justifications to the agenda. As AI programs become more complex and their knowledge bases grow, this becomes a particularly significant advantage.

3.4 PROBLEM REDUCTION

So far, we have considered search strategies for OR graphs through which we want to find a single, path to a goal. Such structures represent the fact that we will know how to get from a node to a goal state if we can discover how to get from that node to a goal state along any one of the branches leaving it.

3.4.1 AND-OR Graphs

Another kind of structure, the AND-OR graph (or tree), is useful for representing the solution of problems that can be solved by decomposing them into a set of smaller problems, all of which must then be solved. This decomposition, or reduction, generates arcs that we call AND arcs. One AND arc may point to any number of successor nodes, all of which must be solved in order for the arc to point to a solution. Just as in an OR graph, several arcs may emerge from a single node, indicating a variety of ways in which the original problem might be solved. This is why the structure is called not simply an AND graph but rather an AND-OR graph. An example of an AND-OR graph (which also happens to be an AND-OR tree) is given in Fig. 3.6. AND arcs are indicated with a line connecting all the components.

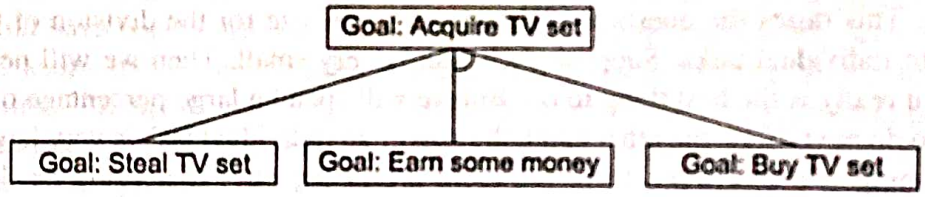

Fig. 3.6 *A Simple AND-OR Graph*

In order to find solutions in an AND-OR graph, we need an algorithm similar to best-first search but with the ability to handle the AND arcs appropriately. This algorithm should find a path from the starting node of the graph to a set of nodes representing solution states. Notice that it may be necessary to get to more than one solution state since each arm of an AND arc must lead to its own solution node.

To see why our best-first search algorithm is not adequate for searching AND-OR graphs, consider Fig. 3.7(a). The top node, A, has been expanded, producing two arcs, one leading to B and one leading to C and D. The numbers at each node represent the value of f' at that node. We assume, for simplicity, that every operation has a uniform cost, so each arc with a single successor has a cost of 1 and each AND arc with multiple successors has a cost of 1 for each of its components. If we look just at the nodes and choose for expansion the one with the lowest f' value, we must select C. But using the information now available, it would be better to explore the path going through B since to use C we must also use D, for a total cost of 9 (C + D + 2) compared to the cost of 6 that we get by going through B. The problem is that the choice of which node to expand next must depend not only on

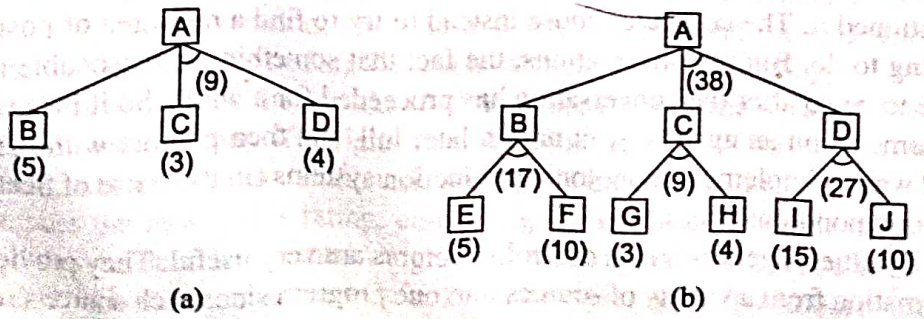

Fig. 3.7 *AND-OR Graphs*

the f' value of that node but also on whether that node is part of the current best path from the initial node. The tree shown in Fig. 3.7(b) makes this even clearer. The most promising single node is G with an f' value of 3. It is even part of the most promising arc G-H, with a total cost of 9. But that arc is not part of the current best path since to use it we must also use the arc I-J, with a cost of 27. The path from A, through B, to E and F is better, with a total cost of 18. So we should not expand G next; rather we should examine either E or F.

