Units

mmf ⟶ Ampere turns

ϕ ⟶ wb.

Reluctance ⟶ Ampere turns/wb

(B) mag. flux density — wb/m^2.

magnetising force — AT/m.

$\mu_0 = 4\pi \times 10^{-7}$

CHAPTER 2

MAGNETIC CIRCUITS

2.1 Magnet and Magnetism

Any body which possesses the power of attracting pieces of iron is known as a magnet and the property of a body by virtue of which this attraction takes place is known as magnetism. Magnetism is one of nature's forms of energy. Some other forms are heat, light, sound and electricity. Natural magnetism is found in lodestone, a type of iron ore, and the earth's vast magnetic field. Lodestone possesses magnetism when it is taken out from the earth, so it is known as a natural magnet. Lodestone was found in Magnesia (Upper part of Greece). Chinese later discovered that a slender piece of lodestone, if suspended freely, would orient itself approximately in the direction of the earth's geographical North-South direction.

2.2 Magnetic Materials

Magnetic materials are classified into three categories. (1) Diamagnetic (2) Paramagnetic and (3) Ferromagnetic.

1. Diamagnetic materials

A diamagnetic material, when placed in a magnetic field, gets a feeble induced magnetic field in a direction opposite to that of the external magnetic field. It has a tendency to move from the region where the field is strong to the region where the field is week. Its permeability is less than unity. Temperature has no effect on the diamagnetic properties of a material.

E.g. Bismuth, Lead, Copper, Zinc, Silver.

2. Paramagnetic materials

If a paramagnetic material is placed in a magnetic field, the magnetic field within the material gets enhanced. When placed in a non uniform external field, it tends to move from the low to the high field region. The behaviour of paramagnetic material is opposite to that of a diamagnetic material. These materials have a permanent magnetic dipole moment. Paramagnetism decreases with rise in temperature.

E.g. Aluminium, Sodium, Copper Chloride

must two note.

3. Ferromagnetic Materials

A ferromagnetic material gets magnetized even by weak magnetic field. A large magnetization occurs in the direction of the external magnetic field. When placed in a non uniform field, it is attracted towards the stronger magnetic field regions. Ferromagnetism decreases with rise in temperature.

E.g., Iron, Nickel, Cobalt

2.3 Classification of Magnets

Magnets can be broadly classified into following categories.

1. Permanent magnets
2. Electromagnets

Permanent magnets retain their magnetism after the magnetic field used to magnetize them has been removed. These are of two types-natural and artificial.

Natural magnets, are comparatively weak, of irregular shape and of arbitrary size. These cannot be effectively employed to serve useful purposes.

The materials such as special steel, hardened steel and alloys can be transformed into permanent magnets. These are termed as artificial magnets.

If the magnetization is done by passing electric current in a coil surrounding the material, the magnet is termed as electromagnet.

2.4 Magnetic Induction

The phenomenon by which a magnetic substance becomes magnet when it is placed near a magnet is called magnetic induction.

note

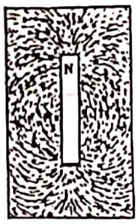
A magnet has two distinctive properties; (i) Attraction and repulsion for magnetic substances and other magnets (ii) The two ends of a magnet are called north (N) and south (S) poles.

ii A magnet will point towards North and South when freely suspended in air.

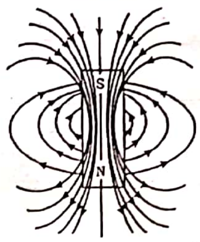
2.5 Magnetic field

It is the space or region around a magnet with in which the magnetic effect can be detected. Suppose that a bar magnet is placed on a plane surface and iron-filings are sprinkled on the surface around the magnet. On tapping the plane gently iron filings are found to arrange themselves in a certain pattern. This proves that in the space around the magnet the influence of the magnet exists. It is further important to observe that the iron filings arrange themselves along certain lines as shown in figure. These lines are called magnetic field lines from closed loops, leaving the magnet at its north pole and entering at its south pole.

A magnet field line is a imaginary line in space. The field lines are closed loops running from North pole to South pole through the air and from South pole to North Pole through the magnet.



(a) Pattern of iron fillings under the influence of a magnet



(b) Magnetic field lines for a bar magnet

2.6 Magnetic flux

Magnetic flux represents the total number of magnetic lines of force in a magnetic field. Magnetic flux is denoted by Greek letter phi (ϕ). The unit of magnetic flux is Weber[Wb].

A north pole of pole strength m wb would radiate a magnetic flux of m wb.

