2.4 Magnetic Induction

The phenomenon by which a magnetic substance becomes magnet when it placed near a magnet is called magnetic induction.

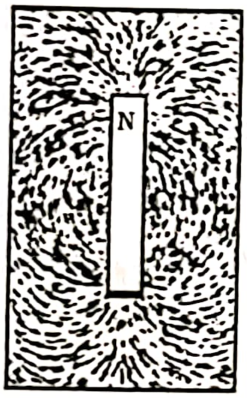
A magnet has two distinctive properties; (i) Attraction and repulsion to magnetic substances and other magnets (ii) The two ends of a magnet are called north (N) and south (S) poles.

A magnet will point towards North and South when freely suspended in air.

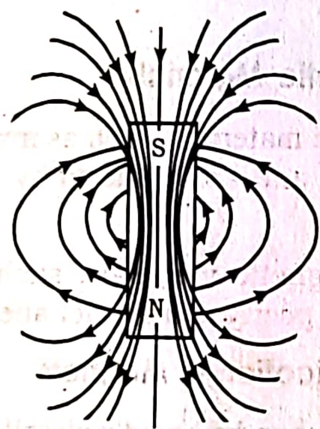
2.5 Magnetic field

It is the space or region around a magnet with in which the magnetic effect can be detected. Suppose that a bar magnet is placed on a plane surface and iron filings are sprinkled on the surface around the magnet. On tapping the plane gently iron filings are found to arrange themselves in a certain pattern. This proves that in the space around the magnet the influence of the magnet exists. It is further important to observe that the iron filings arrange themselves along certain lines as shown in figure. These lines are called magnetic field lines from closed loops, leaving the magnet at its north pole and entering at its south pole.

A magnet field line is a imaginary line in space. The field lines are closed loops running from North pole to South pole through the air and from South pole to North Pole through the magnet.



(a) Pattern of iron fillings under the influence of a magnet



(b) Magnetic field lines for a bar magnet

2.6 Magnetic flux

Magnetic flux represents the total number of magnetic lines of force in a magnetic field. Magnetic flux is denoted by Greek letter phi (ϕ). The unit of magnetic flux is Weber[Wb].

A north pole of pole strength m wb would radiate a magnetic flux of m wb.

2.7 Magnetic flux density

The flux passing through a unit area through a plane at right angles to the plane is defined as magnetic flux density. It is denoted by the letter B. The unit of magnetic flux density is Tesla (T) or wb/m^2.

If ϕ webers of magnetic flux is passing normally through an area of A $metre^2$, the magnetic flux density B is given by

$$B = \frac{\phi}{A} \ wb/m^2$$

2.8 Permeability

The ability of a material to conduct magnetic flux through it is called permeability (Absolute permeability) of that material. It is denoted by μ (mu). The permeability of free space or air is denoted by μ_0 and its value in S.I. unit is $4\pi \times 10^{-7}$ Henry/metre.

2.9 Relative permeability

Relative permeability of a material relative to that of free space or air. It is denoted by μ_r.

$$\text{Relative permeability} = \frac{\text{Absolute permeability of the material}}{\text{Permeability of air}}$$

$$\text{i.e., } \mu_r = \frac{\mu}{\mu_0}$$

$$\text{or } \mu = \mu_0 \mu_r$$

For air, relative permeability is 1.

2.10 Magnetic field Intensity or Magnetizing force

Magnetic field Intensity at any point in a magnetic field is the force experienced by a unit north pole placed at that point. It is denoted by the letter H. The unit of H is N/wb.

Relation between B, H and μ

Consider a bar of magnetic material placed in a uniform magnetic field of strength H N/wb resulting in magnetic flux density B wb/m^2 in the bar. Then the absolute permeability of the material of the bar is

$$\mu = \frac{B}{H}$$

$\mu = B/H$

$$\text{Or } B = \mu H = \mu_0 \mu_r H$$

2.11 Magneto motive force (MMF)

The magnetic pressure which sets up magnetic flux in a magnetic circuit is called Magneto motive force.

Consider a coil N turns carrying a current of 1 amperes.

The mmf is given by

M.M.F = Number of turns in the coil×Current = NI Amp. turns.

The unit of mmf is ampere-turns (AT).

2.12 Reluctance

The opposition offered to the magnetic lines of force in a magnetic circui called reluctance. It is denoted by the letter S.

Reluctance is analogous to resistance in an electric circuit. The uni reluctance is AT/wb.

$$\text{Reluctance (S)} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$$

Where

l = Length of the magnetic path

μ_0 = Permeability of free space

μ_r = Relative permeability

A = Cross-sectional area.

2.13 Permeance

It is the reciprocal of reluctance. It is expressed in wb/AT.

$$\text{Permeance} = \frac{1}{\text{reluctance}} = \frac{\text{Magnetic flux}}{\text{mmf}}$$

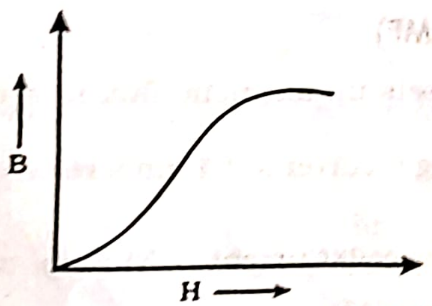
$$\text{Or} \quad \text{Reluctance} = \frac{\text{MMF}}{\text{Magnetic flux}}$$

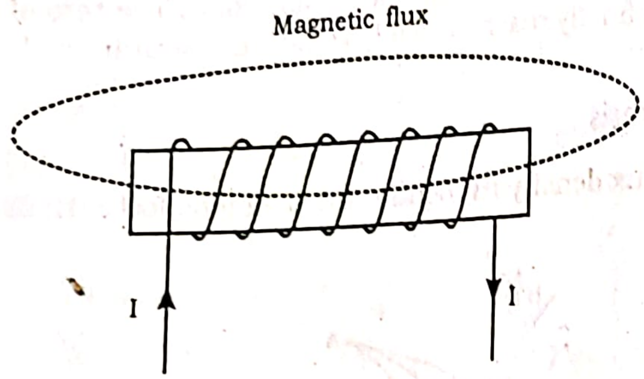
2.14 Comparison between Magnetic and Electric Circuits

Magnetic circuit	Electric Circuit
1. Magnetic flux $= \dfrac{\text{MMF}}{\text{Reluctance}}$	Current $= \dfrac{\text{EMF}}{\text{Resistance}}$
2. Reluctance (s) $= \dfrac{l}{\mu A}$	Resistance (R) $= \dfrac{\rho l}{A} = \dfrac{l}{\sigma A}$
3. Permeance $= \dfrac{1}{\text{Reluctance}}$	Conductance $= \dfrac{1}{\text{Rsistance}}$

2.15 Magnetization curve or B.H curve

It is the graph between the flux density (B) and magnetizing force (H) of a magnetic material. The following figure shows the B-H curve for cast iron.



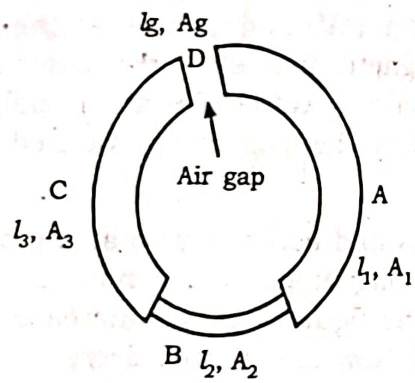


The length of the iron core is l metre and cross-sectional area A m^2.

$$\text{Reluctance} = \frac{l}{\mu A}$$

$$\text{Field intensity inside the solenoid(H)} = \frac{NI}{l}$$

2.18 Composite Magnetic Circuit



In several cases, the magnetic circuit consists of several regions of differer materials and different dimensions. The figure shows four regions A, B, C, D (lengths l_1, l_2, l_3 and l_g respectively. The area of cross-sections are A_1, A_2, A_3 and A respectively. Each of these regions has different reluctance. Then the tota reluctance of the circuit is the sun of individual reluctances.

