Topic Thirteen: Quantum Physics and Nuclear Physics

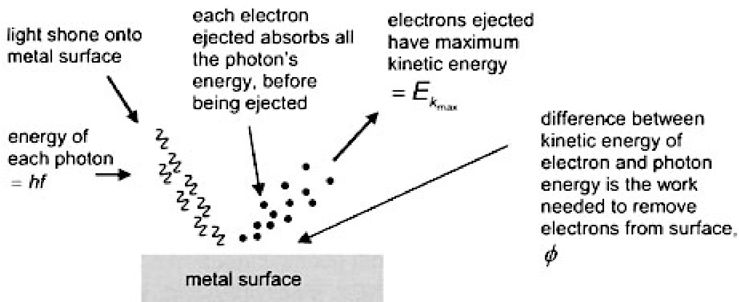
Quantum Physics

This topic discusses the fact that light can be considered as particles (each with a quantum of energy called a photon) and also as waves, where the energy is continuous and unbroken).

In more general terms, the so called "wave particle duality" considers that what we normally consider as waves (like light, X rays etc.) can behave like particles and what we usually consider as particles (like electrons) can behave like waves.

The photoelectric effect is one experimental effect that shows light behaves like particles:

The Photoelectric Effect



This is the effect whereby if light is shone onto a metal surface, electrons (called photoelectrons) are emitted.

- electrons are only emitted if the light is above a certain frequency, called the threshold frequency (ultra-violet light is needed for some metals)
- the value of this threshold frequency depends on the metal being used

  and, assuming the frequency of the light is above threshold frequency:

- the number of photoelectrons emitted depends on the intensity of the light
- the photoelectrons emitted have a range of kinetic energies, up to a certain maximum value (and this maximum kinetic energy depends on the metal used and the frequency of light).

This is explained by the fact that free electrons each require a certain amount of energy to escape from the surface of the metal.

- the minimum energy required to release an electron from a certain metal is called the work function (ϕ) of the metal

Further, it is necessary to consider that the light hitting the metal behaves as particles (photons), rather than continuous waves.

- if light behaved as a wave then, even at low frequency the light would eventually provide enough energy to release an electron. This is not the case

So the energy comes from a particle of light. When a "light particle", (called a photon) hits an electron, it gives up all of its energy to the electron

- the energy of each photon depends on the frequency of the light

Each photon has energy given by the equation: $E = hf$

- E is energy, in joules, h is Planck's constant, $6.63 \times 10^{-34}\,Js$ and f is the frequency of the light

If the work function of the metal is greater than the photon energy (ie. if light frequency is too low), then no photoelectrons will be emitted

- $hf < \phi \Rightarrow no\ emission$

The minimum frequency required to just release photons from the metal is called the threshold frequency - f_0, and the photon energy at this frequency is therefore equal to the work function of the metal

- $hf_0 = \phi$

If the frequency is above threshold frequency, then electrons (with kinetic energy) are emitted

- $f > f_0 \Rightarrow hf > hf_0 \Rightarrow hf > \phi \Rightarrow emission\ results$

Since an electron absorbs all of a photon's energy then if photon energy is greater than energy required to release electron (work function), electron will not only be released, but will have kinetic energy. The kinetic energy will be equal to the amount of energy provided by the photon, over and above the energy required to release it. This is the maximum kinetic energy of the electron since it may have required a little more energy than the work function, (work function is minimum energy required) to escape.

