Topic 11: Wave Phenomena

Standing Waves

Standing waves are created by the interference of a wave and its reflection. Water waves, waves on strings and standing sound waves in open or closed tubes are some examples of standing waves that can easily be observed.

With water waves for example, waves approaching a harbour can combine with waves that have reflected off the harbour wall to result in large waves that are approximately stationary.

If a rope or rubber tube is waved up and down with one end fixed, a standing wave may be produced.

Standing waves are only formed at certain frequencies for certain strings (or pipes) at certain tensions.

Standing waves are an example of resonance.

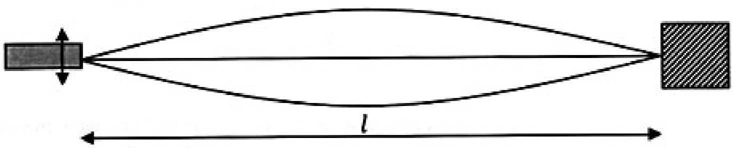
Resonance can be achieved at more than one particular frequency.

The wave produced by the lowest resonant frequency is called the fundamental resonance, or the first harmonic.

The other resonances are called second, third, fourth etc. harmonics.

Standing Wave on String

The diagram shows a string being vibrated rapidly up and down from the left end. At the appropriate frequency (which depends on string length, tension and string mass per unit length) the string vibrates as the diagram shows, with zero-amplitude positions (nodes) at each end and the maximum amplitude position (antinode) half-way along the string.



This is a side view of the string – note that at all points it is vibrating up and down – the three positions shown correspond to maximum positive and negative displacement and zero displacement. The diagram is a realistic picture of what is actually observed but if a snap-shot photograph is taken, the string would lie on a curve somewhere between these extremes – but only along one line (it is only one piece of string!)

Note that this wave section actually represents one-half of a complete wavelength. So, if the (horizontal) length of the string is l, $l = \lambda/2$ and so $\lambda = 2l$

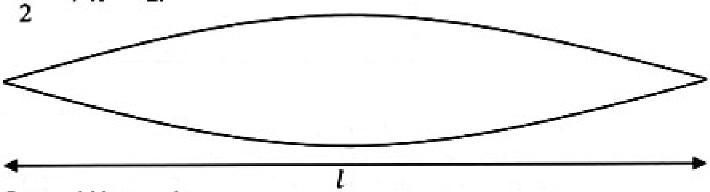
If the vibration frequency is changed slightly, no particular pattern is observed, and the amplitude is significantly reduced (resonance is no longer being achieved). However, other positions of resonance are achievable at higher frequencies. All such standing waves are called harmonics.

We shall now consider simplified diagrams showing some of the harmonics produced by a wave on a string of length l and consider the wavelength of standing wave produced for each.

To construct these harmonics simply remember that there must be a node at each end of the string. For a string of length l (as shown on diagram):

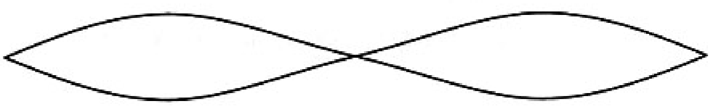
i) Fundamental (First Harmonic)

$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$



ii) Second Harmonic

$\lambda = l$



Example T11.1 Draw the third harmonic. Label nodes, antinodes and state the wavelength in terms of the string length

Notes

- The wavelength of the successive harmonics decrease in a pattern, such that $\lambda = \frac{2l}{n}$ where n is the number of the harmonic (e.g. for first harmonic, $n = 1$, so $\lambda = \frac{2l}{1} \Rightarrow \lambda = 2l$
- Since the speed of the waves in the string (the waves moving along the string and interfering to form the standing wave) is constant for each set-up and string, the frequency is inversely proportional to the wavelength, (using the wave equation; $v = f\lambda$) so if, for example, the frequency of the first harmonic (fundamental) is f, then the second is $2f$, the third; $3f$ etc.

Standing Sound Wave in a Closed Pipe

If air is blown over an empty bottle at certain speeds, vibrations are set up inside the bottle and the resonant frequencies can be heard.

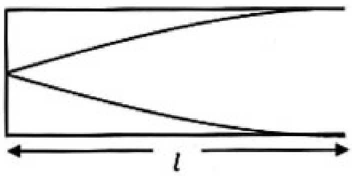
This is the situation with closed pipes: sound travels down the tube and reflects off the closed end, and back. At certain (resonant) frequencies standing waves are produced.

