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Linear Reg

↓

Regression

Logistic Reg

Classification

Sigmoid function

$$\boxed{\frac{1}{1+e^{-y}}} \rightarrow \boxed{[0,1]}$$

$y = mx + c$

Similarities

House Price $\Rightarrow$ 20L

$\Rightarrow$ 20.4L

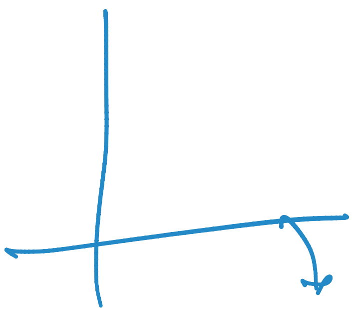
$\rightarrow$ Regression

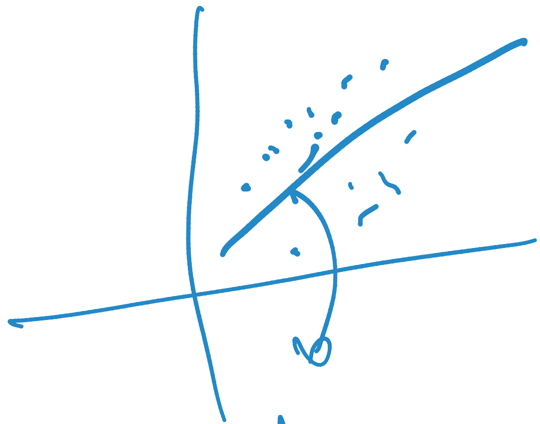
Classification → Classify

Class → Children → Classification

Good

Bad





Probability

$y = mx + c$

0, 50

0

50% → 1

0 NO

1 Yes

Diabetes → Yes → 1

→ No → 0

0.5

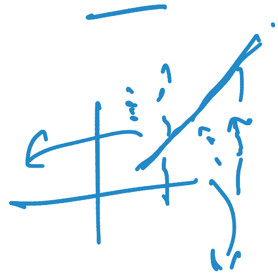
0.72 → Yes

0.17 → No

0 1

$$\frac{1}{1+e^{-y}}$$

$y = mx + c$



SVM → support vector Machines

SVR

SVC

from ~~[illegible]~~ sklearn.svm import

SVR

SVC

Dual form

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cost = $\underbrace{\min\left(\frac{\|w\|}{2}\right)}_{\text{street width}} + C \underbrace{\min\left(\frac{1}{m}\sum_{i=1}^{m}(\xi_i)\right)}_{\text{minimize error}}$ → lagrange formation

$\min\left[+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_i \alpha_j y_i y_j \boxed{x_i^T x_j} - \sum_{i=1}^{m} \alpha_i \right]$

linear $K(a, b) = a^T b$

poly $K(a, b) = (a^T b + r)^d$

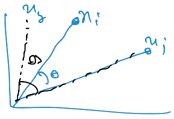
RBF

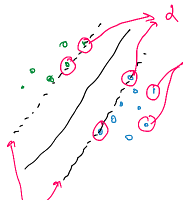
sigmoid

☆ $\boxed{f(x_q) = w^T x + b \;,\; f(x_q) = \sum_{i=1}^{m} \alpha_i y_i x_i^T x_q + b}$

☆ α for support vectors > 0

else $= 0$





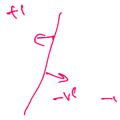
x_i $\begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} d \\ e \end{bmatrix}$