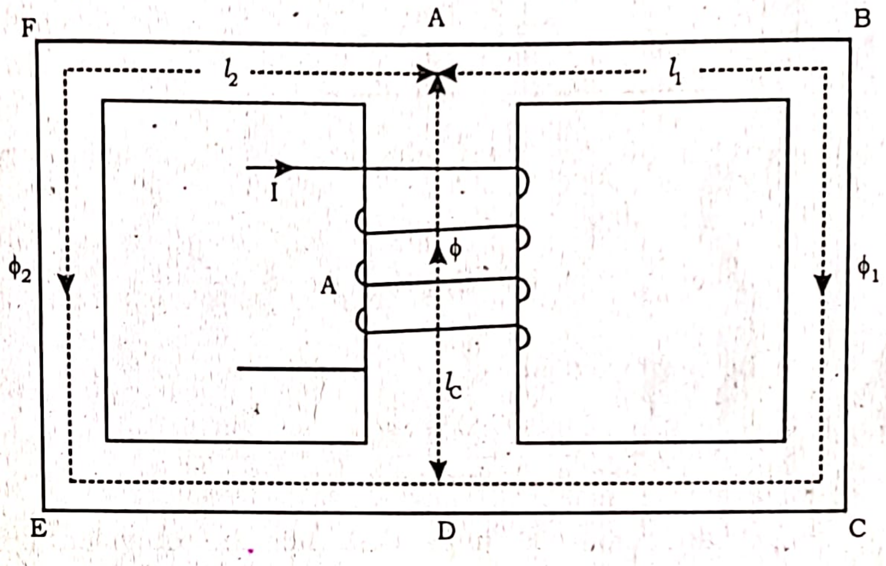
$$\text{Total reluctance} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3} + \frac{l_g}{\mu_0 A_g}$$

where μ_1, μ_2, μ_3 and μ_0 are the permeability of A, B, C and air respectively.

$$\text{Flux } (\phi) = \frac{\text{MMF}}{\text{Total reluctance}}$$

2.19 Parallel Magnetic Circuit

A magnetic circuit which has two or more than two paths for the magnetic flux is called a parallel magnetic circuit.



The figure shows a magnetic circuit excited by a current of 1 ampere passing through N turns of a coil placed in the central limb. The MMF = NI creates a flux of ϕ weber in the central limb of length l_C metre. The flux gets divided into two fluxes ϕ_1 and ϕ_2 in the outer limbs of lengths l_1 and l_2 metre, respectively.

Thus $$\phi = \phi_1 + \phi_2$$

The two magnetic paths ABCD and AFED are in parallel. The ampere turns required for this parallel circuit is equal to the ampere turns required for any one of the paths.

Total MMF required = M.M.F. required for path AD + M.M.F. required for path ABCD or AFED

$$= \phi S + \phi_1 S_1 \quad \text{or} \quad \phi S + \phi_2 S_2$$

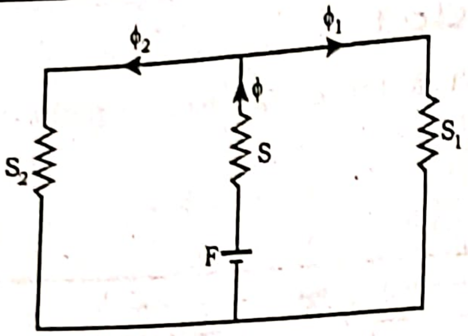
where S is the reluctance of the central limb

S_1 is the reluctance of the path ABCD

S_2 is the reluctance of the path AFED

$$S = \frac{l_C}{\mu_0 \mu_{rC} \cdot A_C}, \quad S_1 = \frac{l_1}{\mu_0 \mu_{r1} \cdot A_1} \quad \& \quad S_2 = \frac{l_2}{\mu_0 \mu_{r2} \cdot A_2}$$

The equivalent circuit of the magnetic circuit shown in figure is as follows.



From the equivalent circuit, the total reluctance of the magnetic path may be computed as

$$S_{total} = S + S_1 \parallel S_2 = S + \frac{S_1 S_2}{S_1 + S_2}$$

and the flux ϕ may be computed as

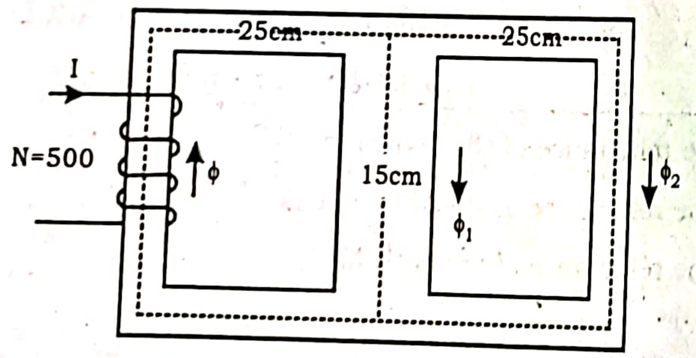
$$\phi = \frac{MMF}{Reluctance} = \frac{NI}{S_{total}}$$

If S_1, S_2 and S_3 are three reluctances in parallel, then the equivalent reluctance S_{eq} is given by

$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3}$$

Example 1

A cast steel magnetic structure made for a bar of section 8 cm × 2 cm is shown in figure. Determine the current that the 500 turn magnetising coil on the left limb should carry so that a flux of 2 m Wb is produced in the right limb. Take μ_r = 600 and neglect leakage.



Solution :

Magnetic circuits through middle and right limb are parallel. Therefore the M.M.F. across the two is the same.

$$\phi_1 S_1 = \phi_2 S_2$$

$$\phi_2 = 2 \text{ m wb} = 2 \times 10^{-3} \text{ wb}$$

$$\phi_1 \times \frac{15 \times 10^{-2}}{\mu A} = 2 \times 10^{-3} \times \frac{25 \times 10^{-2}}{\mu A}$$

$$\therefore \quad \phi_1 = 3.33 \text{ m wb}$$

$$\phi = \phi_1 + \phi_2 = 3.33 + 2 = 5.33 \text{ m Wb}$$

Total ampere turns (AT) required for the whole circuit equal to the sum of

(i) AT required for the left limb and

(ii) AT for middle or right limb

$$\text{AT required for the left limb} = \phi S = \phi = \frac{l}{\mu_0 \mu_r A}$$

$$= 5.33 \times 10^{-3} \times \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$$

$$= 4420.9$$

$$\text{AT required for the right limb} = 2 \times 10^{-3} \times \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$$

$$= 1657.86$$

$$\text{Total AT} = 4420.9 + 1657.86 = 6078.76$$

$$\text{Current, } I = \frac{6078.76}{500} = 12.15 \text{ A}$$

Example 2

An iron ring having cross-sectional area of 400m m² and mean circumference of 500mm carries a coil of 250 turns wound uniformly around it. Calculate (a) Reluctance of the ring. (b) Current required to produce a flux of 1000 μ wb in the ring. Relative permeability of iron is 400.

Solution:

$$l = 500\text{mm} = 500 \times 10^{-3}\text{m}, \; N=250$$

$$A = 400\text{mm}^2 = 400 \times 10^{-6} \text{ m}^2, \; \mu_r = 400$$

$$\text{Reluctance (S)} = \frac{l}{\mu_0 \mu_r A} = \frac{500 \times 10^{-3}}{4\pi \times 10^{-7} \times 400 \times 400 \times 10^{-6}} = 2487 \times 10^3 \text{ AT/wb}$$

$$\text{MMF} = \text{Flux} \times \text{Reluctance}$$

$$NI = \phi S$$

Hence $I = \dfrac{\phi S}{N} = \dfrac{1000 \times 10^{-6} \times 2487 \times 10^{-3}}{250} = \mathbf{9.948 \times 10^{-6}\ Amp}$

Example 3

A mild steel ring has a mean circumference of 500mm and a uniform cr sectional area of 300mm^2. Calculate the mmf required to produce a flux of 500μ Assume $\mu_r = 1200$.

Solution:

$$l = 500mm = 500 \times 10^{-3}m$$
$$A = 300mm^2 = 300 \times 10^{-6}m^2$$
$$\phi = 500\ \mu wb = 500 \times 10^{-6} wb$$
$$\mu_r = 1200$$

$$\text{Reluctance (S)} = \frac{l}{\mu_0 \mu_r A} = \frac{500 \times 10^{-3}}{4\pi \times 10^{-7} \times 1200 \times 300 \times 10^{-6}}$$

$$S = 1105242.AT/wb$$

MMF = Flux × Reluctance $= \phi \times S = 500 \times 10^{-6} \times 1105242.6 = 552.62$ AT

Example 4

Six ampere current is flowing in a solenoid wound with 1200 turns of wire the length of the solenoid is 160cm. Calculate field strength of the solenoid?

Solution: I = 6 A, N = ~~1200, l~~= 160cm = 1.6m

$$\text{Field Strength (H)} = \frac{NI}{l} = \frac{1200 \times 6}{1.6} = 4500\ AT/m$$

Example 5

An iron ring of mean length 60cm has an air gap of 2mm and a winding of 3 turns. If the relative permeability of iron used in the ring is 400 when a curre of 1.5 A flows through it, find the flux density?

Solution:

Left the flux density be B wb/m^2

MMF required for the iron ring is given by

$$MMF_1 = H_1 l_1 = \frac{B l_1}{\mu_0 \mu_r} = \frac{B \times 0.6}{\mu_0 \times 400}$$

> always find MMF in terms of 'B' when we have to find to or more MMF's.

