PS-SMaRT — Landslide Hazard Toolkit

Persistent Scatterer–Soil Moisture Analysis for Risk & Triggering

Theory & Methods — PS-SMaRT

Persistent Scatterer–Soil Moisture Analysis for Risk & Triggering

0) Notation & Coordinate Frames

- ENU = local East–North–Up right-handed frame.
- Heading (h) in degrees (clockwise from North). Incidence (heta) measured **from vertical**.
- Slope (S) in degrees **from horizontal**; Aspect (A) in degrees **clockwise from North**.

$$\phi = \deg^{-1}((h + 90) \bmod 360)$$

$$\theta = \deg^{-1}(\text{incidence from vertical})$$

$$A_r = \deg^{-1}(A), \qquad S_r = \deg^{-1}(S)$$

Step 0 — LOS → Downslope Projection (optional)

(a) LOS unit vector in ENU (toward sensor)

$$\mathbf{l} = \begin{bmatrix} -\sin\phi\, \sin\theta \\ -\cos\phi\, \sin\theta \\ \cos\theta \end{bmatrix}$$

(b) Downslope unit vector from aspect/slope

$$\mathbf{d} = \begin{bmatrix} \sin A_r \ \cos S_r \\ \cos A_r \ \cos S_r \\ -\sin S_r \end{bmatrix}$$

(c) Sensitivity and projection

$$s = \mathbf{l} \cdot \mathbf{d}$$

$$v_{\parallel} = \frac{v_{\mathrm{LOS}}}{\max(|s|,\ \varepsilon)}$$

$$y_{\parallel}(t) = \frac{y_{\mathrm{LOS}}(t)}{\max(|s|,\ \varepsilon)}$$

(d) Circular (aspect) bilinear averaging

$$\bar{A} = \arg\left(\sum_{i=1}^{4} w_i\, e^{\,jA_i}\right), \quad w_i \ge 0,\ \sum_i w_i = 1$$

(e) Acceptance criteria

$$S \ge S_{\min}, \qquad |s| \ge s_{\min}$$

Step A — Filtering of Projected Points

$$S(x,y) \ge S_{\min} \quad \text{and} \quad |v_{\parallel}(x,y)| \ge v_{\min}$$

Step B — Spatial Clustering (DBSCAN)

Let (\mathcal{P}={(x_i,y_i)}) in a metric CRS (meters). For distance (d) and parameters (\varepsilon,\ \text{min_samples}):

$$N_\varepsilon(p) = \{\, q \in \mathcal{P} \ :\ d(p,q) \le \varepsilon \,\}$$

$$\mathrm{core}(p) \iff |N_\varepsilon(p)| \ \ge\ \mathrm{min\_samples}$$

Clusters are maximal density-connected sets; label (-1) denotes noise.

Step C — Cluster Polygons & Statistics

Convex hull of cluster (C):

$$H_C = \mathrm{hull}(\{(x_i, y_i) \in C\})$$

Polished hull (buffer–union, radius (r)):

$$H'_C = \left(\bigcup_{i \in C} B_r(p_i) \right)^{\circ}$$

Descriptive statistics over (v_{\parallel}): mean, std, min, max; polygon area.

Step D — Wet-Anomaly Overlap (optional)

Contingency table on sampled valid pixels:

	Inside slopes	Outside slopes
Anomaly	A	B
No anomaly	C	D

Chi-square statistic

$$\chi^2 = \sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}, \quad E_{ij} = \frac{(\mathrm{row}_i)(\mathrm{col}_j)}{A + B + C + D}$$

Matthews correlation / (\phi)

$$\phi = \mathrm{MCC} = \frac{AD - BC}{\sqrt{(A+B)(A+C)(B+D)(C+D)}}$$

Step E — TWI Inside vs Outside (optional)

Welch's t-test (unequal variances):

$$t = \frac{\bar{T}_{\text{in}} - \bar{T}_{\text{out}}}{\sqrt{\dfrac{s_{\text{in}}^2}{n_{\text{in}}} + \dfrac{s_{\text{out}}^2}{n_{\text{out}}}}}$$

$$\nu \approx \frac{\left(\frac{s_{\text{in}}^2}{n_{\text{in}}} + \frac{s_{\text{out}}^2}{n_{\text{out}}}\right)^2}{\frac{s_{\text{in}}^4}{n_{\text{in}}^2(n_{\text{in}}-1)} + \frac{s_{\text{out}}^4}{n_{\text{out}}^2(n_{\text{out}}-1)}}$$

Step F — Hazard Index & Classes

Robust normalization (per layer (X))

$$X' = \mathrm{clip}\left(\frac{X - \mathrm{P5}(X)}{\mathrm{P95}(X) - \mathrm{P5}(X) + \varepsilon},\ 0,\ 1\right)$$

Blend (mean over available layers)

$$HI = \frac{1}{K}\sum_{k=1}^{K} X'_k$$

Class thresholds

$$\text{Low: } HI \le 0.20, \qquad \text{Moderate: } 0.20 < HI \le 0.40, \qquad \text{High: } HI > 0.40$$

Zonal (polygon) classification

$$\overline{HI}_\Omega = \frac{1}{|\Omega|} \iint_\Omega HI(x, y)\, dx\, dy$$

Workflow (End-to-End)

```
 (Raw PS LOS CSV) + Slope + Aspect [+ Heading/Incidence rasters]
                   │
                   ▼
[0] LOS→downslope (s = l·d), filter by |s| & slope
                   │
                   ├──► (Projected points CSV)  (if provided, skip [0])
                   ▼
[A] Filter by slope ≥ S_min and |v∥| ≥ v_min
                   ▼
[B] DBSCAN clustering in metric CRS
                   ▼
[C] Cluster polygons + stats
              ┌──────────────┴──────────────┐
              ▼                             ▼
  [D] Anomaly overlap (χ², MCC)   [E] TWI in/out t-test
              └──────────────┬──────────────┘
                             ▼
[F] Hazard index (robust blend) + Hazard class raster
                             ▼
  Zonal hazard per polygon + zonal stats + provenance/logs
```

Practical Guidance

- **Units/CRS:** clustering uses the slope raster CRS; it should be projected (meters). Incidence is measured **from vertical**; velocities in mm/yr.
- **Starting values:** (S_{\min}!=10^\circ); (|v_{\min}|!=5)–(10) mm/yr; DBSCAN (�arepsilon) ≈ 1–2× mean PS spacing; `min_samples` 5–10.
- **Numerics:** use (�arepsilon�pprox 10^{-3}) to stabilize division by (s); use circular averaging for aspect to avoid wraparound at (360^\circ).
- **Caveat:** downslope projection assumes dominantly downslope motion; complex kinematics may deviate.