x_j c f

$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

$y_i(w^T x + b) \geq 1$ correctly

$y_i(w^T x + b) < 1$ wrong



X

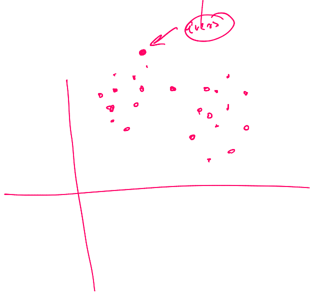
α_1 x_1

α_2 x_2

α_3 x_3

y_i

y_1



logistic

x_1 x_2 x_3

distance = $\boxed{x_i m_1 + x_2 m_2 + x_3 m_3} + b$

Kernel Trick

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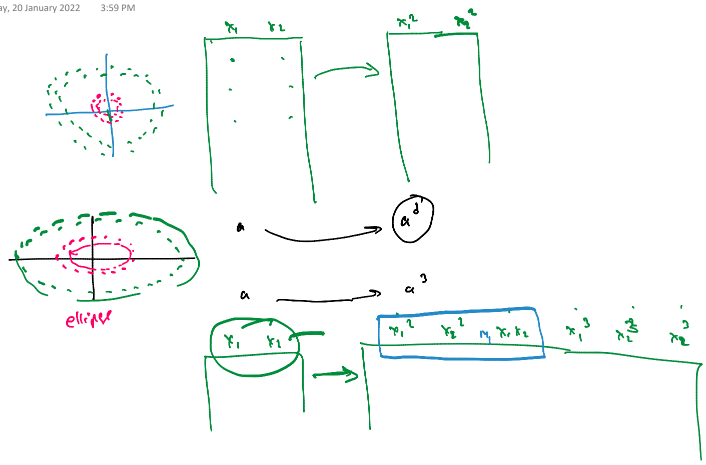


$$\phi(x) = \phi\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 \\ \sqrt{2}\, x_1 x_2 \\ x_2^2 \end{bmatrix}$$

$a^T b$

$$\phi(a)^T \phi(b) = \begin{bmatrix} a_1^2 \\ \sqrt{2}\, a_1 a_2 \\ a_2^2 \end{bmatrix}^T \begin{bmatrix} b_1^2 \\ \sqrt{2}\, b_1 b_2 \\ b_2^2 \end{bmatrix}$$

$a = [a_1 \ \ a_2]$

$$[a \ b \ c] \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

$ad + be + cf$

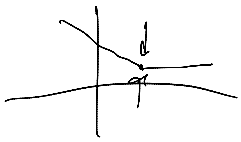
$$= a_1^2 b_1^2 + 2 a_1 a_2 b_1 b_2 + a_2^2 b_2^2$$

$$= (a_1 b_1)^2 + 2 a_1 b_1 a_2 b_2 \quad (a_2 b_2)^2$$

$$= (a_1 b_1 + a_2 b_2)^2$$

$$= \left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}^T \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right)^2$$

$$= (a^T \cdot b)^2$$



Mercer's Theorem

$k(a, b)$ — continuous, symmetric

$k(a,b) = k(b,a)$

$\phi(a, b)$ exists $\Rightarrow$ ( ) high dimension

$\phi(a, b)$ exists $\Rightarrow$ $(\qquad)^{\text{high dimension}}$

ϕ_n

Poly

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$$k(a,b) = (a^T b + \gamma)^d$$

$$\left(a \times b + \frac{1}{2}\right)^2 = a^2 b^2 + \frac{1}{4} + ab$$

$$= \begin{bmatrix} a^2 & \frac{1}{2} & a \end{bmatrix}^T \cdot \begin{bmatrix} b^2 & \frac{1}{2} & b \end{bmatrix}$$

$$\begin{bmatrix} a^2 \\ \frac{1}{2} \\ a \end{bmatrix}^T \begin{bmatrix} b^2 \\ \frac{1}{2} \\ b \end{bmatrix} = \boxed{a^2 b^2 + \frac{1}{4} + ab}$$

$$\begin{bmatrix} a, & a^2, & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} b & b^2 & \frac{1}{2} \end{bmatrix}$$

Rbf

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$$k(a,b) = \exp\left(-\frac{(a-b)^2}{2\sigma^2}\right)$$

$$= \exp\left(-\frac{1}{\lambda}(a-b)^2\right)$$

$\lambda = -2$

$$= \exp -\frac{1}{\lambda}\left(a^2+b^2+2ab\right) = e^{-\frac{1}{\lambda}(a^2+b^2)}\, e^{\left(-\frac{1}{\lambda}\right)2ab}$$

$$= e^{\frac{1}{2}(a^2+b^2)}\, e^{ab} \rightarrow$$ infinite dimension

Taylor series $\left[e^a = 1 + \frac{1}{1!}a^1 + \frac{1}{2!}a^2 + \frac{1}{3!}a^3 \dots \frac{1}{\infty!}a^\infty\right]$

$$e^{ab} = 1 + \frac{1}{1!}(ab) + \frac{1}{2!}(ab)^2 + \frac{1}{3!}(ab)^3 \dots \frac{1}{\infty!}(ab)^\infty$$

