# Electrical Conductivity

**Electrical conductance (C or G)**:
Electrical conductance is the ability of an object (metallic wire, plate, block, etc.) to permit the flow of electricity through it. It depends on the size and shape of the object.

Conductance depends on
1. Temperature
2. Material of the object and
3. Size and shape of the object

$C = \frac{1}{R}$, where R is the resistance.
The unit of conductance is mho or $\Omega^{-1}$ or siemens (S)

**Electrical conductivity ($\sigma$):**
$C \propto \frac{A}{\ell}$ , $C = \sigma\frac{A}{\ell}$ ; $\sigma = \frac{C\ell}{A}$
$\sigma = \frac{1}{\rho}$ , where $\rho$ is the resistivity.

Electrical conductivity is the ability of a material to permit the flow of electricity through it, regardless of its size and shape.
The unit of conductivity is $\Omega^{-1}m^{-1}$ or $Sm^{-1}$.

**Classical Free Electron Theory of Metals (Lorentz -Drude Theory)**:
It is the classical theory used to explain the electrical conductivity of metals.

The **postulates of Drude-Lorenz theory** are
1. Metals contain a large number of randomly moving free electrons (like gas molecules of an ideal gas in a container) in a uniform background of positive ions.
2. Free electrons in the metal are moving in a completely uniform potential field due to the ions fixed in the lattice. In the absence of an external electric field, $V = 0$
3. *The thermal speed of free electrons due to the random motion is of the order of $10^5$ m/s.*
4. During the random motion, the free electrons collide with each other and also with the lattice atoms/ions. All the collisions are elastic i.e., there is no loss of energy.
5. The thermal energy and thus the thermal speed of free electrons increases with temperature.
6. The movement of free electrons obey the classical kinetic theory of gases and Maxwell-Boltzmann distribution of velocities. The average kinetic energy of free electrons is directly proportional to the absolute temperature. $K.E = \frac{3}{2}k_BT$, where $k_B$ is the Boltzmann's constant.
7. In the absence of external electric field, the average velocity of electrons in any direction is zero and so the current density (J) in the metal is also zero.
8. Electrical conductivity arises when an external electric field accelerate free electrons in a specific direction.
9. When an external electric field is applied, the electrons move with some additional velocity in the direction opposite to the direction of electric field.
   The average additional velocity acquired by electrons in an external electric field is called **drift velocity** ($v_d$). Drift velocity is of the order of $10^{-3}$ m/s.
10. The drift velocity is superimposed on the random motion of the electrons.
11. Drift velocity is very small compared to the thermal velocity.
12. In the external electric field, the collision between free electrons and positive ions of the metal becomes inelastic such that the whole energy gained by electrons from the external electric field is completely transferred to the positive ion during the collision. Therefore, their drift velocity

becomes zero after each collision. It means that they do not retain any memory of their previous motion.

13. The average distance travelled by free electrons between two successive collisions is called mean free path ($\lambda$).
14. The average time between two successive collisions is called relaxation time ($\tau$).

**Equation for electrical Conductivity**:

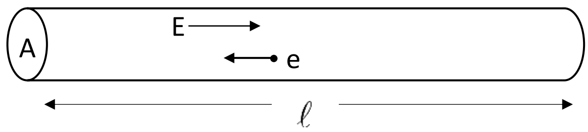
Let us consider a conductor of length $\ell$ and area of cross section A. then, the volume of the conductor $= A\ell$

If n is the number of free electrons per unit volume, then the total number of electrons in the conductor $= nA\ell$

The total charge due to all free electrons in the conductor $= enA\ell$

When an electric field is applied, free electrons acquire additional velocity ($v_d$) and move opposite to the direction of electric field. Let t be the average time taken by the free electrons to move from one end to the other end, then

$$I = \frac{q}{t} = \frac{enA\ell}{t}$$




We know that $\frac{\ell}{t} = v_d$, the drift velocity.

