$+$ $\times$

$-$ $\div$

- **Natural number**: (no fraction or decimal)

  Sometime, $N = 1, 2, 3, 4 \ldots \infty$

  Generally, $N_0 = 0, 1, 2, 3, 4, \ldots \infty$

(no fraction or decimal)

- **Integers**: Integers, $Z = -\infty, \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots, \infty$

- **Factors**: It is combination of multiple which lead to end number.

  factor of 12 : (1, 12), (2, 6), (3, 4)

  (36) → (1, 36), (2, 18), (3, 12), (4, 9), (6)

- **Prime number**: It is the number which as only 2 factor. (1 and itself) (no decimal/fraction)

  1 is not prime only factor = 1
  2 is prime bc2 factor = 1, 2
  3 is prime bc2 factor = 1, 3
  5 is prime bc2 factor = 1, 5

- **Composite number**: It is number has more than 2 factor.

  Eg → 4 has factor 1, 2, 4
  6 has factor 1, 2, 3

(no fraction or decimal)

  1 is neither prime nor composite.

- **Rational number**: Thes are those number which which can be written as p/q.

  & $q \neq 0$.

Terminating decimal : 0·375 (Rational no)
Non-terminating recurring decimal : 0·66666...
Non-terminating & non-recurring decimal : $\sqrt{2} = 1.4142...$ (Irrational)

P, Q → integers
Rational number, Q = $\frac{2}{3}$, $\frac{5}{8}$

- **Irrational number** : These are numbers which are non-terminating or non-recuring decimal. It is square root of no. which is not not perfect square.
  eg :- $\sqrt{2} = 1.414213562...$
  like that $\sqrt{3}$, ; $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ are irrational no.

- **Whole number** : Whole no. are number from (0 to $+\infty$) (no fraction or decimal) It is if natural no. starts from 1.

- **Real number** : Any number which can be plotted on number line is real number. It contain both rational & irrational number.

- **Imaginary number** : Imaginary number are those which can't be solved and we imagine no.
  like, $\sqrt{-1} = i$
  than $i^2 = i \times i = \sqrt{-1} \times \sqrt{-1} = -1$
  here we imagine i is no. whose $\sqrt{\ }$ is -1.
  Eg → i, $3i$, bi, etc.

- **Complex number** : It is combination of real no. and imaginary number.
  Eg :- $\underbrace{3}_{\text{Real}} + \underbrace{7i}_{\text{imaginary}}$ = Complex no.

(Standard infinite set)

- Notations : (All numbers)
  - $N \rightarrow$ Natural number
  - $W \rightarrow$ Whole number
  - $R \rightarrow$ Real number (R^+ = +ve real & R^- = -ve real)
  - $Q \rightarrow$ Rational number
  - $I/Z \rightarrow$ Intigers (I^+/Z^+ = +ve intigers & I^-/Z^- = -ve intigers)
  - $R-Q \rightarrow$ Irrational numbers

○ GCD (Great common divisor):

GCD (18, 60):

$18 = 2 \cdot 3 \cdot 3$

$60 = 2 \cdot 2 \cdot 3 \cdot 5$

= Comon = $2 \times 3 = \underline{6}$

Perfect square:	1, 4, 9, 16, 25...
Square roots:	1, 2, 3, 4, 5...

○ Set: → Set is a collection of item.

→ Set can be infinite.
→ Uniform type not required.
→ Sequence don't matter.
→ Duplicate don't matter. (It auto remove)
→ Not every collection of item is set. (∞ set)

It has → Finite set, and → Infinite set

eg: factor of 24 : {1, 2, 3, 4, 6, 8, 12, 24}
Prime below 15 : {2, 3, 5, 7, 11, 13}
Players : {Kohli, Dhoni, Kholi}

□ Elements:

→ Item in set is called element

□ $x \in X$ means x is an element of X

Hence, we write:

$x \in X$

Element	Not element
$x \in X$	$x \notin X$

□ likewise, $5 \in Z$, $\sqrt{2} \notin Q$

Here,

Z = (intiger) means 5 is intiger.

X = (Rational no') means $\sqrt{2}$ is not rational number.

Subset:

- If X is a subset of Y.

Subset means every element of X is also an element of Y.

Hence, we write notation:

$$\boxed{X \subseteq Y}$$

Subset	Not Subset
$X \subseteq Y$	$X \not\subseteq Y$
$X \subset Y$	

	Subsets	
$B = \{1\} \Rightarrow n(B) = 1$	$\{\}, \{1\}$	2 subset $= 2^1$
$C = \{1,2\} \Rightarrow n(C) = 2$	$\{\}, \underbrace{\{1\}, \{2\}}_{\text{Proper set}}, \{1,2\}$	4 Subset $= 2^2$
$D = \{1,2,3\} = n(D) = 3$	$\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}$	8 subset $= 2^3$

Number of subset can be created by set is 2^n

Example: [n = cardinal no.] or [n = no. of element in set]

- → Kohli $\in$ {Dhoni, Kohli, Pujara}
- → {Kohli, Dhoni} $\subseteq$ {Dhoni, Kohli, Pujara}
- → Primes $\subseteq N$, $N \subseteq Z$, $Z \subseteq Q$, $Q \subseteq R$

- Every set is subset of itself.
- Empty set has no element. ϕ, $\{\}$, (null set)

  empty element is subset of all sets.

  $\phi \in X$

- A set can contain other set.
- Powerset → It is set of subset of a set.

  $X = \{a, b\}$ (no. of element $= 2^{n(a)}$)

  Powerset $= \{\phi, \{a\}, \{b\}, \{a, b\}\}$

- Singleton set = Set with only 1 element.

Subset: $a = \{1, 2\} = \{\}, \{1\}, \{2\}, \{1,2\}$ (Subset

Powerset: $a = \{1, 2\} = \{\{\}, \{1\}, \{2\}, \{1,2\}\}$ (Set of subset)

& set become element

Symbols

$\in$ → Belongs to

$\forall$ → For all / For every

$\exists$ → There exist

$|$, ; , : → Such that, ; → and

$\mathbb{N}$ → Natural no.

$\mathbb{Z}$ → integers

$\mathbb{Q}$ → Rational no.

$\mathbb{R}$ → Real no

$\in$ = Element ($5 \in \mathbb{Z}$)

$\subset$ = Subset ($\{\mathbb{Z}\} \subset \{\mathbb{R}\}$)

$\subseteq$ = Subset (or equal to) ($\{\mathbb{N}\} \subseteq \{\mathbb{Z}\}$)

$\cup$ = union

$\cap$ = Intersection

$A^c, \bar{A}, A', \sim A$ = A complement

$\setminus$ = Set difference

- **Equal set** = When element of set A match element of set B. is equal set.

Eg = $A = \{1, 2\}$

$B = \{x : x^2 - 3x + 2 = 0,\ x \in R\} \Rightarrow B = \{1, 2\}$

Hence, $A = B$ (equal set)

- **Equivalant set** = When cardinal no. are same in 2 set. (means no. of element in set C is equal to no. of element in set D)

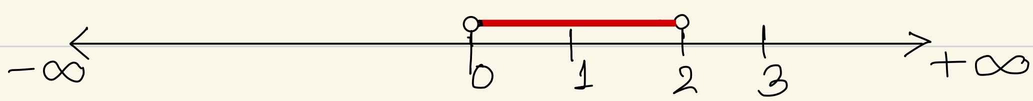
$C = \{1, 2\}$, $d = \{d, f\}$

$n(c) = n(d)$ (equivalant set)

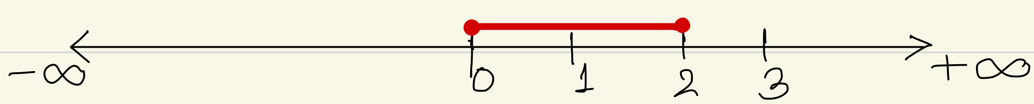
- **Universal set** = The set that has all the element relevent to our question.

- **Intervals** = It is of 2 type:

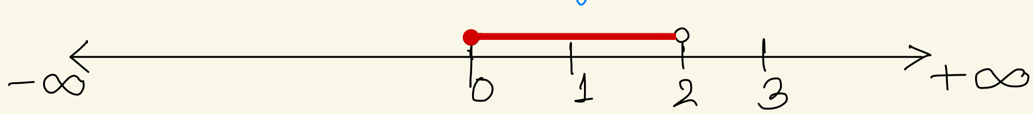
  - **Open intervals** :- It is interval where end point are not included. Ex = $(0, 2)$ or $]0, 2[$



  - **Close interval** :- It is interval where end point are included. Eg = $[0, 2]$



  - **Semi open/close interval** :- In this type of interval one end point is included & 1 end is excluded. Eg = $[0, 2)$ or $[0, 2[$



after -6 to 6 $= -6 < z \leq 6$
from -6 to before $6 = -6 \leq z < 6$

Eg:-

Integers from -6 to 6
$\{z \mid z \in \mathbb{Z}, -6 \leq z \leq 6\}$

- Set of n element has 2^n element.
- Subset of binary number:
  - → $X = \{x_1, x_2, x_3, x_4, \dots x_n\}$
  - → n bit binary no. It also has 2^n element:
    - 3 bit: 000, 001, 010, 100, 011, 110, 111, 101
  - → Digit i represent wether x_i is subset.
    - $X = \{a, b, c, d\}$
    - 0101 is $\{b, d\}$
    - 0000 is $\emptyset$, 1111 is X.
  - → 2^n is n bit number.

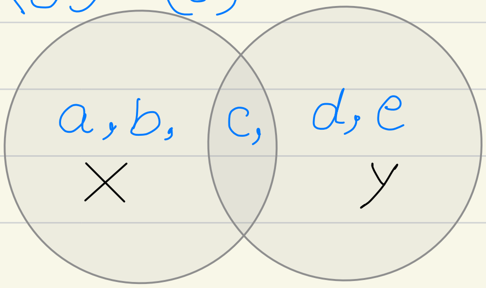
- Set comprehension:
  - The set of even integers.
    $\{x \mid x \in \mathbb{Z}, x \bmod 2 = 0\}$
    - → Begin with existing set $\mathbb{Z}$.
    - (means divided by 2) → Apply condition is $x \in \mathbb{Z}$ so, $x \bmod 2 = 0$
    - → Collect all element that match cond..
  - The set of perfect square.
    $\{m \mid m \in \mathbb{N}, \sqrt{m} \in \mathbb{N}\}$
  - Set of rational in reduced form.
    $\{m/n \mid m, n \in \mathbb{Z}, \gcd(m, n) = 1\}$

Cardinality means no. of element in a set.

- **Union** → Combine X and Y, X∪Y (∪)

  Can be written A∪B, A or B, A+B

  $\{a,b,c\} \cup \{c,d,e\}$
  $= \{a,b,c,d,e\}$

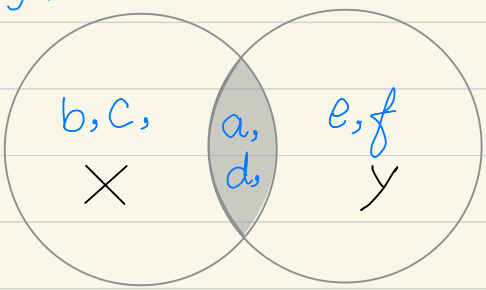


- **Intersection** → Element common in X and Y. (∩)

  written as $X \cap Y$.

  Can be written as A∩B, A·B, A and B

  $\{a,b,c,d\} \cap \{a,d,e,f\}$
  $= \{a,d\}$



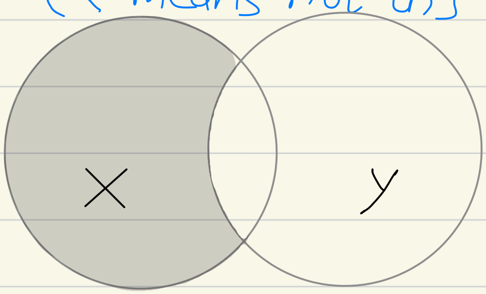
- **Set diffrence** → element of X not in element Y.

  Can writteas $X \setminus Y$ or $X-Y$ (\ means not in)

  $\{a,b,c,d\} \setminus \{a,d,e,f\}$
  $= \{b,c\}$

  In set $x-y \neq y-x$

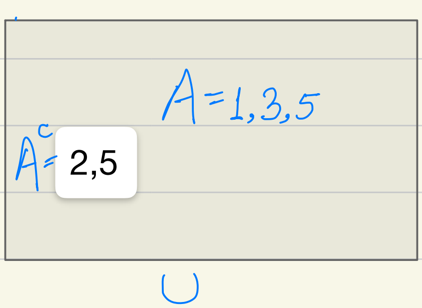
  Also written as : $A-(A\cap B)$, $A\cap B^c$



- **Complement** → Element not in X, $\bar{X}$ or X^c.