MMF required for the air gap is given by,

$$MMF_2 = H_2 l_2 = \frac{B l_2}{\mu_0 \mu_r} = \frac{B \times 2 \times 10^{-3}}{\mu_0 \times 1} = \frac{B \times 2 \times 10^{-3}}{\mu_0}$$

$$\text{Total MMF} = MMF_1 + MMF_2 = \frac{B \times 0.6}{\mu_0 \times 400} + \frac{B \times 2 \times 10^{-3}}{\mu_0} = \frac{B}{\mu_0}\left[\frac{0.6}{400} + 2 \times 10^{-3}\right]$$

$$\text{Total MMF} = \frac{B}{\mu_0}\left[\frac{0.6}{400} + 2 \times 10^{-3}\right] \quad \text{..........(1)}$$

$$\text{AT provided by current} = 1.5 \times 300 \quad \text{..........(2)}$$

Equating (1) and (2) we get

$$1.5 \times 300 = \frac{B}{\mu_0}\left[\frac{0.6}{400} + 2 \times 10^{-3}\right]$$

Hence, $\quad$ **B = 0.1615 wb/m²**

Example 6

A circular iron ring has circular cross sectional area of 12 cm² and length 15 cm in iron. An air gap of 1 mm is made by a sawcut. Find the ampere-turns needed to produce a flux of 24.84 μ wb. Relative permeability of iron is 800. Neglect leakage and fringing?

Solution:

$$\phi = 24.84\ \mu\text{wb} = 24.84 \times 10^{-6}\ \text{wb}$$

$$A = 12\ \text{cm}^2 = 12 \times 10^{-4}\ \text{m}^2$$

$$B = \frac{\phi}{A} = \frac{24.84 \times 10^{-6}}{12 \times 10^{-4}} = 0.0207\ \text{wb/m}^2$$

(a) For air gap

$$H_a = \frac{B}{\mu_0} = \frac{0.0207}{4\pi \times 10^{-7}}$$

$$H_a = 16472.5\ \text{AT/m}$$

$$MMF_a = H_a \cdot l_a = 16472.5 \times 1 \times 10^{-3} = \mathbf{16.4725\ AT}$$

(b) Iron ring

$$H_i = \frac{B}{\mu_0 \cdot \mu_r} = \frac{0.0207}{4\pi \times 10^{-7} \times 800}$$

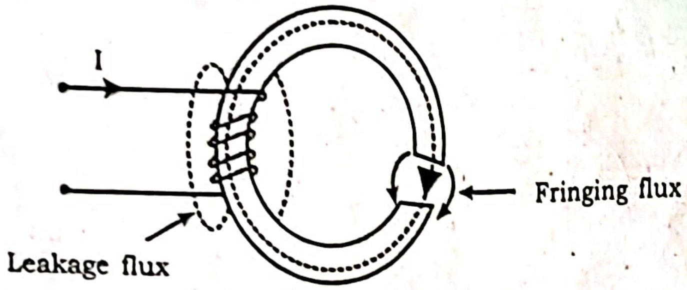
$$H_i = 20.59\ \text{AT/m}$$

$$\text{MMF}_i = H_i l_i = 20.59 \times 0.15 = \mathbf{3.088\ AT}$$

$$\text{Total ampere turns needed} = \text{MMF}_a + \text{MMF}_i = 16.4725 + 3.088 = \mathbf{19.56}$$

2.20 Leakage and Fringing in Magnetic circuit

The flux which does not follow the desired path in a magnetic circuit is call Leakage flux.



Consider an iron ring with air gap as shown in figure. Let a coil be wound on portion of the ring. Let a current I flows through the coil. The complete flux produce in the ring by the current does not flow through the core. A small quantity of flu leaks through the air surrounding the iron ring. This flux is called Leakage flu This leakage flux cannot be utilized for any purpose.

$$\text{Total flux set up} = \text{Useful flux} + \text{Leakage flux}$$

2.21 Leakage factor

It is defined as the ratio of total flux to the useful flux.

$$\text{Leakage factor } \lambda = \frac{\text{Total flux}}{\text{Useful flux}}$$

2.22 Fringing flux

Out of the total flux in air gap, a part of the flux diverges outside the main ai gap. As a result, effective cross-sectional area of the air gap increases and th flux density in the air gap decreases. The diverging flux is called the **Fringin flux.**

Example

A circular iron ring having cross-sectional area of 20 cm^2 and length 30 cm ir iron has an air gap of 2 mm made by a saw cut. Relative permeability of iron i 900. The ring is wound with a coil of 2500 turns and the current in the coil is 3A Determine the air gap flux. Given that the leakage coefficient is 1.1.

Solution

Let the flux density through the iron be B. Then the flux density through the air is B/1.1

(a) Air gap

$$H_a = \frac{B}{\mu_0} = \frac{B/1.1}{4\pi\times10^{-7}} \text{ AT/m}$$

$$l_a = 2 \times 10^{-3} \text{ m}$$

$$MMF_a = H_a l_a = \frac{B\times2\times10^{-3}}{1.1\times4\pi\times10^{-7}}$$

$$MMF_a = 1446\,B$$

(b) Iron ring

$$H_i = \frac{B}{\mu_0\mu_r} = \frac{B}{4\pi\times10^{-7}\times900}$$

$$l_i = 0.3 \text{ m}$$

$$MMF_i = H_i l_i = \frac{B\times0.3}{4\pi\times10^{-7}\times900}$$

$$MMF_i = 265\,B$$

$$\text{Total MMF required} = MMF_a + MMF_i$$

$$= 1446\,B + 265\,B = 1711\,B$$

$$\text{But AT applied} = NI = 2500 \times 3 = 7500$$

$$\text{Hence } 1711\,B = 7500$$

$$B = \frac{7500}{1711} = 4.38 \text{ wb/m}^2$$

$$\text{Area of cross section, } A = 20 \text{ cm}^2 = 20\times10^{-4} \text{ m}^2$$

$$\text{Flux through the air gap} = \frac{BA}{1.1} = \frac{4.38\times20\times10^{-4}}{1.1} = \mathbf{7.96 \times 10^{-3}\ wb}$$

Magnetic Materials

Magnetic materials are classified into three categories. (1) Diamagnetic (2) Paramagnetic and (3) Ferromagnetic.

2.23 Diamagnetic materials

A diamagnetic material, when placed in a magnetic field, gets a feeble induced magnetic field in a direction opposite to that of the external magnetic field. It has a tendency to move from the region where the field is strong to the region where the field is week. Its permeability is less than unity. Temperature has no effect on the diamagnetic properties of a material.

E.g. Bismuth, Lead, Copper, Zinc, Silver.

2.24 Paramagnetic materials

If a paramagnetic material is placed in a magnetic field, the magnetic fi within the material gets enhanced. When placed in a non uniform external fie it tends to move from the low to the high field-region. The behaviour paramagnetic material is opposite to that of a diamagnetic material. The materials have a permanent magnetic dipole moment. Paramagnetism decreas with rise in temperature.

E.g. Aluminium, Sodium, Copper Chloride

2.25 Ferromagnetic Materials

A ferromagnetic material gets magnetized even by weak magnetic field. large magnetization occurs in the direction of the external magnetic field. Wh placed in a non uniform field, it is attracted towards the stronger magnetic fie regions. Ferromagnetism decreases with rise in temperature.

E.g., Iron, Nickel, Cobalt

2.26 Force Experienced by Current Carrying Conductor in a Magnetic Field

When a conductor of length 'l' metre is placed at right angles to a magnet field, it experiences a force F given by,

$$F = BIl \text{ Newton}$$

Where I = Current through the conductor in ampere

B = Flux density in Wb/m^2.

If the conductor is placed at an angle 'θ' to the direction of the magnetic fiel the force is given by

$$F = BIl \sin\theta$$

Example 1

A straight conductor 2m long carries 30 A current and lies perpendicular to uniform field of 0.5 wb/m^2. Find the force on the current carrying conductor?

Solution:

$$l = 2m, I = 30A, B = 0.5 wb/m^2$$

$$F = BIl = 0.5 \times 30 \times 2 = \mathbf{30N}$$

Example 2

A straight conductor 1.5m long carries a current of 80A and lies at right angle to a uniform field of flux density 2 wb/m^2. Find the force on the conductor when (It lies in the given position (ii) It lies a position such that it is inclined at a angle of 30^0 to the direction of the filed?

