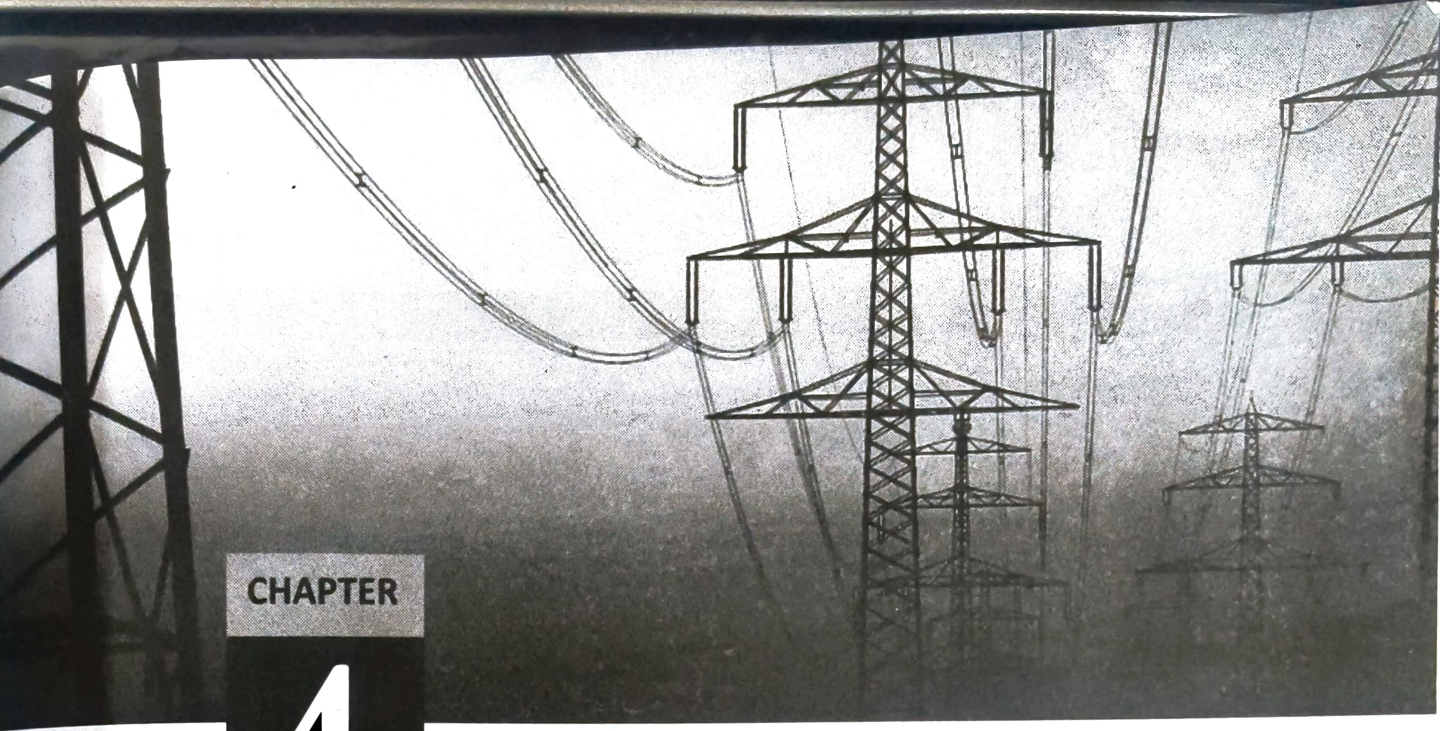

CHAPTER

4

LEARNING OBJECTIVES

AC FUNDAMENTALS

When the direction of current through a circuit continuously changes, such current is called alternating current (AC). Contrary to DC, an AC will have different magnitude of current (or voltage) at different time instants. Thus it becomes a time dependent.

At present electricity is generated in the form of Alternating Current. AC is produced by an electric generator. The generated electricity is transmitted, distributed and mostly utilized in the form of AC. In this chapter, the fundamentals of alternating current have been discussed.

After reading this chapter, the reader should be familiar with the following concepts.

- Generation of alternating voltage
- Representation of sinusoidal waveforms
- Basic terminologies including frequency, time period etc.
- RMS value
- Average value
- Form factor
- Peak factor
- RMS and average values of different wave shapes.

1. INTRODUCTION

The term **A.C** means Alternating Current. At present, electrical energy is universally generated, transmitted and distributed in the form of alternating current.

Advantages of A.C. over D.C.

1. The voltages in A.C. system can be raised or lowered with the help of a device called transformer. In D.C. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible. Now, higher the voltage, lesser is the current flowing through transmission line. Less the current, lesser are the copper losses and lesser is the conducting material required. This makes a.c. transmission always economical and efficient.
3. It is possible to build up high A.C. voltage; high speed A.C. generators of large capacities. The construction and cost of such generators are very low. High A.C. voltages of about 11 kV can be generated and can be raised up to 220 kV for transmission purpose at sending end, while can be lowered down at 400 V at receiving end. This is not possible in case of D.C.
4. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.
5. Whenever it is necessary, A.C. supply can be easily converted to obtain D.C. supply. This is required as D.C. is very much essential for the applications like canes, printing process, battery charging, telephone system, etc. But, such requirement of D.C. is very small compared to a.c.