To construct these diagrams, just remember that there is always a node at the closed end and an antinode at the open end

For a closed pipe of length l

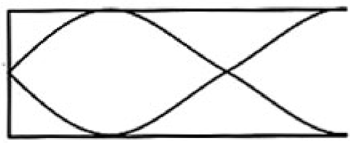
Fundamental (first harmonic)

$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l$



next harmonic

$l = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4l}{3}$



Note that the diagrams actually show how much the air particles are vibrating. At the open end of the tube, we can see that the air particles vibrate with maximum amplitude, and at the closed end, they are stationary. Note also that the air particles are actually vibrating, according to the diagram, left and right – since sound waves are reflected into and back out of the pipe. Sound waves are longitudinal and vibrate parallel to the direction in which the wave is trvelling.

Example T11.2 Draw the next harmonic and state expression for wavelength

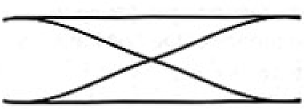
Standing Sound Wave in an Open Pipe

Similarly, resonance and standing waves can be set up in an open pipe – waves, surprisingly, can reflect off the open end. The important difference here is that there are antinodes at both ends. Hence:

For an open ended pipe of length l

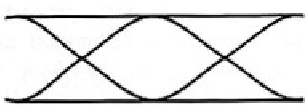
Fundamental

$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$



Second harmonic

$l = \lambda$



Example T11.3

Draw the ***fourth*** harmonic and state its wavelength in terms of the pipe length.

Notes:

For double open-ended pipes,

- General wavelength of n_{th} harmonic: $\lambda = \dfrac{2l}{n}$

In all standing wave situations,

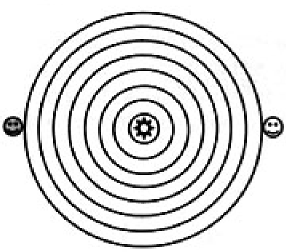
- nodes are separated by a distance of $\dfrac{\lambda}{2}$ (this can be useful – e.g. in above example, there are 3 node-node spaces, making up $\dfrac{3\lambda}{2}$ wavelengths, add on two half node-node separations at the ends gives an extra $\dfrac{\lambda}{2}$. So total number of waves in the length shown is 2. Hence $l = 2\lambda \Rightarrow \lambda = \dfrac{l}{2}$

The Doppler Effect

When an ambulance speeds past a stationary observer, the note heard by the observer changes pitch. When approaching, the pitch is higher than usual and when moving away, the pitch is lower. This is an example of the Doppler Effect: when there is relative motion between a source of sound and an observer, the observed frequency is different from the emitted frequency. Hence a similar effect to that described above would be observed if an observer sped towards a stationary ambulance – the pitch would be greater as the observer approaches, and lower when departing.

To explain this, consider the wavefronts emitted from a sound source, marked ✲ and consider what the observers, marked ☻ and ☺ , will hear (Note that sound waves travel much faster than observers!):

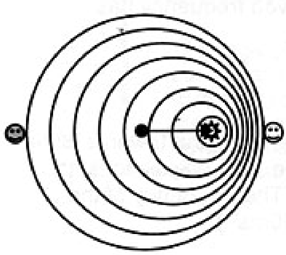
Situation 1 – Observers and source stationary:



Wavefronts are all emitted from same point and thus form concentric circles with equally spaced circumferences

Both observers hear the sound at the same frequency as it is emitted

Situation 2: Source moving towards ☺ and away from ☻

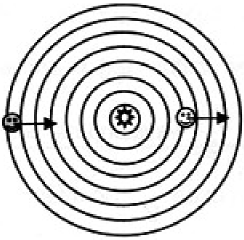


Source moves to right so center of each circular wavefront keeps moving to right
The effect is for wavefronts to bunch-up on the right and to spread apart on the left

Hence ☺ hears a higher than usual frequency and ☻ hears a lower than usual frequency.

Situation 3: Source stationary, both observers moving to right

(☻ towards ☼ and ☺ away from ☼)



Observer ☻ moves towards the wavefronts so observes them more frequently

Observer ☺ moves away from the wavefronts (the ones on his right have already passed him) so observes them less frequently

Therefore the approaching observer hears sound of a higher frequency and the departing observer hears sound of a lower frequency

The Doppler effect is also observed with light (ie electromagnetic waves) – this is why stars moving away from an observer show a "red shift" and stars moving towards an observer show a "blue shift". (red light is the lowest frequency in the visible range and blue/violet, the highest).

