

# ESC201T : Introduction to Electronics

## Lecture 17: LCR Filters (Resonance)

B. Mazhari  
Dept. of EE, IIT Kanpur

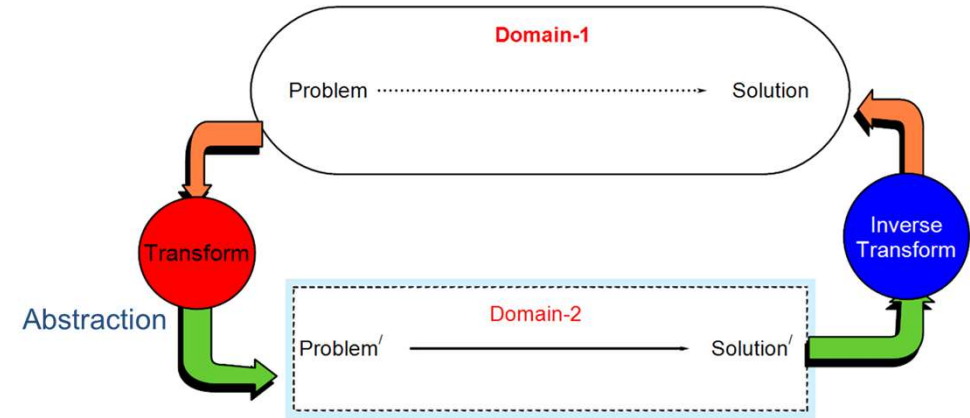
# Wireless Transmission of Speech signal

$$x(t) = x_m \sin(2\pi \times 10^3 t)$$



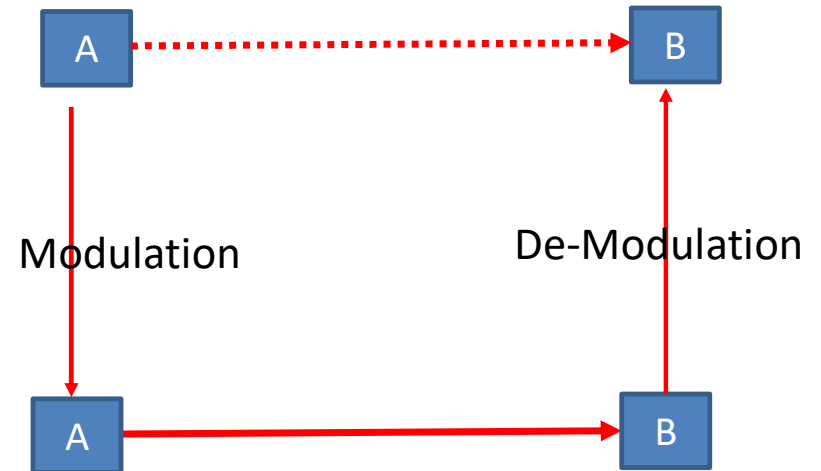
$$y(t) = y_m \sin(2\pi \times 450 \times 10^3 t)$$

Carrier wave



$$x(t) = x_m \sin(2\pi \times 10^3 t)$$

$$x(t) = x_m \sin(2\pi \times 10^3 t)$$

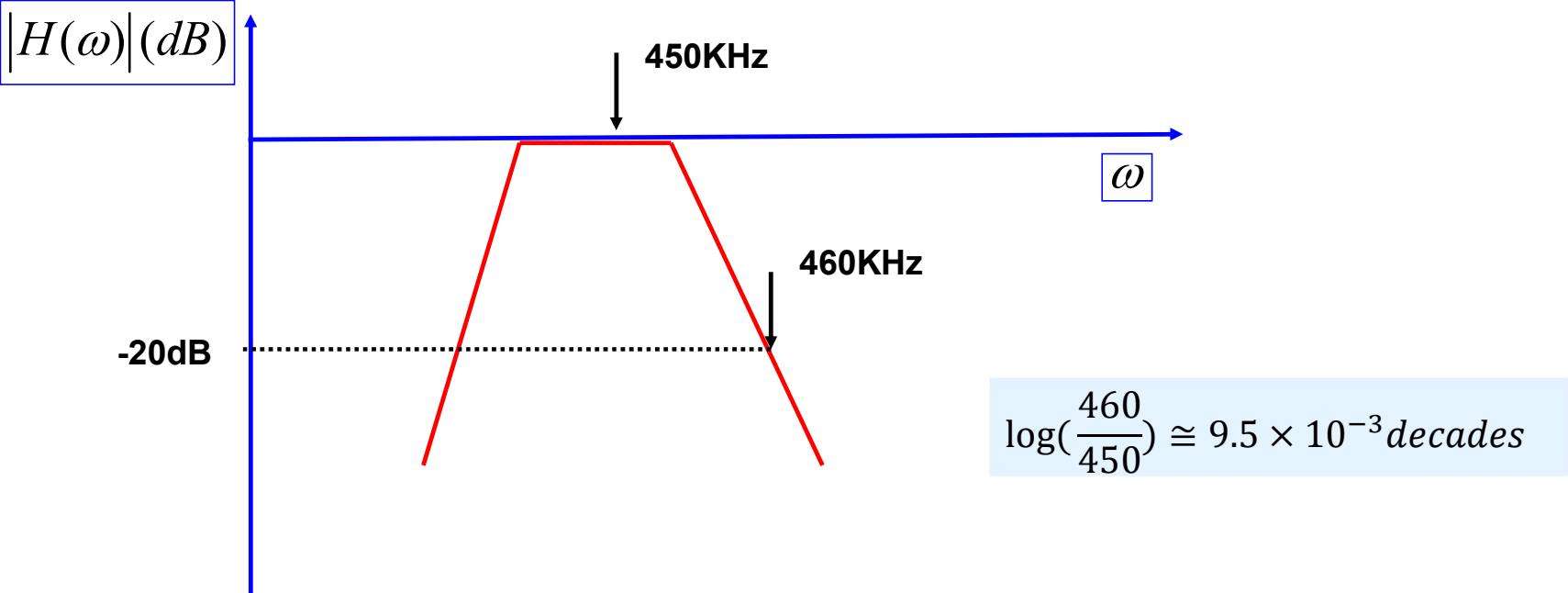


$$Z(t) = (y_m + m \times x(t)) \sin(2\pi \times 450 \times 10^3 t) C$$

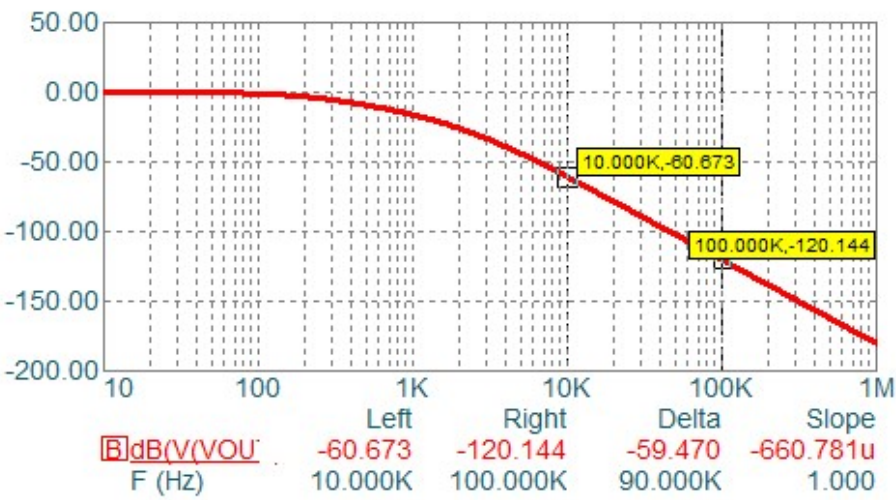
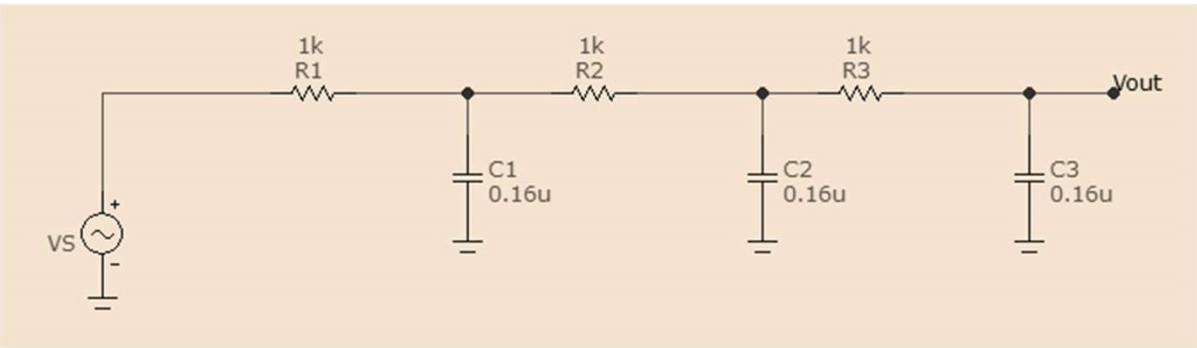
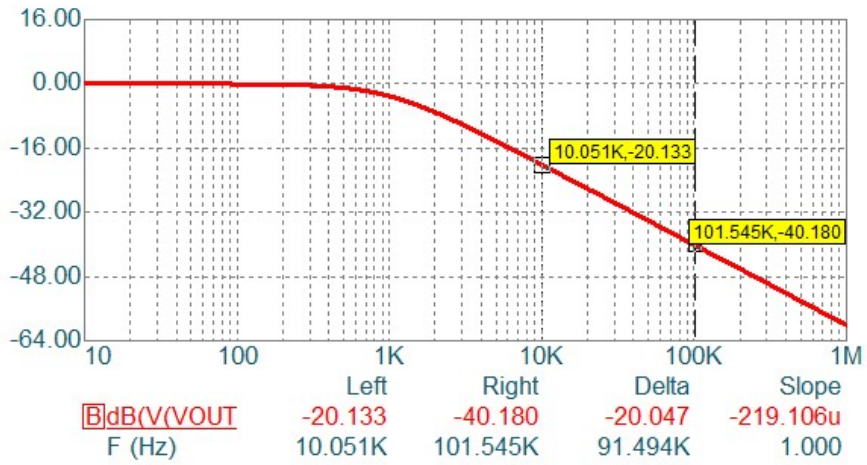
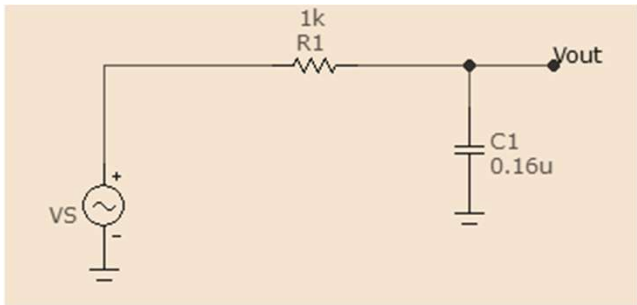
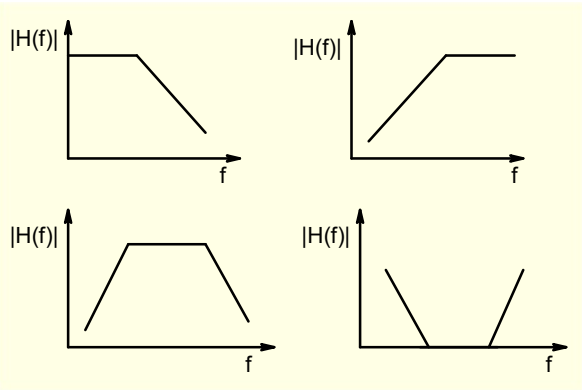
# Amplitude Modulated (AM) Radio

Different radio channels are separated by very narrow frequency interval.

For example, one may want to receive a 450KHz signal but reject 460KHz or 440KHz



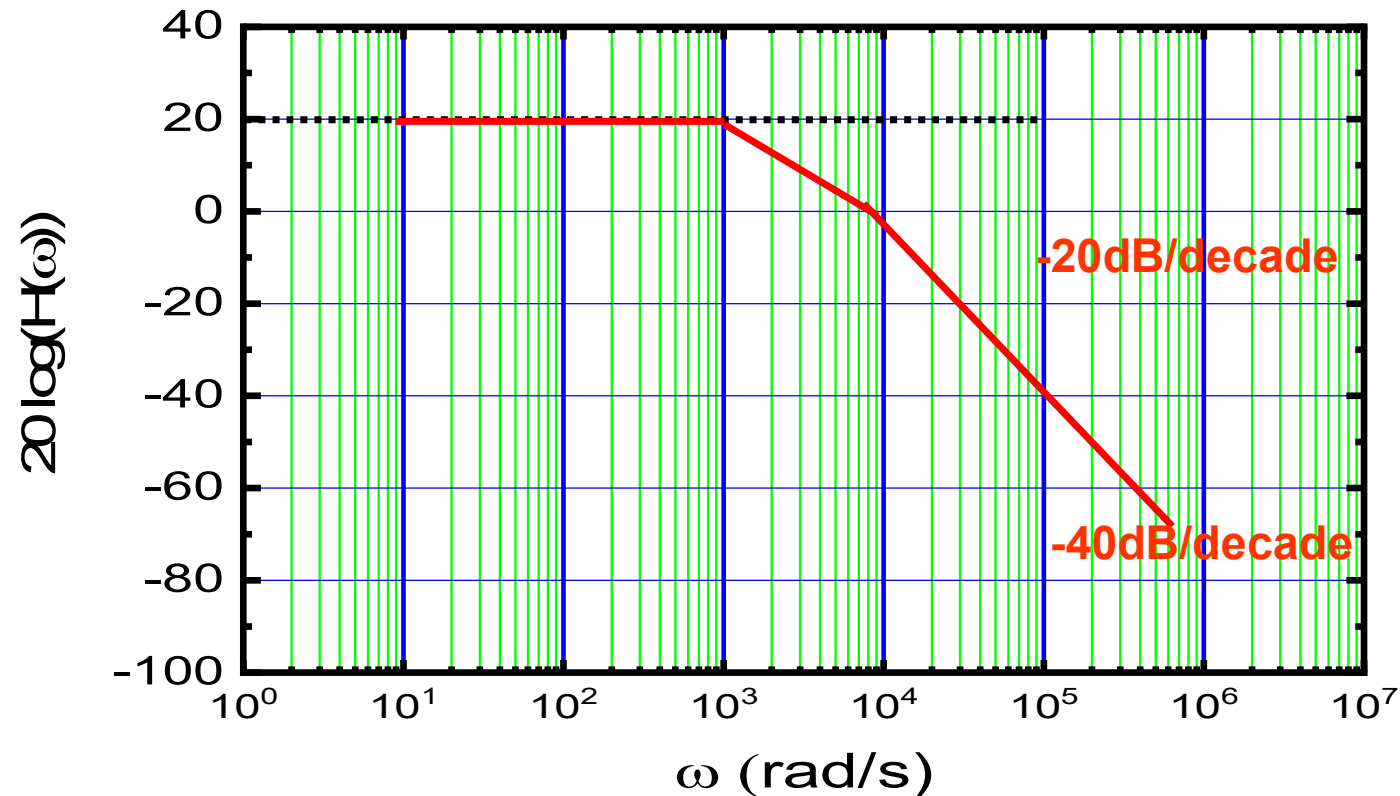
This implies an attenuation of  $\sim -2000 \text{ dB/decade}$