(i) $F = BIl = 2 \times 80 \times 1.5 = 240N$

(ii) $F = BIl \sin\theta = 2 \times 80 \times 1.5 \times \sin 30^0 = \mathbf{120N}$

2.27 ENERGY STORED IN MAGNETIC FIELD

Consider a coil having a constant inductance of L henry, in which the current grows at a uniform rate from zero to I amperes in 't' seconds. The induced emf in the coil is given by,

$$e = -L \times \frac{(I-0)}{t} = -\frac{LI}{t} \quad ....(1)$$

The component of applied voltage required to neutralise this induced emf will be equal to L I/t.If we assume that the current increases by di in dt seconds, then Eq.(1) becomes

$$e = -L\frac{di}{dt}$$

The applied voltage must balance the voltage dropacross resister R and neutralize the above indused emf, thus,

$$V = iR + L\frac{di}{dt}$$

Multiplying throughout by i.dt

$$V\,i\,dt = i^2R\,dt + L\,i\,di$$

where,

V i dt, is the energy supplied by the source in time dt.

$i^2 R\,dt$, the energy dissipated in the form of heat.

Li di, the energy absorbed by the inductance of the coil in building up the magnetic field.

Thus, energy absorbed by the magnetic field during the time dt second

$$= Li\,di \text{ Joules}$$

Hence, total energy absorbed by the magnetic field when the current increases from zero to I amperes

$$= \int_0^I Li \cdot di = L\int_0^I i\,di$$

$$\text{Energy stored} = \frac{1}{2}LI^2$$

PROBLEMS

1. Find the inductance of an air - cored solenoid having a diameter of 6 cm. Ar length of 40 cm, wound with 3000 turns.

Ans. Length of air-cored solenoid $= 40 \times 10^{-2}$ m

Diameter of solenoid $= 6 \times 10^{-2}$ m

No. of turns $= 3000$

Then,

Cross-sectional area, $A = \pi r^2$

$$= \pi \times \left(\frac{6 \times 10^{-2}}{2}\right)^2 = 2.827 \times 10^{-3}\ m^2$$

Slef inductance of solenoid 'L' $= N^2/S$

where 'N' is the number of turns and 'S' is the reluctance

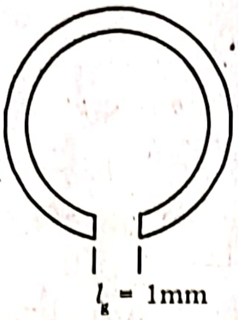
$$S = \frac{l}{\mu_0 \mu_r \times A} \qquad \mu_r = 1$$

$$\text{Inductance } L = \frac{3000^2 \times 4\pi \times 10^{-7} \times 1 \times 2.827 \times 10^{-3}}{40 \times 10^{-2}} = \mathbf{0.0794\ H}$$

2. 12. A mild steel ring of 30 cm mean circumference has a cross sectional area o cm² and has a winding of 500 turns on it. The ring is cut through at a point 50 to provide an air gap of 1 mm in the magnetic circuit. It is found that a current 4A in the winding produces a flues density of 1 T in the air gap. Find

(i) The relative permeability of the mild steel and

(ii) Inductance of the winding.



Ans. $l = 30 \times 10^{-2}$ m, $A = 6 \times 10^{-4}\ m^2$

$N = 500$, $l_g = 1 \times 10^{-3}$ m, $I = 4$ A

$$\text{Air gap reluctance } S_g = \frac{l_g}{\mu_0 \mu_r \times A}$$

$$= \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 6 \times 10^{-4}} = 1326291.19 \text{ AT/Wb}$$

$$\text{Core flux } \phi = B \times A = 1 \times 6 \times 10^{-4} = 6 \times 10^{-4} \text{ Wb}$$

$$\text{Total reluctance } S = \frac{mmf}{\phi} = \frac{500 \times 4}{6 \times 10^{-4}} = 3333333.3 \text{ AT/Wb}$$

$$\text{Reluctance of iron core, } S_i = S - S_g = 2007042.11 \text{ AT/Wb}$$

$$2007042.11 = \frac{30 \times 10^{-2} - 1 \times 10^{-3}}{4\pi \times 10^{-7} \times \mu_r \times 6 \times 10^{-4}}$$

$$\mu_r = 197.58$$

$$\text{Inductance of the winding} = \frac{N^2}{S} = 0.075 \text{ H}$$

3. An iron ring of mean diameter 100 cm and a cross sectional area of 6 cm^2 is wound with 200 turns of wire. Calculate the current required to produce a flux of 0.6 mWb in the ring, if the relative permeability of iron is 2000. If now a radial cut of 2 mm is made in the iron ring, find the new value of current required to produce the same flux in the air gap. Neglect fringing and leakage flux.

Ans. Mean diamter of the ring (D) $= 100\text{ cm} = 100 \times 10^{-2}$ m

Length (l) $= \pi D = \pi \times 100 \times 10^{-2}$ m

Cross sectional area (A) $= 6\text{ cm}^2 = 6 \times 10^{-4}\text{ m}^2$

$N = 200, \mu_r = 2000, \phi = 0.6\text{ m Wb} = 0.6 \times 10^{-3}\text{ Wb}$

$$\text{Reluctance (s)} = \frac{l}{\mu_0\mu_r A} = \frac{\pi \times 100 \times 10^{-2}}{4\pi \times 10^{-7} \times 2000 \times 6 \times 10^{-4}} = 2083.3 \times 10^3\text{ AT/Wb}$$

MMF = Flux × Reluctance

NI = $\phi \times S$

$$\text{Hence } I = \frac{\phi \times S}{N} = \frac{0.6 \times 10^{-3} \times 2083.3 \times 10^3}{200} = 6.249\text{ A}$$

If now a radial cut of 2 mm is made in the iron ring. Find the new value of current required to produce the same flux in the air gap?

Mean length of the iron ring (l_i) $= \pi \times 100 \times 10^{-2} - 2 \times 10^{-3}$

$= 3.139$ m

Length of the air gap (l_a) $= 0.002$ m

$$\text{Reluctance of iron ring } (S_i) = \frac{l_i}{\mu_0\mu_r A} = \frac{3.139}{4\pi \times 10^{-7} \times 2000 \times 6 \times 10^{-4}}$$

$= 2081.614 \times 10^3$ AT/Wb

$$\text{Reluctance of air gap } (S_a) = \frac{l_a}{\mu_0\mu_r A} = \frac{0.002}{4\pi \times 10^{-7} \times 1 \times 6 \times 10^{-4}}$$

$= 2652.58 \times 10^3$ AT/ Wb

Total Reluctance (S) $= S_i + S_a$

$= 2081.614 \times 10^3 + 2652.58 \times 10^3$

$= 4734.19 \times 10^3$ AT/Wb

MMF = Flux × Reluctance

NI = $\phi \times S$

$$\text{Hence } I = \frac{\phi \times S}{N} = \frac{0.6 \times 10^{-3} \times 4734.19 \times 10^3}{200}$$

$= 14.2$ A

4. A flux of 0.5 mWb produced by a coil of 900 turns wound on a ring with a current of 3A in it. Calculate (i) the inductance of the coil (ii) the emf induced in the coil, when a current of 5A is switched off, assuming the current to fall to zero 1 millisecond.

Ans. $\phi = 0.5 \times 10^{-3}$ Wb, N = 900, I = 3A

$$\text{Inductance, } L = \frac{N\phi}{I}$$

$$= \frac{900 \times 0.5 \times 10^{-3}}{I} = 0.15\ H$$

If a current of 5 A changes to zero in 1 millisecond,

$$\text{emf induced, } e = L \times \frac{di}{dt} = 0.15 \times \frac{(5-0)}{1 \times 10^{-3}} = 750\ V$$

5. A magnetic core in the form of a closed ring has a mean length of 20 cm an cross section of 1 cm^2. The relative permeability of iron is 2400. What direct curr will be needed in a coil of 2000 turns uniformly wound round the ring to creat flux of 0.2 mwb in the iron.

Ans.

Mean length $= 20 \times 10^{-2}$ m

Area of cross section $= 1 \times 10^{-4}\ m^2$

$\mu_r = 2400$

$N = 2000$

$\phi = 0.2$ mWb

$$\text{Reluctance 'S' of the core} = \frac{l}{\mu_0 \mu_r \times A}$$

$$\text{Inductance of the core} = N^2/S$$

$$= \frac{N^2 \times \mu_0 \mu_r A}{l}$$

$$= \frac{2000^2 \times \mu_0 \times 2400 \times 1 \times 10^{-4}}{20 \times 10^{-2}}$$

$$= 6.03\ H$$

$$\text{Then, } I = \frac{N\phi}{L}$$

$$= \frac{2000 \times 0.2 \times 10^{-3}}{6.03}$$

$$= 0.066\ A$$

CHAPTER 3

ELECTROMAGNETIC INDUCTION

3.1 Introduction

On August 29, 1831, a British scientist Michael Faraday (1771-1867) discovered the phenomenon of electromagnetic induction. After a variety of experiments he found that a moving magnetic field does give rise to an emf. Most commercial apparatus like motors, generators and transformers are based upon the principle of electromagnetic induction.