  Universal set → It is the set from your set come from.

$U = \{1,2,3,4,5\}$
$A = \{1,3,5\}$ $A^c = \{2,4\}$
$B = \{1,2,3,4,5\}$ $B^c = \{\}$
$C = \{\}$ $C^c = \{1,2,3,4,5\}$

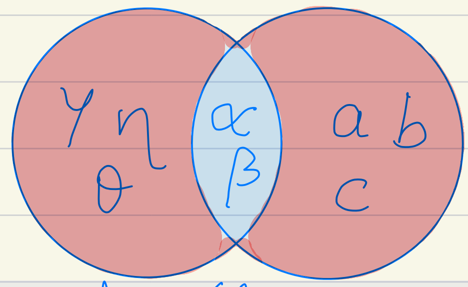


$(A^c)^c = A$, $n(A^c) + n(A) = n(U)$

Symmetric difference of 2 sets:

- Can be written as $A \Delta B$ or $A \oplus B$ or $(A \setminus B) \cup (B \setminus A)$

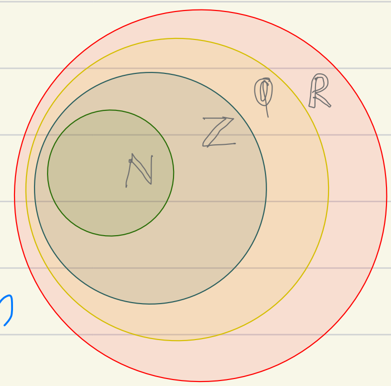
$A = \{\alpha, \beta, \gamma, \eta, \theta\}$
$B = \{\alpha, \beta, a, b, c\}$



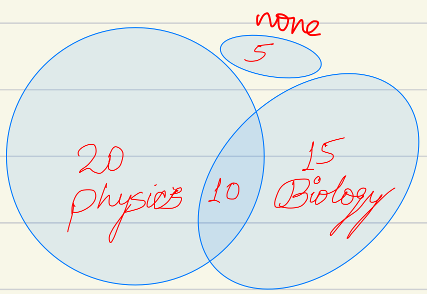
Can be written as:
$(A/B) \cup (B/A)$ or $(A-B) \cup (B-A)$ or $(A \cap B^c) \cup (B \cap A^c)$
$= (A \cup B) - (A \cap B)$

- **Membership:** $5 \in$ Prime, $5 \notin$ Prime.
Subset : Prime $\subset \mathbb{N}$, $\mathbb{N} \subset \mathbb{Z}$, $\mathbb{Z} \subset \mathbb{Q}$, $\mathbb{Q} \subset \mathbb{R}$

$\mathbb{Q}$ = Rational no.
$\mathbb{R}$ = Real no.



Euler-venn diagram



Total student = 50

$P \cup B = 45$ (Taken phy or bio or both)
$P \cap B = 10$ (Both phy & Bio taken)
$P \setminus B = 20$ (Only phy)
$B \setminus P = 15$ (Only Bio)
$\overline{P \cup B} = 5$ (taking neither) ↳ complement

In a class of 60 student, 35 took physics, 30 took bio & 10 took neither. How many took both Ph&B.

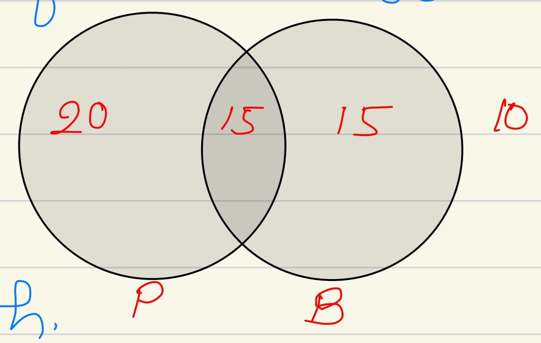
⇒ $|Y|$ = Cardinality of Y (no. of element) = 60

⇒ $|P| + |B| = 35 + 30 = 65$

If 10 ~~took~~ neither than:

⇒ $|P \cup B| = 60 - 10 = 50$

So, $65 - 50 = 15$ taken both.



- Cartesian product :-
  - $A \times B = \{(a,b) \mid a \in A, b \in B\}$

    It is pair of element from A & B.

  - $A = \{0, 1\}$, $B = \{2, 3\}$
  - $A \times B = \{(0,2), (0,3), (1,2), (1,3)\}$

→ In pair order is Important :-
- $(0,1) \neq (1,0)$

- Combine cartesian product with set comprehension :-

  $\{(m,n) \mid (m,n) \in \mathbb{N} \times \mathbb{N}, n = m+1\}$

  $\{(0,1), (1,2), (2,3) \ldots (17,18), \ldots\}$

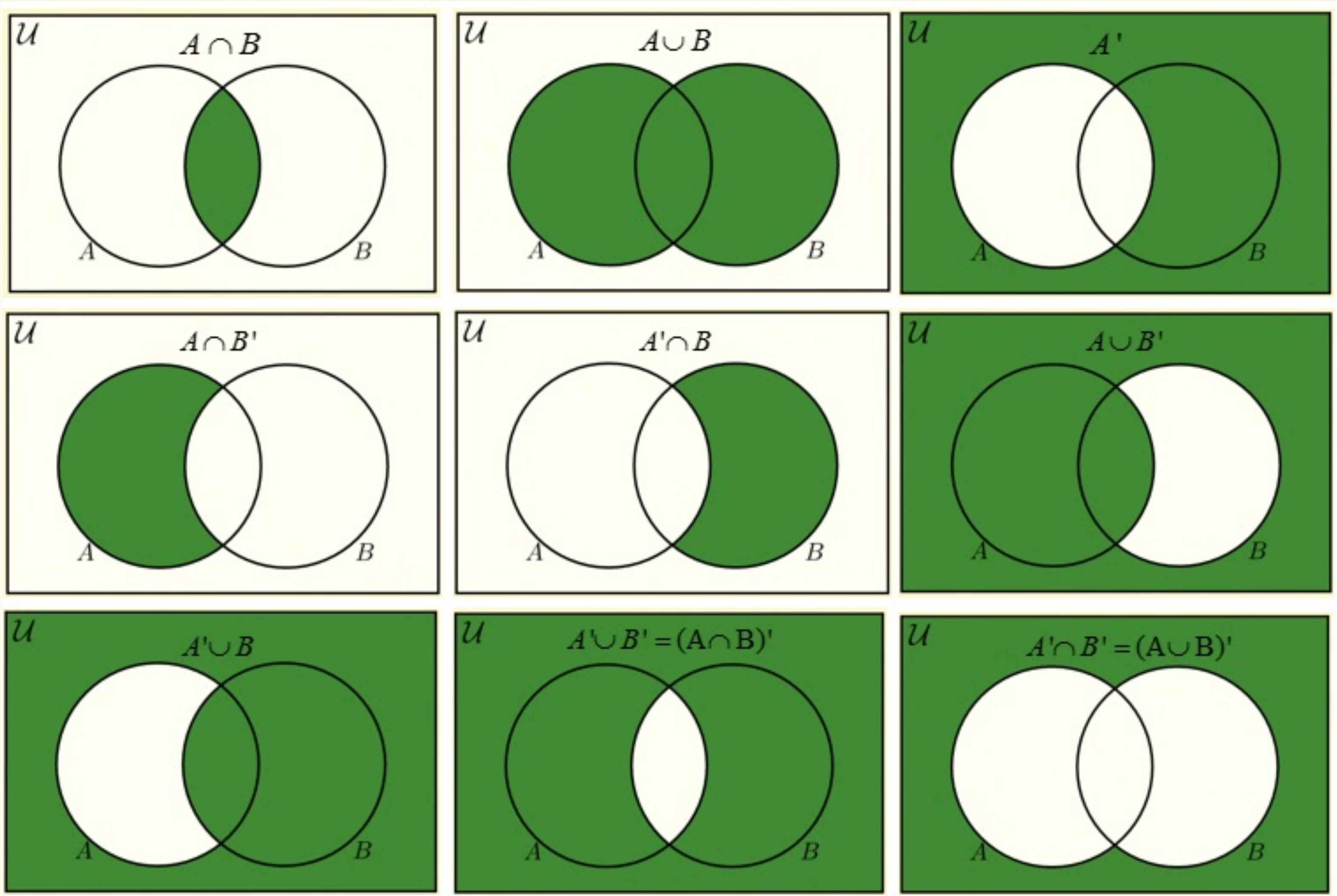
- Pairs (d,n) wher d is factor of n

  – $\{(d,n) \mid (d,n) \in \mathbb{N} \times \mathbb{N}, d \mid n\}$

  $\{(1,1), \ldots\ldots, (2,82), \ldots, (14,56), \ldots\}$

- Binary relation = $R \subseteq A \times B$

  notation, $(a,b) \in R$, $a R b$



U
A ∩ B
A
B
U
A ∪ B
A
B
U
A'
A
B
U
A ∩ B'
A
B
U
A' ∩ B
A
B
U
A ∪ B'
A
B
U
A' ∪ B
A
B
U
A' ∪ B' = (A ∩ B)'
A
B
U
A' ∩ B' = (A ∪ B)'
A
B

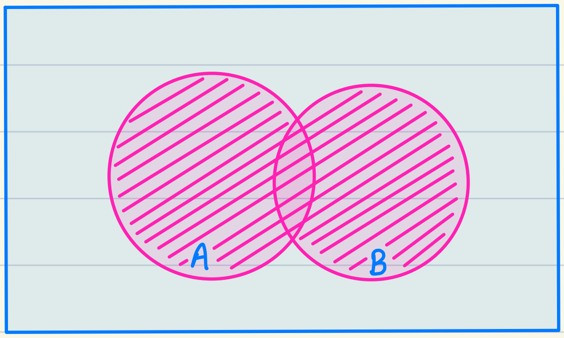
○ DE-MORGAN'S LAW :-

→ Law of complement
→ law of union & intersection

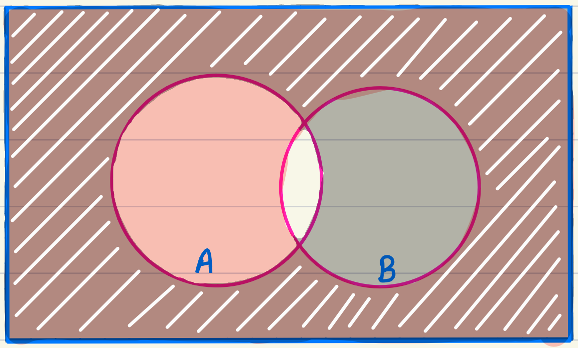
→ Law of complement :

① $(A \cup B)^c = A^c \cap B^c$
② $(A \cap B)^c = A^c \cup B^c$

① $(A \cup B)^c$ = $A^c \cup B^c$

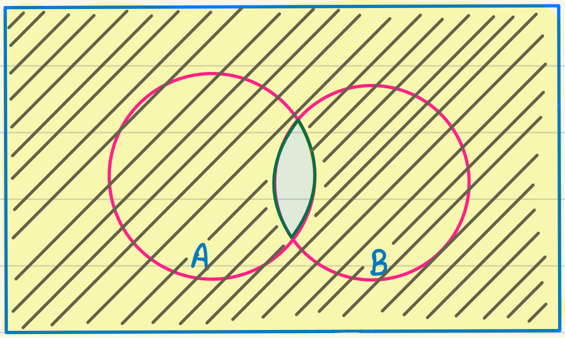


Blue area is $(A \cup B)^c$

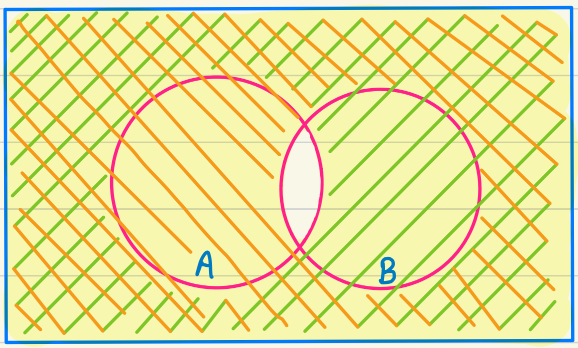


Area with both black & Red sode combine is $A^c \cup B^c$ (white line)

② $(A \cap B)^c$ = $A^c \cup B^c$



$A \cup B$ is green shaded part its intersection is grey line area



A^c is green line area and B^c is orange line area and $A^c \cup B^c$ is both orange green line combine yellow.

