

ESC 201T: Introduction to Electronics

Lecture 6: Toolbox For Circuit Analysis-3

Equivalent Resistance and Superposition

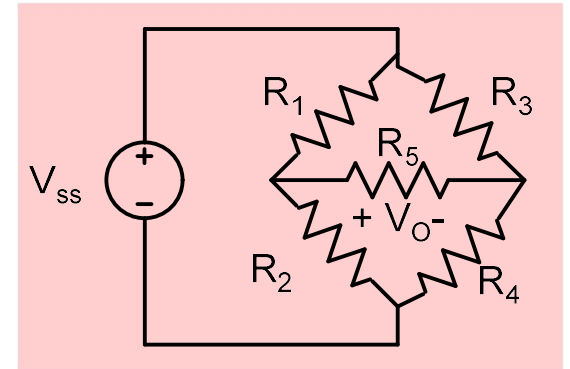
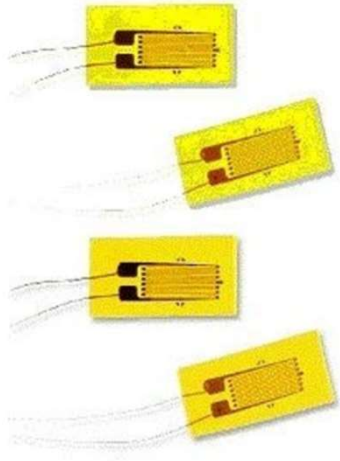
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The Five laws of library science by S. R. Ranganathan:

- ❖ Books are for use.
- ❖ Every reader his or her book.
- ❖ Every book its reader.
- ❖ Save the time of the reader.
- ❖ The library must be a growing organism

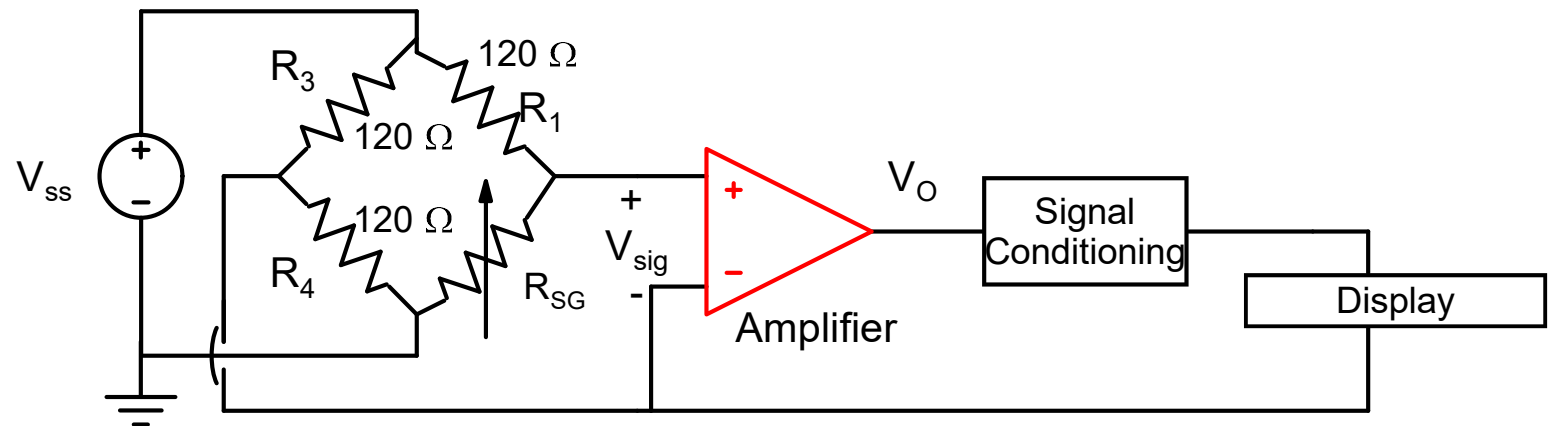
“Analysis is for understanding”

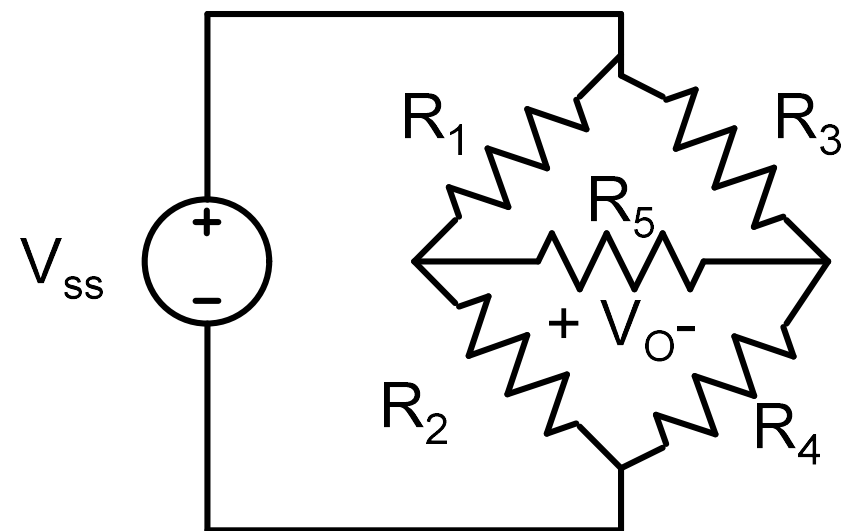
Strain Gauge



NIE Strain Gauge,
 $120 \pm 0.3 \Omega$, 10.0
 mm, G.F. $-2.11 \pm 1\%$

$$\frac{\Delta R_{SG}}{R_{SG}} = \frac{1}{50} \cdot E (\%)$$





Loop 2: $-i_1 R_3 + i_2 R_5 + (I - i_1) R_1 = 0$
 Loop 3: $-i_2 R_5 - (i_1 + i_2) R_4 + (I - i_1 - i_2) R_2 = 0$

$\Rightarrow \Delta V = V_S (R_3 R_2 - R_4 R_1) R_5$
 $R_1 R_3 R_2 + R_1 R_4 R_3 + R_1 R_2 R_3 + R_1 R_5 R_4 + R_2 R_5 R_3 + R_2 R_4 R_3 + R_2 R_5 R_4 + R_1 R_3 R_4$