## Second Order System

$$H(\omega) = \frac{10}{1 + j\frac{\omega}{10^3}} \times \frac{1}{1 + j\frac{\omega}{10^4}}$$

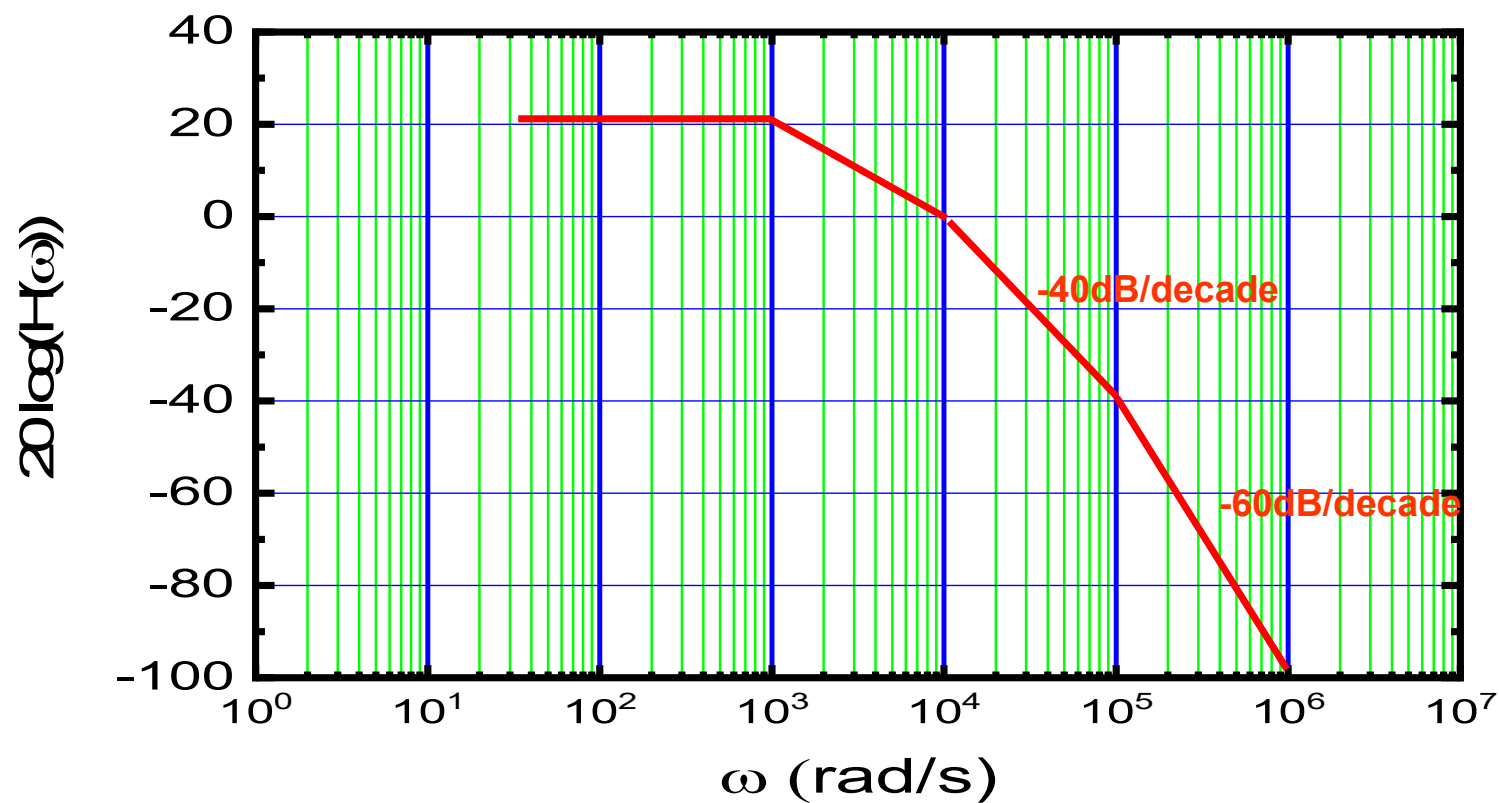
$$20\text{Log}_{10}(|H(\omega)|) = 20 - 10\text{Log}_{10}(1 + (\frac{\omega}{10^3})^2) - 10\text{Log}_{10}(1 + (\frac{\omega}{10^4})^2)$$



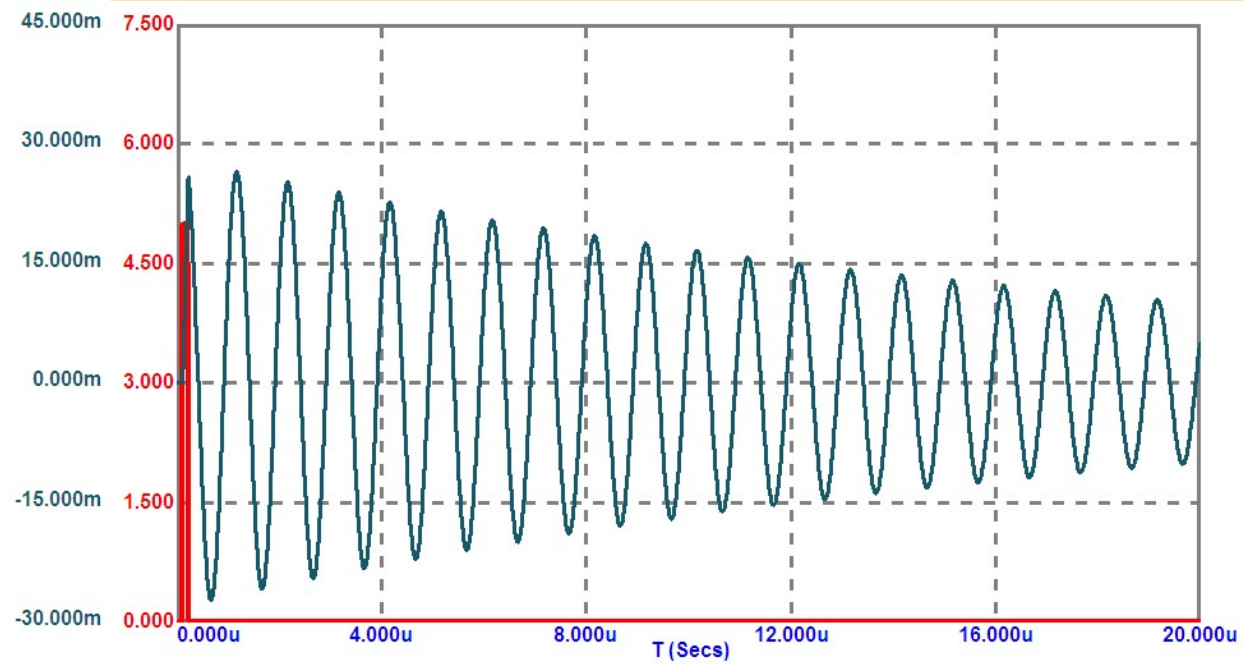
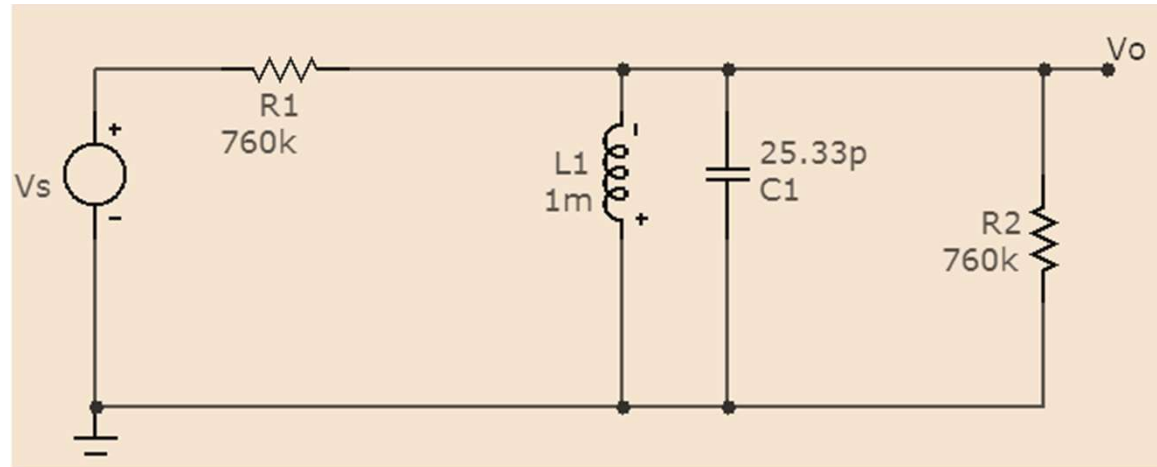
# Third Order System

$$H(\omega) = \frac{10}{1 + j \frac{\omega}{10^3}} \times \frac{1}{1 + j \frac{\omega}{10^4}} \times \frac{1}{1 + j \frac{\omega}{10^5}}$$

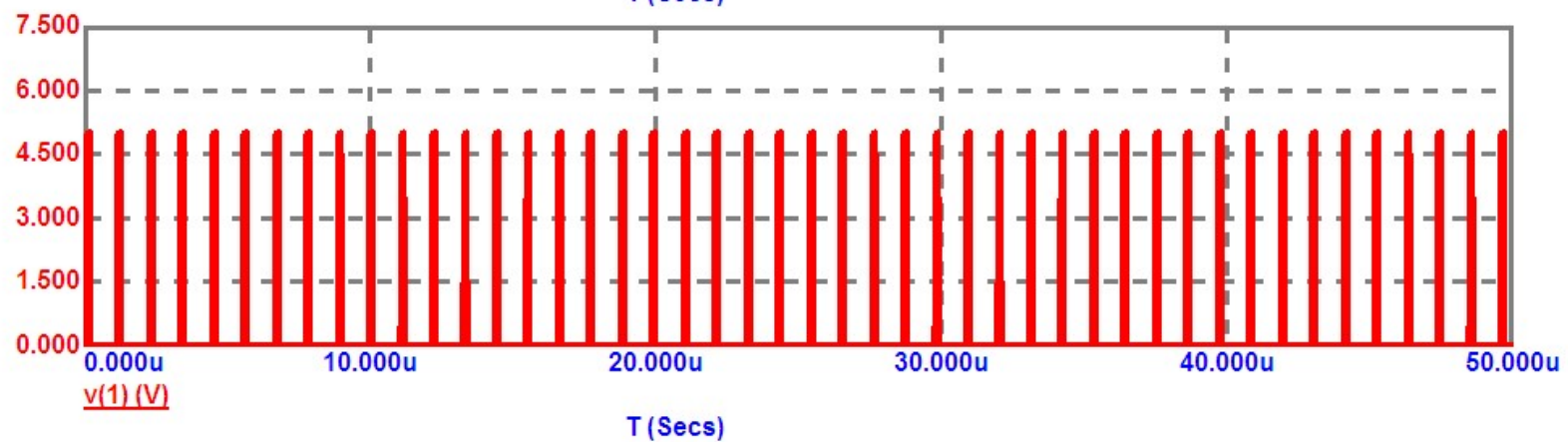
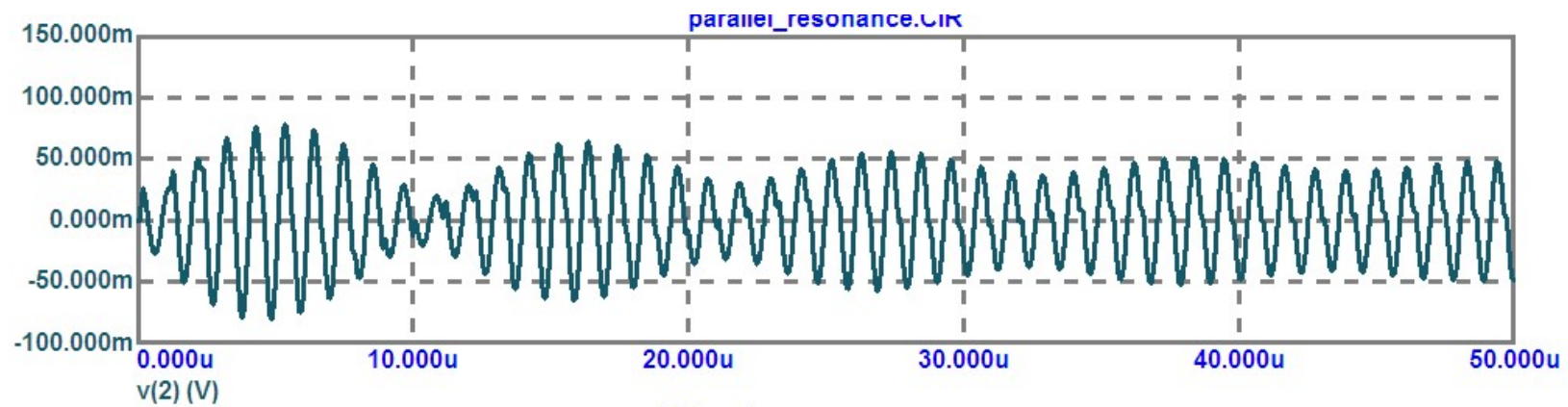
$$20\text{Log}_{10}(|H(\omega)|) = 20 - 10\text{Log}_{10}(1 + (\frac{\omega}{10^3})^2) - 10\text{Log}_{10}(1 + (\frac{\omega}{10^4})^2) - 10\text{Log}_{10}(1 + (\frac{\omega}{10^5})^2)$$