1.1. ALTERNATING VOLTAGE AND CURRENT

- ***A voltage which changes its polarity at regular intervals of time is called an alternating voltage.***

When an alternating voltage is applied in a circuit, the current flows first in one direction and then in the opposite direction; the direction of current at any instant depends upon the polarity of the voltage. It is possible to produce alternating voltages and currents with an endless variety of waveforms like square waves, triangular waves, rectangular waves etc., yet the engineers choose to adopt sine waveform.

Why electric supply companies all over the world generate sinusoidal alternating voltages and currents?

Due to following advantages, electric supply companies all over the world generate sinusoidal alternating voltages and currents.

1. Mathematically, it is very easy to write the equations for purely sinusoidal waveform.

2. Any other type of waveform can be resolved into a series of sine or cosine waves of fundamental and higher frequencies, sum of all these waves gives the original wave form. Hence, it is always better to have sinusoidal waveform as the standard waveform.
3. The sine and cosine waves are the only waves which can pass through linear circuits containing resistance, inductance and capacitance without distortion. In case of other waveforms, there is a possibility of distortion when it passes through a linear circuit.
4. The integration and derivative of a sinusoidal function is again a sinusoidal function. This makes the analysis of linear electrical networks with sinusoidal inputs, very easy.

2. GENERATION OF ALTERNATING VOLTAGE AND CURRENT

- ***The machines which are used to generate electrical voltages are called generators. The generators which generate purely sinusoidal a.c. voltages are called alternators.***

The basic principle of an alternator is the principle of electromagnetic induction. The sine wave is generated according to Faraday's law of electromagnetic induction. It says that whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f. gets induced in the conductor. The relative motion may exist because of movement of conductors with respect to magnetic field or movement of magnetic field with respect to conductor. Then such an induced e.m.f. can be used to supply the electrical load.

An alternating voltage may be generated:

1. ***By rotating a coil at constant angular velocity in a uniform magnetic field***

OR

2. ***By rotating a magnetic field at a constant angular velocity within a stationary coil.***

Now, let us see how an alternator produces a sine wave, with the help of simplest form of an alternator called single turn or single loop alternator. It consists of a permanent magnet of two poles. A single turn rectangular coil is kept in the vicinity of the permanent magnet. The coil is made up of same conducting material like copper or aluminium. The coil is made up of two conductors namely A-B and C-D. Such two conductors are connected at one end to form a coil. This is shown in the figure1.

The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction. The remaining two ends of the coil are connected to the rings mounted on the shaft called slip rings. Slip rings are also rotating members of the alternator. The two brushes P and Q are resting on the slip rings. The brushes are stationary and just making contact with the slip rings. The slip rings and brush assembly is necessary to collect the current induced in the rotating coil and make it available to the stationary external resistance. The overall construction is shown in the figure 1.

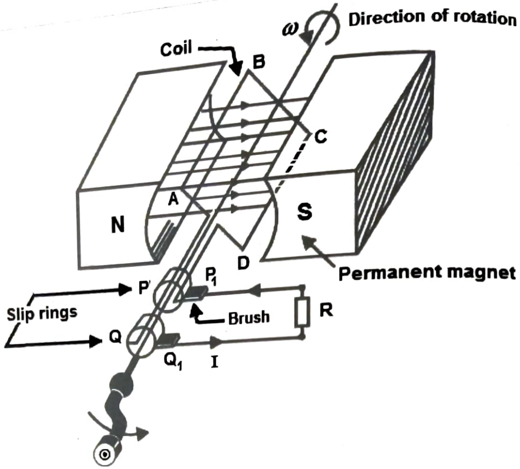

Figure 1: Generation of AC Voltage

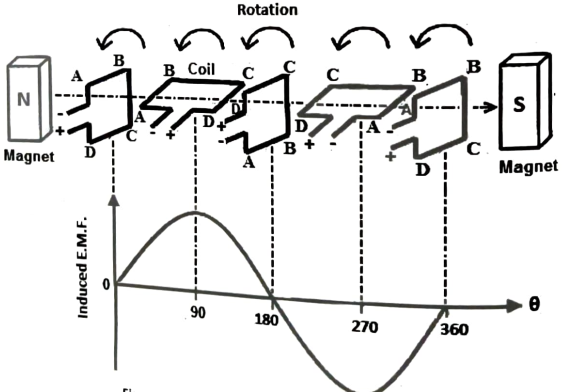

Figure 2: Position of the coil and induced e.m.f.

Working

The coil is rotated in anticlockwise direction. While rotating, the conductors AB and CD cut the lines of flux of the permanent magnet. Due to Faraday's law of electromagnetic induction, an e.m.f. gets induced in the conductors. This e.m.f. drives a current through resistance R connected across the brushes P and Q. The magnitude of the induced e.m.f. depends on the position of the coil in the magnetic field.

