Topic Two – Mechanics

Motion

The motion of an object is usually defined using three measurements: displacement, velocity and acceleration.

Less informative/useful quantities about motion are speed and distance travelled. These quantities are defined at certain points in time.

distance travelled length of path taken

displacement the position of an object relative to a defined starting position. This is described by giving the distance and direction from the starting point.

speed rate at which distance is covered/changes (with time)

velocity rate at which displacement changes (with time)

velocity is also referred to as "instantaneous velocity" since it is a measurement at an instant in time, rather than over a period of time. The velocity of an object is always the same as the speed, but direction of motion must also be included.

acceleration - rate at which velocity changes (with time)

Strictly, direction of acceleration should also be given – and in linear motion + represents velocity increasing in the "forwards" direction and – represents velocity increasing in the "reverse" direction.

average speed $= \dfrac{total\ distance\ travelled}{total\ time\, taken}$

Average speed can also be thought of as the constant speed in the given time to cover the same distance in question in that time.

average velocity $= \dfrac{total\ displacement}{total\ time\, taken}$

Average velocity can also be thought of as the constant velocity in a given time to cover the same displacement in question in that time.

relative velocity - the velocity of one object as seen from another

Relative velocity of A as seen from B = $V_A - V_B$ where V_A and V_B are the velocities of A and B

Example T2.1

A particle moves at constant speed in a anticlockwise semicircle, radius 12m, taking 3.5 seconds to travel to a point due north of the starting point.

(a) find the average speed
(b) find the average velocity
(c) find the speed and velocity after 1 second
(d) is the object accelerating?

Interpreting Motion Graphically

Graphs provide a very powerful way of describing any type of motion, whether it be uniformly accelerated, starting-and-stopping, or whatever.

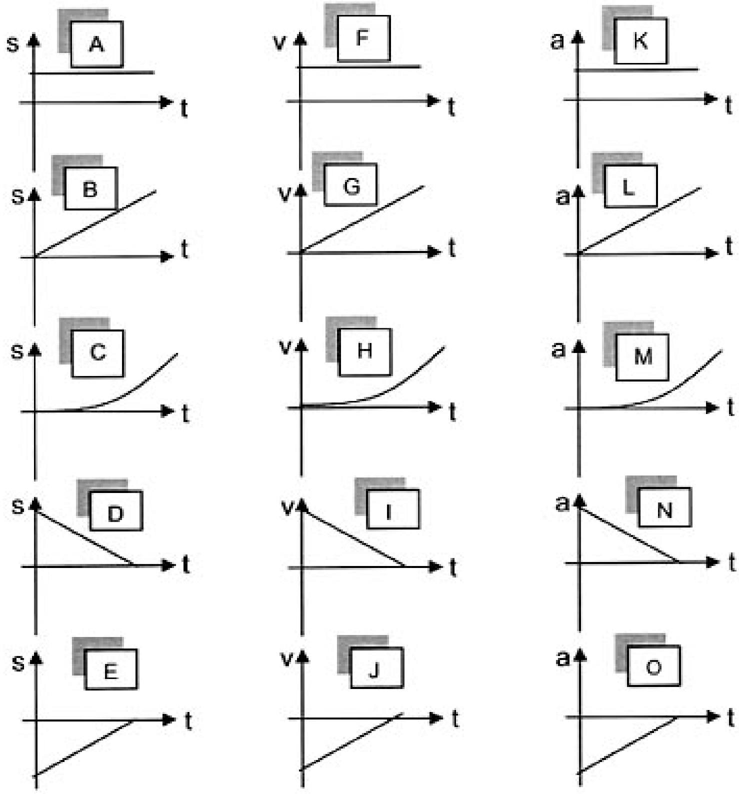
In all cases:

- The gradient of a displacement versus time graph at any point in time gives the velocity of the object
- The gradient of a velocity versus time graph at any point in time gives the acceleration of the object
- The area under a velocity versus time graph from one point in time to another gives the displacement of the of the object at the second point relative to the first (it is often more useful to find area from t=0 to any particular point in time – to give displacement from starting point)

These simple rules lead to the derivation of the equations of motion (done later).