→ law of union & intersection:

→ $\cup/\cap$ is distributive over $\cap/\cup$:-

① $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

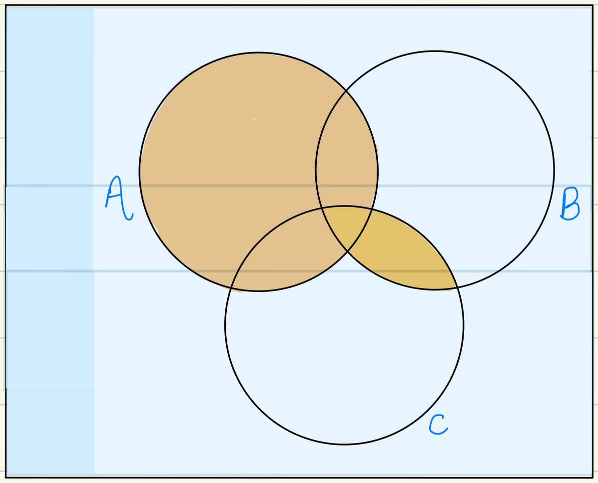
② $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

like removing().

$= a(b+c)$

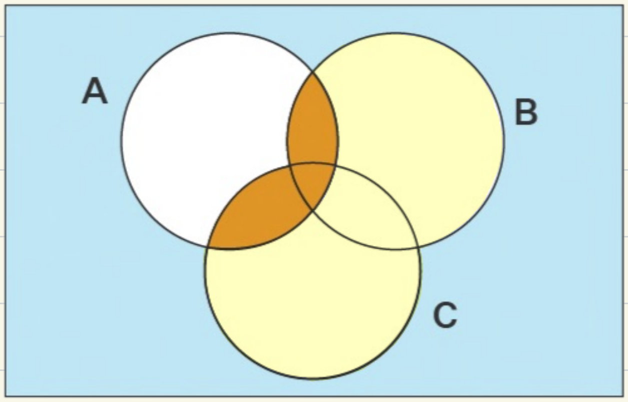
$\Rightarrow ab+ac$

1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



$A \cup (B \cap C) =$ ▭

2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

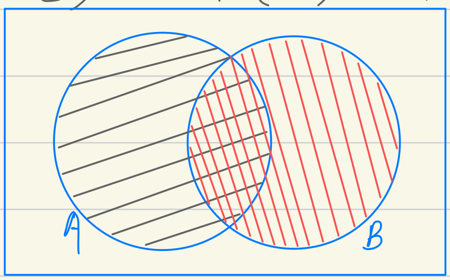


$A \cap (B \cup C)$ — ▭

$(B \cup C)$ — ▭

Addition theorem in set:-
(Inclusion & exclusion principle)

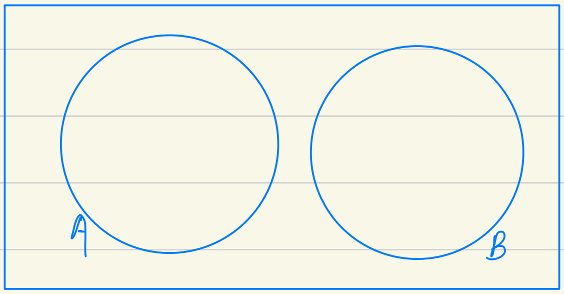
→ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Bcz, to add {A} and {B} we have to remove red-black check line once bcz, it is added twice.

→ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Disjoint set →



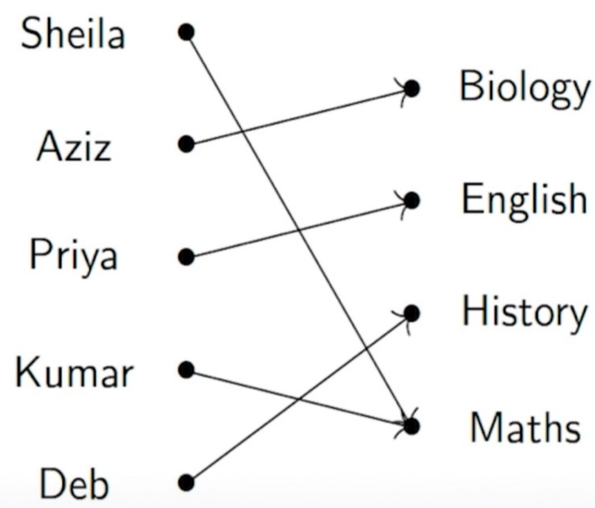
→ $n(A \cup B) = n(A) + n(B)$

- $x = 2, 4$ (If we write like this than order of 2,4 are not important
  $x = (2, 4)$ (In this case order is important like point in graph)
  # means : $2, 4 = 4, 2$ and $(2,4) \neq (4,2)$

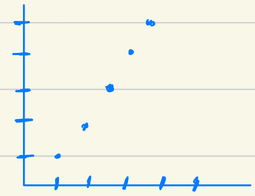
- Cartition product of sets:
  $A = \{1, 2\}$ $B = \{A, B\}$
  $A \times B = \{(1,A), (1,B), (2,A), (2,B)\}$ [Here $A \times B \neq B \times A$]
  $B \times A = \{(A,1), (A,2), (B,1), (B,2)\}$
  $n(A \times B) = n(A) \times n(B)$ & $n(A \times B) = n(B \times A)$

- Teachers and courses
  - T, set of teachers in a college
    C, set of courses being offered
  - $A \subseteq T \times C$ describes the allocation of teachers to courses
  - $A = \{(t, c) \mid (t, c) \in T \times C, t \text{ teaches } c\}$
- Mother and child
  - P, set of people in a country
  - $M \subseteq P \times P$ relates mothers to children
  - $M = \{(m, c) \mid (m, c) \in P \times P, m \text{ is the mother of } c\}$

A relation as a graph



- Identity relation:



  - $I \subseteq A \times A$
  - $I = \{(a,b) \mid (a,b) \in A \times A, a = b\}$

- Reflexibe relation:
  - $R \subseteq A \times A$, $I \subseteq R$
  - $\{(a,b) \mid (a,b) \in N \times N, a,b > 0, a \mid b\}$

- Symetric relation: (a,b) can change place.
  - $(a,b) \in R$ if & only if $(b,a) \in R$
  - $\{(a,b) \mid (a,b) \in N \times N, \gcd(a,b) = 1\}$

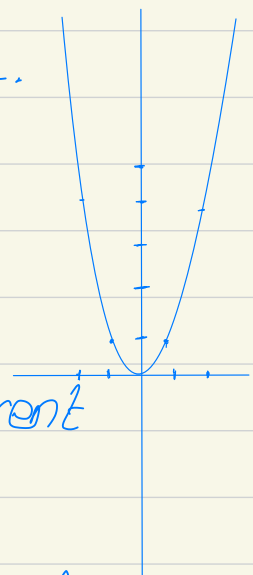
- Transitive relation:
  - If $(a,b) \in R$ and $(b,c) \in R$ than $(a,c) \in R$
  - $= \{(a,b) \mid (a,b) \in \mathbb{R} \times \mathbb{R}, a < b\}$

- **Antisymmetric relation:**
  - If $(a,b) \in R$ and $a \neq b$, than $(b,a) \notin R$
  - $\Rightarrow M \subseteq P \times P$ relates mother to child then, $(P,C) \in M$ than $(C,P) \notin M$

- **Reflexive, symmetric transitive:**
  (Equavalance relation)
  - In equavalance relation partition a set. (AM/PM)
  - Group of equavalant element is called equivalance class.

Functions:

It is rule to map input to output.



Conv. x to x^2

$x \longmapsto x^2$, $\boxed{f(x) = x^2}$

Function is injective: It means different input produces diff. output.

- $A \times B$ – Cartision product, all pair (a,b), $a \in A$ & $b \in B$
  - $A = \{1, 4, 8\}$ $B = \{2, 8, 9\}$
  - $A \times B = \{(1,2), (1,8), (1,9), (4,2), (4,8), (4,9), (8,2), (8,8), (8,9)\}$
  - $B \times A = \{(2,1), (8,1), (9,1), (2,4), (8,4), (9,4), (2,8), (8,8), (9,8)\}$

  Cartision product can take more than 2 sets.
  - $S \subseteq A \times B = \{(1,1), (4,16), (7,49)\}$
  - $S = \{(a,b) \mid (a,b) \in A \times B, b = a^2\}$

- **Divisiblity:**

$$D = \{(d,n) \mid (d,n) \in N \times N,\ d \mid n\}$$
$$D = \{(d,n) \mid (d,n) \in Z \times N,\ d \mid n\}$$

+ve

+ve & -ve

- **Prime power:-**

They are number which are multiple of itself many time.

eg : $\{(3,1), (5,625), (7,343)\ldots$

3^0 5^4 7^3

$$P = \{P \mid P \in N,\ factor(P) = \{1,P\},\ P \neq 1\}$$
$$P_{power} = \{(P,n) \mid (P,n) \in P \times N,\ n = P^m \text{ for some } m \in N\}$$

- **Function:**

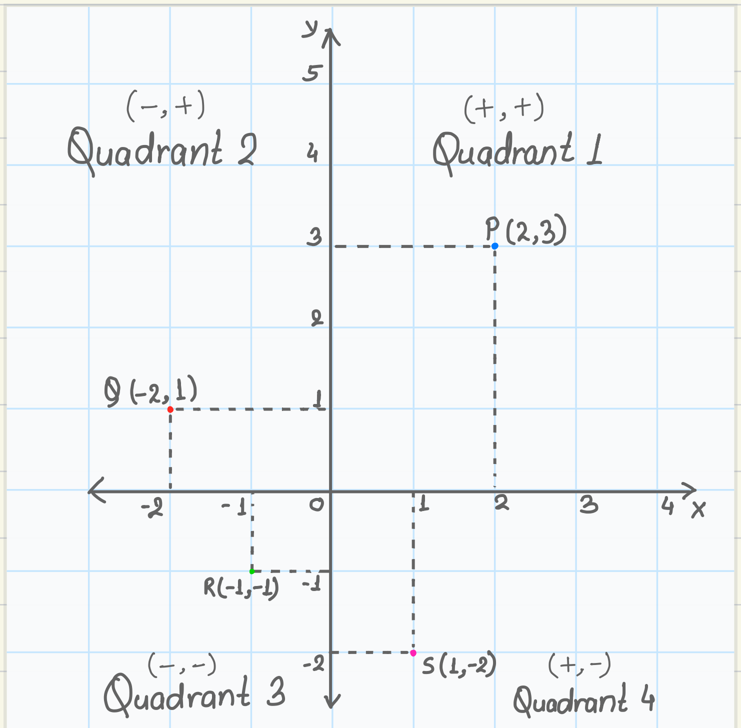
It is a rule to map input to output.

$x \to x^2$, $g(x) = x^2$

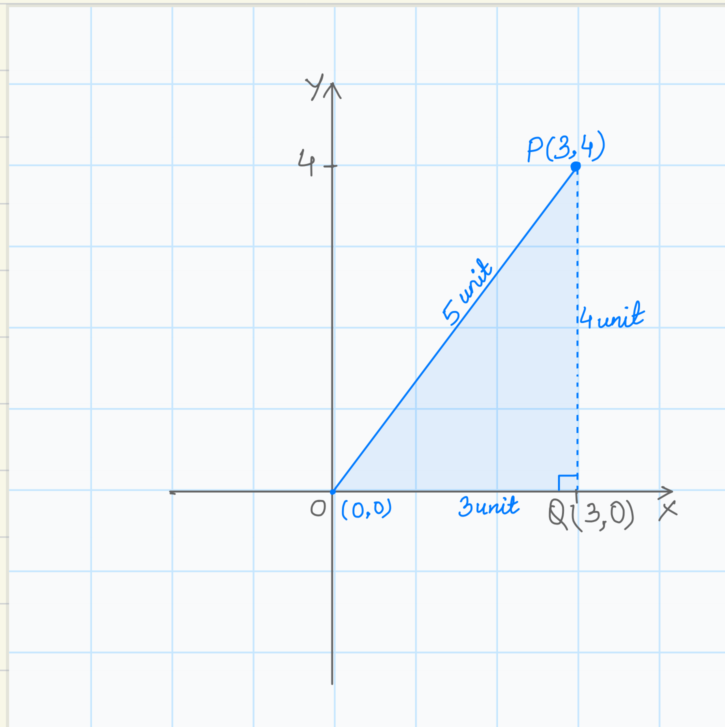
- If a num. divide a sum & it divide 1 part of sum than it must also divide other part of the sum.

