Extra Credit Exercises from Module VI

Exercise 8) b) Let $d = \text{ord}(\delta)$.

Now, since $M_\nu(x)$ is irreducible , monic and $M_\nu(\delta) = 0$, $M_\nu = M_\delta$.

Hence

$$M\nu \mid x^d - 1$$

and so

$$\nu^d = 1$$

Thus ord $\nu \mid d$. Let $d' = \text{ord}(\nu)$. Then

$$M_\delta \mid x^{d'} - 1$$

so that

$$\delta^{d'} = 1$$

Thus $d \mid d' = \text{ord}\,\nu$.

Exercise 9) If $a + b \neq 0$ then $\exists\, 0 \leq t < p^k - 1$ such that

$$a + b = \alpha^t$$

i.e.
$$\alpha^i + \alpha^j - \alpha^t = 0$$

Set $f(x) = x^i + x^j - x^t$ and realize $f(\alpha) = 0$. Thus

$$M_\alpha \mid f$$

and so by Lemma 2 (2)

$$0 = f(\beta) = \beta^i + \beta^j - \beta^t$$

Hence

$$\Psi(a + b) = \beta^t = \beta^i + \beta^j = \psi(a) + \psi(b)$$