2.7 Magnetic flux density

The flux passing through a unit area through a plane at right angles to the plane is defined as magnetic flux density. It is denoted by the letter B. The unit of magnetic flux density is Tesla (T) or wb/m^2.

If ϕ webers of magnetic flux is passing normally through an area of A metre^2. the magnetic flux density B is given by

$$B = \frac{\phi}{A} \text{ wb/m}^2$$

2.8 Permeability

The ability of a material to conduct magnetic flux through it is called permeability (Absolute permeability) of that material. It is denoted by μ (mu). The permeability of free space or air is denoted by μ_0 and its value in S.I. unit is $4\pi \times 10^{-7}$Henry/metre.

2.9 Relative permeability

Relative permeability of a material relative to that of free space or air. It is denoted by μ_r.

$$\text{Relative permeability} = \frac{\text{Absolute permeability of the material}}{\text{Permeability of air}}$$

$$\text{i.e., } \mu_r = \frac{\mu}{\mu_0}$$

$$\text{or } \mu = \mu_0 \mu_r$$

For air, relative permeability is 1.

2.10 Magnetic field Intensity or Magnelizing force / magnetising field

Magnetic field Intensity at any point in a magnetic field is the force experienced by a unit north pole placed at that point. It is denoted by the letter H. The unit of H is N/wb.

Relation between B, H and μ

Consider a bar of magnetic material placed in a uniform magnetic field of strength H N/wb resulting in magnetic flux density B wb/m^2 in the bar. Then the absolute permeability of the material of the bar is

$$\mu = \frac{B}{H}$$

$$H = \frac{NI}{l} = \frac{mmf}{l}$$

$$\text{Or } B = \mu H = \mu_0 \mu_r H$$

2.11 Magneto motive force (MMF)

The magnetic pressure which sets up magnetic flux in a magnetic circuit is called Magneto motive force.

Consider a coil N turns carrying a current of I amperes.

The mmf is given by

M.M.F = Number of turns in the coil$\times$Current = NI Amp. turns.

The unit of mmf is ampere-turns (AT).

2.12 Reluctance

The opposition offered to the magnetic lines of force in a magnetic circuit is called reluctance. It is denoted by the letter S.

Reluctance is analogous to resistance in an electric circuit. The unit of reluctance is AT/wb.

$$\text{Reluctance (S)} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$$

Where

l = Length of the magnetic path
μ_0 = Permeability of free space
μ_r = Relative permeability
A = Cross-sectional area.

2.13 Permeance

It is the reciprocal of reluctance. It is expressed in wb/AT.

$$\text{Permeance} = \frac{1}{\text{reluctance}} = \frac{\text{Magnetic flux}}{\text{mmf}}$$

$$\text{Or Reluctance} = \frac{\text{MMF}}{\text{Magnetic flux}}$$

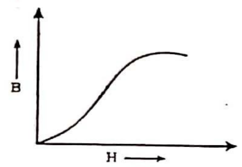
$R = \frac{V}{I}$

2.14 Comparison between Magnetic and Electric Circuits

Magnetic circuit	Electric Circuit
1. Magnetic flux = $\frac{\text{MMF}}{\text{Reluctance}}$	Current = $\frac{\text{EMF}}{\text{Resistance}}$
2. Reluctance (s) = $\frac{l}{\mu A}$	Resistance (R) = $\frac{\rho l}{A} = \frac{l}{\sigma A}$
3. Permeance = $\frac{1}{\text{Reluctance}}$	Conductance = $\frac{1}{\text{Rsistance}}$

2.15 Magnetization curve or B.H curve

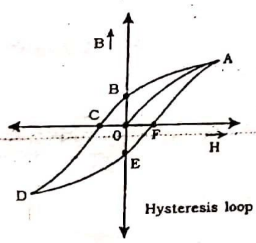
It is the graph between the flux density (B) and magnetizing force (H) of a magnetic material. The following figure shows the B-H curve for cast iron.



Initially H increase rapidly with B upto a certain point. The rate of increase of B becomes very low and finally reaches steady value. i.e., Saturation of the material takes place.

2.16 Magnetic Hysteresis

It is the lagging of flux density (B) behind the magnetic force (H) in a magnetic material.



Hysteresis loop

Let a magnetizing force H be applied to an unmagnified bar of iron. Let the magnetizing force H be increased from zero to a certain maximum value and then gradually reduced to zero. The curve OAB is obtained. At the point B, H becomes zero. But there is still some magnetization left in the material. The flux density B does not fall to zero. The remaining value of B at the point is called residual magnetism. The power of retaining the magnetism is called 'retentivity' of the material.