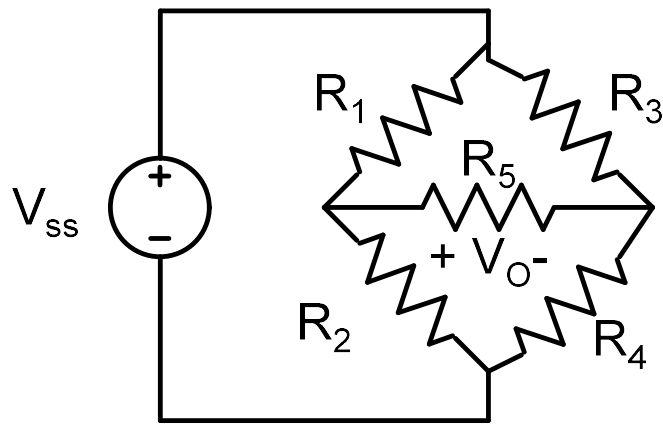
From L2: $I = \frac{i_1 (R_3 + R_1) - i_2 R_5}{R_1}$
 Putting in L3: $\left(\frac{i_1 (R_3 + R_1) - i_2 R_5}{R_1} \right) R_2 = i_1 (R_4 + R_2) + i_2 (R_5 + R_4 + R_2)$
 $\Rightarrow i_1 \left(\frac{R_3 + R_1}{R_1} R_2 - (R_4 + R_2) \right) = i_2 \left(\frac{R_5 + R_4 + R_2}{R_1} \right)$
 $\Rightarrow i_1 \left(\frac{R_3 R_2 + R_1 R_2 - R_4 R_1 - R_2 R_1}{R_1} \right) = i_2 \left(\frac{R_5 R_1 + R_4 R_1 + R_2 R_1}{R_1} \right)$

New inverse of above 3×3 matrix:
 From L2: $I = \frac{i_1 (R_3 + R_1) - i_2 R_5}{R_1}$
 Putting in L3: $\left(\frac{i_1 (R_3 + R_1) - i_2 R_5}{R_1} \right) R_2 = i_1 (R_4 + R_2) + i_2 (R_5 + R_4 + R_2)$
 $\Rightarrow i_1 \left(\frac{R_3 + R_1}{R_1} R_2 - (R_4 + R_2) \right) = i_2 \left(\frac{R_5 + R_4 + R_2}{R_1} \right)$
 $\Rightarrow i_1 \left(\frac{R_3 R_2 + R_1 R_2 - R_4 R_1 - R_2 R_1}{R_1} \right) = i_2 \left(\frac{R_5 R_1 + R_4 R_1 + R_2 R_1}{R_1} \right)$

Unknown: I, i_1, i_2

$\Rightarrow i_1 = i_2 \left(\frac{R_5 R_1 + R_4 R_1 + R_2 R_1 + R_5 R_2}{R_3 R_2 - R_4 R_1} \right)$ (2)
 $\Rightarrow I = \frac{i_2 (R_3 + R_1) (R_5 R_1 + R_4 R_1 + R_2 R_1 + R_5 R_2)}{R_3 R_2 - R_4 R_1} - i_2 R_5$
 $I = i_2 \left(\frac{R_3 R_5 R_1 + R_3 R_4 R_1 + R_3 R_2 R_1 + R_5 R_2 R_1 + R_2 (R_5 + R_4 + R_2) + R_1 R_5 R_2}{R_1 (R_3 R_2 - R_4 R_1)} - R_5 \right)$
 $I = i_2 \left(\frac{R_3 R_5 R_1 + R_3}{R_1 (R_3 R_2 - R_4 R_1)} \right)$
 Now, $i_2 \left(\frac{R_1 (R_3 R_2 + R_4 R_3 + R_2 R_3 + R_3 R_2 R_1 + R_5 R_2) + R_1^2 (R_5 + R_4 + R_2)}{R_1 (R_3 R_2 - R_4 R_1)} \right) (R_1 + R_2) =$
 $i_2 \left(\frac{R_5 R_1 + R_4 R_1 + R_2 R_1 + R_5 R_2}{R_3 R_2 - R_4 R_1} \right) (R_1 + R_2) - i_2 R_2 = V_S$
 $\Rightarrow i_2 \left(\frac{R_1 (R_1 + R_2)}{R_3 R_2 - R_4 R_1} \right) (R_5 R_3 + R_4 R_3 + R_2 R_3 + R_5 R_2 + R_4 R_2 + R_2 R_1 + R_5 R_1 + R_4 R_1 - R_4 R_1 - R_5 R_1) - i_2 R_2 = V_S$

$\frac{i_2 (R_1 + R_2)}{R_3 R_2 - R_4 R_1} (R_5 R_3 + R_4 R_3 + R_2 R_3 + R_5 R_2 + R_4 R_2 + R_2 R_1 + R_5 R_1 + R_4 R_1 - R_4 R_1 - R_5 R_1) - i_2 R_2 = V_S$ (3)
 $i_2 \left(\frac{R_1 R_5 R_3 + R_1 R_4 R_3 + R_1 R_2 R_3 + R_1 R_5 R_2 + R_1 R_4 R_2 + R_1 R_2 R_1 + R_1 R_5 R_1 + R_1 R_4 R_1 - R_1 R_4 R_1 - R_1 R_5 R_1}{R_3 R_2 - R_4 R_1} \right) - i_2 R_2 = V_S$
 $\Rightarrow i_2 \left(\frac{R_1 R_5 R_3 + R_1 R_4 R_3 + R_1 R_2 R_3 + R_1 R_5 R_2 + R_1 R_4 R_2 + R_1 R_2 R_1 + R_1 R_5 R_1 + R_1 R_4 R_1 - R_1 R_4 R_1 - R_1 R_5 R_1}{R_3 R_2 - R_4 R_1} \right) - i_2 R_2 = V_S$
 $\Rightarrow i_2 = \frac{V_S (R_3 R_2 - R_4 R_1)}{R_1 (R_5 R_3 + R_4 R_3 + R_2 R_3 + R_5 R_2 + R_4 R_2 + R_2 R_1 + R_5 R_1 + R_4 R_1 - R_4 R_1 - R_5 R_1) + R_2 (R_5 R_3 + R_4 R_3 + R_2 R_3 + R_5 R_2 + R_4 R_2 + R_2 R_1 + R_5 R_1 + R_4 R_1 - R_4 R_1 - R_5 R_1)}$
 $i_2 = \frac{V_S (R_3 R_2 - R_4 R_1)}{R_1 (R_4 (R_3 + R_5 + R_2) + R_3 (R_1 + R_5)) + R_2 (R_3 (R_5 + R_4) + R_5 R_4)}$



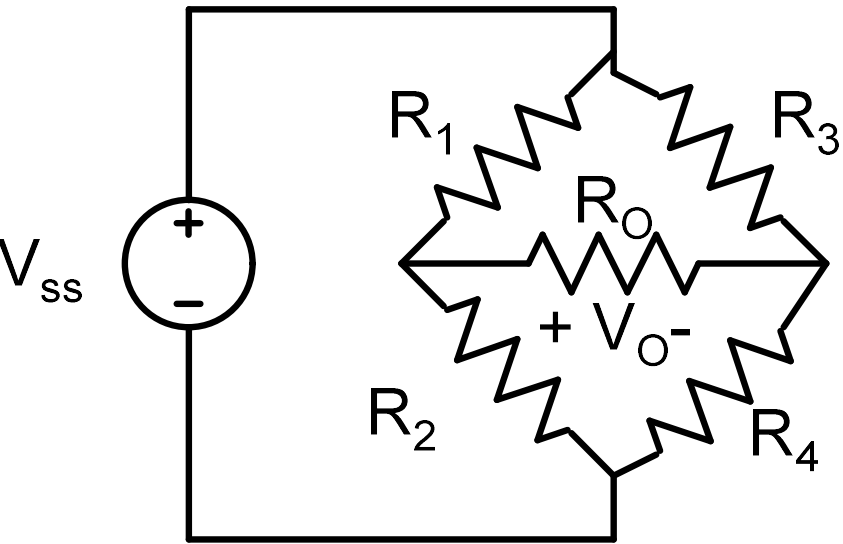
Loop 2: $-i_1 R_3 + i_2 R_3 + (I - i_1) R_1 = 0$
 Loop 3: $-i_2 R_4 - (i_1 + i_2) R_4 + (I - i_1 - i_2) R_2 = 0$