- $E_{k_{max}} = hf - \phi \Rightarrow hf = \phi + E_{k_{max}}$

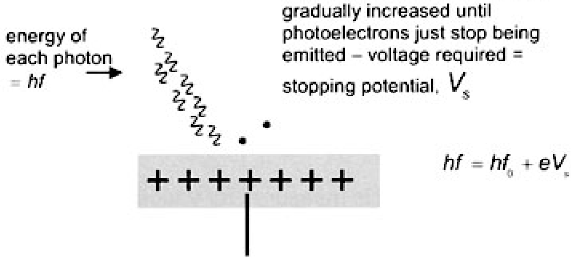
(This equation was derived by Einstein who explained the photoelectric effect, as above)

Example T13.1

Light is shone onto a potassium surface and the frequency is gradually increased until, when the frequency is just above $5.38 \times 10^{14}\,Hz$, photoelectrons are emitted from the surface. The frequency is then increased to $5.95 \times 10^{14}\,Hz$.

(a) calculate the work function of potassium, in electron volts.
(b) calculate the maximum speed of the photoelectrons when the higher frequency light is applied.

Stopping Potential



If the metal surface is given a positive charge it will attract back the electrons, making it more difficult for them to escape.

The energy required to "move" an electron through a potential difference of V volts, or put another way, the energy required to increase the potential of an electron by V volts, is given by the equation $W = qV$ Since we are talking about electrons, $q = e$ and therefore, $W = eV$.

So, in order to remove an electron from a metal surface with a (positive) potential, V, the energy of the photon has to be sufficient to overcome the work function of the metal and to increase the potential of the electron, for it to escape the attraction of the metal.

Conversely, if a frequency above threshold frequency is used, so that, under normal circumstances, electrons would escape with additional kinetic energy and the potential of the metal is gradually raised until photoelectric emission just stops, then this potential is called stopping potential – symbol V_s

Summary:

energy available by photons	= hf (photon energy)
minimum energy required to release electrons (no potential)	= ϕ (work function) = hf_0
where minimum frequency of light	= f_0 (threshold frequency)
additional energy available for E_k	$= hf - \phi = hf - hf_0$

If stopping potential, V_s applied to metal, $eV = hf - hf_0 \Rightarrow hf = hf_0 + eV_s$ and electrons can no longer escape (kinetic energy is reduced to zero)

Equations (given in data book)

$E = hf$	to find photon energy of incident radiation (light)
$hf = \phi + E_{k\,max}$	to find kinetic energy of photoelectrons
$hf = hf_0 + eV_s$	to find stopping potential ($E_{k_{max}} = 0$)

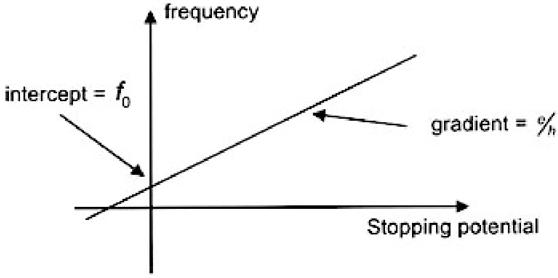
This last equation tells us that stopping potential depends on the threshold frequency (which depends on the metal surface) and on the frequency of the incident radiation.

Example T13.2

(a) Calculate the stopping potential for the potassium surface, as in T12.1, when the high frequency light, as in (b) is used.
(b) calculate the stopping potential when ultra-violet light, of wavelength 375nm is used as the incident radiation.

Graphical

If, for a certain metal, the stopping potential is measured for various frequencies of incident radiation, the following graph is obtained:



We get a straight line graph, because the relationship between stopping potential and frequency is linear – as shown by the equation:

$$hf = hf_0 + eV_s$$
$$\Rightarrow f = \frac{hf_0 + eV_s}{h} = f_0 + {}^{e}\!/_{h}V_s$$
$$\therefore f = {}^{e}\!/_{h}V_s + f_0$$

Comparing this with the formula of a general straight line, $y = mx + c$, we see that the gradient of the line is equal to ${}^{e}\!/_{h}$ and the y-axis intercept, f_0

Wave-particle duality

We have seen that waves behave as particles (e.g. photoelectric effect). It has also been shown that what we usually consider to be particles show wave-like behaviour.

de Broglie's Hypothesis

de Broglie suggested that all (moving) particles have a wave like nature. He linked the wavelength (so called de Broglie wavelength) with the momentum of the particle in the following equation:

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}, \text{ where, } \begin{aligned} &p = momentum\ of\ particle \\ &h = Planck's\ constant \\ &\lambda = (de\ Broglie)\ wavelength\ of\ particle \end{aligned}$$

Validation

de Broglie's theory was tested by directing electrons at a thin metal foil. It was shown that the electrons were diffracted to form a pattern of rings on a screen – the same pattern as that produced by X-rays.

