MSO-203 B ASSIGNMENT 3

IIT, KANPUR

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Multiple choice questions may have more than one correct answers. You have to submit all the problems except 3 (second and third part).

1. Choose the correct answer(s):

   a) $u_x + u_y = u$ is a linear PDE.

   b) $|\nabla u| = 1$ is a non linear PDE.

   c) $|\nabla u| = 1$ is a semilinear PDE.

   d) $uu_x + u_y = \sin(x)$ is a quasilinear PDE.

2. Classify the following PDE and also determine their order.

   a) $\triangle u = \sin^2(x)$ (Poisson Equation)

   b) $u_{tt} - u_{xx} = 0$ (Wave Equation)

   c) $u_t = \triangle u$ (Heat Equation)

   d) $div\left(|\nabla u|^{p-2}\nabla u\right) = 0$ (p-Laplace equation) for $p > 1$.

   e) $det(D^2 u) = 1$ (Monge Ampere equation)

   f) $u_t + uu_x = 0$ (Burgers Equation)

   g) $xu_{xxy} + u_{yyy} = 1$.

3. Solve the following problem:

$$\begin{cases} u_x - 2u_y = u \quad \textbf{in } \mathbb{R}^2, \\ u(x,0) = 1. \end{cases}$$

$$\begin{cases} u_x - 2u_y = u \quad , \\ u(0,y) = y. \end{cases}$$

   and

$$\begin{cases} u_x - 2u_y = u \\ u(x,x) = x. \end{cases}$$

4. Consider the following semilinear equation:

$$\begin{cases} u_x + u_y = u^{\frac{1}{2}}, \\ u(x,0) = 0. \end{cases} \tag{1}$$

   Show that solution of this problem is not unique. Try to explain a possible reason for this. Does it contradict the Existence uniqueness theorem provided in lectures?

5. Solve the following semilinear equation:

$$\begin{cases} uu_x + u_y = u^2 & , \\ u(x,0) = 1. \end{cases} \tag{2}$$

6. Consider the following problem:

$$\begin{cases} u_x + u_y = 0 & , \\ u(x,x) = 1. \end{cases} \tag{3}$$

Then the above problem has

a) infinitely many solutions

b) no solution

c) atmost finitely many solutions

d) unique solution.

7. Does the projected characteristics in Problem 4 and 5 intersects?

8. Does the non-characteristics condition is satisfied for the problem 4 and for the Toy problem 2 in the lecture video.

[After doing the second part of the above problem, try to connect the information on the number of solution of the problem and the main theroem]