# Resonance

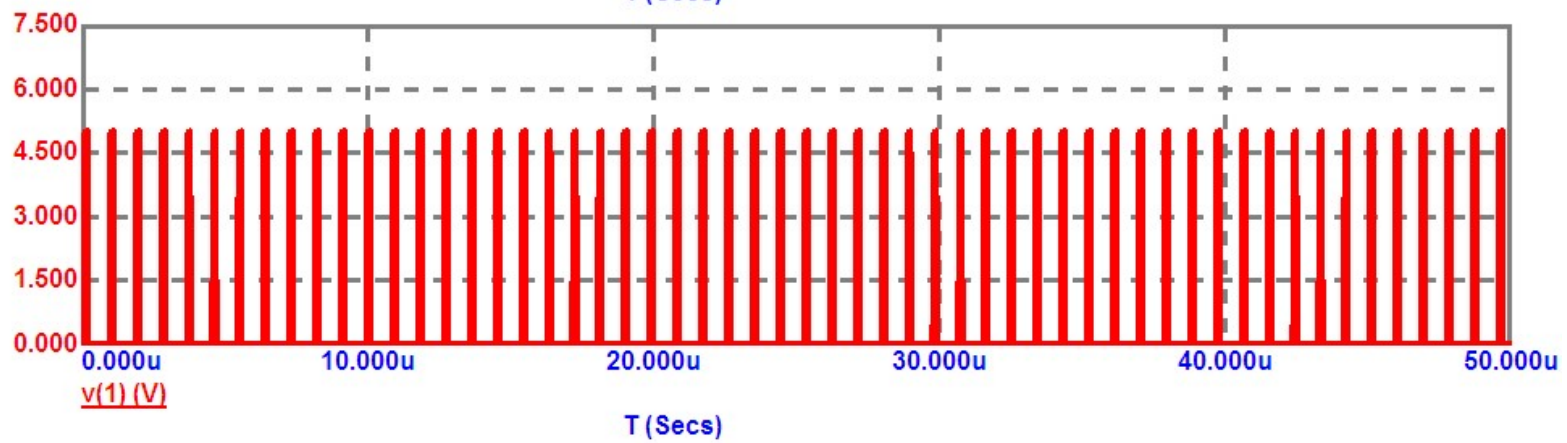
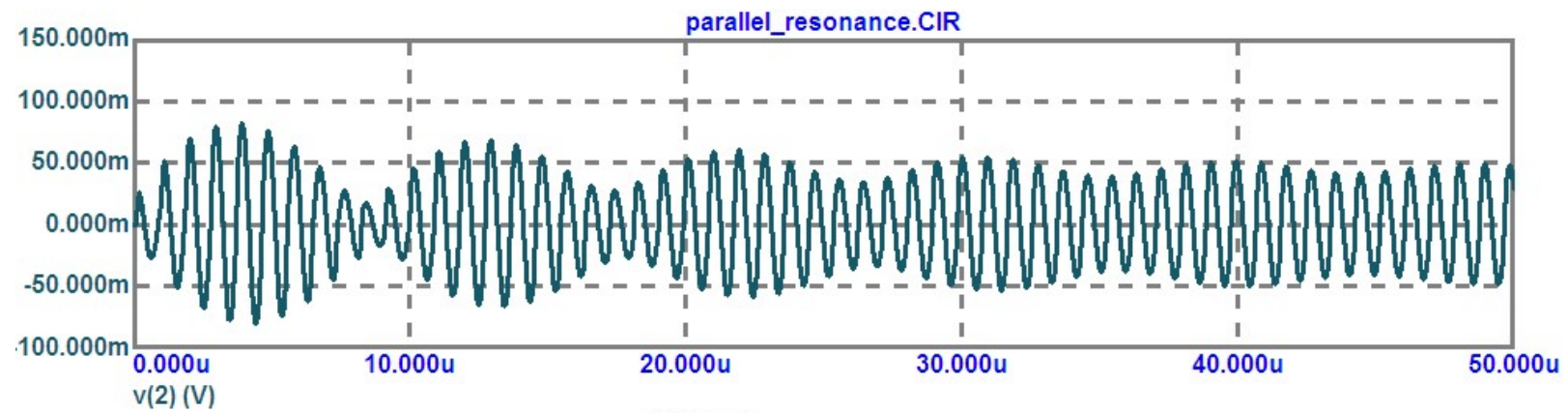


A small disturbance leads to oscillatory behavior

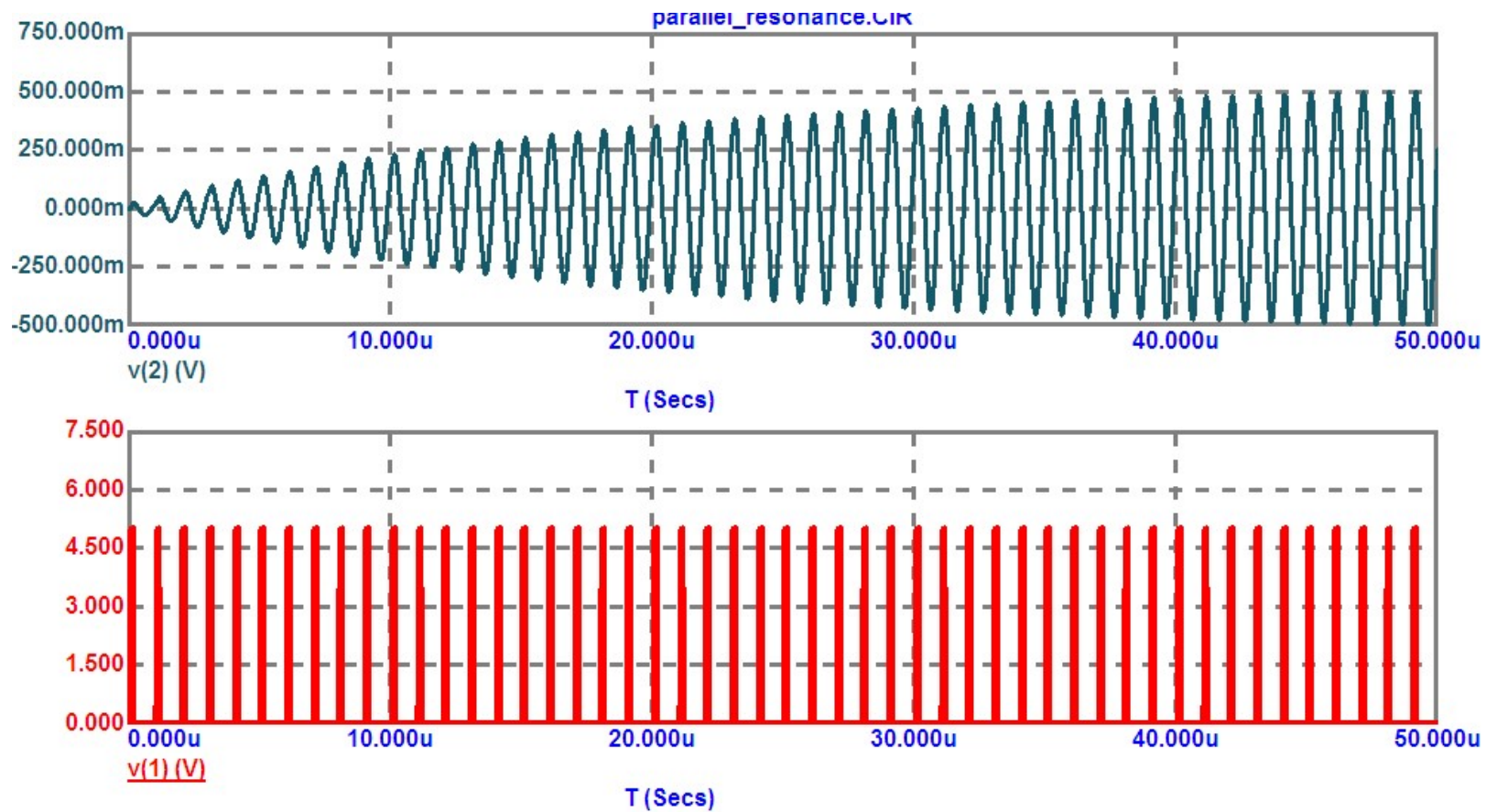


$$T = 1.1\mu\text{s}$$



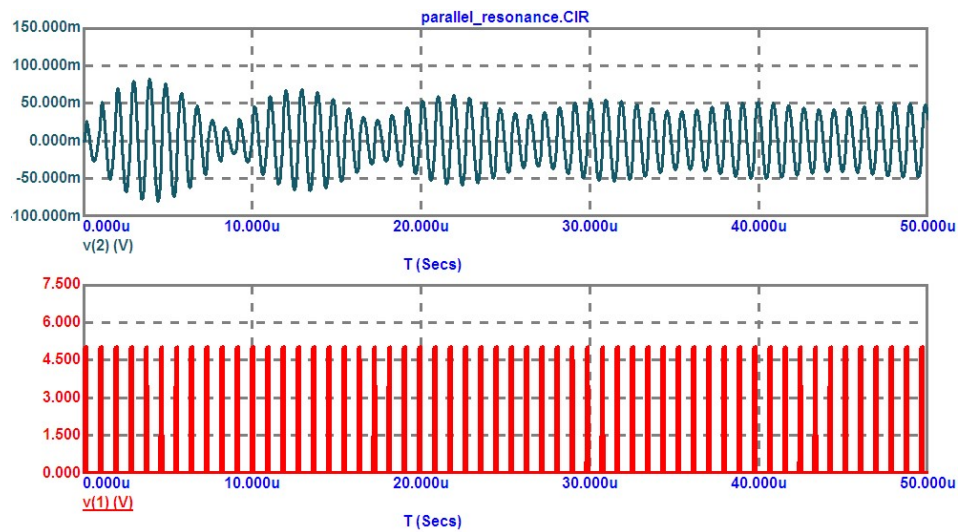


$$T = 0.9\mu\text{s}$$

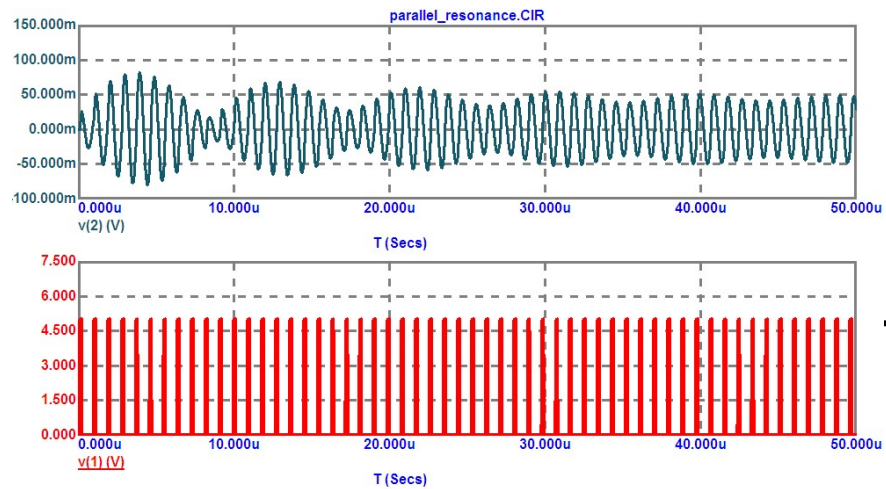


$$T = 1\mu\text{s}$$

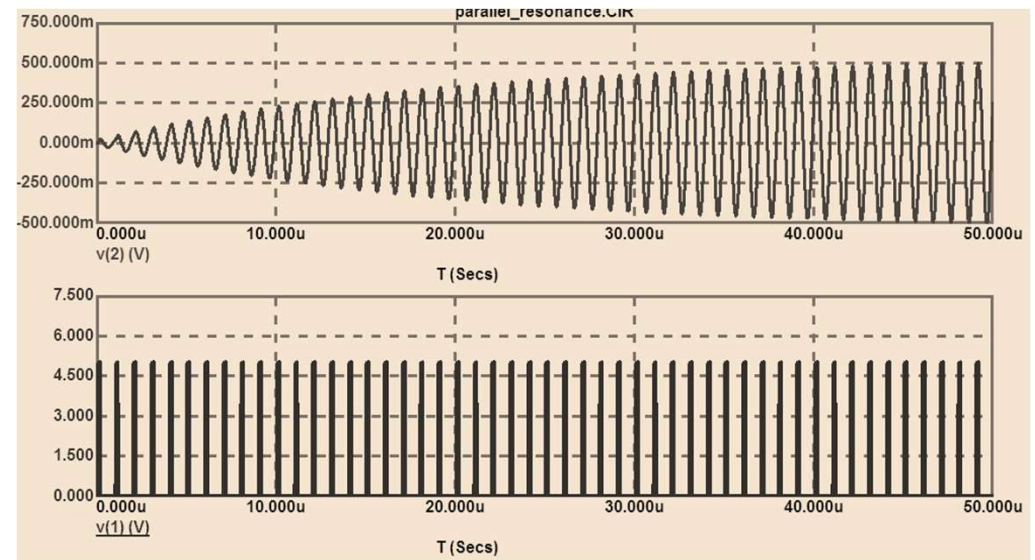
The amplitude is 10 times larger even though input magnitude is same !



