Topic 5 – Electric Circuits

Electric Potential Difference, Current and Resistance

Overview

In order to make an electric appliance work, an electric circuit is needed. Essentially, electric devices work when electrons contained in the wires of the device are caused to move. This movement generally results in the production of a magnetic field and a heating effect. Both of these results are very useful: the heating effect can produce useful heat, light and other forms of radiation, and the magnetic field can be used to produce force and movement – and can therefore be used in a huge variety of electric motor devices, ranging from hair driers to stereo systems, to electric/hybrid cars.

So the electric circuit basically consists of the wiring in the actual device, attached to connecting external wires, attached to a battery or other electrical supply, attached back to the device. The whole affair thus forms a loop. Essentially, the power supply provides a force field. The force field is not magnetic but electric: it exerts force on charged particles – like electrons. The external connecting wires thus transmit the force field to the wiring in the electrical device and the result is that electrons within the device (wiring) move. As stated earlier, this then causes a heating effect and/or a magnetic effect, and the device does work (converts energy).

This topic addresses how and why electrons in circuits move and ways to measure "electricity" and the energy changes that occur.

Electric Current

An electric current is said to flow when charged particles move. For this part of the course we consider only circuit electricity where the electric current flows in metallic wires. Since, in metals, the only particles that can move around are free electrons these are the charges responsible for current flow.

Electric current is defined to be the rate of flow of charge (electrons)
Electric current is measured in amps (A).
Charge is measured in coulombs (C)

Therefore, 1A = 1C per second $1A = 1Cs^{-1}$

Equation: $I = \frac{q}{t}$ where $I = current, \quad q = charge, \; t = time$

As given in the data booklet, a single electron has a charge of $1.6\times10^{-19}C$ (actually we should, strictly, include a minus sign since an electron is negatively charged). We can therefore deduce that to give a current of 1A, we need 6.25×10^{18} flowing past a point (in the wire) per second.

The unit of current, the ampere (or amp), is defined as the current needed when flowing through two parallel wires placed 1 metre apart in a vacuum, in order to produce a certain magnetic force between the wires.

Electric Potential Difference

Electric potential difference is also referred to simply as voltage.

Electric potential difference is measured in volts.

To understand electric potential difference at a simplistic level, consider one coulomb of electrons moving around a circuit. They come out of the power supply freshly fuelled with energy (electric potential energy). It is a 6 volt (6V) power supply, so the coulomb of electrons has exactly 6 joules (6J) of energy available. As they move through the thick connecting wires they do so effectively without losing any energy. The electric potential energy drop is therefore zero, so far. Next they move through the appliance. Assuming that this is the only appliance in the circuit, they will lose all their energy as they pass through the appliance. The potential energy difference of the electrons, from the point where they enter the appliance to the point where they leave, will therefore be 6J. The voltage across the appliance will therefore be 6V. When the electrons return to the power supply, they will be 6J lower in energy than when they started. The potential difference across the appliance is therefore 6V.

From the above passage it would appear that potential difference is energy difference. The subtle difference is that potential difference is potential energy difference per unit charge (coulomb).

Before considering the following definition, remember that energy difference = energy converted = work done.

Electric Potential Difference - Definition

The electric potential difference between two points is the work done per unit charge in moving a charge from one point to the other:

$$\Delta V = \frac{work\ done}{charge} = \frac{W}{q}$$ (equation NOT GIVEN in Data booklet)

This is the same as saying that potential difference is the potential energy difference per unit charge, when the charge moves from one point to another.

Potential difference is measured in volts.

Explanatory example:

Suppose a power supply is connected up. The + side will attract electrons and the – side will repel them. Electrons will therefore experience a force. This force causes the electrons to move and so we have work being done (work done = force x distance). (Recall that if we have work done, we have a conversion of energy – this energy may be heat or movement, for example)

Suppose that the work done by each electron is $3.8\times10^{-17}J$ (energies are very small with electrons)

We know that the charge of one electron is $1.6\times10^{-19}C$

$$\Delta V=\frac{W}{q}=\frac{3.8\times10^{-17}}{1.6\times10^{-19}}=237.5\approx240V$$

So, if a circuit is connected up and the result is that each electron releases $3.8\times10^{-17}J$ when passing from one point to another, then the potential difference between these two points must be 240 volts.

