

## Topic 4 – Oscillations and Waves

### Introduction

This topic is concerned with the nature and application of waves.

Waves are one means by which energy is transferred from one place to another.

Although a wave is defined as a transfer of energy, not matter, wave-transfer always involves oscillations – and these oscillations can be oscillations of an electromagnetic field (in the case of electromagnetic radiation (EMR) waves, such as light, x-rays etc. or oscillations of particles, such as water waves and sound waves.

In this guide, we shall refer to the oscillations in waves as “particle” oscillations – but must recognise that they can be particle or field oscillations.

### **Examples of oscillations**

A mass bouncing up and down on a spring

A girl bouncing on a trampoline

A teacher pacing back and forth across the room

A child swinging on a playground swing

The air molecules as a sound wave passes

The molecules in any solid material

### Simple Harmonic Motion (SHM)

Simple harmonic motion is a type of motion that accurately describes the oscillating motion of the “particles” in a wave.

It is important to realise that SHM is not merely oscillating motion: it is a specific kind of oscillating motion. Not all the examples of oscillations listed above are described by SHM

SHM involves motion where the acceleration of the particle is always proportional to and in the opposite direction to the displacement of the particle

The first, fourth and fifth examples on the list above are common examples of SHM

All waves involve SHM oscillations

An easy way to visualise SHM is to consider a pendulum.

Pendulum bobs swing with SHM

The central position of the bob (where it hangs when left to stop) is the equilibrium (zero displacement) position.

When the pendulum is set in motion, as it swings outwards its displacement increases, its velocity decreases and the force pulling it back to the centre increases.

This force tells us that the acceleration is always acting towards the centre.

The motion of particles in a wave is exactly the same. A cork on the surface of a water wave moves up and down (showing direction of "particles" the same way that a pendulum bob swings back and forth).

### Some Terms to understand / learn

#### **oscillation**

- the term used to describe the movement of a "particle" (can also be field) from a position "to and fro" back to its original position. Vibration is an alternative term.

#### **medium**

- the "material" through which a wave travels (can also be a vacuum, for light and other forms of electro-magnetic radiation)

#### **displacement**

- the distance of a particle from its undisturbed (equilibrium) position
- usually determined by y axis position on wave graph/diagram
- measured in metres (but not normally stated or calculated)

#### **amplitude**

- the maximum displacement of a particle from equilibrium position
- represented by height (from middle/equilibrium position) of wave diagram
- measured in metres, although not a normally useful measurement

#### **period**

- the time taken for one complete oscillation of a particle ie. time taken for production of one complete wave
- measured in seconds
- symbol for period is T

#### **frequency**

- the number of waves/oscillations produced/observed per second
- measured in hertz
- symbol for frequency is f
- note that  $f = 1/T$

#### **Phase difference**

- The phase difference between two particles along a wave is the fraction of a cycle by which one moves behind the other
- The phase difference between two sources is the fraction of a cycle by which one source moves behind the other
- One cycle corresponds to  $2\pi$  radians or  $360^\circ$

## Defining Simple Harmonic Motion

SHM is defined as motion where the acceleration of the particle is proportional to but in the opposite direction to the displacement of the particle.

i.e.  $a \propto -x$  where:  $a = \text{acceleration}$ ,  $x = \text{displacement}$

The defining equation for simple harmonic motion is as follows:

$$a = -\omega^2 x \quad \text{where:} \quad \omega \text{ is the angular frequency of the oscillating particles}$$

***You must learn this equation; it is not given in the data booklet.***

## Further equations for the motion of particle describing SHM

(given in data booklet, but you must understand and know how to apply)

$$x = x_0 \sin \omega t; \quad x = x_0 \cos \omega t$$

$$v = v_0 \cos \omega t; \quad v = -v_0 \sin \omega t$$

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

$$\omega = \frac{2\pi}{T}$$

$x = \text{displacement of particles at time } t$

$x_0 = \text{maximum displacement (amplitude)}$

where  $v = \text{velocity of particles at time } t$

$v_0 = \text{maximum velocity}$

$\omega = \text{angular frequency}$

## **Angular frequency**

The angular frequency of an oscillating particle is very similar to the frequency – but frequency gives oscillations per second and angular frequency gives angle per second (radians per second –  $\text{rads}^{-1}$ )

Remember,  $360^\circ = 2\pi \text{ radians}$

$$\text{frequency} = 1\text{Hz} \Rightarrow \text{angular frequency} = 2\pi \text{rads}^{-1}$$

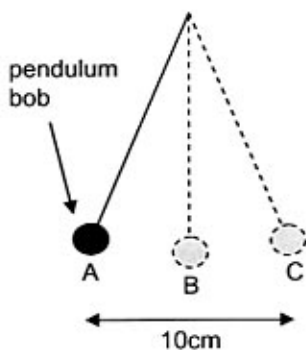
Hence:  $\text{frequency} = 25\text{Hz} \Rightarrow \text{angular frequency} = 50\pi \text{rads}^{-1}$

$$T = 0.1\text{s} \Rightarrow f = \frac{1}{0.1} \text{Hz} = 10\text{Hz}, \quad \omega = \frac{2\pi}{0.1} \text{rads}^{-1} = 20\pi \text{rads}^{-1}$$

### Example T4.1

A pendulum is set in motion so that it swings back and forth, from A through B to C, then back through B to A and so on, as shown in the diagram shown below. Other information is given on the diagram.

We shall assume that it does not lose any energy as it swings, so it keeps swinging to equal amplitudes.



The pendulum bob is timed and it takes 1.25 seconds to move from  $A \rightarrow B \rightarrow C \rightarrow B \rightarrow A \rightarrow B \rightarrow C \rightarrow B \rightarrow A \rightarrow B \rightarrow C$ , so that it has moved a total distance of 50cm in this time.

