MODULE-3 - QUANTUM MECHANICS & NANOTECHNOLOGY.

QUANTUM MECHANICS :

Classical Physics couldnot properly explain many physical phenomena , because it deals with macroscopic particles. Max Planck in 1900 put forward the quantum theory to explain blackbody radiation. Einstein introduced the idea of light quantum or photon. Particle nature of radiation was stressed in these theories. But wave nature of radiation was essential to explain interference, diffraction etc. In 1924 ~~d~~ Louis de Broglie suggested wave-particle duality. In 1926, Schrodinger developed the wave mechanics. P.A.M. Dirac unified wave mechanics and matrix mechanics to set up a general formalism called Quantum mechanics. It deals with microscopic particles.

* Wave nature of particles :

In 1924 ~~De~~ de Broglie predicted that ~~par~~ like radiation, particle has a dual nature - particle and wave nature.

de Broglie hypothesis :

All moving particle is associated with a wave whose wavelength is given by

$$\boxed{\lambda = \frac{h}{p}} \quad \text{--- (1)}$$

wave → Known as matter wave or De Broglie wave

h = planck's constant $= 6.625 \times 10^{-34}$ JS.

p = momentum = mv.

According to mass-energy relation

$$E = mc^2$$

m = mass
c : velocity of light

$$\frac{E}{c} = mc = m \times \text{velocity}$$

① → wave nature
$E = mc^2$ → particle

ie, $\frac{E}{c} = P$ —— ②

we have $E = h\nu$

ν : frequency
λ : wavelength

and $c = \nu\lambda$

or $\nu = \frac{c}{\lambda}$

then $E = h\nu = \frac{hc}{\lambda}$

or $\frac{E}{c} = \frac{h}{\lambda}$ —— ③

$E = h\nu$ —— ①
$E = mc^2$ —— ②
$mc^2 = h\nu$ (dual nature)
$\cancel{mc = h\nu}$
$\cancel{p = h\nu}$
$mc = \frac{h\nu}{c}$
$P = \frac{h}{\lambda}$
$c = \nu\lambda$
$V = \nu\lambda$
$\lambda = \frac{V}{\nu}$
$\lambda = \frac{h}{p}$

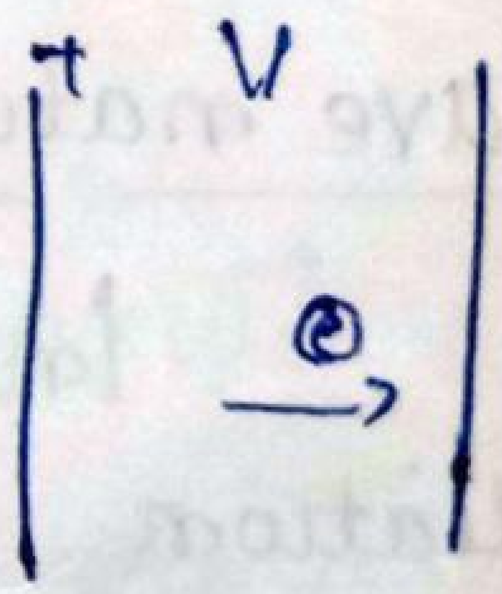
Comparing ② and ③ we get

$$P = \frac{h}{\lambda} \text{ or}$$

$$\boxed{\lambda = \frac{h}{P}}$$

* Calculate the wavelength of an electron accelerated by a p.d of V volt.

ans)



Energy of electron = eV —— ①

where e = charge of e
$= 1.6 \times 10^{-19}$ J.

V = ~~volt~~ p.d.

$KE = \frac{1}{2}mv^2$; v = velocity of e.

ie, $\frac{1}{2}mv^2 = eV$ or

$$v^2 = \frac{2eV}{m}$$

$$\text{or } v = \sqrt{\frac{2eV}{m}}$$

then momentum $p = mv$

$$p = m\sqrt{\frac{2eV}{m}} = \sqrt{\frac{m^2 2eV}{m}}$$

$$p = \sqrt{2meV}$$

then $\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}}$ — (4)

$h \rightarrow$ planck's const

$m \rightarrow$ mass of e
9.1×10^{-31} kg

$e \rightarrow$ charge of e
1.6×10^{-19} J.

$V \rightarrow$ p.d. in Volt.