Coordinate Geometry

○ Rectangular coordinate system :



Coordinate Plane

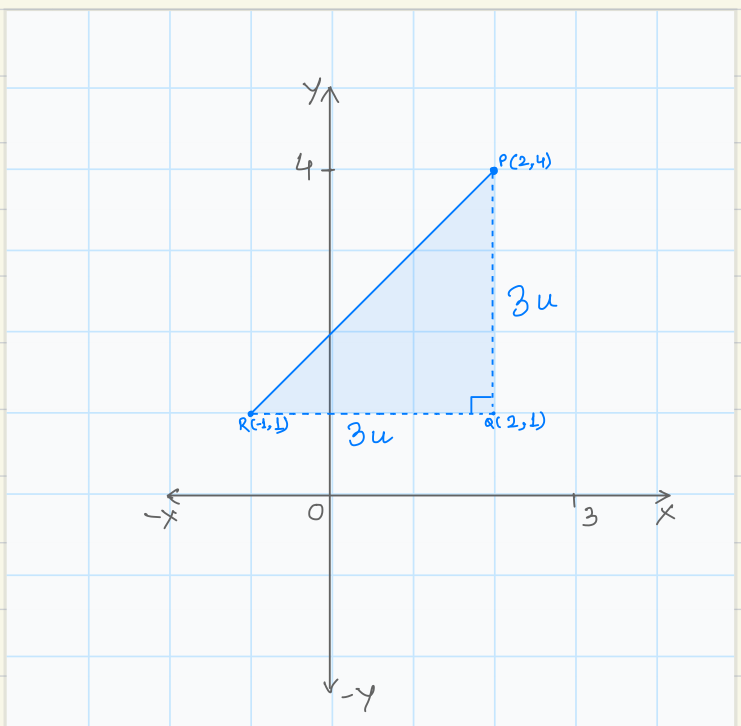


find distance (OP)

By pythagorean we know:

$$OP^2 = OQ^2 + QP^2$$

$$OP = \sqrt{3^2+4^2} = 5$$



By same formulae :

$$PR = \sqrt{3^2+3^2} = \sqrt{18}$$

$$= 3\sqrt{2}$$

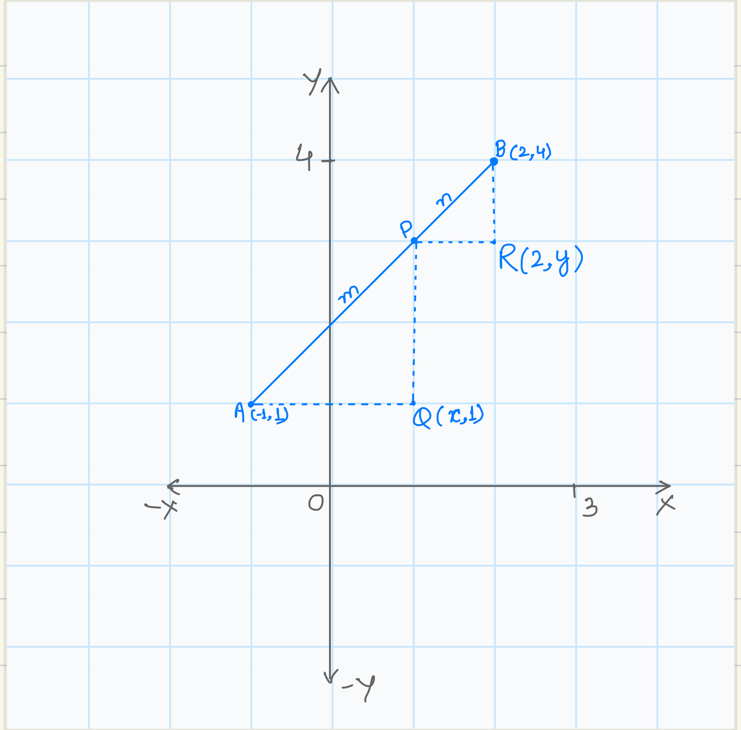
- Point
- line
- circle
- Parabola
- Elypse
- Hyperbola

All are conic section

Section formula :

P cuts line AB in m:n ratio. Find P coordinate.

→ Observe $\triangle AQP \sim \triangle PRB$.



$$\Rightarrow \frac{m}{n} = \frac{AP}{PB} = \frac{AQ}{PR} = \frac{PQ}{BR}$$

$$\Rightarrow \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

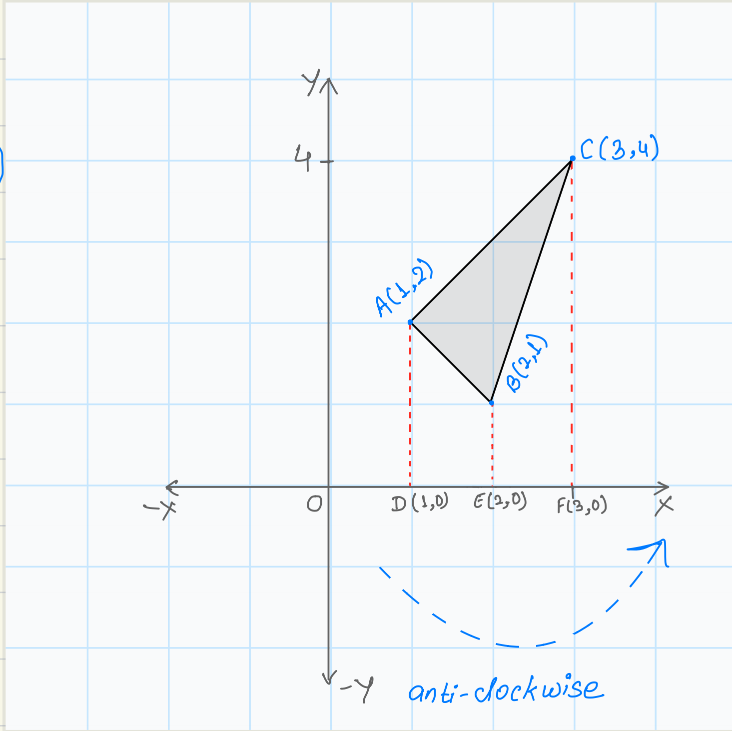
from this we will get:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

Area of a triangle using co-ordinate :-

By constructing red ∥ line we created 3 trapisium. So,

$$A(\triangle ABC) = A(\square ADFC) - A(\square ADEB) - A(\square BEFC)$$



Area of trapisium is $= \frac{1}{2}$(Sum of ∥ sides) × height

$$A(\square ADFC) = \frac{1}{2}(AD + FC) \times DF = \frac{1}{2}(y_1 + y_3)(x_3 - x_1)$$

$$A(\square ADEB) = \frac{1}{2}(AD + EB) \times DE = \frac{1}{2}(y_1 + y_2)(x_2 - x_1)$$

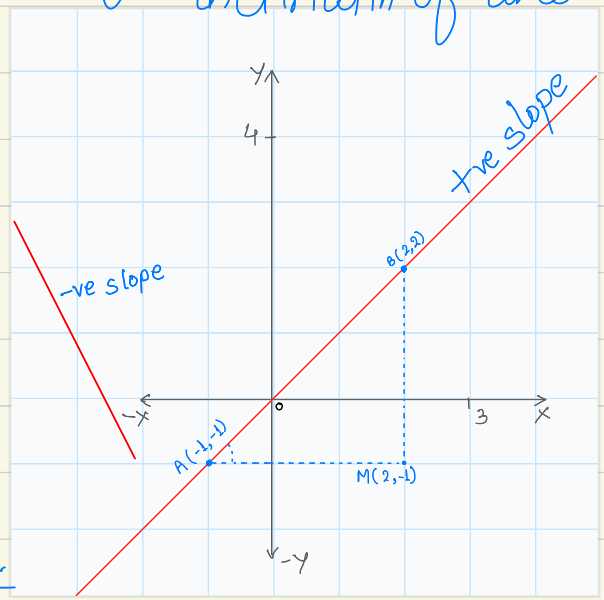
$$A(\square BEFC) = \frac{1}{2}(BE + CF) \times EF = \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

After calculation:-

$$A(\triangle ABC) = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Slop of a line :

θ = inclinatin of line



To find slope of line identify 2 pts. $A(x_1, y_1)$ and $B(x_2, y_2)$

Construct Δ with pt. $M(x_2, y_1)$

$\tan\theta = \frac{\Delta y}{\Delta x}$

$m = \frac{MB}{AM} = \frac{y_1 - y_2}{x_1 - x_2} = \tan\theta$

→ Vertical line slope = not defined '⊥'

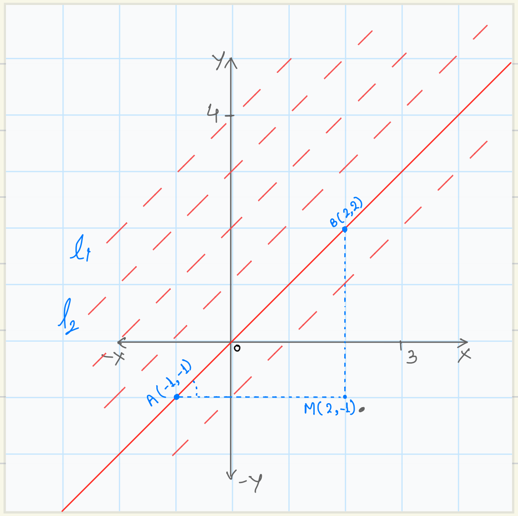
→ Horizontal " = 0. '∥'

Slope of a line $= \frac{\Delta \text{ in } y}{\Delta \text{ in } x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3}{3} = 1$

(not define) $\tan\theta = \frac{\pi}{2} = \perp$ x-ax

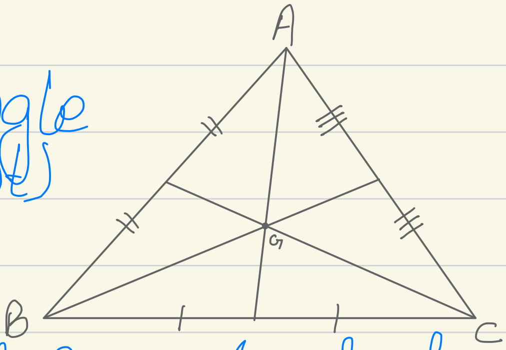
There can be many line with same slope.

Here we can see many dashed line with same inclanation.



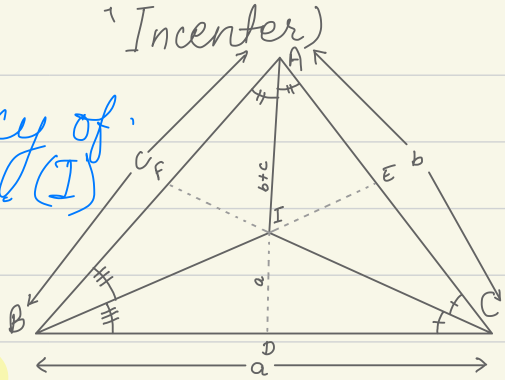
- If two line are parallel in graph than slope of both line are equal and vice-versa

- Centroid : 3 median of a trangle is always concurrent. (met at a point)
  This point is called centroid (G).
  'G' divides median in 2:1 (ratio).



$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$, If all 3 coordinate of Δ is given.

- **Incenter:** Point of concurrency of internal ∠ bisector of Δ is incenter (I)

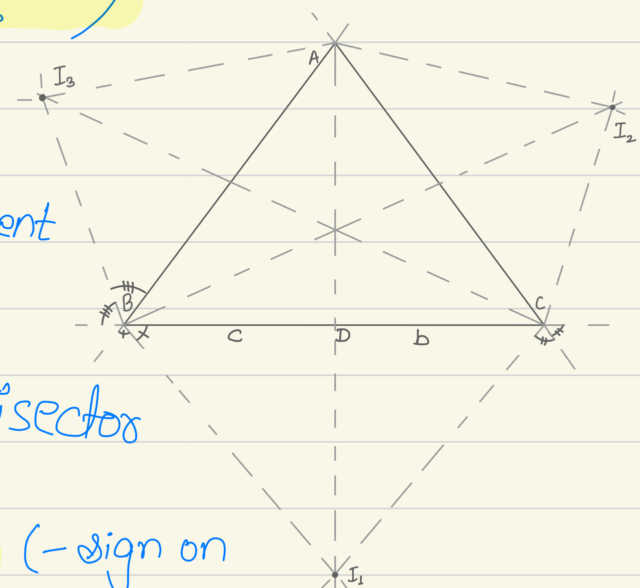


→ $BD : CD = c : b$

→ $AI : ID = b + c : a$

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

- **Excenter:** I_1, I_2, I_3 are excenter of ΔABC. It is point of concurrent of 2 exterior angle bisector and 1 interior angle bisector.



