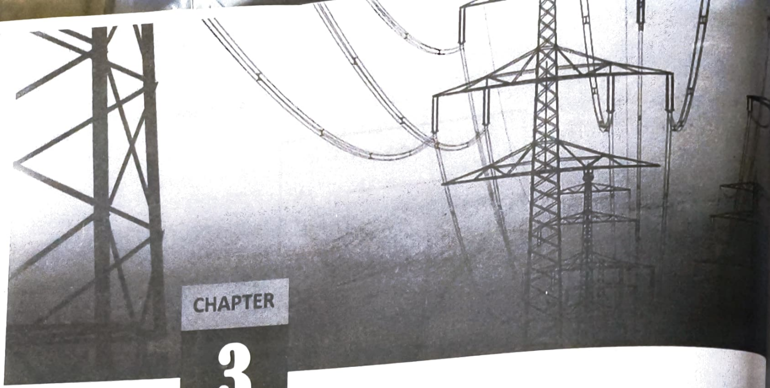


CHAPTER

3

LEARNING OBJECTIVES

The phenomenon of electromagnetic induction was discovered by the British physicist Michael Faraday in 1831. James clerk Maxwell mathematically described it as Faraday's law of induction. Electromagnetic induction is the phenomenon of generation of an electromotive force or voltage across an electrical conductor in a changing magnetic field. The direction of the induced emf can be discovered through Lenz's law. Electromagnetic induction underlies the operation of all electrical machines. Some of the most basic electrical components such as generators, motors and transformers make use of electromagnetic induction.

ELECTROMAGNETIC INDUCTION

After reading this chapter, the reader should be familiar with the following concepts.

- Faraday's laws
- Lenz's law
- Fleming's right and left hand rule
- Statically induced and dynamically induced emfs
- Self-inductance and mutual inductance
- coefficient of coupling

1. FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

In 1831, Michael Faraday, the famous English scientist demonstrated that when magnetic flux linking a conductor changes, an e.m.f is induced in the conductor. This phenomenon is known as electromagnetic induction.

- **Electromagnetic induction of emf (or voltage) is basic to the operation of transformers, generators (ac or dc) and motors (ac or dc).**

Faraday performed a series of experiments to demonstrate the phenomenon of electromagnetic induction. He summed up his conclusions into two laws, known as Faraday's laws of electromagnetic induction.

First Law:

It tell us about the condition under which an e.m.f. is induced in a conductor or coil and may be stated as under.

When a magnetic flux linking a conductor or coil changes, an e.m.f. is induced in it.

OR

Whenever a conductor cuts the magnetic flux, an e.m.f. is induced in that conductor.

Second Law:

It gives the magnitude of the induced e.m.f in a conductor or coil and may be stated as under.

The magnitude of the e.m.f. induced in a conductor or coil is directly proportional to the rate of change of flux- linkages.

Explanation

- The product of number of turns (N) of the coil and magnetic flux (ϕ) linking the coil is called flux linkages
- **Flux linkages = Flux × Number of turns of the coil =** $N\phi$

Consider a coil having N turns. The initial flux linking with a coil is ϕ_1

$$\text{Initial flux linkages } = N\phi_1$$

In time interval t, the flux linking with the coil changes from ϕ_1 to ϕ_2

$$\text{Final flux linkages } = N\phi_2$$

$$\text{Rate of change of flux linkages } = \frac{N\phi_2 - N\phi_1}{t}$$

As per faradays second law, the magnitude of the e.m.f. induced in a conductor or coil is directly proportional to the rate of change of flux- linkages.

$$\text{Induced e.m.f., } e \ \alpha \ \frac{N\phi_2 - N\phi_1}{t}$$

$$e = k\frac{N\phi_2 - N\phi_1}{t}$$

The value of k is unity. Then $e = \frac{N\phi_2 - N\phi_1}{t} = \frac{N(\phi_2 - \phi_1)}{t}$

In differential form, we have $e = N\frac{d\phi}{dt}$

Here $d\phi$ is the change in magnetic flux linkage and dt is the change in time.