~~Classical mechanics~~

*) Calculate the wavelength associated with an e under a potential difference of 100 V.

ans) for an e $\lambda = \frac{h}{\sqrt{2meV}}$

$h = 6.625\times10^{-34}$ Js

$m = 9.1\times10^{-31}$ kg

$e = 1.6\times10^{-19}$ J

$V = 100$ V.

then $\lambda = \frac{6.625\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times100}}$

$$\lambda = 1.2\times10^{-10}\ m$$

* Estimate the de Broglie wavelength of an e moving with a KE of 100 eV.

ans) $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$.

for an electron

$$\frac{1}{2}mv^2 = eV$$

$$= 100\,eV = 100 \times 1.6 \times 10^{-19}\,J.$$

then $$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}$$

$$\lambda = \qquad m$$

* Calculate the de Broglie wavelength of e whose KE is 10 keV.

ans) $$\lambda = \frac{h}{p}$$

$$KE = \frac{p^2}{2m}$$

$$p = \sqrt{2mKE}$$

$$p = \sqrt{2 \times 9. \times 10^{-31} \times 10 \times 10^3 \times 1.6 \times 10^{-19}}$$

$$KE = 10\,keV$$

$$= 10 \times 10^3 \times 1.6 \times 10^{-19}\,J$$

then $$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\lambda = \qquad m$$

*

C.M	Q.M
→ It is applied for macroscopic particles like, earth, moon etc	→ It is applied for microscopic particles like nucleus, atoms etc.
→ Newton's laws of motion and gravitation are imp	→ Heisenberg uncertainty principle and De-Broglie concept are important

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* Heisenberg uncertainty principle :

It is impossible to have an accurate measurement of two conjugate variables simultaneously ie,

It is impossible to know both the exact position and exact momentum of an object at the same time.

Let the error or uncertainty in position $= \Delta x$

uncertainty in momentum $= \Delta P_x$

then according to uncertainty principle

$$\Delta x \Delta P_x \geq \hbar \quad \text{where } \hbar = \frac{h}{2\pi}$$

Similarly uncertainty in energy $= \Delta E$

uncertainty in time $= \Delta t$

then $\Delta E \Delta t \geq \hbar$

* Applications of uncertainty principle :

1. Absence of electron inside the nucleus :

Let the electron resides in the nucleus.

nucleus is of the order of 10^{-14}m

ie, $\Delta x \approx 10^{-14}$m (possibility of seeing the e in nucleus is of the order 10^{-14}m)

By uncertainty principle

$$\Delta x \Delta P_x \geq \hbar$$

Consider the equality

$$\Delta x \Delta P_x = \hbar$$

$$\Delta x \Delta P_x = \frac{h}{2\pi}$$

then $\Delta P_x = \frac{h}{2\pi \times \Delta x} = \frac{6.626 \times 10^{-34}}{2 \times 3.14 \times 10^{-14}}$

$= 1.1 \times 10^{-20}$

momentum contributes to the energy.

$\therefore$ energy of nucleus $= 1.10 \times 10^{-20}$ J

energy of single e ≈ 20 MeV

$\approx 20 \times 10^{6} \times 1.6 \times 10^{-19}$ J

$\simeq 3.2 \times 10^{-12}$ J.

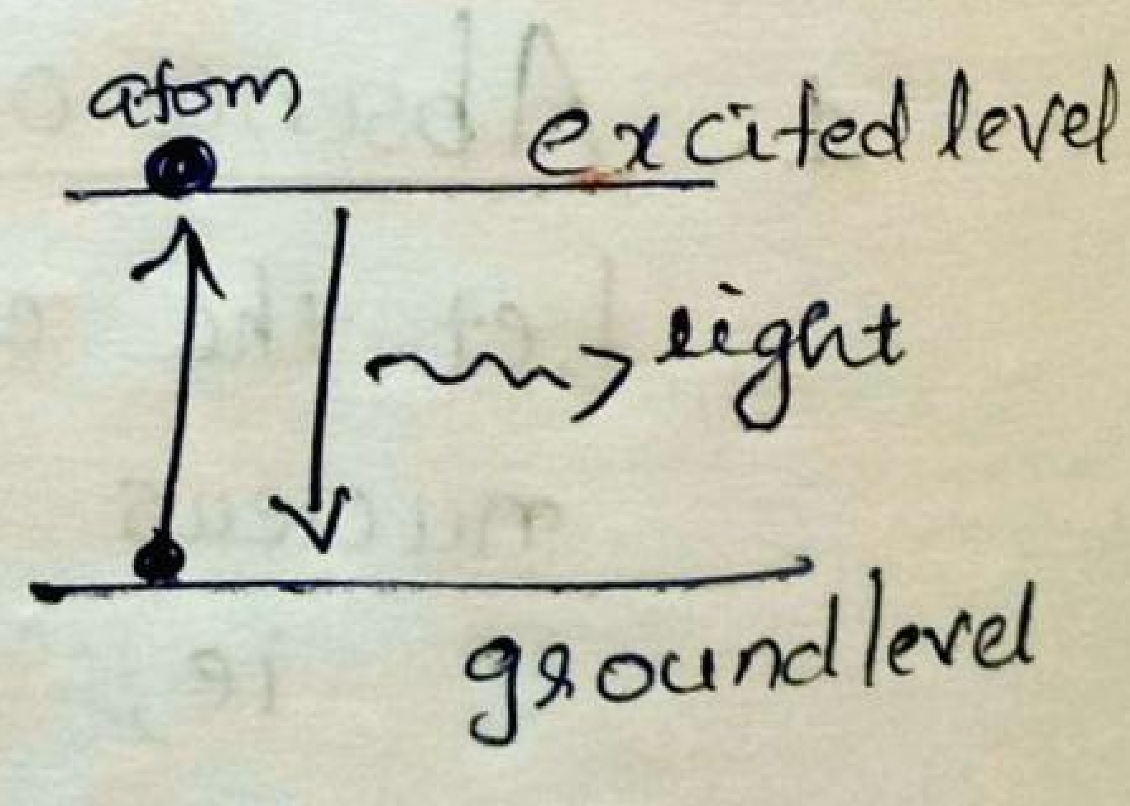
$\Rightarrow$ energy of nucleus $\neq$ energy of single e.