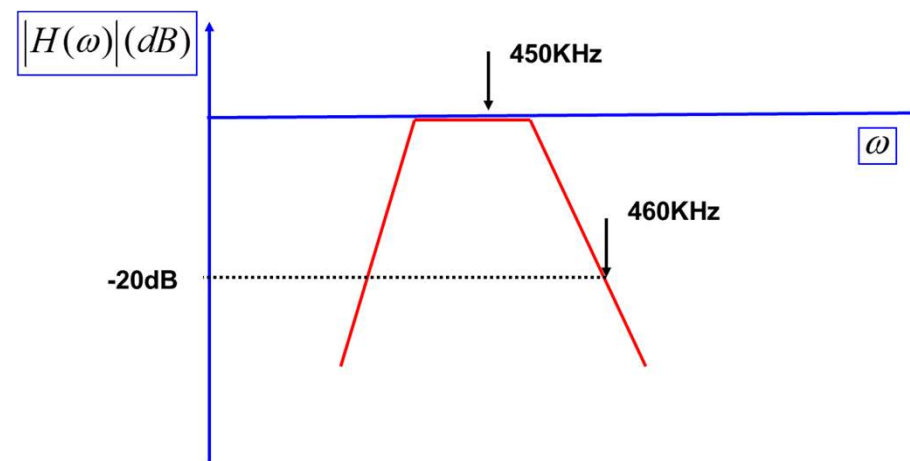
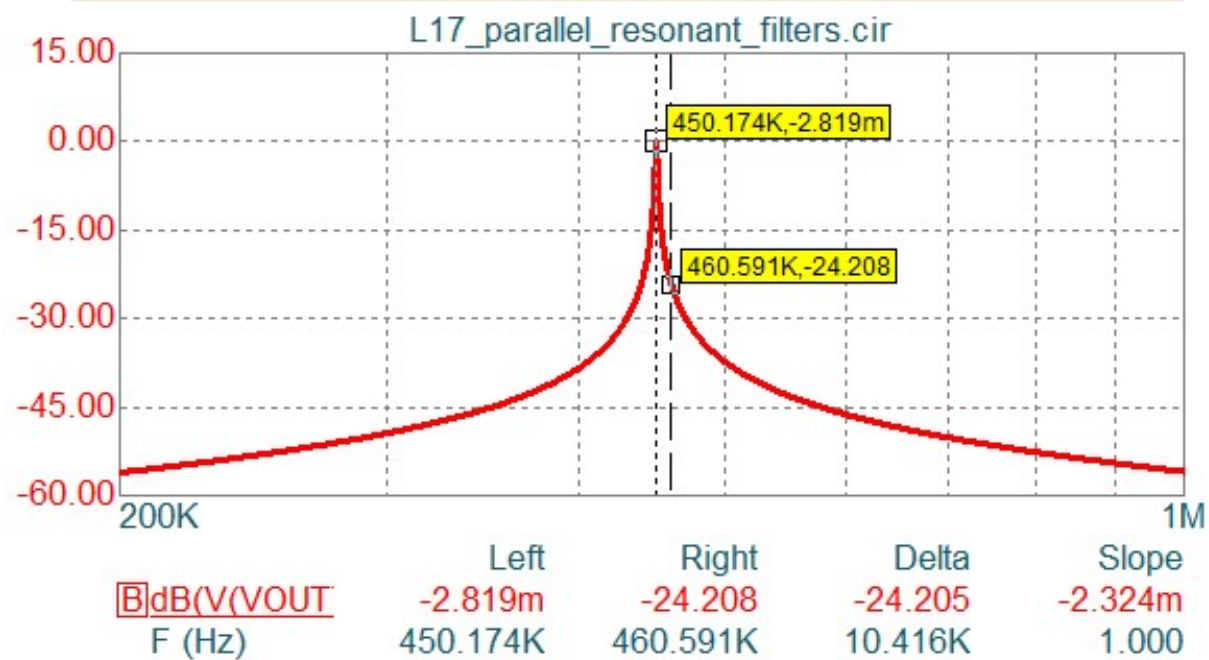
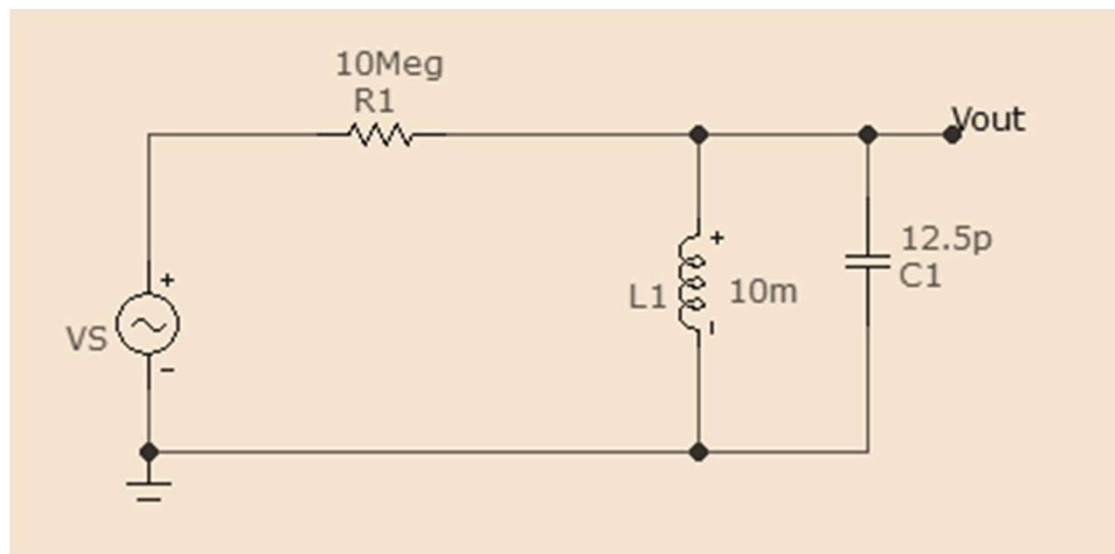
$T = 0.9\mu s$

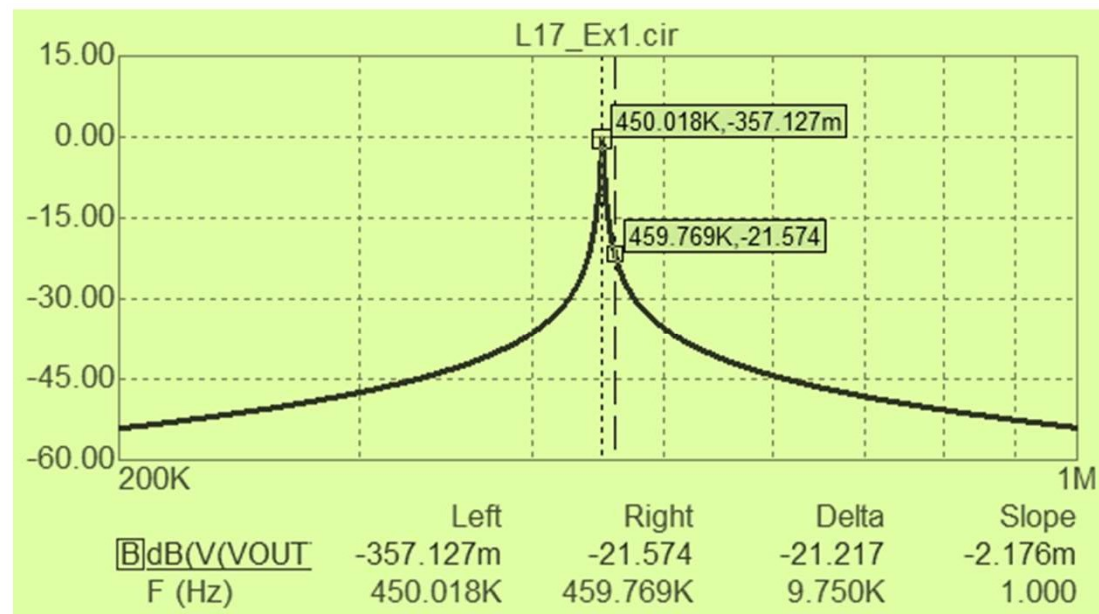
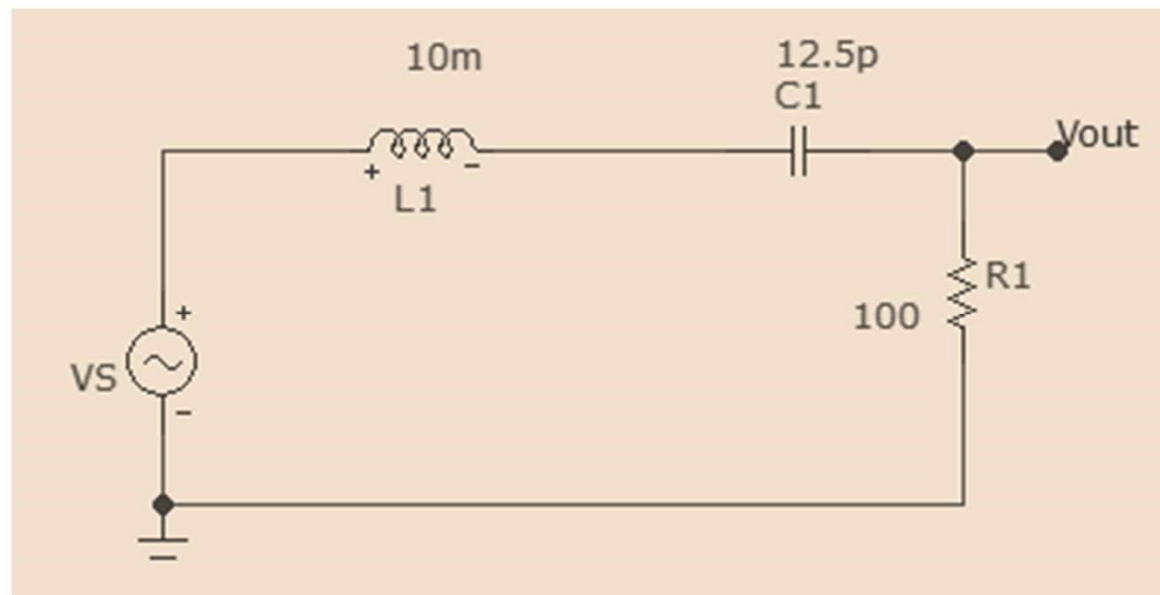


$T = 0.9\mu s$



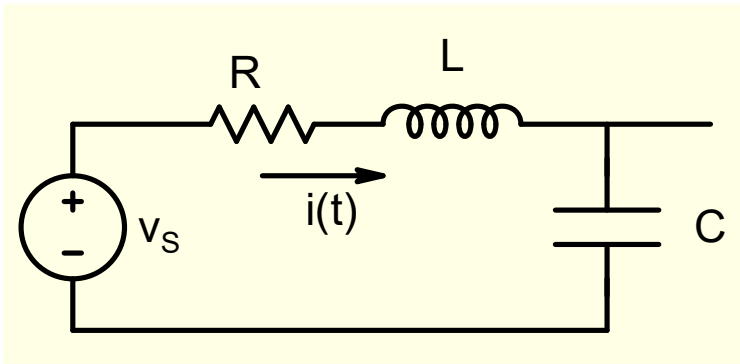
$T = 1\mu s$





## Series Resonant Circuit

In this series resonant circuit, current reaches a peak at a condition in which capacitive and inductive reactance cancel each other to give rise to a purely resistive circuit



$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

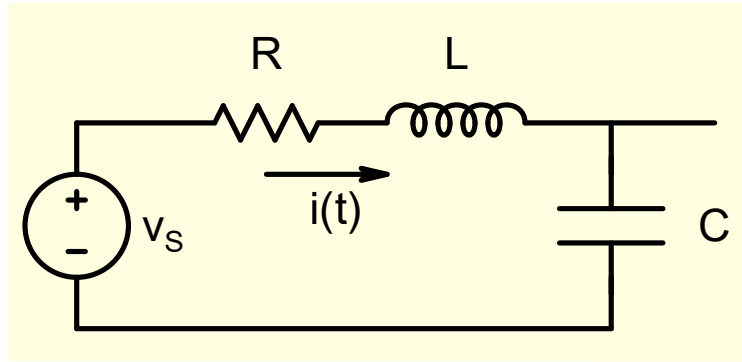
Resonant frequency:

$$j\omega_o L - j\frac{1}{\omega_o C} = 0 \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

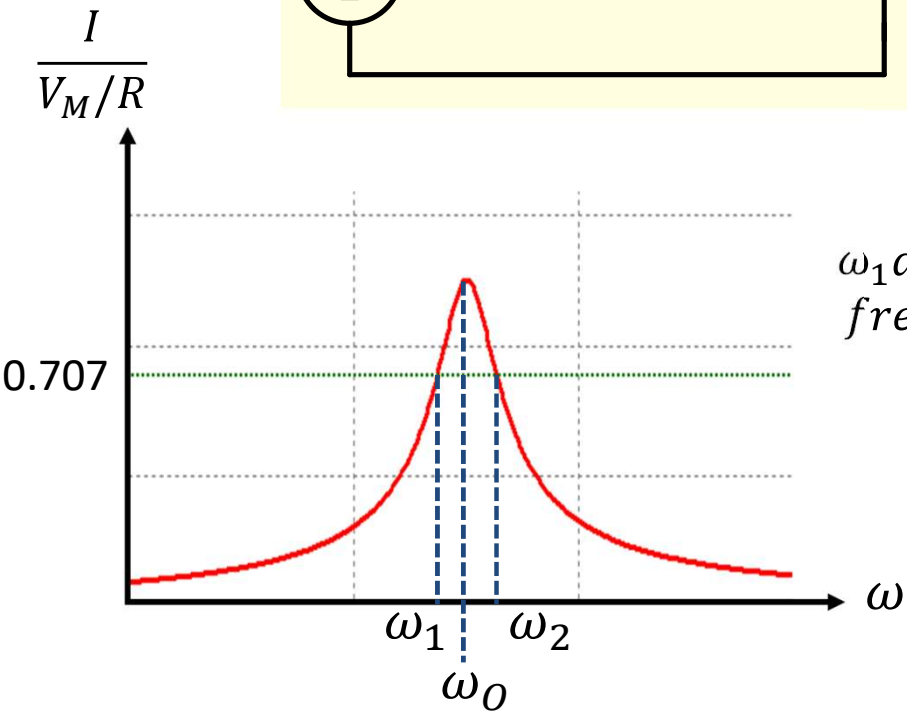
$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_{eq} = R$$

Current and voltage are in phase (power factor is unity) and current is maximum !



$$|I(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$



$$|I(\omega_1)| = \frac{V_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$|I(\omega_2)| = \frac{V_m}{\sqrt{R^2 + (\omega_2 L - \frac{1}{\omega_2 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

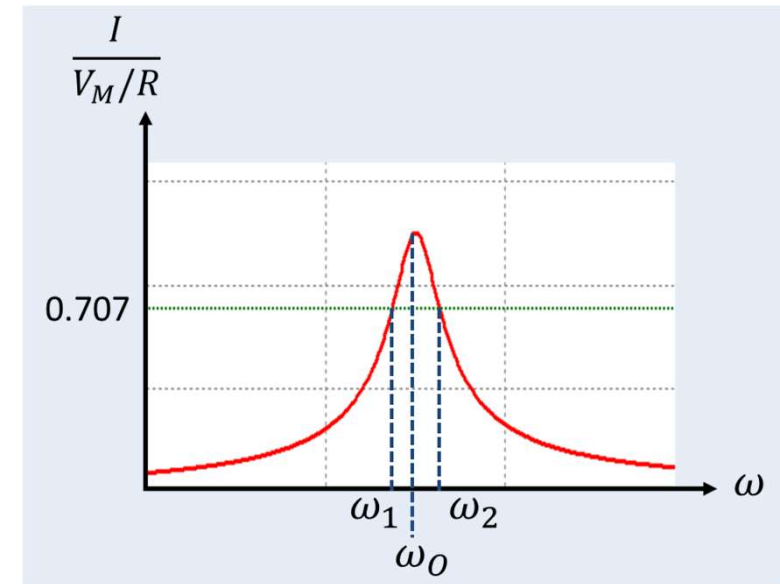


$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$



**Quality (Q) factor: Sharpness of resonance**

$$Q = \frac{\omega_o}{B} = \frac{\omega_o}{\Delta\omega} = \frac{\omega_o L}{R}$$

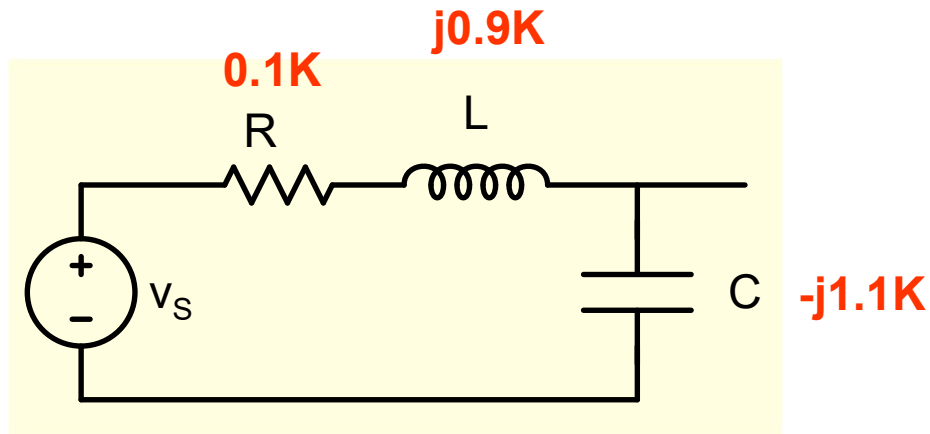
$$Q = 2\pi \frac{\text{Peak Stored Energy}}{\text{Energy dissipated in one period at resonance}}$$

$$Q = 2\pi \times \frac{\frac{1}{2} L \times I_m^2}{\frac{1}{2} I_m^2 R \times T_o} = \frac{\omega_o L}{R}$$

