
KTUNOTES

FIRST SEMESTER B.TECH DEGREE EXAMINATION, NOVEMBER 2017

MODEL QUESTION PAPER

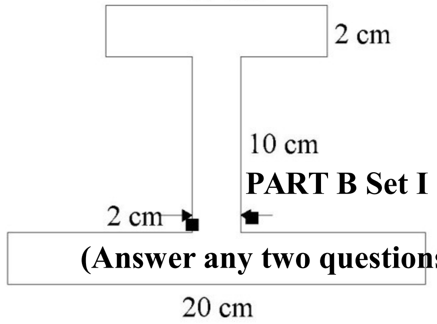
BE 100 ENGINEERING MECHANICS

Time : 3 Hour Maximum Marks :100

PART A (Answer all questions)

1. Explain free body diagram with the example of a ladder resting against a smooth wall and a rough floor.
2. Find the support reactions of a cantilever beam of span 6m carrying a UDL of 6kN/m.
3. State and prove Pappus Guldinus theorem
4. Explain laws of friction
5. A car is moving with a velocity of 15 m/s.The car is brought to rest by applying breaks in 5s.Determine the retardation and distance travelled by the car after applying breaks
6. Distinguish Free vibration and Forced vibration
7. Explain instantaneous centre of rotation
8. A body is vibrating with simple harmonic motion of amplitude 120mm and frequency 5cps. Calculate the maximum velocity and maximum acceleration of the body

(8x5=40)

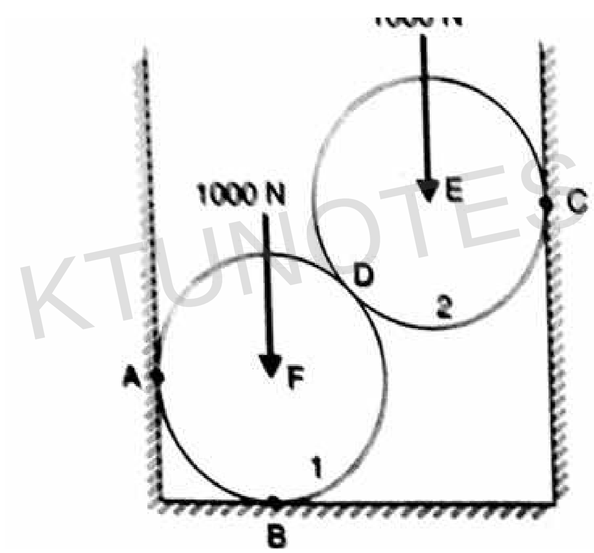


PART B Set I

(Answer any two questions) (2x10=20)

9. ABCD is a rectangle in which AB=30mm, BC= 20mm. ‘E’ is the middle point of ‘AB’. Forces of magnitude 16,14,18,8,10 and 20N act along ‘AB’, ‘BC’, ‘CD’, ‘DA’, ‘EC’ and ‘DE’ respectively. Find the magnitude, direction and position with respect to ‘ABCD’ of single force to keep the body in equilibrium. ‘B’ is to right of ‘A’ and taken in anticlockwise direction (10)

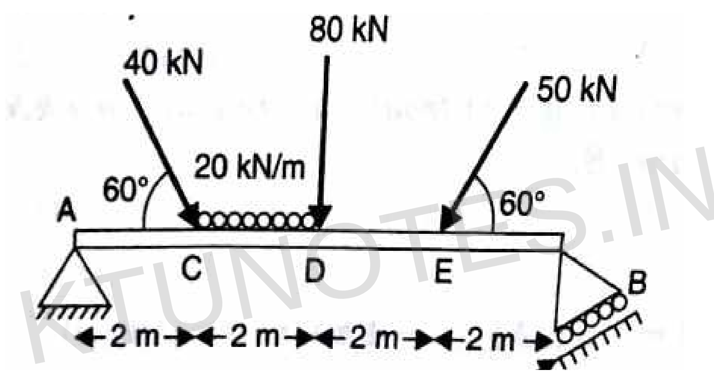
10. Two spheres ,each of weight 1000N and of radius 25cm rest in a horizontal channel of width 90cm as shown in fig. Find the reactions on the points of contact A,B and C (10)