$\therefore$ no e can exist inside the nucleus.

2) Natural line broadening mechanism –

uncertainty in the frequency of light emitted by an atom.

excitation time of an atom is the time spent by the atom in the excited level.



excitation time of an atom is 10^{-8} sec

ie, $\Delta t \approx 10^{-8}$ sec

According to the uncertainty principle

$$\Delta E \Delta t \geq \hbar$$

$$\Delta E \Delta t = \hbar$$

$$\Delta E = \frac{h}{2\pi \times \Delta t} = \frac{6.626\times10^{-34}}{2\pi\times10^{-8}}$$

$$\Delta E \approx 1.1\times10^{-26} J.$$

we have $E = h\nu$

then $\Delta E = h\Delta\nu$ (h is a constant)

$\therefore$ the uncertainty in frequency of light emitted by the atom

$$\Delta\nu = \frac{\Delta E}{h} = \frac{1.1\times10^{-26}}{6.626\times10^{-34}}$$

$$\Delta\nu \approx 0.16\times10^{-8} Hz.$$

* Wave function and its characteristics :

Wave associated with a particle can be represented by a function called wave function ψ. A quantity that makes periodic changes with the particle is called wavefunction.

Eg:- in soundwave varying quantity is pressure.

In light wave varying quantities are electric and magnetic field

Characteristics

→ Continuous and single valued function
→ Complex in nature
→ used to predict the state of the system.
→ function of position r and time t

$$\psi = \psi(r,t)$$

→ it is a probability and can be normalised.

Let $\psi = \psi(r,t)$

Normalisation can be written as

$$\int \psi(r,t)\,\psi^*(r,t)\,d\tau = 1 \quad \text{(for particle)}$$

$$\int \psi(r,t)\,\psi^*(r,t)\,d\tau = 0 \quad \text{(no particle)}$$

where $\psi^*(r,t)$: complex of ψ.

$d\tau$ = volume element
$= dx\,dy\,dz$.

$$\psi\psi^* = |\psi|^2$$

$\cancel{\psi\psi^* = |\psi|^2}$

* Schrodinger time dependent wave equation:

Let the wave function

$$\psi(r,t) = Ae^{i(kx-\omega t)} \quad \text{--- (1)}$$

A: Amplitude
K: propagation const
ω: ang. freq.

where $K = \frac{2\pi}{\lambda}$

we know $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{p} \times \frac{2\pi}{2\pi}$$

$$= \frac{h}{2\pi}\,\frac{2\pi}{p}$$

$$p\,\lambda = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k.$$

$$\text{or } k = \frac{p}{\hbar} \quad \text{——— (2)}$$

We have $E = h\nu$

angular frequency $\omega = 2\pi\nu$

but $\nu = \frac{E}{h}$

$$\hbar = \frac{h}{2\pi}$$

$$\therefore \omega = 2\pi \frac{E}{h}$$

$$\omega = \frac{E}{\hbar} \quad \text{——— (3)}$$

Substituting (2) and (3) in (1)

(1) $\Rightarrow$

$$\psi(x,t) = Ae^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)}$$

$$\psi = Ae^{\frac{i}{\hbar}(Px - Et)} \quad \text{——— (4)}$$

differentiating (4) w.r.t. x

$$\frac{\partial\psi}{\partial x} = Ae^{\frac{i}{\hbar}(Px - Et)} \times \frac{iP}{\hbar}$$

$$\left\{ y = e^{ax} \quad \frac{dy}{dx} = ae^{ax} = ay \right.$$

$$\frac{\partial\psi}{\partial x} = \frac{iP}{\hbar}\psi$$

again differentiating w.r.t x

$$\frac{\partial^2\psi}{\partial x^2} = \frac{iP}{\hbar} Ae^{\frac{i}{\hbar}(Px - Et)} \times \frac{iP}{\hbar}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{i^2}{\hbar^2}P^2\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{p^2\psi}{\hbar^2}$$

$i = \sqrt{-1}$

$i^2 = -1$

or $$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = p^2\psi \quad \text{--- (5)}$$

$$\left(-i\hbar\frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi = p \times p \times \psi$$

$$\Longrightarrow \boxed{p = -i\hbar\frac{\partial}{\partial x}} \quad \text{--- (6)}$$

p is called momentum operator.