- The direction of induced e.m.f. is given by Lenz's law. The magnitude and direction of induced e.m.f. should be written as

$$e = -N\frac{d\phi}{dt}$$

The minus sign is introduced in accordance with Lenz's law which is discussed in the next section.

As per Lenz's law, the induced e.m.f. sets up a current in such a direction so as to oppose the very cause producing it. Mathematically this opposition is expressed by a negative sign.

2. LENZ'S LAW

The Lenz's law states that, ***"the direction of an induced e.m.f. produced by the electromagnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the e.m.f.***

In short the induced e.m.f. always opposes the cause producing it, which is represented by a negative sign, mathematically in its expression., $e = -N\frac{d\phi}{dt}$

Explanation for Lenz's law

Consider a solenoid as shown in the figure1. Let a bar magnet is moved towards coil such that N-pole of magnet is facing a coil which will circulate the current through the coil.

According to Lens Law, the direction of current due to induced e.m.f. is so as to oppose the cause. The cause is motion of bar magnet towards coil. So e.m.f. will set up a current through coil in such a way that the end of solenoid facing bar magnet will become N-pole. Hence two like poles will face each other experiencing force of repulsion which is opposite to the motion of bar magnet as shown in the figure 1.

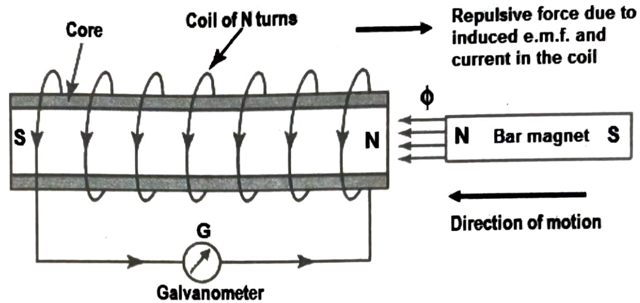


Figure 1: Lenz's law

If the same bar magnet is moved away from the coil, then induced e.m.f. will set up a current in the direction which will cause, the end of solenoid facing bar magnet to behave as S-pole. Because of this two unlike poles face each other and there will be force of attraction which is direction of magnet, away from the coil. The galvanometer shows deflection in other direction as shown in the figure 2.

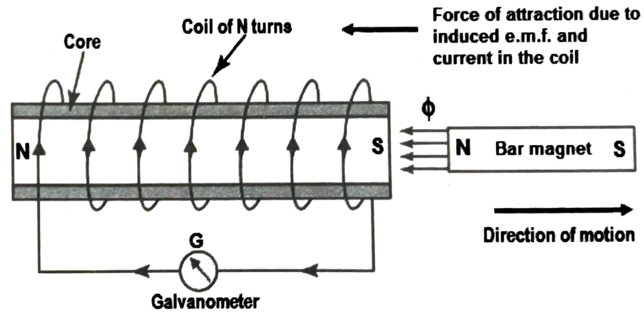


Figure 2 : Lenz's law

3. INDUCED E.M.F

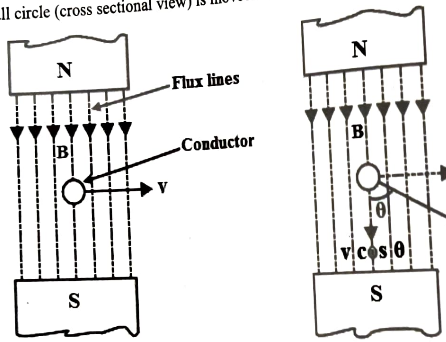
When the magnetic flux linking a conductor (or coil) changes, an e.m.f. is induced in it. This change in flux linkages can be brought about in the following two ways:

1. The conductor is moved in a stationary magnetic field in such a way that the flux linking it changes in magnitude. The e.m.f. induced in this way is called dynamically induced e.m.f. (as in a D.C. generator). It is so called because e.m.f. is induced in the conductor which is in motion.