The value of H is reversed , B reduces and becomes zero at the point C. The value of H required to wipe off residual magnetism is known as 'Coercive force'. OC is the coercive force as shown in the figure. Further increase of H in the negative direction curve CD is obtained. Now the gradual decrease of H to zero and then a build up in the original direction is described by the curve DEFA.

As H is increased and then decreased to its original values, the flux density B does not return to its original value. This fact is called 'hysteresis'. The curve ABCDEFA is called hysteresis loop.

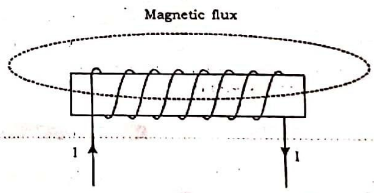
2.17 Magnetic Circuit

A magnetic circuit is a closed path followed by magnetic lines of force.

In electromagnet there are two circuits or path, one for the current through the coil and one for the flux through the core.

Simple Magnet Circuit

The figure shows a solenoid having iron core and N turns of winding. The current through the winding is I. The mmf in the circuit is NI and this mmf sets up a magnetic field in the solenoid.



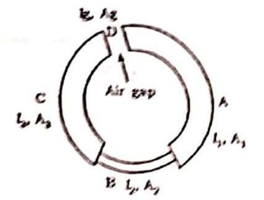
The length of the iron core is l metre and cross-sectional area A m^2.

Reluctance = $\frac{l}{\mu A}$

$\frac{B}{H} = \mu$

Field intensity inside the solenoid(H) = $\frac{NI}{l}$

2.18 Composite Magnetic Circuit



In several cases, the magnetic circuit consists of several regions of different materials and different dimensions. The figure shows four regions A, B, C, D of lengths l_1, l_2, l_3 and l_g respectively. The area of cross-sections are A_1, A_2, A_3 and A_g respectively. Each of these regions has different reluctance. Then the total reluctance of the circuit is the sun of individual reluctances.

$$\text{Total reluctance} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3} + \frac{l_g}{\mu_0 A_g}$$

where μ_1, μ_2, μ_3 and μ_0 are the permeability of A, B, C and air respectively.

$$\text{Flux } (\phi) = \frac{\text{MMF}}{\text{Total reluctance}}$$

Example 1

An iron ring having cross-sectional area of 400m m^2 and mean circumference of 500mm carries a coil of 250 turns wound uniformly around it. Calculate (a) Reluctance of the ring. (b) Current required to produce a flux of 1000 μ wb in the ring. Relative permeability of iron is 400.

Solution:

$$l = 500\text{mm} = 500 \times 10^{-3}\text{m},\ N=250$$

$$A = 400\text{mm}^2 = 400\times10^{-6}\ \text{m}^2,\ \mu_r = 400$$

$$\text{Reluctance (S)} = \frac{l}{\mu_0\mu_r A} = \frac{500\times10^{-3}}{4\pi\times10^{-7}\times400\times400\times10^{-6}} = 2487\times10^3\ \text{AT/wb}$$

$$\text{MMF} = \text{Flux} \times \text{Reluctance}$$

$$NI = \phi S$$

$\mu = \frac{B}{H}$

$$\text{Hence} \quad I = \frac{\phi S}{N} = \frac{1000\times10^{-6}\times2487\times10^3}{250} = 9.948\ \text{Amp}$$

Example 2

A mild steel ring has a mean circumference of 500mm and a uniform cross-sectional area of 300mm². Calculate the mmf required to produce a flux of 500μwb? Assume $\mu_r = 1200$.

Solution:

$$l = 500\text{mm} = 500\times10^{-3}\text{m}$$

$$A = 300\text{mm}^2 = 300\times10^{-6}\text{m}^2$$

$$\phi = 500\ \mu\text{wb} = 500\times10^{-6}\text{wb}$$

$$\mu_r = 1200$$

$$\text{Reluctance (S)} = \frac{l}{\mu_0\mu_r A} = \frac{500\times10^{-3}}{4\pi\times10^{-7}\times1200\times300\times10^{-6}}$$

$$S = 1105242.\text{AT/wb}$$

$$\text{MMF} = \text{Flux}\times\text{Reluctance} = \phi\times S = 500\times10^{-6}\times1105242.6 = 552.62\ \text{AT}$$

Example 3

Six ampere current is flowing in a solenoid wound with 1200 turns of wire. If the length of the solenoid is 160cm. Calculate field strength of the solenoid?

