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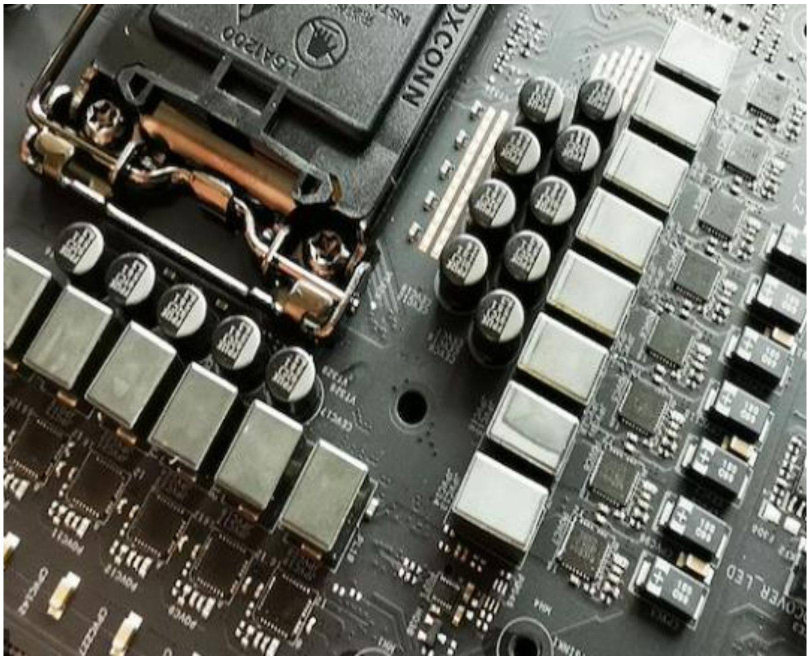

CONTROL SYSTEMS LAB REPORT

Group Number:55

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LAB V4: Design and performance evaluation of PID control

Experiment Performed on: 17.05.2022

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1. Introduction

Several combinations of P, I and D controllers can be used to build a suitable controller for a system which results in an almost ideal characteristic behavior. This experiment deals with P, PI and PIDT1 controllers which are further explained below.

- **P-controller** or a proportional controller whose transfer equation is described below, where K_R is known as the controller gain.

$$\boldsymbol{Transfer\ function}\ G_R(s) = K_R$$

- **PI controller** or the proportional-integral controller transfer equation is described below, where $K_R$is the P-controller gain and TI is the integrator time constant.

$$\boldsymbol{Transfer\ function}\ G_R(s) = K_R\left(1+\frac{1}{T_I s}\right)$$

- **PIDT1 controller** or the proportional-integral-derivative controller with a first order low pass filter whose transfer function is described below, where K_R is the P-controller gain, T_I is the integrator time constant; T_D is the derivative time constant and $T_V$is the filter time constant.

$$\boldsymbol{Transfer\ function}\ G_R(s) = K_R\left(1+\frac{1}{T_I s}+\frac{T_D s}{T_V s+1}\right)$$

In order to test the above controllers, the following plant models were used:

1. **Plant A**: is a second order plant with one pole at the origin. An example of this plant is the positioning of a DC motor. Its transfer function is given by,

$$\boldsymbol{G_{SA}(s)} = \frac{\mathbf{1.5}}{(\mathbf{1+0.1}\boldsymbol{s})\boldsymbol{s}}$$

2. **Plant B**: is a plant with three real poles. It is an overdamped system. These plants can be found in chemical and process industries. Its transfer function is given by,

$$\boldsymbol{G_{SB}(s)} = \frac{\mathbf{1.5}}{(\mathbf{1+0.1}\boldsymbol{s})(\mathbf{1+0.5}\boldsymbol{s})(\mathbf{1+2}\boldsymbol{s})}$$

3. **Plant C**: is an under-damped third order system. This kind of systems are found in vehicle drivelines, flexible arm robot manipulators or electrical transmission lines. Its transfer function is given by,

$$G_{SC}(s) = \frac{1.5}{(1+0.1s)\left(\left(\frac{s}{5}\right)^2+2\cdot 0.4\frac{s}{5}+1\right)}$$

This experiment is divided into two parts. The first part deals with the derivation of the controller parameters and the second deals with the implementation of the values on the plant. The implementation of the control parameters analyzes the setpoint tracking and disturbance rejection characteristics of the controller-plant loop.