Let us see the relation between magnitude of the induced e.m.f. and the positions of the coil. Consider different instants and the different positions of the coil. See the initial position of the coil be as shown in the figure 2.The plane of the coil is perpendicular to the direction of the magnetic field. The instantaneous component of velocity of conductor AB and CD, is parallel to the magnetic field as shown and there cannot be the cutting of the flux lines by the conductors. Hence, no e.m.f. will be generated in the conductor AB and CD and no current will flow through the external resistance R.

So, as θ varies from $0°$ to $360°$, the e.m.f. in a conductor AB or CD varies in an alternating manner. i.e. zero, increasing to achieve maximum in one direction, decreasing to zero, increasing to achieve maximum in other direction and again decreasing to zero. This set of variation repeats for every revolution as the conductors rotate in a circular motion with a certain speed. This variation of e.m.f. in a conductor can be graphically represented which is shown in figure2.

2.1 EQUATION OF ALTERNATING VOLTAGE AND CURRENT

Consider a rectangular coil of N turns rotating in anticlockwise direction with an angular velocity of ω rad/sec in a uniform magnetic field as shown in figure 3. The emf induced in the coil will be sinusoidal.

Let the time be measured from the instant the plane of the coil coincides with OX axis. In this position of the coil [see figure 3 (a)] the flux linking with the coil has its maximum value ϕ_{max}

Let the coil turn through an angle $\theta = \omega t$ in anticlockwise direction in t seconds and assume the position shown in figure 3 (b). In this position the maximum flux ϕ_{max} acting vertically downward can be resolved into two perpendicular components.

1 Component $\phi_{max} \sin \omega t$ parallel to the plane of the coil. This component induces no emf in the coil.

2 Component $\phi_{max} \cos \omega t$ perpendicular to the plane of the coil. This component induces emf in the coil.

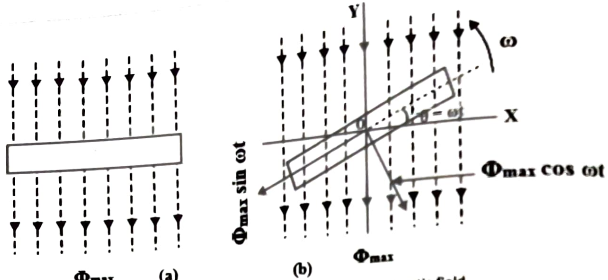

Figure 3: A rectangular coil in a uniform magnetic field

∴ Flux linkages of the coil at the considered instant (i.e. at 0^0) = No. of turns × Flux linking

$$= N\phi_{max}\cos\omega t$$

According to Faraday's laws of electromagnetic induction, the emf induced in the coil is equal to the rate of change of flux linkages of the coil. Hence the emf v at the considered instant is given by

$$v = -\frac{d}{dt}(N\phi_{max}\cos\omega t) = -N\phi_{max}\omega(-\sin\omega t)$$

$$v = N\phi_{max}\omega\sin\omega t \qquad (1)$$

The value of v will be maximum (call it V_m) when $\sin\omega t = 1$.

i.e. when the coil has turned through 90^0 in anticlockwise direction from the reference axis (i.e. OX axis. Now the equation (1) becomes

$$V_m = N\phi_{max}\omega$$

$$v = V_m\sin\omega t \text{ where } V_m = N\phi_{max}\omega$$

$$v = V_m\sin\theta$$

It is clear that emf induced in the coil is sinusoidal i.e. the instantaneous value of emf varies as the sine function of time angle (θ or ωt) If this alternating voltage $v = V_m\sin\omega t$ is applied across a load, alternating current flows through the circuit which would also vary sinusoidally. The equation of alternating current is given by $i = I_m\sin\omega t$ provided the load is resistive.

3. REPRESENTATION OF SINUSOIDAL WAVEFORMS

An alternating voltage or current changes continuously in magnitude and alternates in direction at regular intervals of time. It rises from zero to maximum positive value, falls to zero,

and increases to a maximum in the reverse direction and falls back to zero again as shown in figure 4. From this point on indefinitely, the voltage or current repeats the procedure. The important A.C. terminology is defined below:

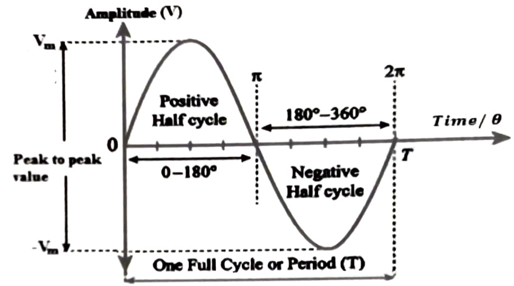

Figure 4: A sinusoidal AC waveform

3.1 WAVEFORM

- The shape of the curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abscissa is called its waveform or wave shape.

3.2 INSTANTANEOUS VALUE

- The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltage and current are represented by **v** and **i** respectively. As an example, the instantaneous values of voltage (See figure 4) at 0°, 90° and 270° are 0, + V_m, $-V_m$ respectively.

3.3 CYCLE

- One complete set of positive and negative values of an alternating quantity is known as a cycle. Figure 4 shows one cycle of an alternating voltage. A cycle can also be defined in terms of angular measure. One cycle corresponds to 360° electrical or 2π radians.