2. The conductor is stationary and the magnetic field is moving or changing. The e.m.f. induced in this way is called statically induced e.m.f. (as in a transformer). It is so called because the e.m.f. is induced in a conductor which is stationary.

3.1 DYNAMICALLY INDUCED E.M.F.

Consider a single conductor of length l metres moving at right angles to a uniform magnetic field of B Wb/m^2 with a velocity of v m/s . See figure 3 (a). Here a single conductor is represented by a small circle (cross sectional view) is moved in a magnetic field.



(a) (b)

Figure 3: Dynamically induced e.m.f. in a current carrying conductor placed in a magnetic field

Suppose the conductor moves through a small distance dx in dt seconds. Then area swept by the conductor is = $l \times dx$

$$\text{Flux cut, } d\phi = Flux\,density \times Area\,swept = B\,l\,dx \text{ Wb}$$

According to Faraday's laws of electromagnetic induction, the magnitude of e.m.f. **e** induced in the conductor is given by;

$$e = N\frac{d\phi}{dt} = \frac{B\,l\,dx}{dt} \qquad [\because N = 1]$$

$$e = B\,l\,v \text{ volts} \qquad \left[\because \frac{dx}{dt} = v\right]$$

- If the conductor moves at angle θ to the magnetic field [See figure 3(b)], then the velocity at which the conductor moves across the field is $v\sin\theta$

$$e = B\,l\,v\sin\theta$$

The direction of the induced e.m.f can be determined by Fleming's right-hand rule.

3.1.1 FLEMING'S RIGHT HAND AND LEFT HAND RULES

Fleming's Right Hand rule is mainly applicable for electric generator and **Fleming's Left Hand rule** is mainly applicable for electric motor.

Fleming's right hand rule states that if we stretch the thumb, middle finger and the forefinger of our right hand mutually perpendicular to each other, if the forefinger indicates the direction of the magnetic field and the thumb indicates the direction of motion of conductor, then the (central) middle finger indicates the direction of induced current in conductor placed in the magnetic field.

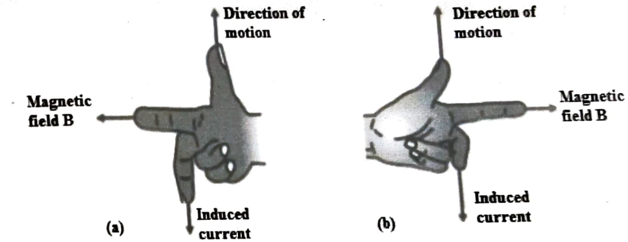


Figure 4: (a) Fleming's right hand rule (b) Fleming's left hand rule

Fleming's left hand rule states that if the thumb, forefinger and the middle finger of the left hand are stretched mutually perpendicular to each other, then the forefinger represents the direction of magnetic field, the (central) middle finger represents the direction of the induced current, and then the thumb represents the direction of force exerted on the conductor.

Memory Code

In both rules – Fleming's right hand rules and Fleming left hand rule –the fore finger, central finger and the thumb represent the same quantities. It may be helpful to associate

Fore finger	with	Field;
Central finger	with	Current; and
thuMb	with	Motion of the conductor

Often, students get confused which rule to apply where. In electrical engineering, we come across two types of situations. One may be called generator action, and the other motor action. In generator action, the induced emf is the result when a conductor is moved in a magnetic field (this is what happens in a dynamo). Whereas, in motor action, the motion of a conductor is the result when a current is passed through the conductor placed in a magnetic field (this is what happens in an electric fan).

You can easily remove the confusion by noting that it is your right hand that (usually) generates most of the things (like writing, painting, tightening of a screw, etc.). Thus, for the sake of remembering, the right-hand rule can be associated with the generator action of the right hand. The other rule (i.e., Fleming's left-hand rule) then applies to the motor action.