3.2 Faraday's Laws of electromagnetic induction

First law: 1 This law states that whenever there is a change in magnetic flux linked with a coil, an emf is induced in it.

Second law: 2 This law states that the magnitude of the induced emf is equal to the rate of change of magnetic flux linking with the circuit.

Consider a coil of 'N' turns. Let the flux linked with the coil change from ϕ_1 to ϕ_2 in 't' seconds.

$$\text{Initial flux linkages} = N\phi_1$$

$$\text{Final flux linkages} = N\phi_2$$

According to faraday's second law, The induced e.m.f,

$$|e| = \frac{(N\phi_2 - N\phi_1)}{t} \text{ volt}$$

Putting the above equation in its differential form,

$$|e| = N\frac{d\phi}{dt} \text{ volt}$$

3.3 Lenz's Law

The direction of the induced emf (or current) is given by Lenz's law. It states:

The direction of the induced current is such that it opposes the very cause, which produces it.

In the view of Lenz's law we can state Faraday's laws as,

$$e = -N\frac{d\phi}{dt}$$

Example 1

A coil of 1200 turns gives rise to a magnetic flux of 3mwb, when carryin certain current. If the current is reversed in 0.2 seconds, what is the aver value of emf induced in the coil.

Solution:

No. of turns of the coil, $N = 1200$

Change in flux linkages, $d\phi = 2\times3 = 6\text{mwb} = 6\times10^{-3}\text{wb}$

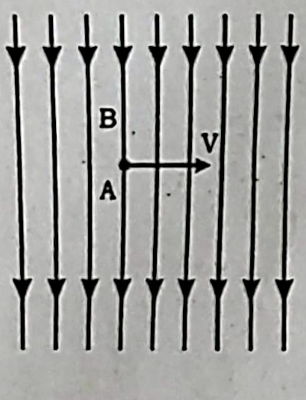
E.M.F induced in the coil, $e = N\left(\frac{d\phi}{dt}\right) = 1200\left(\frac{6\times10^{-3}}{0.2}\right) =$ **36 Volt**

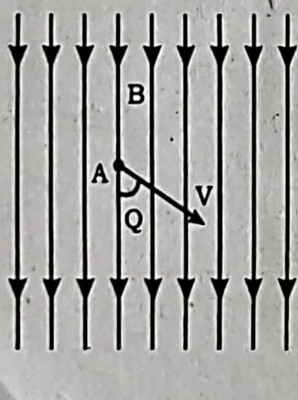
3.4 Induced E.M.F

Induced e.m.f can either be (i) dynamically induced or (ii) statically induc In the first case the field is stationary and the conductors cut across it. But in t second case the conductor remains stationary and the flux linked with it changed.

3.5 Dynamically induced E.m.f

Consider a conductor of length 'l' meter lying within a uniform magnetic fie of flux density B wb/m^2. Let it move through a distance dx in time dt.





Then the area swept by the conductor $= l \times dx$

Hence, flux cut by the conductor = flux density×area $= B \times l \times dx$ weber

According to Faraday's law, the induced emf in the conductor is given by

e = rate of change of flux linkages $= Bl(dx/dt) = BlV$ Volt

Where $V = (dx/dt)$ = velocity.

Thus dynamically induced e.m.f., $e = Blv$ volt.

If the conductor moves at an angle θ to the direction of the flux, the induced emf is given by

$$e = Blv \sin\theta \text{ Volt}$$

Example 2

A straight conductor of length 2 meters moves at right angle to a uniform magnetic field of flux density $2 wb/m^2$ with a uniform velocity of 40 meters/second. Calculate the induced emf in the conductor. Find the value of induced emf when the conductor moves at an angle of 45^0 with the direction of the field?

Solution:

$$B = 2wb/m^2 \; l=2 \text{ meters, } v=40m/s$$

$$\text{Induced e.m.f } e = B \times l \times V \times \sin\theta = 2 \times 2 \times 40 \times 1 = 160 \text{ volt}$$

When $\theta = 45^0$

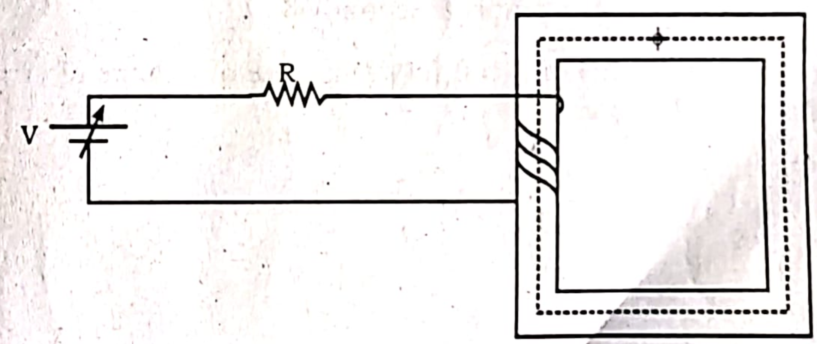
$$\text{Induced emf, } e = Blv \sin\theta = 2 \times 2 \times 40 \sin 45^0 = \mathbf{113.13 \text{ volt}}$$

3.6 Statically induced EMF

Statically induced emf may be of two types. (a) self induced emf and (b) mutually induced emf.

(A) Self induced emf and self-inductance.

This is the emf induced in a coil due to the change of its own flux linked with it.



As the current in the coil changes due to the change in the applied voltage (V), flux linking with a coil also changes producing an emf. This emf is called self induced emf. As per Lenz's law, the direction of induced emf is such as to oppose any change in flux i.e., opposite to the flux producing it. Hence often it is referred to as the counter emf of self- induction. The property of the coil, which opposes any change of current or flux through it, is called its self-inductance and is denoted by letter L.

Expression of Self-inductance of a coil

Consider a coil of N turns carrying a current I amperes. When a current in coil changes, the flux linking with the coil also changes. The emf induced in coil is given by

$$e = -N\frac{d\phi}{dt} \quad \text{.................(1)}$$

$$= -\frac{d}{dt}(N\phi)$$

$$\alpha -\frac{d}{dt} \quad (\because N\phi \propto I)$$

$$e = -L\frac{dI}{dt} \quad \text{.................(2)}$$

Where 'L' is a constant called the self-inductance of the coil or coefficient self-induction. Rewriting the equation (2) we get

$$L = -\frac{e}{(dI/dt)} \quad \text{.............(3)}$$

The unit of inductance is henry (H).

From equation (3)

$$\text{I henry} = \frac{\text{1 volt}}{\text{(1 ampere/second)}}$$

Thus one henry is the inductance of a coil in which rate of change of curre of one ampere induces an emf one volt.

Comparing (1) & (2)

$$N\frac{d\phi}{dt} = L\frac{dI}{dt}$$

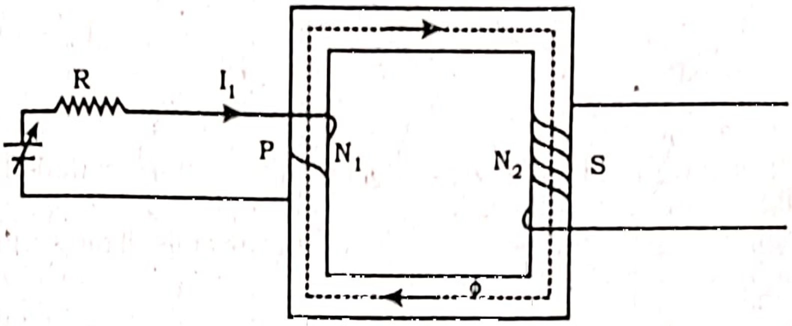
Integrating we get, $\quad N\phi = LI$

$$\therefore \quad L = \frac{N\phi}{I}$$

Thus self-inductance of a coil is the flux linkages per ampere.

3.7 Mutually Induced emf

The phenomenon of generation of induced emf in a circuit by changing th current in the neighbouring circuit is called mutual induction.



If there are 2 coils P & S close to each other, then any change of current in coil P results in mutually induced emf in coil S. Thus the 2 coils are mutually coupled. Any change in current I_1 of coil P causes changing flux to link with both the coils. The emf induced in coil P forms the self-induced emf while emf induced in coil S forms the mutually induced emf. Coil in which change in current takes place is usually called the primary while the other coil is called the secondary.

Mutual inductance between the two coils is the property by which change of current in one coil (primary) induces voltage in the other coil (secondary)

3.8 Expression for mutual inductance (M)

Consider two magnetically coupled coils P &S having turns N_1 & N_2 respectively as shown in figure. Let a current I_1 flowing in the coil P produces flux ϕ_1 webers in it. Let whole this flux links with the turns of the secondary coils S. Thus flux ϕ_2 linking with the coil S is equal to ϕ_1.