Solution: $I = 6$ A, $N = 1200$, $l = 160\text{cm} = 1.6\text{m}$

$$\text{Field Strength (H)} = \frac{NI}{l} = \frac{1200\times6}{1.6} = 4500\ \text{AT/m}$$

Example 4

An iron ring of mean length 60cm has an air gap of 2mm and a winding of 300 turns. If the relative permeability of iron used in the ring is 400 when a current of 1.5 A flows through it, find the flux density?

Solution:

Left the flux density be B wb/m²

MMF required for the iron ring is given by

$$\text{MMF}_1 = H_1 l_1 = \frac{B l_1}{\mu_0\mu_r} = \frac{B\times0.6}{\mu_0\times400}$$

MMF required for the air gap is given by,

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$$MMF_2 = H_2 l_2 = \frac{B l_2}{\mu_0 \mu_r} = \frac{B \times 2 \times 10^{-3}}{\mu_0 \times 1} = \frac{B \times 2 \times 10^{-3}}{\mu_0}$$

$$\text{Total MMF} = MMF_1 + MMF_2 = \frac{B \times 0.6}{\mu_0 \times 400} + \frac{B \times 2 \times 10^{-3}}{\mu_0} = \frac{B}{\mu_0}\left[\frac{0.6}{400} + 2 \times 10^{-3}\right]$$

$$\text{Total MMF} = \frac{B}{\mu_0}\left[\frac{0.6}{400} + 2 \times 10^{-3}\right] \quad \text{..........(1)}$$

$$\text{AT provided by current} = 1.5 \times 300 \quad \text{..........(2)}$$

Equating (1) and (2) we get

$$1.5 \times 300 = \frac{B}{\mu_0}\left[\frac{0.6}{400} + 2 \times 10^{-3}\right]$$

Hence, $B = \mathbf{0.1615\ wb/m^2}$

Example 5

A circular iron ring has circular cross sectional area of 12 cm² and length 15 cm in iron. An air gap of 1 mm is made by a sawcut. Find the ampere-turns needed to produce a flux of 24.84 μ wb. Relative permeability of iron is 800. Neglect leakage and fringing?

Solution:

$$\phi = 24.84\ \mu wb = 24.84 \times 10^{-6}\ wb$$

$$A = 12\ cm^2 = 12 \times 10^{-4}\ m^2$$

$$B = \frac{\phi}{A} = \frac{24.84 \times 10^{-6}}{12 \times 10^{-4}} = 0.0207\ wb/m^2$$

(a) For air gap

$$H_a = \frac{B}{\mu_0} = \frac{0.0207}{4\pi \times 10^{-7}}$$

$$H_a = 16472.5\ AT/m$$

$$MMF_a = H_a \cdot l_a = 16472.5 \times 1 \times 10^{-3} = \mathbf{16.4725\ AT}$$

(b) Iron ring

$$H_i = \frac{B}{\mu_0 \cdot \mu_r} = \frac{0.0207}{4\pi \times 10^{-7} \times 800}$$

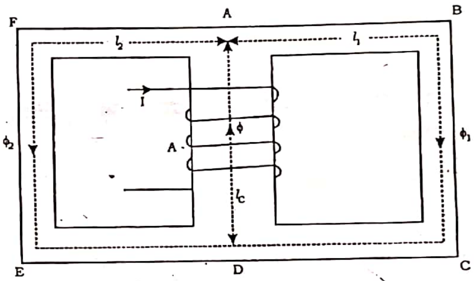
$$H_i = 20.59\ AT/m$$

$$MMF_i = H_i l_i = 20.59 \times 0.15 = \mathbf{3.088\ AT}$$

Total ampere turns needed $= MMF_a + MMF_i = 16.4725 + 3.088 = \mathbf{19.56\ AT}$

2.19 Parallel Magnetic Circuit

A magnetic circuit which has two or more than two paths for the magnetic flux is called a parallel magnetic circuit.



The figure shows a magnetic circuit excited by a current of 1 ampere passing through N turns of a coil placed in the central limb. The MMF = NI creates a flux of ϕ weber in the central limb of length l_c metre. The flux gets divided into two fluxes ϕ_1 and ϕ_2 in the outer limbs of lengths l_1 and l_2 metre, respectively.

Thus $\boxed{\phi = \phi_1 + \phi_2}$

The two magnetic paths ABCD and AFED are in parallel. The ampere turns required for this parallel circuit is equal to the ampere turns required for any one of the paths.