$\therefore$ $\mathbf{I = neAv_d}$ $---$ (1)

Current density $\mathbf{J} = \frac{I}{A} = \mathbf{nev_d}$ $---$ (2)

The force experienced by the free electrons due to the external electric field, E is given by $F = eE$.

The acceleration produced, $a = \frac{F}{m} = \frac{eE}{m}$ $---$ (3)

If $\tau$ is the relaxation time, then the drift velocity acquired by free electrons,

$$v = u + at \quad \text{or} \quad v_d = 0 + \frac{eE}{m}\tau \qquad \mathbf{v_d = \frac{eE}{m}\tau} --- (4)$$

substituting eqn (4) in eqn (2) $\quad J = ne\frac{eE}{m}\tau = \frac{ne^2\tau}{m}E$

from Ohm's law, we know that $\quad J = \sigma E$

thus, electrical conductivity $\boldsymbol{\sigma} = \mathbf{\frac{ne^2\tau}{m}}$ $---$ (5) $\quad$ or resistivity, $\boldsymbol{\rho} = \mathbf{\frac{m}{ne^2\tau}}$

**conductivity ($\sigma$) and mobility ($\mu$)**:

mobility is the drift velocity produced per unit electric field, $\mu = \frac{v_d}{E} = \frac{e\tau}{m}$

or, $\quad \boldsymbol{\sigma} = \mathbf{ne\mu}$ $---$ (6)

**relaxation time ($\tau$):**

Under the influence of an external electric field, free electrons acquire an additional velocity known as drift velocity. When the field is switched off, the drift velocity begins to decrease exponentially as a result of collisions with the positive ions. This process, which tends to restore the equilibrium for free electrons, is called the relaxation process.

If $v_d(0)$ is the velocity at $t = 0$ at which the field is switched off, then

The velocity at any time is given by $v_d(t) = v_d(0)e^{-t/\tau}$

When $t = \tau$ (relaxation time), then $v_d(t) = v_d(0)e^{-1} = \frac{v_d(0)}{e} = \frac{v_d(0)}{2.718} = 0.37\, v_d(0)$

Thus, relaxation time is the time taken for the drift velocity to decrease to $\frac{1}{e}$ of its initial value. Its value is of the order of $10^{-14}$s .

*It gives the time taken by free electrons in a metal to return from non-equilibrium condition to the equilibrium condition, after the applied field is turned off.*

**Drawbacks of classical free electron theory**:

1. It could not explain the electrical conductivity of semiconductors and insulators.

2. The phenomena such as photo electric effect, Compton Effect and black body radiation could not be explained by classical free electron theory.
3. It could not explain the temperature variation of electrical conductivity. *According to classical free electron theory, Resistivity* $\rho \propto \sqrt{T}$ *,but experimentally observed that conductivity* $\rho \propto T$.
4. It does not explain why the metals prefer only certain structures.
5. It could not explain the specific heat capacity of solids. *According to classical theory the value of specific heat capacity of metals is given by 4.5R (R = Universal gas constant) whereas the experimental value is nearly 3R (Dulong Petit law).*
6. Ferromagnetism could not be explained by this theory. *The theory also fails to explain the temperature independence of paramagnetic materials.*
7. According to classical theory, the mean free path is about 2.85nm. But its experimental value is about 10 times above this value.

**Quantum free electron theory (Drude Sommerfeld model):**

- Electrons are assumed as identical and indistinguishable particles
- They obey Fermi Dirac statistics (FD statistics) *and hence electrons can be considered as fermions*
- Electrons obey Pauli's exclusion principle. Hence only one particle can be accommodated in a single energy state (*two electrons can be accommodated in an energy level*).
- The electrons have dual nature i.e., particle and wave natures.
- The allowed energy levels of an electron are quantized *or the energies of free electrons are quantized.*
- Electrons move under a constant potential field inside the metal.
- The electrons move freely within the metal and they are not allowed to leave the metal *due to the existence of a potential barrier at its surfaces*.