Note that the power supply is used to provide this potential difference, via the electric field that it produces – so a 240V power supply would be needed in the above example.

Example T5.1

The potential energy difference between an electron at point A and electron at point B is 1.08 x 10^{-17} J. Find the potential difference between the points.

<u>Electric Potential</u>

Voltage is the quantity that measures the potential energy *difference* (per unit charge) across two points in a circuit. It is also the quantity that measures the actual potential energy per coulomb of a charge. This quantity is called, simply, potential. For example the negative plate of a 2V cell ("battery") may have a potential of –1V and the positive plate; +1V. The potential difference across the cell is, therefore 2V (1V – – 1V = 1V + 1V = 2V) and the cell can deliver 2J per coulomb of electrons that pass through the cell.

<u>The Electronvolt</u>

A useful unit of energy in atomic and nuclear physics (with such small energies often involved) is the electronvolt.

One electronvolt is the energy associated with an electron moving through a potential difference of one volt.

This energy is the same as the work done on or by the electron when the field is applied.

Thus, using $\Delta V=\frac{W}{q}\Rightarrow W=q\Delta V=1.6\times10^{-19}C\times1V=1.6\times10^{-19}J$

So, $1\ electronvolt=1.6\times10^{-19}\ joules$ or: $1eV=1.6\times10^{-19}J$

In simple terms, to convert joules to electronvolts divide by 1.6×10^{-19}

To convert electronvolts to joules, multiply by 1.6×10^{-19}

Example T5.2

Find the energy released when an electron moves through a potential difference of magnitude 500V

Note that the symbol, e, is often used for the charge of an electron, 1.6×10^{-19} C.

Electrical Resistance

Electrical resistance is a property of a material that measures its ability to allow current to flow. High resistance materials thus hinder current flow and low resistance materials allow easy passage of current. A block of wood, being an electrical insulator, effectively has infinite resistance, whilst a lump of iron, being a conductor, has effectively zero resistance. Other materials have resistance somewhere in between.

Electrical resistance is measured in ohms
The symbol for ohms is Ω

Resistance – Definition

The resistance of a material (or device) is the ratio of the voltage across the material to the current flowing through the material.

Defining equation: $R = \dfrac{V}{I}$

where $R = resistance, \;\; V = voltage, \;\; I = current$

So, to measure the resistance of an electrical device or material we can connect it up to a power supply (using any safe voltage) and measure this voltage and the current through the device. Resistance is simply voltage divided by current.

Current, Resistance and Resistivity

The rate at which charge flows depends on the velocity of the charges. The velocity of the charges is called drift velocity

(For ohmic conductors – see below) the speed (drift velocity) of electrons in a conductor depends on:

1) cross-sectional area of conductor
2) length of conductor
3) material from which conductor is made
4) temperature of conductor

resistance of conductor

5) potential difference (voltage) across conductor

The resistivity of a material is a measure of the resistance of a certain material. We can find the resistivity of various materials in data-books or on the internet and we can then find the electrical resistance of, for example, a wire made of this material using the following formula:

$R = \frac{\rho L}{A}$ where $R = resistance\ (\Omega)$ $\rho(\text{"rho"}) = resistivity\ (\Omega m)$
$L = length\ (m)$ $A = cross\ sectional\ area\ (m^2)$

As we can see above, resistivity is measured in ohm-metres (Ωm)

Example T5.3

Given that copper has a resistivity of $1.72 \times 10^{-8}\Omega m$ at 20°C, calculate the resistance of a piece of copper wire with a diameter of 1.00mm and a length of 5cm at 20°C.

Note that, generally, as the temperature of a metallic material increases its resistivity (and therefore resistance) increases. This can be explained in terms of the atoms in the metal vibrating at an increasing rate at higher temperatures and hindering the progress of passing electrons.

Ohm's Law:

"The current through a wire is proportional to the potential difference across it, provided the temperature is unchanged"

Ohm's law equation:

$$R = \frac{V}{I}$$

Ohm's law effectively states that the resistance, R, is constant - and does not change if current (or voltage) is increased as long as the material is not allowed to heat up.