Electrons are usually considered as particles and diffraction is wave behaviour. Therefore de Broglie was proven right: particles do behave as waves.

Example T13.3

Find the de Broglie wavelength of a beam of electrons moving at $2.5 \times 10^5 ms^{-1}$

Example T13.4

Find the speed of electrons necessary for an electron beam to behave like x-rays, of frequency $3.4 \times 10^{18} Hz$

Atomic Spectra and Energy States – Bohr's Postulates

We saw in Topic 7 how atomic emission and absorption spectra arise. We shall now look in more detail at the processes involved.

Remember, the spectra arise because of electron energy level jumps from one shell (allowed electron energy state) to another. Bohr made some suggestions (postulates) that can be used to quantify some of these energy transitions.

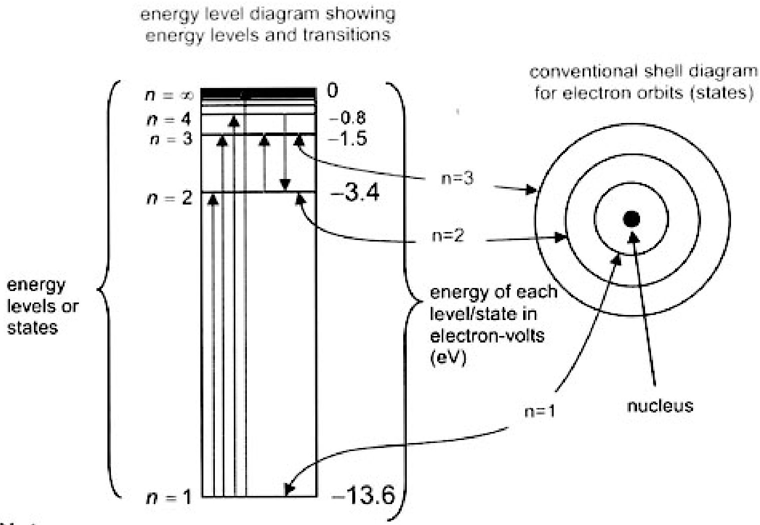
Bohr's Postulates

1. Only certain levels - orbits/energies are allowed for the electrons in an atom – called stationary states or quantum levels.
2. The electron will not radiate (or absorb) energy whilst in one of these states.
3. During a (quantum) jump, energy transfer is given by, $E = hf$, where E is the energy, in joules, f is the frequency of radiation emitted or absorbed and h is Planck's constant.

Consider, for example, the hydrogen atom.

The lowest energy state for its electron is the first shell, called $n = 1$
This lowest, most stable energy state, is called the ground state.
In order to jump from the ground state to higher states, energy is needed. If hydrogen is illuminated with the whole spectrum of frequencies of electro-magnetic radiation, the photons with the correct energy to cause this excitation from the ground state to the next state ($n = 2$) will be absorbed. Other photons will be absorbed when they pass on their energies to promote the electron between other energy states (shells). This is how an absorption spectrum arises.

The energy levels for the hydrogen atom are as follows:



Notes:

- The blue lines represent some of the possible transitions from one state (or level) to another.
- Transitions can occur from any state to any other state
- States $n = 5, 6, etc$ have not been shown, for lack of space!
- $n = \infty$ corresponds to the electron entirely leaving the atom – ionization
- For a transition of the electron from $n = 1$ to a higher level, energy must be put in (absorbed from electromagnetic radiation, if available)
- If the electron drops, atom will emit energy (in the form of electromagnetic radiation)
- The type of electromagnetic radiation absorbed or emitted depends on the size of the energy transition (and hence frequency of photon)
- Different atoms have different energy levels – this is why absorption and emission spectra can be used to identify materials.