Doppler Effect Equations for Sound

We have 4 equations: two for a moving source and two for a moving observer, as follows:

$$f' = f\left(\frac{v}{v \pm u_s}\right) \qquad \textit{moving source}$$

$$(+\ \textit{for source moving away from observer}, -\textit{for towards})$$

$$f' = f\left(\frac{v \pm u_o}{v}\right) \qquad \textit{moving observer}$$

$$(+\ \textit{for observer moving towards source}, -\textit{for away})$$

Where:

$f' = \textit{frequency of sound heard by observer}$
$f = \textit{frequency of sound emitted by source}$
$v = \textit{speed of sound (in air)}$
$u_s = \textit{speed of source}$
$u_o = \textit{speed of observer}$

When using these equations, always check that your observed frequency has changed as expected.

Example T11.4

A car sounds its horn as it travels at a speed of $30ms^{-1}$ past Asaf and towards Buha, two people standing at the edge of a straight road. Describe, with calculations, the difference in the sound as observed by Asaf and by Buha. (The frequency of the sound emitted is 510Hz, and the speed of sound in air is 340ms^{-1})

Doppler Equation for Electromagnetic Waves

$\Delta f = \frac{v}{c} f$ where $\Delta f = change\ in\ frequency\ (frequency\ shift)$
$v = relative\ velocity\ between\ source\ and\ observer$
$c = speed\ of\ light$
$f = frequency\ of\ electromagnetic\ wave\ (light) emitted$

The equation is an approximation that can only be used when v is much less than c

Example T11.5

A star emits light of wavelength 610nm. Light received at the Earth from this star is analysed as having a wavelength of 645nm. How fast is the star moving relative to the Earth and in what direction?

Examples of use of Doppler effect:

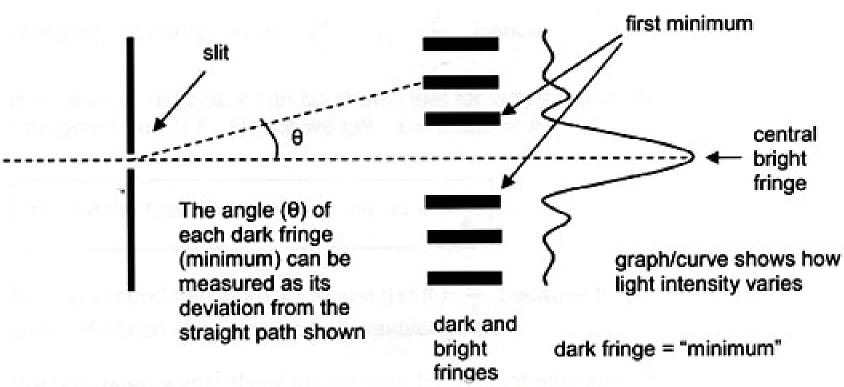
Blood flow rate – ultra-sound signals are directed into the body. The ultrasound machine can both emit and receive/detect the ultrasound signal. The machine can also measure the time delay and hence the depth of reflecting particle and can measure the frequency of the reflected signal. If the reflected signal has a higher frequency, then the reflecting particles must be moving towards the machine, and vice-versa for a lower frequency. The machine can thus calculate the speed of blood flow.

Police speed cameras also work on this principle but use microwaves or infrared radiation rather than ultrasound.

Diffraction

I described and explained diffraction in topic 4. We now consider diffraction at a single slit.

When light is directed towards a very narrow slit (like a single scratch on a painted glass slide) such that the width of the slit (this measurement is called b) is of the order of only a few wavelengths of the light, it emerges in a particular way, due to diffraction. If the light is projected onto a screen a diffraction pattern is observed.



The light emerges in light and dark "fringes". The above diagram shows a simplified version of what is seen. The graph (usually see rotated 90° anti-clockwise) shows how the intensity varies with position. Thus, the central fringe has the greatest intensity and is the brightest.

It is possible to explain this pattern and to predict the positions of the dark fringes, as follows.

When a light-wave enters the slit, the wave is not a single ray, but a beam. Each crest that passes through the slit is called a wave-front. (The wave-fronts are at 90° to the beam).