- One-half cycle of an alternating quantity is called an alternation. An alternation spans 180° electrical. Thus in figure 4, the positive or negative half of alternating voltage is the alternation.

3.4 TIME PERIOD

The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.

3.5 FREQUENCY

The number of cycles that occur in one second is called the frequency (f) of the alternating quantity. It is measured in cycles/sec (C/s) or Hertz (Hz). One Hertz is equal to 1C/s.

The frequency of power system is low; the most common being 50 C/s or 50 Hz. It means that alternating voltage or current completes 50 cycles in one second. The 50 Hz frequency is the most popular because it gives the best results when used for operating both lights and machinery.

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

3.6 AMPLITUDE

The maximum value (positive or negative) attained by an alternating quantity is called its amplitude or peak value. The amplitude of an alternating voltage or current is designated by V_m (or E_m) or I_m

4 AVERAGE VALUE

- **The average value of a waveform is the average of all its values over a period of time.**

$$\text{Average value} = \frac{\text{Total (net) area under curve for time T}}{\text{Time T}}$$

$$\text{Average value of symmetrical wave} = \frac{\text{Area of one alternation}}{\text{Base length of one alternation}}$$

$$\text{Average value of asymmetrical wave} = \frac{\text{Area over one cycle}}{\text{Base length of one cycle}}$$

- ***The average value of an alternating current is that the steady direct current which transfers the same charge across any circuit, that transferred by the alternating current during the same time period.***

The average value of current and voltage is found by the following methods.

1. Mid -ordinate method
2. Analytical method

1. **Mid ordinate method:** The waveform is divided in to number of small stripes of equal width. The average of each strip is taken as the mid value of that strip i_1, i_2, i_3 etc. as shown in figure 5. Average value is found by dividing the sum of the mid values by the number of strips.

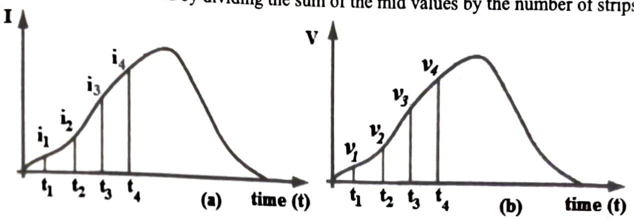

Figure 5: Current and voltage wave forms for mid-ordinate method

$$I_{avg} = \frac{i_1 + i_2 + i_3 + \dots\dots + i_n}{n}$$

$$V_{avg} = \frac{v_1 + v_2 + v_3 + \dots\dots + v_n}{n}$$

2. **Analytical method**

The average value can also found out mathematically by

$$I_{avg} = \frac{1}{T}\int_0^T i(t)\,dt \qquad V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$\boxed{I_{avg} = \frac{2I_m}{\pi} = 0.637 \times I_m \qquad V_{avg} = \frac{2V_m}{\pi} = 0.637 \times V_m}$$

Here v(t) and i(t) are instantaneous values and current respectively.

4.1 AVERAGE VALUE OF SINUSOIDAL CURRENT

The average value of alternating current (or voltage) over one cycle is zero. It is because the waveform is symmetrical about time axis and positive area exactly cancels the negative area. However, the average value over a half-cycle (positive or negative) is not zero. Therefore, average value of alternating current (or voltage) means half-cycle average value unless stated otherwise.

- ***The half-cycle average value of A.C. is that value of steady current (D.C.) which would send the same amount of charge through a circuit for half the time period of A.C. as is sent by the A.C. through the same circuit in the same time.***

It is represented by I_{av} This can be obtained by integrating the instantaneous value of current over one half cycle (i.e. area over half-cycle) and dividing the result by base length of half-cycle (= π).

The equation of a sinusoidal alternating current is $i = I_m \sin\theta$. Consider an elementary strip of thickness $d\theta$ in the first half-cycle of current wave as shown in figure 6.

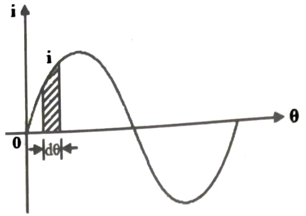

Figure 6: Sinusoidal wave

Let i be the mid-ordinate of this strip.

$$\text{Area of the strip} = i\,d\theta$$

$$\text{Area of the half cycle} = \int_0^{\pi} i\,d\theta$$

$$= \int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= I_m\left[-\cos\theta\right]_0^{\pi} = 2I_m$$

$$\text{Average value, } I_{av} = \frac{\text{Area of half cycle}}{\text{Base length of half cycle}} = \frac{2I_m}{\pi} = 0.637\,I_m$$

Hence, the half-cycle average value of A.C. is 0·637 times the peak value of A.C.

5 R.M.S. OR EFFECTIVE VALUE

The average value cannot be used to specify a sinusoidal voltage or current. It is because its value over one-cycle is zero and cannot be used for power calculations. Therefore, we must search for a more suitable criterion to measure the effectiveness of an alternating current (or voltage). The obvious choice would be to measure it in terms of direct current that would do work (or produce heat) at the same average rate under similar conditions. This equivalent direct current is called the root-mean-square (R.M.S.) or effective value of alternating current.