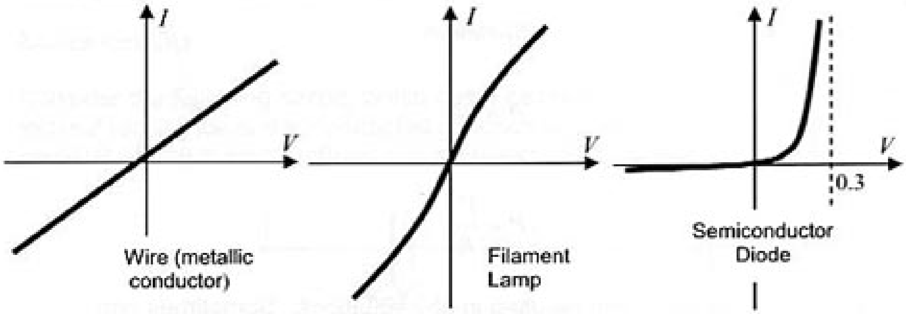
Generally, metallic conductors obey Ohm's Law.

Ohmic and non-ohmic behaviour

A conductor is said to be ohmic if the current flowing through the conductor is proportional to the potential difference across the conductor (i.e. one for which resistance remains constant).

An easy and effective way to observe this behaviour is to plot current versus potential difference for a range of voltages.

Examples



The only graph for which $V \alpha I$ is the first. The wire is thus ohmic and the other two are non-ohmic. Note that if a very large current is forced through the wire, it will become non-ohmic (like the lamp).

A lamp is non- ohmic because the filament gets very hot as current is increased. This causes resistance to increase. The gradient of the IV graph hence decreases.

A diode is non-ohmic because if the potential difference is reversed, no current flows: resistance is infinite. For the wire and lamp the direction of the current (and potential difference) makes no difference to the way they behave, as conductors. Semiconductor diodes generally have an effective resistance of zero at 0.3V

<u>**Electrical Power:**</u>

Power is defined as the rate of doing work

$Power = \dfrac{work\ done}{time\ taken}$, assuming work is done at a constant rate

But $work\ done, W = q\Delta V$ as stated earlier

Hence, Power = $\dfrac{qV}{t}$

but, for a constant current, $q = It \Rightarrow$ Power = $\dfrac{ItV}{t} \Rightarrow$ Power = IV

So, electrical power delivered by a resistor (conductor) is equal to the product of the potential difference across the resistor and the current flowing through the resistor.

$P = VI$

Other expressions for power – derivations:

$P = V\,I, \; but \; V = I\,R,$

$\Rightarrow P = (I\,R)I = I^2 R$

$P = V\,I, but \; I = \dfrac{V}{R}$

$\Rightarrow P = V\left(\dfrac{V}{R}\right) = \dfrac{V^2}{R}$

Summary

$P = VI$

$P = I^2 R$

$P = \dfrac{V^2}{R}$

Note that any of these equations can be used in any situations. Sometimes one yields a quicker answer than another.

Example T5.4

A circuit has a single resistor, with a resistance of 30Ω, connected to a 12V power supply

a) Find the current flowing through the resistor
b) Find the power delivered through the resistor

<u>**EMF (ElectroMotive Force)**</u>

Refers to a source of electrical energy.

The emf of a source is equal to the electrical energy produced per unit charge inside the source. Unit: Volt.

In simple terms, the emf of a source is the maximum possible voltage of the source. Some of this voltage is "lost" because the source itself has a resistance – so the apparent voltage of the power supply, and the voltage across the external resistor, will be a little lower than the emf of the power supply (source)

So – not all energy is converted to electrical energy outside the source since some is lost inside the source due to internal resistance.

Internal resistance - the resistance of a source

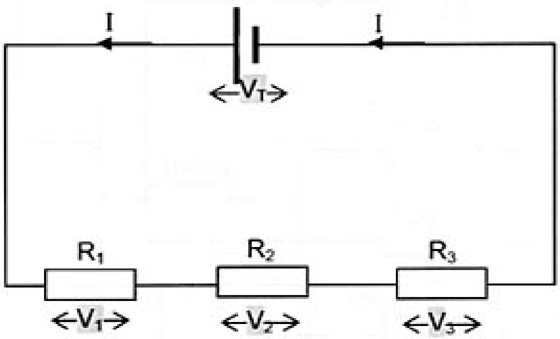
Example T5.5

An 8.0V power supply is connected to a 12Ω resistor. The voltage across the resistor is measured as 7.68V and the current; 0.64A. Explain the voltage drop, and explain (with any necessary calculations) what the emf is and the what internal resistance of the supply is.