As shown, the "swing distance" is 10cm

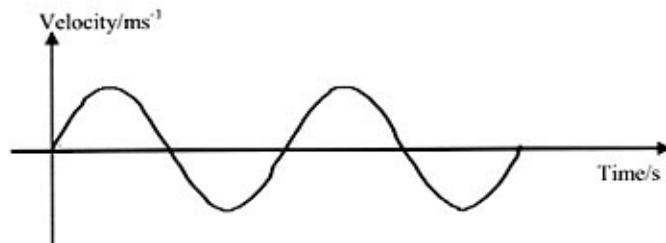
Take rightwards as the positive direction

- What is the amplitude of the motion of the bob?
- What is the period of the motion?
- What is the angular frequency ( $\omega$ ) of the (motion of the) bob?
- What is the maximum velocity of the bob?
- Find the displacement, velocity and acceleration of the bob 0.125s after release
- Find the displacement, velocity and acceleration of the bob 0.25s after release
- Find the displacement, velocity and acceleration 0.375s after release
- Find the displacement, velocity and acceleration 0.6s after release

All the examples in example T4.1 could be applied, exactly the same way as a particle in a sound or water wave, or any other wave.

### Example T4.2

The graph below shows how the velocity of a nitrogen molecule (in air), describing SHM, changes as a sound passes:



- (a) Mark on the diagram with the letter A, positions where the displacement of the molecule is zero
- (b) Mark on the diagram with the letter B, positions where the acceleration has maximum magnitude.
- (c) Explain how the displacement can be found from the graph
- (d) Explain how the acceleration can be found from the graph at any point in time
- (e) Explain how the graph shows that the motion is (i) oscillating (ii) SHM
- (f) Mark on the diagram with the letter C the point when the molecule has made one complete oscillation and explain how the displacement of the molecule can be found at this point in time. State this displacement.

### **Equations for the Kinetic Energy of a particle describing SHM**

$$E_k = \frac{1}{2} m \omega^2 (x_0^2 - x^2) \quad \text{where } E_k = \text{kinetic energy when displacement is } x$$

$$E_{k(\max)} = \frac{1}{2} m \omega^2 x_0^2 \quad \text{where } E_{k(\max)} = \text{maximum kinetic energy}$$

#### **Example T4.3**

Referring to the pendulum bob in example T4.1 and assuming that the bob has a mass of 30g:

- (a) Describe how the energy changes (and the energy forms involved) as the bob swings back and forth (assume no energy loss due to heat/sound as a result of resistance to motion)
- (b) Calculate the maximum kinetic energy of the bob
- (c) Calculate the kinetic energy when its displacement is 3cm
- (d) Calculate the kinetic energy of the bob when its displacement is -2 cm
- (e) Calculate the kinetic energy of the bob after 0.1s from time of release

### **Energy changes during SHM**

The energy changes for SHM in general will always be an oscillation (variation) from kinetic energy to some other form. For the pendulum bob, for example it is KE → GPE → KE etc. (KE = kinetic energy, GPE = gravitational potential energy) For a vibrating atom it would be KE → electrostatic potential energy → KE etc. The electrostatic potential is, provided by the force of attraction between atoms.

## Forced Oscillations and Resonance

### Damping

In the above pendulum oscillation examples, we assumed no energy loss out of the "system" so that the system keeps oscillating forever. However in reality there will be energy losses as a result of resistive forces. **Damping is the process by which energy is removed** from an oscillating system. The result is that the amplitude of oscillation decreases with time until eventually the amplitude is zero and the oscillation has stopped completely.

Forces involved in damping are dissipative because they dissipate (remove) energy. It is logical that the forces always oppose the motion of the oscillating particle.

### Examples of damping

- **Child on a swing** – air resistance and friction in the swing hinges are both damping forces (similar for pendulum)
- **Sound wave** – absorption of vibrations in surrounding materials (so energy transfer is sound → heat causing dissipation of sound (when you speak, the sound can carry – and can often be heard as an echo – but not forever))
- **Water wave** – wind forces and water resistance forces are both causes of wave amplitude decreasing. When a stone, for example, is dropped into a pond and ripples circulate outwards with decreasing amplitude and energy.

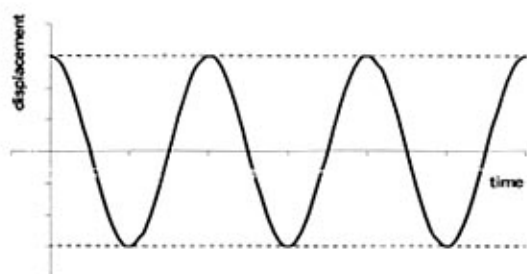
In most cases the transfer of energy out of a damped system is to heat energy (but can also be via sound energy or other forms) and this results in a slight temperature increase of the surroundings and of the system itself.

### Extent of damping

Where there is no energy transfer out of a system the system is said to be undamped. Where energy transfer out of the system is small, damping is said to be light. An example would be a pendulum swinging in air. However, if the pendulum was placed in very thick oil or treacle it could take a long time just to fall to its equilibrium position (and not therefore actually oscillate). This is called heavy damping. If it is placed in oil thick enough to return quickly to its equilibrium position without oscillating, this is called critical damping.

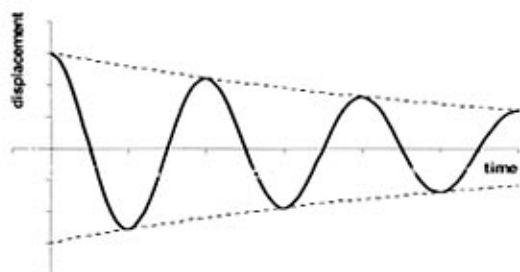
Displacement / time graphs are shown below for these 4 situations.