**Fermi Dirac distribution function:** For a system in thermal equilibrium, Fermi Dirac function describes the probability distribution of fermions on various energy levels.

The probability of occupancy of an energy state E with a fermion (here electron) is given by

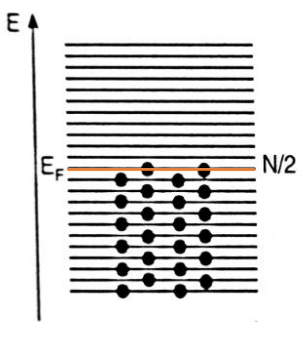
$f(E) = \frac{1}{1+e^{\alpha+\beta E}}$ , where $\alpha = \frac{-E_F}{k_B T}$ and $\frac{1}{k_B T}$ . Here $E_F$ is a constant called fermi energy and $k_B$ is the Boltzmann's constant.

Or $$\boxed{f(E) = \frac{1}{1 + e^{\frac{E-E_F}{k_B T}}}}$$

This function is called Fermi Dirac function.

**Fermi level and fermi energy**:

At absolute zero, free electrons occupy energy levels in pairs, starting from the lowest level up to a specific energy level known as the Fermi level. The energy corresponding to fermi level is called Fermi energy. Thus, **Fermi level** is defined as the uppermost filled energy level by free electrons in a conductor at absolute zero. **Fermi energy** is the highest energy that a free electron can have in a conductor at absolute zero.




**Variation of Fermi Dirac function with temperature**:

(a) **At T = 0K** (absolute zero):

**(i)** For energy levels lying below the fermi level ($E_i < E_F$), $E_i - E_F$ is a negative quantity.

$$f(E) = \frac{1}{1+e^{\frac{E-E_F}{k_B T}}} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0} = 1$$

$f(E) = 1$ indicates that all the energy levels lying below the fermi level are completely occupied by electrons

**(ii)** For energy levels lying above the fermi level ($E > E_F$), $E_i - E_F$ is a positive quantity.

$$f(E) = \frac{1}{1+e^{\frac{E-E_F}{k_B T}}} = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty} = 0$$

$f(E) = 0$ indicates that all the energy levels lying above the fermi level are completely empty at absolute zero.

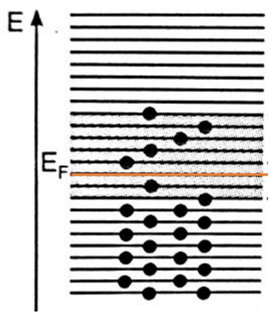
**(iii)** For $E = E_F$, the quantity $E - E_F = 0$

$$f(E) = \frac{1}{1+e^{\frac{0}{0}}} = \text{indeterminate.}$$

It implies that the probability of occupancy of fermi level by an electron at 0K ranges from 0 to 1.

**(b) At T > 0K**

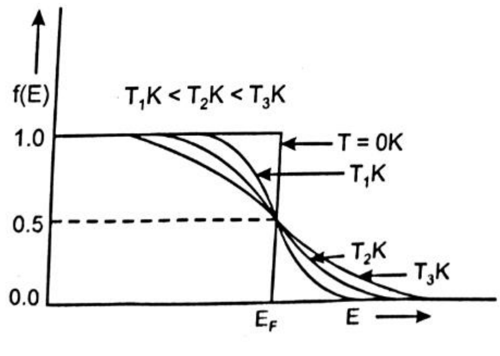
As temperature increases, electrons just below the fermi level absorb energy and are excited to the energy levels just above the fermi level. Thus, the probability of finding electrons in energy levels just below the fermi level will decrease and at the same time the probability of finding electrons in energy levels just above the fermi level will increase.




When $T > 0K$ and $E = E_F$, then $f(E) = \frac{1}{1+e^{\frac{0}{k_B T}}} = \frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2}$.