$\Delta V = V_S (R_3 R_2 - R_4 R_1) R_5$
 $R_1 R_3 R_2 + R_1 R_4 R_3 + R_1 R_2 R_3 + R_1 R_5 R_4 + R_2 R_3 R_3$
 $+ R_2 R_4 R_3 + R_2 R_5 R_4 + R_1 R_2 R_4$

From L_2 : $I = \frac{i_1 (R_3 + R_1) - i_2 R_5}{R_1}$
 Putting in L_3 : $\left(\frac{i_1 (R_3 + R_1) - i_2 R_5}{R_1} \right) R_2 = i_1 (R_4 + R_2) + i_2 (R_5 + R_4 + R_2)$
 $\Rightarrow i_1 \left(\frac{R_3 + R_1}{R_1} R_2 - (R_4 + R_2) \right) = i_2 \left(\frac{R_5 + R_4 + R_2}{R_1} \right)$
 $\Rightarrow i_1 \left(\frac{R_3 R_2 + R_2 R_2 - R_4 R_1 - R_5 R_1}{R_1} \right) = i_2 \left(\frac{R_5 R_1 + R_4 R_1}{R_1} \right)$

Unknown: I, i_1, i_2

$$V_O = \frac{V_{SS} \times R_5 \times (R_2 R_3 - R_1 R_4)}{R_2 R_4 R_5 + R_2 R_3 R_5 + R_1 R_4 R_5 + R_1 R_3 R_5 + R_2 R_3 R_4 + R_1 R_3 R_4 + R_1 R_2 R_4 + R_1 R_2 R_3}$$

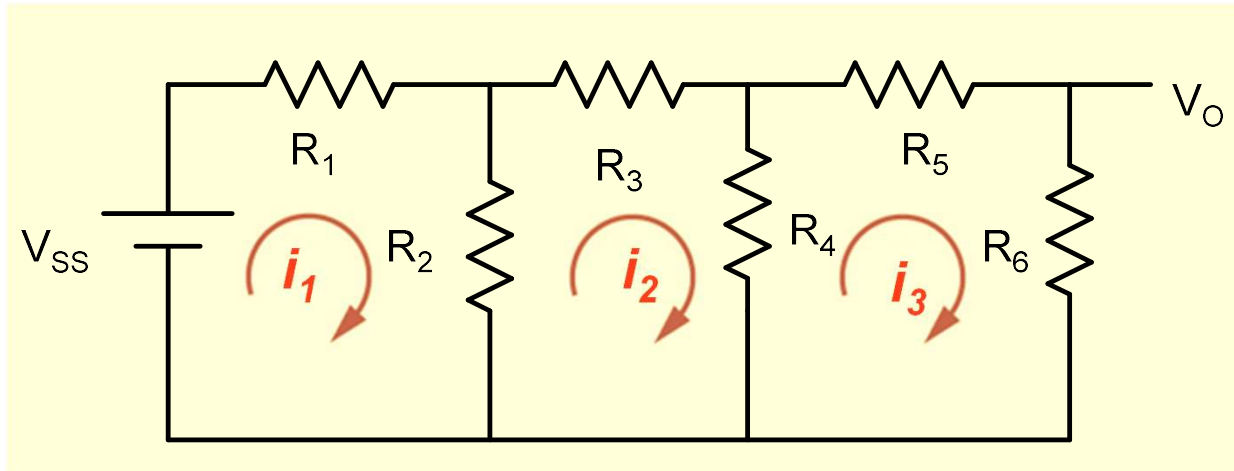


$$V_O = V_{SS} \times \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) \times \left\{ \frac{R_O}{R_O + \{(R_1 \parallel R_2) + (R_3 \parallel R_4)\}} \right\}$$

A Structured expression that reveals role of each element

$$V_O = \frac{V_{SS} \times R_O \times (R_2 R_3 - R_1 R_4)}{R_2 R_4 R_O + R_2 R_3 R_O + R_1 R_4 R_O + R_1 R_3 R_O + R_2 R_3 R_4 + R_1 R_3 R_4 + R_1 R_2 R_4 + R_1 R_2 R_3}$$

A Disordered expression that does not provide insight



$$V_o = \frac{V_{SS} \times R_2 R_4 R_6}{R_2 R_4 R_6 + R_2 R_4 R_5 + R_2 R_3 R_6 + R_2 R_3 R_5 + R_2 R_3 R_4 + R_1 R_4 R_6 + R_1 R_4 R_5 + R_1 R_3 R_6 + R_1 R_3 R_5 + R_1 R_3 R_4 + R_1 R_2 R_6 + R_1 R_2 R_5 + R_1 R_2 R_4}$$

- ❖ Mesh and Nodal analysis are “brute-force” techniques that are not only time consuming and error prone for us to use but also yield **unstructured** expressions that are often unsuitable for gaining **insight** into operation of circuits and **modifying or designing** them.
- ❖ Need techniques that yield relatively simpler structured expressions that **reveal the role of different components** and that require less **effort** and is less **error prone**

Analysis using REUSE Methodology

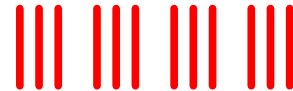
Do not carry out analysis from scratch !

Analyze, Remember and Reuse

Example: we do not carry out multiplication from scratch using repeated addition !