Example T2.2

The graphs below (labelled A to O) give information about the motion of 15 different objects. t represents time, s represents displacement, v; velocity and a, acceleration. Use the graphs to answer the question that follows.



Complete the table below by inserting $0 \; or \; + \; or \; -$ in the sign columns – to show the sign of the displacement, velocity or acceleration throughout the time on the graph and inc (for increasing) or dec (for decreasing) or con (for constant) in the change columns, to show how displacement, velocity or acceleration change as time elapses.

(for the velocity graphs (F-J), assume that the object starts (when t=0) at zero displacement and for the acceleration graphs (K-O), assume that the object starts at zero displacement and velocity)

	displacement		velocity		acceleration	
graph	sign	change	sign	change	sign	change
A						
B						
C						
D						
E						
F						
G						
H						
I						
J						
K						
L						
M						
N						
O						

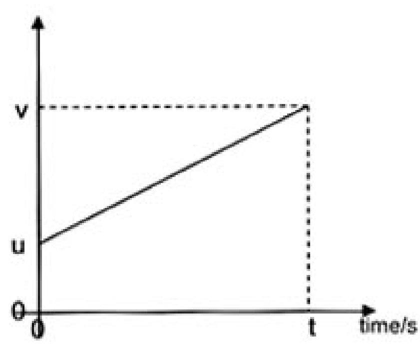
Equations of Motion – Derivation

These equations apply only to motion that is uniformly accelerated.

Consider a general example of such motion

initial velocity (at time t = 0) = u
final velocity (at time t = t) = v
acceleration = a (constant, over all time)
displacement = s

The velocity versus time graph for such motion would look as in the diagram on the right:



Derivation of Equations of Motion from graph

Example T2.3

Derive the equations of motion from the graph shown just previously

Hint: start by finding the gradient (slope) and noting that acceleration = slope, then find area (=displacement)

Falling Objects

When solving problems involving objects falling under the influence of the Earth's gravitational field, we usually assume:

- that the gravitational field strength, g, (which is equal to, and also called, acceleration due to gravity) is constant (and taken to be $9.81ms^{-2}$)
- that the weight of the object is the only force acting on it

If these two conditions do indeed apply it means that the acceleration of the object is constant and the object is said to be in free-fall

$$weight\,(F) = mg$$

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{mg}{m} = g$$

so, for objects in free fall $a = g = 9.81ms^{-2}$ $(on\ surface\ of\ Earth)$

These conditions do not strictly apply in reality. The first condition is very close to true for objects very close to the surface of the Earth (within a few km, certainly). However the second condition ignores air resistance – a force whose magnitude (size) depends upon:

- the size, shape and texture of the surface
- the speed of the object

As the speed of an object increases the air resistance also increases until at a certain speed, the force of air resistance is equal to the weight of the object, the resultant force is then zero and so the object no longer accelerates. The constant speed reached is called terminal speed or terminal velocity.

Example T2.4

A stone is projected vertically into the air, from the ground, at 25 ms^{-1}
(a) Find the maximum height reached
(b) Find the total time taken to hit the ground.

Example T2.5

If an iron ball and a plastic ball are dropped in air from the same height, explain what would be observed and whether the equations of motion would be useful in each case

Forces

Some Examples of Forces (list is not exhaustive)

normal reaction (contact force)
applied force
friction (between solid surfaces)
air or fluid resistance (also called drag) (between solids and liquids or gases)
electric/magnetic/gravitational
weight (often loosely called gravity)
tension

Free body diagrams

A free body diagram shows all the forces acting on the body (and NOT the resultant force). The resultant force acting on the body is thus the vector sum of all these forces.

Examples T2.6

Draw free body diagrams for the following, showing relative size, direction and names of forces.

1) A woman standing on the floor

2) A man falling through the air, ignore air resistance

3) A mass sliding down frictionless slope

4) A parachutist falling at terminal speed

Newton's First law

A body will continue in its current state of motion (velocity) unless acted on by a resultant force.

$$\text{i.e.: } a = 0 \ if \ sum of \ forces (resultant \ force), \sum F = 0$$

If it is known that the resultant force on an object is zero, we can therefore conclude that the acceleration of the object is zero. The converse is also often useful; if it is known that the acceleration is zero, we can conclude that the resultant force is zero.