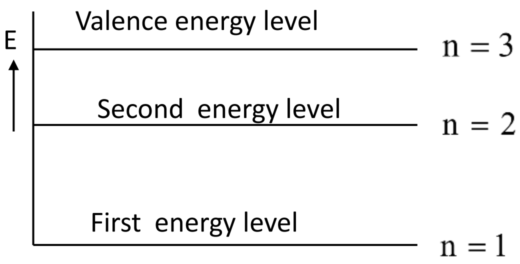
Thus, Fermi level is also defined as the energy level at which the probability of occupancy is $\frac{1}{2}$ $\left(f(E) = \frac{1}{2}\right)$ at any non zero temperature ($T \neq 0K$).

The variation of $f(E)$ with temperature can be represented as below.

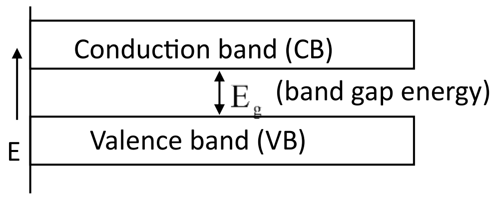



**Energy bands**:

According to the Bohr atom model, there are well defined discrete energy levels for all electrons in an isolated atom. But when atoms come together to form a solid, the energy of electrons in an atom gets modified by the presence of neighbours. Each discrete energy level now split up into a number of energy levels and this group of closely spaced energy levels constitute an energy band.




*Energy levels of an isolated atom*



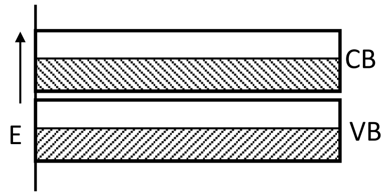

*Energy bands in a crystal*

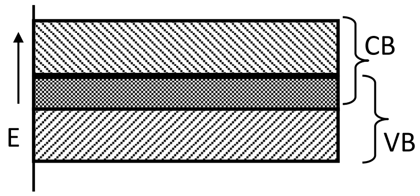
The range of energy (group of energy levels) possessed by electrons in a solid is called **energy band**.
The **range of energy possessed by electrons in the valence shell is called valence band**.
The **range of energy possessed by free electrons (conduction electrons) is called conduction band**.
The **energy gap between** top of the **valence band and** bottom of the **conduction band is called** ***band gap energy*** or ***forbidden energy gap (*** $E_g$).

**<u>Classification of Solids on the basis of their energy band structure</u>:**

<u>Conductors (metals)</u>:

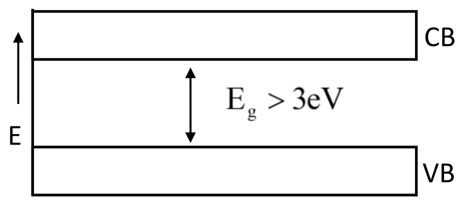
The substances which readily permit the passage of electric current through them are called conductors (iron, copper, silver, etc.). In conductors **either the conduction and valence bands are partially filled or they are overlapped**. At room temperature, conductors have large number of free electron and they easily conduct electricity. As temperature increases conductivity of conductors decreases. This shows that conductors have positive temperature coefficient of resistivity.



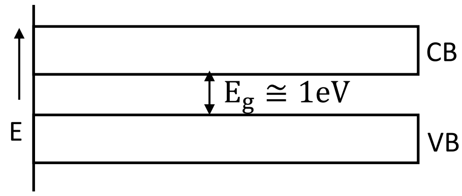

<u>Insulators</u>:

The substances which do not permit the passage of electric current through them are called insulators (wood, plastic, glass, etc.). In the case of insulators, **a large band gap** ($E_g > 3eV$) **exists**. At room temperature, the valence band is completely filled, while the conduction band is empty, or the substance does not conduct electricity. As temperature increases some electrons will succeed in reaching the conduction band and conductivity increases slightly. This shows that insulators have negative temperature coefficient of resistivity.