$$\begin{array}{r} 34 \\ \times 3 \\ \hline 102 \end{array}$$

$$3 \times 4 = 12$$



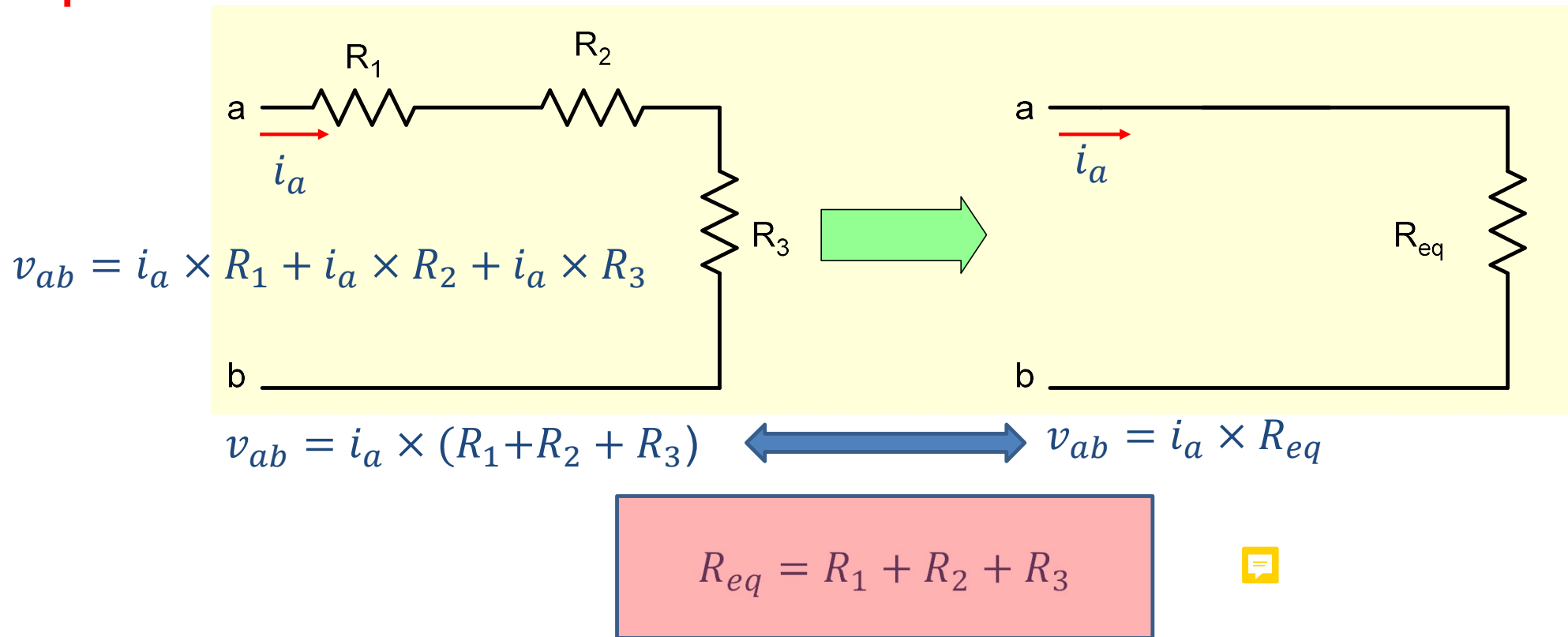
You cannot carry out complex multiplication with ease using the first principle

$$\begin{array}{l} 4 \times 1 = 4 \\ 4 \times 2 = 8 \\ 4 \times 3 = 12 \\ 4 \times 4 = 16 \end{array}$$

.....

Memorize multiplication table and use it again and again

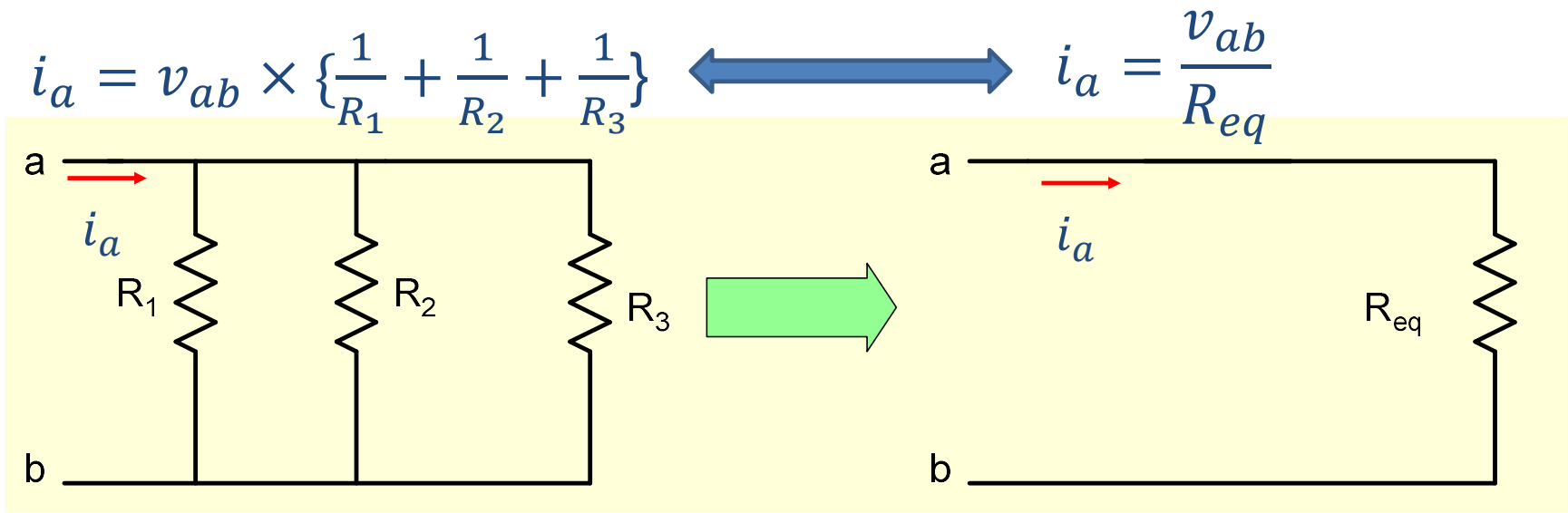
Equivalent Series Resistances



Both circuits are equivalent as far as terminal **V vs. I** relation is concerned.

Once derived, this is a useful result that can be Re-Used at many places

Parallel Resistances



$$i_a = \frac{v_{ab}}{R_1} + \frac{v_{ab}}{R_2} + \frac{v_{ab}}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