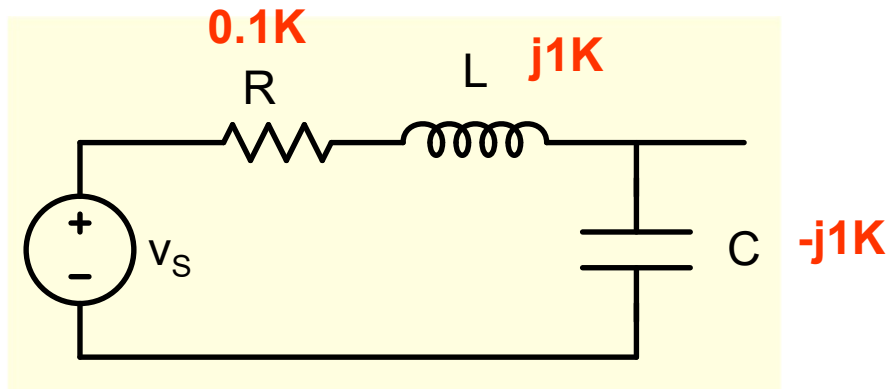
$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow Q = \frac{1}{\omega_o C R}$$



$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$$



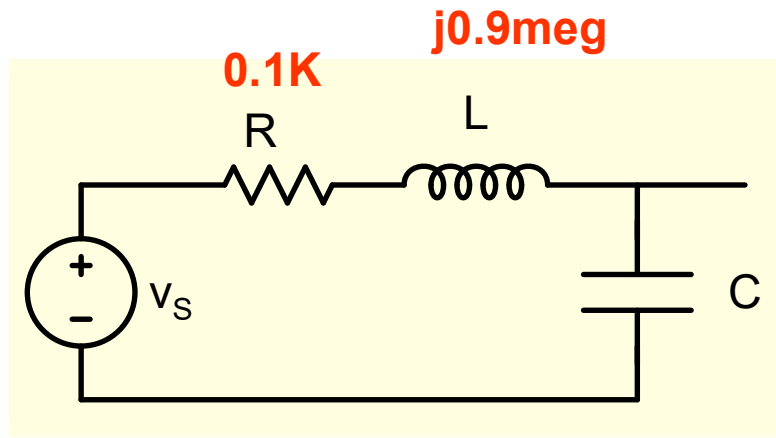
$$Z = 0.1K - j0.1K$$



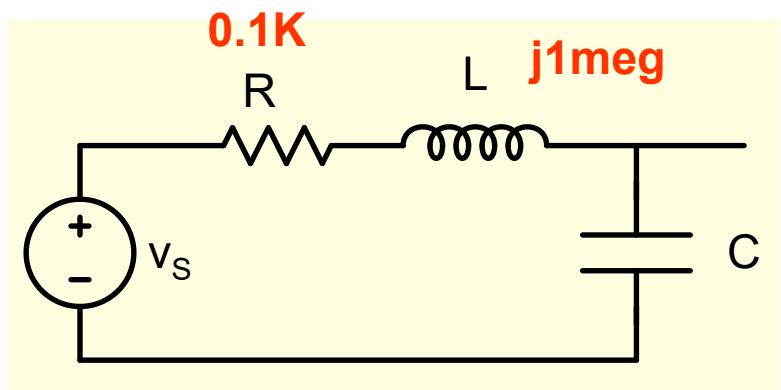
$$Z = 0.1K$$

Not very large change in impedance as we approach resonance !

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$$



$Z = 0.1K - j0.1meg$

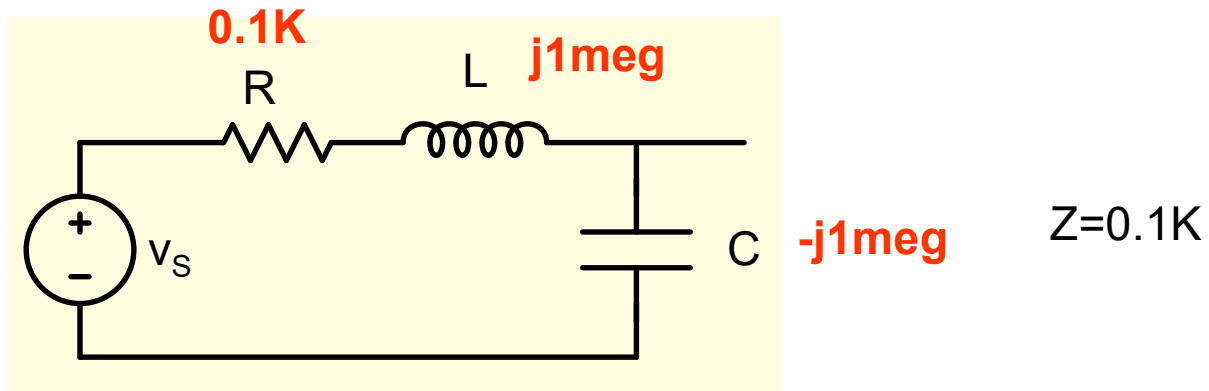
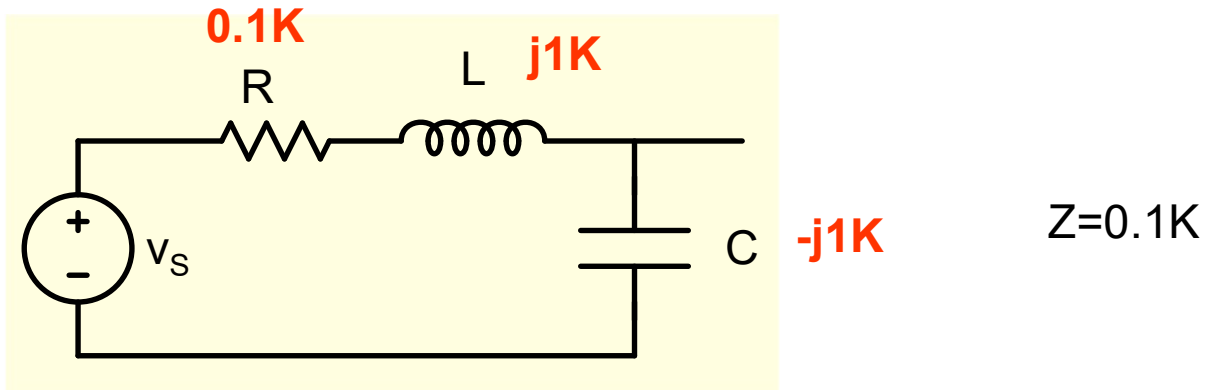


$Z = 0.1K$

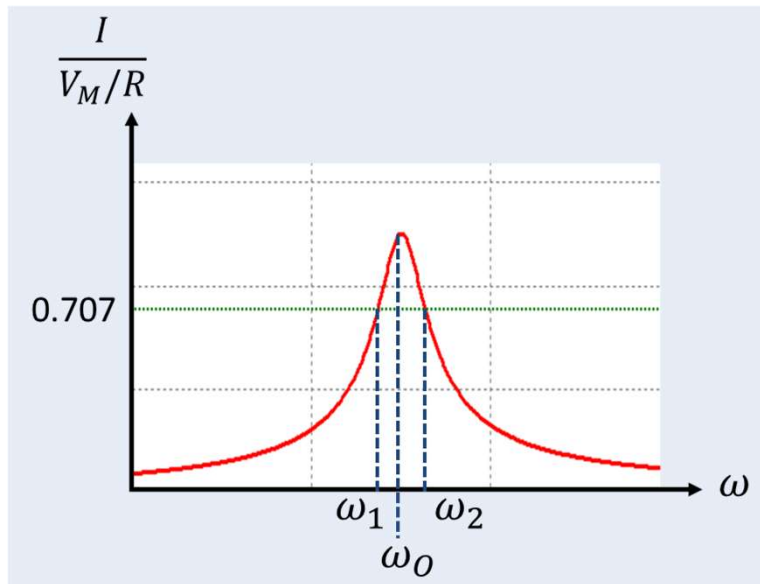
very large change in impedance as we approach resonance ! Implying high quality factor



## Quality factor Q



$$Q = \frac{\omega_o L}{R} \text{ or } Q = \frac{1/\omega_o C}{R}$$



$$Q = \frac{\omega_0 L}{R}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

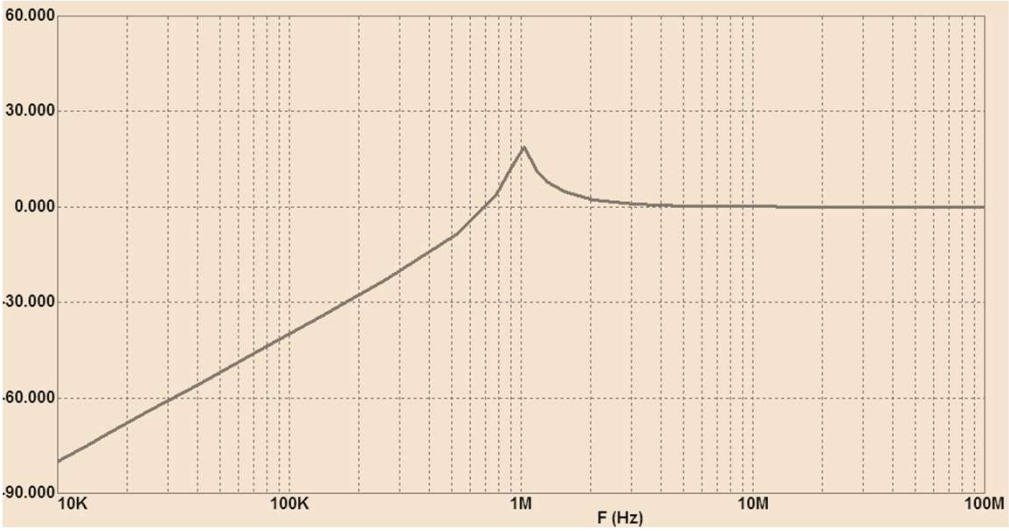
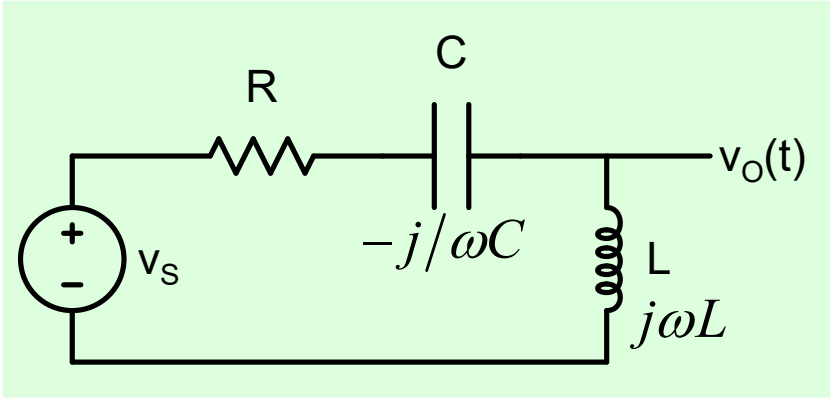
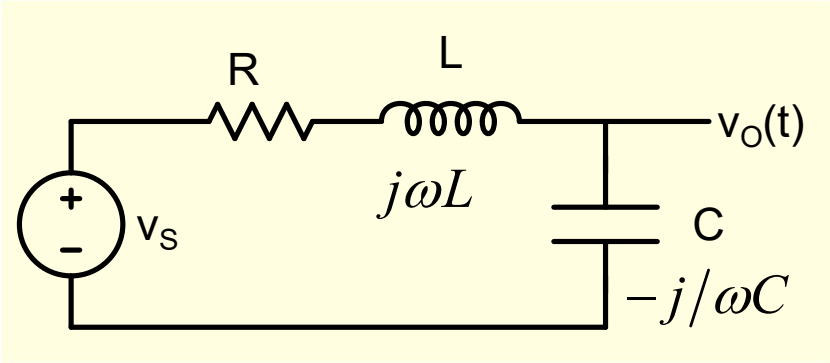
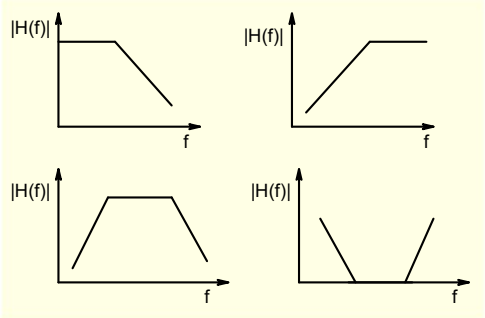
$$Q = \frac{\omega_0}{B} = \frac{\omega_0}{\Delta\omega}$$

