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Assignment - 04.

1). Rank = ?

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 6R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_4} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & -4 & -11 & 5 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_4} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$\xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = No of non-zero rows

$\boxed{\text{Rank} = 3}$

2. $\omega \to$ vector space of all sym 2×2 matrices.

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + \cancel{(c-d)}\,(b-c)x + (c-a)x^2.$$

dimension of ω -
Symmetric 2×2 matrix has 2 independent value-
The diag. elem. & off-diag. elem. $[b=c]\,[a]\,[d]$

$\therefore \dim(\omega) = 3$

Rank of T :-
T has all polynomials in P that can come by applying
T mapping to all polynomial of degree 2.
$\therefore$ Rank of $T = 3$

Using Rank-Nullity Theorem,

$$\text{Nullity} = \dim(\omega) - \text{Rank}(T) = 3 - 3 = 0.$$

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③ $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ Eigenvalues & E vectors of A^{-1} & $A+4I$

char eqn $(A) = \left|\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}\right|$

$(2-\lambda)^2 - 1 = 0$

$\lambda^2 + 4 - 4\lambda - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda-3) - 1(\lambda-3) = 0$

$(\lambda-1)(\lambda-3) = 0$

$\boxed{\lambda = 1 \\ \lambda = 3}$

Eigen values of $A = 1, 3$ Property
→ Eigen values of $A^{-1} = 1, \frac{1}{3}$

→ Eigen values of $A+4I = 5, 7$

→ If A in invertible,
Eigen vector of A^{-1} = Eigen vector of A
& Eigen vector of $(A+4I)$ = Eigen vector of A

for $\lambda = 1$,

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\left.\begin{array}{l} x - y = 0 \\ -x + y = 0 \end{array}\right\} \rightarrow \begin{bmatrix} K \\ K \end{bmatrix}$

$\lambda = 3$,

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\left.\begin{array}{l} -x - y = 0 \\ -x - y = 0 \end{array}\right\} \rightarrow \begin{bmatrix} K \\ -K \end{bmatrix}$

④ $\left.\begin{array}{l} 3x - 0.1y - 0.2z = 7.85 \\ 0.1x + 7y - 0.3z = -19.3 \\ 0.3x - 0.2y + 10z = 71.4 \end{array}\right\}$ → diagonal dominance ✓

$z = \frac{(71.4 - 0.3x - 0.2y)}{10}$

$y = \frac{(-19.3 - 0.1x + 0.3z)}{7}$

$x = \frac{(7.85 + 0.1y + 0.2z)}{3}$

for first iterⁿ,

$x = \frac{7.85}{3} = \boxed{2.6167}$

$y = \left(\frac{-19.3 - 0.1(2.6167)}{7}\right) = \boxed{19.5617}$ ⇒ $z = \frac{71.4 - 0.3(2.6167) - 0.2(19.56)}{10}$

$z = \frac{71.4 - 0.785 - 3.912}{10}$

$\boxed{z = 6.67}$

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for second iteration,

base case $(x, y, z) = (2.6167, 19.5617, 6.6703)$

$$x = \frac{7.85 + 0.1(19.5617) + 0.2(6.6703)}{3} = \boxed{7.716}$$

$$y = \frac{-19.3 + 0.1(7.716) + 0.3(6.6703)}{7} = \boxed{-2.36}$$

$$z = \frac{71.4 - 0.3(7.716) + 0.2(-2.36)}{10} = \boxed{6.956}$$

for third iteration,

base case $(x, y, z) = \{7.716, -2.36, 6.956\}$

$$x = \frac{7.85 + 0.1(-2.36) + 0.2(6.956)}{3} \Rightarrow \boxed{x = 3.002}$$

$$y = \frac{-19.3 + 0.1(3.002) + 0.3(6.956)}{7} \Rightarrow \boxed{y = -2.455}$$

$$z = \frac{71.4 - 0.3(3.002) - 0.2(-2.455)}{10} \Rightarrow \boxed{z = 7.099}$$

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(5). Consistent equations → Equations which have solutions

Inconsistent equation → Equations which don't have solutions.

$$\begin{cases} x + 3y + 2z = 0 \\ 2x - y + 3z = 0 \\ 3x - 5y + 4z = 0 \\ x + 17y + 4z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ / $R_3 \to R_3 - 3R_1$ / $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_2]{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(AB) \neq n$