Resistance in Series and Parallel Circuits

Series circuits

Consider the following circuit, which has a cell with an emf of V_T volts, negligible internal resistance and is connected in series to 3 resistances, of R_1, R_2 and R_3 ohms respectively. A current, I, flows around the circuit, as shown:



The same current, I, flows through all resistors

Let the total resistance of the circuit be R_T

The total voltage is therefore: $V_T = IR_T$

But also, the total potential difference (pd) across resistors in series is equal to the sum of the pds across the individual resistors, i.e..

$$V_T = V_1 + V_2 + V_3$$

But, $V_1 = IR_1, V_2 = IR_2, V_3 = IR_3$

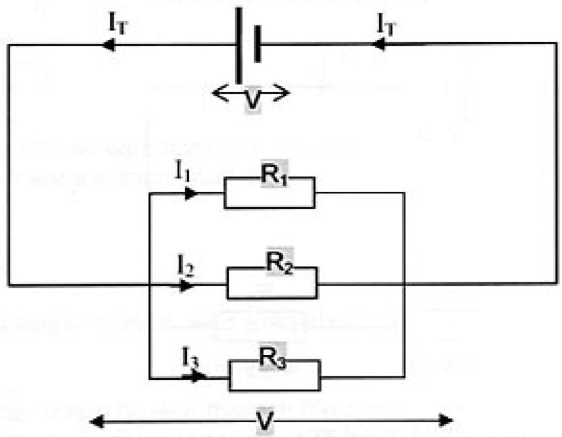
Hence, $$\begin{aligned} V_T &= IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) \\ V_T &= I(R_1 + R_2 + R_3) \end{aligned}$$

But, $V_T = IR_T$

Hence $\boxed{R_T = R_1 + R_2 + R_3}$

Parallel circuits

Consider the following circuit, which has a cell with an emf of V_T volts, negligible internal resistance and is connected to 3 resistances in parallel to one another, of resistance R_1, R_2 and R_3 ohms respectively. A current, I, flows around the circuit, as shown:



The potential difference across any branch of a parallel circuit is equal.

Therefore, V(across R_1)= V(across R_2)=V(across R_3)=V(of cell)

Let the total resistance of the circuit be R_T The total voltage is: $V = I_T R_T$

But also, the total current around the circuit (and through the cell) is equal to the sum of the currents through each branch of the parallel part of the circuit. I.e.,

$$I_T = I_1 + I_2 + I_3$$

But, $V = I_1 R_1, V = I_2 R_2, V = I_3 R_3$

$$\Rightarrow I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

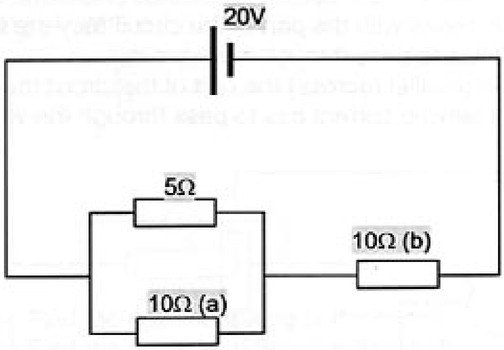
$$I_T = I_1 + I_2 + I_3 \Rightarrow I_T = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

But also: $V = I_T R_T \Rightarrow I_T = \dfrac{V}{R_T}$

Hence: $\dfrac{V}{R_T} = \dfrac{V}{R_1} + \dfrac{V}{R_2} + \dfrac{V}{R_3} \Rightarrow$ $\boxed{\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$

Example T5.6

Consider the following circuit, and complete the table below:



	5Ω	10Ω (a)	10Ω (b)	Total circuit
Voltage				
Current				
Resistance				
Power				
Charge passed in 1 sec				

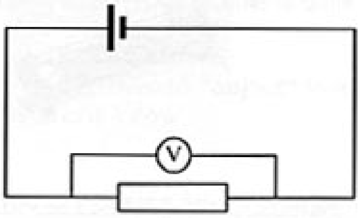
Some tips to start you off!

- start by finding the total circuit resistance
- then find total circuit current (from total resistance and voltage)
- next find pd across 10Ω resistance in series
- find current through each of the two resistors in parallel, using V=IR
- find power dissipated by each resistor using P=VI (or one of other formulae), charge passed using Q=It

(**note**: there are usually many different ways of solving circuit problems – they all (obviously) lead to the same solutions!)