As per Lenz's law,

$$e_2 = -N_2 \frac{d\phi}{dt}$$

But $\frac{d\phi}{dt}$ is proportional to $\frac{dI_1}{dt}$

$$e_2 = -N_2 \frac{d\phi}{dt} \quad \text{..........(1)}$$

$$e_2 = -M \frac{dI_1}{dt} \quad \text{..........(2)}$$

$M = \frac{N\phi}{I_1}$

Comparing eqn. (1) and (2) we get

$$N_2 \frac{d\phi}{dt} = M \frac{dI_1}{dt}$$

$$N_2\phi = MI_1$$

$$\therefore M = \frac{N_2\phi}{I_1}$$

$$\text{Similarly} \quad M = \frac{N_1\phi}{I_2}$$

Hence $e = -M\frac{dI_1}{dt}$ where M is the constant of proportionally called the mutual inductance or coefficient of mutual induction between the coils. The unit of mutual inductance is henry.

3.9 Coefficient of coupling

Consider 2 coils P and S wound on the same core as shown in the figure. We have assumed earlier that the complete flux, ϕ_1 setup by the first coil P links with the second coil S. In practice it is not true. Let only a fraction k_1 ($k<1$) of the flux ϕ_1 setup with the first coil P link with the second coil S. Then the flux linked with the second coil is $K_1\phi_1$.

$$M = \frac{N_2K_1\phi_1}{I_1}$$

Similarly complete flux ϕ_2 produced in the second coil S does not linked with the first coil P. Let only a fraction $K_2(K_2<1)$ of ϕ_2 link with coil P.

$$\text{Then} \quad M = \frac{N_1K_2\phi_2}{I_2}$$

Multiplying equation 1&2 we get

$$M^2 = \frac{N_2K_1\phi_1}{I_1} \times \frac{N_1K_2\phi_2}{I_2} = \frac{K_1K_2N_1\phi_1}{I_1} \times \frac{N_2\phi_2}{I_2}$$

$$= K_1K_2L_1L_2 \qquad \left[\begin{array}{l} \frac{N_1\phi_1}{I_1} = L_1 \\ \frac{N_2\phi_2}{I_2} = L_2 \end{array}\right]$$

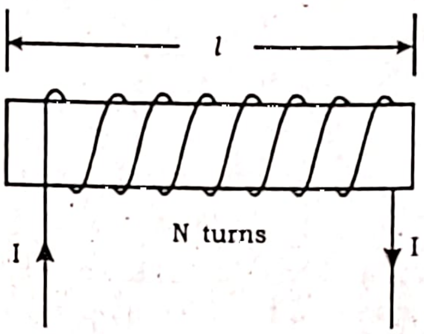
Where L_1 & L_2 are the self-inductance of the first and second coil respectively. But both the coils P and S are wound on the same frame. Hence $K_1 = K_2 = K$.

$$\text{Hence} \quad M^2 = K^2L_1L_2$$

Or $K = \frac{M}{\sqrt{L_1L_2}}$ constant K is called the coefficient of coupling.

3.10 Self Inductance of a Solenoid

Consider an iron-cored solenoid of dimension as shown in figure.



$\frac{N_2\phi}{I_1}$

l = Length of the solenoid

N = Number of turns

I = Current through the solenoid

A = Area of cross-section of the solenoid

L = Self inductance of the solenoid

$$\text{Flux } (\phi) = \frac{\text{MMF}}{\text{Reluctance}}$$

$$\phi = \frac{NI}{(l/A\mu_0\mu_r)} \quad \text{..........(1)}$$

$$\text{But } L = \frac{N\phi}{I}$$

Unit of

$$\phi = \frac{LI}{N} \quad \text{........(2)}$$

Comparing equation (1) and (2)

$$\frac{LI}{N} = \frac{NI}{(l/A\mu_0\mu_r)}$$

mutual inductance unity

$$L = \frac{N^2 A\mu_0\mu_r}{l}$$

$$\text{But Reluctance (S)} = l/A\mu_0\mu_r$$

$$\text{Hence } L = \frac{N^2}{S}$$

PROBLEMS

1. Two identical coils of 400 turns each lie in parallel plane and produced the flux of 0.04 weber. If current of 8 amp is flowing in one coil, find out the mutual inductance between coils?

Solution:

$N_2 = 400$ turns, $\phi = 0.04$ weber, $I_1 = 8$ A

$$M = \frac{N_2\phi}{I_1} = \frac{400 \times 0.4}{8} = \textbf{2 Henry}$$

2. A coil is wound with 220 turns and has a resistance of 50 ohms. If existi voltage is 200 volts and magnetic flux is 0.08 weber, Calculate the s inductance of the coil?

Solution:

$N_2 = 220$ turns, $\phi = 0.08$ Weber

$$I = \frac{V}{R} = \frac{200}{50} = 4A$$

$$L = N\phi/I = \frac{220 \times 0.08}{4} = \textbf{4.4 Henry}$$

3. A coil of 200 turns carries a current of 4A. The magnetic flux linkage wi the coil is 0.02 Wb. Calculate the inductance of the coil. If the current uniformity reserved in 0.02 seconds, calculate the self induced emf in tl coil?

Solution:

$N = 200$ turns, $\phi = 0.02$ wb, $I = 4A$

Inductance of the coil,

$$L = \frac{N\phi}{I} = \frac{200 \times 0.02}{4} = 1H$$

As the current is reversed, change in current

$$dI = 4 - (-4) = 8A$$

$$\text{Induced emf } |e| = L\frac{dI}{dt} = \frac{1 \times 8}{0.02} = \textbf{400 Volts}$$

4. Two identical coils P and S each having 500 turns lie in parallel plane: Current in coil P changing at the rate of 500 A/ second induces emf of 1 Volts in coil S. Calculate the mutual inductance between the two coils. the self inductance of each coil is 50mH. Calculate the flux produced in co P per ampere of current and coefficient of coupling between the two coils?

$$e_1 = \frac{MdI_1}{dt}$$

$$\text{Hence} \quad M = \frac{e_2}{dI_1/dt} = \frac{12}{500} = 0.024 \text{ H}$$

$$L_1 = \frac{N_1\phi_1}{I_1}$$

$$\text{Hence } \frac{\phi_1}{I_1} = \frac{L_1}{N_1} = \frac{50\times10^{-3}}{500} = 10^{-4}\ \text{wb/A}$$

$$K = \frac{M}{\sqrt{L_1L_2}} = \frac{0.024}{\sqrt{50\times10^{-3}\times50\times10^{-3}}} = \mathbf{0.48\ or\ 48\%}$$

5. An air solenoid has 300 turns, its length is 25 cm and cross-sectional area 3 cm^2. Calculate its self inductance. If the coil current of 10A is completely interrupted in 0.04 second, calculate the induced emf in the coil.

Solution

$N = 300$, $l = 25cm = 0.25m$, $A = 3cm^2 = 3 \times 10^{-4}m^2$

$$\mu_r = 1 \text{ (Air core)}$$

$$L = \frac{N^2A\mu_0\mu_r}{l} = \frac{300^2\times3\times10^{-4}\times4\pi\times10^{-7}\times1}{0.25}$$

$$L = 1.357\times10^{-4}\ H$$

$$\text{Induced emf (e)} = \frac{Ldi}{dt} = \frac{1.357\times10^{-4}\times10}{0.04} = \mathbf{0.034\ volt}$$

6. An air-cored toroidal coil has 450 turns and a mean diameter of 30cm and a cross-sectional area of 3cm^2. Calculate (i) the inductance of the coil (ii) the average emf induced if a current of 2A is reversed in 0.04 Second.

Solution.

$$N = 450,\ d = 30cm = 0.3\ m$$

$$l = \pi d = \pi\times0.3m$$

$$A = 3cm^2 = 3\times10^{-4}cm^2$$

$$\mu_r = 1 \text{ (Air core)}$$

(i) $$L = \frac{N^2A\mu_0\mu_r}{l} = \frac{450^2\times3\times10^{-4}\times4\pi\times10^{-7}\times1}{\pi\times0.3}$$

$$L = \mathbf{8.1\times10^{-5}\ H}$$

(ii) $$e = \frac{LdI}{dt} = \frac{8.1\times10^{-5}\times4}{0.04} = \mathbf{8.1\times10^{-3}\ volt}$$

7. An inductive circuit is carrying a current of 4 Amps. If its inductance is 0.15 Hen Find the value of the self induced e.m.f. when the current is reduced to zero in ms?