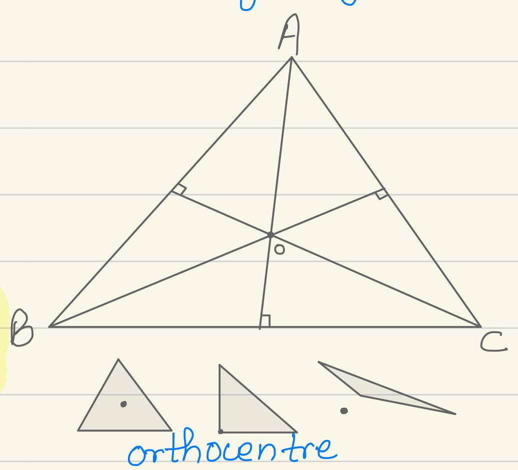
→ I_1 divides the internal angle bisector AD in $b+c : a$. (means, $A : I_1 : AD$)

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

(− sign on begining of 'a' bcz I_1 is front of A)

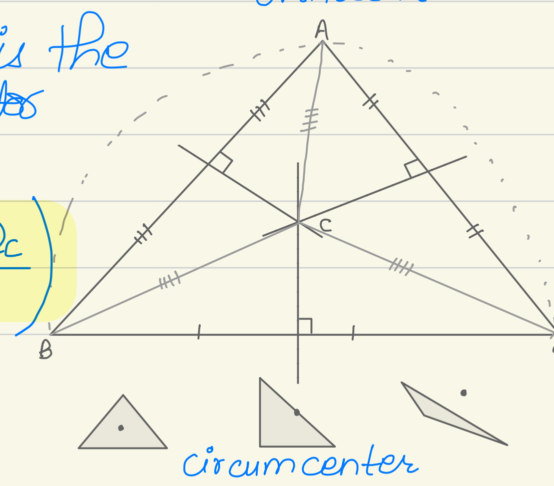
I_2 & I_3 vice-versa of I_1.

- **Orthocenter:-** Point of concurrency of 3 altitude is orthocenter. (can be outside Δ)



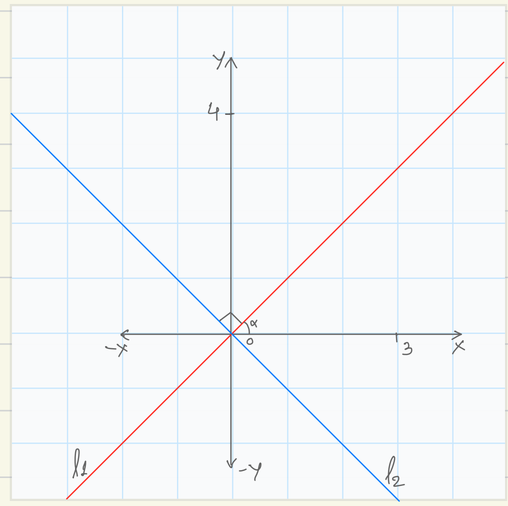
$$O = \left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C} \right)$$

- **Circumcenter:-** Circumcenter is the point of concurrency of three ⊥ bisector of sides of Δ.



$$C = \left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C} \right)$$

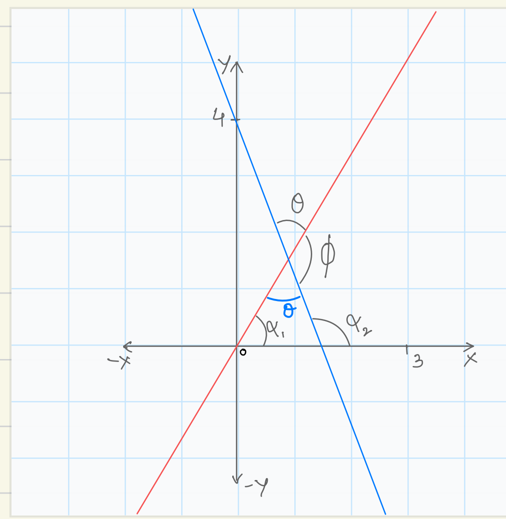
- 3 points are collinear if:
  - → $AB + BC = AC$
  - → area$(\Delta ABC) = 0$
  - → If 'B' divides AC in $m:n$



If $l_1 \perp l_2$, than product of slop of $l_1 \cdot l_2 = -1$ and vice. versa.

Here suppose:
- $\alpha_1 \& \alpha_2 \neq 90°$

&, $\theta = \alpha_2 - \alpha_1$



θ or $\angle$ between 2 line :-

$$\tan\theta = \left|\frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \cdot \tan\theta_1}\right| = \left|\frac{m_2 - m_1}{1 + m_2 \cdot m_1}\right|$$

- Inclination: It is the angle made by straight line with the direction of x-axis in anticlock wise direction.
  Eg $= \alpha_1, \alpha_2$ range $= \pi > \alpha \geq 0$

- In case of verticle line all the cordinate of x-axis are same in respcect to y-axis.
  if $x = b$ than coordinates of that line are (b, y)
- In case of horizontal line all the cordinate of y-axis are same in respcect to x-axis.
  if $y = a$ than coordinates of that line are (x, a)

○ Equation of line : In point slope form:-

For a non-vertical line l, with slope m and a fix point $P(x_0, y_0)$ on a line, to find equation of line through this point:-

1) 1st we have to take a point $Q(x, y)$ collinier or on the same line and find slope m.

by, $m = \frac{\Delta y}{\Delta x}$ ⇒ $m = \frac{y - y_0}{x - x_0}$

⇒ $y - y_0 = m(x - x_0)$ is in point slope form.

Q Find equation of line through point P(5,6) and slope -2.

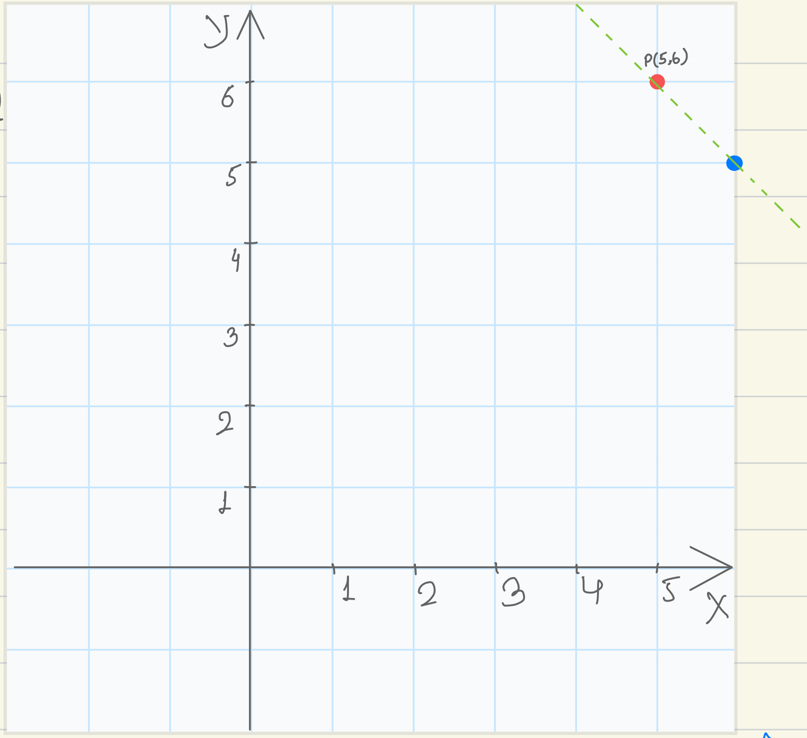
→ let's take arbitary pt. on this line $Q(x, y)$

→ Slope of line is -2 so,

⇒ $-2 = \frac{(y-6)}{(x-5)}$

⇒ $-2(x-5) = y - 6$

⇒ $y = 16 - 2x$



let say, if $x = 5$, than $y = 6$, so line look somthing like ↑

Equation of a line (two point form):

A line l pass from $P(x_1, y_1)$ & $Q(x_2, y_2)$

Assume $R(x, y)$ is arbitary pts on l line.

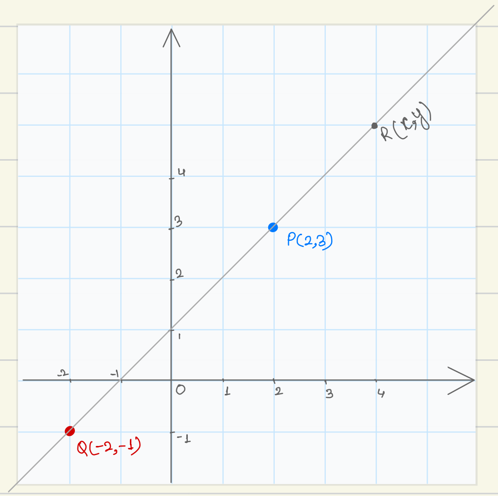
Then point P, Q, R are collinear.

Hence, slope of PR = slope of PQ.

Therefore, $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Q. Find equation for P & Q pts.



→ putting these point in 2 pt. form equation.

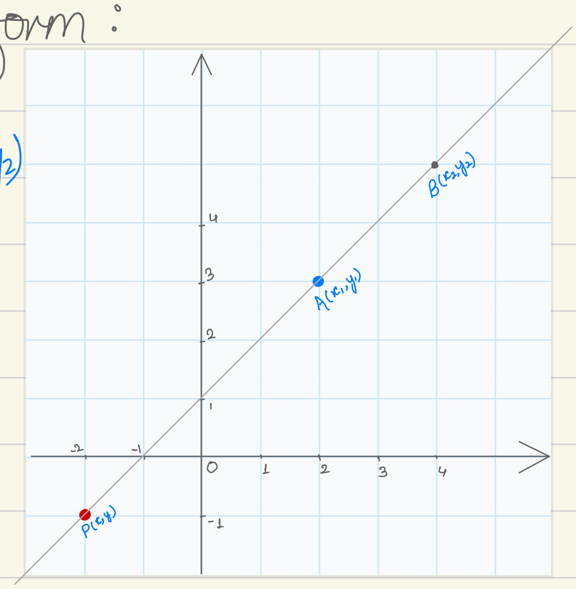
$$(y - 3) = \frac{-1 - 3}{-2 - 2}(x - 2)$$

Equation of line in determinant form:

→ Eq. of line passing through (x_1, y_1) & (x_2, y_2)

→ If P is on that line than it must be collinear.

→ So, area ΔPAB should be 0.

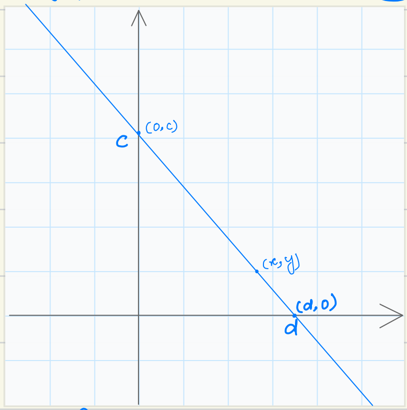


$$\text{so, } \frac{1}{2}\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Equation of straight line in determinant form.

- Equation of line (Slope-intercept form)

When a line l with slope m cuts y-axis at c. Than c is called y-intercept of line l.



$$m = \frac{y-c}{x-0} \Rightarrow y - c = mx$$

$$\Rightarrow \boxed{y = mx + c}$$

And, when line l with slope m cuts x-axis at d. than x-axis is called x-intercept of line l

So, $m = \dfrac{y-0}{x-d} \Rightarrow \boxed{y = m(x-d)}$

Q find equation of line with slope 1/2 & y-intercept = −3/2

$\Rightarrow y = mx + c \qquad y = \frac{1}{2}x + \left(-\frac{3}{2}\right) \Rightarrow y = \frac{1}{2}x - \frac{3}{2}$

Q find eq. of line with slope 1/2 and x-intercept 4.

$\Rightarrow \quad y = \frac{1}{2}(x-4) \quad$ or $\quad 2y - x + 4 = 0$

- Equation of line : Intercept form

When a line makes x-intercept at a and y-intercept at b than two point of line are (a,0) and (0,b)

Using 2-point form:-

$\rightarrow (y-0) = \dfrac{b-0}{0-a}(x-a)$ or $\dfrac{x}{a} + \dfrac{y}{b} = 1$

Q Find equation of line with x-intercept at -3 and y- intercept at 3.