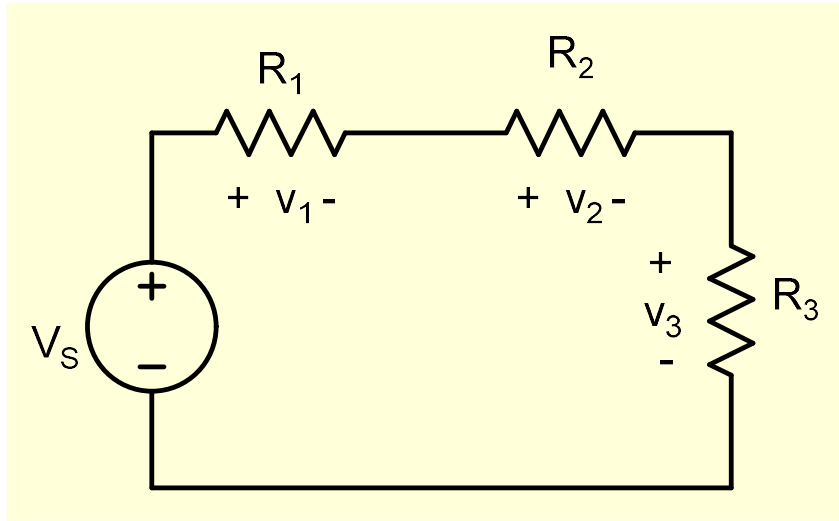


Once derived, this is a useful result that can be Re-Used at many places

Voltage division

Another example of Analysis Reuse

A voltage applied to resistors connected in series will be divided among them



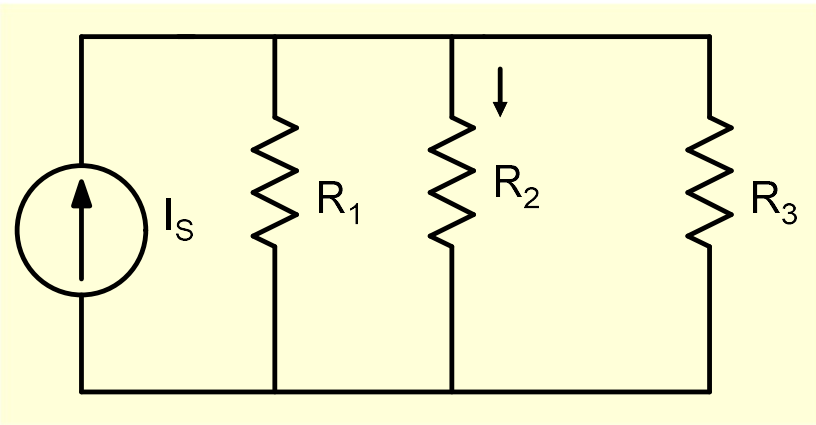
$$v_J = v_S \times \frac{R_J}{\sum R_i}$$

$$v_2 = v_S \times \frac{R_2}{R_1 + R_2 + R_3}$$

Once derived using first principles, this is a useful result that can be Re-Used at many places

Current Division

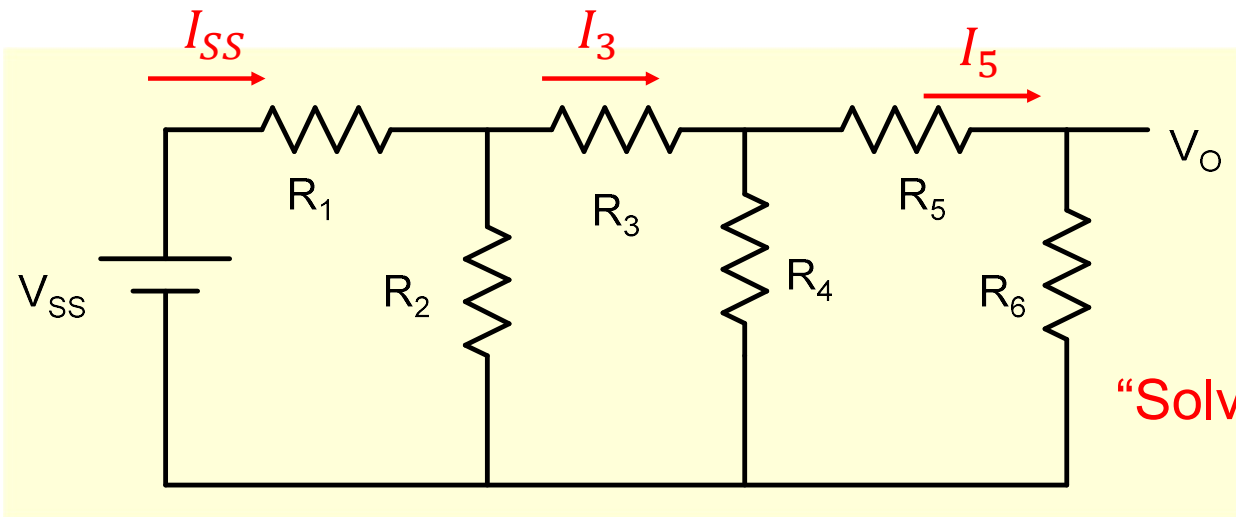
The total current flowing into a parallel combination of resistors will be divided among them



$$I_2 = I_S \times \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3}$$

$$I_2 = I_S \times \frac{R_1}{R_1 + R_2} \text{ for two resistors}$$

Example

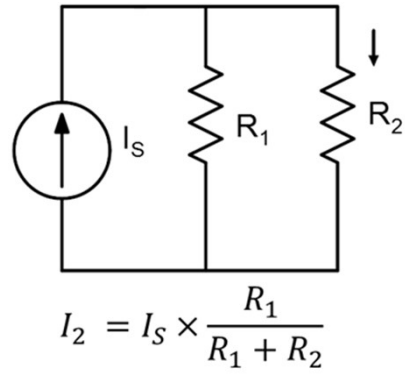


$$V_O = I_5 \times R_6$$

“Solve circuits by inspection”

Re-use series and parallel results

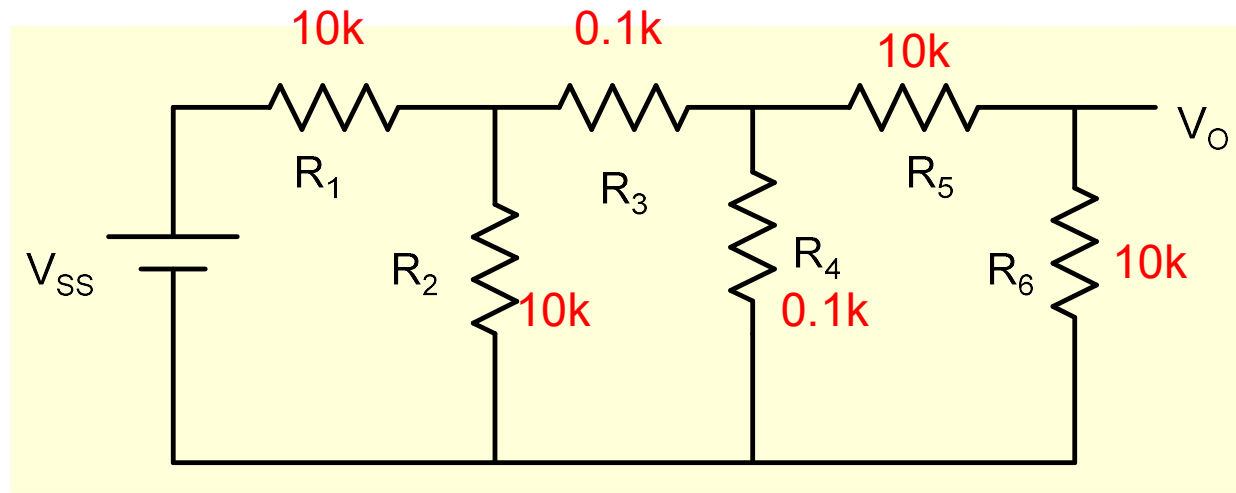
$$R_{eq} = (\{(R_5 + R_6) \parallel R_4\} + R_3) \parallel R_2 + R_1 \quad I_{SS} = \frac{V_{SS}}{R_{eq}}$$



Re-use current division result to calculate I_3 and I_5

$$I_3 = I_{SS} \times \frac{R_2}{R_2 + \{(R_5 + R_6) \parallel R_4\} + R_3} \quad I_5 = I_3 \times \frac{R_4}{R_4 + R_5 + R_6}$$