Example T2.7

A 10 kg mass is on a slope elevated at 40° to the horizontal. The mass remains stationary.

(a) Draw a free body diagram for the mass
(b) Resolve the weight of the mass into components perpendicular to the slope
(c) State the size and direction of the frictional force acting on the mass

We should note that zero acceleration does not imply that a body is motionless – only that its velocity (speed and direction) is constant (unchanging). However, if a body is (and remains) motionless this does imply that the resultant force acting on the body is zero. We need to be very careful here. If a body stops momentarily (for an instant), it can be in a state of acceleration (i.e. speeding up, in a certain direction) – in which case, the resultant force on the body is not zero.

Example T2.8

A 5 kg mass is thrown vertically into the air. The table below describes its state of motion immediately after it is thrown, half way up during its ascent, at the highest point, half way down during its descent and just before it hits the ground. Complete the table by writing + (positive), – (negative) or 0 (zero) to show the direction of displacement, velocity, acceleration and force at each stage. Take the ground level as zero displacement and take the upwards direction as positive direction.

Motion	Displacement	Velocity	Acceleration	Force
Immediately after thrown				
Half - way up				
At highest point				
Half - way down				
just before hits ground				

Equilibrium

A body is said to be in translational equilibrium if the vector sum of all the forces acting on the body is equal to zero. (Note that such a body may not be in rotational equilibrium – net *rotational* forces may also exist). Translational motion is motion from one place to another. If a body is ***not*** in translational equilibrium, the effect of the unbalanced forces will be to accelerate it from one place to another.

Newton's Second law

The acceleration of a body is proportional to the resultant force acting on the body (and is in the same direction)

i.e.. $\sum F = ma$ ($\sum F$ = sum of forces or resultant force)

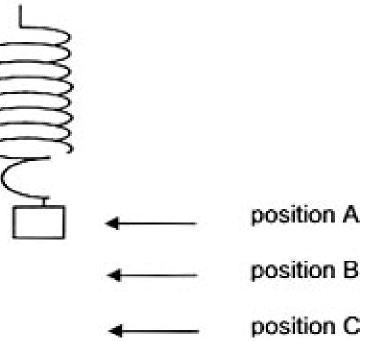
Example T2.9

If the slope in example T2.7 were frictionless, calculate the resultant force on the mass and acceleration of the mass, whilst on the slope.

An important follow-on from Newton's second law is that not only is the size of the acceleration proportional to the resultant force, but also that the direction of the acceleration is the same as the direction of the resultant force. To find the direction of acceleration, it is often useful to first find the direction of the resultant force – i.e. to deduce acceleration from force rather than from change in velocity (thinking about velocity and direction of motion often confuses direction of acceleration. For example, in Example T2.8, finding direction of acceleration is easy if you simply consider the direction of resultant force - weight)

Example T2.10

Mark on with arrows the direction of the acceleration of a mass oscillating on a spring (as shown in the diagram below) at positions A, B and C. The mass is shown at the highest position – position A. Position B is the position where the mass will eventually come to rest and position C is the lowest position of the mass.



Newton's third law of motion

When two particles interact, they always exert equal and opposite forces on each other.

Forces are therefore often described as (equal and opposite) pairs ("Newton's 3rd Law pairs"), since it is not possible to have an unpaired force.

Note that Newton 3rd Law pairs are always:

- ✓ equal to each other in size – so if one changes, the other must change also
- ✓ the same type of force (so if one is tension, so must the other be etc.)
- ✓ acting on two different objects ie. not on the same object (since they are the forces on the two particles when the two particles interact) (so the weight of a brick and the normal reaction force acting on it, whilst equal and opposite, do not constitute a Newton's 3rd Law pair)

Example T2.11

Jessica attempts to lift a 500N weight off the ground by applying an upwards force of 300N to a string attached to the weight. Draw a free-body diagram of the weight and identify/describe all the (Newton's 3rd law) pairs of equal and opposite forces.