Ans. $L = 0.15\text{ H}, I = 4\text{A}$

$$\text{Induced emf} = L\frac{dI}{dt} = 0.15 \times \frac{(4-0)}{10 \times 10^{-3}} = \mathbf{60\ V}$$

8. A conductor of 2 m long moves at right angles to a magnetic field of flux densit Tesla with a velocity of 12.5 m/sec. What will be the induced e.m.f. in the conduct

Ans. Induced e.m.f. $= Blv = 1 \times 2 \times 12.5 = 25$ Volt

9. Two coils having 150 and 200 turns respectively are wound side by side or closed magnetic circuit of cross section. 1.5×10^{-2} m^2 and mean length 3 m. T relative permeability of the magnetic circuit is 2000. Calculate (i) the mutu inductance between the coils (ii) the voltage induced in the second coil if t current changes from 0 to 10 A in the first coil in 20 m sec?

Ans. $N_1 = 150, N_2 = 200, A = 1.5 \times 10^{-2}\text{ m}^2$

$l = 3\text{ m}, \mu_r = 2000$

$$\text{Mutual inductance between the coils (M)} = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 2000 \times 150 \times 200 \times 1.5 \times 10^{-2}}{3} = 0.3768\text{ H}$$

$$\frac{dI}{dt} = \frac{10-0}{20 \times 10^{-3}}$$

Voltage induced in the second coil,

$$e = M \cdot \frac{dI}{dt} = \frac{0.3768 \times 10}{20 \times 10^{-3}} = \mathbf{188.4\ volt}$$

10. An emf of 16 volts is induced in a coil of inductance 4H. What must be the rate change of current?

Ans. $$|e| = L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{16}{4} = 4\text{A/S}$$

Rate of change of current = 4 A/S

11. A coil induces 350 mV when the current changes at the rate of 1 A/sec. What the value of inductance?

Ans. Induced voltage (e) $= 350 \times 10^{-3}$ V

$$\text{Rate of change of current} = \frac{di}{dt} = 1\text{A / Sec}$$

$$\text{Equation for induced voltage } e = L\frac{di}{dt}$$

$$\text{Then, inductance L} = \frac{350 \times 10^{-3}}{1} = \mathbf{350\ H}$$

12. Two coils having 150 and 200 turns respectively are wound side by side on a closed magnetic circuit of cross section 1.5×10^{-2} m² and mean length 3 m. The relative permeability of the magnetic circuit is 2000. Calculate

(i) the mutual inductance between the coils.

(ii) the voltage induced in the second coil if the current changes from 0 to 10 A in the first coil in 20 ms.

Ans. $N_1 = 150$, $N_2 = 200$, $A = 1.5 \times 10^{-2}$ m², $l = 3$ m

$$\mu_r = 2000$$

$$\text{Mutual inductance, } M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 2000 \times 150 \times 200 \times 1.5 \times 10^{-2}}{3}$$

$$= 0.376\ \text{H}$$

$$\frac{dI}{dt} = \frac{10-0}{20 \times 10^{-3}}$$

Voltage induced in the second coil

$$e = M \times \frac{di}{dt}$$

$$= 0.376 \times \frac{10}{20 \times 10^{-3}}$$

$$= 188.4\ \text{V}$$

13. A coil consists of 750 turns and a current of 10A on the coil gives rise to a flux of 1.2m Wb. Calculate the inductance of the coil. If the current is reversed in 0.01 second, find the voltage induced in the coil.

Ans. No. of turns = 750

$$I = 10\ \text{A},\ \phi = 1.2 \times 10^{-3}\ \text{Wb}$$

$$\text{Inductance L} = \frac{N\phi}{I} = \frac{750 \times 1.2 \times 10^{-3}}{10} = 90\ \text{mH}$$

$$\frac{di}{dt} = 0.01\ \text{second}$$

$$e = L \times \frac{di}{dt} = 90 \times 10^{-3} \times 0.01 = 0.9\ \text{mV}$$

14. A square coil of 10 cm, side and with 100 turns is rotated at a uniform speed of 500 r.p.m about an axis at right angles to a uniform field of 0.5 Wb/m². Calculate the instantaneous value of induced e.m.f when the plane of the coil is (i) at right angle to the plane of the field and (ii) at 45° with the field direction

Ans.

Length of the coil 'l' = 10×10^{-2} m

No. of turns 'N' = 100

Speed in rpm = 500

Flux density 'B' = $0.5\ Wb/m^2$

Emf induced in one side of coil having 'N' turns

$$e = N \times Blv \sin\theta$$

Total emf induced in both sides of coil is

$$e = 2BlNv \sin\theta \text{ volt}$$

where v is peripheral velocity in m/sec

and $\quad v = \pi \times f \times b$

where 'b' is the width of coil in meters and 'f' is the frequency of rotation c coil in Hz.

(i) Coil is right angled to the plane of the field.

So, here $\theta = 90^0$ $\quad$ Sin 90 = 1

$$e = 2\pi \times \frac{500}{60} \times 0.5 \times 0.1^2 \times 100 = 26.17\ V$$

(ii) Here $\theta = 45^0$ $\quad$ sin 45 = 0.7071

$$e = 2\pi \times \frac{500}{60} \times 0.5 \times 0.1^2 \times 100 \times 0.7071 = 18.51\ V$$

15. A conductor of length 1 metre moves at right angles to a uniform magnetic field of flux density of $1.5\ Wb/m^2$ with a velocity of 50 metre/second. Calculate e.m.f. induced in it.

Ans. $l = 1m$ $\quad B = 1.5\ Wb/m^2$ $\quad v = 50\ m/s$

Emf induced $\quad e = Blv \sin\theta$ volts

Here $\quad \sin\theta = 1 \quad \therefore \theta = 90^0$

So $\quad e = 1.5 \times 1 \times 50 \times 1 = 75\ V$

16. Two coupled coils connected in series have an equivalent inductance of 0.725 when connected in aiding and 0.425 when connected in opposing. Find self and mutual inductance, if coefficient of coupling is 0.42.

Ans. $\quad L' = 0.725$ (additive)

$L'' = 0.425$ (substractive)

Coefficient of coupling $K = 0.42$

$$L' = L_1 + L_2 + 2M$$

$$L'' = L_1 + L_2 - 2M$$

$$0.725 = L_1 + L_2 + 2M \quad \text{.....................(1)}$$

$$0.425 = L_1 + L_2 - 2M \quad \text{.....................(2)}$$

Substracting eqn (2) from eqn(1)

We get, $\quad 0.3 = 4M$

$$\text{Mutual inductance } M = \frac{0.3}{4} = 0.075$$

Put value of M on Eqn.(1)

$$0.725 = L_1 + L_2 + 0.15$$
$$L_1 + L_2 = 0.575\ \text{H} \qquad \text{.................(3)}$$

In general
$$M = K\sqrt{L_1L_2}$$

$$0.075 = 0.42\sqrt{L_1L_2}$$
$$L_1L_2 = 0.0318 \qquad \text{...................(4)}$$
$$L_1 = \frac{0.0318}{L_2} \qquad \text{...............(4a)}$$

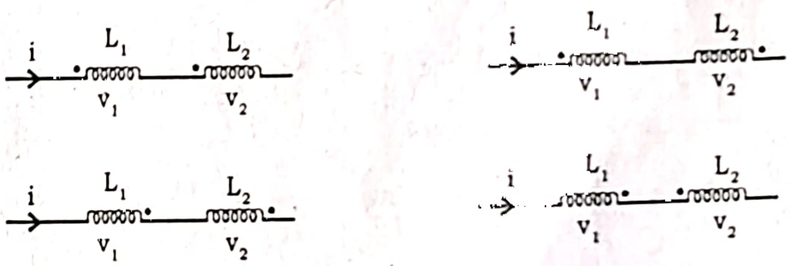
Put this value of L_1 on eqn. (3)

$$\frac{0.0318}{L_2} + L_2 = 0.575$$

Then, get an equation
$$L_2^2 - 0.575\, L_2 + 0.0318 = 0$$
$$L_1 = 0.513\ \text{H}$$
$$\&\ L_2 = 0.575 - 0.513 = 0.0625\ \text{H}$$

17. Derive the expression for effective inductance when two coils are connected (i) in series and (ii) in parallel.

Ans. Two coils can be connected in series so that their fluxes at any instant are in the same direction or in opposite direction. These two connections are known as series aiding and series opposing.



Series aiding connection   Series opposing connection

In series aiding connection, the total voltage induced in each of the two coils is partly due to its self inductance and partly due to mutual inductance. Therefore,

$$V_1 = L_1\frac{di}{dt} + M\frac{di}{dt} = (L + M)\frac{di}{dt}$$

$$V_2 = (L_2 + M)\frac{di}{dt}$$

$$V = V_1 + V_2 = \frac{di}{dt}(L_1 + L_2 + 2M)$$

But $\dfrac{V}{di/dt}$ is the total inductance L_a.