$\Downarrow$

Inconsistent solution

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$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

$\left\{ \begin{array}{l} x_0 = 1 \\ y_0 = 1 \\ z_0 = 1 \end{array} \right\}$ initial case values

$$\left(x = \frac{23 + 6y - 2z}{3}\right)$$

$$\left(y = -15 + 4x + z\right)$$

$$\left(z = \frac{16 - x + 3y}{7}\right)$$

iteration 1.

$$x_1 = \frac{23 + 6 - 27}{3} = \boxed{0.67}$$

$$y = -15 + 4\left(\frac{2}{3}\right) + 1$$

$$y = -14 + \frac{8}{3} = \frac{-34}{3}$$

$$\boxed{y_1 = -11.33}$$

$$z = \left(16 - \frac{2}{3} + \left(\frac{-34}{}\right)\right)\frac{1}{3} \Rightarrow z_1 = \frac{-56}{3.3} = \boxed{-6.22}$$

iteration 2.

$$x_2 = \left(23 + \cancel{6}^{2}\left(\frac{-34}{\cancel{3}}\right) + 2\left(\frac{56}{3.3}\right)\right)\frac{1}{3} \Rightarrow \boxed{x_2 = -10.85}$$

$$y_2 = \left(-15 + 4\left(-10.85\right) + -6.22\right) \Rightarrow \boxed{y_2 = -64.62}$$

$$z_2 = \frac{\left(16 + 10.85 + 3\left(-64.62\right)\right)}{7} \Rightarrow \boxed{z_2 = -37.77}$$

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iteration (3)

$x_2 = -10.85$

$y_2 = -64.62$

$z_2 = -37.77$

$x_3 = \frac{23 + 6(-64.62) + 2(37.77)}{3} \Rightarrow \boxed{x_3 = -289.18}$

$y_3 = -15 + 4(-289.18) - 37.77 \Rightarrow \boxed{y_3 = -1209.49}$

$z_3 = \cancel{3}\ \frac{16 - (-289.18) + 3(-1209.49)}{3}$

$\boxed{z_3 = -3323.29}$

Q(2). $T: W \rightarrow P \qquad T: P_2 \longrightarrow P_2$

(6) $T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

Additivity : $T(u+v) = T(u) + T(v)$

Homogenity : $T(cu) = c(T(u))$

(1). $T(u) = (a+1) + (b+1)x + (c+1)x^2$

$T(v) = (a'+1) + (b'+1)x + (c'+1)x^2$ {Let}

$T(u+v) = (a+a'+2) + (b+b'+2)x + (c+c'+2)x^2$

$\therefore$ satisfies additivity

(2) Let $du = da + dbx + dcx^2$

$T(du) = T(da + dbx + dcx^2)$

$d\,T(u) = d(a+1) + d(b+1)x + d(c+1)x^2$

$\therefore$ satisfies homogenity

Fun^n T satisfies both required conditions to be a linear transformation.

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Linear transformation for comp. vision for rotating images.

When rotating an image, a linear transformation matrix is applied to each and every pixel to achieve the desired rotation angle.
The transformation involves multiplying the original pixel coordinates by the rotation matrix, which alters the position of each pixel based on the rotation angle specified. This process preserves the image quality and structure while repositing the pixels to create the rotated effect.
By utilizing linear transformation in computer vision tasks like image rotation, precise adjustments can be made to images without distorting their content.

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(9)
10.(b). Explain one application of matrix operations in image processing ē example

Let's consider a real-life example where you want to rotate a digital photograph. The photograph is ~~slight~~ slightly tilted. To correct this, you want to rotate the image by a certain angle. This task can be accomplished by using matrix operations.

Given an angle of rotation θ, the rotation matrix for a 2D image rotation is:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

To rotate an image represented by a matrix I (where each element of the matrix corresponds to a pixel value), you perform matrix multiplication with the rotation matrix R:

$$I_{Rotated} = R . I$$

Let us say image has to rotate anti cw by 30°, then

$$R = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \Rightarrow R = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$$

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we then multiply each pixel value in the original image matrix by this rotation matrix to obtain the rotated image.
This process effectively applies a geometric transformation to the image, resulting in the desired rotation. Once rotated, the image will appear corrected, with the horizon level.