Momentum and Impulse

Defining Equations

momentum = mass x velocity ($p = mv$)
impulse = change in momentum ($Impulse = \Delta p = m\Delta v$)

note that momentum is a vector quantity, so a change in direction is a change in momentum

Units of impulse are usually written as Ns but can also be written as the same as units of momentum ie. $kgms^{-1}$ (read kilograms metres per second)

Link to force

$$force = mass \times acceleration \ (Newton's\ 2^{nd}\ law)$$
$$= mass \times \frac{(v-u)}{t} \quad where\ v = final\ velocity, u = initial\ velocity$$
$$= \frac{m(v-u)}{t} = \frac{mv - mu}{t} = \frac{change\ in\ momentum}{t}$$
$$in\ symbols,\ F = \frac{\Delta p}{t} = \frac{impulse}{t} \quad (impulse = change\ in\ momentum,\ p = momentum)$$

$$Ft = impulse \ (written\ Impulse = F\Delta t)$$

Example T2.12

A road vehicle has a mass of 1.2 tonnes (1,200 kg) and is initially stationary. The engine imparts an impulse of 14.4kNs on the vehicle.

(a) At what speed is the vehicle moving after the engine force has been applied?
(b) If the acceleration of the vehicle was $0.8ms^{-2}$ find the force exerted by the engine
(c) Find the time taken for the vehicle to reach the speed calculated in part (a)

Impulse for non-constant forces

If force is not constant, the impulse (change in momentum) can be found in two possible ways. Either use impulse = average force x time or find the impulse from the area under a force versus time graph (but note that the force units must be newtons and the time must be seconds.

Conservation of momentum

In a closed system, the total momentum of all the masses in the system remains constant.

key points - the total momentum of a closed system always remains constant (i.e. is conserved)
if one object gains momentum, the other must gain an equal amount of momentum in the opposite direction (negating the gain of first object)

Types of collision

elastic — kinetic energy is also conserved
- objects bounce without conversion of kinetic energy to other forms (speed of approach = speed of separation)

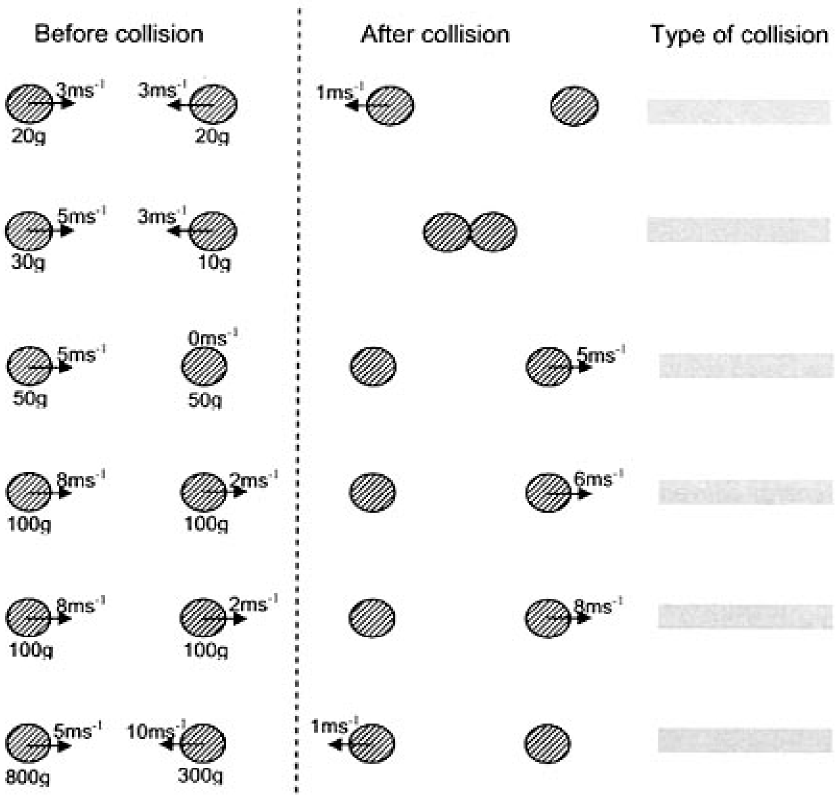
inelastic — - there is a maximum loss of kinetic energy (but the momentum must still be conserved)
objects stick together on impact

partially inelastic — there is some loss of kinetic energy, but objects do not stick together on impact
momentum, as always, is still conserved

explosion — like a collision, where momentum is conserved, but energy is converted from an external source (to cause the explosion) into kinetic energy. Objects move away from each other, rather than towards each other.