**Undamped oscillation** (dotted lines show how amplitude varies)



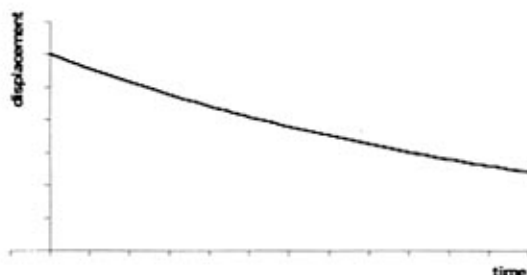
amplitude remains constant

**Lightly damped oscillation** (dotted lines show how amplitude varies)



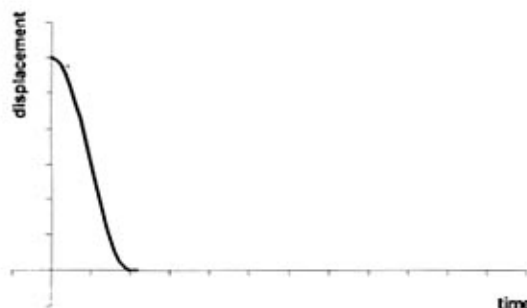
amplitude slowly decreases

**Heavily damped oscillation**



Particle no longer oscillates – amplitude line is same as actual displacement, which reduces gradually

**Critically damped oscillation**



System is returned to equilibrium in shortest possible time and does not overshoot (into negative displacement and another oscillation) – again, amplitude line is same as actual displacement

## Importance of damping

Sometimes it is necessary to dampen oscillating systems. A good example is a car on its suspension. If you push down hard on the front bonnet of a car, it will spring back up again. This is due to the suspension spring (shock absorber) which is necessary to absorb any sudden jolts when, for example, the car is driven rapidly over a bump. A shock absorber is an oscillating system that, when working correctly, has been critically damped. This critical damping is brought about by a cylinder of fluid inside the spring that is compressed. Without this fluid there would not be sufficient damping of the oscillating spring system and driving the car would be very dangerous. When going over a bump, for example, the car would be thrown into a bouncing motion. Part of the regular safety testing of motor cars involves checking that the suspension (shock absorber) system is correctly damped.

### Summary:

**Light damping** – when the oscillations gradually die away

**Heavy damping** – when the system returns slowly towards equilibrium without oscillating

**Critical damping** – when the system returns to equilibrium as quickly as possible without overshooting (and oscillating)

## Natural Frequency

Many objects (systems) can be set oscillating by applying some initial force. The system then oscillates naturally, without applying any further driving force. The frequency that the system oscillates naturally is called the natural frequency of the system.

### Examples:

Attach a **ruler sticking out from the table**, and then twang it. It vibrates at a certain natural frequency. E.g. up and down 5 times per second would be 5 hertz or 5Hz (5 oscillations per second). The natural frequency of a ruler like this depends on the length sticking out.

**Child on swing** – pull back a child then release her – she will swing at a natural frequency (again, dependent on the length of the swing rope)

**Mechanical Constructions such as bridges** – must be designed to be slightly flexible, otherwise there is always a risk of breakage (and allowing slight flexing prevents this). If a bridge is jolted in some way it can oscillate (although damping usually minimises this) and good bridges are designed to be critically damped

Guitar strings, tuning forks, brass instruments, (all instruments in fact) are just a few other examples.

Essentially, everything that is able to oscillate also has its own natural frequency.



## **Forced Oscillations**

A forced oscillation is simply an oscillation that is brought about and maintained by the application of a force

An example, again, is a child on a swing. If the child is pushed each time she swings, the oscillation is called a forced oscillation. The frequency at which the force is applied is called the frequency of the driving force, or the frequency of the forced oscillation.

## **Resonance**

Resonance is a condition, or a description of an oscillating system. This condition is achieved when the frequency of the driving force is exactly equal to the natural frequency of the system being oscillated. When resonance is achieved, the amplitude of the system is maximised and will keep increasing unless it is dampened.

### **Example**

Again, we shall refer to the child on a swing.

When the child is left to swing naturally suppose she does so with a frequency of 1 swing (out and back) every 2 seconds – so the period is equal to 2 seconds and the frequency, 0.5Hz

Suppose now a boy begins to push the child.

He starts by matching the natural frequency at which the child swings, i.e. 0.5Hz – so he applies the force once every 2 seconds (remember  $f = \frac{1}{T} \Leftrightarrow T = \frac{1}{f}$ )

He is able, without increasing the size of the force, to make the child swing higher and higher – amplitude keeps increasing. The boy's applied force (driving force) and the oscillating swing are said to be in resonance.

Now the boy decides to close his eyes and stand at an appropriate place where he can still push the girl. He applies a force, periodically and the same size as before. However this time, since he has his eyes closed, he does not quite match the natural frequency of the child swinging. The result will be that sometimes he will push too early, against the child and sometimes too late and will have reduced or no effect. The result will be that the amplitude of the oscillation will be reduced.

