpass filters, then neglecting the nonideal effects of the D/A and the A/D converters, we can assume

$y[n] = x[n]$