Therefore $L_a = L_1 + L_2 + 2M$

In the series opposing connection, the mutually induced voltage oppose self induced voltage.

Therefore, $$V_1 = L_1\frac{di}{dt} - M\frac{di}{dt} = (L_1 - M)\frac{di}{dt}$$

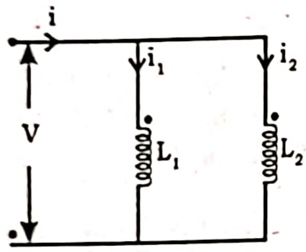
$$V_2 = (L_2 - M)\frac{di}{dt}$$

$$V = V_1 + V_2 = (L_1 + L_2 - 2M)\frac{di}{dt}$$

Therefore, the total inductance L_b of the series opposing connection is

$$L_b = L_1 + L_2 - 2M$$

Coils are connected in parallel



$$V = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

From the above equations

$$\frac{di_1}{dt} = \frac{(L_2 - M)V}{L_1L_2 - M^2}$$

$$\frac{di_2}{dt} = \frac{(L_1 - M)V}{L_1L_2 - M^2}$$

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{(L_1 + L_2 - 2M)V}{L_1L_2 - M^2}$$

If L_p is the equivalent inductance of parallel arrangement.

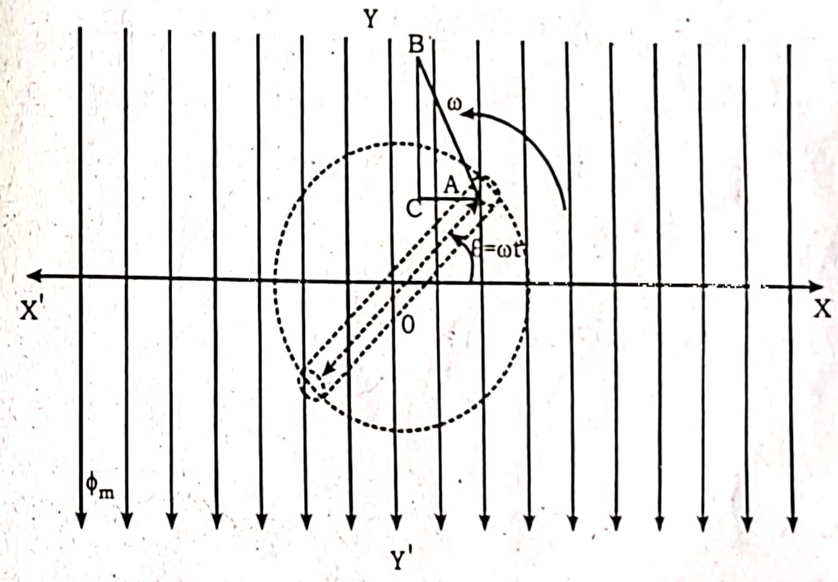
$$\frac{di}{dt} = \frac{1}{L_p}V$$

Therefore, $$L_p = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

CHAPTER 4

FUNDAMENTALS OF ALTERNATING CURRENT

4.1 Production of Alternating Emf



Consider a single turn rectangular coil rotating with a constant angular velocity of ω radian/second in a uniform magnetic field. The axis of rotation is being perpendicular to the magnetic lines of force. Let the time be measured from the instant the coil lies in the plane of reference XOX'. The angle θ swept by the coil in a time t seconds is given by $\theta = \omega t$. Where ω is the angular velocity of the coil in rad/second. Linear velocity v of the coil sides,

$v = \omega r$ m/s where 'r' is the radius of the path in meters.

Linear velocity of coil side at right angles to the magnetic field = AD = v sin θ = v sin ωt.

When the coil continues its motion in the direction AC,

The flux cut per second = Blv sin ωt

where

'l' is the length of the coil side parallel to the axis in meters

'B' is the flux density in tesla (weber/m^2)

Thus the emf generated in the coil side at time t = $Blv \sin\omega t$

Emf generated in the coil at time $t = 2\,Blv \sin \omega t = 2\,Bl\omega r \sin \omega t$ (v = (

But maximum flux linking the coil, $\phi = B \times 2lr$.

Thus emf generated in the coil at any instant 't' = $\phi\omega \sin \omega t$ ($\phi = 2\,Blr$) ....

Thus emf will be maximum when $\sin \omega t = 1$

Hence maximum value of emf generated, $E_m = \phi\omega$

Substituting it in equation (1)

The instantaneous value of emf generated at any time t is

$$e = E_m \sin \omega t \quad \text{.....................(2)}$$

Similarly the expression for induced alternating current is given by,

$$i = I_m \sin \omega t \quad \text{.....................(3)}$$

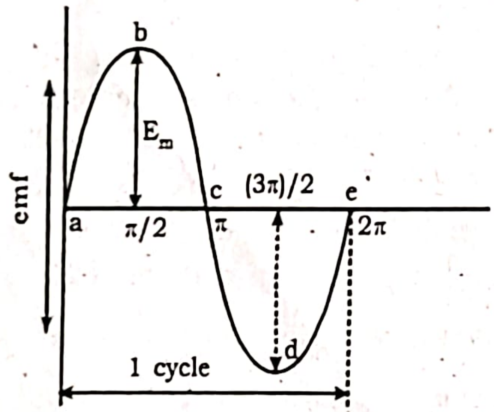
If 'f' is the frequency of rotation of the coil. i.e., no.of cycles passed through p second, then $\omega = 2\pi f$

Substituting for ω in equation (2) & (3)

$$e = E_m \sin (2\pi f)\, t$$

$$i = I_m \sin (2\pi f)\, t$$

If the emf values as given by equation (2) from instant to instant are plotte along the Y-axis and time along x-axis, the graph will be as shown in the figur



Sine Wave of emf

The graph of voltage shown in the figure is called a sinusoidal alternating emf The trace abcde of the graph completes one cycle and consists of two alternatives one positive and other negative. Such a wave will complete a certain number of cycles in one second, which is called the frequency of the wave and is expressed in Hertz(Hz). In practice while drawing the ac waves, horizontal axis is marked in radians or degrees instead of seconds.

4.2 Important Terms

Cycle :- One complete set of positive and negative values of an alternating quantity is called a cycle.

Periodic time The time taken for one cycle is known as time period or periodic time (T). The relationship between frequency and time period is given by,

$$T = \frac{1}{f}$$

Frequency : The number of cycles completed in one second is called the frequency of an alternating quantity. Frequency is expressed in cycles/second or Hertz.

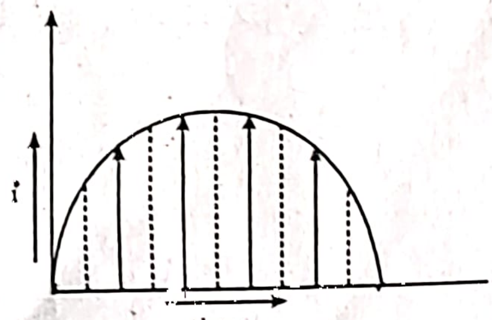
Amplitude : This is the magnitude of the maximum positive or negative value of alternating quantity. It is often referred to as the peak value.

Instantaneous value : The value of voltage or current at a particular instant is known as instantaneous values.

Average value : The average value of the current or voltage of an alternating wave shape is the arithmetic mean of the ordinates at equal intervals over a half cycle of the wave. However, if the arithmetic mean is found out over the complete cycle, it will be zero for sinusoidal as well as for non-sinusoidal wave, provided the wave shape is symmetrical.

4.3 Determination of Average Value

i) Mid Ordinate Method :- The average value of an alternating wave can be determined graphically by taking the arithmetic mean of the ordinates at equal intervals over a half cycle of the wave.



Let the wave from be split into equal parts. The middle values are measured and the average value is calculated.

$$I_{av} = \frac{i_1 + i_2 + i_3 + \ldots\ldots + i_n}{n}$$

ii) Analytical method :- The total area is measured over the half cycle and divided by the base gives the average value.

Let us assume a sine wave of current $i = I_m \sin\theta$

The area of the sine curve from 0^0 to 180^0 may be formed out by integ[rating]
the curve.

$$\text{Area} = \int_0^{180} i\,dt = \int_0^{180} I_m \cdot \text{Sin}\,\theta\, d\theta$$

$$= I_m\,[-\cos\theta]_{0^0}^{180^0} = 2I_m$$

Base of the wave from $= \pi$ radian

$$\therefore I_{av} = \frac{\text{Area}}{\pi} = \frac{2I_m}{\pi}$$

$$= \frac{2}{\pi} \times I_m = 0.637\,I_m$$

The area of the sine curve from 0^0 to 180^0 may be formed out by integ
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Base of the wave from = π radian

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