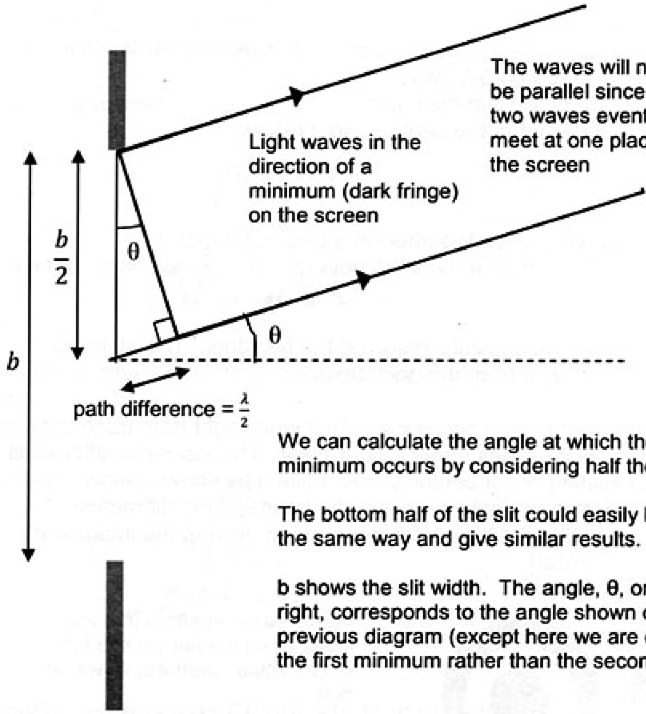
Huygen's Principle tells us that we can consider a wave-front as a line of point sources of light. So effectively, when a wave-front enters the slit, we can think of this as an infinite number of light-rays.

These light rays can interact (interfere) with each other. At different positions along the screen there will be different types of interference.

In the centre a crest from one "point source" or ray will always be met by a crest from another, due to the symmetry. Similarly a trough from one wave at that point will be met by a trough from another. So in the centre constructive interference results. When two crests meet, a larger crest is formed; when two troughs meet, a larger trough is formed. The wave amplitude (and hence intensity) increases.

At a certain point a little way along from the centre, a crest from one wave will meet a trough from another, due to the slightly different distances the two waves have to travel and the asymmetry. Here there will be destructive interference and a dark fringe (minimum) will be observed.

The diagram overpage helps to explain this and allows us to derive a formula to predict the positions of each dark fringe.



We can calculate the angle at which the first minimum occurs by considering half the slit.

The bottom half of the slit could easily be treated the same way and give similar results.

b shows the slit width. The angle, θ, on the bottom right, corresponds to the angle shown on the previous diagram (except here we are considering the first minimum rather than the second).

Simple geometry tells us that the other angle shown on the diagram is the same; hence we can label it θ. An approximation here is that the two waves are indeed parallel – which is very close to true since the screen is so far away in comparison to the slit width.

For the first minimum to occur at the screen, one wave must meet the other exactly half a wavelength ahead. The path difference of the two waves must therefore be $\frac{\lambda}{2}$.

This path difference corresponds to the section of the triangle opposite θ shown in the diagram.

Using trigonometry, $sin\theta = \frac{opp}{hyp} = \frac{\lambda/_2}{b/_2} = \frac{\lambda}{b}$ Hence: $\lambda = bsin\theta$

If we work in radians, it can be shown that for very small angles (which we are dealing with here) $sin\theta \simeq \theta$. Hence we get: $\lambda = bsin\theta = b\theta$

So, for the first minimum: $\lambda = b\theta \ \Rightarrow \theta = \frac{\lambda}{b}$

For the second minimum we would get $\theta = \frac{3\lambda}{b}$ because to get this we would need a path difference of one and a half wavelengths.

We only need worry about the formula for the first minimum on this course.

Example T11.6

Light with a wavelength of 500nm is shone through a slit of width 0.04mm so that the diffraction pattern falls on a screen 4.5m away.

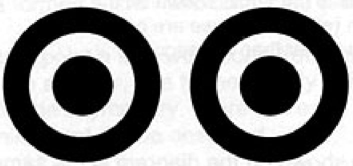
(a) Calculate the position of the first minimum
(b) Find the approximate width of the central bright fringe.