2.1 Plant A

- P-controller

The controller gain K_R is found by generating a Bode plot for the $G_R \cdot G_{SA}$, with the parameter K_R = 1, then the magnitude is found at the gain crossover frequency ω_{gc} with the phase margin of ϕ_m equal to 60°. Figure 1 shows the Bode plot for the plant with ω_{gc}.

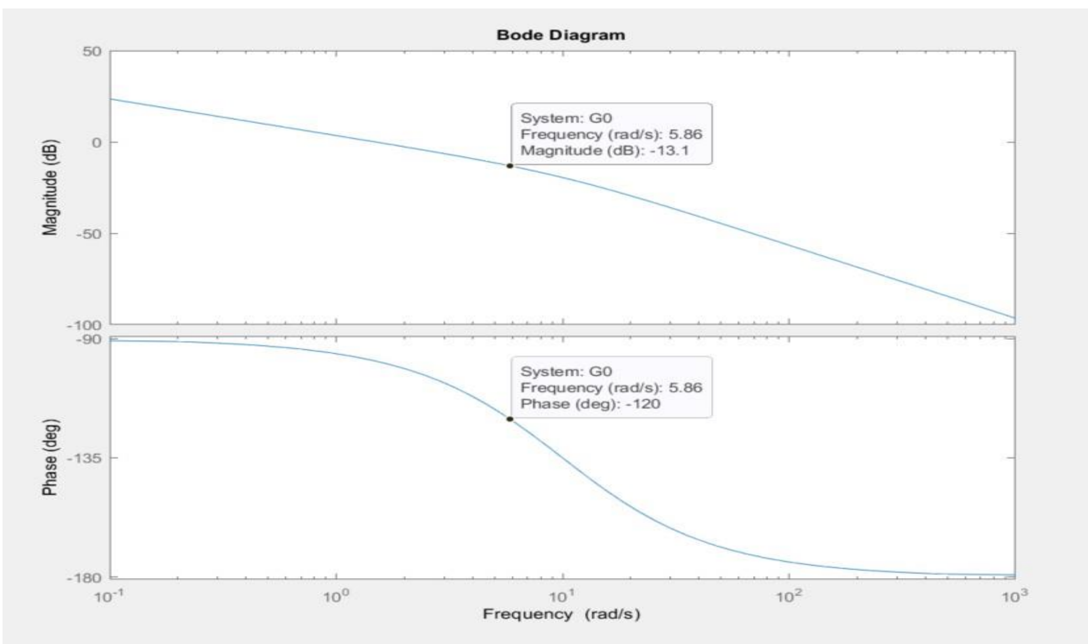

Figure 1: Bode Diagram for Plant A with K_R = 1

The magnitude A_{dB} at the gain crossover frequency is used to find the K_R,

$$K_R = 10^{\frac{-A_{dB}}{20}}$$

K_R = 4.510

- PI-controller

Since, one of the poles coincides with the origin, the symmetrical optimum method is used to calculate the T_I and $K_R$and this controller uses the transfer function as per the equation above.

The value of T_I is calculated using the trial-and-error method as long as the phase curve reaches a maximum value of -135◦ and T_I is found to be 0.59s.

The controller gain, K_R is derived the same way but with a phase margin ϕ_m of 45◦ by plotting the Bode plot of $G_R \cdot G_{SA}$

2.2 Plant B

- P controller

The controller gain, K_R, is calculated by plotting the Bode plot of transfer function $G_R \cdot G_{SB}$ with an initial Kc value of 1 for a phase margin ϕm of 60◦ whose gain was derived earlier.

- PI controller

The integrator time constant is found such that the controller zero cancels the slowest pole of the plant. The slowest pole is the non-zero pole that lies closest to the origin which in this case is -0.5. For the slowest pole to get cancelled,

$$-\frac{1}{T_I s} = -\frac{1}{2s}$$

So, $T_I = 2s$

With the above T_I, we can calculate K_R by reading the magnitude at a frequency of 0.931 and use equation for K_R at a phase margin of 60◦ from the Bode plot resulting using $G_R \cdot G_{SB}$.

- PIDT1 controller

The parameters T_I, $T_D$and $T_V$must be determined to calculate the K_R parameter value. The time constants are to be determined such that the controller zeros cancel the two slowest poles of the plant. The two slow poles in this plant are -2 and -0.5.