Example T13.5

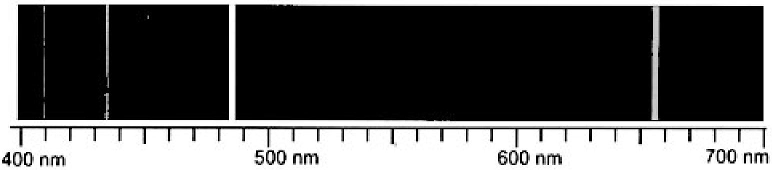
(a) Using the diagram above page find the energy required, in joules, to ionize a hydrogen atom from its ground state
(b) Hence, find the frequency of electromagnetic radiation to effect this transition.

Example T13.6

A hydrogen line emission spectrum has a line of wavelength $1.78 \times 10^{-6} m$.
(a) Find the photon energy, in eV, of light emitted for this line
(b) Using the energy level diagram above, identify the transition causing this line

The diagram below shows the emission spectrum for hydrogen. The lines are all in the visible range. They correspond to transitions to the level $n = 2$, from other levels. The highest energy light emitted is the blue light on the far left, and the lowest is the red (red has lower frequency than blue light). Hence the lines on the left correspond to larger electron transitions than those on the right.

Emission spectrum for hydrogen



Example T13.7

Using the energy level diagram for hydrogen, shown two pages previously, identify the transitions for two of the lines in the hydrogen emission spectrum shown above (do all for more practice, if you wish!!) [Note that the energy levels shown are approximate so your answers will not agree exactly to the lines shown on the emission spectrum].

<u>Electron in a Box Model</u>

The electron in a box model considers that an electron has certain possible wavelengths, determined by standing waves that fit in the box exactly like standing waves on a string. It explains why atomic electrons only have certain discreet energies. The model provides the following formula for energies:

$E_K = \frac{n^2h^2}{8m_eL^2}$ where: $n = 1,2,3,\ldots$ $h = Plank's\ constant,$ $m_e = mass\ of\ an\ electron,$

$L = length\ of\ the\ box$

<u>Schrodinger's Model of electrons within the atom</u>

Schrodinger's model basically did not predict exactly where an electron is in an atom, but it provided a mathematical probability function defining the region where it is, say 95%, likely to be. It theorized that in fact an electron position cannot be found, or predicted with certainty, but the region for 95% certainty is actually quite small. These "probability regions" are called orbitals and his theory is still used to define various shapes of electron orbitals. His model has helped scientists understand chemical bonding and molecular structure.

<u>Heisenberg's Uncertainty Principle</u>

This principle states that it is impossible to know with certainty:

- both the position and the momentum of a particle
- both the time elapsed and the energy of a particle

Note that it is possible to know with certainty one of the pairs, for example position, but if this is the case the other (e.g. momentum) is not known at all.

Nuclear Physics

Nuclear Size

One method of finding the approximate size of a nucleus is from the results of Gieger & Marsden's alpha scattering experiment – as described in Topic 7. Please review this experiment before continuing.