Resolution

Try drawing two small dots, very close together on a piece of paper.
As you move the paper away from your eyes the dots will, at a certain point, appear as a single dot.

If you see both dots you have successfully resolved the two dots. The ability to resolve decreases as the distance from the dots increases.

The reason you see the two dots as a single dot is that when light from each dot enter your eye the light diffracts as it passes through your pupil. This causes a diffraction pattern, for each dot, consisting of concentric circles (disks) as shown below. As the distance between your eye and the dots increase, the images (and diffraction patterns) begin to move together. When there is too much overlap the images are perceived as a single image (dot).



The diffraction pattern that will result when the human eye (or other circular aperture) views two dots.

Note that in this case (with the two patterns separated) the image will appear as two dots.

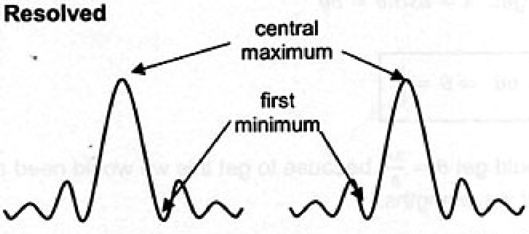
The Rayleigh Criterion

The Rayleigh criterion allows us to know if two points are resolvable (distinguishable as two separate points.

It states that **two points will be resolvable if the central maximum of the diffraction pattern formed of one point coincides with the first minimum of the other** (and this is the closest that the two diffractions can get to each other before they appear to be one point.

Thus, the following diagrams show diffraction patterns to illustrate resolved, just resolved and unresolved image:

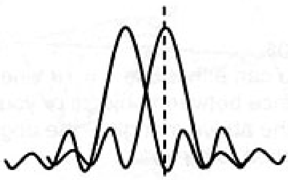
(a) Resolved



There is effectively no overlap between the two diffraction patterns
Blue and red patterns are shown here to aid the argument that follows.

The two diffractions can move together until they reach the limit prescribed by the Rayleigh Criterion. Hence:

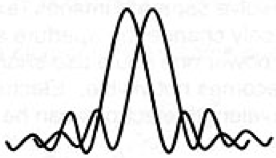
(b) Just resolved



The central maximum from the red pattern has overlapped to the point that it overlaps the first minimum of the blue pattern

If the two patterns overlap any further they will be appear as a single image. Hence:

(c) Unresolved



The blue and red diffraction patterns now go beyond the level of overlap as prescribed by the Rayleigh Criterion: they lines will thus appear as a single line.

Applying the Rayleigh Criterion to human vision

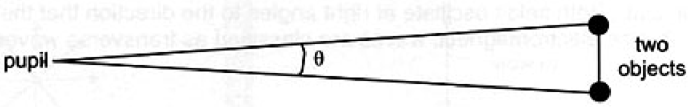
For human vision, the aperture through which the light passes is the pupil of the eye. This circular aperture causes diffraction and the diffraction pattern falls on the retina of the eye as an image.

We have seen that the position for the first minimum for a single slit is given by the formula: $\theta = \frac{\lambda}{b}$

It has been shown (beyond this syllabus) that for a pupil of the human eye (a circular aperture) a similar formula can be applied to determine the minimum distance apart that two objects need to be to be resolvable (still seen as two objects). It is:

$$\theta = 1.22\frac{\lambda}{b},$$

where b is the diameter of the aperture (e.g. pupil) and θ is the angle, as shown in the diagram below:



Maximum resolution power (minimum angle) given by $\theta = 1.22\frac{\lambda}{b}$ where b is pupil diameter (or other lens aperture, such as microscope).

Example T11.7

Two black dots with centres exactly 2mm apart, are drawn on a piece of white paper. How far back from the eyes can the paper be moved until the two dots are seen as one? (assume that the wavelength of light is 550 nm and that the pupil diameter is 2.5mm)

- Note that the angle, θ, must be in radians
- Note also that to solve the distance, you can either use $s = r\theta$ where r is distance from eyes to objects, s is distance between objects or you can use your own geometric solution based on the above triangle – the angles at the left end on above diagram can be assumed to be 90°.

Once you have solved this problem, try it out!

Practical Importance of resolution

Microscope and telescopes must be able to resolve separate images (example: cell walls). To change the resolving power one can only change the aperture size or the wavelength of light used. To increase resolving power one could use shorter wavelengths but one is limited since too short becomes not visible. Electron microscopes can be used – high energy, low wavelength electrons can be detected by sensors thus making them visible.