The value of K_R can be obtained using $G_R \cdot G_{SB}$for a phase margin of 60◦.

$$G_{SB} = \frac{1.5}{(1+0.1s)(1+0.5s)(1+2s)}$$

$T_V = 0.1$

$(T_D + T_V)\, T_I$=1 ; $(T_D + 0.1)$2.4=1; T_D=0.3166s;

$(T_I + T_V) = 2.5$; $T_I = 2.5 - 0.1 = 2.4s$;

2.3 Plant C

- P controller

The controller gain K_R for a phase margin ϕ_m of 60◦ calculated from the Bode plot of G_R·G_{SC} and is mentioned in table 1.

- PI controller

The integral time constant T_I, is calculated using the natural frequency of the conjugate complex pole-pair of the plant, which is,

$$\left(\frac{s}{5}\right)^2 + 2 \cdot 0.4 \frac{s}{5} + 1$$

where the natural frequency ω_0 is 5 Hz as it is in the form

$$\left(\frac{s}{\omega_0}\right)^2 + 2 \cdot D \frac{s}{\omega_0} + 1$$

Therefore,

$$T_I = \frac{1}{\omega_0} = 0.20s$$

The gain K_R can be calculated from the Bode plot of G_R·G_{SC} for a phase margin ϕm of 60◦.

- PIDT1 controller

The conjugate complex poles of the plant are to be cancelled out by the controller zeros.

Where T_V is,

$$T_V = \frac{0.2}{\omega_0} = 0.040s$$

$$(T_D + T_V)\, T_I = 1/25$$

$$T_I = \frac{2 \times 0.4}{5} - 0.04 = 0.1200$$

$$T_D = 0.2940$$

The control gain K_R for a phase margin of 60◦ from the Bode plot resulting from G_R·G_{SC}

PLANT	Controller	Φ_m[◦]	ω_{gc}	K_R	T_I[s]	T_D[s]	T_V[s]
A	P	60	5.86	4.510	-	-	-
	PI	45	4.28	2.886	0.59	-	-
B	P	60	1.58	2.857	-	-	-
	PI	60	0.931	1.374	2.00	-	-
	PIDT1	60	2.64	4.516	2.40	0.3166	0.100
C	P	60	5.12	0.6166	-	-	-
	PI	60	3.39	0.3020	0.20	-	-
	PIDT1	60	3.81	0.3316	0.12	0.2940	0.040

Table 1: Controller Parameters

3. Implementation of PID Control

A Simulink model is set up for the control loop with the three plants and simulated for a period of 8s for setpoint tracking and disturbance rejection .A figure of the Simulink model is given below(shown for C PIDT1):

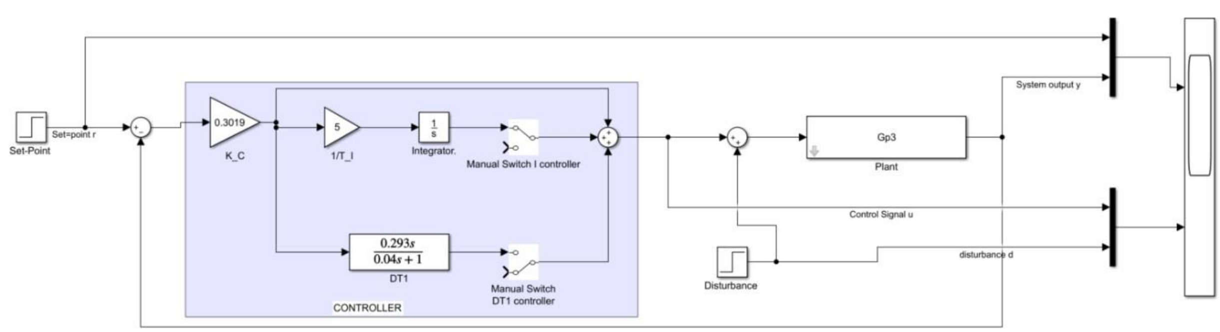

Figure 2: Simulink Model of the Control Loop

3.1 Plant A

The setpoint tracking and disturbance rejection results are given below with both P and PI controllers.