Differentiating (4) wrt. time

$$\frac{\partial\psi}{\partial t} = Ae^{\frac{i}{\hbar}(px-Et)} \times -\frac{iE}{\hbar}$$

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}\psi$$

$$\hbar\frac{\partial\psi}{\partial t} = -iE\psi$$

xing by i on both sides

$i^2 = -1$

$$i\hbar\frac{\partial\psi}{\partial t} = -i^2E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \quad \text{--- (7)}$$

comparing we get

$$\boxed{E = i\hbar\frac{\partial}{\partial t}} \quad \text{--- (8)}$$

E is called Energy operator.

Total Energy

$$E = KE + PE$$

$$E = \frac{1}{2}mv^2 + V$$

$$E = \frac{p^2}{2m} + V$$

$$\frac{p^2}{2m} = \frac{m^2v^2}{2m} = \frac{1}{2}mv^2$$

xing by φ on both sides.

$$E\varphi = \left(\frac{p^2}{2m} + V\right)\varphi \quad \text{—— (9)}$$

$$E\varphi = H\varphi \quad \text{where } H = \frac{p^2}{2m} + V$$

H = Hamiltonian

Substituting ~~for~~ (5) and (7) in (9)

$$i\hbar\frac{\partial\varphi}{\partial t} = \frac{p^2\varphi}{2m} + V\varphi$$

$$i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2} + V\varphi$$

$$\boxed{i\hbar\frac{\partial\varphi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)\varphi}$$

=> 1D time dependent Schrodinger wave equation

If $V=0$

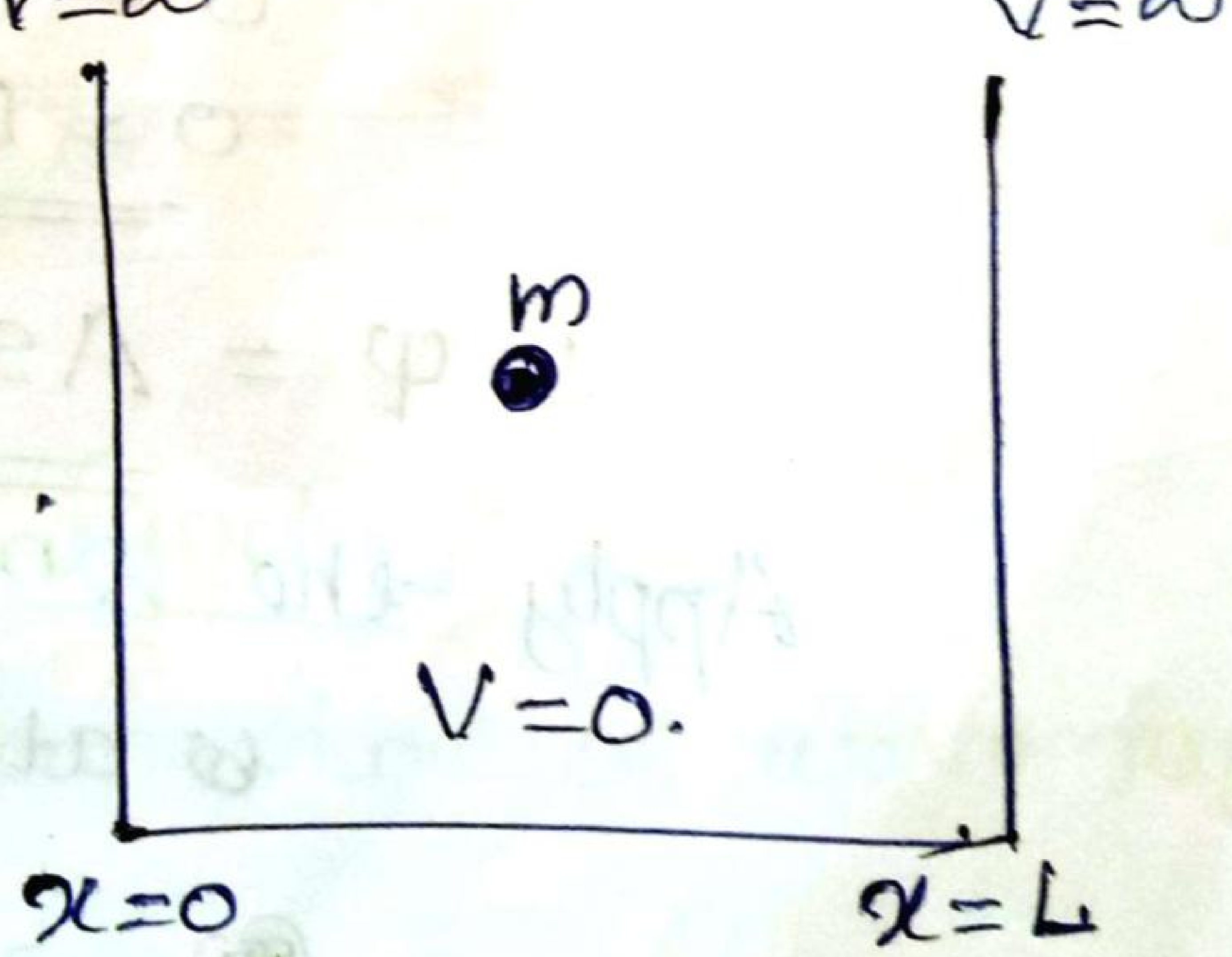
$$\boxed{i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2}}$$