3.2 STATICALLY INDUCED EMF

- ***When the conductor is stationary and the field is moving or changing, the e.m.f. induced in the conductor is called statically induced e.m.f.***

A statically induced e.m.f. can be further sub-divided into two

1. **Self-induced e.m.f.**
2. **Mutually induced e.m.f.**

3.2.1 SELF-INDUCED E.M.F.

- ***The e.m.f. induced in a coil due to the change of its own flux linked with it is called self-induced e.m.f.***

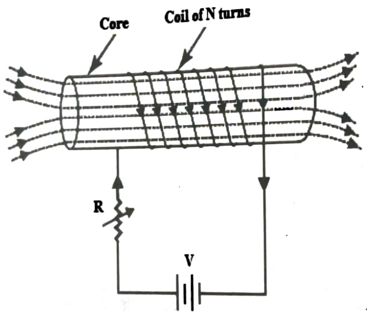


Figure 5: Self-induced e.m.f.

When a coil is carrying current, a magnetic field is established through the coil. If current in the coil changes, then the flux linking the coil also changes. Hence an e.m.f. is induced in the coil. This is known as self-induced e.m.f. It is given by $e = N\frac{d\phi}{dt}$ .The direction of this e m f. (by Lenz's law) is such so as to oppose the cause producing it, namely the change of current (and hence field) in the coil. The self-induced e.m.f. will persist so long as the current in the coil is changing.

Self-inductance

- ***When current in a coil changes, the self-induced e.m.f. opposes the change of current in the coil. This property of the coil is known as its self-inductance or inductance.***

Consider a coil of N turns carrying a current of I amperes as shown in figure 5. If current in the coil changes, the flux linkages of the coil will also change.

This will set up a self-induced e.m.f. e in the coil given by $e = N\frac{d\phi}{dt} = \frac{d}{dt}(N\phi)$

Since flux is due to current in the coil, it follows that flux linkages $(N\phi)$ will be proportional to I

$$e = \frac{d}{dt}(N\phi) \propto \frac{dI}{dt}$$

$$e = \text{Constant} \times \frac{dI}{dt}$$

$$e = L\frac{dI}{dt} \qquad (\text{ in magnitude})$$

Here L is a constant called self-inductance or inductance of the coil. The unit of inductance is henry (H).

- ***Hence a coil (or circuit) has an inductance of 1 henry if an e.m.f. of 1 volt is induced in it when current through it changes at the rate of 1 ampere per second.***

The magnitude of self-induced e.m.f. is $e = L\frac{dI}{dt}$. However, the magnitude and direction of self-induced e.m.f should be written as $e = -L\frac{dI}{dt}$. The minus sign is because the self-induced e.m.f. tends to send current in the coil in such a direction so as to produce magnetic flux which opposes the change in flux produced by the change in current in the coil. In fact, minus sign represents Lenz's law mathematically.

Info Plus

- The greater the self-induced voltage, the greater the self-inductance of the coil and hence larger is the opposition to the changing current. **The inductance of a coil depends upon these factors, viz:**
  1. **Shape and number of turns.**
  2. **Relative permeability of the material surrounding the coil.**
  3. **The speed with which the magnetic field changes.**
- In fact, anything that affects magnetic field also affects the inductance of the coil. Thus, increasing the number of turns of a coil increases its inductance. Similarly, substituting an iron core for air core increases its inductance.
- The self-inductance of a coil opposes the change of current (increase or decrease) through the coil. This opposition occurs because a changing current produces self-induced e.m.f. (e) which opposes the change of current. For this reason, **self-inductance of a coil is called electrical inertia of the coil.**

3.2.1.1 EXPRESSIONS FOR SELF INDUCTANCE

- If the magnitude of self-induced e.m.f. e and the rate of change of current $\frac{dI}{dt}$ are known, then inductance can be determined from the following relation

$$e = L\frac{dI}{dt} \qquad \text{Then} \quad L = \frac{e}{\left(\frac{dI}{dt}\right)}$$

- If the flux linkages of the coil and current are known, then inductance can be determined as under

$$e = L\frac{dI}{dt} = \frac{d}{dt}(LI)$$

$$e = N\frac{d\phi}{dt} = \frac{d}{dt}(N\phi)$$

From the above two expressions, we have

$$LI = N\phi \qquad \text{Then} \quad \text{Self inductance } L = \frac{N\phi}{I}$$

- Also the inductance of a magnetic circuit can be found in terms of its physical dimensions. Consider an iron-cored solenoid with area of cross section a ,length l and number of turns N.