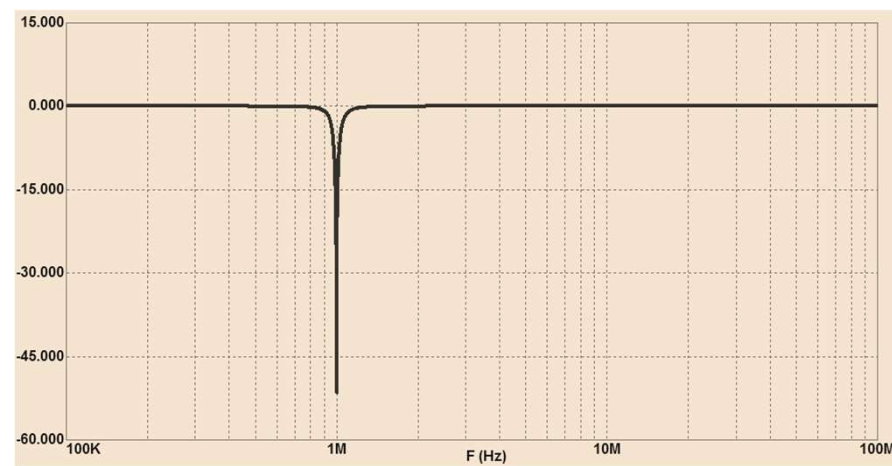
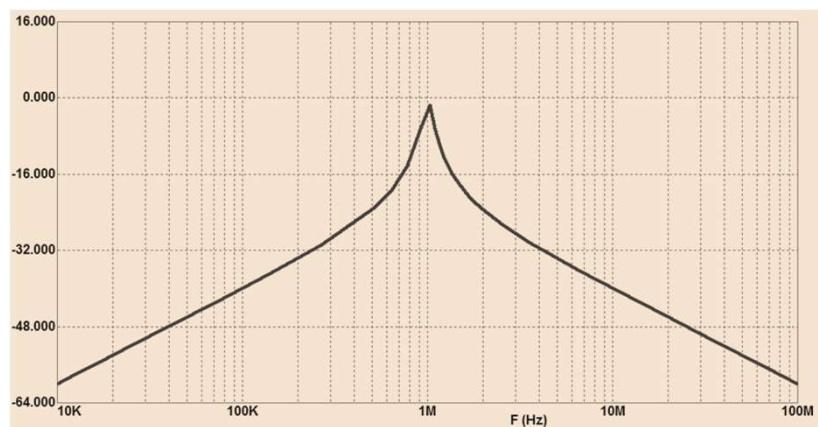
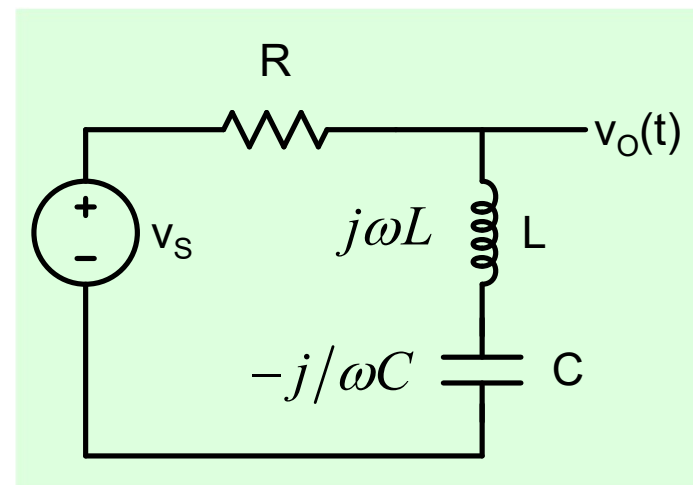
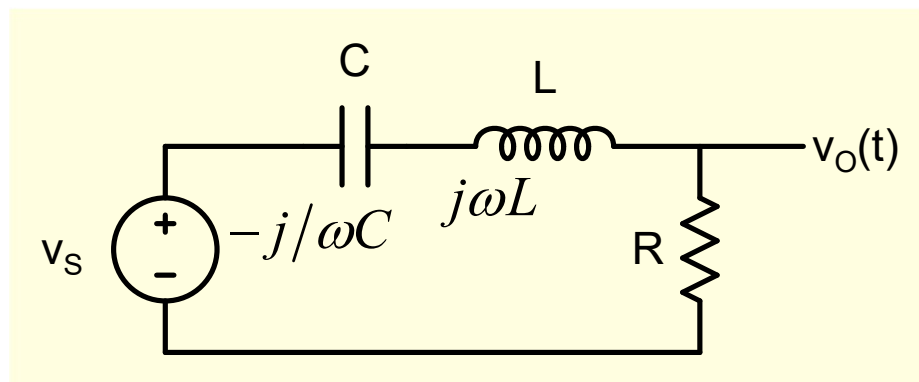
Hence Q represents sharpness of resonance

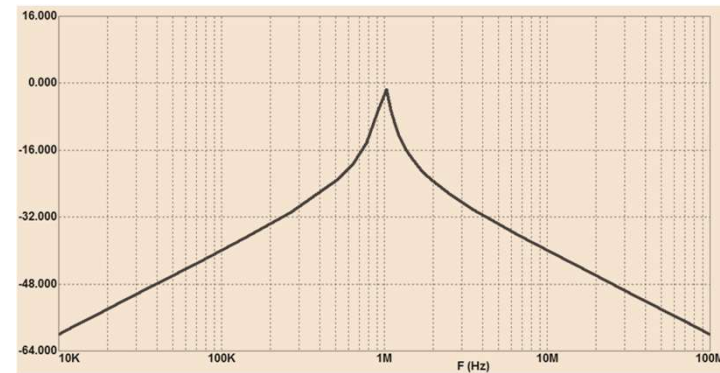
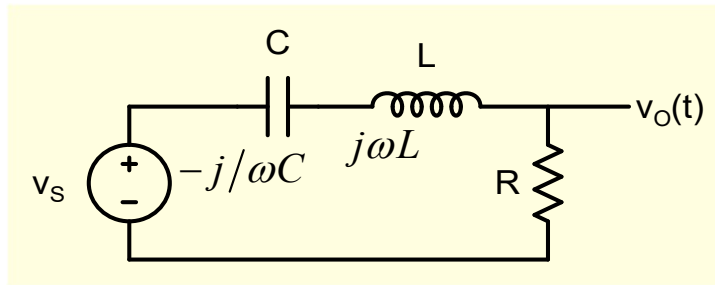
For high Q circuits:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \quad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

R-L-C filters







How much Q do we need to pass 450KHz but reject 460KHz by 20dB?

$$|H(\omega)| = \left| \frac{V_o(\omega)}{V_{IN}(\omega)} \right| = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Assuming  $V_{IN} = 1V$  and noting that  $Q = \omega_0 L/R$

$$|V_o(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega_0^2}{\omega^2} Q^2 \left(\frac{\omega^2}{\omega_0^2} - 1\right)^2}}$$

For  $\omega = \omega_0$ ,  $V_o = 1$  so the signal simply passes through !

$$\omega_0 = 2 \times \pi \times 450 \times 10^3 = 2.8 \times 10^6 \text{ rad / s}$$

$$|V_o(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega_o^2}{\omega^2} Q^2 (\frac{\omega^2}{\omega_o^2} - 1)^2}}$$

$$|V_o(\omega)| \cong \frac{\omega_o^2}{Q \times (\omega^2 - \omega_o^2)}$$

$$|V_o(\omega)| \cong \frac{\omega_o}{2Q \times (\omega - \omega_o)}$$

For an attenuation of -20dB or  $10^{-1}$  at  $\omega - \omega_o = 62.8$  Krad/s

$$Q \cong 226.3$$

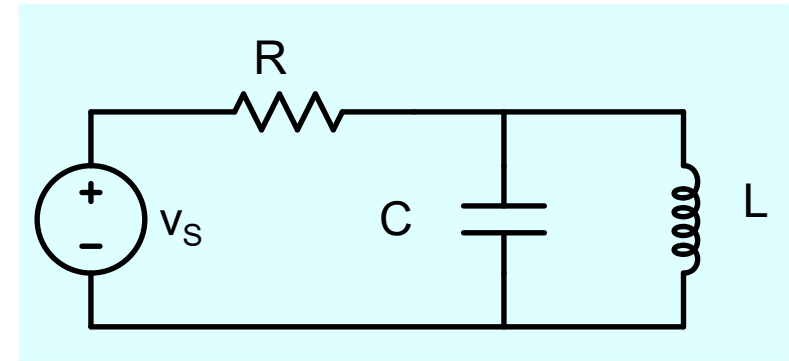
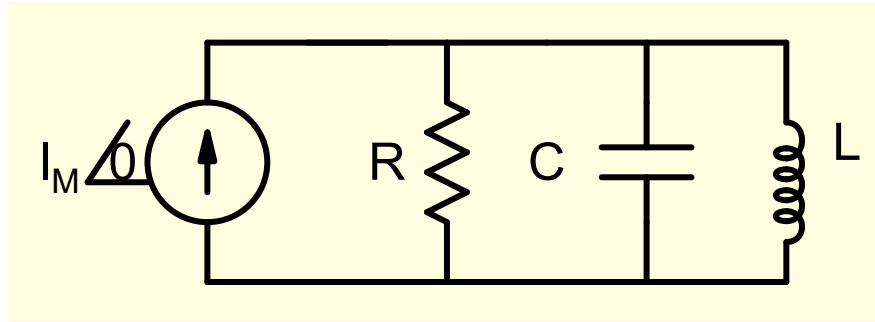
Example: for  $Q = 226.3$  at 450KHz

$$Q = \frac{\omega_o L}{R}$$

$$\text{Suppose } L = 10^{-3} H; \Rightarrow R = 12.5 \Omega; \Rightarrow C = 125 pF$$



## Parallel Resonance



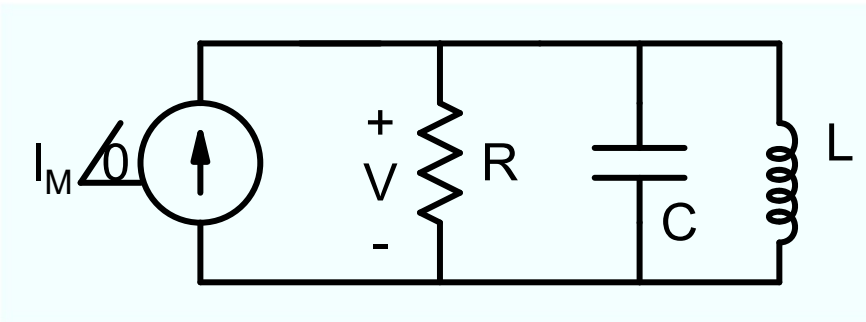
$$Y_{eq} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$$

Resonant frequency:

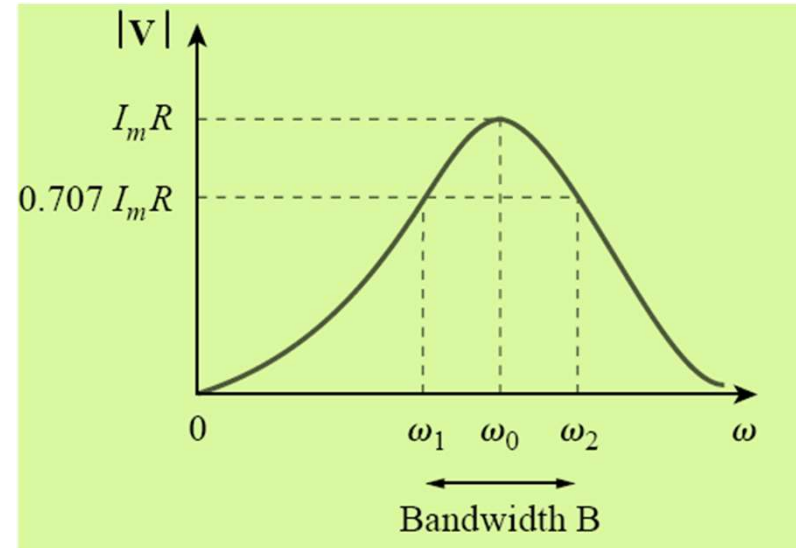
$$j\omega_o C - j\frac{1}{\omega_o L} = 0 \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_{eq} = R$$



$$|V(\omega)| = \frac{I_m R}{\sqrt{1 + \frac{R^2 C^2}{L^2} \left( \omega L - \frac{1}{\omega C} \right)^2}}$$



$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

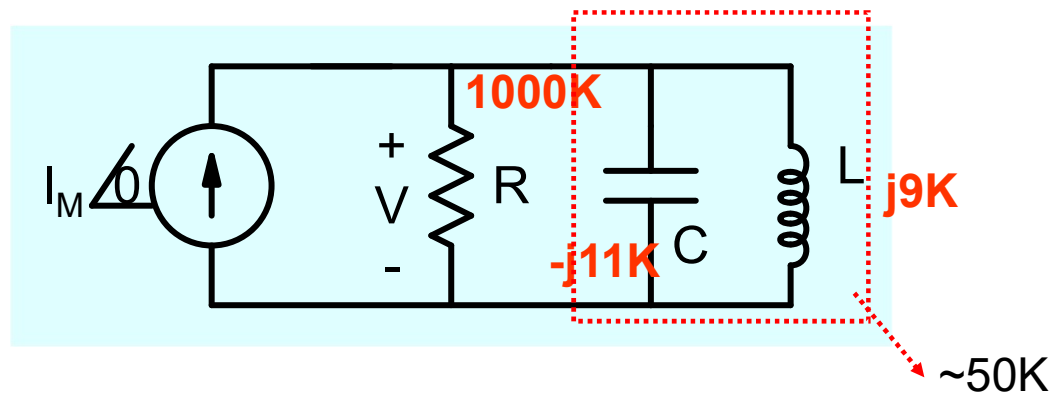
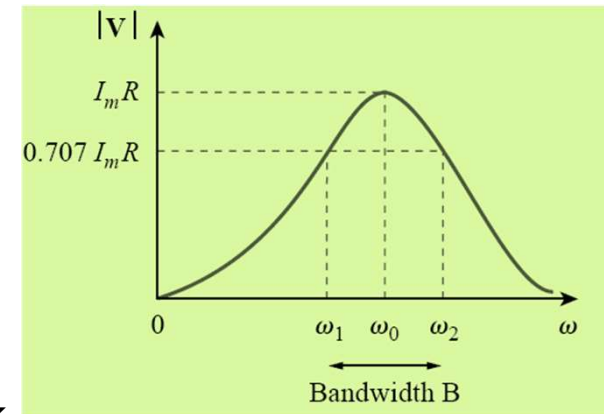
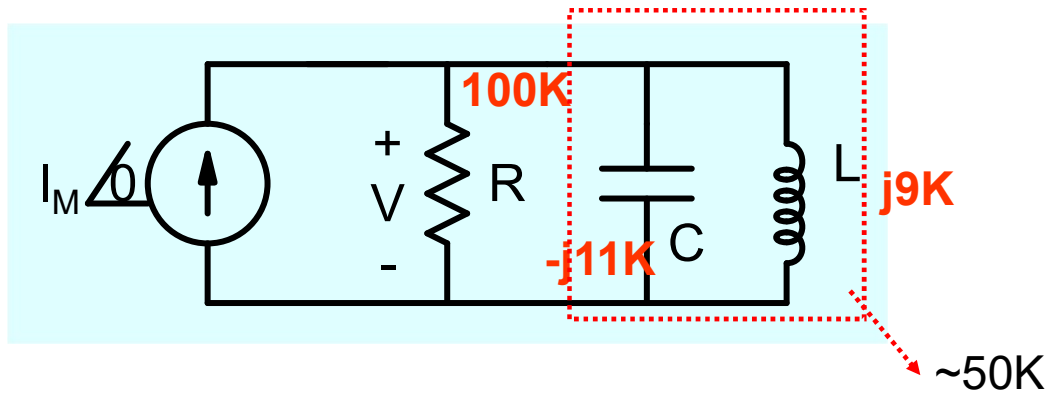
$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

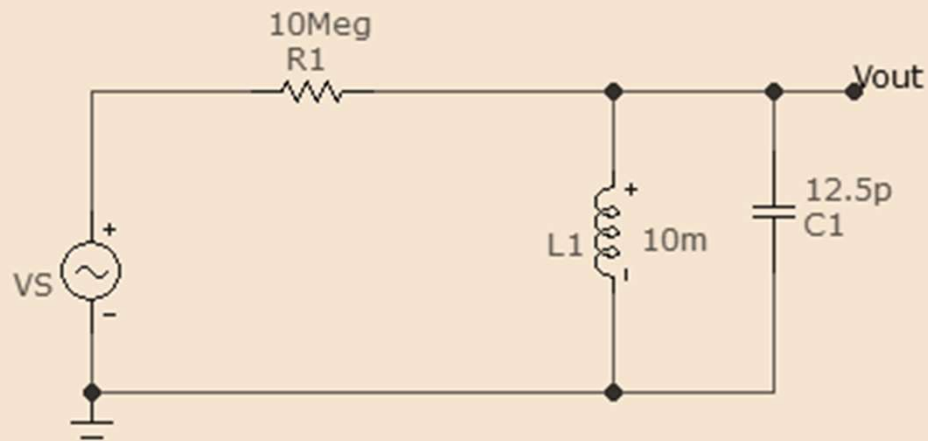
For high Q:

$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}, \quad \omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

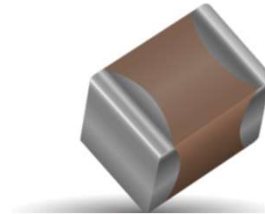
$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \quad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

Why is  $Q = \frac{R}{\omega L}$  for parallel resonance ?