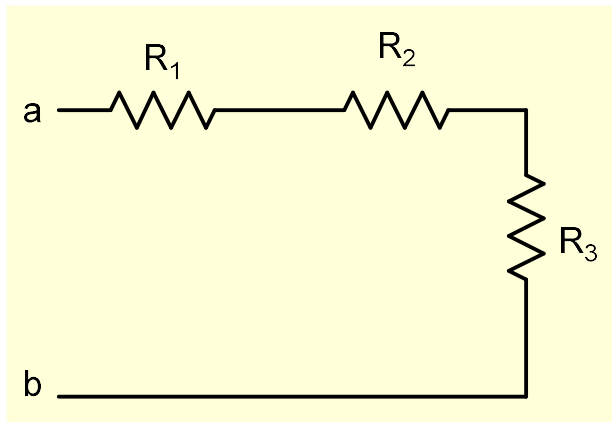
$$V_O = \frac{V_{SS}}{(\{(R_5 + R_6) \parallel R_4\} + R_3) \parallel R_2 + R_1} \times \frac{R_2}{R_2 + \{(R_5 + R_6) \parallel R_4\} + R_3} \times \frac{R_4}{R_4 + R_5 + R_6} \times R_6$$



$$V_O = \frac{V_{SS} \times R_2 R_4 R_6}{R_2 R_4 R_6 + R_2 R_4 R_5 + R_2 R_3 R_6 + R_2 R_3 R_5 + R_2 R_3 R_4 + R_1 R_4 R_6 + R_1 R_4 R_5 + R_1 R_3 R_6 + R_1 R_3 R_5 + R_1 R_3 R_4 + R_1 R_2 R_6 + R_1 R_2 R_5 + R_1 R_2 R_4}$$

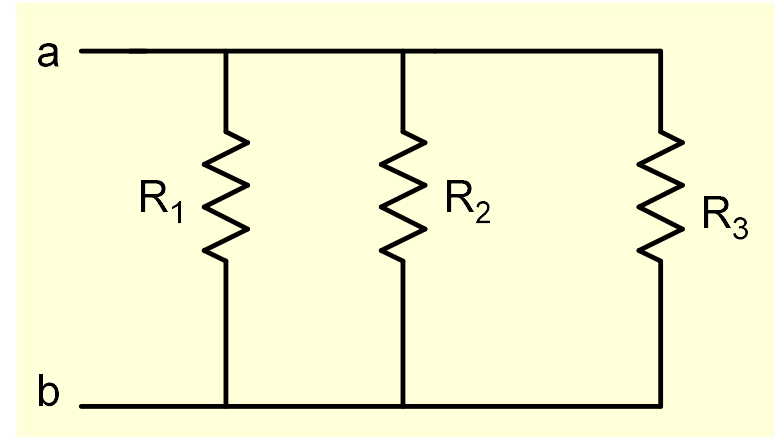
$$V_O = \frac{V_{SS}}{(\{(R_5 + R_6) \parallel R_4\} + R_3) \parallel R_2 + R_1} \times \frac{R_2}{R_2 + \{(R_5 + R_6) \parallel R_4\} + R_3} \times \frac{R_4}{R_4 + R_5 + R_6} \times R_6$$

$$V_O \approx \frac{V_{SS}}{R_1} \times 1 \times \frac{R_4}{2}$$



$$R_{eq} = R_1 + R_2 + R_3$$

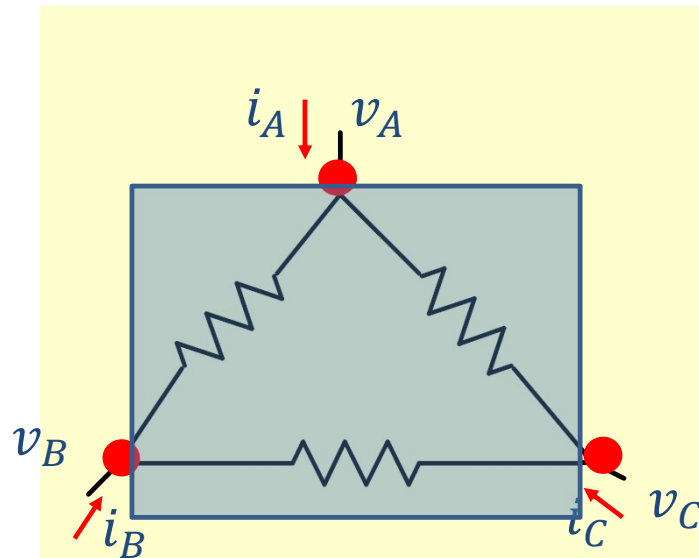
$$v_2 = v_s \times \frac{R_2}{R_1 + R_2 + R_3}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$I_2 = I_s \times \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3}$$

Equivalent Circuits

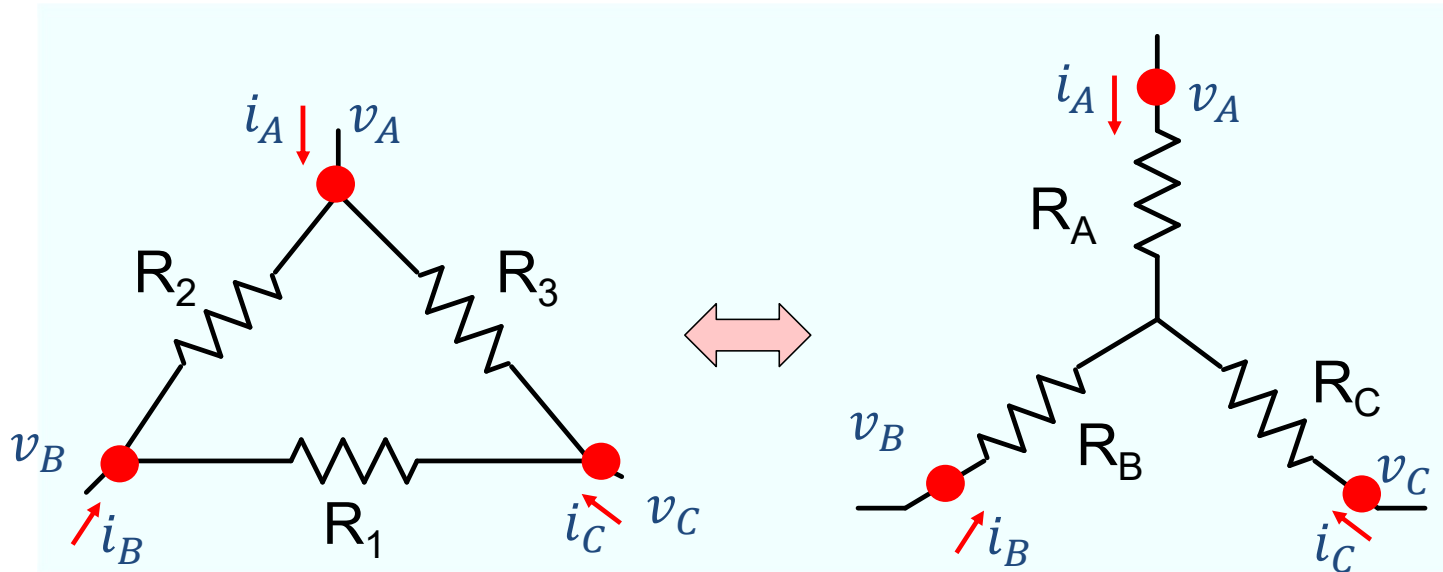


Circuit-A

Circuit-B

B is equivalent to A as far as terminal current voltage characteristics is concerned

