Topic Twelve: Electromagnetic Induction

<u>**Electromotive force (emf)**</u> – an emf is a potential difference resulting from a source of electrical power: For example, the internal voltage, or potential difference supplied by a battery is referred to as its emf.

In this topic, the potential difference resulting from a conductor moving through a magnetic field is referred to as emf.

Induced emf or current – the word induced can be replaced with produced, but conventionally induced is the term used.

Emf (voltage) is induced in a wire if there is relative motion between the wire and a magnetic field. This can happen in one of several ways:

- if the wire/coil moves
- if the magnet or electromagnet moves
- if the electromagnet is switched on or off or is alternating

Note that emf (voltage) is only induced if field lines are "cut" by the wire/coil and that the rate of cutting determines the size of the emf induced.

Definition: **The induced emf is the amount of mechanical energy converted into electrical energy per unit charge.**

The unit of emf is the volt

Magnetic flux – a measure of the ***amount*** of field lines passing through an area, at right angles to that area – can be increased by increasing the area or the concentration (flux density) of field lines and is maximum when field lines are at right angles to area (component at right angles to the area decreases as field lines deviate from this angle).

Think of magnetic flux as the number of field lines and think of magnetic flux density as field strength

To understand the definition of magnetic flux density, remember that field strength is defined as the force per unit current length on a wire placed perpendicularly to the magnetic field. So it is essentially defined in terms of force.

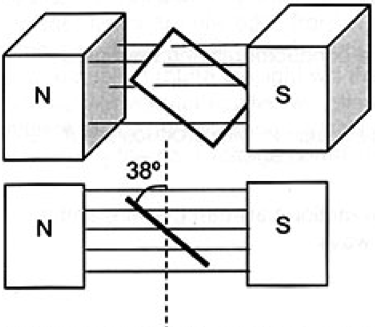
Definition: **Magnetic Flux (Φ) (formula given in Data Book):**

$$\Phi = BA\cos\theta, \quad \text{where } \begin{array}{l} B = \textit{magnetic field strength} - \textit{Tesla } (T) \\ A = \textit{area} - \textit{square metres} \\ \theta = \textit{angle between flux and line normal to area} \end{array}$$

Unit: The weber (Wb) $1\,Wb = 1\,Tm^2$

Example T12.1

A coil of wire measuring 7.5cm by 11.5cm is placed between the poles of a magnet as follows:



Find the magnetic flux passing through the coil, if the magnetic field between the poles of the magnet is 75mT

Magnetic Flux Linkage

Magnetic Flux Linkage - is a measure of the number of turns of wire "linked" to (passing through) magnetic flux. Can be increased by increasing magnetic flux or number of turns of wire.

Definition: $Flux\ linkage = flux \times number\ of\ turns = \Phi \times N$

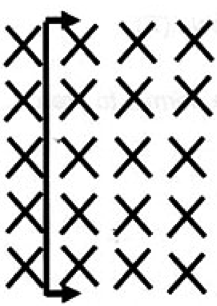
Example T12.2

If, in the previous example, the coil has 120 turns, find the flux linkage of the magnetic field through the coil

Note: Flux linkage has the same units as flux

Induced emf

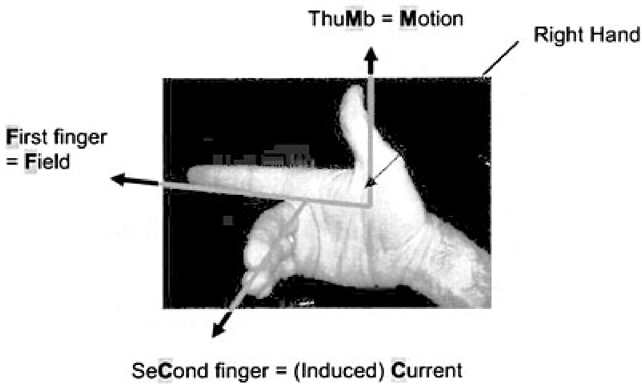
Consider a wire moving through a uniform magnetic field, as follows:



- The wire contains electrons free to move (free electrons).
- The movement of the wire from left to right means that the electrons within the wire also move from left to right.
- From previous work (chapter 5) we saw that charges moving within a magnetic field experience a force.
- The movement of the electrons, with the wire, from left to right, can be thought of as an electric current, from right to left (conventional)
- Using Fleming's Left Hand Rule (FLHR), we can predict a force on each electron down the wire (from top to bottom)
- This force on the electrons is the induced emf and a current will be induced in the wire, if allowed, from bottom to top (conventional current)