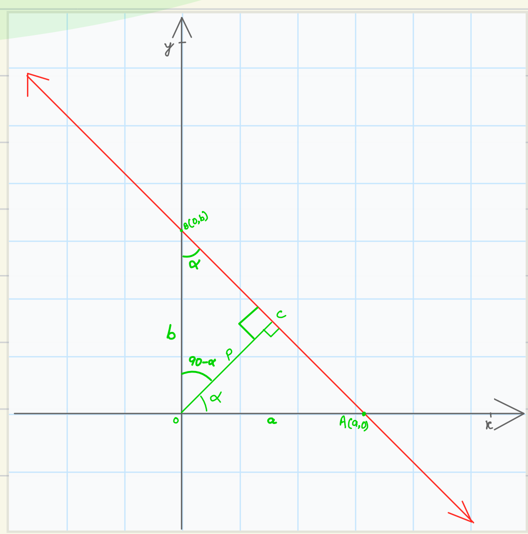
$\Rightarrow \frac{x}{-3} + \frac{y}{3} = 1$

General equation of a line:

Diff. form of equation of line	Representation	General form $Ax+By+C=0$
Slope-point form	$(y-y_0) = m(x-x_0)$	$m = -\frac{A}{B}$, $y_0 - mx_0 = -\frac{C}{B}$
Slope-intercept form	$y = mx+c$ or $y = m(x-d)$	$m = -\frac{A}{B}$, $c = -\frac{C}{B}$ or $d = -\frac{C}{A}$
Two point form	$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$	$\frac{y_2-y_1}{x_2-x_1} = -\frac{A}{B}$, $y_1 + \frac{A}{B}x_1 = -\frac{C}{B}$
Intercept form	$\frac{x}{a} + \frac{y}{b} = 1$	$a = \frac{-C}{A}$, $b = -\frac{C}{B}$

○ Normal/⊥ form of line:-

Eq. of line at distance 'p' units from origin and ⊥ line from origin is making ∠'α' with x-axis.



$\cos\alpha = \frac{\text{base}}{\text{hypotenus}} = \frac{p}{a}$, $\sin\alpha = \frac{\Delta BCO \perp}{\text{hypotenus}} = \frac{p}{b}$

$a = p\sec\alpha$ (x-inter) $b = p\operatorname{cosec}\alpha$ (y-inter)

Hence, eq. of line:

$\frac{x}{p\sec\alpha} + \frac{y}{p\operatorname{cosec}\alpha} = 1$

$\boxed{x.\cos\alpha + y.\sin\alpha = p}$

General form = $Ax + By + C = 0$

Any equation in form $Ax + By + C = 0$, where $A, b \neq 0$ simultinously, is called general linear equation equation of line. (This line will not be vertical.

Q. Equation of line is $3x - 4y + 12 = 0$. find slope, x-intercept & y-intersept on the line.

⇒ Here, $A = 3$, $B = -4$ and $C = 12$

Using intersept form, $a = -C/A = -4$
& , $b = -C/B = 3$ } Intersepts

Slope intercept form = $y = mx + C$

Changing $3x - 4y + 12 = 0$ in that form.

$3x + 12 = 4y$

$y = \frac{3x}{4} + 3$

So, $m = \frac{3}{4}$

Slope $= 3/4$.

If line are parallel:

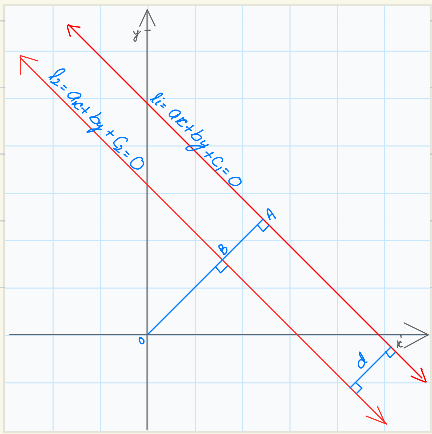
- $a_1 b_2 = a_2 b_1$
- $a_1 a_2 + b_1 b_2 = 0$

→ In ∥ line their slope are equal.

→ In ⊥ line product of their slope is -1.

□ Distance b/2 2 ∥ line

Distance is same every whr.

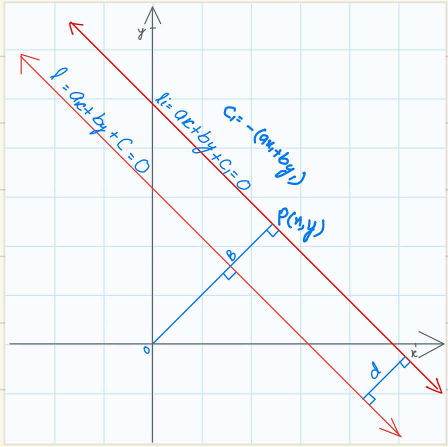


d o, $OA = \left| \frac{C_1}{\sqrt{a^2 + b^2}} \right|$ $OB = \left| \frac{C_2}{\sqrt{a_2 b_2}} \right|$ & , $AB = OA - OB$

$AB = \left| \frac{C_1 - C_2}{\sqrt{a^2 + b^2}} \right|$

- Coff. must be same.
- C_1 & C_2 on same side.

Distance b/w point (x_1, y_1) and a line $ax + by + c = 0$



$\Rightarrow$ Construct a l_1 $\parallel$ line from l on point P.

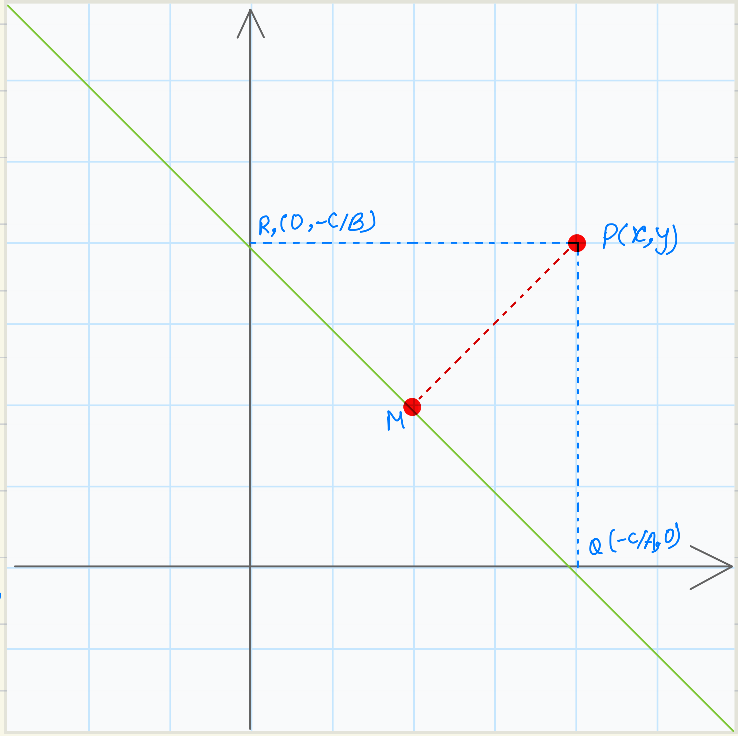
Putting x_1, y_1 in line formula to find C_1.

$\Rightarrow ax_1 + by_1 + C_1 = 0$

$\Rightarrow C_1 = -(ax_1 + by_1)$

$\Rightarrow$ Puttin in distance of $\parallel$ line formula $\Rightarrow d = \left| \dfrac{ax_1 + by_1 + C}{\sqrt{a^2 + b^2}} \right|$

Distance of a point $P(x_1, y_1)$ from line l having equation $Ax + By + C = 0$



$\Rightarrow$ For $A, B \neq 0$

By intercept form:-

$\Rightarrow \dfrac{x}{a} + \dfrac{y}{b} = 1$

$\Rightarrow Ax + By + C = 0$

$\Rightarrow Ax + By = -C$

$\Rightarrow \dfrac{Ax}{-C} + \dfrac{By}{-C} = \dfrac{-C}{-C}$

that form

$\Rightarrow x = \dfrac{-C}{A} \quad y = \dfrac{-C}{B}$

$A(\Delta PQR) = \frac{1}{2} QR \times PM$. Hence, $PM = 2A(\Delta PQR)/QR$

By, area of Δ by guardine formula:

$A(\Delta ABC) = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$

$\Delta PQR = \frac{1}{2} \left| x_1 \left(\frac{-C}{B} \right) - \frac{C}{A} \left(y_1 + \frac{C}{B} \right) \right| = \frac{1}{2} \frac{|C|}{|AB|} |Ax_1 + By_1 + C|$

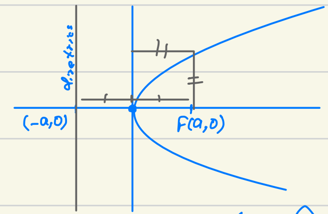
Quadratic Function

A quadratic function is described as equation in the form of: (Where square is present)

- $f(x) = \underline{ax^2} + \underline{bx} + \underline{c}$, (Quadratic fn) where $a \neq 0$

Linear term

Quadratic term Constant



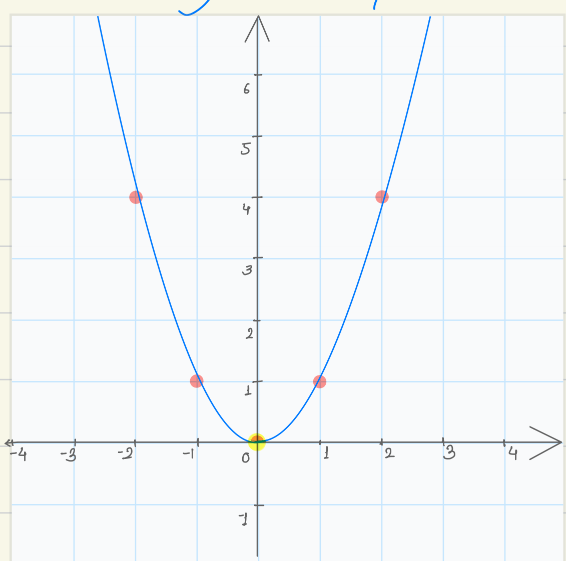
- $f(x) = mx + c$, (linear fn)

Parabola is a locus of point (x,y) which move such that its dist. from line is same as dist. from point.
The graph of any quadratic fn is parabola.

Making suppose b & $c = 0$

$f(x) = ax^2 + bx + c$

if $x = 1$ $y = 1$
$x = 2$ $y = 4$
$x = -1$ $y = 1$
$x = -2$ $y = 2$

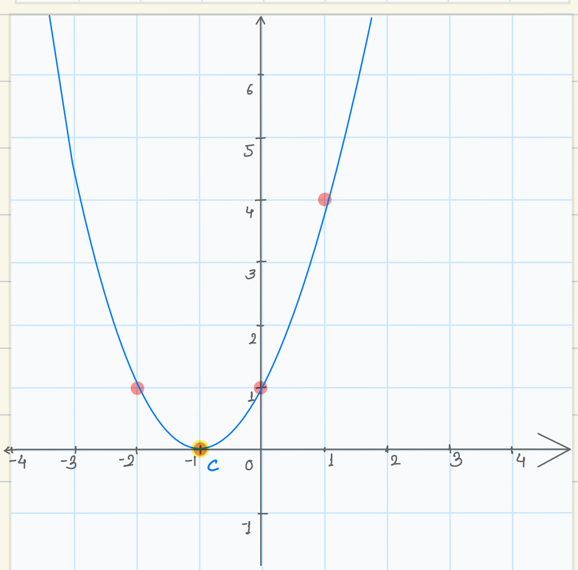


- Plot graph for $f(n) = x^2 + 2x + 1$

We give value of x and get y.

x	y
-2	1
-1	0
0	1
1	4

→ vertex (Minimum/maximum)



→ All parabola has axis of symmetry. (both side match)
→ Y intercept of quadratic function is c.
like:

- putting 0 in $x \Rightarrow a(0)^2 + b(0) + c = c$
- Axis of symmetry equation: $x = -b/(2a)$
- X-coordinate of vertex: $-b/(2a)$
- If $a > 0$ graph will be upward & vice-versa.

