Module-3

Myhill-Nerode Theorem (MNR) [minimization of DFA]

* neccessary and sufficient theorem for prove the Language is not regular.

* Based on equivalence relation (MNR)

⇒ given a Language L and x and y are the strings on Σ^*. if for every string $z \in \Sigma^*$

$xz, yz \in L$ or $xz, yz \notin L$

then x and y are said to be indistingusiable over L by the equivalence relation $\boxed{x \equiv_L y}$

Here z is said to be indistinguisiable extension.

MNR gives finite no: of partition. [if it regular] And the finite no: of partition is the no: of states in minimization of DFA.

eg:

$L = \{ w, w$ contains even no: of 0's over {0,1}

take 2 strings x and y $\in L$

$x = 00$ $y = 0000$

$z \in \Sigma^*$ $\rightarrow$ odd no of 0's

$\rightarrow$ even no of 0's

odd

$z = 0$

$xz = 000 \notin L$, $yz = 00000 \notin L$

even

$z = 00$

$xz = 0000 \in L$, $yz = 000000 \in L$

Here 2 equivalence classes. It is finate.

∴ It is regular language with 2 state in DFA

eg: $L = \{a^n b^n, n \geq 0\}$

$x, y \in \Sigma^*$

$x = a^2$ $y = a^4$

$z = b^2$

$xz \in L$, $yz \notin L$

$z = b^m$

m any natural number

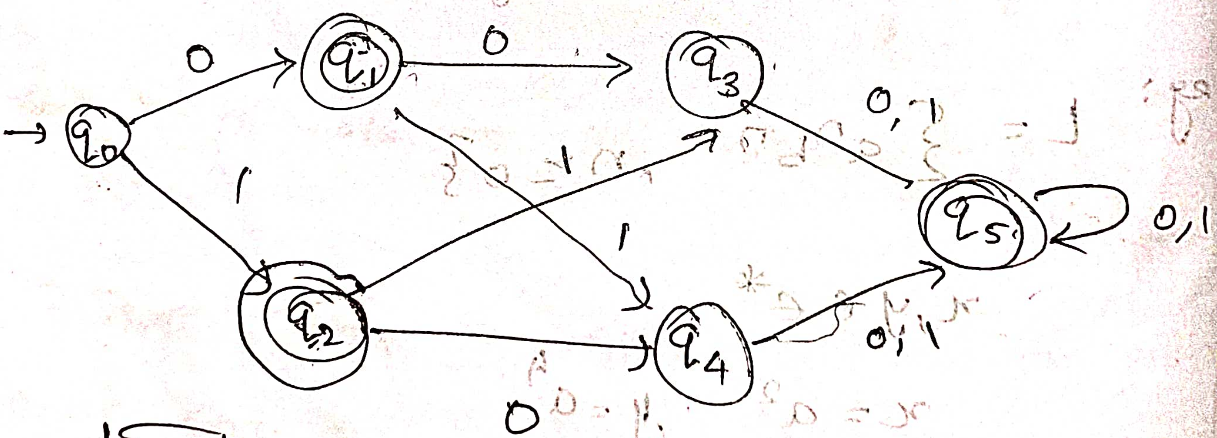
So infinite partition

So L is not regular.

Algorithm ⇒ minimization of DFA with MNT (Table filling method)

1. ~~Draw a tab~~ Draw a table for all pairs of states (P, q)
2. mark each pairs (P, q) $P \in F$, $q \in NF$
3. If there are any unmarked pairs (P, q) such that $(\delta(P, x), \delta(q, x))$ it is marked where x is an input symbol. then mark (P, q)

   $F \Rightarrow$ Final State, $q \Rightarrow$ Non Final State
4. Repeat this until no more marking possible
5. Combine all unmarked pairs and make them single state in your minimize DFA.