So, whenever a wire moves through a magnetic field, an emf is induced in the wire. The direction of the induced current can be determined using the method described above, but it is easier using Fleming's Right Hand Rule (FRHR)

The thumb and fingers of the ***Right Hand*** are arranged as follows:



Notes

- if conductor moves through a magnetic field parallel to the field lines, no emf is induced; there must be a component of the field perpendicular to the movement of the conductor.
- As a conductor moves through a magnetic field, field lines (flux) are said to be cut.
- The size of the induced emf is dependent on the rate at which field lines are cut

Example T12.3

An emf is induced across the wings of an aircraft as it takes off and continues on its flight-path. Give two reasons why the induced emf may be greater during the aircraft's flight-path than during take-off.

<u>Faraday's Law</u>

The induced emf (ε) in a circuit is equal to the rate of change of flux linkage through the circuit.

$$\varepsilon = -\frac{\Delta(N\Phi)}{\Delta t} = -N\frac{\Delta\Phi}{\Delta t}$$ *(given in data book)*

gradient of $N\Phi$ versus t graph

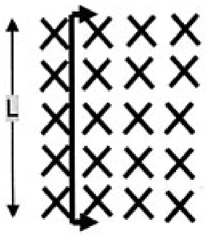
gradient of Φ versus t graph

Note;

- The minus sign indicates that the emf is always induced so as to oppose the change causing it – see Lenz's law.
- Flux linkage changes if a wire or a coil moves through a magnetic field (area cut multiplied by turns) or if a coil spins or moves in a magnetic field.

Derivation: emf induced in a straight conductor moving through a uniform magnetic field

Faraday's law can be applied to a straight conductor moving through a uniform magnetic field at right angles to the field, as follows:



Let:
Strength of field = B
Area swept out per second = A
Length of wire in contact with field = L,
Wire move through field at speed v

$$\frac{\Delta\Phi}{\Delta t}=\frac{\Delta(BA)}{\Delta t}\ Since\ \Phi = BA\cos 90 = BA$$

$$B\frac{\Delta A}{\Delta t}\ (since\ B\ is\ constant)$$

$$= B\frac{L\times d}{\Delta t}\ (where\ d = distance\ moved)$$

$$= BL\frac{\Delta d}{\Delta t}\ (since\ L\ is\ constant)$$

$$= BLv\left(\frac{\Delta d}{\Delta t} = speed\right)$$

For a wire moving through a magnetic field, at speed v, at right angles to the field, strength B, the induced emf, ε, is given by:

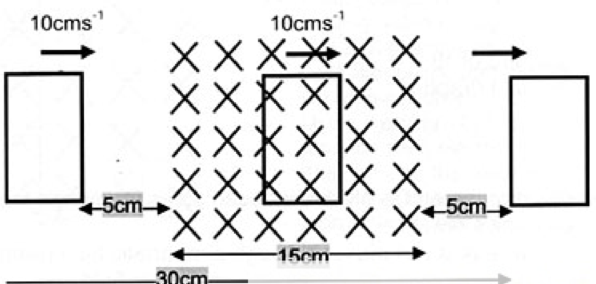
$$\varepsilon = BLv$$

Example T12.4

A horizontal steel rod of length 235mm is dropped horizontally through a magnetic field of strength 45mT and direction horizontal and at right angles to the rod. Find the size of the induced emf after (a) 0.1 second (b) 0.5 seconds, assuming g=10ms^{-2} and that the rod remains in the field as it drops, and remains horizontal.

Example T12.5

A 35 turn rectangular coil, measuring 5.0cm by 8.5cm is moved through a uniform 35mT magnetic field at a speed of 10 cms^{-1}, as shown in the diagram below:



(a) Describe the motion of the electrons and the current in the coil as the coil:

(i) begins to enter the magnetic field
(ii) moves totally within the magnetic field
(iii) begins to move out of the magnetic field, on the right hand side

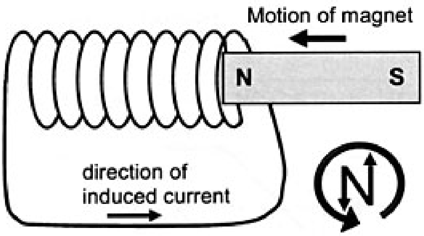
(b) Calculate the maximum current in the coil, assuming that the resistance of the coil is 0.01Ω and explain where the coil is in order to attain this maximum current

(c) Sketch a graph showing how the current on the bottom edge of the coil varies with distance moved by the coil, as it moves forward 30cm, as shown in the diagram.