Example



$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_2 R_1}{R_1 + R_2 + R_3}$$

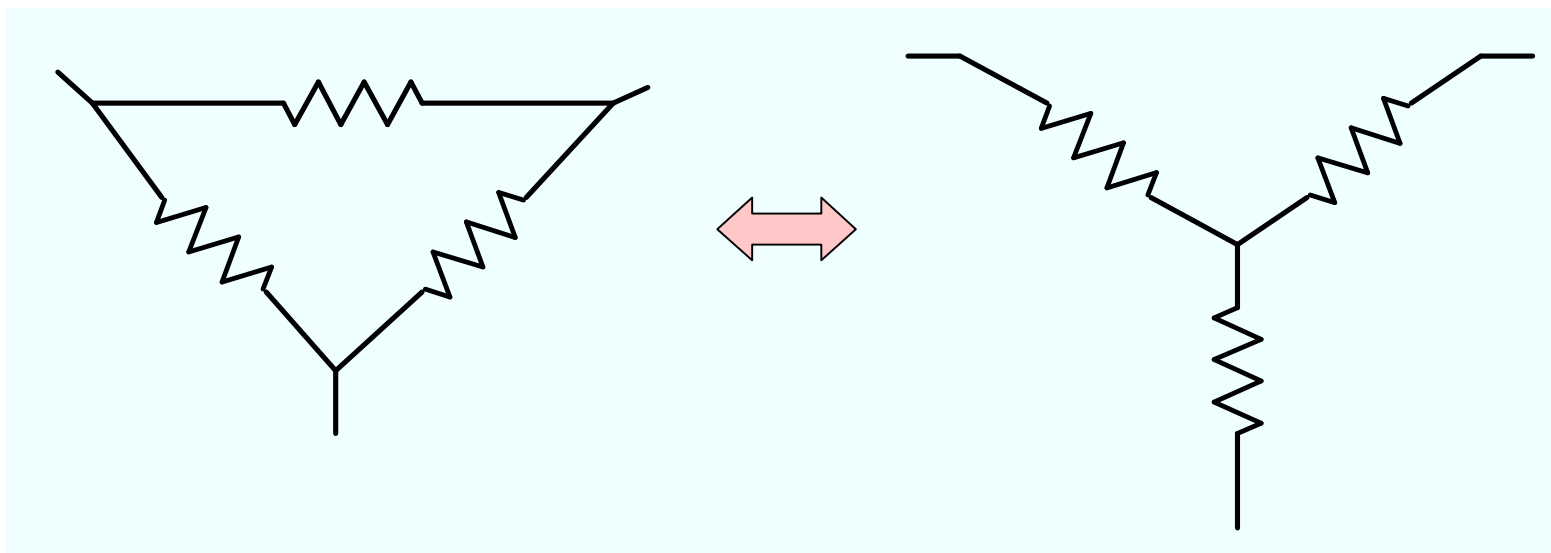
$$R_C = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C}$$

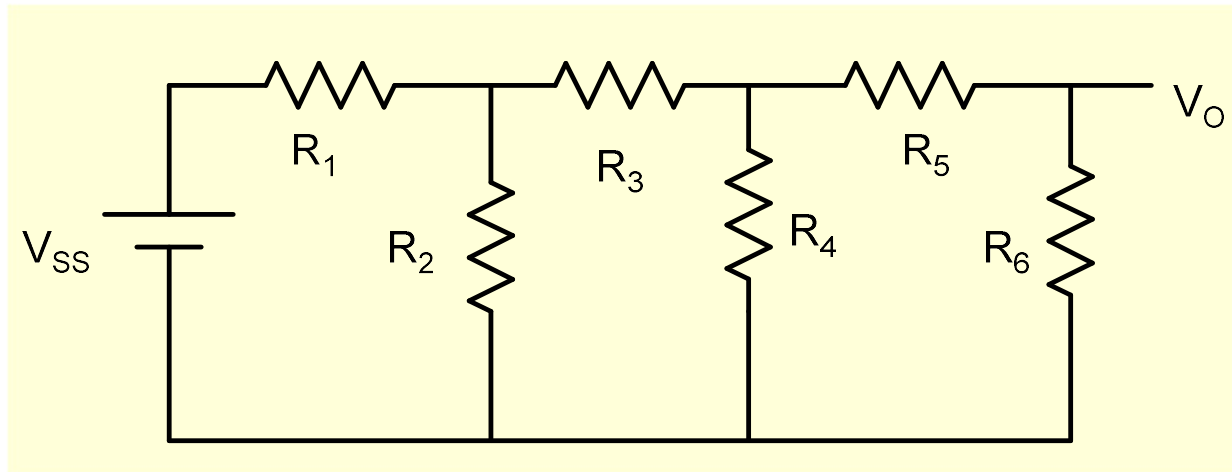
$$R_1 = R_C + R_B + \frac{R_C R_B}{R_A}$$

$$R_3 = R_A + R_C + \frac{R_A R_C}{R_B}$$

$\Delta - Y$ Transformation



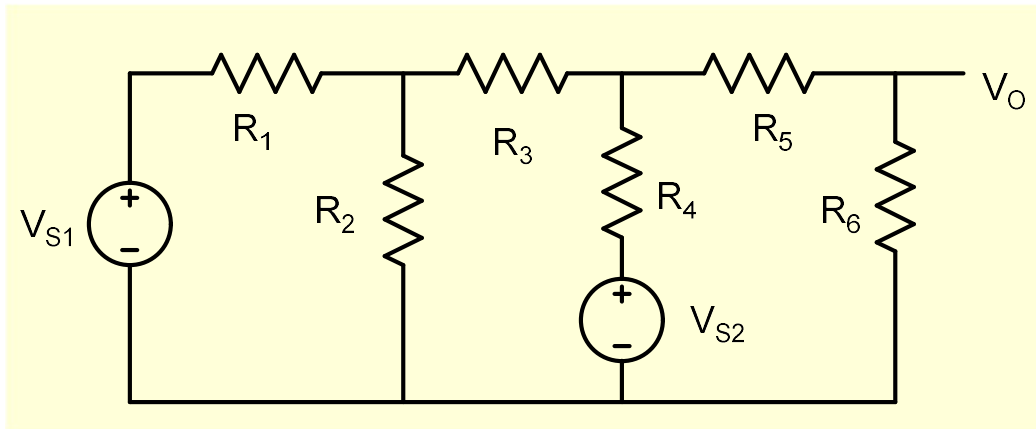
Solve a Complex Problem by Breaking it into Simpler problems.