The effective or R.M.S. value of an alternating current is that steady current (D.C.) which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistance for the same time.

For example, when we say that the R.M.S. or effective value of an alternating current is 5A, it means that the alternating current will do work (or produce heat) at the same rate as 5A direct current under similar conditions.

The R.M.S. or effective value of alternating current (or voltage) can be determined as follows.

1. **Mid ordinate method:** Consider the waveforms shown in figure 5. The waveform is divided in to number of small stripes of equal width. Let $i_1, i_2, i_3, \ldots\ldots i_n$ be the currents of small intervals $t_1, t_2, t_3, \ldots\ldots t_n$ and $v_1, v_2, v_3, \ldots\ldots v_n$ be the voltages of small intervals $t_1, t_2, t_3, \ldots\ldots t_n$. Here of small intervals n.

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots\ldots + i_n^2}{n}}$$

$$= \sqrt{\text{mean value of } i^2}$$

= Square **Root** of the **Mean** of the **Squares** of the current

= **Root-Mean-Square** (R.M.S.) value

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots\ldots + v_n^2}{n}}$$

2. **Analytical method:** Let v(t) and i(t) be the instantaneous values and current respectively and T be the time period. Then the rms value is obtained by

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2\,dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 \times I_m$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 \times V_m$$

- Mid- ordinate method is ***useful for random or irregular shaped waveforms.***
- Analytical method is more suitable for wave forms which can be expressed using mathematical expressions such as ***sine wave, square wave, triangular wave etc.***

Info Plus

An alternating voltage or current is always specified in terms of R.M.S. values.

- For example, common household appliances are rated at 230V A.C. This is an R.M.S. value. The domestic A.C. supply is 230V, 50 Hz. It is the R.M.S. or effective value. It means that alternating voltage available has the same heating effect as 230V D.C.
- When we say that alternating current in a circuit is 5A, we are specifying the R.M.S. value. It means that the alternating current flowing in the circuit has the same heating effect as 5A D.C
- AC ammeters and Volt meters records RMS values.

6 PEAK FACTOR AND FORM FACTOR

There exists a definite relation among the peak value, average value and R.M.S. value of an alternating quantity. The relationship is expressed by two factors, namely; form factor and peak factor.

1. FORM FACTOR

The ratio of rms value to the average value of an alternating quantity is called form factor.

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}}$$

For a sinusoidal voltage or current, $\text{Form factor} = \dfrac{0.707\times\text{Max. value}}{0.637\times\text{Max. value}} = 1.11$

2. PEAK FACTOR

The ratio of maximum value to the rms value of an alternating quantity is called its peak factor.

$$\text{Peak factor} = \frac{\text{Max. value}}{\text{R.M.S value}}$$

For a sinusoidal voltage or current, $\text{Peak factor} = \dfrac{\text{Max. value}}{0.707\times\text{Max. value}} = 1.414$

Peak factor is also called crest factor or amplitude factor.

7 SOLVED NUMERICAL PROBLEMS

1. Obtain the form factor and peak factor of a sinusoidal waveform.

Consider a sinusoidal wave given below in figure 7. Here the maximum value of the sine wave is V_m and time period 2π.

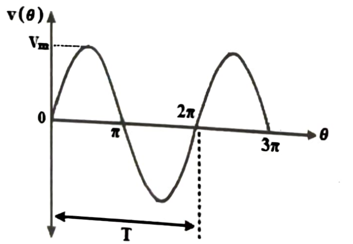

Figure 7: Sine wave

We know that $\text{Form factor} = \dfrac{\text{R.M.S value}}{\text{Average value}}$ and $\text{Peak factor} = \dfrac{\text{Max. value}}{\text{R.M.S value}}$

R.M.S value

$v(t) = V_m \sin\theta$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(V_m\sin\theta)^2 d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}V_m^2\sin^2\theta\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}V_m^2\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta} = \sqrt{\frac{V_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}[2\pi - 0]} = \sqrt{\frac{V_m^2}{4\pi}[2\pi]} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Similarly $I_{rms} = \dfrac{I_m}{\sqrt{2}}$

Average Value

- Average value of voltage or current of a sinusoidal waveform over one period is zero. Therefore, the average value of positive half cycle is calculated.

$$V_{avg} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{\pi}\int_0^{\pi} v(\theta)d\theta = \frac{1}{\pi}\int_0^{\pi} V_m\sin\theta\, d\theta = \frac{V_m}{\pi}\int_0^{\pi}\sin\theta\, d\theta = \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$V_{avg} = \frac{2V_m}{\pi}$$

Similarly $I_{avg} = \dfrac{2I_m}{\pi}$

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$\text{Peak factor} = \frac{\text{Max. value}}{\text{R.M.S value}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = 1.414$$

2. Obtain the form factor and peak factor of a square waveform.

Consider a square wave shown in figure 8 with maximum value V_m and time period T.