<u>Semiconductors</u>:

The substances whose electrical conductivity is in between that of conductors and insulators are called semiconductors (silicon, germanium, etc.). In the case of semiconductors, **a small band gap exists** ($E_g \cong 1eV$). At room temperature, the valence band is almost filled, while the conduction band is almost empty, or the substance show a small conductivity. As temperature increases more and more electrons will reach conduction band and the conductivity increases. This shows that semiconductors have negative temperature coefficient of resistivity.




At low temperatures the valence band is completely filled and conduction band is empty. Hence a semiconductor acts as an insulator at low temperature.

Problems:

1. Find the mobility of electrons in copper wire if there are $9 \times 10^8$ valence electrons /m$^3$ and the conductivity of copper is $6 \times 10^7$ S/m.

   Ans: $4.16 \times 10^{-3}\ \mathrm{m^2/V\,s}$

2. Find the relaxation time of conduction electrons in a metal it its resistivity is $1.54 \times 10^{-8}\Omega$m and it has $5.8 \times 10^{28}$ conduction electrons/m$^3$.
   Ans: $3.9 \times 10^{-14}$s

3. An aluminium wire of diameter 0.8mm which carries 10A. Calculate the drift velocity of electrons through this wire, assume that $4.5 \times 10^{28}$ electrons/m$^3$ are available for conduction.
   Ans: $2.76 \times 10^{-3}\ \mathrm{m/s}$
4. What is the current density in copper wire of diameter 0.1cm carries a steady current of 5A. Also calculate the drift velocity of electrons in copper. Assume that only one electron of an atom takes part in conduction. (density of copper is $8.92 \times 10^3\ \mathrm{kg/m^3}$, atomic weight is 63.5)
   Ans: $\mathrm{J} = 6.37 \times 10^6\ \mathrm{A/m^2}$, $\mathrm{v_d} = 4.7 \times 10^{-3}\mathrm{m/s}$
5. Evaluate the fermi function for energy kT above the fermi energy.
   Ans: 0.269
6. In a solid, consider the energy level lying 0.01eV above Fermi level. What is the probability of this level being occupied by an electron at 200K.
   Ans: 0.359
7. In a solid, consider the energy level lying 0.01eV below Fermi level. What is the probability of this level being occupied by an electron at 300K.
   Ans: 0.595
8. In a solid, consider the energy level lying 0.01eV above Fermi level. What is the probability of this level being not occupied by an electron at 300K.
9. Ans: 0.595
10.

Questions:

1. Explain the terms (i) drift velocity and (ii) carrier mobility
2. Write a note on mean free path and relaxation time of conduction electrons.
3. Discuss the important postulates of classical free electron theory of metals.
4. Explain classical free electron theory of electrical conductivity in metals.
5. Using the free electron model derive an expression for electrical conductivity in metals.
6. Write any three drawbacks of classical free electron theory of metals.
7. Explain relaxation time in terms of the drift velocity of free electrons.
8. Define Fermi level and Fermi energy.
9. Explain the variation of Fermi function with temperature.
10. Explain the classification of materials based on band theory.
11. Write short note on (a) valence band, (b) conduction band and (c) forbidden energy gap.
12. A rectangular block of a solid is connected to a DC voltage source. Obtain the expression for the (i) current density flowing through the block and (ii) conductivity of the material in terms of the concentration of free electrons in it.
13. (i)Describe the formation of energy bands in a crystalline solid.
    (ii)Define valence band, conduction band and band gap energy in the energy band structure. Based on this classify solids into conductors, semiconductors and insulators
14. What is the difference between energy level and an energy band.
15. Write and explain fermi function. Explain with the help of a diagram how it varies with change of temperature.
16. Write down the Fermi Dirac equation for the probability of occupancy of an energy level E by an electron. Show that the probability of occupancy by an electron is zero if $\mathrm{E} > \mathrm{E_F}$ and unity if $\mathrm{E} < \mathrm{E_F}$ at temperature 0K.
17. Define Fermi distribution function. Show that at all temperatures ($\mathrm{T} > 0\mathrm{K}$) probability of occupancy of fermi level is 50%.

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