Total MMF required = M.M.F. required for path AD + M.M.F. required for path ABCD or AFED

$= \phi S + \phi_1 S_1$ or $\phi S + \phi_2 S_2$

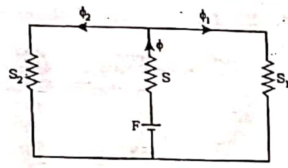
where S is the reluctance of the central limb

S_1 is the reluctance of the path ABCD

S_2 is the reluctance of the path AFED

$$\boxed{S = \frac{l_c}{\mu_0 \mu_{rc} \cdot A_c}, \quad S_1 = \frac{l_1}{\mu_0 \mu_{r1} \cdot A_1} \quad \& \quad S_2 = \frac{l_2}{\mu_0 \mu_{r2} \cdot A_2}}$$

The equivalent circuit of the magnetic circuit shown in figure is as follows.



From the equivalent circuit, the total reluctance of the magnetic path S_{total} may be computed as

$$S_{total} = S + S_1 \| S_2 = S + \frac{S_1 S_2}{S_1 + S_2}$$

and the flux ϕ may be computed as

$$\phi = \frac{\text{MMF}}{\text{Reluctance}} = \frac{NI}{S_{total}}$$

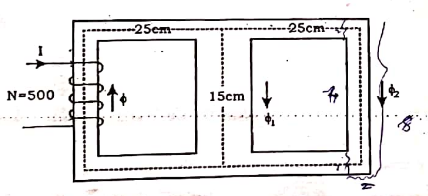
If S_1, S_2 and S_3 are three reluctances in parallel, then the equivalent reluctance S_{eq} is given by

$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3}$$

Example 1

A cast steel magnetic structure made for a bar of section 8 cm × 2 cm is shown in figure. Determine the current that the 500 turn magnetising coil on the left limb should carry so that a flux of 2 m Wb is produced in the right limb. Take μ_r = 600 and neglect leakage.

2 2 2'



Solution :

Magnetic circuits through middle and right limb are parallel. Therefore the M.M.F. across the two is the same.

$$\phi_1 S_1 = \phi_2 S_2$$

$$\phi_2 = 2 \text{ m wb} = 2 \times 10^{-3} \text{ wb}$$

$$\phi_1 \times \frac{15 \times 10^{-2}}{\mu A} = 2 \times 10^{-3} \times \frac{25 \times 10^{-2}}{\mu A}$$

$$\therefore \quad \phi_1 = 3.33 \text{ m wb}$$

$$\phi = \phi_1 + \phi_2 = 3.33 + 2 = 5.33 \text{ m Wb}$$

Total ampere turns (AT) required for the whole circuit equal to the sum of

(i) AT required for the left limb and

(ii) AT for middle or right limb

$$\text{AT required for the left limb} = \phi S = \phi = \frac{l}{\mu_0 \mu_r A}$$

$$= 5.33 \times 10^{-3} \times \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$$

$$= 4420.9$$

$$\text{AT required for the right limb} = 2 \times 10^{-3} \times \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$$

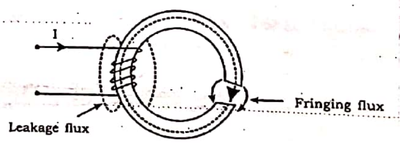
$$= 1657.86$$

$$\text{Total AT} = 4420.9 + 1657.86 = 6078.76$$

$$\text{Current, I} = \frac{6078.76}{500} = 12.15 \text{ A}$$

2.20 Leakage and Fringing in Magnetic circuit

The flux which does not follow the desired path in a magnetic circuit is called Leakage flux.



Consider an iron ring with air gap as shown in figure. Let a coil be wound on a portion of the ring. Let a current I flows through the coil. The complete flux produced in the ring by the current does not flow through the core. A small quantity of flux leaks through the air surrounding the iron ring. This flux is called Leakage flux. This leakage flux cannot be utilized for any purpose.

$$\text{Total flux set up} = \text{Useful flux} + \text{Leakage flux}$$

Leakage factor

It is defined as the ratio of total flux to the useful flux.

$$\text{Leakage factor } \lambda = \frac{\text{Total flux}}{\text{Useful flux}}$$

Fringing flux

Out of the total flux in air gap, a part of the flux diverges outside the main air gap. As a result, effective cross-sectional area of the air gap increases and the flux density in the air gap decreases. The diverging flux is called the **Fringing flux.**

Example 1

A circular iron ring having cross-sectional area of 20 cm^2 and length 30 cm in iron has an air gap of 2 mm made by a saw cut. Relative permeability of iron is 900. The ring is wound with a coil of 2500 turns and the current in the coil is 3A. Determine the air gap flux. Given that the leakage coefficient is 1.1.