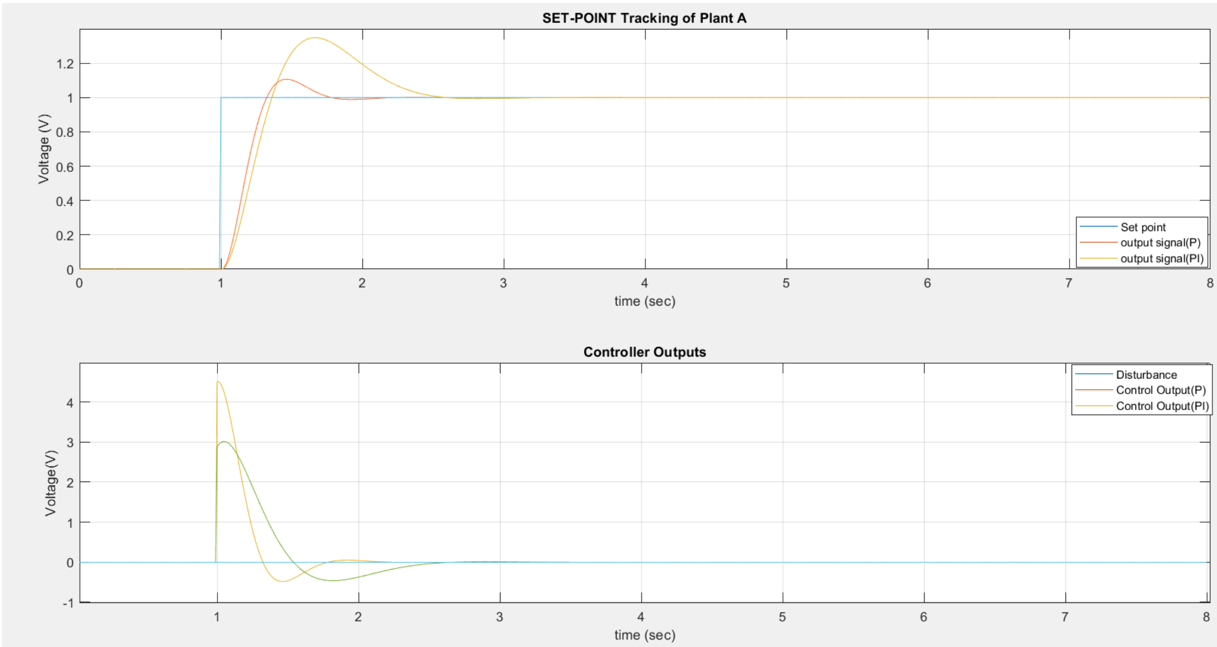

Figure 3: Setpoint Tracking for Plant A

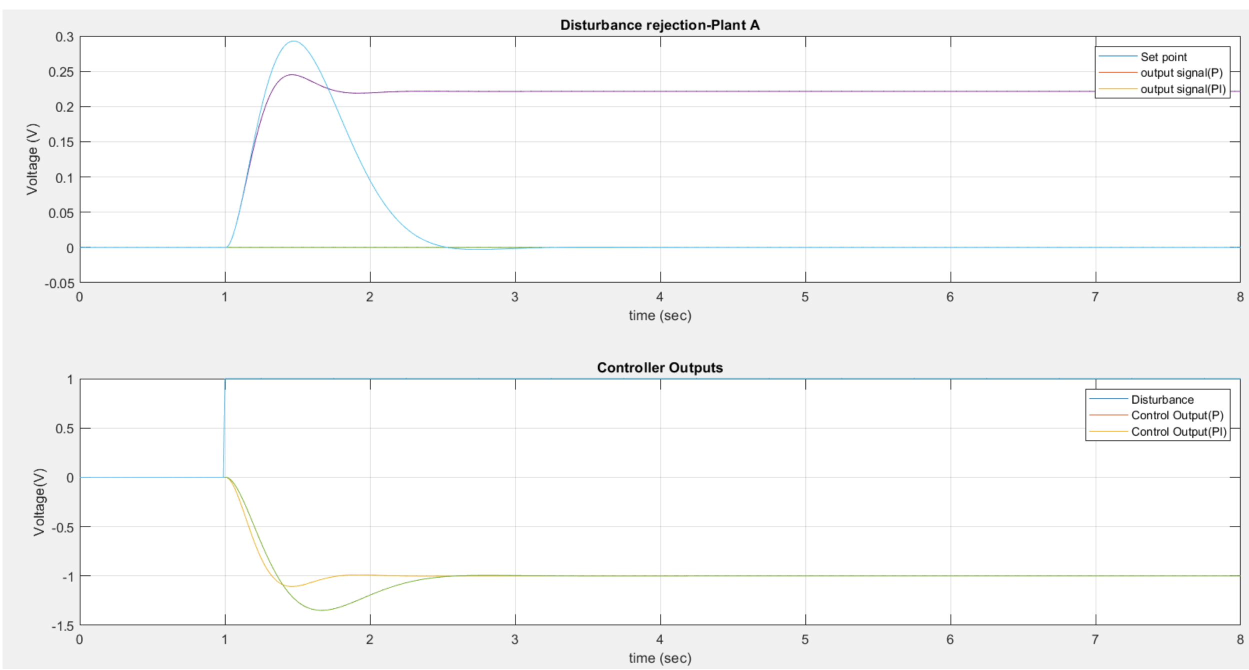

Figure 4: Disturbance Rejection for Plant A

3.2 Plant B

The setpoint tracking and disturbance rejection results are given below with P, PI and PIDT1 controllers.