Example T5.7

A battery has an internal resistance of 10Ω and is connected to an external resistance of 100Ω. A voltmeter is connected across the external resistor, as shown below. It gives a reading of 10V:



(a) Find the current flowing in the circuit
(b) Find the potential difference across the internal resistance
(c) Hence, state the emf of the battery
(d) What assumption about the voltmeter have you made in your calculations?
(e) If a different voltmeter was placed in the circuit, and this new voltmeter had a resistance of 60Ω, what reading would the voltmeter now give?

Notes:

The internal resistance of a source is approximately constant, BUT the voltage "lost" across the internal resistance depends on the current flowing in the circuit and through the supply, which depends on the size of the total external resistance.

The smaller the external resistance, the greater the pd across the internal resistance. (for example, in the above example – (a) to (c), the emf of the supply is 11V, but only 10V is available to the external resistor. If however, the internal resistance is equal to the external resistance, only half the emf value would be available for the external resistor)

Potential Dividers

A potential divider is a circuit that is able to vary (divide) the potential difference of the supply.

This is particularly useful for students doing experiments involving varying the voltage (potential difference) when they do not have a variable voltage power supply.

A common potential divider circuit involves a rheostat (wound wire resistor – as in the picture. The resistance is varied by changing the length of the wire that the current passes through via the sliding terminal in the middle.

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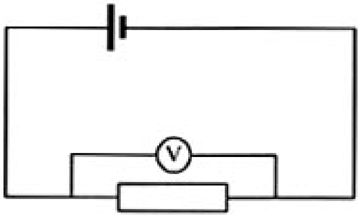

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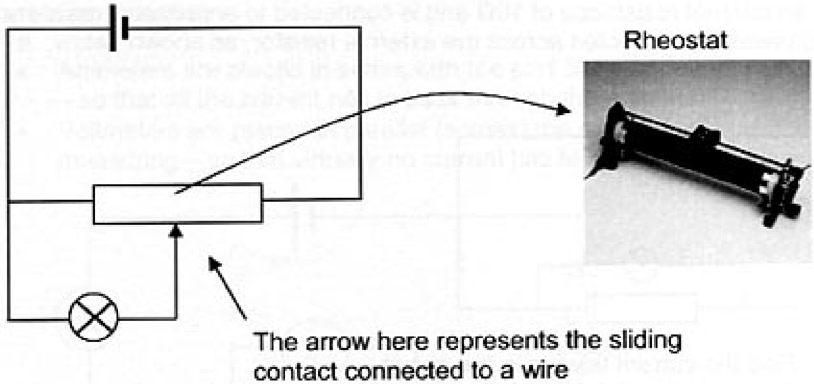
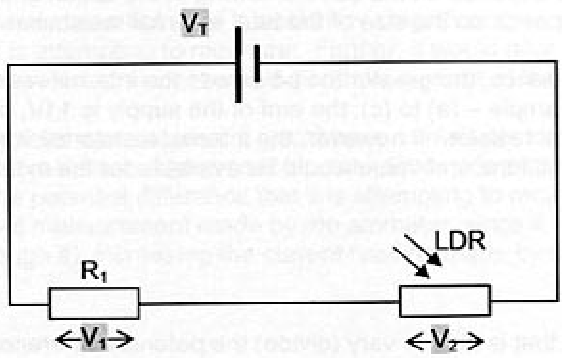
The circuit below shows potential divider being used to vary the voltage across a bulb.



Potential Dividers as sense/response circuits

Potential Divider Circuits can also involve the use of two resistors in series, rather than using a rheostat. This set-up is useful for electronic sensor circuits – since one of the resistors is a sensor, and its resistance controls the split of the total potential difference across both resistors.

Example: LDR potential divider circuit



An LDR is a light dependent resistor: its resistance changes as the level of light changes.

If light increases, the resistance of the LDR decreases, V_2 decreases and V_1 increases

The fixed resistor, R_1, can be attached to a meter or can form part of an alarm system, or any other device. The essential point here is that the voltage across the fixed resistor, R_1 is controlled by the LDR.

The sensor resistor could also be a temperature dependent resistor (thermistor) or strain gauge (resistance depends on physical strain-force)