In order to describe an algorithm for searching an AND-OR graph we need to exploit a value that we call *FUTILITY*. If the estimated cost of a solution becomes greater than the value of *FUTILITY*, then we abandon the search. *FUTILITY* should be chosen to correspond to a threshold such that any solution with a cosflibove it is too expensive to be practical, even if it could ever be found. Now we can state the algorithm.

Algorithm: Problem Reduction

1. Initialize the graph to the starting node.
2. Loop until the starting node is labeled *SOLVED* or until its cost goes above *FUTILITY*:
   (a) Traverse the graph, starting at the initial node and following the current best path, and accumulate the set of nodes that are on that path and have not yet been expanded or labeled as solved.
   (b) Pick one of these unexpanded nodes and expand it. If there are no successors, assign *FUTILITY* as the value of this node. Otherwise, add its successors to the graph and for each of them compute f' (use only h' and ignore g, for reasons we discuss below). If f' of any node is 0, mark that node as *SOLVED*.
   (c) Change the f' estimate of the newly expanded node to reflect the new information provided by its successors. Propagate this change backward through the graph. If any node contains a successor arc whose descendants are all solved, label the node itself as *SOLVED*. At each node that is visited while going up the graph, decide which of its successor arcs is the most promising and mark it as part of the current best path. This may cause the current best path to change. This propagation of revised cost estimates back up the tree was not necessary in the best-first search algorithm because only unexpanded nodes were examined. But now expanded nodes must be reexamined so that the best current path can be selected. Thus it is important that their f' values be the best estimates available.

This process is illustrated in Fig. 3.8. At step 1, A is the only node, so it is at the end of the current best path. It is expanded, yielding nodes B, C, and D. The arc to D is labeled as the most promising one emerging from A, since it costs 6 compared to B and C, which costs 9. (Marked arcs are indicated in the Fig.s by arrows.) In step 2, node D) is chosen for expansion. This process produces one new arc, the AND arc to E and F, with a combined cost estimate of 10. So we update the f' value of D to 10. Going back one more level, we see that this makes the AND arc B-C better than the arc to D, so it is labeled as the current best path. At step 3, we traverse that arc from A and discover the unexpanded nodes B and C. If we are going to find a solution along this path, we will have to expand both B and C eventually, so let's choose to explore B first. This generates two new arcs, the ones to G and to H. Propagating their f' values backward, we update f' of B to 6 (since that is the best we think we can do, which we can achieve by going through G). This requires updating the cost of the AND arc B-C to 12 (6 + 4 + 2). After doing that, the arc to D is again the better path from A, so we record that as the current best path and either node E or node F will be chosen for expansion at step 4. This process continues until either a solution is found or all paths have led to dead ends, indicating that there is no solution.

In addition to the difference discussed above, there is a second important way in which an algorithm for searching an AND-OR graph must differ from one for searching an OR graph. This difference, too, arises from the fact that individual paths from node to node cannot be considered independently of the paths through other nodes connected to the original ones by AND arcs. In the best-first search algorithm, the desired path