Lenz's Law

The direction of the induced current (or emf) is always so as to oppose the change causing it.

Lenz's law is an application of the principle of conservation of energy. If Lenz's law did not apply, energy could be made from nothing.
Consider, for example, moving a bar magnet into a cylindrical coil, with the North end entering first, as follows:



- The magnet is pushed into the coil.
- This movement is the change causing the induced current.
- The direction of the induced current will be so as to oppose this motion
- A north pole will therefore be induced on the right hand end of the coil (to repel the magnet)
- This corresponds to an anti-clockwise current (see diagram below magnet) as viewed from this end of the coil.
- The current in the coil is therefore as indicated on the diagram.

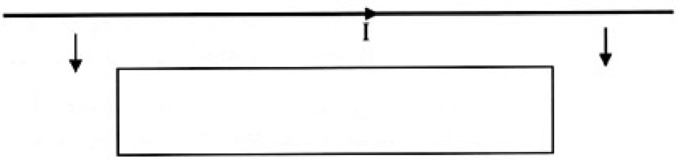
Note that if a S pole was induced on the coil-end, the magnet would be attracted into the coil, so it would move faster into the coil, a greater induced current would result, and electrical energy would be created out of nothing. As it is, in order to induce current, a force has to be applied to overcome repulsion and work done against this force equals electrical energy induced in the coil. Energy is conserved.

Using similar arguments, as the coil is pulled out, an attractive S pole will be induced on the right hand end of the coil, and current will flow in the opposite direction.

In many instances Lenz's law or FRHR can be used to predict the direction of the induced current. However, the Lenz's Law method is often simpler and easier to use.

Example T12.6

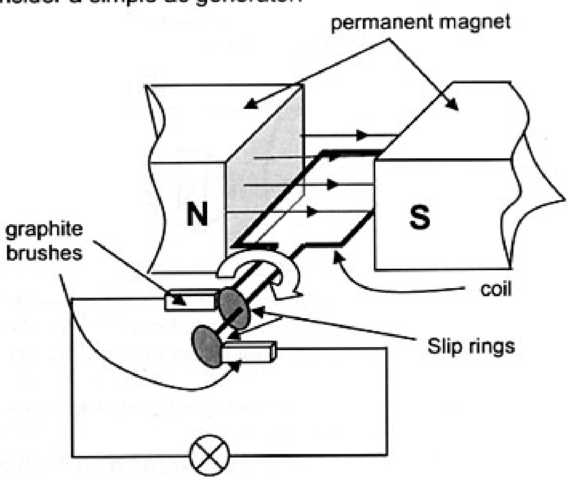
A long straight wire is in the same plane as a rectangular loop of metallic wire. The straight wire carries a current, I as shown, and is moved towards the rectangular loop. What happens to the loop of wire?



Alternating Current

Electricity is generated by using a primary source of energy to turn turbines which in turn rotates coils of wire in magnetic fields. This form of electricity generation results in an alternating emf, that varies sinusoidally. Hence our domestic electricity is not dc (direct current) but alternating, with a frequency of 50 Hz (50 cycles per second) in the United Kingdom and many other countries.

To understand this, consider a simple ac generator:



The ac generator is very much like a motor, except that instead of current being fed into the coil and the coil interacting with the magnetic field and turning, the coil is made to turn through a magnetic field and this results in current being induced in the coil.

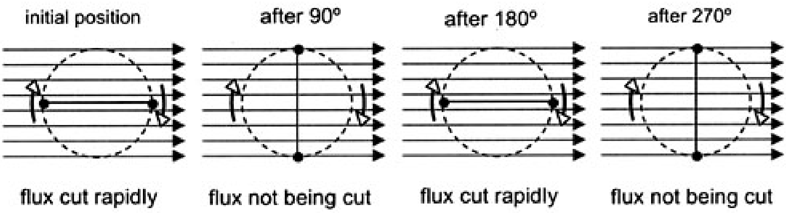
Consider the coil in its current position:

- If it is made to rotate clockwise (as viewed from in front of the page)
- The left hand side of the coil moves upwards
- Using FRHR, current is induced in the wire on this side into the page
- The right hand side of the coil moves downwards
- Using FRHR, current is induced in the wire on this side out of the page, towards the observer.
- The current through the bulb will be moving from right to left (←)

However, as the coil continues to rotate, the direction of the current changes. As it is moving at 90° from its current position, there will in fact be zero current induced in the coil. At this position, the wires (at top and bottom of coil, in this position) are not cutting field lines, but moving along them.