We know that $\text{Form factor} = \dfrac{\text{R.M.S value}}{\text{Average value}}$ and $\text{Peak factor} = \dfrac{\text{Max. value}}{\text{R.M.S value}}$

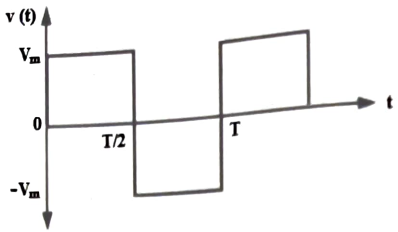

Figure 8: Square wave

$$v(t)=\begin{cases} V_m & t=0 \text{ to } \frac{T}{2} \\ -V_m & t=\frac{T}{2} \text{ to } T \end{cases}$$

R.M.S value

$$V_{rms}=\sqrt{\frac{1}{T}\int_0^T v(t)^2dt}=\sqrt{\frac{1}{T}\int_0^{T/2} V_m^2dt+\int_{T/2}^{T}(-V_m)^2dt}=\sqrt{\frac{1}{T}\int_0^T V_m^2dt}$$

$$=\sqrt{\frac{V_m^2}{T}\int_0^T dt}=\sqrt{\frac{V_m^2}{T}[t]_0^T}=V_m$$

$$V_{rms}=V_m$$

Average value

- Average value of voltage or current of a symmetrical (about x- axis) waveform over one period is zero. Therefore, the average value of positive half cycle is calculated.

$$V_{avg}=\frac{1}{T}\int_0^T v(t)dt=\frac{1}{T/2}\int_0^{T/2}V_m dt$$

$$=\frac{V_m}{T/2}[t]_0^{T/2}=V_m$$

$$V_{avg}=V_m$$

$$\text{Form factor}=\frac{\text{R.M.S value}}{\text{Average value}}=\frac{V_m}{V_m}=1$$

$$\text{Peak factor}=\frac{\text{Max. value}}{\text{R.M.S value}}=\frac{V_m}{V_m}=1$$

3. Obtain the form factor and peak factor of a triangle waveform.

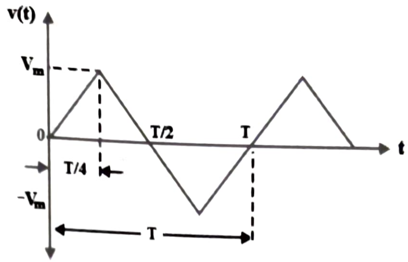

Figure 9 : Triangle wave

Consider a triangle wave shown in figure 9 with peak value V_m and time period T. One quarter of the wave during 0 to T/4 is expressed as $v(t)=\frac{V_m}{T/4}t$. The R.M.S value of the first quarter of the wave is same for the complete wave.

R.M.S value

$$V_{rms}=\sqrt{\frac{1}{T/4}\int_0^{T/4}v(t)^2dt}=\sqrt{\frac{4}{T}\int_0^{T/4}\left(\frac{V_m}{T/4}t\right)^2dt}=\sqrt{\frac{4}{T}\int_0^{T/4}\left(\frac{V_m^2 4^2}{T^2}t^2\right)dt}$$

$$=\sqrt{\frac{4\times16V_m^2}{T^3}\int_0^{T/4}t^2\,dt}=\sqrt{\frac{64V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^{T/4}}=\sqrt{\frac{64V_m^2}{3T^3}\left[\left(\frac{T}{4}\right)^3-0\right]}=\sqrt{\frac{64V_m^2T^3}{3T^3 64}}=\frac{V_m}{\sqrt{3}}$$

Average value

$$V_{avg}=\frac{1}{T}\int_0^T v(t)dt=\frac{1}{T/4}\int_0^{T/4}\frac{V_m}{T/4}t\,dt=\frac{4}{T}\frac{4}{T}V_m\int_0^{T/4}t\,dt=\frac{16}{T^2}V_m\int_0^{T/4}t\,dt$$

$$=\frac{16}{T^2}V_m\left[\frac{t^2}{2}\right]_0^{T/4}=\frac{16V_m}{T^2\times2}\frac{T^2}{4^2}=\frac{16V_m}{T^2 2}\frac{T^2}{16}=\frac{V_m}{2}$$

$$\text{Form factor}=\frac{\text{R.M.S value}}{\text{Average value}}=\frac{V_m/\sqrt{3}}{V_m/2}=\frac{2}{\sqrt{3}}$$

$$\text{Peak factor}=\frac{\text{Max. value}}{\text{R.M.S value}}=\frac{V_m}{V_m/\sqrt{3}}=\sqrt{3}$$

4. Obtain the form factor and peak factor of a saw tooth waveform.

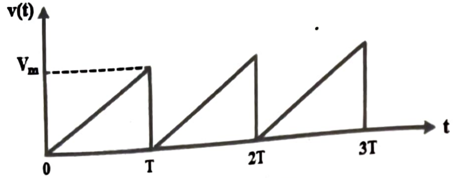

Figure 10 : Saw tooth wave

The saw tooth wave v (t) is shown in figure 10. The equation representing v (t) can be obtained using the general equation for a straight line passing through the origin $y = mx$. Here m is the slope of the line, x and y represent the coordinates.