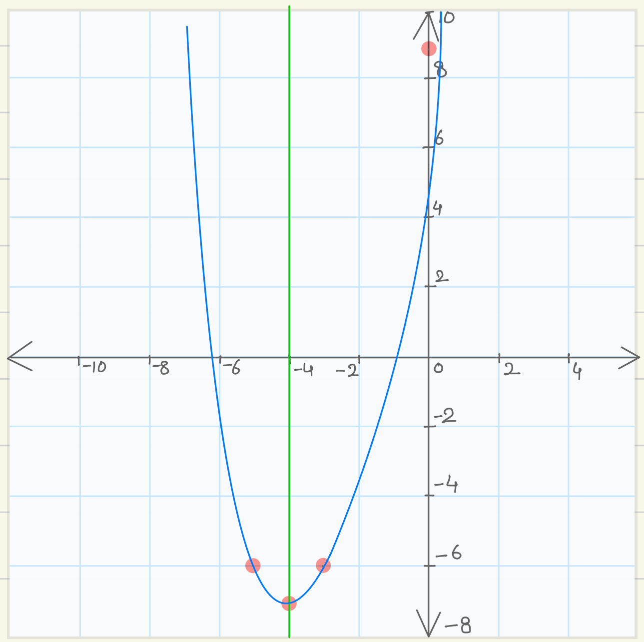
Eg: Graph a function: $f(x) = x^2 + 8x + 9$ $a > 0$

Y intercept by $(x=0) = 9$

Axis of symmetry $= \frac{-8}{2(1)} = -4$

The vertex: $(-4, -7)$

X	Y
0	9
-3	-6
-4	-7
-5	-6



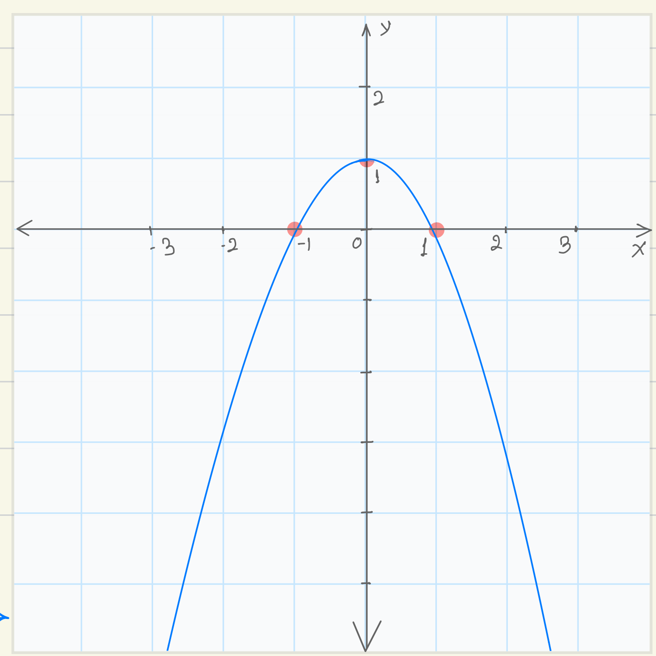
Graph eq. $f(x) - x^2 + 1$.

Y intercept $= 1$

Axis of symmetry $= 0$

vertex $= (0, 1)$

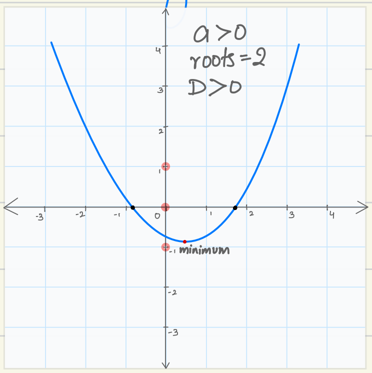
X	Y
-1	0
0	1
1	0

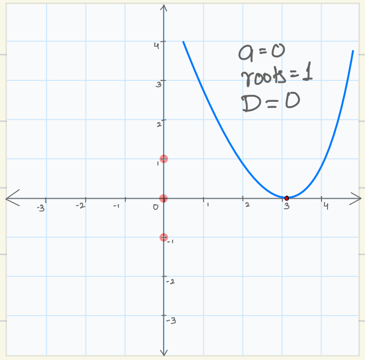


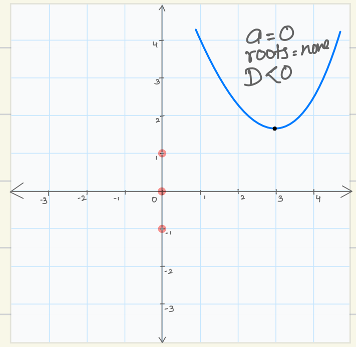
$a < 0 \rightarrow$

- When $a > 0$ it make upward graph & vice versa.

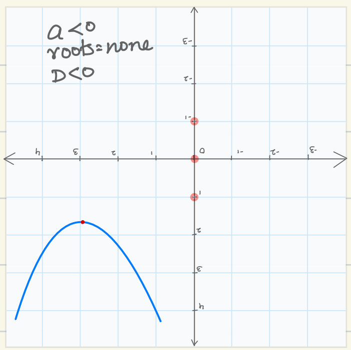
→ No. of roots determine how many time it cuts.

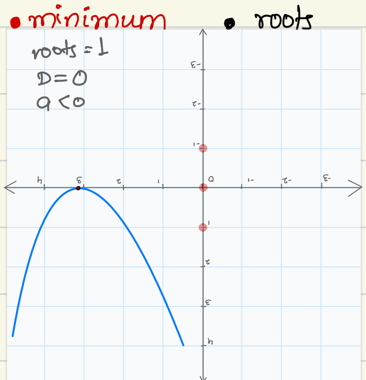


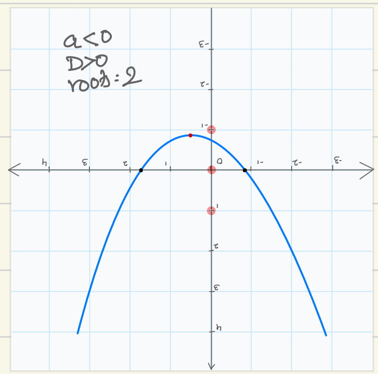




• minimum • roots

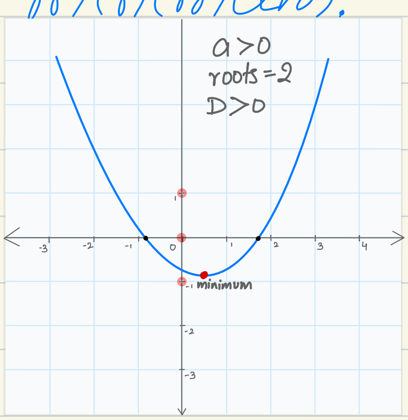






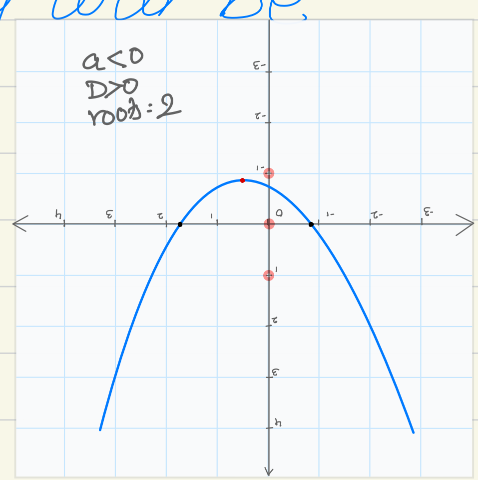
- putting 0 in $x \Rightarrow a(0)^2 + b(0) + c = c$
- Axis of symmetry equation: $x = -b/(2a)$
- X-coordinate of vertex: $-b/(2a)$
- If $a > 0$ graph will be upward & vice-versa.

- When x will be $\frac{-b}{2a}$ than y will be minimum.



for:

$ax^2 + bx + c$



minimum $x = -b/2a$ max $= \infty$

minimum $y = -D/4a$

Range: $\left[\frac{-D}{4a}, \infty\right)$

min $= \infty$

maximum $x = -b/2a$

maximum $y = -D/4a$

Range: $\left(-\infty, \frac{-D}{4a}\right]$

Eg. Let $f(x) = x^2 - 6x + 9$
- Determine whether f has min or max value. If so, than what is value?
- State domain and range of f.

→ Domain of rang is entire line.

$f(x) = x^2 - 6x + 9$

here, $a = 1$, $b = -6$ & $c = 9$

since $a > 0$ function open up. so, It has min-value

- min-value is vertex of parabola.

$= -b/2a = 6/(2 \times 1) = 3$

$\cancel{s}$o, $f(3) = 9 - 18 + 9 = 0$

Q A tour bus in chennai has 500 customer/day. Charge = 40 rs. Owner estamate it lose 10 passanger/day for each ₹4 hike. How much should fare to maximise income?

→ Let x = no. of ₹4/- fare hike. than price/pasanger $= 40 + 4x$ & no. of passanger is $(500 - 10x)$. So, income is:

$I(x) =$

$\Rightarrow (500 - 10x)(40 + 4x) \Rightarrow 40x^2 + 1600x + 20000$

Maximum is possible bcz A is -ve ∩

vertex $= x = -b/2a$

$= \frac{+1600}{+80} = 20$ so, $I(20) = 36000$

This mean company could make 20 time hike $: 40 + 4 \times 20 = 120$ rs.

- Slop of quadratic equation :

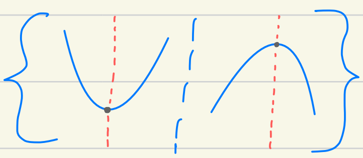
Given quadratic fn , $f(x) = ax^2 + bx + c$, where $a \neq 0$
how to determine slope of function.

Slope of parabola or curve line is :

$$\boxed{m = 2ax + b}$$

If $f(x) = ax^2 + bx + c$ $\quad a \neq b$
then, $g(x) = 2ax + b$

If somehow slope of quadratic fn = 0, than
$g(x) \Rightarrow 2ax + b = 0$
where, $x = \frac{-b}{2a}$ vertex =



Means, the slope of a quadratic f(x) at vertex will be zero.

$m = 2ax + b$

- Quadratic function : Intercept form

- Common roots :

→ Both roots are common = $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

→ One common root = If: $(a_1c_2 - c_1a_2)^2 = (a_1b_2 - b_1a_2)(b_1c_2 - c_1b_2)$
If this condition match.

→ When no roots are common → Above cond. not satisfy.

Quadratic formula:

$\Rightarrow ax^2 + bx + c = 0$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$\Rightarrow x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{2a}}$

$\Rightarrow x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{2a}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\alpha = \frac{-b + \sqrt{d}}{2a}, \quad \beta = \frac{-b - \sqrt{d}}{2a}$

Discriminant :- $d = b^2 - 4ac$

If, $b^2 - 4ac = 0$ (1 root)
$b^2 - 4ac < 0$ (no real root)
$b^2 - 4ac > 0$ (2 real root)

Roots of Quadratic equation property: $= \alpha, \beta$

$\alpha + \beta = \frac{-b}{a}, \quad \alpha \times \beta = \frac{c}{a}$

Diff. of roots, : $|\alpha - \beta| = \sqrt{\frac{b^2 - 4ac}{a^2}} = \frac{\sqrt{D}}{|a|}$

⇒ If ⇒ $D = 0$, than roots will be equal and real.
⇒ If ⇒ $D > 0$, than roots will be real & distinct.
⇒ If ⇒ $D < 0$, than roots will be imaginary & distinct. it always lie in a pair.

POLYNOMIALS

- Layman's prespective :

A polynomial is a kind of methomatical expression which is a sum of several methematical terms. Mathematical expression can be number, variable or product of saveral variables.

- Mathematician prespective :

It is a algebric expression in which only arthematic in addition, substraction, multiplication and 'natural' (non -ve) exponents of variables.

- Degree of a polynomial :

→ The exponent on the variable in term is called degree of variable in that term.

Eg → $4x^2y^2$ $deg(x)=2$, $deg(y)=2$

→ Degree of a term is sum of degree of variable in that term.

Eg = $4x^2y^2 = deg(x) + deg(y) = 2+2=4$

→ Degree of polynomyal is largest degree of any one of the term with non-zero cofficient.

Eg → $3x^2 + 4x^2y^2 + 10y + 1$ = degree(p) = 4.

Power should be whole no.