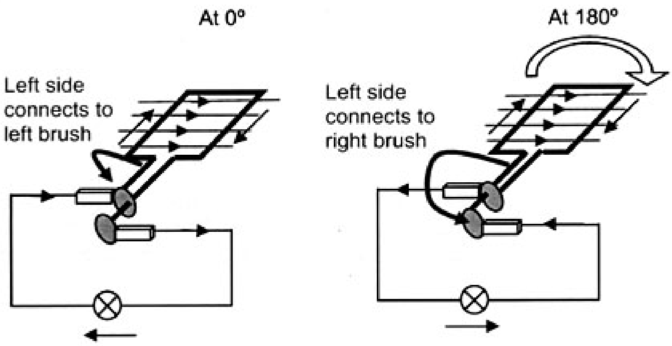
Consider the following end-on diagrams of the side of the coil moving through the magnetic flux:

The sides of the coil are represented by the marks (•). These wires are the ones that move through the magnetic field and where emf is induced. The size of the induced emf depends on the rate at which the flux is cut.

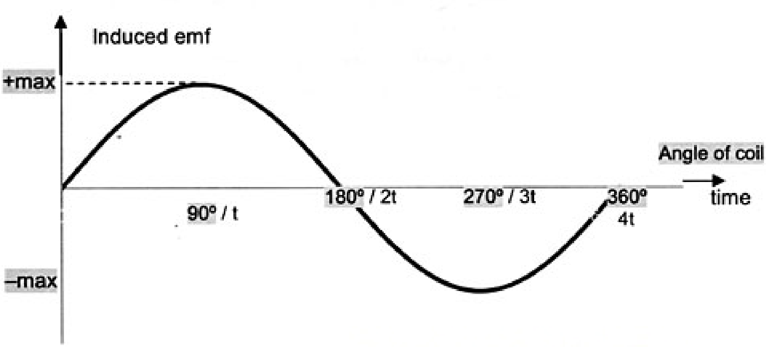


As viewed from the front (generator diagram), current is always induced into the page. However, every 180°, the wires swap over the brush that they feed into. This results in a reversal of direction of current every 180°.

To show this, we can look at the relevant parts of the generator as it turns through 180°:



A current-time graph thus reveals the sinuisoidal emf (and current) that results in the coil: Time corresponds to angle of coil, since the coil rotates at a constant speed.



Note that the maximum flux change (rate of cutting flux) is when the coil is parallel to the field – this may seem confusing, but refer to the four diagrams above and count how many field lines are cut in the top 45° sector of the coil's rotation path (≈ 0) and compare this with the number cut in a 45° sector at either the left or right of the centre of the circle (≈ 3 *or* 4).

Note also that if the rotation frequency of the coil is doubled, the peaks on the above graph occur twice as often and the maximum (peak) emf is twice as high – so the graph will have twice the amplitude

<u>Power of a.c. current & voltage.</u>

Direct current (d.c.) is current that always flows in the same direction. Electrical supplies that produce d.c. current usually produce current that is not only in a constant direction, but also of a constant size.

For example, $3A\ d.c \Rightarrow$ current is a constant value of $3A$.
So what does $3A\ a.c$ mean?

To run many electrical appliances, either a.c. or d.c. electricity may be used. For example, a 60 watt light bulb appears to be identical in brightness whether run on a.c. or d.c. (as long as the correct voltage is used in both instances)

The problem is that the voltage of an a.c. supply varies from negative to zero to positive. One could quote the maximum positive voltage, as the voltage of an a.c. supply. If we do this, then compare running a bulb with 50V d.c. with 50V a.c. Clearly the 50V d.c. supply would supply a higher amount of energy to the bulb, since it remains at 50V, whilst the a.c. supply varies, producing a lower average rate of energy-supply, and resulting in a dimmer bulb.

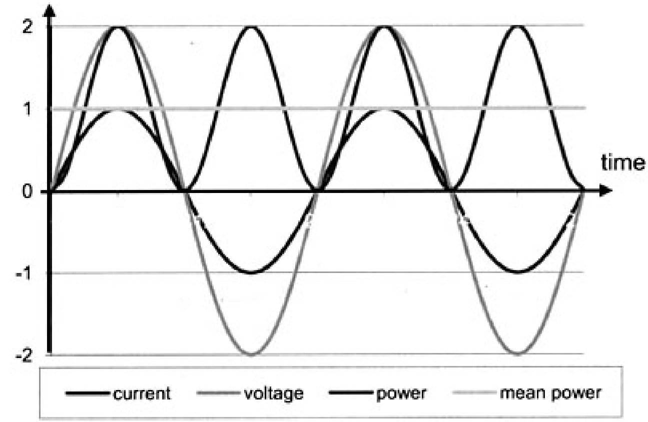
There is another, more useful way of measuring a.c. voltage and current.