## **Useful examples of resonance**

**Violin** – string is strummed with the bow so that resonance is achieved and sound is emitted

**Microwave ovens** – the frequency of the microwaves used match the natural frequency of water molecules and this causes amplitude of vibration to increase and heating to occur

**Ultrasound** to destroy (break up) kidney stones – ultrasound (high frequency sound) with a frequency that matches the natural frequency of the kidney stone crystals causing large amplitude vibration of particles and shattering of the stones

### Examples where resonance is a nuisance

**Aircraft shattering windows** – frequencies of sounds emitted from certain jet engines can match the natural frequency of glass sheets (which would depend upon the size of the sheet). This causes large amplitude vibration and shattering if damping is insufficient

**Bridge collapse** – soldiers marching over a bridge always break step to avoid a continuous and significantly large oscillating driving force. If not, there is a chance that they could set up a vibration matching the natural frequency of the bridge and cause damage if this oscillation became too large. Famously, in 1940 a gusting wind set a Tacoma Bridge vibrating which resulted in its collapse. The designers had not considered correctly that it is not only the size of the force of a wind that needs to be taken into account, but also the resonance effect and the frequency of a gusting wind. Do a search on youtube to see this impressive bridge collapse.

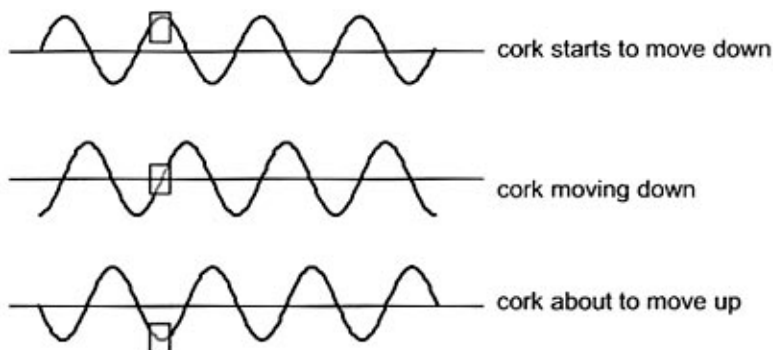
**Vibrations in cars** – at certain speeds some cars (particularly older ones) or parts on the car, such as mirrors or the steering wheel, begin to vibrate. This is when the vibration caused by the engine (driving force) matches the natural frequency of the system. Such systems need damping mechanisms (e.g. rubber mounts) to counter this effect.

### Wave Characteristics

Waves are the means by which energy is transferred. All waves also involve oscillations.

- Note:**
- energy, not matter, is transferred
  - there is actually no net (overall) transfer of energy resulting from a standing wave – energy is effectively reflected back and forth between two points
  - where there is a net transfer of energy, the wave is called a progressive wave.

To help understand the concepts of wave motion and particle motion within a wave, consider a side view of part of a water-wave with a cork floating on it, as follows:



- The wave shows 4 complete cycles, or oscillations
- Wavelength is, therefore, the total horizontal length of the wave shown divided by 4
- The wave is moving from left to right - this can be noted by the position of the crest that starts at the same position as the cork, and has moved forwards half a wavelength on the third wave shown
- The straight horizontal lines show the equilibrium position of the wave – this corresponds to the position of undisturbed water
- The energy movement corresponds to the wave movement: from left to right
- The motion of the particles (water molecules) is up and down – shown by the movement of the cork
- If the next 3 corresponding diagrams are drawn (as above) the wave would have moved forwards by one complete wavelength. The cork would have also moved through one complete oscillation (up and down). Hence, a wavelength corresponds to an oscillation of the vibrating particles.
- This (water wave) is an example of a transverse wave

### Types of wave:

- 1) Longitudinal- "particle" oscillations (vibrations) are parallel to direction of wave.
- 2) Transverse - "particle" oscillations are perpendicular to direction of wave motion

### Note – particles are

air atoms/molecules in case of sound waves (in air)  
 water molecules in case of water waves  
 electromagnetic field in case of electromagnetic radiation

### More Terms to understand / learn

#### Wavelength

- the distance along the axis of the wave from one part of the wave to the next occurrence of this part (e.g. crest to crest or trough to trough)
- measured in metres
- symbol for wavelength is  $\lambda$

## Wave speed

- the speed that a wave travels. Can be found by finding the speed of a particular point on a wave – e.g. crest for a water wave. Can also be found by finding the rate at which energy is transferred – e.g. sound waves: measure how fast the sound travels

### crest

- the point on a wave with maximum positive displacement

### trough

- the point on a wave with maximum negative displacement

### compression (maximum pressure)

- term used to describe region where particles (e.g. air molecules) are closer together than they would be in their normal equilibrium state.

### Rarefaction (minimum pressure)

- term used to describe region where particles (e.g. air molecules) are further apart than they would be in their normal equilibrium state.

## Intensity

- term used to describe the energy of a wave
- more intense light is brighter; more intense sound is louder
- $\text{intensity} \propto \text{amplitude}^2$  so, if the amplitude of a wave doubles, the intensity (energy) of the wave multiplies by 4 (increases by a factor of 4)

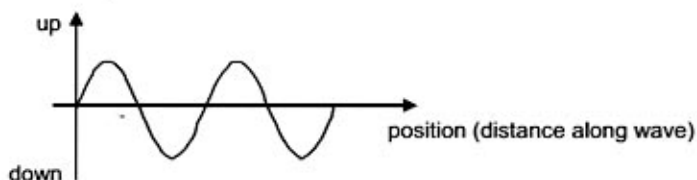
## Wave models (graphs)

The conventional sine-curve shaped wave is actually a mathematical graph showing the displacement of the particles within a wave, as follows. Pay particular attention to the description of the horizontal axes of each of the four graphs shown.

### Transverse Waves

We should be familiar with two different graph representations:

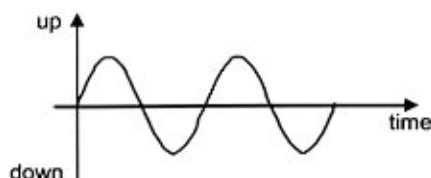
#### displacement – at one instant in time



This wave shows how displacement varies along the wave. It is a good visual representation of a real transverse wave – for example, a water wave. Horizontal distance from peak to peak gives wavelength and the maximum height of the wave, from the wave axis (x axis on graph) gives us the amplitude of the wave – which corresponds to the maximum displacement of the particles in the wave. A picture of

a water wave would give us this graph – with correct displacement; distance coordinates, measured directly off the picture.

