Capacitance:

A capacitor is any pair of conductors separated by an insulating material. When the conductors have equal and opposite charges Q and the potential difference between the two conductors is V_{ab} , then the definition of the capacitance of the two conductors is

$$C = \frac{Q}{V_{ab}}$$

The energy stored in the electric field is

$$U = \frac{1}{2}CV^2$$

If the capacitor is made from parallel plates of area A separated by a distance d, where the size of the plates is much greater than d, then the capacitance is given by

$$C = \epsilon_0 A/d$$

Capacitors in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

Capacitors in parallel:

$$C_{\text{eq}} = C_1 + C_2 + \ldots$$

If a dielectric material is inserted, then the capacitance increases by a factor of K where K is the dielectric constant of the material

$$C = KC_0$$

Current:

When current flows in a conductor, we define the current as the rate at which charge passes:

$$I = \frac{dQ}{dt}$$

We define the current density as the current per unit area, and can relate it to the drift velocity of charge carriers by

$$\vec{J} = nq\vec{v}_d$$

where n is the number density of charges and q is the charge of one charge carrier.

Ohm's Law and Resistance:

Ohm's Law states that a current density J in a material is proportional to the electric field E. The ratio $\rho = E/J$ is called the *resistivity* of the material. For a conductor with cylindrical cross section, with area A and length L, the *resistance* R of the conductor is

$$R = \frac{\rho L}{A}$$

A current I flowing through the resistor R produces a potential difference V given by

$$V = IR$$

Resistors in series:

$$R_{\text{eq}} = R_1 + R_2 + \ldots$$

Resistors in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

Power:

The power transferred to a component in a circuit by a current I is

$$P = VI$$

where V is the potential difference across the component.

Kirchhoff's rules:

The algebraic sum of the currents into any junction must be zero:

$$\sum I = 0$$

The algebraic sum of the potential differences around any loop must be zero.

$$\sum V = 0$$

RC Circuits:

When a capacitor C is charged by a battery with EMF given by $\mathcal{E}$ in series with a resistor R, the charge on the capacitor is

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

where $t = 0$ is when the the charging starts.

When a capacitor C that is initially charged with charge Q_0 discharges through a resistor R, the charge on the capacitor is

$$q(t) = Q_0 e^{-t/RC}$$

where $t = 0$ is when the the discharging starts.

Physics 2 — Formula Sheet for Exam 1

Force on a charge:
An electric field $\vec{E}$ exerts a force $\vec{F}$ on a charge q given by:

$$\vec{F} = q\vec{E}$$

Coulomb's law:
A point charge q located at the coordinate origin gives rise to an electric field $\vec{E}$ given by

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\hat{r}$$

where r is the distance from the origin (spherical coordinate), $\hat{r}$ is the spherical unit vector, and ϵ_0 is the permittivity of free space:

$$\epsilon_0 = 8.8542 \times 10^{-12}\,\mathrm{C^2/(N\cdot m^2)}$$

Superposition:
The principle of superposition of electric fields states that the electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges

$$\vec{E} = \sum_i \vec{E}_i$$

To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements and integrate all these elements:

$$\vec{E} = \int d\vec{E} = \int_q \frac{dq}{4\pi\epsilon_0 r^2}\,\hat{r}$$

Electric flux:
Electric flux is a measure of the "flow" of electric field through a surface. It is equal to the product of the area element and the perpendicular component of $\vec{E}$ integrated over a surface:

$$\Phi_E = \int E\cos\phi\, dA = \int \vec{E}\cdot\hat{n}\, dA = \int \vec{E}\cdot d\vec{A}$$

where ϕ is the angle from the electric field $\vec{E}$ to the surface normal $\hat{n}$.

Gauss' Law:
Gauss' law states that the total electric flux through any closed surface is determined by the charge enclosed by that surface:

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$$

Electric conductors:

The electric field inside a conductor is zero. All excess charge on a conductor resides on the surface of that conductor.

Electric Potential:

The electric potential is defined as the potential energy per unit charge. If the electric potential at some point is V then the electric potential energy at that point is $U = qV$. The electric potential function $V(\vec{r})$ is given by the line integral:

$$V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{l} + V(\vec{r}_0)$$

Beware of the minus sign. This gives the potential produced by a point charge q:

$$V = \frac{q}{4\pi\epsilon_0 r}$$

for a collection of charges q_i

$$V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}$$

and for a continuous distribution of charge

$$V = \int_q \frac{dq}{4\pi\epsilon_0 r}$$

where in each of these cases, the potential is taken to be zero infinitely far from the charges.

Field from potential:

If the electric potential function is known, the vector electric field can be derived from it:

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

or in vector form:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Beware of the minus sign.