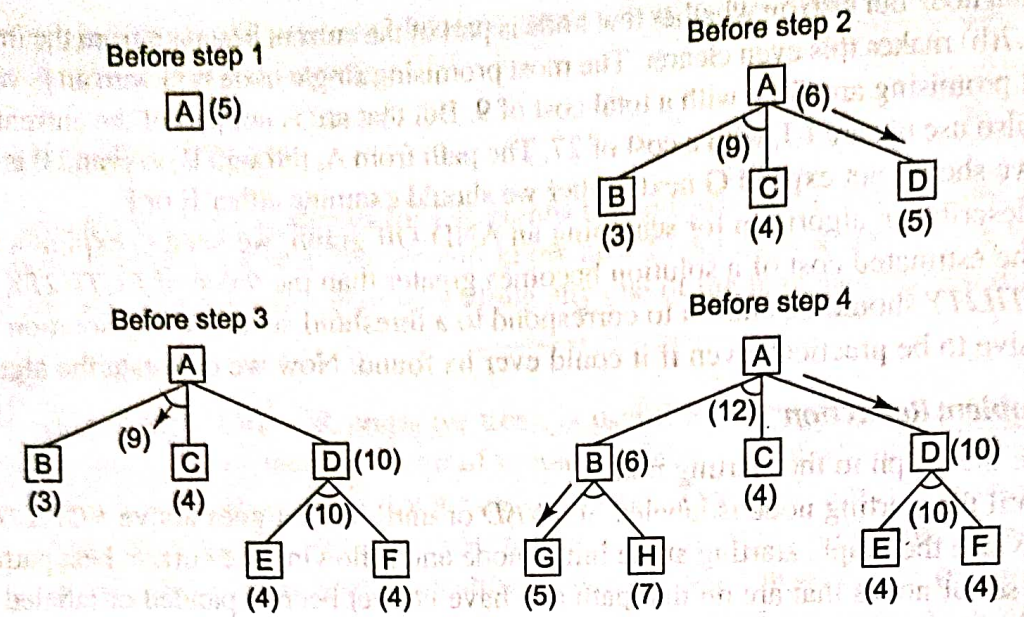

Fig. 3.8 *The Operation of Problem Reduction*

from one node to another was always the one with the lowest cost. But this is not always the case when searching an AND-OR graph.

Consider the example shown in Fig. 3.9(a). The nodes were generated in alphabetical order. Now suppose that node J is expanded at the next step and that one of its successors is node E, producing the graph shown in Fig. 3.9(b). This new path to E is longer than the previous path to E going through C. But since the path through C will only lead to a solution if there is also a solution to D, which we know there is not, the path through J is better.

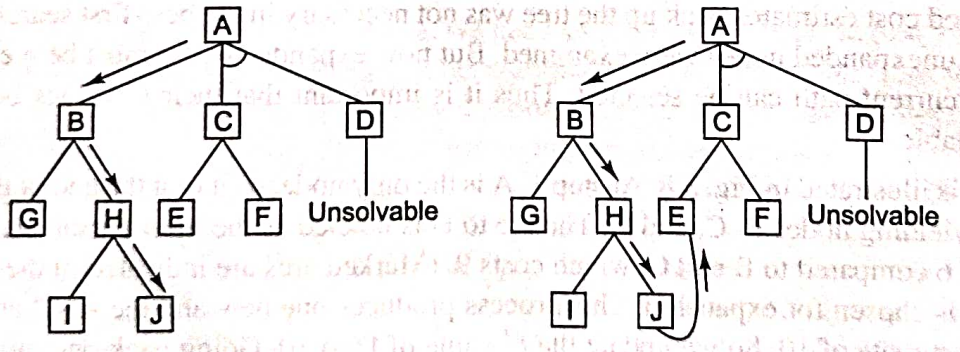

Fig. 3.9 *A Longer Path May Be Better*

There is one important limitation of the algorithm we have just described. It fails to take into account any interaction between subgoals. A simple example of this failure is shown in Fig. 3.10. Assuming that both node C and node E ultimately lead to a solution, our algorithm will report a complete solution that includes both of them. The AND-OR graph states that for A to be solved, both C and D must be solved. But then the algorithm considers the solution of D as a completely separate process from the solution of C. Looking just at the alternatives from D, E is the best path. But it turns out that C is necessary anyway, so it would be better also to use it to satisfy D. But since our algorithm does not consider such interactions, it will find a nonoptimal path. In Chapter 13, problem-solving methods that can consider interactions among subgoals are presented.

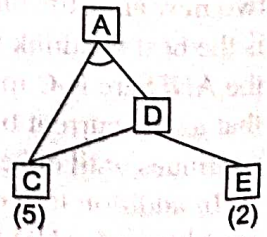

Fig. 3.10 *Interacting Subgoals*