Then for 3 Dimension

$$\boxed{i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi + V\varphi}$$

Particle in an infinite square well potential / Particle in a box :

Consider a particle of mass 'm' confined in a potential well of infinite depth and finite width L. V=0 everywhere within the well and V=∞ outside the well.



x=0, is the initial position
x=L is the final position

probability of finding the particle outside the well is zero.

The situation can be represented by Schrodinger time independent equation ($\because V$ doesn't depend on time)

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{--- (1)}$$

$V=0$ then

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0.$$

Let $\frac{2mE}{\hbar^2} = K^2$ ~~then~~ --- (A) then

$$\frac{\partial^2\psi}{\partial x^2} + K^2\psi = 0 \quad \text{--- (2)}$$

Solution of the dif-eqn :

Let $\psi = A\sin Kx + B\cos Kx$ --- (3)

A and B are amplitudes ~~constan~~

Apply the boundary condition

at $x=0$, $\psi=0$

(3) $\Rightarrow$

$$0 = A\sin K\times 0 + B\cos K\times 0$$

$$0 = B$$

$$\therefore \psi = A\sin Kx \quad \text{--- (4)}.$$

Apply the 2nd boundary condition

at $x=L$, $\psi=0$.

(4) $\Rightarrow$

$$0 = A\sin KL \Rightarrow$$

$A=0$ or $\sin KL=0$

but $A\neq 0 \Rightarrow \sin KL=0$

ie, $KL = n\pi$

or $K = \frac{n\pi}{L}$

$$K^2 = \frac{n^2\pi^2}{L^2} \quad \text{—— (B)}$$

equating (A) and (B)

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

ie, $$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}} \quad \text{—— (5)}$$

$n = 1, 2, 3 \ldots$ = quantum no

E_n are called energy eigon values.

$$\left.\begin{aligned} E_1 &= \frac{\pi^2\hbar^2}{2mL^2} \\ E_2 &= \frac{4\pi^2\hbar^2}{2mL^2} \\ E_3 &= \frac{9\pi^2\hbar^2}{2mL^2} \end{aligned}\right\} \text{energy eigon values.}$$

E_1, E_2, E_3 are called first 3 discrete energy values.

Calculation of wave function :

The normalisation condition of wave function

$$\int_0^L \psi\psi^* dx = 1$$

but $\psi\psi^* = |\psi|^2$

$$\therefore \int_0^L \psi\psi^* dx = \int_0^L |\psi|^2 dx = 1 \quad \text{—— (6)}$$

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we have $\psi = A \sin kx$

but $k = \frac{n\pi}{L}$

$$\therefore \psi_n = A \sin\left(\frac{n\pi x}{L}\right) \longrightarrow (7)$$

$\cancel{\text{6}}$' then (5) $\Longrightarrow$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$A^2 \int_0^L \frac{1 - \cos 2\left(\frac{n\pi x}{L}\right)}{2} dx = 1$$

$$\frac{A^2}{2}\left[\int_0^L 1\, dx - \underbrace{\int_0^L \cos 2\frac{n\pi x}{L} dx}_{0}\right] = 1$$

$$\frac{A^2}{2}\int_0^L dx = 1 \Rightarrow \frac{A^2}{2}[x]_0^L = 1$$

$$\frac{A^2}{2}[L-0] = 1$$

$$\frac{A^2}{2} L = 0$$

$$A^2 = \frac{2}{L}$$

$$\text{or } A = \sqrt{\frac{2}{L}}$$

Substitute this in (6)

$$\boxed{\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}} \longrightarrow (8)$$

$\Rightarrow$ wave function or eigen fn

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$$\psi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

$$\psi_2 = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$

$$\psi_3 = \sqrt{\frac{2}{L}} \sin \frac{3\pi x}{L}$$

$$\int_0^L \cos 2\frac{n\pi x}{L}\, dx = \left[\frac{\sin 2\frac{n\pi x}{L}}{\frac{2n\pi}{L}}\right]_0^L$$