Here $y = v(t)$ and $x = t$

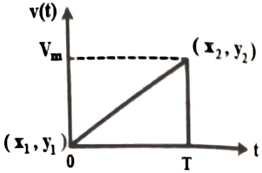

Figure 11

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{V_m - 0}{T - 0} = \frac{V_m}{T}$$

We have $y = mx$

$$v(t) = \frac{V_m}{T} t$$

R.M.S value

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T} t\right)^2 dt} = \sqrt{\frac{V_m^2}{T^3}\int_0^T t^2\, dt} = \sqrt{\frac{V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T} = \frac{V_m}{\sqrt{3}}$$

Average value

$$V_{avg} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{T}\int_0^T \frac{V_m}{T} t\, dt = \frac{V_m}{T^2}\int_0^T t\, dt = \frac{V_m}{T^2}\left[\frac{t^2}{2}\right]_0^T = \frac{V_m}{2}$$

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{V_m/\sqrt{3}}{V_m/2} = \frac{2}{\sqrt{3}}$$

$$\text{Peak factor} = \frac{\text{Max. value}}{\text{R.M.S value}} = \frac{V_m}{V_m/\sqrt{3}} = \sqrt{3}$$

5. Find the average value, R.M.S value, form factor and peak factor for a full wave rectified sine wave.

The full wave rectified sine wave is shown in figure 12. Its period is π.

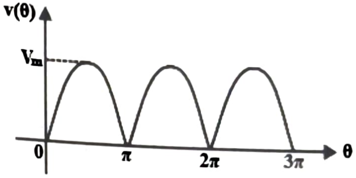

Figure 12 : Full rectified sine wave

R.M.S value

$v(t) = V_m \sin\theta$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \sqrt{\frac{1}{\pi}\int_0^\pi (V_m \sin\theta)^2 d\theta} = \sqrt{\frac{1}{\pi}\int_0^\pi V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{1}{\pi}\int_0^\pi V_m^2\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^\pi (1-\cos 2\theta)\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi}$$

$$= \sqrt{\frac{V_m^2}{2\pi}[\pi - 0]} = \sqrt{\frac{V_m^2}{2\pi}[\pi]} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Similarly $I_{rms} = \frac{I_m}{\sqrt{2}}$

Average Value

$$V_{avg} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{\pi}\int_0^\pi v(\theta)d\theta = \frac{1}{\pi}\int_0^\pi V_m \sin\theta\, d\theta = \frac{V_m}{\pi}\int_0^\pi \sin\theta\, d\theta = \frac{V_m}{\pi}[-\cos\theta]_0^\pi$$

$$V_{avg} = \frac{2V_m}{\pi}$$

Similarly $I_{avg} = \frac{2I_m}{\pi}$

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$\text{Peak factor} = \frac{\text{Max. value}}{\text{R.M.S value}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = 1.414$$

6. Find the average value, R.M.S value, form factor and peak factor for a half wave rectified sine wave.

The half wave rectified sine wave is shown in figure 13. Its period is 2π. The mathematical representation of the given waveform is

$$v(t) = \begin{cases} V_m & \theta = 0 \text{ to } \pi \\ 0 & \theta = \pi \text{ to } 2\pi \end{cases}$$

Figure 13: Half wave rectified sine wave

R.M.S value

$v(t) = V_m \sin\theta$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(V_m\sin\theta)^2 d\theta} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi}(V_m\sin\theta)^2 d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\int_0^{\pi}(1-\cos 2\theta)\, d\theta} = \sqrt{\frac{V_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{4\pi}[\pi - 0]} = \sqrt{\frac{V_m^2}{4\pi}[\pi]} = \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2} \qquad \text{Similarly } I_{rms} = \frac{I_m}{2}$$

Average Value

$$V_{avg} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)d\theta = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right] = \frac{V_m}{2\pi}\int_0^{\pi}\sin\theta\, d\theta = \frac{V_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$V_{avg} = \frac{V_m}{2\pi}\times 2 = \frac{V_m}{\pi}$$

Similarly $I_{avg} = \dfrac{I_m}{\pi}$

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{V_m/2}{V_m/\pi} = 1.57$$

$$\text{Peak factor} = \frac{\text{Max. value}}{\text{R.M.S value}} = \frac{V_m}{V_m/2} = 2$$

7. Find the average and rms value for the given wave form. [KTU JULY 2016]

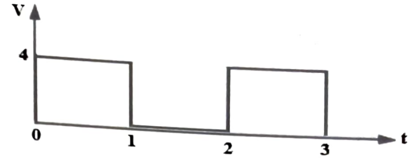

Figure 14

Average Value

$$V_{avg} = \frac{1}{T}\int_0^T V dt = \frac{1}{2}\left[\int_0^1 4\,dt + \int_1^2 0\,dt\right] = \frac{4}{2}\left[\int_0^1 dt\right] = 2[t]_0^1 = 2\times 1 = 2$$