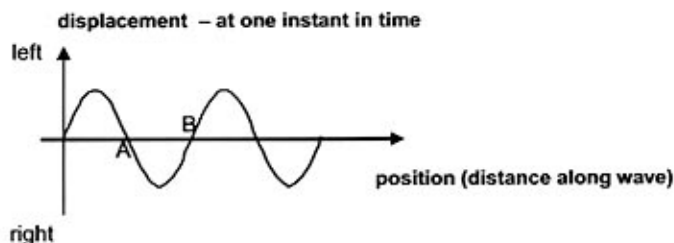
### displacement – at one position on the wave



This wave shows how displacement at a single point on the wave varies as time progresses. It is therefore not a good visual representation of the wave – since if we can see the particles (e.g. water wave) we see the whole wave, not just a single point on the wave. This type of wave can easily be misinterpreted – horizontal “distance” from peak to peak on the graph does not give wavelength – because it is not a distance, it is a time. The time from peak to peak therefore gives the period of the wave and, as before, the maximum height of the wave, from the wave axis (x axis on graph) gives us the amplitude of the wave – which corresponds to the maximum displacement of the particles in the wave.

### Longitudinal Waves

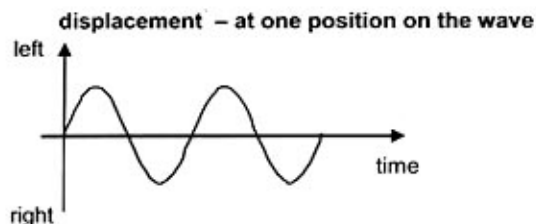
Again, we should be familiar with two different graph representations:



This wave shows how displacement varies along the wave. It is not a good visual representation of a real longitudinal wave – for example, a sound wave – since the graph shows left displacement upwards, visually and right displacement down, visually. In reality we would see the particles moving to the left and right, parallel to the axis of the wave, and not up and down, perpendicular to the direction of wave propagation. Distance from peak to peak gives wavelength and the maximum height of the wave, from the wave axis (x axis on graph) gives us the amplitude of the wave – which corresponds to the maximum displacement (left or right) from the usual equilibrium position of the particles in the wave.

Longitudinal waves are often called compression waves because they consist of a series of compressions – where particles are closer together than usual, (compressed) – separated by rarefactions. In the wave-graph above, point A, where the graph first crosses the x-axis, corresponds to a rarefaction, since particles to the left are displaced leftwards (as shown by the upwards part of graph) and particles to the right are displaced to the right (as shown by the downwards part of graph). Using

a similar explanation, point B corresponds to a compression, as does the origin point of the graph.



This wave shows how displacement at one point on the wave varies as time progresses. Using similar arguments to those used in the previous two graphs, it is not a good visual representation of a real longitudinal wave. Distance from peak to peak gives period and the maximum height of the wave, from the wave axis (x axis on graph) gives us the amplitude of the wave – which corresponds to the maximum displacement (left or right) of the particles in the wave.

### Wave Relationships

One wavelength on a displacement / time graph represents the period of the wave  
 One wavelength on a displacement / position graph represents the wavelength of the wave

For all waves: wave speed = frequency x wavelength

$$\{ v = f \times \lambda \}$$

#### **Derivation:**

- distance travelled by wave per second = number of waves passing a point per second x length of each wave
- but speed of wave = distance travelled by wave per second
- speed of wave = number of waves passing a point per second x length of each wave (wavelength)
- but number of waves passing a point per second = frequency

**$\therefore$  speed of wave = frequency x wavelength**

#### **Example T4.4**

- (a) calculate the frequency of gamma rays with a wavelength of  $3.8 \times 10^{-13} \text{ m}$
- (b) calculate the speed of sound in air given that the wavelength of a middle C note has a frequency of 256Hz and a wavelength of 1.30m
- (c) a canoeist riding waves is at one moment at the peak of a wave and 0.4 seconds later is at the next trough of the wave. Given that the wave peaks are 2.5 metres apart, calculate the speed of these waves.

## Electromagnetic Spectrum (EMS)

The EMS is a group of waves that are all forms of electromagnetic radiation: rather than oscillating matter, they consist of oscillating electric and magnetic fields.

All waves in the EMS travel at the same speed (speed of light,  $c$ ) ie.  $3.0 \times 10^8 \text{ ms}^{-1}$  (in a vacuum)

You need to know the waves in the electromagnetic spectrum, and their approximate wavelengths. This information is as follows:

Use the mnemonic: GAXUVIMR to remember the waves in order of wavelength, shortest first

$10^{-15} \text{ m}$	Ga	gamma rays
$10^{-12} \text{ m}$	X	X rays
$10^{-9} \text{ m}$	Uv	Ultra-Violet radiation
$10^{-7} \text{ m}$	V	Visible radiation (light)
$10^{-5} \text{ m}$	Ir	Infra-red radiation
$10^{-2} \text{ m}$	M	Microwaves
$10 \text{ m}$	R	Radio waves

It should be noted that each type of wave in the list above actually has a range of wavelengths. The different wavelengths, for example, of visible light give us the different colours of the visible spectrum (ROYGBIV)

### **Example T4.5**

Using the fact that audible sound has a frequency range of 20 Hz to 20 kHz and travels at  $330 \text{ ms}^{-1}$  and using the data given above, do some calculations and to describe how sound wave frequencies compare to those of waves in the EMS