Consider a supply that produces an alternating voltage, with maximum (peak) value, 2V. Suppose that this results in a current of peak value 1A. The current produced by a sinuisoidal a.c. voltage is also sinuisoidal, as long as the conductor is ohmic (in which case, $V \propto I$)

This is represented on the graph below – the red line represents current and the green line; voltage.

The average current, if one wished to find it, is clearly zero, and the average voltage is zero. However, since $power = current \times voltage$ and $negative \times negative = positive$, power is always positive. When the current and voltage as shown in the graph are multiplied together, the result is the blue line, representing power.

Current, voltage and power in an a.c. circuit - graph



One can see intuitively from the graph that the average power is half peak (maximum) power, 1 watt, in this case. Maximum power is 2 watts, and minimum power is 0 watts, so power also varies. With mains appliances, voltage and current change direction so rapidly that the maximum and zero power are not observed – one does not see the bulb dim and brighten (in fact, the bulb would not have time to cool down). The average power is observed.

So average power of an a.c. supply is equal to half peak power.
This leads us to the concept of root-mean-square current and voltage:

I'll start by defining the problem: imagine an a.c. circuit , of resistance R, that has a peak supply voltage equal to V_0 which results in a peak a.c. current of I_0. We want to find the equivalent d.c. voltage and equivalent d.c. current that will deliver the same power as the a.c. circuit.

$$power = current \times voltage$$
$$peak\ power(P_0) = peak\ current \times peak\ voltage = I_0V_0$$
$$average\ power = \tfrac{1}{2}peak\ power = \tfrac{1}{2}(I_0V_0)$$
$$But\ V = I \times R \Rightarrow V_0 = I_0R \ \Rightarrow\ I_0 = \frac{V_0}{R}$$
$$\therefore average\ power = \tfrac{1}{2}(I_0(I_0R)) = \tfrac{1}{2}I_0^2R = \tfrac{1}{2}\left(\frac{V_0}{R}\right)V_0 = \tfrac{1}{2}\frac{V_0^2}{R}$$

so we have four possible expressions for average power:

$$average\ power = \tfrac{1}{2}P_0$$
$$average\ power = \tfrac{1}{2}(I_0V_0)$$
$$average\ power = \tfrac{1}{2}I_0^2R$$
$$average\ power = \tfrac{1}{2}\frac{V_0^2}{R}$$

Now to find the d.c. voltage and current. Suppose the voltage required to produce the same power as the average power calculated above is V and the current, I

$Power = V \times I$, but $V = I \times R$ $\therefore Power = (I \times R) \times I = I^2R$

$or, I = V/R$ $\therefore Power = V \times V/R = \dfrac{V^2}{R}$

Equating d.c. power to a.c. power, we get:

$$I^2\cancel{R} = \tfrac{1}{2}I_0^{\,2}\cancel{R} \Rightarrow I^2 = \tfrac{1}{2}I_0^{\,2} \Rightarrow I = \sqrt{\tfrac{1}{2}I_0^{\,2}} = \frac{I_0}{\sqrt{2}}$$

$$\text{and,}\quad \frac{V^2}{\cancel{R}} = \tfrac{1}{2}\frac{V_0^{\,2}}{\cancel{R}} \Rightarrow V^2 = \tfrac{1}{2}V_0^{\,2} \Rightarrow V = \sqrt{\tfrac{1}{2}V_0^{\,2}} = \frac{V_0}{\sqrt{2}}$$

We call the d.c. voltage and current required to give the same power as the average (mean) power produced by peak voltage and current of V_0 *and* I_0 the rms (root mean square) voltage and current, and give them the symbols, V_{rms} *and* I_{rms}

Hence: $I_{rms} = \dfrac{I_0}{\sqrt{2}}$ *and* $V_{rms} = \dfrac{V_0}{\sqrt{2}}$

Example T12.7

A bulb is usually powered with an a.c. supply with a peak emf of 12V. What d.c. voltage would be required to power the bulb with equal brightness?

Example T12.8

In the U.K. mains electricity is alternating at 50Hz, and has rms voltage of 240V. What is the maximum voltage produced by a U.K mains supply?