Example T2.13

Each example below (next page) shows two-body collisions. Complete the diagrams by drawing labelled velocity vectors (not to scale) on the bodies where they are incomplete. (Masses of objects are omitted on the right – they are the same as masses before collision!) By calculating the kinetic energy before and after each collision, indicate in each case whether the collision is elastic, inelastic or partially elastic.



Work, Energy, Power

Key Point: Energy is never gained or lost, only converted from one form to another

Quantifying types of energy:

Kinetic ("movement" energy): $E_k = \frac{1}{2}mv^2$

where $m = mass, v = velocity$

work done (energy converted by the "force provider"): $W = Fd$

where $F = force\ applied,\ d = distance\ moved\ in\ direction\ of\ force$

Note that work done can also be found by finding the area under a force (N) versus distance (s) graph – and is particularly useful when force is variable

Heat (energy emitted or absorbed): $Q = mc\Delta T$

where $m = mass, c = specific\ heat\ capacity, \Delta T = temperature\ change$

Electric (energy used or converted): $E = VIt = Vq$

where $V = potential\ difference, I = current, t = time, q =$ charge

Latent Heat (thermal energy absorbed or released) $Q = ml$

where $m = mass, l = specific\ latent\ heat\ of\ fusion / vaporization$

Gravitational (increase in energy due to height): $\Delta E_p = mg\Delta h$

where $m = mass, g = gravity, \Delta h = change\ in\ height$

Other useful equations

Work done = energy converted

Power = work done / time taken

Efficiency (%) = $\dfrac{useful\ work\ done}{total\ energy\ used} \times 100$

Example T2.14

A 2kW electric motor is used to run a water pump and in 3 hours pumps approximately 100 tonnes (1 tonne=1000kg) of water, lifting the water into a canal 5m higher.

(a) Find the work done by the pump in this time
(b) Find the potential energy gained by the water in this time
(c) Calculate the efficiency of the pump and explain how the principal of conservation of energy applies in this example.

Example T2.15

A man of mass 90 kg walks along a horizontal road, covering a distance of 3 km. Find the work done by the man in covering this distance.

Non Uniformly Accelerated Motion

For the purposes of this course there are two instances to deal with concerning non-uniformly accelerated motion.

- ✓ recognizing deviations from constant acceleration (in real situations). For example, recognizing that objects falling in air do not really accelerate at a uniform rate – air resistance, a non-uniform force, affects the motion
- ✓ circular motion

Circular Motion

We consider only uniform circular motion – that is: motion of objects moving in a circle at a constant (uniform) speed.

For such motion:

- the acceleration is always directed towards the centre of the circle
- the acceleration is called centripetal acceleration
- the resultant force on the object must be towards the centre of the circle
- this resultant force is called centripetal force
- to describe circular motion there must be a centripetal force acting on the object
- the size of the acceleration depends on the speed of the object and on the radius of the circle described by the object, thus the centripetal acceleration on an object describing uniform circular motion is given by:

  $a = \frac{v^2}{r}$ where v is the speed and r is the radius
- the size of the centripetal force required depends also on the size of the mass. As usual, $F = ma$ Hence the centripetal force required to maintain the uniform circular motion of such an object is given by:

  $$F = ma = m\frac{v^2}{r} \text{ or } F = \frac{mv^2}{r}$$

Example T2.16

A string is used to twirl a 300g mass in a horizontal circle of radius 25 cm, in zero gravity conditions, so that the mass moves at a speed of 2 ms^{-1}

(a) find the tension in the string

(b) if the mass is now twirled in a vertical circle **with gravity** at the same speed, draw a free body diagram showing the mass at the lowest point and calculate the tension in the string at this point.

Note

This example illustrates the point that centripetal force is the required resultant force that must act on a mass if the mass is to describe a circle. This resultant force can be from a single force or the result of two or more forces.