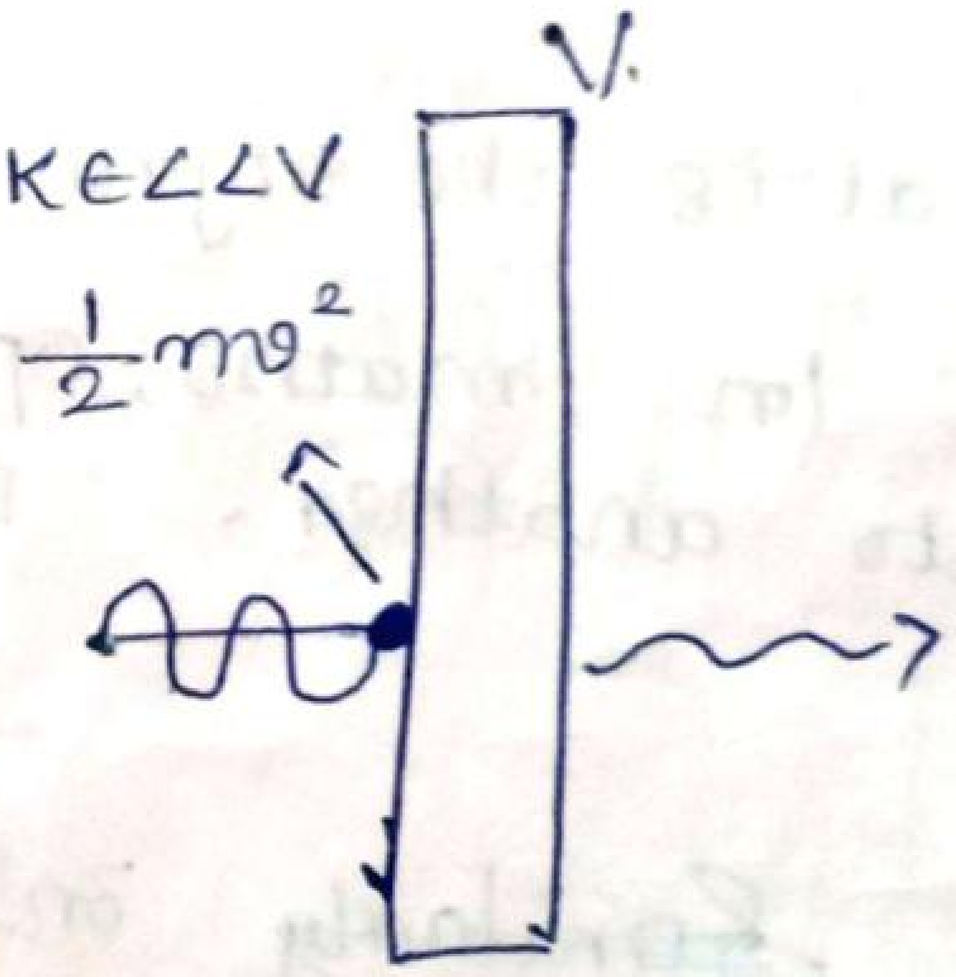
$$\frac{1}{\frac{2n\pi}{L}}\left[\sin 2n\pi \cancel{L}/\cancel{L} - \sin 0\right]$$

$$[0-0] = \underline{0}$$

problems = next page

QM Tannelling / Tunnel Effect :

Consider a wall having PE V and a particle of KE $\frac{1}{2}mv^2$ colliding against the wall. classically the particle will rebound. The probability of finding the particle on other side of the wall $=0$.