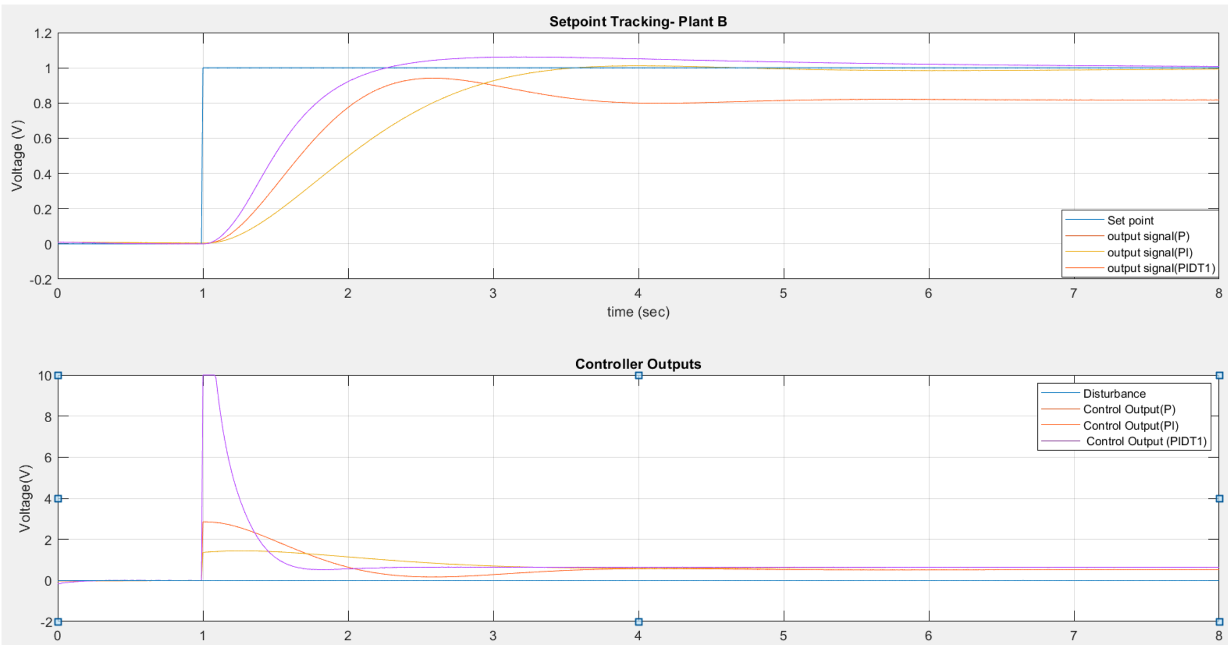

Figure 5: Setpoint Tracking for Plant B

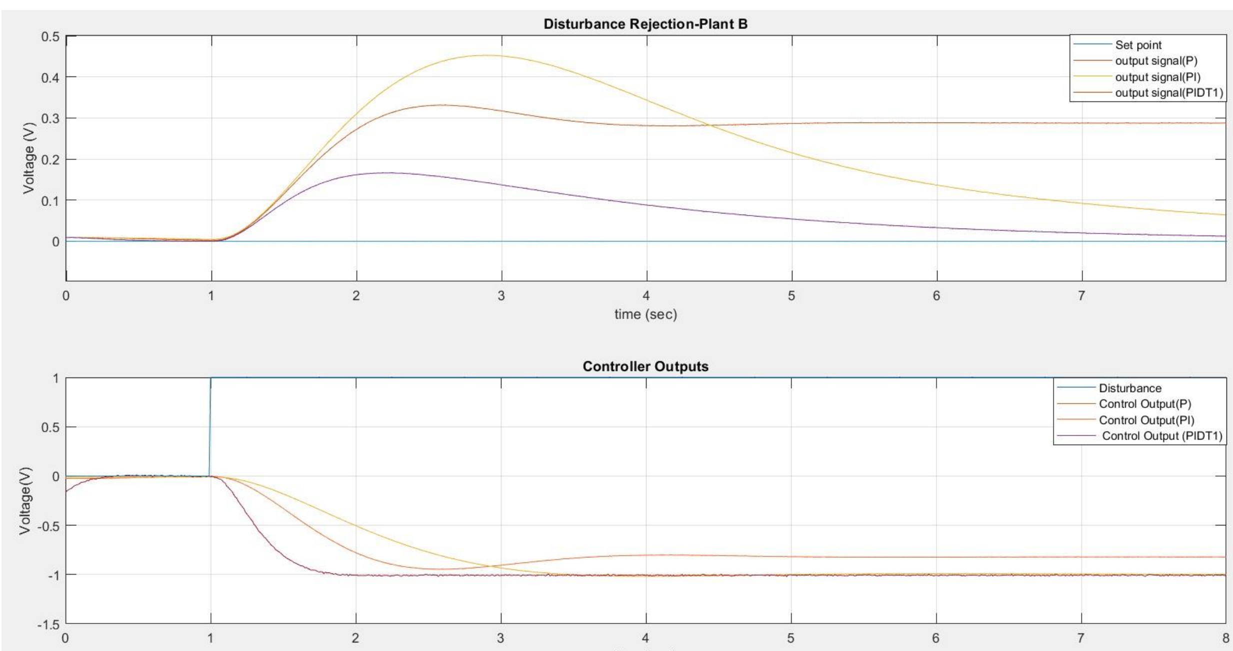

Figure 6: Disturbance Rejection for Plant B

3.3 Plant C

The setpoint tracking and disturbance rejection results are given below with P, PI and PIDT1 controllers.