Solution

Let the flux density through the iron be B. Then the flux density through the air is B/1.1

(a) Air gap

$$H_a = \frac{B}{\mu_0} = \frac{B/1.1}{4\pi \times 10^{-7}} \text{ AT/m}$$

$$l_a = 2 \times 10^{-3} \text{ m}$$

$$MMF_a = H_a l_a = \frac{B \times 2 \times 10^{-3}}{1.1 \times 4\pi \times 10^{-7}}$$

$$MMF_a = 1446 B$$

(b) Iron ring

$$H_i = \frac{B}{\mu_0 \mu_r} = \frac{B}{4\pi \times 10^{-7} \times 900}$$

$$l_i = 0.3 \text{ m}$$

$$MMF_i = H_i l_i = \frac{B \times 0.3}{4\pi \times 10^{-7} \times 900}$$

$$MMF_i = 265 B$$

$$\text{Total MMF required} = MMF_a + MMF_i$$

$$= 1446 B + 265 B = 1711 B$$

$$\text{But AT applied} = NI = 2500 \times 3 = 7500$$

$$\text{Hence } 1711 B = 7500$$

$$B = \frac{7500}{1711} = 4.38 \text{ wb/m}^2$$

$$\text{Area of cross section, A} = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$$

$$\text{Flux through the air gap} = \frac{BA}{1.1} = \frac{4.38 \times 20 \times 10^{-4}}{1.1} = \mathbf{7.96 \times 10^{-3} \text{ wb}}$$

2.21 Force Experienced by Current Carrying Conductor in a Magnetic Field

When a conductor of length 'l' metre is placed at right angles to a magnetic field, it experiences a force F given by,

$$F = BIl \text{ Newton}$$

Where

I = Current through the conductor in ampere

B = Flux density in Wb/m^2.

If the conductor is placed at an angle 'θ' to the direction of the magnetic field, the force is given by

$$\boxed{F = BIl \sin\theta}$$

Example 1

A straight conductor 2m long carries 30 A current and lies perpendicular to a uniform field of 0.5 wb/m^2. Find the force on the current carrying conductor?

Solution:

$$l = 2\text{m}, I = 30\text{A}, B = 0.5 \text{wb/m}^2$$

$$F = BIl = 0.5 \times 30 \times 2 = \mathbf{30N}$$

Example 2

A straight conductor 1.5m long carries a current of 80A and lies at right angles to a uniform field of flux density 2 wb/m^2. Find the force on the conductor when (i) It lies in the given position (ii) It lies a position such that it is inclined at an angle of 30° to the direction of the filed?

(i) $F = BIl = 2 \times 80 \times 1.5 = 240N$

(ii) $F = BIl \sin\theta = 2 \times 80 \times 1.5 \times \sin 30^\circ = \mathbf{120N}$

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$e = -L\frac{di}{dt}$

$v = iR + L\frac{di}{dt}$

$vi\,dt = i^2R + iL\,di$

2.22 ENERGY STORED IN MAGNETIC FIELD

Consider a coil having a constant inductance of L henry, in which the current grows at a uniform rate from zero to I amperes in 't' seconds. The induced emf in the coil is given by,

$$e = -L \times \frac{(I-0)}{t} = -\frac{LI}{t} \quad \text{.........(1)}$$

The component of applied voltage required to neutralise this induced emf will be equal to L I/t.If we assume that the current increases by di in dt seconds, then Eq.(1) becomes

$$e = -L\frac{di}{dt}$$

The applied voltage must balance the voltage dropacross resister R and neutralize the above indused emf, thus,

$$V = iR + L\frac{di}{dt}$$

Multiplying throughout by i.dt

$$V\,i\,dt = i^2R\,dt + L\,i\,di$$

where,

V i dt, is the energy supplied by the source in time dt.

i^2 R dt, the energy dissipated in the form of heat.

Li di, the energy absorbed by the inductance of the coil in building up the magnetic field.

Thus, energy absorbed by the magnetic field during the time dt second

= Li di Joules

Hence, total energy absorbed by the magnetic field when the current increases from zero to I amperes

$$= \int_0^I Li \cdot di = L\int_0^I i\,di$$

$$\text{Energy stored} = \frac{1}{2}LI^2$$

PROBLEMS

1. Find the inductance of an air - cored solenoid having a diameter of 6 cm. And a length of 40 cm, wound with 3000 turns.

