Lecture-35

Recap:

Solving, linear, homogeneous, recurrence relations with constant coefficients

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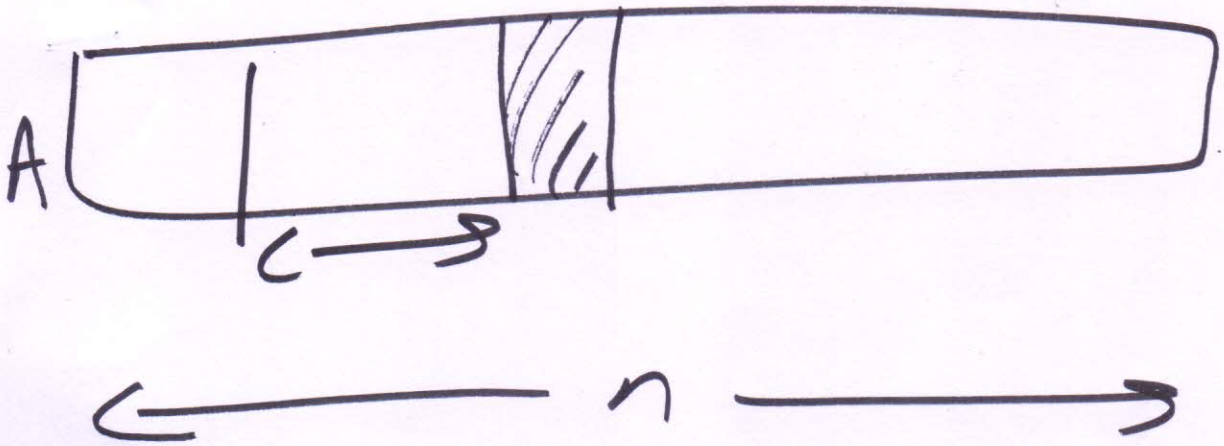
eg what is the time complexity of binary search?

$O(\log n)$

Divide and conquer

$f(n)$ = no. of steps required to do binary search on an array of length n.



$$f(n) = f\left(\frac{n}{2}\right) + 2$$

$$f\left(\frac{n}{2}\right) = f\left(\frac{n}{4}\right) + 2$$

$$f\left(\frac{n}{4}\right) = f\left(\frac{n}{8}\right) + 2$$

Take a problem of size n, break it into "a" subproblems of size n/b, with $g(n)$ additional steps.

Binary search: $a = 1$, $b = 2$

$$f(n) = a\, f\left(\frac{n}{b}\right) + g(n)$$

(3)

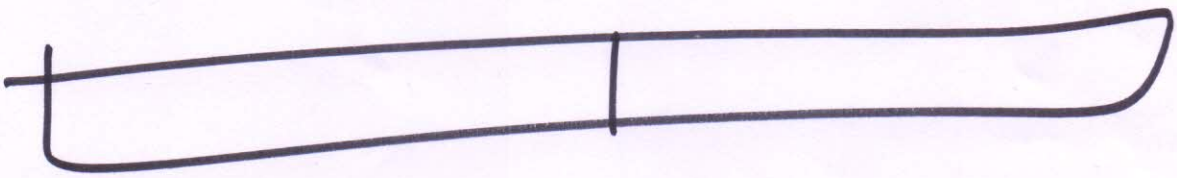
$\downarrow$ divide.

$\downarrow$ additional work for conquer

$$f(n) = 1 \cdot f\left(\frac{n}{2}\right) + 2$$

$O(\log n)$

Merge Sort:



$$f(n) = 2 \cdot f\left(\frac{n}{2}\right) + n$$

$$f(n) = O(n \log n)$$

Master Theorem

$$f(n) = a\,f\left(\frac{n}{b}\right) + c \cdot n^d$$

$$f(n) = \begin{cases} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

Binary Search: $a = 1, b = 2, c = 2, d = 0$

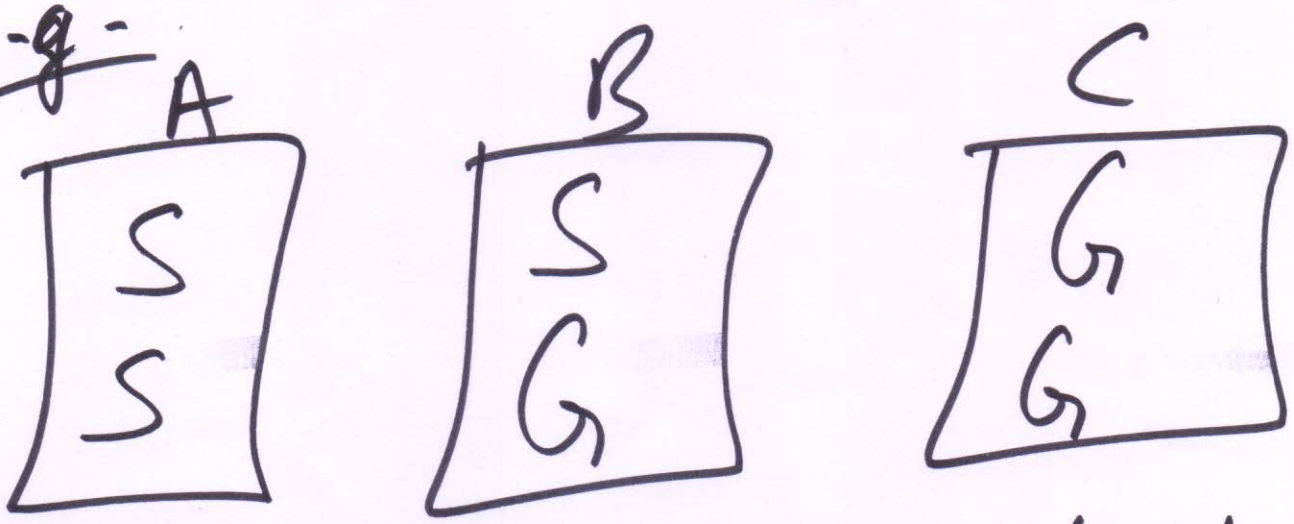
$O(\log n)$

Merge Sort $a = 2, b = 2, c = 1, d = 1$

$O(n \log n)$

Probability Theory

e.g.:



i) choose a box randomly

ii) choose a coin randomly

iii) it turns out to be a Gold coin

what is the probability that the other coin in the same box is also a Gold coin.? H.W.

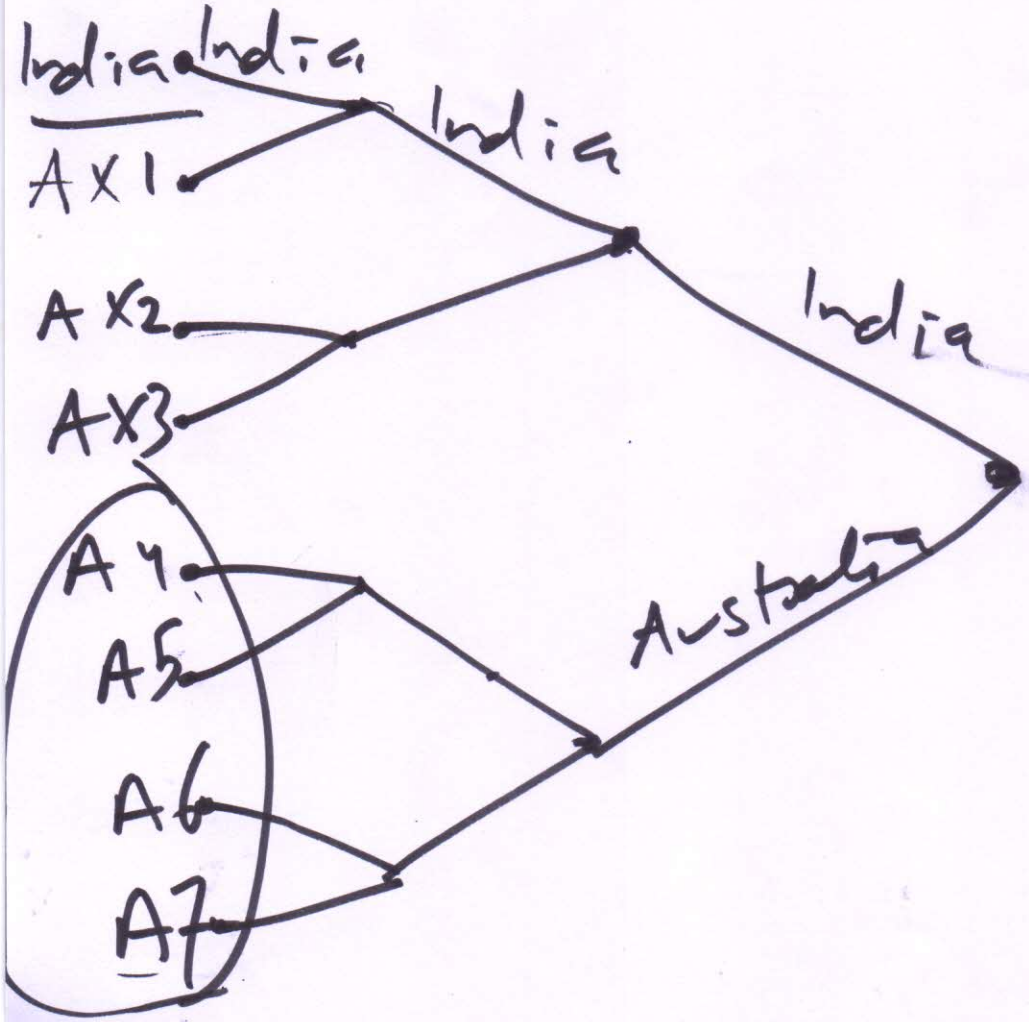
e.g. These are 8 teams ⑥ in the quarter finals of the world cup.

Ranking	Team
1	India
2	Australia
3	England
4	N.Z.
5	S.A.
6	Pakistan
7	Bangladesh
8	W.I.

What is the probability (7) that the top 2 teams meet in the finals?

OR

What is the probability that Australia is the runner-up?



$\frac{4}{7}$

Sample Space:

Set of all possible outcomes.

Coin toss: $S = \{H, T\}$

dice throw: $S = \{1, 2, 3, 4, 5, 6\}$

toss 2 coins: $S = \{HH, HT, TH, TT\}$

toss a coin & a dice together

$$S = \begin{Bmatrix} H1, H2, \cdots, H6, \\ T1, T2, \vdots, T6 \end{Bmatrix}$$

Event: any subset of the sample space.

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$ events

$B = \{2, 4, 6\}$

$C = \{2, 3, 5\}$

$D = \{1, 6\}$

No. of possible events is $2^6 = 64$ = no. of subsets.