$$e^{-\frac{1}{\lambda}(a-b)^2} = e^{\frac{1}{2}(a^2+b^2)}\left[1 + \frac{1}{1!}ab + \frac{1}{2!}(ab)^2 + \frac{1}{3!}(ab)^3 \dots \frac{1}{\infty!}(ab)^\infty\right]$$

$$= s + s\frac{1}{1!}ab + s\frac{1}{2!}(ab)^2 + \frac{1}{3!}(ab)^3 \dots \frac{1}{\infty!}(ab)^\infty$$

$$= \left[\sqrt{s}, \sqrt{s}\sqrt{\frac{1}{1!}ab}\ \sqrt{s\frac{1}{2!}(ab)^2}\dots\right] \cdot \left[\sqrt{s}, \sqrt{s\frac{1}{1!}(ab)}\ \sqrt{s\frac{1}{2!}(ab)^2}\dots\right]$$

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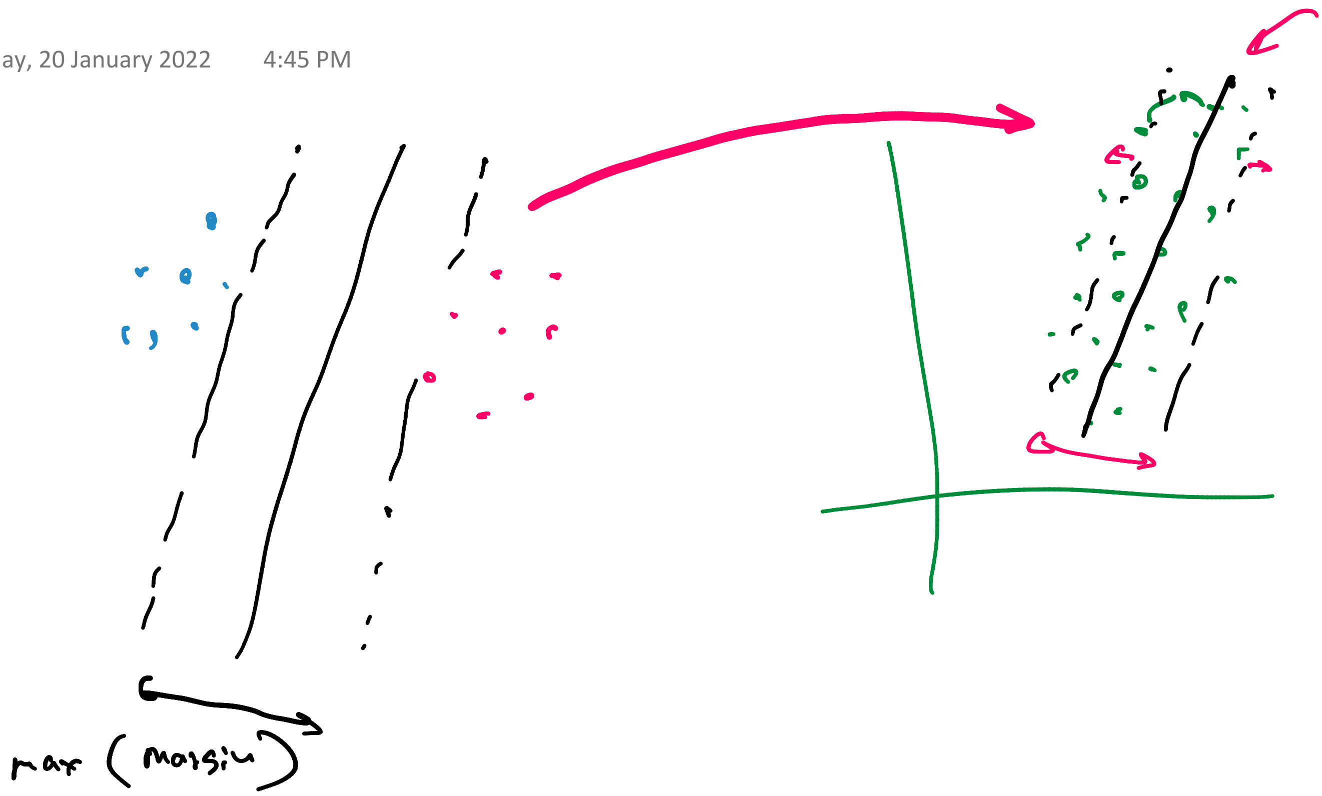

classification