11(a) A force acts at the origin of a co-ordinate system in a direction defined by the angles α_x=69.3^0 and α_z=57.9^0 .Knowing that the ‘Y’ component of the force is -174N, determine the (i) angle α_y and (ii) the other components and the magnitude of the force

(5)

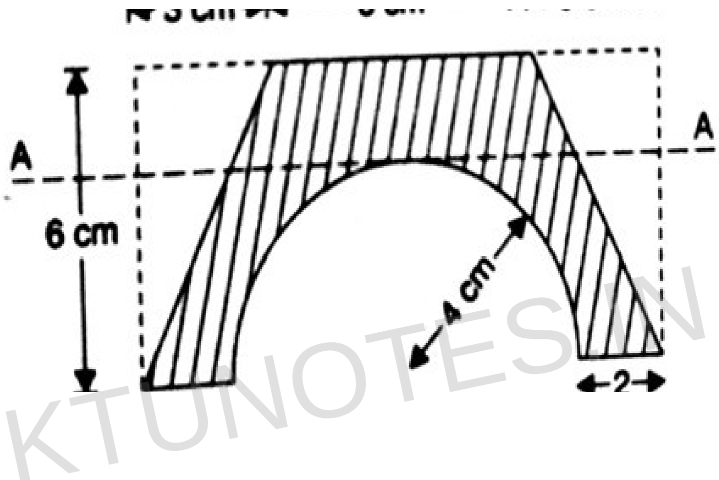
(b) Find the reactions at the support A and B of the beam as shown in fig. (5)



PART B Set II

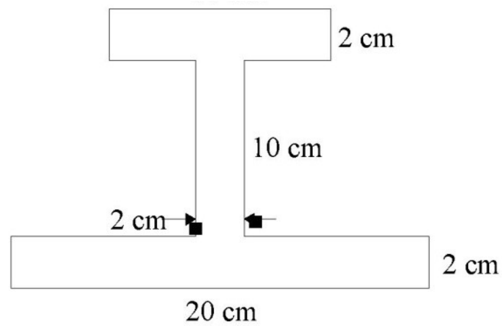
(Answer any two questions) (2x10=20)

12 (a) Calculate the moment of inertia of the section shown in fig about the centroidal axis.



(6)

b) Calculate the centre of gravity of the section shown in fig about the centroidal axis.



(4)

13) Two blocks A& B weighs 500N & 1000N are placed on an inclined plane. The blocks are connected by a string parallel to the inclined plane. The coefficient of friction between the inclined plane and block A is 0.25 and that for the block B is 0.3. Find the inclination of plane when the motion is about to take place. Also calculate tension in the string.

(10)

14 .A simply supported beam of span 10 m is loaded with two concentrated loads of 15kN and 20kN at 4m and 6m from the end A respectively.. Find the reactions at the supports of the beam using principle of virtual work.

(10)

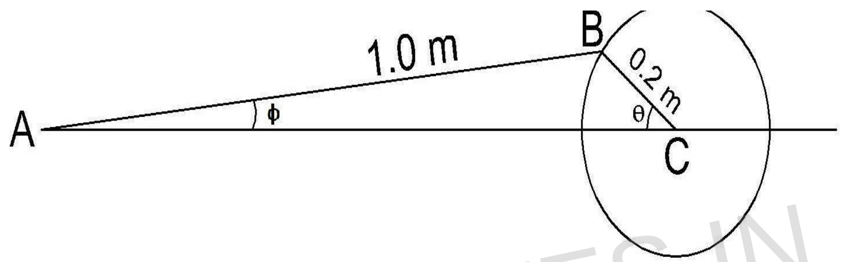
11.