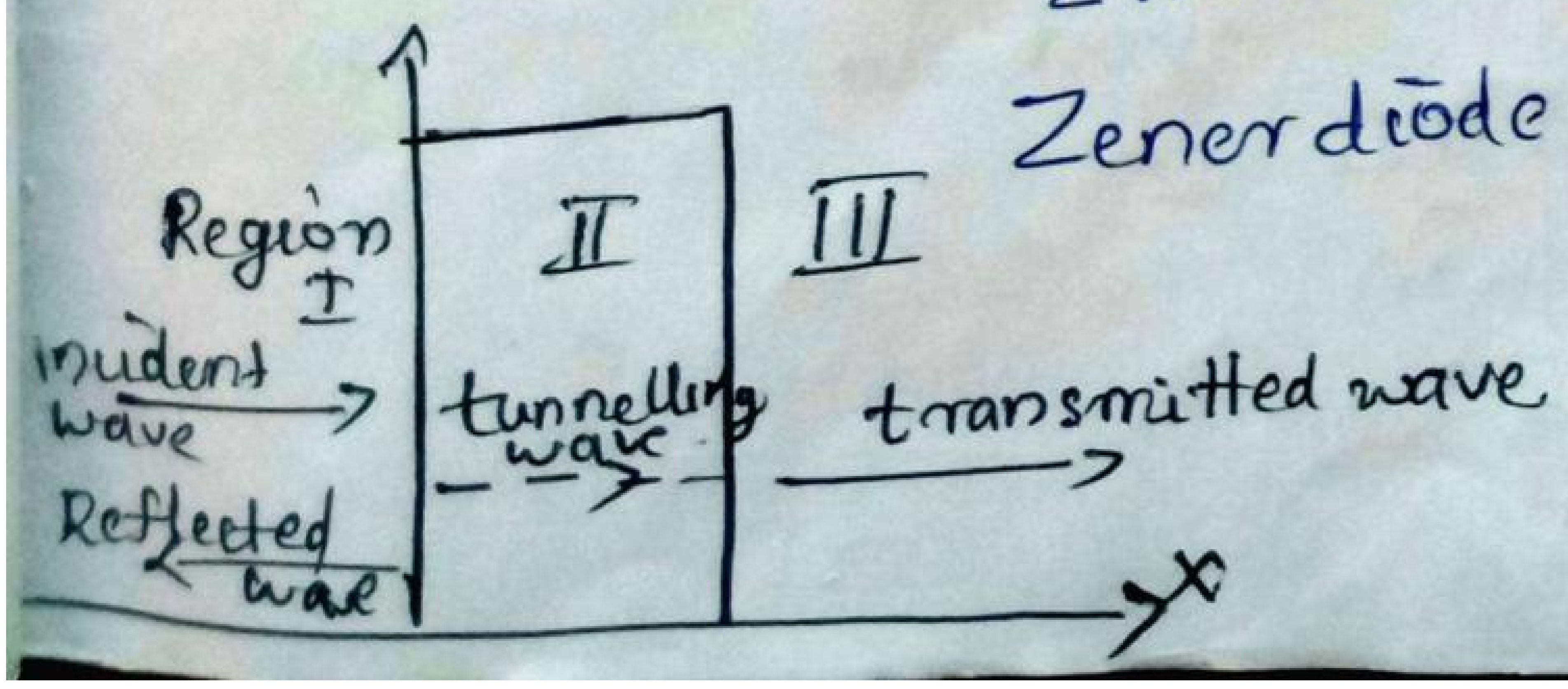


But quantum mechanically there is a finite probability of penetrating the particle through the walls and seeing on the other side. This is known as QM Tannelling

Eg : Josephson's effect - Tunnelling of cooper pairs across the insulator.

Emission of particle from nucleus.

Zener diode breakdown.



Problems :

* What are QM operators ?

ans) 1) momentum operator $p = -i\hbar \frac{\partial}{\partial x}$ or

$$p = i\hbar \nabla.$$

2) Energy operator $E = i\hbar \frac{\partial}{\partial t}$

3) Hamiltonian operator $H = \frac{p^2}{2m} + V$

$$H = \frac{-\hbar^2 \nabla^2}{2m} + V.$$

* What is the significance of operators in QM ?

ans) In maths, operator transform one function into another. Eg: differential operator $\frac{d}{dx}$

$$\frac{d}{dx} f(x) = f'(x)$$

~~Similarly~~ in QM, each dynamical variable is represented by an operator, which acts on a wavefn to give new wavefn. Eg: energy operator, mom. operator etc.

In QM operators are linear ie,

$$A(\varphi_1 + \varphi_2) = A\varphi_1 + A\varphi_2$$

and $A(c\varphi_1) = cA\varphi_1$

* Signifiance of normalisation condition.

a) $\varphi^*\varphi$ or $|\varphi|^2$ represents the probability density or probability of finding the particle in unit volume. If a particle exists in a given region of space the total probability of finding the particle in that region is one. This is mathematically written as normalisation condition

$$\int \varphi^* \varphi \, d\tau = 1$$

* An e and proton are moving with same KE which one has shorter wavelength?

a) we have $\lambda = \frac{h}{p}$ or

$$\lambda = \frac{h}{mv}$$

for e $\lambda_e = \frac{h}{m_e v}$ and for proton $\lambda_p = \frac{h}{m_p v}$

$$m_e = 9.1 \times 10^{-31} \, kg$$

$$m_p = 1.6 \times 10^{-27} \, kg.$$

$$m_p > m_e \Rightarrow \lambda_p < \lambda_e$$

$\Rightarrow$ proton has shorter wavelength.