R.M.S value

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 dt} = \sqrt{\frac{1}{2}\left[\int_0^1 4^2 dt + \int_1^2 0\,dt\right]} = \sqrt{\frac{1}{2}\left[\int_0^1 16\,dt\right]} = \sqrt{\frac{16}{2}\left[\int_0^1 dt\right]} = \sqrt{8[t]_0^1} = \sqrt{8\times 1} = 2.828\text{V}$$

8. Calculate the rms and average value of the voltage wave form shown in figure. [KTU JAN 2016]

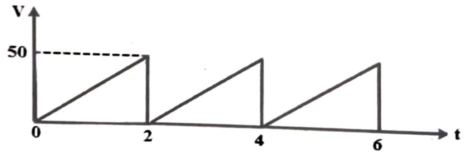

Figure 15

The equation representing saw tooth wave can be obtained using the general equation for a straight line passing through the origin $y = mx$. Here m is the slope of the line, x and y represent the coordinates.

Here $y = V$ and $x = t$

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50-0}{2-0} = \frac{50}{2} = 25$$

We have $y = mx$

$V = 25t$

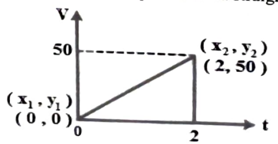

Figure 16

R.M.S value

$$V_{rms}=\sqrt{\frac{1}{T}\int_0^T V^2dt}=\sqrt{\frac{1}{2}\left[\int_0^2 25t^2dt\right]}=\sqrt{\frac{1}{2}\left[\int_0^2 625t^2\,dt\right]}=\sqrt{\frac{625}{2}\left[\int_0^2 t^2dt\right]}$$

$$=\sqrt{\frac{625}{2}\left[\frac{t^3}{3}\right]_0^2}=\sqrt{\frac{625}{2\times3}[2^3]}=28.86V$$

Average value

$$V_{avg}=\frac{1}{T}\int_0^T Vdt=\frac{1}{2}\int_0^2 25t\,dt=\frac{25}{2}\int_0^2 t\,dt=\frac{25}{2}\left[\frac{t^2}{2}\right]_0^2=\frac{25}{2\times2}\times\left[2^2\right]=25V$$

9. Calculate the rms and average value of the voltage wave form shown in figure.

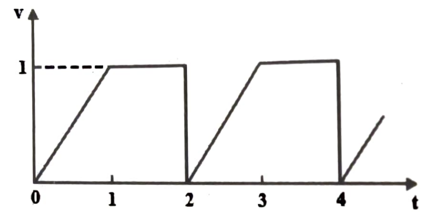

Figure 17

The wave can be represented mathematically as

$$v=t \text{ for } 0<t<1; \quad V=1 \text{ for } 1<t<2$$

Average value, $Vav=\frac{1}{2}\left[\int_0^1 t\,dt+\int_1^2 dt\right]=\frac{1}{2}\left[\frac{t^2}{2}\right]_0^1+\frac{1}{2}[t]_1^2=0.25+0.5=0.75$

$$V_{rms}=\sqrt{\frac{1}{T}\int_0^T V^2dt}=\sqrt{\frac{1}{2}\int_0^1 t^2dt+\frac{1}{2}\int_1^2 dt}=\sqrt{\frac{1}{2}\left[\frac{t^3}{3}\right]_0^1+\frac{1}{2}[t]_1^2}=\sqrt{\frac{1}{6}+\frac{1}{2}}=\frac{4}{6}=0.8165V$$

10. The equation of an alternating current is $i=141.4\sin 314t$. What is rms value of current and frequency?

Maximum value of current, I_{max} = coefficient of the sine of the time angle $=141.4$ A

RMS value of current, $I_{rms}=\frac{I_{max}}{\sqrt{2}}=100\,A$

Frequency, $f=\frac{\text{Coefficient of time}}{2\pi}=\frac{314}{2\pi}=50\,Hz$

11. An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of 20A. (a) Write down the equation for instantaneous current. (b) Find the instantaneous value at 0.0025s. (c) Find the instantaneous value of current 0.125s after passing through a positive maximum value. (d) At what time, measured from a positive maximum value, will the instantaneous current be 14.14A? [KTU DEC 2019]

(a) $I_{rms}=20A$, $f=50$ Hz, $\omega=2\pi f=314$ radian

$$I_m=\sqrt{2}\times I_{rms}=\sqrt{2}\times20A=28.28$$

$$I=I_m\sin\omega t=28.28\sin314t \quad \text{------------- (1)}$$

$$\omega=314 \text{ radian}=314\times\frac{180}{\pi}=18000 \text{ degree}$$

(b) when t=0.0025 s

$$I=28.2\sin(18000\times0.0025)=20A$$

(c) Since the time values are given from the point where the current has positive maximum value, the equation (1) modified to $I=28.28\sin\left[314t+\frac{\pi}{2}\right]=28.28\cos314t$

$$314 \text{ radian}=314\times\frac{180}{\pi}=18000 \text{ degree}$$

At $t=0.125s$

$$I=28.28\cos(18000\times0.125)=0A$$

(d) when i=14.14 A

$$14.14=28.28\cos(18000\times t)$$

$$t=\frac{1}{300}=0.0033s=3.335\text{ ms}$$