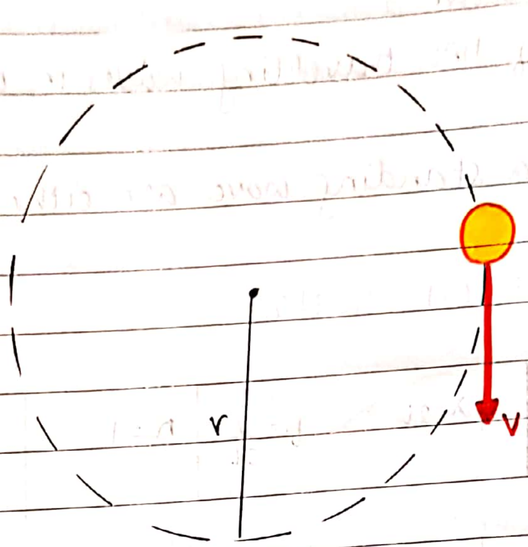
CIRCULAR MOTION & GRAVITATION

CIRCULAR MOTION & ANGULAR SPEED



Total distance = $2\pi r$
Period = T.
∴ Linear speed:

$$v = \frac{2\pi r}{T}$$

$$\text{Angular speed } (\omega) = \frac{\text{angle swept}}{\text{time taken}} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

The velocity vector is at tangent to the circle.
In a short time Δt, the body travels a distance $v\Delta t$.
The distance travelled is an arc of the circle which is equal to $r\Delta\theta$.

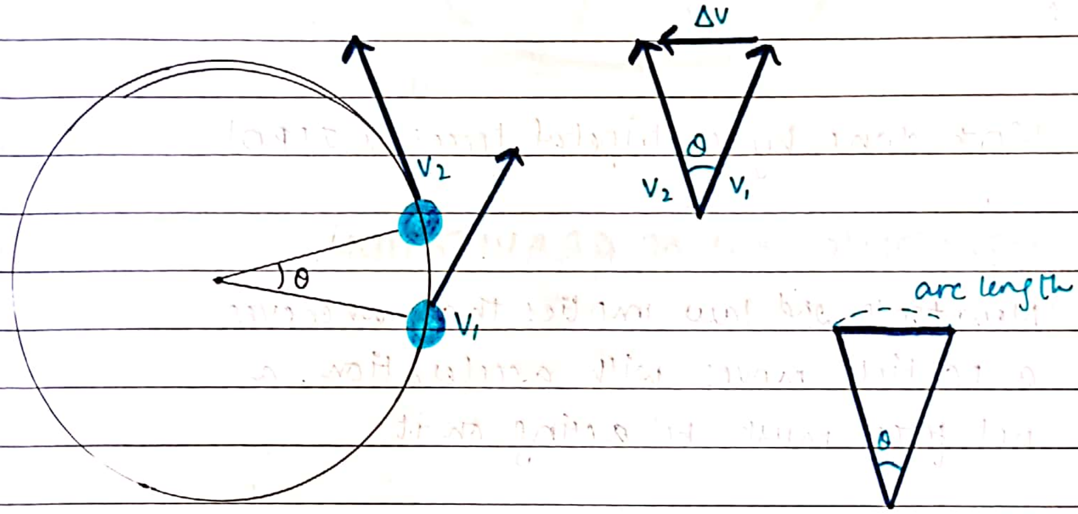
Therefore, $v\Delta t = r\Delta\theta$

$$v = \frac{r\Delta\theta}{\Delta t}$$

$$v = r\omega$$

CENTRIPETAL ACCELERATION

In circular motion, even though the linear speed is constant, the velocity is not. Thus, there is acceleration.



If $\Delta\theta$ is very small, then the distance Δv is approximately an arc of a circle of radius v, & subtending an ~~a~~ angle $\Delta\theta$

$$\Delta v = v\Delta\theta$$

$$a = \frac{v\Delta\theta}{\Delta t} = v\omega$$

$$a = \frac{v^2}{r} \qquad a = 4\pi^2 r f^2 \qquad a = \frac{4\pi^2 r}{T^2} \qquad a = \omega^2 r$$

The centripetal acceleration is directed towards the centre.
There is no such force as centrifugal force
A body in circular motion cannot be in equilibrium & so no force pushing away from the centre is required.

CENTRIPETAL FORCES

$$F = \frac{mv^2}{r} = m\omega^2 r$$

Work done by centripetal forces is ZERO!

NEWTON'S LAW OF GRAVITATION

Newton's 2nd law implies that whenever a particle moves with acceleration, a net force must be acting on it.

Force of gravitation between two point masses is:

point masses → masses that are very small compared to their seperation.

$$F = \frac{GM_1M_2}{r^2}$$

Applies to planets & sun because they are spherical & of uniform density, so one can assume that the entire mass of the body is concentrated at its centre — as if it is a point mass.

A mass M is said to create gravitational feild in the space around it. Another mass placed at some point near M 'feels' the gravitational field in the form of a gravitational force.

Gravitational feild strength(g) at a certain point is the gravitational force per unit mass experienced by a small point mass 'm' at that point.

$$g = \frac{F}{m}$$

acceleration of free fall.

The force experienced by a small point mass m placed at a distance from mass M is.

$$F = \frac{Mm}{r^2} \cdot G$$

$$g = \frac{M}{r^2}$$

Gravitational feild strength is a vector quantity. Around a single point or spherical mass, it is radial. The feild is not uniform - the feild lines get farther apart with increasing distance from the mass

ORBITAL MOTION

The force of gravitation provides a centripetal force on the particle.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \frac{2\pi r}{T}$$

$$\rightarrow v^2 = \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Period of planets going around the sun is proportional to the 3/2 power of the orbit radius.