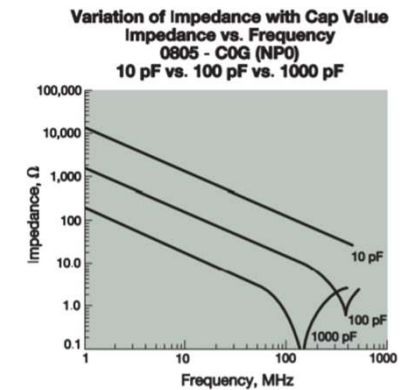
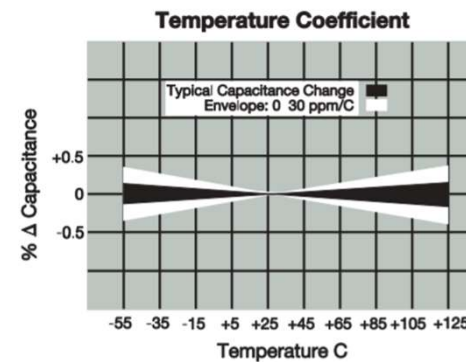
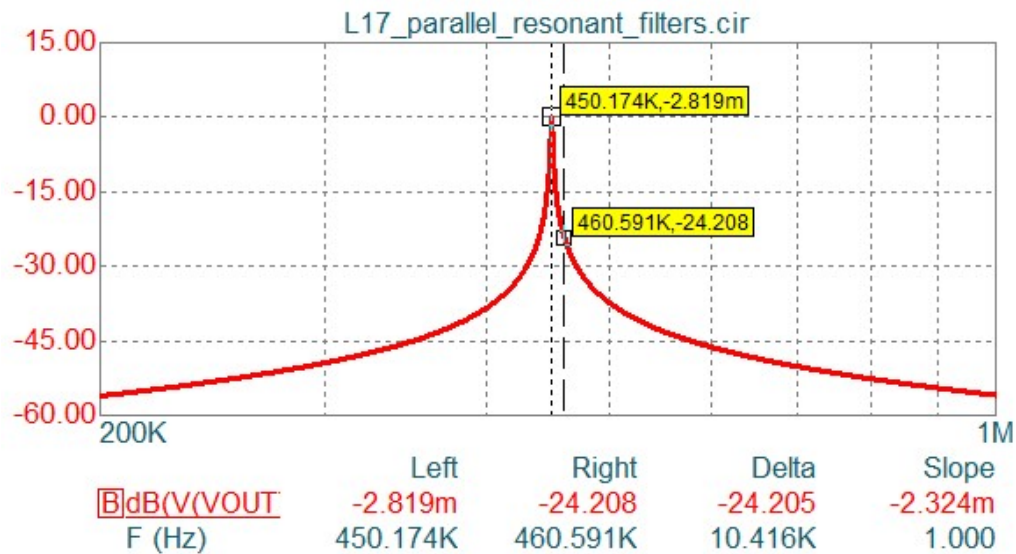


## C0G (NP0) Dielectric General Specifications

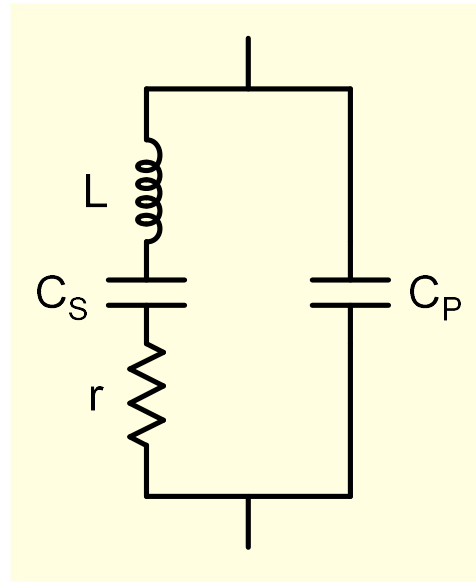


C0G (NP0) is the most popular formulation of the "temperature-compensating," EIA Class I ceramic materials. Modern C0G (NP0) formulations contain neodymium, samarium and other rare earth oxides.

C0G (NP0) ceramics offer one of the most stable capacitor dielectrics available. Capacitance change with temperature is  $0 \pm 30 \text{ ppm}/^\circ\text{C}$  which is less than  $\pm 0.3\%$  C from  $-55^\circ\text{C}$  to  $+125^\circ\text{C}$ . Capacitance drift or hysteresis for C0G (NP0) ceramics is negligible at less than  $\pm 0.05\%$  versus up to  $\pm 2\%$  for films. Typical capacitance change with life is less than  $\pm 0.1\%$  for C0G (NP0), one-fifth that shown by most other dielectrics. C0G (NP0) formulations show no aging characteristics.

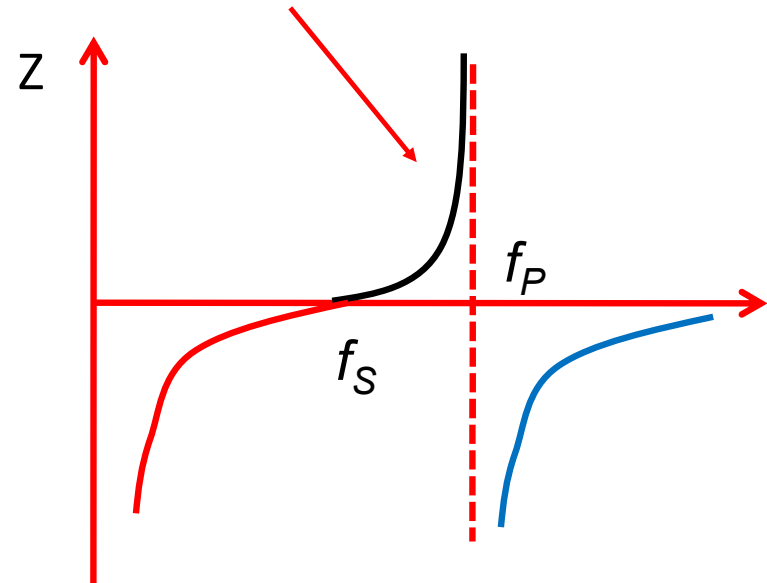


## Piezoelectric Quartz Crystal



$$C_S \gg C_P$$

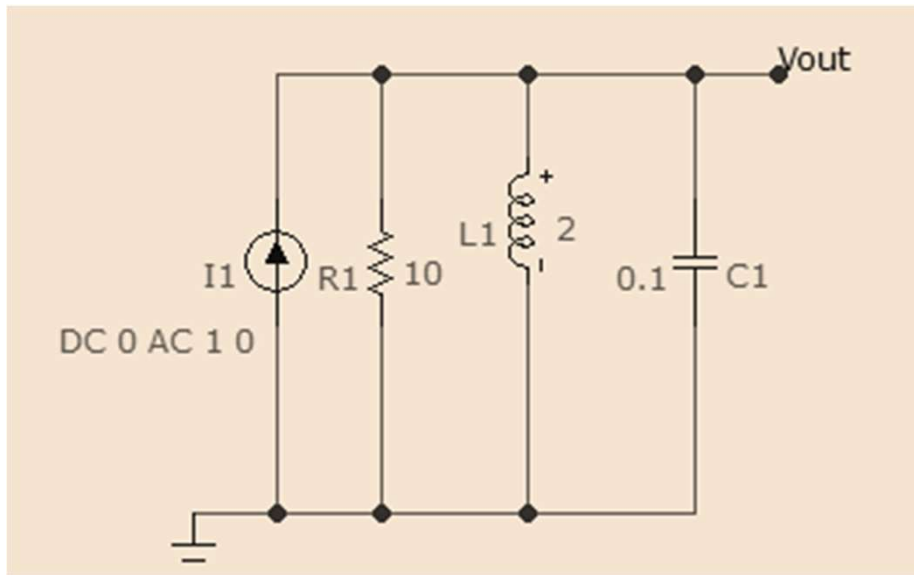
Inductive over a very narrow range



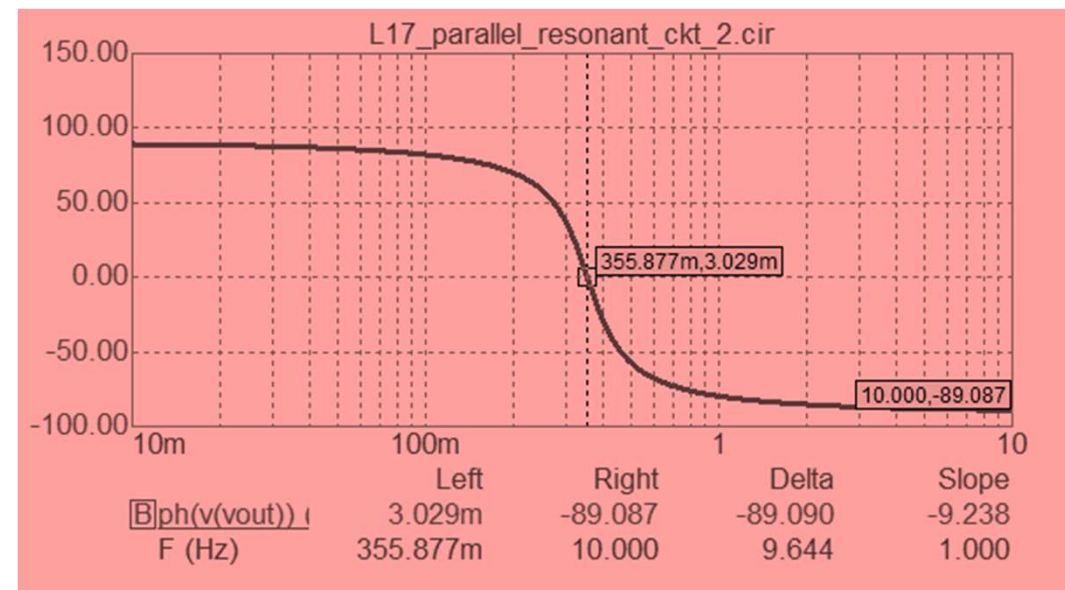
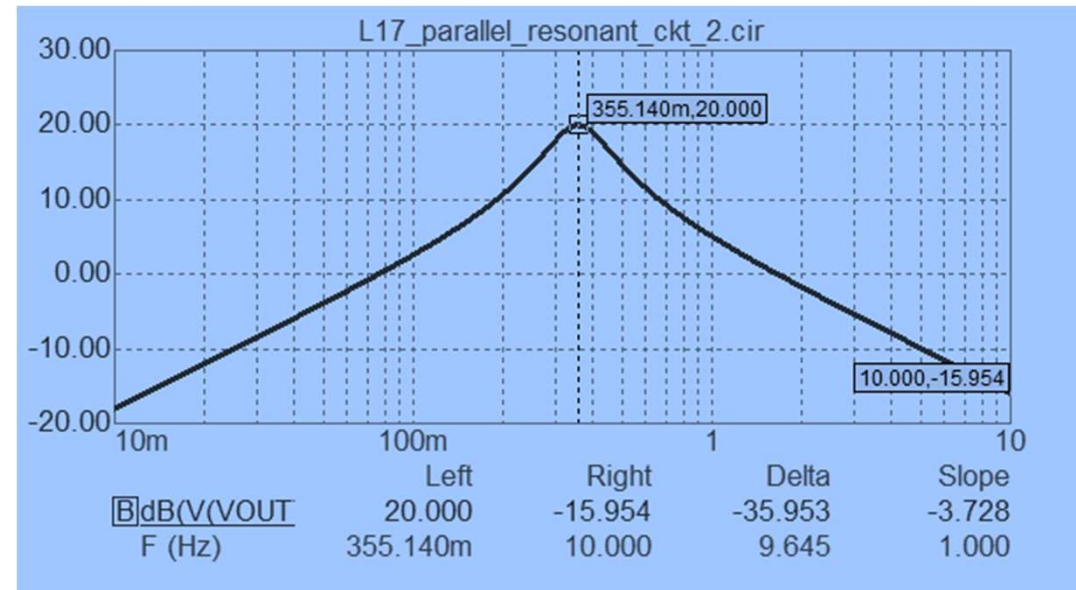
Q can be very high  $\sim 10^4 - 10^6$

PARAMETERS	
ABRACON P/N	ABLS Series
Frequency	3.579545 MHz to 75 MHz
Operation Mode	AT cut (Fundamental or 3rd OT) or BT cut (See options) 3.579545MHz - 24.0MHz (Fundamental) (Standard) 24.01 - 75.00MHz (3rd- Overtone) (Standard) 24.01MHz - 50.00MHz (Fund. AT or BT) (Option. See 2.1)
Operating Temperature	0°C to + 70°C (see options)
Storage Temperature	- 55°C to + 125°C
Frequency Tolerance at +25°C	± 50 ppm max. (see options)
Frequency Stability over the Operating Temp. (Ref to +25°C)	± 50 ppm max. (see options)
Equivalent Series Resistance	See Table 1
Shunt Capacitance $C_0$	7pF max.
Load Capacitance $C_L$	18pF (see options)
Drive Level	1 mW max., 100µW typical
Aging at 25°C ± 3°C Per Year	± 5ppm max.
Insulation Resistance	500 MΩ min at 100Vdc ± 15V
Spurious Responses	-3dB max.
Drive level dependency (DLD)	from 1µW to 500µW (minimum 7 points tested)