## The Behaviour of Waves

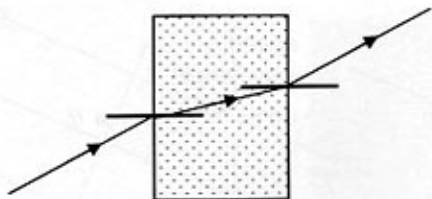
- Reflection**
- when a wave meets a surface it may be reflected  
angle of incidence = angle of reflection
  - wave can be reflected, transmitted or absorbed by a medium
  - the wave may refract as well as reflect on entering the new medium

- Refraction**
- wave enters a new medium
  - speed changes
  - wavelength changes
  - frequency remains the same
  - direction changes (unless incident angle =  $0^\circ$ )

If wave enters a medium in which it travels slower:

- its speed decreases
- its wavelength decreases
- its frequency remains the same
- it bends towards the normal

Example – light passing through air, then a glass prism



#### Notes:

- The light bends towards the normal (line perpendicular to surface-boundary) as it enters the “slower” medium (glass)
- Waves never bend past the normal line
- The amount of bending (ie. the angles) can be calculated using Snell’s Law (covered later)
- The light bends away from the normal as it passes from the glass to the “faster” medium (air)
- The ray entering the glass prism is parallel to the ray emerging from the prism because the two faces of the prism are parallel and the amount of bending at the first surface is the same as the amount of “unbending” at the second.

#### Snell’s law

Snell’s law connects the speed, wavelength and direction of an incident wave with that of the same wave once it has been refracted.

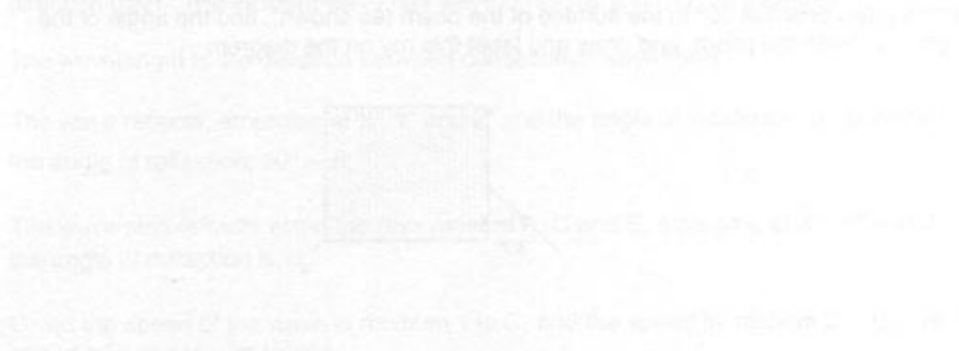
Snell’s Law states that, for a certain boundary (e.g. air  $\rightarrow$  glass) the following ratios are always the same, whatever the incident angle:

speed in medium 1    speed in medium 2

wavelength in medium 1    wavelength in medium 2

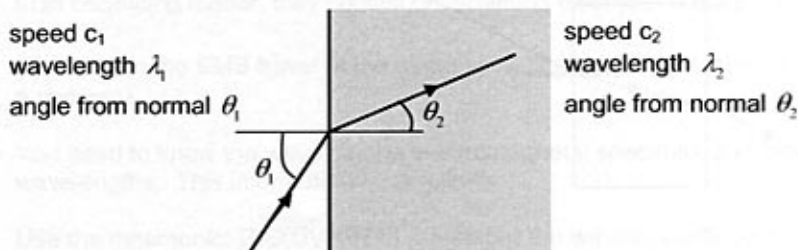
sine of angle in medium 1    sine of angle in medium 2

The following diagram explains this in more detail.





For a ray refracted as follows:



$$\frac{c_1}{c_2} = \frac{\lambda_1}{\lambda_2} = \frac{\sin \theta_1}{\sin \theta_2} = a \text{ constant (refractive index)}$$

If medium 1 is a vacuum or air, the constant is called the refractive index of the second substance. The refractive index therefore tells us how much faster light is in air than in the second medium (e.g. glass).

#### Example T4.6

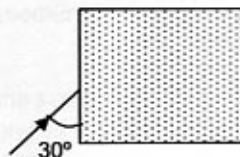
Light is shone into a glass block (prism) at an angle of  $47^\circ$  to the normal (i.e. angle of incidence =  $47^\circ$ ). It emerges at  $29^\circ$  to the normal.

(Note: light can be taken to travel at the same speed in air as in a vacuum – and the speed in a vacuum is given in the data booklet)

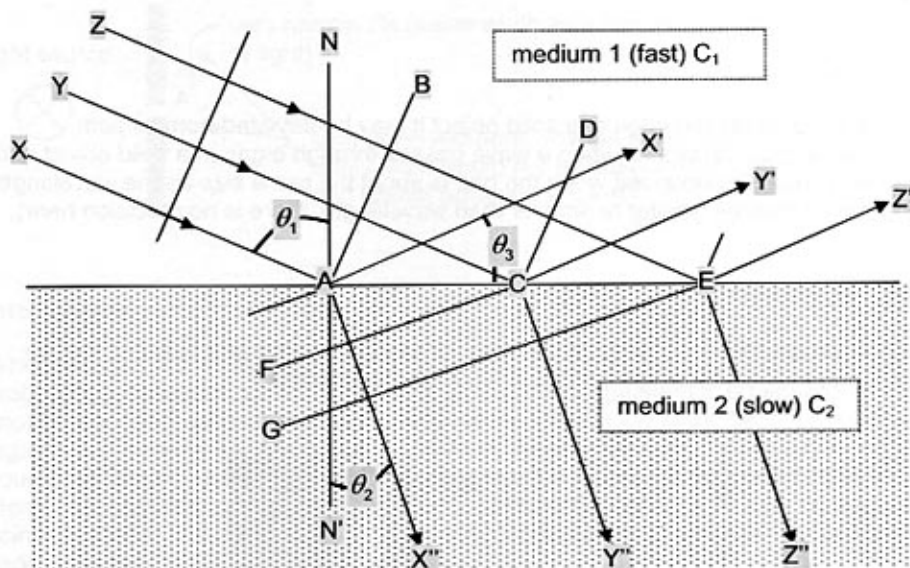
- find the refractive index of the glass used in the prism
- find the speed of light in this glass
- if the wavelength of light in the glass is 600nm, calculate the wavelength of the light in the air
- calculate the frequency of this light in air
- calculate the frequency of this light in the glass