Inductance of the solenoid is given by $L = \frac{N\phi}{I}$

Now $\phi = \frac{\text{mmf}}{\text{reluctance}} = \frac{NI}{\left[\frac{l}{a\mu_0\mu_r}\right]}$

Differentiating ϕ w.r. t. I , we get $\frac{d\phi}{dI} = \frac{\text{N}a\mu_0\mu_r}{l}$ ------(A)

If ϕ and I have a linear relationship $\frac{d\phi}{dI}$ can be written as $\frac{\phi}{I}$

Then equation (A) becomes $\frac{\phi}{I} = \frac{\text{N}a\mu_0\mu_r}{l}$ ------(B)

Multiply equation (B) with N on both sides we get $\frac{N\phi}{I} = \frac{\text{N}^2 a\mu_0\mu_r}{l}$

$$L = \frac{N^2 a\mu_0\mu_r}{l}$$

$$\text{Self inductance } L = \frac{N^2}{l/a\mu_0\mu_r} = \frac{N^2}{\text{Reluctance}} = \frac{N^2}{\text{S}}$$

3.2.2 MUTUALLY-INDUCED E.M.F.

- ***The e.m.f. induced in a coil due to the changing current in the neighbouring coil is called mutually induced e.m.f.***

Consider two coils A and B placed adjacent to each other as shown in figure 6.

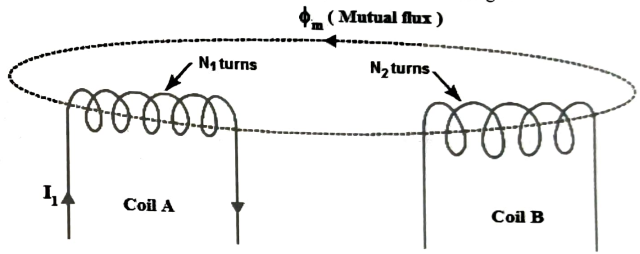


Figure 6: Mutual Inductance

A part of the magnetic flux produced by coil A passes through or links with coil B. This flux which is common to both the coils A and B is called mutual flux (ϕ_m) . If current in coil A is varied, the mutual flux also varies and hence e.m.f. is induced in both the coils. The e.m.f. induced in coil A is called self-induced e.m.f as already discussed. The e.m.f induced in coil B is known as mutually induced e.m.f.

The mutually induced e.m.f. in coil B persists so long as the current in coil A is changing. If current in coil A becomes steady, the mutual flux also becomes steady and mutually induced e.m.f. drops to zero.

- ***The property of two neighbouring coils to induce voltage in one coil due to the change of current in the other is called mutual inductance.***

The larger the rate of change of current in coil A, the greater is the e.m.f. induced in coil B. In other words, mutually induced e.m.f. in coil B is directly proportional to the rate of change of current in coil A i.e.

Mutually induced e.m.f. in coil B $\propto$ Rate of change of current in coil A

$$e_M \propto \frac{dI_1}{dt}$$

$$e_M = M\frac{dI_1}{dt} \qquad \text{(in magnitude)}$$

Here M is called mutual inductance between two coils.

3.2.2.1 EXPRESSIONS FOR MUTUAL INDUCTANCE

- If the magnitude of mutually induced e.m.f. (e_M) in one coil for the given rate of change of current in the other is known, then M between the two coils can be determined from the following relation.

$$e_M = M\frac{dI_1}{dt}$$

$$M = \frac{e_M}{\left[\frac{dI_1}{dt}\right]}$$

- Consider the figure 7.Let there be two magnetically coupled coils A and B having N_1 and N_2 turns respectively Suppose a current I_1 flowing in coil A produces a mutual flux. Note that mutual flux ϕ_{12} is that part of the flux created by coil A which links the coil B.