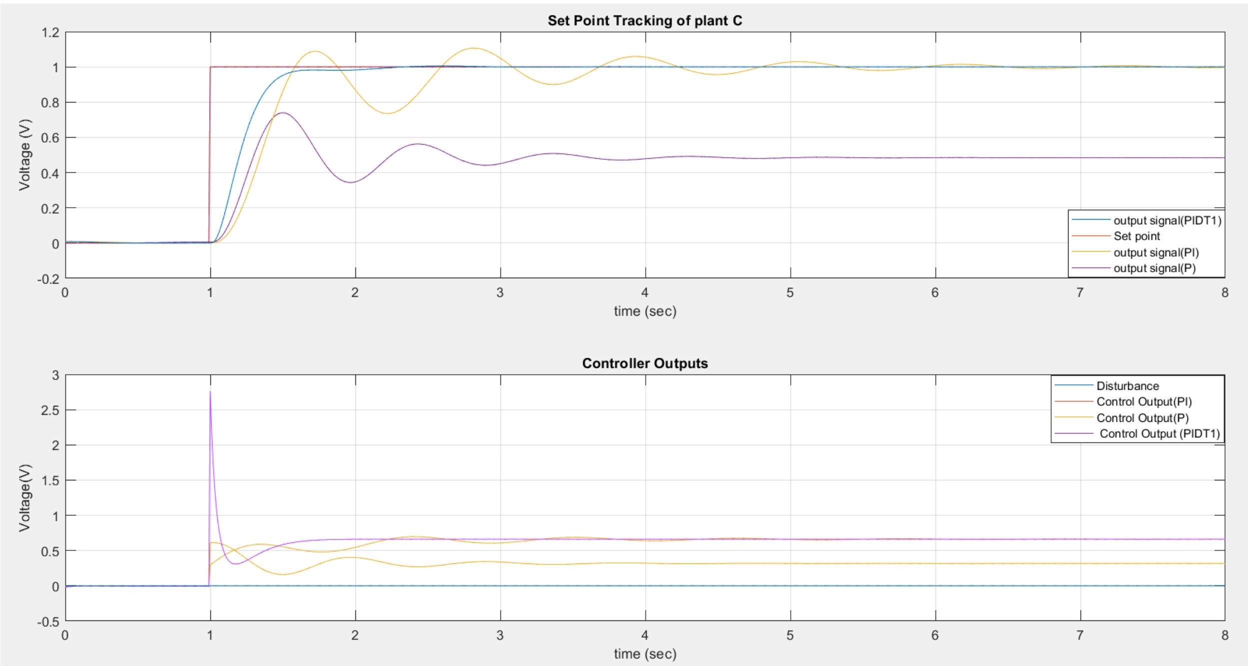

Figure 7: Setpoint Tracking for Plant C

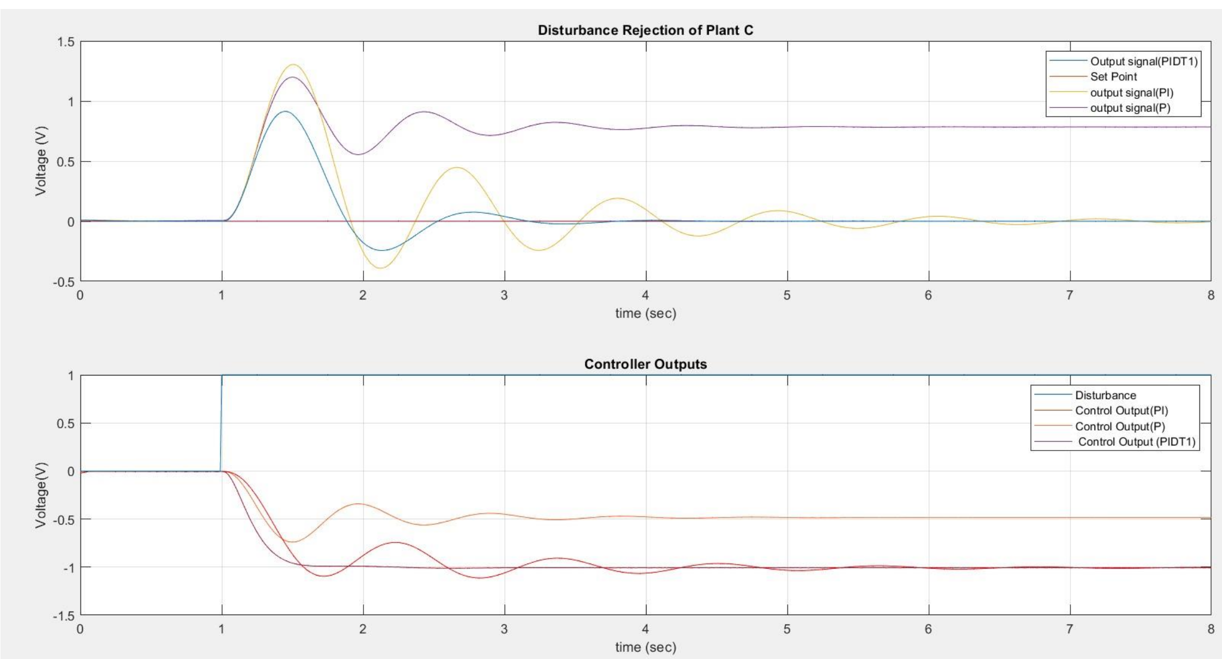

Figure 8: Disturbance Rejection for Plant C

3.4 PIDT1 Controller for Plants B and C

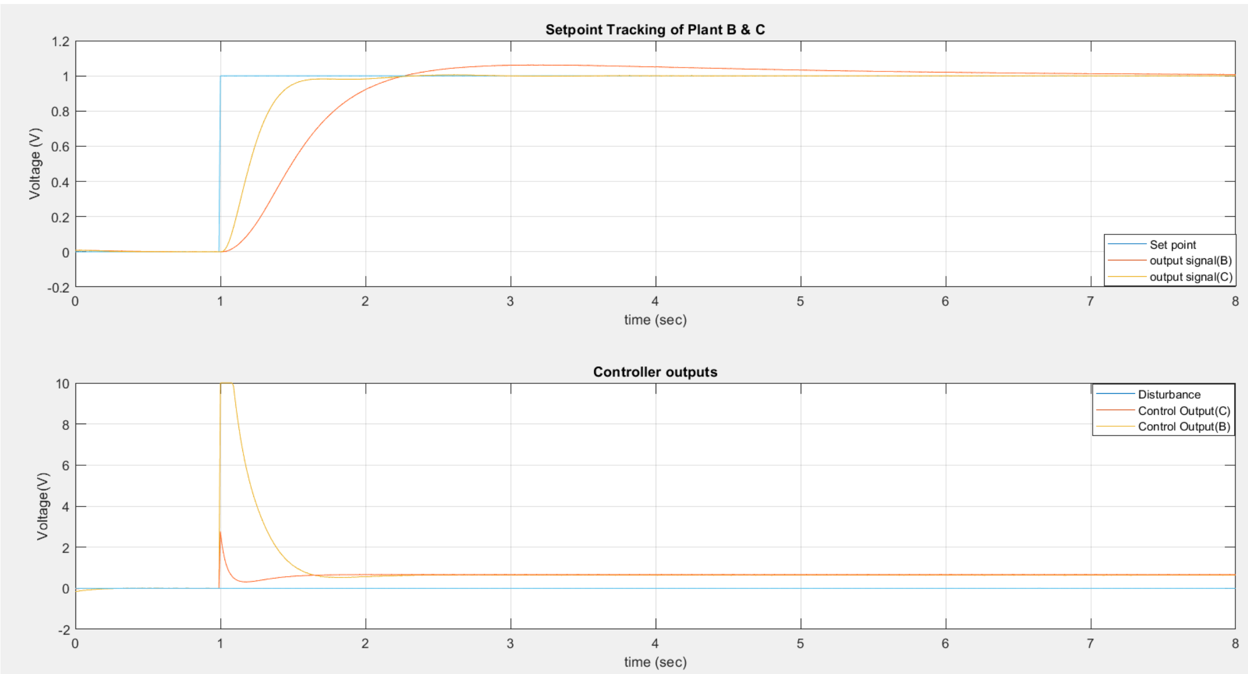

Figure 9: Setpoint Tracking for Plants B and C (with PIDT1 Controller)

The above figure shows the Setpoint tracking of the Plants B and C with a PIDT1 Controller to compare the system characteristics.

4. Results

The characteristics of the implementation of the PID control on the three plants is given in table 2.

Plant	Controller	Setpoint Tracking				Disturbance Rejection	
		e_∞ [%]	M_p [%]	t_r [s]	$t_{s,5\%}$ [s]	y_∞	$t_{s,0.05\%}$ [s]