* Using uncertainty principle, calculate the uncertainty in the frequency of emitted radiation if the uncertainty in time of an excited atom is 5×10^{-8} sec.

ans) Given $\Delta t = 5\times10^{-8}$

we have $\Delta E \Delta t \geq \hbar$

$$\Delta E \Delta t = \hbar$$

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{h}{2\pi \Delta t}$$

$$\Delta E = \frac{6.625\times10^{-34}}{2\times3.14\times5\times10^{-8}}$$

$$\Delta E = 2.1\times10^{-27}\,J.$$

$$E = h\nu$$

$$\Delta E = h\Delta\nu$$

then $\Delta\nu = \frac{\Delta E}{h} = \frac{2.1\times10^{-27}}{6.625\times10^{-34}}$

error in frequency $\Delta\nu = 3169811.32\ Hz$

* Calculate 1st three permitted energy levels of an e in an infinite squarewell potential of 1Å wide. mass of e $= 9.1\times10^{-31}$ kg., $h = 6.625\times10^{-34}$ JS

ans) we have

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \hbar = \frac{h}{2\pi}$$

$$\therefore E_n = \frac{n^2\cancel{\pi^2}h^2}{2mL^2 4\cancel{\pi^2}}$$

$$E_n = \frac{n^2h^2}{8mL^2}$$

here $m = 9.1 \times 10^{-31}$ kg

$L = 1\,\mathring{A} = 10^{-10}$ m.

$$E_1 = \frac{1^2 \times h^2}{8mL^2}$$

$$E_1 = \frac{(6.625 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$E_1 = 6.029 \times 10^{-18}\ J$$

$$E_1 = \frac{6.029 \times 10^{-18}}{1.6 \times 10^{-19}} = 37.68\ eV.$$

$$E_2 = \frac{2^2 \times h^2}{8mL^2} = \frac{4 \times (6.625 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$E_2 =$ ______ J

$E_2 =$ ______ eV.

$$E_3 = \frac{3^2 h^2}{8mL^2} = \frac{9 \times (6.625 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$E_3 =$ ______ J

$E_3 =$ ______ eV.

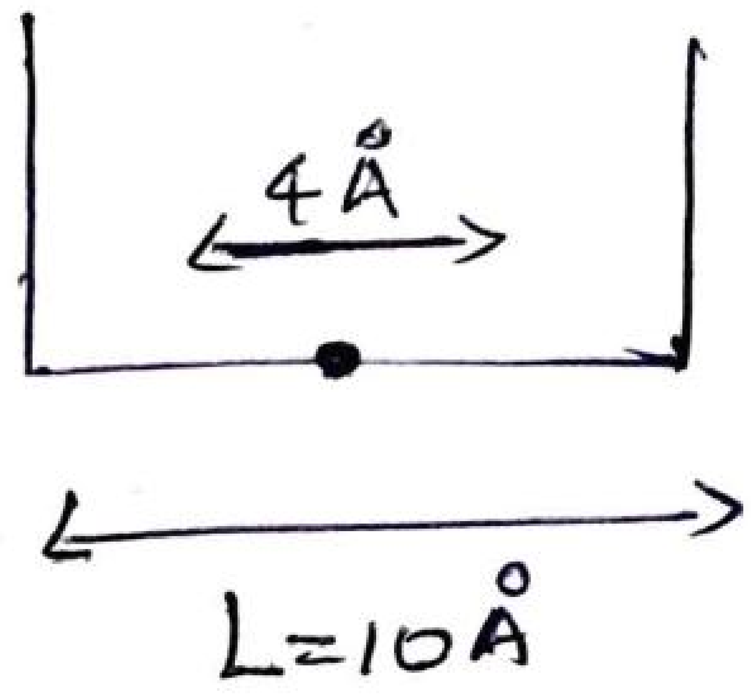
HW * Find the 3 lower energy levels of particle of mass 10^{-8} kg moving with a speed of 10^{7} m/s in an infinite potential well of width 15 mm. {find E_1, E_2, E_3}.

*) A particle is moving in one dimensional box of infinite height and width 10 Å. Calculate the probability of finding the particle within an interval of 4 Å at the centre of box.

ans)

$$L = 10\,\text{Å} = 10 \times 10^{-10}\,m$$

$$\Delta x = 4\,\text{Å} = 4 \times 10^{-10}\,m$$



we have $\psi = \sqrt{\frac{2}{L}} \sin \frac{n \pi x}{L}$

Let $n=1$

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

at the centre $x = \frac{L}{2}$

then $\psi = \sqrt{\frac{2}{L}} \sin \frac{\pi}{\cancel{L}} \frac{\cancel{L}}{2}$

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{\pi}{2} \qquad \sin 90 = 1$$

$$\psi = \sqrt{\frac{2}{L}}$$

Probability of finding the particle at the centre

$$P = \psi\psi^* = |\psi|^2 = \psi^2$$

$$= \left(\sqrt{\frac{2}{L}}\right)^2$$

$$P = \frac{2}{L}$$

Probability of finding the particle at the centre within a distance $\Delta x = 4\,\text{Å}$ is

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$P \times \Delta x$

$= \frac{2}{L} \times 4 \times 10^{-10}$

$= \frac{2}{10 \times \cancel{10^{-10}}} \times 4 \times \cancel{10^{-10}}$

$= \frac{8}{10} = \underline{\underline{0.8}}$