Example T12.9

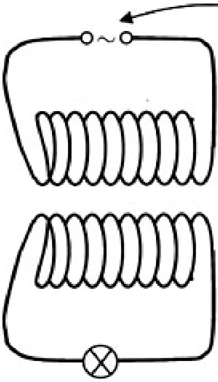
An electric heater designed for use for the U.K. mains supply, (a.c, 240V,rms), is rated as 2000W.

(a) What is the rms current through the heating element when connected to the mains?
(b) What is the maximum current when connect to the mains?
(c) What is the resistance of the heating element?
(d) If the heater is run on an a.c. supply with a peak voltage of 240V, how will this affect the power of the heater?

<u>Transmission of Electrical Power</u>

Transformers

Transformers make use of electromagnetic induction, but also allow voltage to be increased or decreased without significant loss of power. Transformers basically consist of 2 coils of wire. The first coil, called the primary coil, is powered by an a.c. electrical supply. Electro-motive-force (voltage) is induced in the other (secondary) coil. This induced voltage and current is caused by expanding or collapsing field lines caused by the changing current in the primary coil (supply is alternating). The field is therefore continually cut by wires in the secondary coil.



The voltage induced in the secondary coil depends on the turns in the primary and secondary coil and on the voltage across the primary coil so by altering the turns ratio for the two coils, the voltage in the secondary can be changed from that provided by the supply (in the primary coil).

Equation:

$\frac{V_p}{V_s} = \frac{N_p}{N_s}$, where, V_p = *voltage in primary coil*, V_s = *voltage in secondary coil*
N_p = *turns in primary coil*, N_s = *turns in secondary coil*

Transformers which increase the supply voltage are called ***step up transformers*** and those which reduce it are called ***step down transformers***.

Example T12.10

A transformer has been designed to convert voltage from 240V to 12V. If the secondary coil has 125 turns, how many turns should the primary coil have? (assume transformer is 100% efficient)

Obviously, if voltage is increased, there must be a decrease in current, (so that power is same in both coils) otherwise energy would be created.

If transformers are completely efficient in transferring energy from one coil to the other then,

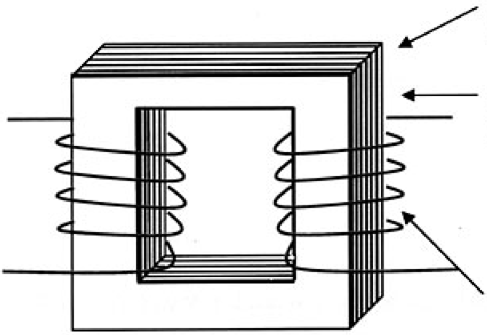
Power used in primary coil = power produced in secondary

$$\Rightarrow V_p I_p = V_s I_s$$

$$\Rightarrow \frac{V_p}{V_s} = \frac{I_s}{I_p},\ and\ \frac{N_p}{N_s} = \frac{I_s}{I_p}$$

Ideal Transformer – construction

Transformers are never 100% efficient, but they are designed to be as close as possible to "ideal" (100% efficient). They are constructed, as follows:



Note: Eddy current are currents induced in the core which try to oppose and reduce the current induced in the wire. Laminating the core makes it difficult to conduct electricity without changing significantly the magnetic linkage properties of the core.

Power Transmission

Transformers are hugely useful in the National Grid system of electricity distribution, since the transfer of electricity is more efficient at high voltage and low current than at high current and low voltage (high current leads to a high level of energy loss by heating – as a consequence of resistance in the transporting wires).

Transformers are thus used to increase voltages at power stations for transport over the grid. They are employed again to decrease voltages to the appropriate level at the users end of the network. The power lost as heat in the wires is given by:

Power Loss = I^2R where I is the current and R is the total resistance of the wires used.

Example T12.11

A factory requires electrical energy at a rate of 20MW. If the resistance of the electricity cable used to transport electricity to the factory is $4.8\,\Omega$, calculate the power losses in the cable if the supply voltage is: (a) 100kV (b) 10kV

The example demonstrates that increasing the current in the cable by a factor of 10 causes a 100-fold increase in power-loss. Therefore using high voltage and low current electricity transmission (particularly over long distances) is far better than using low voltage and high current.

Risks and Concerns

Being in the presence of power lines or electrical appliances can cause the induction of tiny currents in the human body. However, it seems unlikely that low frequency magnetic fields such as these cause any harm to the human body. Further study is necessary: currently we have no conclusive evidence.