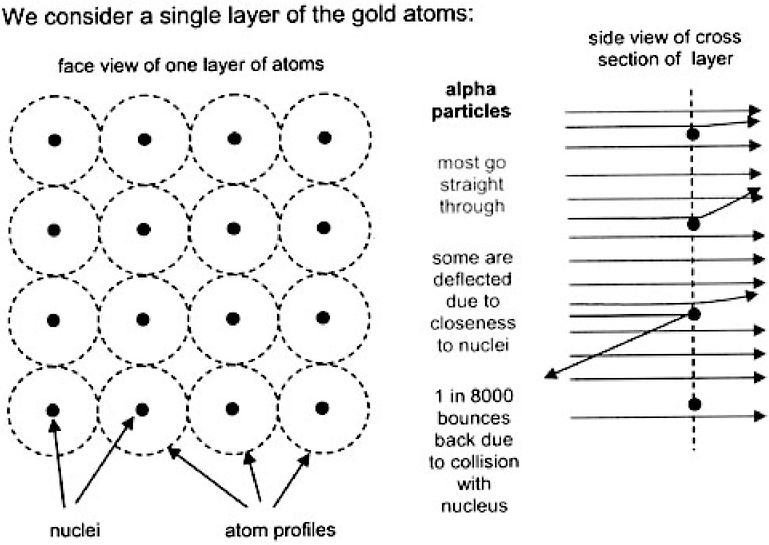
The information we need from this experiment is that if alpha particles are directed at a gold foil about $6.0 \times 10^{-7} m$ thick, one in 8000 are deflected by angles greater than 90°. As explained earlier, we assume that deflections of this kind can only occur when the alpha particle hits one of the gold nuclei and "bounces" back.

For this approximation we also need to know the approximate size of an atom. This can be found by accurate density measurements (details not required) and the diameter of an atom is found to be approximately $3.0 \times 10^{-10} m$

Now let's define some variables:

Let:
$$D_N = \text{diameter of a nucleus}$$
$$D_A = \text{diameter of an atom } (= 3 \times 10^{-10} m)$$
$$A_N = \text{cross sectional area of a nucleus}$$
$$A_A = \text{cross sectional area of an atom}$$

We consider a single layer of the gold atoms:



The number of layers, n, of gold atoms in the $0.6 \mu m$ gold foil is equal to the thickness of the foil divided by the diameter of one atom.

$$n = \frac{6 \times 10^{-7} m}{3 \times 10^{-10} m} = 2000$$

Since alpha particles are only deflected by the gold nuclei (and not the rest of the atom),

the proportion of alpha particles deflected by one layer of gold nuclei $= \dfrac{\textit{area of nuclei in one layer}}{\textit{area of atoms in one layer}}$

(the atoms are so small that they effectively fill up all space in the layer)

$$= \frac{\textit{cross sectional area of a nucleus}}{\textit{cross sectional area of an atom}} \quad \text{(same ratio)}$$

$$= \frac{A_N}{A_A}$$

proportion of alpha particles deflected by 2000 layers (ie by the gold foil) $= 2000 \times \dfrac{A_N}{A_A}$

but we know that the foil deflects 1 alpha particle in 8000 hence:

$$\frac{1}{8000} = 2000 \times \frac{A_N}{A_A}$$

$$\Rightarrow A_N = \frac{A_A}{8000 \times 2000}$$

$$\Rightarrow \pi \frac{D_N^{\ 2}}{4} = \frac{\pi^{D_A^2}/_4}{8000 \times 2000} = \frac{\pi D_A^{\ 2}}{4 \times 8000 \times 2000} \quad \left(\pi r^2 = \pi \frac{D^2}{4}\right)$$

$$\Rightarrow D_N^{\ 2} = \frac{D_A^{\ 2}}{16000000}$$

$$But,\ D_A = 3 \times 10^{-10} m \Rightarrow D_N^{\ 2} = \frac{(3 \times 10^{-10})^2}{16000000} = 5.63 \times 10^{-27}$$

$$\Rightarrow D_N = \sqrt{5.63 \times 10^{-14}} = 7.5 \times 10^{-14}$$

So, using this method, we see that an approximation of the diameter of a nucleus is $7.5 \times 10^{-14} m$, and that an atom is about 10^4 times the size (diameter) of a nucleus.