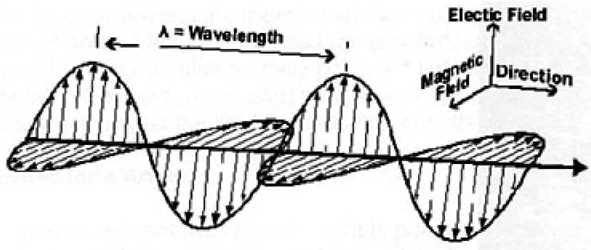
Very large apertures can also be used. Hence good quality telescope lenses typically have very diameters, as do radio telescopes (can be over 50m!).

Reading a CD or DVD involves shining light off pits on the disk. Sensors detect the light received, after reflection, and the information is then used to create sound, music etc. To be able to distinguish information from one pit to another, the pits must be resolvable. To increase the amount of information on a disk, the pits need to be small and close together. However, if they are too close they become unresolvable. Applying the Rayleigh criterion we see that we can increase the resolution power (and therefore be able to minimize pit size) by using light of very short wavelength. Short wavelength laser beams are thus used.

Polarization

Light is a member of the family of the electromagnetic spectrum.
Like all members of this family, it is an energy form that propagates (travels) at approximately $3 \times 10^{8} ms^{-1}$ and consists of oscillating electric and magnetic fields. The oscillating electric field creates an oscillating magnetic field at right angles to the electric field, and the magnetic field creates an electric field and so on. This happens at "the speed of light". Both fields oscillate at right angles to the direction that the wave is travelling in – hence electromagnetic waves are classified as transverse waves.

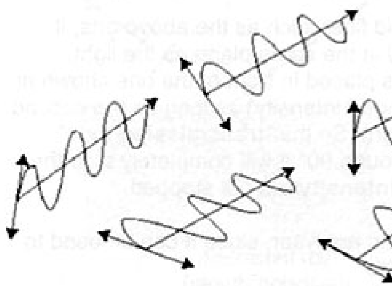
The diagram below shows a simplified view of an electromagnetic wave:



Copyright: ndt-ed.org

However, the diagram in fact shows an electromagnetic wave that has been plane polarized. A natural electromagnetic wave (e.g. light coming from a hot filament) consists of electric field oscillations in all directions perpendicular to the wave direction, and hence magnetic field directions are similarly distributed.

The following diagram shows some possible oscillations, to help illustrate this:

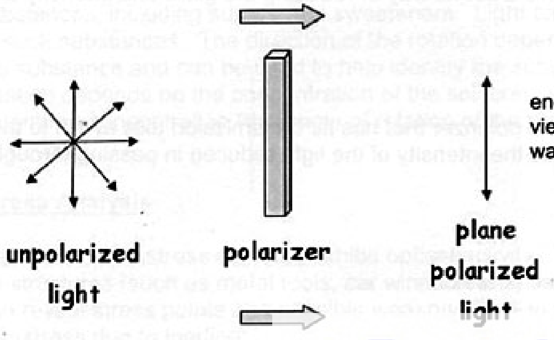


Copyright: iiviinfrared.com

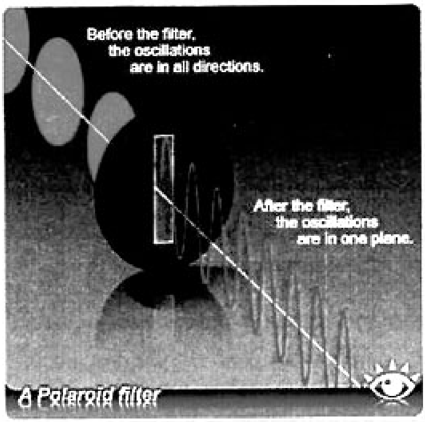
Note that the diagrams only show the electric field oscillations ("vectors") – the magnetic ones would be perpendicular, as shown in the previous diagram.

Light (or any other electromagnetic radiation) that has been polarized has all but one plane of the electric field oscillations removed.