Ans. Length of air cored solenoid $= 40 \times 10^{-2}$ m

Diameter of solenoid $= 6 \times 10^{-2}$ m

No. of turns $= 3000$

Then,

Cross-sectional area, $A = \pi r^2$

$$= \pi \times \left(\frac{6 \times 10^{-2}}{2}\right)^2 = 2.827 \times 10^{-3}\ m^2$$

Slef Inductance of solenoid 'L' $= N^2/S$

where 'N' is the number of turns and 'S' is the reluctance

$$S = \frac{l}{\mu_0\mu_r \times A} \qquad \mu_r = 1$$

$$\text{Inductance } L = \frac{3000^2 \times 4\pi \times 10^{-7} \times 1 \times 2.827 \times 10^{-3}}{40 \times 10^{-2}} = 0.0794\ H$$

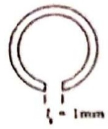
2. 12. A mild steel ring of 30 cm mean circumference has a cross sectional area of 6 cm^2 and has a winding of 500 turns on it. The ring is cut through at a point 50 as to provide an air gap of 1 mm in the magnetic circuit. It is found that a current of 4A in the winding produces a flues density of 1 T in the air gap. Find

(i) The relative permeability of the mild steel and

(ii) Inductance of the winding.

Ans. $l = 30 \times 10^{-2}$ m, $A = 6 \times 10^{-4}\ m^2$

$N = 500$, $l_g = 1 \times 10^{-3}$ m, $I = 4$ A



Air gap reluctance $S_g = \dfrac{l_g}{\mu_0\mu_r \times A}$

$$= \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 6 \times 10^{-4}} = 1326291.19\ AT/Wb$$

Core flux $\phi = B \times A = 1 \times 6 \times 10^{-4} = 6 \times 10^{-4}$ Wb

$$\text{Total reluctance } S = \frac{mmf}{\phi} = \frac{500 \times 4}{6 \times 10^{-4}} = 3333333.3\ AT/Wb$$

Reluctance of iron core, $S_i = S - S_g = 2007042.11$ AT/Wb

$$2007042.11 = \frac{30 \times 10^{-2} - 1 \times 10^{-3}}{4\pi \times 10^{-7} \times \mu_r \times 6 \times 10^{-4}}$$

$$\mu_r = 197.58$$

$$\text{Inductance of the winding} = \frac{N^2}{S} = 0.075\ H$$

3. An iron ring of mean diameter 100 cm and a cross sectional area of 6 cm^2 is wound with 200 turns of wire. Calculate the current required to produce a flux of 0.6 mWb in the ring, if the relative permeability of iron is 2000. If now a radial cut of 2 mm is made in the iron ring, find the new value of current required to produce the same flux in the air gap. Neglect fringing and leakage flux.

Ans.

Mean diamter of the ring (D) = 100 cm = 100×10^{-2} m

Length (l) = πD = $\pi \times 100 \times 10^{-2}$ m

Cross sectional area (A) = 6 cm^2 = 6×10^{-4} m^2

$N = 200$, $\mu_r = 2000$, $\phi = 0.6$ m Wb $= 0.6 \times 10^{-3}$ Wb

$$\text{Reluctance (s)} = \frac{l}{\mu_0\mu_r A} = \frac{\pi \times 100 \times 10^{-2}}{4\pi \times 10^{-7} \times 2000 \times 6 \times 10^{-4}} = 2083.3 \times 10^3 \text{ AT/Wb}$$

MMF = Flux × Reluctance

NI = $\phi \times S$

$$\text{Hence } I = \frac{\phi \times S}{N} = \frac{0.6 \times 10^{-3} \times 2083.3 \times 10^3}{200} = 6.249 \text{ A}$$

If now a radial cut of 2 mm is made in the iron ring. Find the new value of current required to produce the same flux in the air gap?