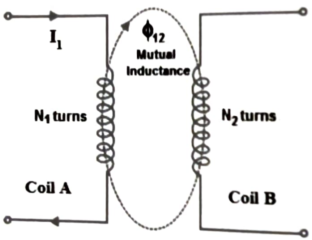


Figure 7: Mutual Inductance

$$e_M = M\frac{dI_1}{dt} = \frac{d}{dt}(MI_1)$$

$$e_M = N_2\frac{d\phi_{12}}{dt} = \frac{d}{dt}(N_2\phi_{12})$$

From these two expressions we have

$$MI_1 = N_2\phi_{12}$$

$$M = \frac{N_2\phi_{12}}{I_1}$$

Mutual inductance between two coils is 1 henry if a current of 1 A flowing in one coil produces flux linkages of 1 Wb-turn in the other.

- The mutual inductance between the two coils can be determined in terms of physical dimensions of the magnetic circuit. Figure 7 shows two magnetically coupled coils A and B having N_1 and N_2 turns respectively. Suppose l and 'a' are the length and area of cross-section of the magnetic circuit respectively. Let μ_r be the relative permeability of the material of which the magnetic circuit is composed.

Mutual flux $\phi_{12} = \dfrac{\text{mmf}}{\text{reluctance}} = \dfrac{NI_1}{\left[\dfrac{l}{a\mu_0\mu_r}\right]}$

$$\frac{\phi_{12}}{I_1} = \frac{\text{N}a\mu_0\mu_r}{l}$$

Now $M = \dfrac{N_2\phi_{12}}{I_1}$

$$M = \frac{N_1N_2a\mu_0\mu_r}{l}$$

$$= \frac{N_1N_2}{\left[l/a\mu_0\mu_r\right]} = \frac{N_1N_2}{\text{Relucatnce (S)}}$$

3.2.3 COEFFICIENT OF COUPLING

The coefficient of coupling (k) between two coils is defined as the fraction of magnetic flux produced by the current in one coil that links the other.

Consider two magnetically coupled coils 1 and 2 having N_1 and N_2 turns respectively.

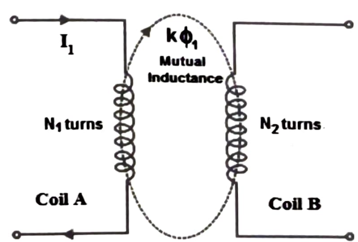


Figure 8: Coefficient of coupling

The current I_1 flowing in coil 1 produces a magnetic flux ϕ_1. Suppose the coefficient of coupling between the two coils is k. It means that flux $k\phi_1$ links with coil 2. Then, by definition,

$L_1 = \frac{\phi_1 N_1}{I_1}$ and $M_{12} = \frac{k\phi_1 N_2}{I_1}$. Here M_{12} represents mutual inductance of coil 1 and coil 2.

The current I_2 flowing in coil 2 will produce flux ϕ_2. Since the coefficient of coupling between the coils is k, it means that flux $k\phi_2$ will link with coil 1. Then,

$L_2 = \frac{\phi_2 N_2}{I_2}$ and $M_{21} = \frac{k\phi_2 N_1}{I_2}$. Here M_{21} represents mutual inductance of coil 2 and coil 1.

Mutual inductance between the two coils is exactly the same i.e. $M_{12} = M_{21} = M$

$$M_{12} \times M_{21} = \frac{k\phi_1 N_2}{I_1} \times \frac{k\phi_2 N_1}{I_2}$$

$$M^2 = k^2 \frac{\phi_1 N_1}{I_1} \times \frac{\phi_2 N_2}{I_2} = k^2 L_1 L_2$$

$$M = k\sqrt{L_1 L_2}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{\text{Actual mutual inductance}}{\text{Maximum possible inductance}}$$

- ***The coefficient of coupling is also defined as the ratio of actual mutual inductance (M) between the two coils to the maximum possible value*** $\left(\sqrt{L_1 L_2}\right)$

4. SOLVED NUMERICAL PROBLEMS

1. A conductor of length 0.5 m is placed in a magnetic field of strength 0.5 Wb/m². Calculate the force experienced by the conductor when a current of 50A flows through it. If the force moves the conductor at a velocity of 20 m/sec, calculate the EMF induced in it.