#### Example T4.7

Light travels at  $3.00 \times 10^8 \text{ ms}^{-1}$  in air and  $2.07 \times 10^8 \text{ ms}^{-1}$  in glass. If a light ray is shone into a glass prism at  $30^\circ$  to the surface of the prism (as shown), find the angle of the light ray inside the prism, and draw and label this ray on the diagram



## Refraction and Reflection at a medium



The above diagram shows a wave travelling in medium 1 and approaching a new medium, medium 2. The wave travels slower in medium 2 than in medium 1. As the wave enters medium 2 it refracts, slowing down and bending towards the normal. It also reflects, bouncing back into medium 1 at an equal speed and the angle of the reflection is equal to the angle of incidence.

This wave could be any kind of wave: water, sound or light, for example. The new medium for water would be, for example, a shallower section of water (where it travels slower). For light the boundary may be air, then glass or perhaps air then water, for example. For sound, the boundary could be air then water (say, in a swimming pool) or warm air, then cold air)

A wave can be considered as a beam. Three rays are shown on this beam: X, Y and Z.

Wavefronts are lines that move forward with the wave, perpendicular to the wave direction (ray). The incident wave has wavefronts AB and CD, for example.

The wavelength is the distance between consecutive wavefronts

The wave reflects, emerging at X' Y' and Z' and the angle of incidence,  $\theta_1$ , is equal to the angle of reflection,  $90^\circ - \theta_3$

The wave also refracts when the rays meet at A, C and E, emerging at X'' Y'' and Z'' the angle of refraction is  $\theta_2$

Given the speed of the wave in medium 1 is  $C_1$  and the speed in medium 2 is  $C_2$ , we can use Snell's law as follows:

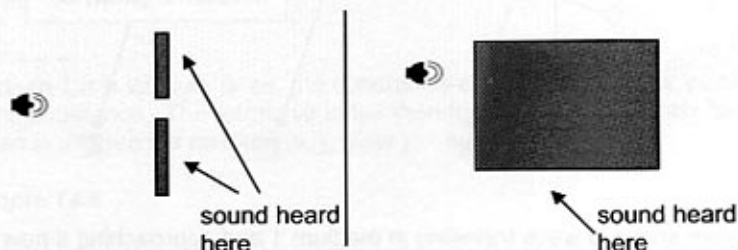
$$\frac{c_1}{c_2} = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} \text{ a constant for this boundary}$$

## Diffraction

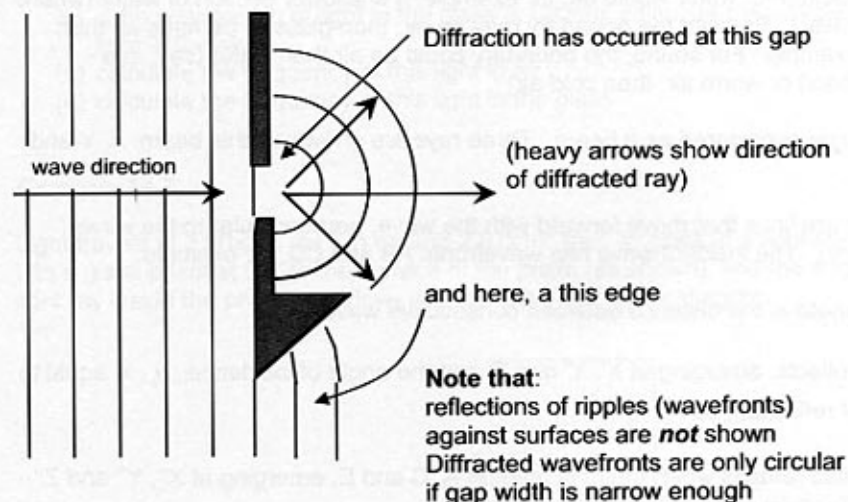
When a wave meets the edge of a solid object it may be deviated from its path. Diffraction is most noticeable when a wave passes through a gap in a solid object and the effect is more pronounced when the gap is about the same size as the wavelength of the waves (can be greater or smaller than wavelength, there is no precision here).

### Examples of Diffraction

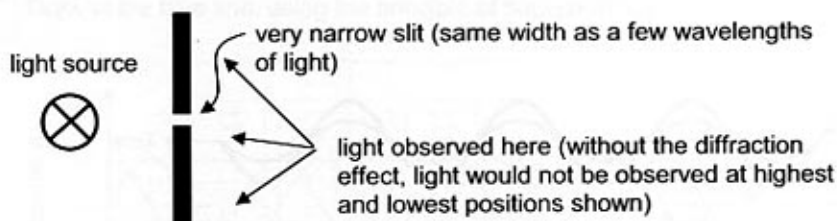
- i) Sound waves passing through a doorway, or past the corner of a building, can be heard "around the corner" (as well as in the expected regions!)