Mean length of the iron ring (l_i) = $\pi \times 100 \times 10^{-2} - 2 \times 10^{-3}$

= 3.139 m

Length of the air gap (l_a) = 0.002 m

$$\text{Reluctance of iron ring } (S_i) = \frac{l_i}{\mu_0\mu_r A} = \frac{3.139}{4\pi \times 10^{-7} \times 2000 \times 6 \times 10^{-4}}$$

$= 2081.614 \times 10^3$ AT/Wb

$$\text{Reluctance of air gap } (S_a) = \frac{l_a}{\mu_0\mu_r A} = \frac{0.002}{4\pi \times 10^{-7} \times 1 \times 6 \times 10^{-4}}$$

$= 2652.58 \times 10^3$ AT/ Wb

Total Reluctance (S) = $S_i + S_a$

$= 2081.614 \times 10^3 + 2652.58 \times 10^3$

$= 4734.19 \times 10^3$ AT/Wb

MMF = Flux × Reluctance

NI = $\phi \times S$

$$\text{Hence } I = \frac{\phi \times S}{N} = \frac{0.6 \times 10^{-3} \times 4734.19 \times 10^3}{200}$$

= 14.2 A

4. A flux of 0.5 mWb produced by a coil of 900 turns wound on a ring with a current of 3A in it. Calculate (i) the inductance of the coil (ii) the emf induced in the coil, when a current of 5A is switched off, assuming the current to fall to zero 1 millisecond.

Ans. $\phi = 0.5 \times 10^{-3}$ Wb, N = 900, I = 3A

$$\text{Inductance, } L = \frac{N\phi}{I}$$

$$= \frac{900 \times 0.5 \times 10^{-3}}{3} = 0.15 \text{ H}$$

If a current of 5 A changes to zero in 1 millisecond,

$$\text{emf induced, } e = L \times \frac{di}{dt} = 0.15 \times \frac{(5-0)}{1 \times 10^{-3}} = 750 \text{ V}$$

5. A magnetic core in the form of a closed ring has a mean length of 20 cm and a cross section of 1 cm^2. The relative permeability of iron is 2400. What direct current will be needed in a coil of 2000 turns uniformly wound round the ring to create a flux of 0.2 mwb in the iron.

Ans.

Mean length = 20×10^{-2} m

Area of cross section = 1×10^{-4} m^2

μ_r = 2400

N = 2000

ϕ = 0.2 mWb

$$\text{Reluctance 'S' of the core} = \frac{l}{\mu_0\mu_r \times A}$$

Inductance of the core = N^2/S

$$= \frac{N^2 \times \mu_0\mu_r A}{l}$$

$$= \frac{2000^2 \times \mu_0 \times 2400 \times 1 \times 10^{-4}}{20 \times 10^{-2}}$$

= 6.03 H

$$\text{Then, } I = \frac{N\phi}{L}$$

$$= \frac{2000 \times 0.2 \times 10^{-3}}{6.03}$$

= 0.066 A

PRACTICE PROBLEMS

1. A cast steel electromagnet has an air gap length of 3 mm and an iron path of length 40 cm. Find the number of ampere turns necessary to produce a flux density of 0.7 Wb/m^2 in the gap. Neglect leakage and fringing. [*Ans.* 1935 AT]
2. An iron ring has a cross-sectional area of 8×10^{-4} m^2 and a mean diameter of 38.2 cm. It is uniformly wound with a coil of 480 turns. When the coil is carrying a current of 2A, the flux set up in the ring is found to be 0.008 Wb. Find out the relative permeability of iron at this flux density. [*Ans.* 1250]
3. The airgap in a magnetic circuit is 1.5 mm long and 2500 mm^2 in cross section. Calculate (a) the reluctance of the air gap and (b) the mmf to send a flux of 800 μWb across the air gap.

   [*Ans.* (a) 0.477×10^6 AT/Wb, (b) 382 AT]
4. A solenoid 1 m in length and 10 cm in diameter has 5000 turns. Calculate the (a)

the approximate inductance (b) the energy stored in the magnetic field when a current of 2 A flows in the solenoid. [*Ans.* (a) 0.246 H, (b) 0.492 J]

5. A current of 2A is flowing through each of the conductors in a coil containing 15 such conductors. If a point pole of unit strength is placed at a perpendicular distance of 10 cm from the coil, determine the field intensity at that point.

[*Ans.* 47.74 AT/m]

6. A coil of 100 turns is wound uniformly over an isolator ring with a mean circumference 2 m and a uniform sectional area of 0.025 cm^2. If the coil carries current of 2A, calculate (a) the mmf of the circuit (b) magnetic field intensity (c)flux density and (d) total flux.

[*Ans.* (a) 200 AT (b) 100 AT/m (c) 0.12565 m Wb/m^2 (d) 3.14×10^{-9} Wb]

7. An iron ring of mean circumference 50 cm has an air gap of 1 mm. It is uniformly wound with a coil having 200 turns. If the relative permeability of iron is 300, when a current of 1A flows through the coil, calculate the flux density.

[*Ans.* 94.2 mWb/m^2]