A	P	0.00	9.58	0.243	0.710	0.222	0.0752
	PI	0.00	34.20	0.256	1.432	0.000	0.1283
B	P	18.9	20.72	0.660	3.737	0.284	0.2742
	PI	0.20	8.30	1.369	4.194	0.033	0.6826
	PIDT1	0.00	7.30	0.476	1.293	0.012	0.6968
	P	50.7	49.68	0.220	3.217	0.779	0.3226

C	PI	0.00	17.40	0.392	4.020	0.000	0.5235
	PIDT1	0.00	7.10	0.330	0.983	0.000	0.2314

Table 2: Control Loop Performance Indicators

Critical comments:

- P Controller - The P Controller has a good rise time but produces a noticeable overshoot and steady state error(except for plant A). It does not have good disturbance rejection characteristics as the system does not reach the setpoint when there is a disturbance signal.

- PI Controller-The PI Controller has a zero steady state error but takes a longer time to settle. The overshoot is not as large as the overshoot with the P controller, but is larger than the overshoot with the PID controller. Comparatively Its disturbance rejection also ranges between that of a P controller and a PID controller .

- PID Controller-From the following experiment we got to know that with the implementation of the PID control parameters for Plants A, B and C, it can be inferred that a PIDT1 controller is best controller for the plants as they produce no steady state errors, have a reasonable rise time and settling time, and, a low overshoot. A PIDT1 also has good Disturbance rejection characteristics with a very low steady state output.