# Resonant Frequency

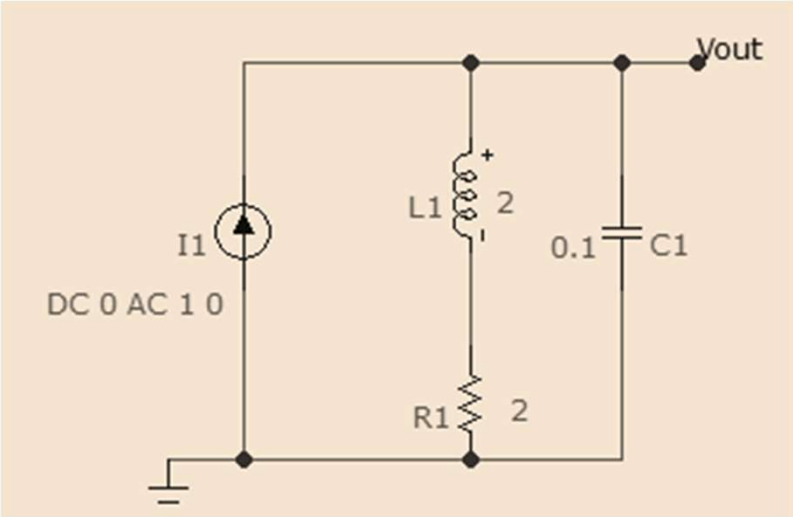


$$f_o = \frac{1}{2\pi\sqrt{LC}}$$





# What is the resonant frequency (unity power factor) ?

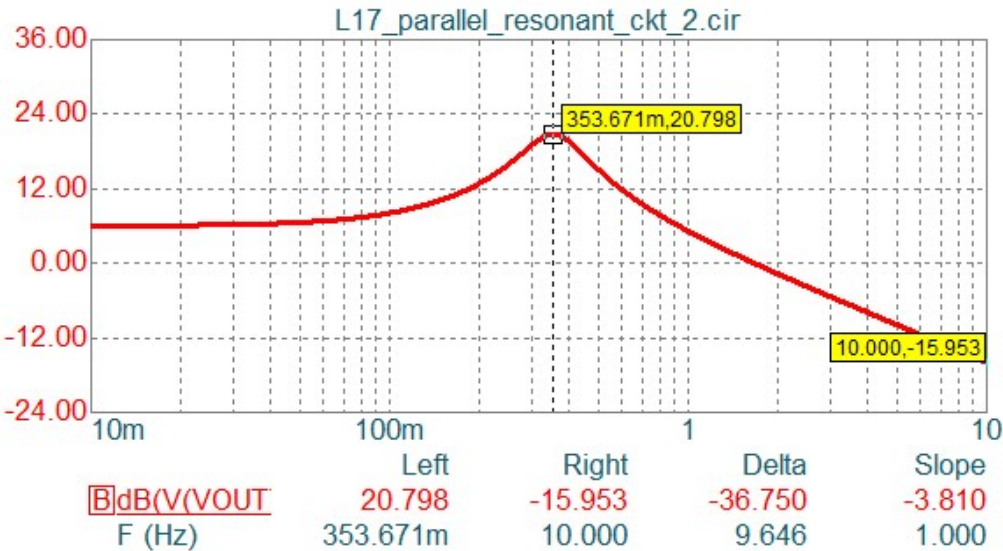
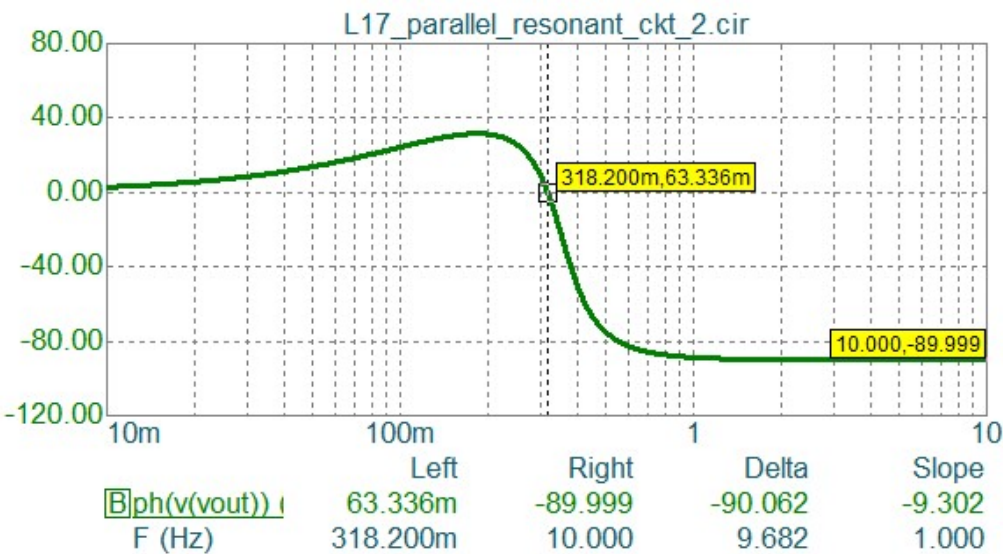


$$Y = j\omega 0.1 + \frac{1}{2 + j2\omega}$$

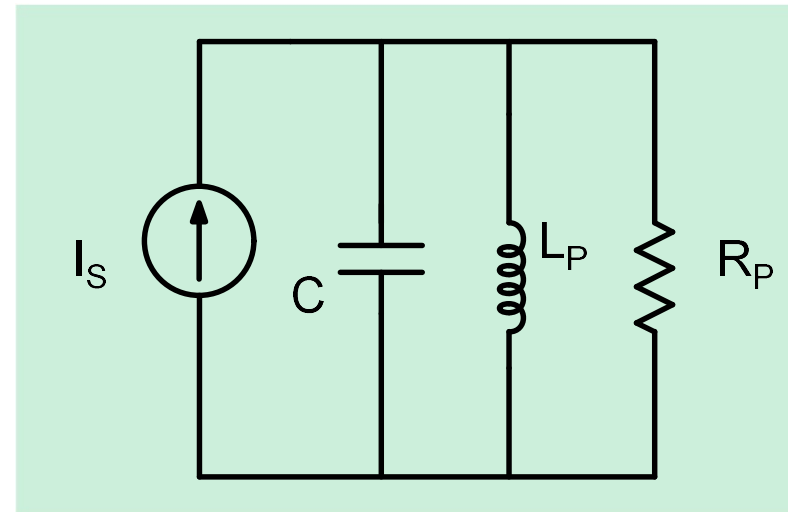
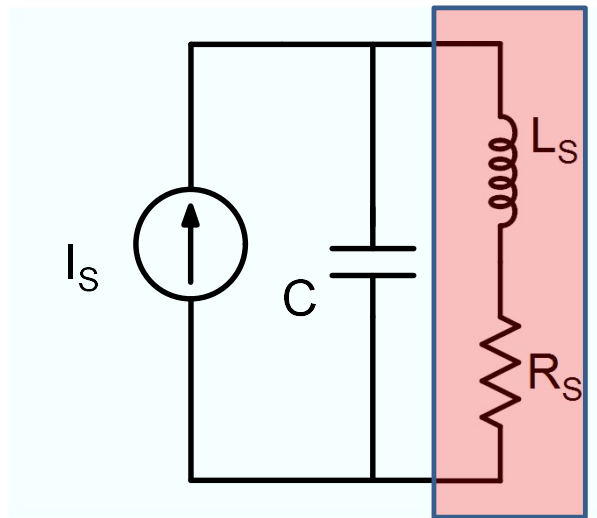
For unity power factor, imaginary part = 0

$$Y = \frac{2}{4 + 4\omega^2} + j\omega 0.1 - \frac{j2\omega}{4 + 4\omega^2}$$

$$0.1\omega_o - \frac{2\omega_o}{4 + 4\omega_o^2} = 0 \Rightarrow \omega_o = 2 \text{ rad/s}$$







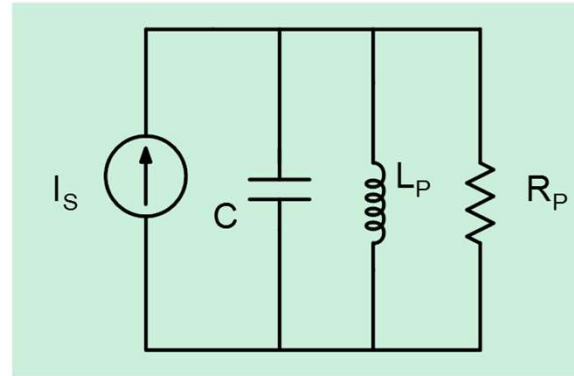
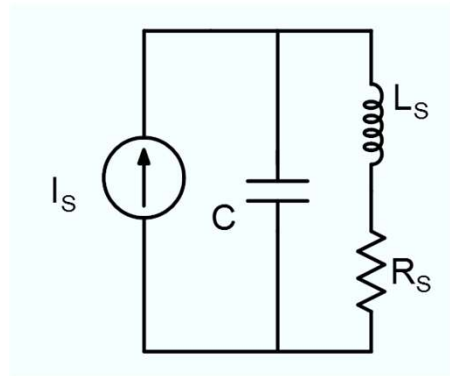
$$Z_S = R_S + j\omega L_S$$

$$Y_S = \frac{1}{Z_S} = \frac{1}{R_S + j\omega L_S}$$

$$Y_S = \frac{R_S - j\omega L_S}{R_S^2 + \omega^2 L_S^2} = \frac{R_S}{R_S^2 + \omega^2 L_S^2} + \frac{1}{j\omega} \times \frac{\omega^2 L_S}{R_S^2 + \omega^2 L_S^2} = \frac{1}{R_P} + \frac{1}{j\omega L_P}$$

$$R_P = R_S + \frac{\omega^2 L_S^2}{R_S}$$

$$L_P = L_S + \frac{R_S^2}{\omega^2 L_S}$$



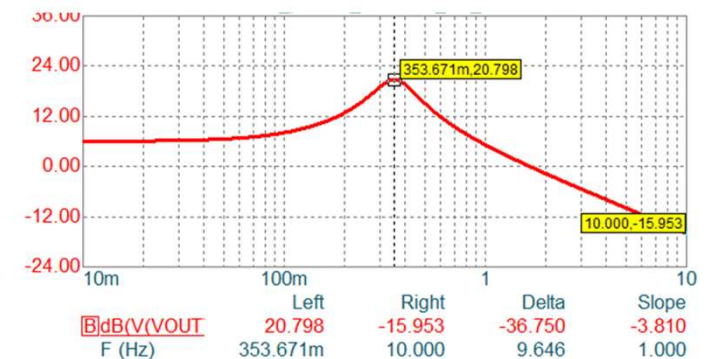
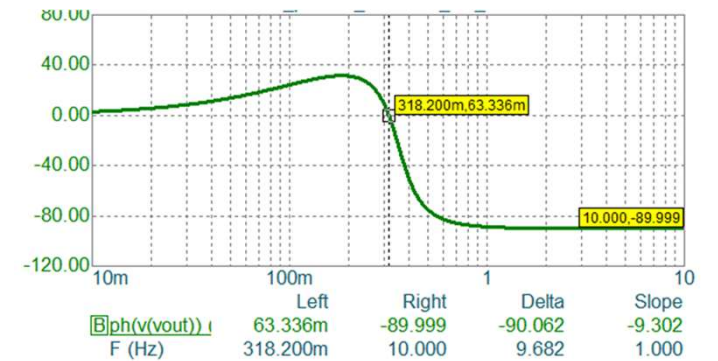
$$R_P = R_S + \frac{\omega^2 L_S^2}{R_S}$$

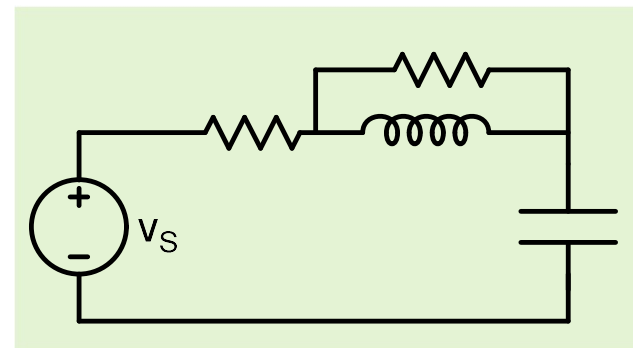
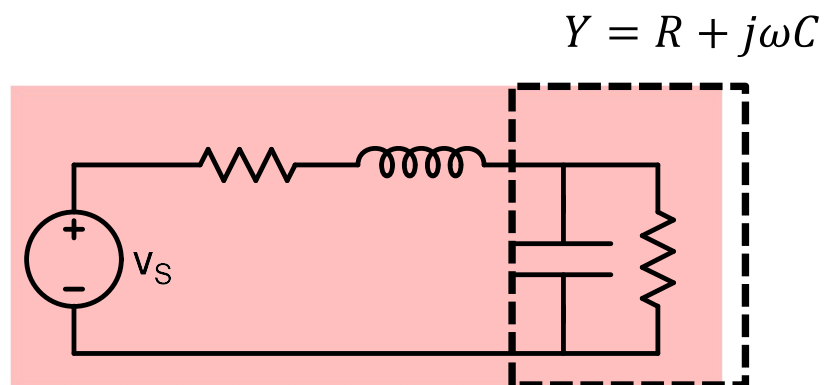
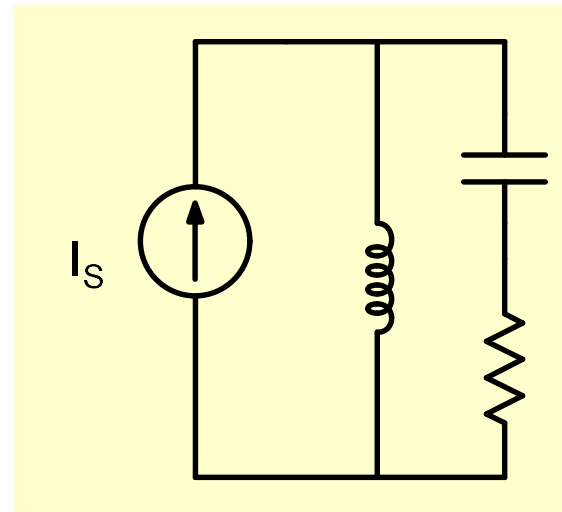
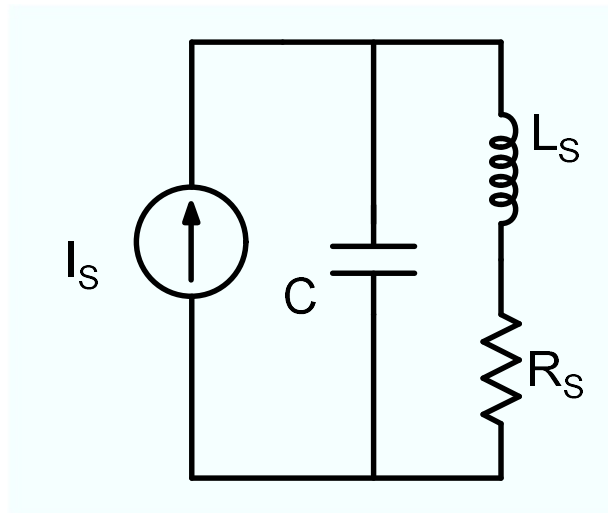
$$L_P = L_S + \frac{R_S^2}{\omega^2 L_S}$$

unity power factor frequency :  $\omega_P L_P = \frac{1}{\omega_P C}$

$$Y = \frac{1}{R_P} + j\omega C - \frac{j}{\omega_P L_P}$$

$Y_{min}$  is reached at a higher frequency than  $f_P$





$$Z_s = \frac{1}{Y} = \frac{1}{R + j\omega C} = \frac{R - j\omega C}{R^2 + \omega^2 C^2}$$

**Resonance Allows Highly Efficient Implementation Of Highly Selective Filters**