Special materials, called filters (polaroid filters) can do this – if light is passed through them it emerges as plane polarized light, as the following diagram shows:



The following diagram shows another illustration of the same concept:



Copyright: learn.uci.edu

Malus's Law

If plane polarized light is passed through a Polaroid filter such as the above one, it passes through unaffected if the filter is orientated in the same plane as the light. Referring to the above diagram, if a second filter is placed in front of the one shown all the light will pass through unaffected (with unchanged intensity) as long as the second filter is still in line with the plane of the polarized light (So the "**transmission axis**" remains vertical). If the second filter is rotated through 90° it will completely stop the light. At other angles, the light will be reduced in **intensity** but not stopped.

The second filter in this example is referred to as an analyser, since it can be used to analyse/identify the plane of polarization of the light.

Malus's Law allows us to predict the intensity of such a beam of plane polarised light of initial intensity I_0 if the transmission axis is at an angle θ to the plane containing the electric field. The intensity of the transmitted light is given by I, where:

$I = I_0 cos^2\theta$ Malus's Law

Malus's law can also be used to show that a Polaroid filter reduces the intensity of natural unpolarised light by 50%.

Polaroid sunglasses are therefore a useful way of reducing the glare and intensity of bright sunlight.

Example T11.8

Vertically polarized light falls on a polarizer that has its transmission axis at 30° to the vertical. By what percentage has the intensity of the light reduced in passing through the polarizer?

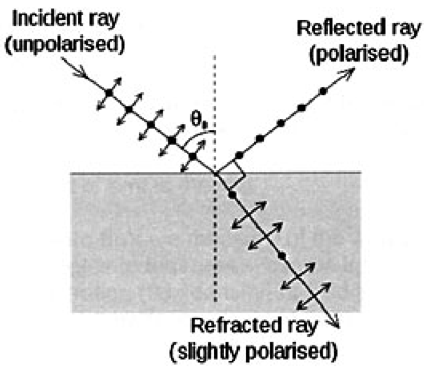
Polarization by reflection

When unpolarized light meets a surface such as water, where the light can both reflect and refract, the reflected ray is polarized to an extent. For example, the glare from the sea contains partially polarized light and this glare can be reduced by wearing Polaroid (polarizing) sunglasses. Depending on the angle of the sunglasses and the way that the light reaches the lens, the intensity of light can thus be cut down.

Brewster's Angle

I have stated that the reflected light is partially polarized. This means that more than one plane of the electric oscillation is still left. The extent of polarization depends on the angle that the incident ray makes with the reflecting surface. Brewster's angle gives the incident angle of the light required in order that the reflected light is totally polarized (i.e. plane polarized – all but one plane of electric field oscillations removed).

Brewsters angle occurs when the refracted ray is at 90° to the reflected ray, as shown in the diagram below:



Brewster's law:

$$tan\theta_B = \frac{n_2}{n_1}$$

If the ray is incident in the air then

$n_1 = 1$ and $tan\theta_B = n_2$

n_2 is the refractive index of the medium (e.g. water)

Example T11.9

Calculate the angle between horizontal and a light ray incident on water if the light ray is to be totally polarized on reflection, given that the refractive index of water is 1.5

<u>Optically active substances</u>

Some substances are able to rotate the plane of plane polarized light. Such substances are called optically active substances. There are many optically active substances, including sugars and sweeteners. Light can be passed through solutions of such substances. The direction of the rotation depends on the chemical bonding in the substance and can be used to help identify the substance. The extent of the rotation depends on the concentration of the solution, and can thus be used to determine concentration (the angle of rotation of the plane polarized light is measured).

<u>Stress Analysis</u>

Materials under stress can also exhibit optical activity. Thus, if polarized light is shone on structures (such as metal tools, car windscreens) the way that the light is reflected can reveal stress points and possible weaknesses – in design or as a result of overstress due to loading.

<u>LCD screens</u>

Liquid Crystal Display (LCD) screens also make use of the polarisation of light. Each liquid crystal forms a pixel on the screen. Plane polarized light is passed through the liquid crystal. These crystals have the special property that they rotate the plane of polarised light by 90° but, if a voltage is applied to the crystal, they do not. Analyser filters are placed in front of the liquid crystals, so that only light that has the same axis of polarization as the incident light passes through. The crystals with no voltage applied rotate the light will therefore not allow the light to pass through and the crystals with voltage applied will allow light to pass through. It is not difficult to imagine that arrays of these crystals can be arranged to form digits (say, 0 to 9) and by applying the right voltage combinations any number can be illuminated.