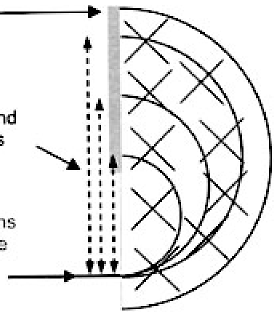
Nuclear mass

The mass spectrometer is particularly useful for comparing masses of various molecular fragments. They are used extensively in organic chemistry to identify organic molecules. The sample is first bombarded with electrons. This causes the molecules to fragment (split) into neutral and positive ions.

For example, if the sample was propane, propane has the formula CH_3–CH_2–CH_3 (C_3H_8). Possible fragments are C^+, CH^+, $CH_2^{\ +}$, $CH_3^{\ +}$, CH_3–$CH_2^{\ +}$, etc. The relative mass of these fragments is 12 (for C^+), 13 (12+1 for CH^+), 14, 15, and 29. The mass spectrometer tells us the mass of these fragments, and from this information (usually with other information aswell) we can deduce that the original sample is propane.

The principle of a mass spectrometer is that the ions are deflected by a magnetic field and the amount of deflection depends on the mass of each fragment (ion) and on the charge.

Mass Spectrometer – schematic diagram



Notes

- the positive ions are accelerated using an electric field and are passed through a velocity selector, which ensures they all have the same velocity when they enter the mass magnetic field, shown above.
- all charged particles travel in circles when moving through, and at right angles to, a uniform magnetic field.
- usually the particles will all have the same charge; +1, but they could for example be ions with a +2 charge.
- Using FLHR we can predict the direction of the force on the positive charges as they enter, and travel through the magnetic field – remember, direction of conventional current is in the same direction as positive charge. On the point of entry, the force is thus upwards, and as they move, the force is always at right angles to the velocity of the particles, towards the centre of the circle that they describe.

Theory

magnetic force on ions = centripetal force required to maintain them in a circle

$\Rightarrow Bqv = \dfrac{mv^2}{r}$, where,
B = magnetic field strength
q = charge on ion
v = speed of ion
m = mass of ion
r = radius of path in magnetic field

$$\Rightarrow \frac{m}{q} = \frac{B\cancel{v}r}{v^{\cancel{2}}} \Rightarrow \frac{m}{q} = \frac{Br}{v} \Rightarrow \frac{m}{q} = \frac{B}{v}r$$

and, since B *and* v *are constant,* $\dfrac{m}{q} \propto r$

for singly charged ions, $q = e$, *and* $m \propto r$

So we can measure the radius of the path taken by the various ions and find their mass ratios. If the field strength and velocity of particles is known, we can find their actual masses (mass:charge ratio)

The mass spectrometer provides evidence for isotopes, since it identifies species of the same element that have atoms of different mass.

Example T13.8

Hydrogen ions are passed through a mass spectrometer and it is found that two paths of different radii are taken to the deflector plate. Explain this observation.

Example T13.9

Two particles, X^+ and Y^{2+}, enter a mass spectrometer. The mass of Y is triple (3x) the mass of X. If the radius of the path taken through the magnetic field in the mass spectrometer by X^+ is R, find, in terms of R, the radius of the path taken by Y^{2+}

Nuclear Energy Levels

Experimental evidence shows that nuclei, not only atoms, have certain energy levels. One example of such evidence is during radioactive decay processes. When a parent nucleus undergoes radioactive decay, the decay product, or daughter nucleus, is usually in an excited state. It relaxes back to its ground state by emitting a gamma-ray photon. The fact that a gamma ray photon is emitted is evidence for the fact that the nucleus can be in an excited, high energy state. Further, certain nuclei only emit photons of a certain frequency (and, therefore, energy) – the photon energy is observed to be characteristic of the nucleus. This is evidence for the fact that nuclei have well-defined energy levels, as do atoms (and their "electron shells").

Radioactive Decay

Beta Decay

There are 3 types of beta decay: electron emission, positron emission and electron capture. We only consider the first two.