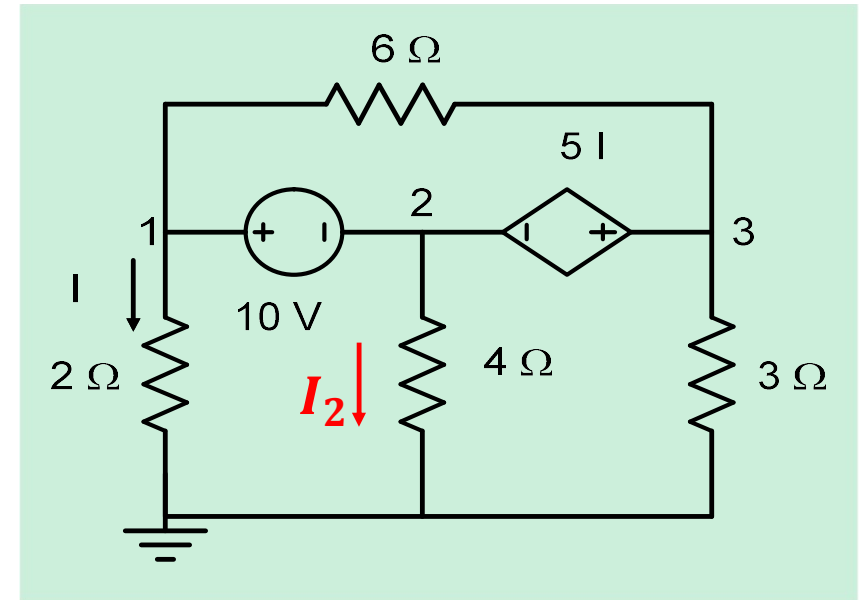
$$V_O = \frac{V_{SS}}{(\{(R_5 + R_6) \parallel R_4\} + R_3) \parallel R_2 + R_1} \times \frac{R_2}{R_2 + \{(R_5 + R_6) \parallel R_4\} + R_3} \times \frac{R_4}{R_4 + R_5 + R_6} \times R_6$$

$$V_O = K \times V_{SS}$$

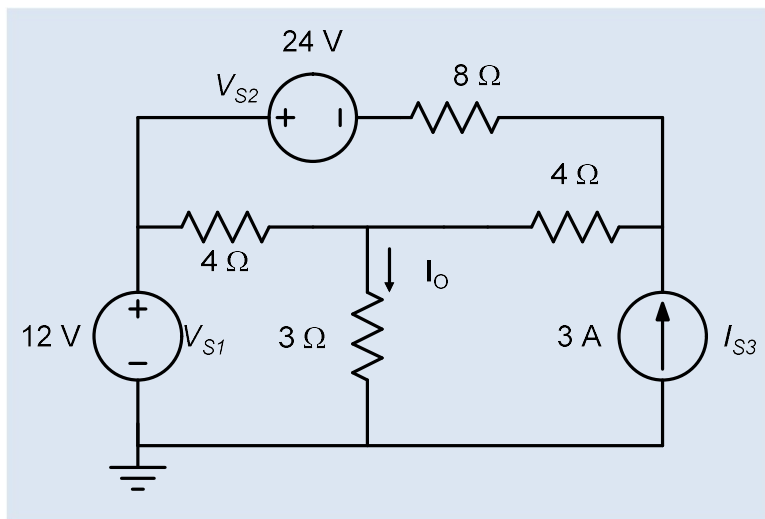
Circuit made of linear components is a linear circuit



$$V_O = K_{O1} \times V_{S1} + K_{O2} \times V_{S2}$$



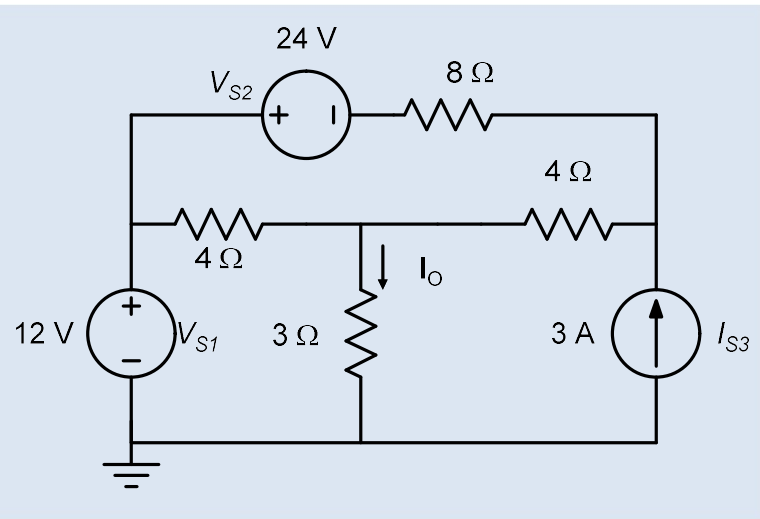
$$I_2 = K_2 \times 10$$



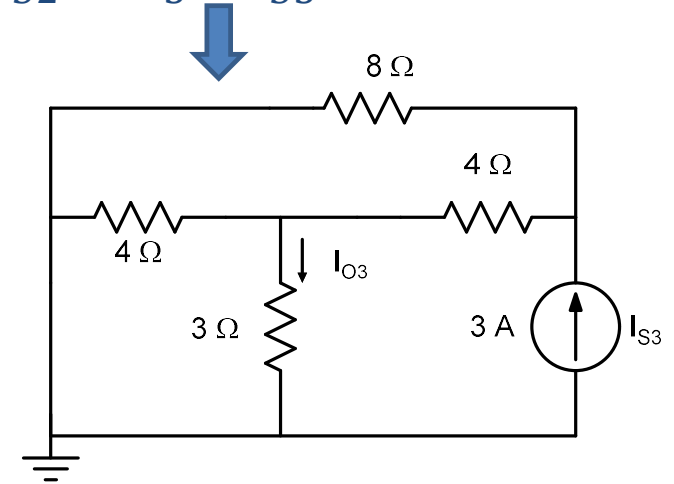
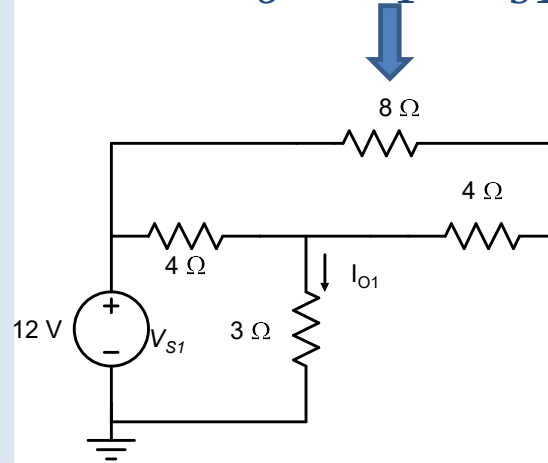
$$I_O = K_1 \times V_{S1} + K_2 \times V_{S2} + K_3 \times I_{S3}$$

The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually.

The Superposition principle allows a complex circuit to be decomposed into simpler circuits



$$I_O = K_1 \times V_{S1} + K_2 \times V_{S2} + K_3 \times I_{S3}$$



$$I_O = I_{O1} + I_{O2} + I_{O3}$$

