- Only part of the problems may be graded. But, you have to submit all the problems.
- **Submit only pdf files.**

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1. $\boxed{\text{3 marks}}$ [You should study the self-study material on the standard forms of LP, available in the Canvas/Files.] Put the following linear programming problem in standard form, that is, standard inequality form. (Do not solve it.)

$$\begin{aligned} \text{minimize} \quad & x_1 - 3x_2 \\ \text{subject to} \quad & x_1 + x_2 = 2 \\ & x_1 \geq 3 \\ & x_2 \text{ unconstrained} \end{aligned} \qquad .$$

2. $\boxed{\text{8 marks}}$ Let $C \subset \mathbf{R}^n$ be a given set and $C \neq \emptyset$. Let $f : \mathbf{R}^n \to \mathbf{R}$ be a function. Consider the following optimization problem:

$$\text{(Prob1)} \qquad \text{Maximize} \qquad f(x)$$
$$\text{under the constraint } x \in C.$$

   a. Suppose $f(x) = c \cdot x$ for a given vector nonzero $c \neq 0 \in \mathbf{R}^n$ and $C = B_r$, the ball of radius $r > 0$ centred a the origin: $B_r = \{x \in \mathbf{R}^n \mid |x| \leq r\}$.
      - Express the optimal solution $\bar{x}$ of (Prob1) in terms of c and r. (Hint: Recall the Cauchy-Schwarz inequality.)
      - Draw an illustration (a picture) that explains this in the two dimensional case ($n = 2$.)

   b. (This part is independent of part a.) Define $F_r := \{x \in \mathbf{R}^n \mid f(x) \geq r\}$, and consider

$$\text{(Prob2)} \qquad \text{Maximize} \qquad r$$
$$\text{under the constraint } C \cap F_r \neq \emptyset$$

   Here recall that C is the given constraint set in (Prob1).
      - Is it possible to have more than one optimal solution $\bar{r}$ of (Prob2)? Justify. (Hint: What is the decision variable and the objective function of (Prob2)?)
      - Suppose $\bar{x}$ is an optimal solution of (Prob1) and $\bar{r}$ is an optimal solution of (Prob2). Does the following equation hold?

$$\bar{r} = f(\bar{x}).$$

      Justify your answer.

3. $\boxed{\text{4 marks}}$ For a given $r > 0$, define $B_r = \{x \in \mathbf{R}^n \mid |x| \leq r\}$, the ball of radius r centered at the origin.

a. Express B_r as the intersection of **infinite** number of half spaces.

b. Can one express B_r as the intersection of **finite** number of half spaces? Explain your answer in the two dimensional ($\mathbf{R}^2$) case, by drawing relevant figures. (Your answer for this problem does not have to be rigorous. For example, a convincing figure and explanation of it would be sufficient.)

4. $\boxed{\text{10 marks}}$ Write an one-page essay about the three founding fathers of linear programming: Kantorovich, Von Neumann, and Dantzig. For example, you can explains their contributions to the development of linear programming and optimization theory in general. The essay should be about all these three people. **You must type your article using font size 12pt, and it should not go beyond one page.** Your essay should be written in your own words, though you must use the available sources, e.g. from the internet; cite your sources.