Solution: Force F on a current carrying conductor placed in a magnetic field is given as $F = BIl\ Newton$

Substituting the values, $F = 0.5 Wb/m^2 \times 50 \times 0.5 m = 12.5 N$

Induced EMF, e in a conductor moving in a magnetic field is given as $e = Blv V$

Substituting the given values, $e = 0.5 Wb/m^2 \times 0.5 m \times 20 m/\sec = 5 Wb/\sec = 5V$

2. A conductor of length 0.5m moves in a uniform magnetic field of density 1.1T at a velocity of 30 m/s. Calculate the induced voltage in the conductor when the direction of motion is inclined at 60° to the direction of the field. **[KTU June 2017]**

Solution: Induced voltage = $B\,l\,v \sin\theta$ = 1.1 × 0.5 × 30 × sin60 = 14.29 V

3. There is mutual magnetic coupling between two coils of number of turns 500 and 2000, respectively. Only 50% of the flux produced by the coil of 500 turns is linked with the coil of 1000 turns. Calculate the mutual inductance of the two coils. Also calculate the EMF induced in the coil of 1000 turns when current changes at the rate of 10A/second in the other coil. The self-inductance of the coil of 500 turns in 200mH.

Solution:

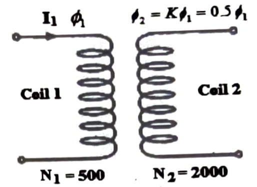


Figure 9

$$\text{Mutual inductance } M = \frac{\text{flux linkage in coil 2}}{\text{current change in coil 1}}$$

$$= N_2 \frac{K\phi_1}{I_1} = 2000 \times \frac{0.5 \times \phi_1}{I_1} = \frac{1000\phi_1}{I_1} \quad \text{----------(1)}$$

$$\text{Self inductance } L_1 = \frac{N_1\phi_1}{I_1}$$

$$\frac{\phi_1}{I_1} = \frac{L_1}{N_1} = \frac{200 \times 10^{-3}}{500} \quad \text{----------(2)}$$

Substitute (2) in (1) we get, $M = \frac{1000 \times 200 \times 10^{-3}}{500} = 400 \times 10^{-3} H$

Induced EMF in the second coil, $e_2 = M\frac{di}{dt} = 400 \times 10^{-3} \times 10 = 4V$

4. A coil having an inductance of 60 mH is carrying a current of 90 A. Calculate the self-induced emf in the coil, when the current is (1) reduced to zero in 0.03 second (2) reversed in 0.03 second.

Solution: Coil inductance, L= 60 mH = 0.06 H

When the current is reduced to zero in 0.03 second

Rate of change of current $\frac{di}{dt} = \frac{90-0}{0.03} = 3000 A/s$

Self-induced emf, $e = L\frac{di}{dt} = 0.06 \times 3000 = 180V$

When current is reversed in 0.03 second,

Rate of change current, $\frac{di}{dt} = \frac{90-(-90)}{0.03} = 6000 A/s$

Self-induced emf, $e = L\frac{di}{dt} = 0.06 \times 6000 = 360\ V$

5. Two coils having 100 and 50 turns respectively are wound on a core with $\mu = 4000\ \mu_0$. Effective core length =60cm and core area =9cm^2. Find the mutual inductance between the coils.

Solution: Number of turns N_1=100 and N_2=50

Core area, $a = 9\ cm^2 = 9 \times 10^{-4} m^2$

Core length $l = 60\,cm = 0.6\,m$

$$\mu = 4000\mu_0 = 4000 \times 4\pi \times 10^{-7} = 5.0265 \times 10^{-3}\ H/m$$

Mutual inductance between the coils, M=

$$M = \frac{N_1 N_2 a\mu}{l} = \frac{100 \times 50 \times 9 \times 10^{-4} \times 5.0265 \times 10^{-3}}{0.6} = 37.7mH$$