Degree	Name	Example
0	Cons. poly	$c, 1, 5$
1	Linear poly	$2x+4$, $ax+b$
2	Quadratic poly	$3x^2+2$, $4xy+2x$
3	Cubic poly	$3x^2+2$, $4x^2y+2y+1$
4	Quarticpoly	$10x^4+y^4$, $x^4+10x+1$

→ Addition of Polynomial :

Degree

$P(x) < Q(x) = Q(x)$

$P(x) = Q(x) = \text{any}$

$P(x) > Q(x) = P(x)$

1. $P(x) = x^2 + 4x + 4$, $q(x) = 10$

$$\Rightarrow \quad P(x) = x^2 + 4x + 4$$
$$q(x) = 0x^2 + 0x + 10$$
$$P(x) + q(x) = x^2 + 4x + 14$$

2. $P(x) = x^4 + 4x$, $q(x) = x^3 + 1$

$\Rightarrow x^4 + x^3 + 4x + 1$

3. $P(x) = x^3 + 2x^2 + x$, $q(x) = x^2 + 2x + 2$

$\Rightarrow x^3 + 3x^2 + 3x + 2$

→ Substraction of Polynomial :

1. $P(x) = x^2 + 4x + 4$, $q(x) = 10$

$$\Rightarrow \quad P(x) = x^2 + 4x + 4$$
$$q(x) = (0x^2 \pm 0x \pm 10) \times -1$$
$$P(x) - q(x) = x^2 + 4x - 6$$

2. $P(x) = x^4 + 4x$, $q(x) = x^3 + 1$

$\Rightarrow x^4 - x^3 + 4x - 1$

3. $P(x) = x^3 + 2x^2 + x$, $q(x) = x^2 + 2x + 2$

$\Rightarrow x^3 + x^2 - x - 2$

Let $P(x) = \sum_{k=0}^{n} a_k x^k$, and $Q(x) = \sum_{j=0}^{m} b_j x^j$ then

$$P(x) + q(x) = \sum_{k=0}^{m \vee n} (a_k + b_k) x^k$$

→ Multiplication of polynomial :

1. $P(x) = x^2 + x + 1$ and $Q(x) = 2x^3$

$$\Rightarrow P(x)\,Q(x) = (x^2+x+1)(2x^3)$$
$$= 2x^{3+2} + 2x^{3+1} + 2x^3$$
$$= 2x^5 + 2x^4 + 2x^3$$

2. $P(x) = x^2 + x + 1$ and $q(x) = 2x + 1$

$$\Rightarrow P(x)\,Q(x) = (x^2+x+1)(2x+1)$$
$$= (x^2+x+1)(2x) + (x^2+x+1)$$
$$= 2x^{2+1} + 2x^{1+1} + 2x + x^2 + x + 1$$
$$= 2x^3 + 2x^2 + 3x + x^2 + 1$$
$$= 2x^3 + 3x^2 + 3x + 1$$

To generalise :

$P(x) = a_2x^2 + a_1x + a_0$ and $Q(x) = b_1x + b_0$

$$\Rightarrow P(x)\,Q(x) = (a_2x^2 + a_1x + a_0)(b_1x + b_0)$$
$$= (a_2x^2 + a_1x + a_0)(b_1x) + (a_2x^2 + a_1x + a_0)(b_0)$$
$$= (a_2b_1x^{2+1} + a_1b_1x^{1+1} + a_0b_1x) + (a_2b_0x^2 + a_1b_0x + a_0b_0)$$
$$= a_2b_1x^3 + a_1b_1x^2 + a_0b_1x + a_2b_0x^2 + a_1b_0x + a_0b_0$$
$$= a_2b_1x^3 + a_1b_1x^2 + a_2b_0x^2 + a_1b_0x + a_0b_1x + a_0b_0$$
$$= a_2b_1x^3 + (a_1b_1 + a_2b_0)x^2 + (a_1b_0 + a_0b_1)x + a_0b_0$$

Let $P(x) = \sum_{k=0}^{n} a_k x^k$, and $Q(x) = \sum_{j=0}^{m} b_j x^j$. then

$$P(x)\,Q(x) = \sum_{k=0}^{m+n} \sum_{j=0}^{k} (a_j b_{k-j}) x^k$$

Eg: $P(x) = x^2+x+1$ and $q(x) = x^2+2x+1$

By above formulae:

K	a_k	b_k
0	1	1
1	1	2
2	1	1

Cofficient

Putting in formula:-

Cofficient	Calculation.
a_0b_0	1
$a_1b_0 + a_0b_1$	$1+2=3$
$a_0b_2 + a_1b_1 + a_2b_0$	$1+2+1=4$
$a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0$	$0+1+2+0=3$
$a_0b_4\, a_1b_3 + a_2b_2 + a_3b_1 + a_4b_0$	$0+0+1+0+0=1$

So, result $= P(x)\,Q(x) = x^4+3x^3+4x^2+3x+1$

→ Division of Polynomial:

Q $\dfrac{3x^2+4x+3}{x} = 3x+4+\dfrac{3}{x}$

○ Division of polynomial by another polynomial:

$= \dfrac{3x^2+4x+1}{x+1} = \dfrac{3x^2+3x+x+1}{x+1} = \dfrac{3x(x+1)+1(x+1)}{x+1}$

$= \dfrac{(3x+1)\cancel{(x+1)}}{\cancel{(x+1)}} = 3x+1$

If it is not factor of $x+1$ so,

$$= \frac{3x^2+4x+4}{x+1} = \frac{3x^2+3x+x+4}{x+1}$$

$$= \frac{3x\cancel{(x+1)}}{\cancel{(x+1)}} + \frac{x+4}{x+1} = 3x + \frac{x+1+3}{x+1} = 3x + \frac{\cancel{(x+1)}^{1}}{\cancel{(x+1)}} + \frac{3}{x+1}$$

$$= 3x + 1 + \frac{3}{x+1}$$

Algorithem →

$P(x) = x^4+2x^2+3x+2$ by $q(x) = x^2+x+1$

$$= \frac{x^4+2x^2+3x+2}{x^2+x+1}$$

$$= \frac{x^4+x^3+x^2-x^3+x^2+3x+2}{x^2+x+1}$$

$$= \frac{x^2\cancel{(x^2+x+1)}}{\cancel{(x^2+x+1)}} + \frac{-x^3+x^2+3x+2}{x^2+x+1}$$

$$= x^2 + \frac{-x^3-x^2-x+2x^2+4x+2}{x^2+x+1}$$

$$= x^2 - x\frac{\cancel{(+x^2+x+1)}}{\cancel{(x^2+x+1)}} + \frac{2\cancel{(x^2+x+1)}}{\cancel{(x^2+x+1)}} \frac{+2x}{(x^2+x+1)}$$

$$= x^2 - x + 2 + \frac{2x}{x^2+x+1}$$

So, $\overbrace{\underbrace{\frac{P(x)}{Q(x)}}_{\text{Divisor}}}^{\text{Dividend}} = \underbrace{x^2-x+2}_{\text{Quotient}} + \frac{2x}{q(x)} \longrightarrow$ Remainder

Graph of polynomials:

→ Graph don't have sharp corners.

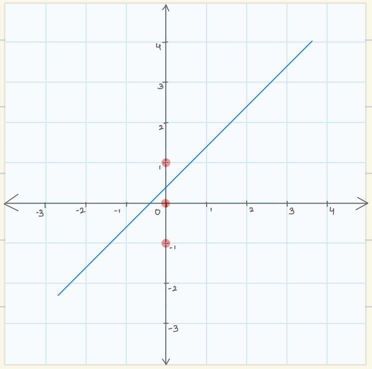
→ Polynomial fn(n) dont have any break.

Zeros of polynomial f(n):-

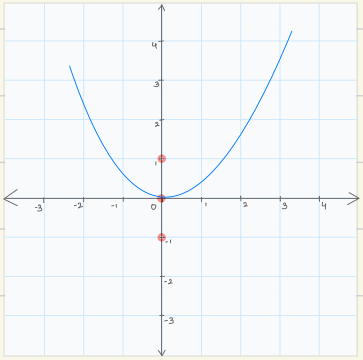
The value of x for which f(x) = 0 are zeros of f.

If zeros are even multiple than graph touch x axis and bounce back or bounceback after touching x-axis.

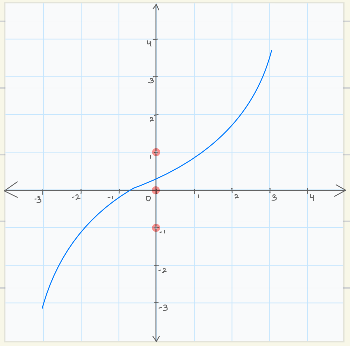
If zeros are odd multiple it cuts directly.



$(x+1)$



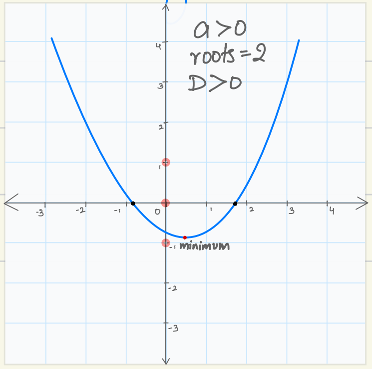
$(x+1)^2$

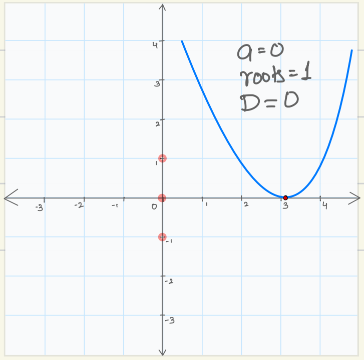


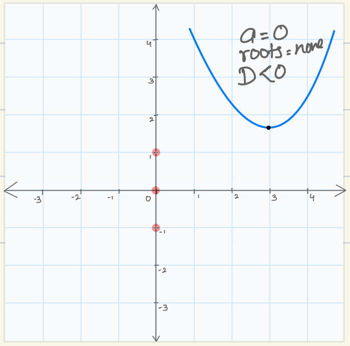
$(x+1)^3$

When $a > 0$ it make upward graph & vice versa.

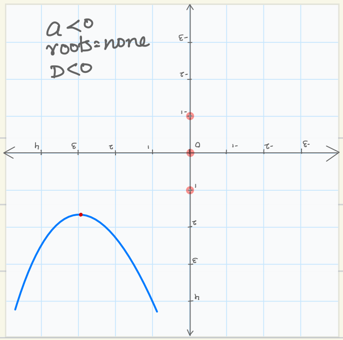
→ No. of roots determine how many time it cuts.

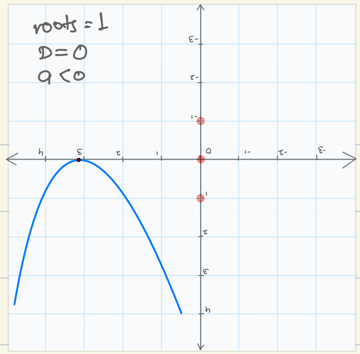


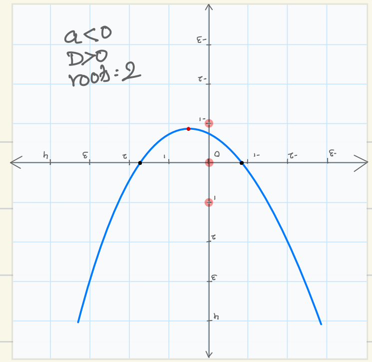




• minimum • roots







☐ End behavior of polynomials:

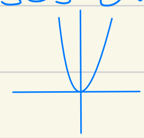
To determine how graph will plot after crossing all its zeros.

So, for:

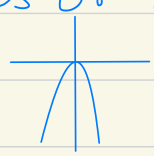
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

Here $a_n x^n$ dominate end behavior of polynomial.

→ If $a_n > 0$ and x^n has even power exponent as x increases or decreases $f(x)$ always go to +ve infinity.



→ If $a_n < 0$ and x^n has even power exponent as x increases or decreases $f(x)$ always go to −ve infinity.



→ If $a_n > 0$ and x^n is of odd power as x increases fx also increases and x decreases fx also decreases. & both go to ∞ on in $+\infty$ & ↓ in $-\infty$.

○ Intermediate value theorem: Let f be polyn. fn)

It states that if $f(a)$ and $f(b)$ have opp. sign, then there exists atleast one value c b/w a and b for which $f(c) = 0$.

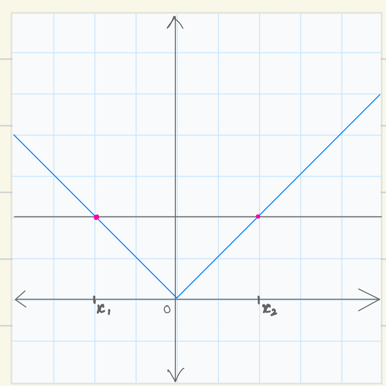
Domain A	Domain B	
1 x	more than 1 $f(x)$	Not a fn.
More than 1 $f(x)$	One $f(x)$	Non-revesable fn.
1 x	one $f(x$	It is fn & reversable.

- One to one function :

A function $f: A \rightarrow B$ is called one-to-one function. If, for any $x_1 \neq x_2 \in A$,

then, $f(x_1) \neq f(x_2)$

- Horizontal line test :- In vertical line test we check if $f(x)$ passes for more than 1 point it is not one-to-one function.



Here $f(x_1) = f(x_2)$

→ If f is an increasing or decreasing fn then f is one to one fn.