- ii) water waves – ripples show wavefronts and wave-direction



### iii) Light passing through a narrow slit



## Interference

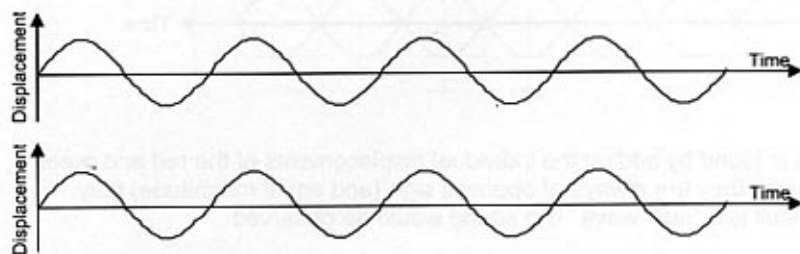
When two or more waves exist at the same time, they may interfere with each other, producing a "resultant" wave. Examples: Sound – two speakers in phase with one another (i.e. producing compressions and rarefactions at the same time) – in different regions in the space surrounding the speakers, sound of low and high volume (loudness) can be observed. Water waves: when two crests from two different waves meet, they combine to form a higher crest (supercrest). Light – light from two different points (but have to be from same initial source) directed on to a screen – bright and dark areas can be observed on the screen.

In each example mentioned above, the change in intensity (loudness for sound, brightness for light etc.) depends on how the waves at the point (place) in question meet each other. The Principle of Superposition can be used to explain these intensity changes:

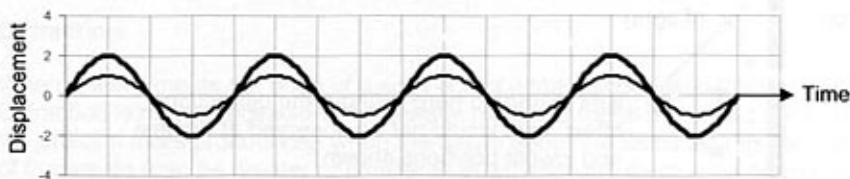
### Principle of Superposition

When two or more waves (have to be the same type of wave) meet, the total displacement at any point is the sum of the displacements that each individual wave would cause at that point

To illustrate this, imagine sound is being observed at a single point. Over time, repeated rarefactions and compressions will be observed (the sound wave). Consider the waves from each source individually:



At all points in time the two waves are in phase (i.e. crests occur at the same time, as do troughs). This results in constructive interference, as follows:



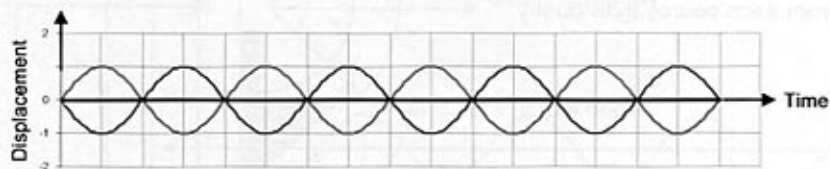
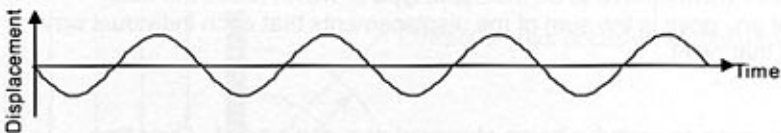
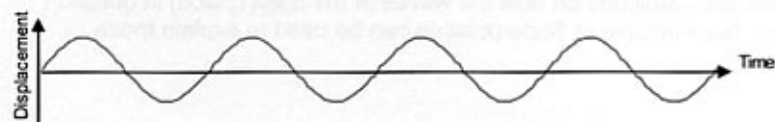
(displacement axis not to same scale as above waves)

The red coloured curve shows the displacement of both the individual waves (they are on top of each other, so they appear as one)

The thick blue curve is obtained by applying the principle of superposition – the displacement at each point along the wave is obtained by adding together the displacements of each red wave

The amplitude (maximum displacement) of the resultant wave is thus double that of either red wave so this is constructive interference.

Now consider two waves that are out of phase (a crest from one source is observed at the same time as a trough from the other):

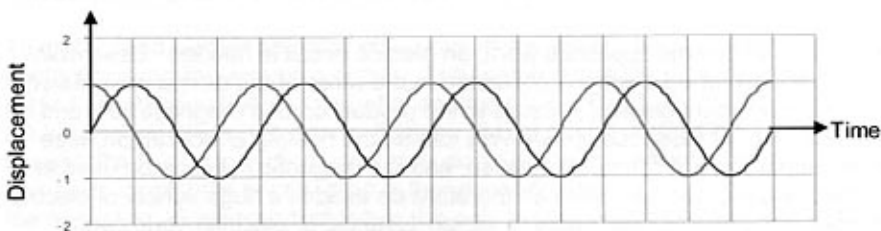


The blue wave is found by adding the individual displacements of the red and green waves – and since they are always of opposite sign (and equal magnitude) they cancel – the result is a "null" wave – no sound would be observed

Finally let us consider two waves that are not in phase but not completely out of phase:

### Example T4.8

Draw in the blue line, using the principle of superposition



The blue wave is the result of the simultaneous existence of the red and the green wave. Consider time 0: green wave displacement + red wave displacement =  $1 + 0 = 1$  Therefore resultant displacement at time, 0 is 1

At time corresponding to 1 unit on the graph: green displacement = 0, red = 1 therefore total = 1, and, after 2 units of time total displacement =  $-1$  (green) + 0 (red) =  $-1$ , etc.

Note that maximum resultant displacement is greater than either individual displacement, so we have constructive interference

This effect (and principle) applies to all types of wave, for example light, water, and sound.