Electron emission

Electron emission occurs when a nucleus is unstable due to the high neutron:proton ratio, and resulting instability A neutron within the nucleus changes into a proton and an electron. The proton remains within the nucleus, but the electron is ejected, as a beta-particle.

Example T13.10

Carbon-14 (atomic number, 6) decays to Nitrogen-14 (atomic, number, 7), with emission of another radiation. Explain this process by reference to the neutron:proton ratio of the two nuclei and write a nuclear reaction equation to illustrate it. Name the radiation emission.

Positron Emission

A positron is the anti-particle of an electron. It has the same mass and charge of an electron, except that the charge is positive and not negative. Symbol: ${}^{0}_{1}e$ or β^+

Positron emission occurs when a nucleus is unstable due to the high proton:neutron ratio, and resulting instability A proton within the nucleus changes into a neutron and a positron. The neutron remains within the nucleus, but the positron is ejected – as a positive beta-particle.

Example T13.11

Complete the nuclear reaction equation by filling in the mass and atomic number for Neon. Explain why the decay has occurred and name the emission.

$${}^{22}_{11}Na \rightarrow \quad Ne + {}^{0}_{1}e$$

Notes

- There are no naturally occurring positron emitters – for example, sodium-22 is not naturally occurring – it must be created by another nuclear reaction.
- Stable neutron:proton ratios change as atomic number increases (neutron:proton ratios increase from 1:1 to a higher proportion of neutrons, as nuclei get bigger)

Neutrino / Antineutrino emission

The anti-neutrino is the anti-particle of the neutrino. They are both identical except that a neutrino has $spin = \frac{1}{2}$ and an antineutrino has $spin = -\frac{1}{2}$ (spin is an angular momentum characteristic of a particle)

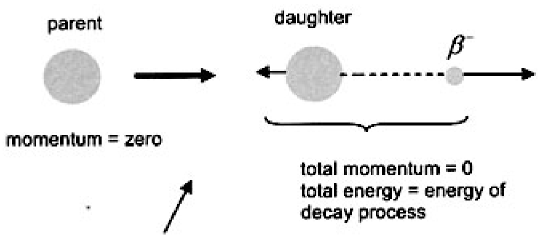
They both:
have zero charge
have zero mass ($\approx \frac{1}{2000} \times mass\ of\ electron$)
travel at speed of light
do not interact with matter (pass straight through, unaffected)
have spin = $\pm\frac{1}{2}$ ($+\frac{1}{2}$ *neutrino*, $-\frac{1}{2}$ *antineutrino*)

symbols: $neutrino: \nu \quad antineutrino: \bar{\nu}$

Neutrino or antineutrino emission was shown to accompany beta-emission, since $\beta - particles$ are emitted with a range of energies. This is not be possible unless the decay also emits a third particle (in addition to the daughter nucleus and a beta-particle).

The diagrams on the next page helps to explain this:

Without third particle:

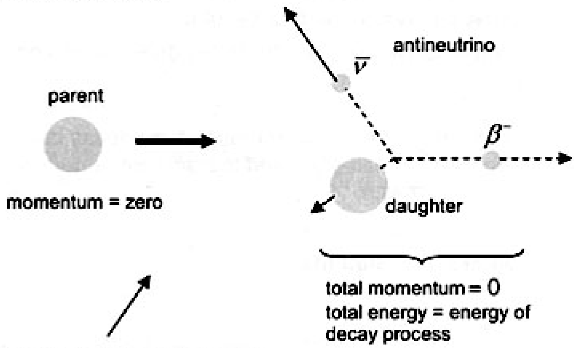


NOT POSSIBLE – BECAUSE:

This kind of two-particle interaction, where momentum is conserved and energy of products is equal to a certain energy, according to the decay process, would mean that the beta particle would have to have a certain energy.