q_1	✓				
q_2	✓				
q_3	✓	✓	✓		
q_4	✓	✓	✓		
q_5	✓	✓	✓	✓	✓
	q_0	q_1	q_2	q_3	q_4

~~$\delta(q_0,0), \delta(q_1,0) = (q_1, q_3)$~~

~~$\delta(q_0,0), \delta(q_2,0) = (q_1, q_4)$~~

$\delta(q_0,0)$, $\delta(q_3,0) = (q_1, q_5)$ ✓

$\delta(q_0,0)$, $\delta(q_4,0) = (q_1, q_5)$ ✓

$\delta(q_0,1)$, $\delta(q_3,1) = (q_2, q_5)$ ✓

$\delta(q_0,1)$, $\delta(q_4,1) = (q_2, q_5)$

$\delta(q_1,0)$, $\delta(q_2,0) = (q_3, q_4)$

$\delta(q_1,1)$, $\delta(q_2,1) = (q_4, q_3)$

$\delta(q_1,0)$, $\delta(q_5,0) = (q_3, q_5)$

$\delta(q_1,1)$, $\delta(q_5,1) = (q_4, q_5)$

$\delta(q_2,1)$, $\delta(q_5,1) = (q_3, q_5)$ ✓

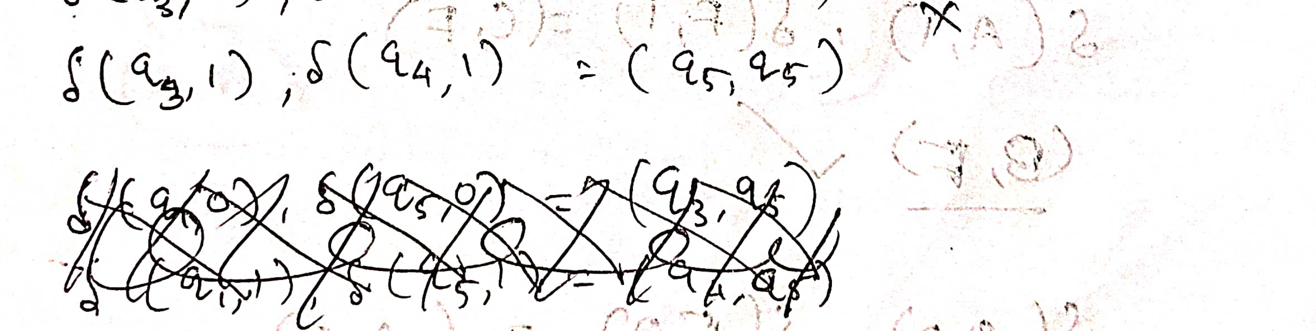
$\delta(q_2,0)$, $\delta(q_5,0) = (q_4, q_5)$ ✓

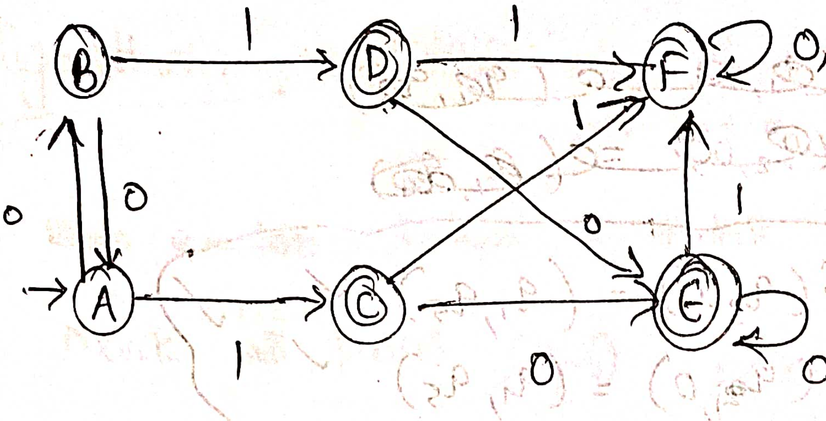
$\delta(q_3,0)$, $\delta(q_4,0) = (q_5, q_5)$

$\delta(q_3,1)$, $\delta(q_4,1) = (q_5, q_5)$

~~$\delta(q_1,0), \delta(q_5,0) = (q_3, q_5)$~~

~~$\delta(q_1,1), \delta(q_5,1) = (q_4, q_5)$~~





	A	B	C	D	E
B					
C	✓	✓			
D	✓	✓			
E	✓	~~X~~ ✓			
F	✓	✓	✓	✓	✓

(A, F) ✓

$\delta(A,0), \delta(F,0) = (B,F)$

$\delta(A,1), \delta(F,1) = (C,F)$

(B, F) ✓

$\delta(B,0), \delta(F,0) = (A,F)$

(C, E) X

$\delta(C,0), \delta(E,0) = (E,E)$

$\delta(C,1), \delta(E,1) = (F,F)$

$\underline{(D,E)}$ ✗

$\delta(D,0), \delta(E,0) = (E,E)$

$\delta(D,1), \delta(E,1) = (F,F)$

$\underline{(C,D)}$ ✗

$\delta(C,0), \delta(D,0) = (E,E)$

$\delta(C,1), \delta(D,1) = (F,F)$

$\underline{(A,B)}$ ✗

$\delta(A,0), \delta(B,0) = (B,A)$

$\delta(A,1)\ \delta(B,1) = (C,D)$

<u>Non-marked</u>

$(A,B), \underbrace{(C,D)(D,E),(C,E)}_{(C,D,E)}$