However, it is observed that beta-particles are emitted with a continuous range of values. This can only be possible if a third particle is also emitted, as follows:

With third particle:



THE CORRECT PROCESS

Since there are an infinite number of ways to draw the three momentum vectors so that resultant momentum = zero (just like there are an infinite numbers of ways of drawing a triangle) it is now possible for the beta-particle emitted to have a range of energies – the neutrino (or antineutrino as shown here) can also have any energy (and momentum).

The full beta-particle decay processes are thus:

$$\beta^- \text{ decay} \quad parent \rightarrow daughter + electron\ (\beta^-) + antineutrino\ ({}^0_0\overline{\nu})$$
$$\beta^+ \text{ decay} \quad parent \rightarrow daughter + positron\ (\beta^+) + neutrino\ ({}^0_0\nu)$$

(try writing equations using previous 2 examples, but include neutrinos/antineutrinos)

Decay Law (for Radioactive decay)

There are two versions of the decay law. The first states that the activity of a radioactive sample is proportional to the number of radioactive nuclei present. This is equivalent to saying that the rate of disintegration of a radioactive sample is proportional to the number of radioactive nuclei present. ie.

$$\frac{dN}{dt} \propto N \text{, where, } N = number\ of\ radioactive\ nuclei\ at\ time,\ t$$
$$\Rightarrow \frac{dN}{dt} = -\lambda N \text{ ,where, } \lambda = radioactive\ decay\ constant,\ \frac{dN}{dt} = rate\ of\ decay$$

The second form can be derived from the first by integration (not required for this course) and is as follows:

$$N = N_0 e^{-\lambda t} \text{, where, } \begin{array}{l} N_0 = number\ of\ radioactive\ nuclei\ present\ initially \\ N = number\ present\ at\ time,\ t \\ \lambda = decay\ constant \end{array}$$

Example T13.12

A tree dies, and, on its death, contains 2.2×10^{19} atoms of carbon-14, which is radioactive and has a half life of 5730 years. The tree is then buried underground and remains for many years. Given that the decay constant for C–14 is $1.21 \times 10^{-4}\, years^{-1}$, calculate the number of radioactive atoms per gram remaining after (a) 2000 years (b) 5730 years

Note that the answer to part (b) does not need to involve the use of the decay law equation, since 5730 years is equal to a whole half-life – and the amount remaining after one half life is half what you had to start with.

Derivation of Equation: $\lambda = \dfrac{\ln 2}{T_{\frac{1}{2}}}$**, where** $T_{\frac{1}{2}}$ **= half life**

Referring to: $N = N_0 e^{-\lambda t}$

After one half life has passed half the amount initially present has decayed, so, $N = {}^{N_0}\!/_2$ when $t = T_{\frac{1}{2}}$. Hence,

$${}^{N_0}\!/_2 = N_0 e^{-\left(\lambda T_{\frac{1}{2}}\right)}$$

$$\Rightarrow \tfrac{1}{2} = e^{-\left(\lambda T_{\frac{1}{2}}\right)} \Rightarrow 2^{-1} = e^{-\left(\lambda T_{\frac{1}{2}}\right)} \Rightarrow \ln 2^{-1} = \ln e^{-\left(\lambda T_{\frac{1}{2}}\right)}$$

$$\Rightarrow -\ln 2 = -\lambda T_{\frac{1}{2}} \ln e \Rightarrow -\ln 2 = -\lambda T_{\frac{1}{2}}$$

$$\Rightarrow \lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$$

Example T13.13

A sample of a radioactive isotope contains 10^{18} atoms initially and has a half life of 10 hours.

(a) sketch a graph showing how the number of radioactive nuclei change for the first 30 hours

(b) Find the amount remaining after 10 hours

(c) calculate the decay constant and hence find the amount remaining after 1 day

Note that λ must be compatible with units of $T_{\frac{1}{2}}$ (e.g. NOT $seconds^{-1}$ *with hours* - convert $T_{\frac{1}{2}}$ to "same" units as λ)