PART B Set III

(Answer any two questions) 2x10

15 a) The crank of a reciprocating engine is rotating at 210 r.p.m. The length of the crank and connecting rod are 20cm & 100cm respectively. Find the velocity of point A (velocity of piston) when crank has turned an angle of $45^o$with the horizontal.

(6)



b) An elevator weighs 5000N is ascending with an acceleration of 3 m/s^2.During this ascend its operator whose weight is 700N is standing on the floor. What will be the reaction produced by the floor on the operator, what will be the total tension in the cable on the elevator.

(4)

16 a). In a system the amplitude of the motion is 1.6m and time period is 4 sec. find the time required for the particle in passing between points which are at a distance of1.2m and 0.6m from the centre of force and are on the same side of the system .

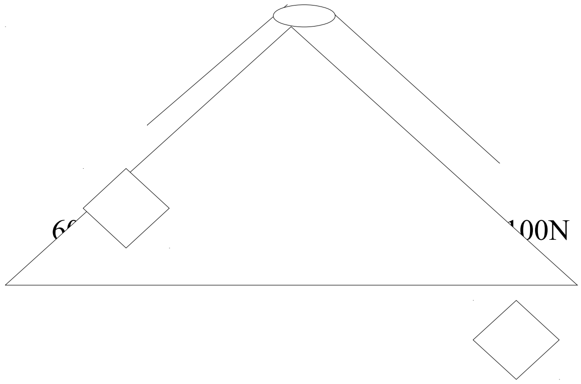
(5)

b) The strength of a spring is such that a load of 50 N is required to elongate it by 10mm. When a certain load W is suspended from one end and caused to perform SHM, the complete oscillations per minute is 100. Calculate the stiffness of the spring and the value load W.

(5)

17. Two rough planes inclined at 45^0 and 60^0 to horizontal are placed as shown in **Figure**

Two blocks of weights 60 N and 100 N respectively are placed on the faces and are connected by a string and passing over a frictionless pulley. If the coefficient of friction planes and blocks is 0.35, find the resulting acceleration and tension in the string. (10)



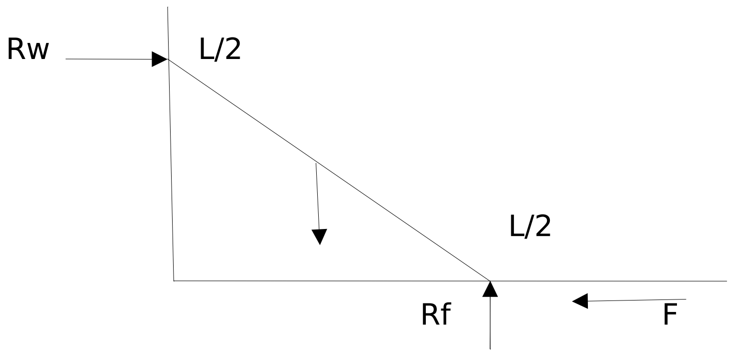
454545ww

2x10=20

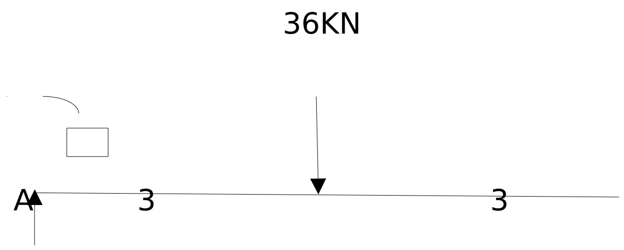
BE100 ENGINEERING MECHANICS

ANSWER KEY

1.A body is isolated from all its supports and is represented by all the forces acting on it including self weight and reaction from support.



2.   free body diagram of cantilever beam



M_A=108KNm

R_A=36KN

3.

The theorems of Pappus - Guldinus were first set forth by Pappu about 300 A.D and then restated by the swiss mathematician Guldinus about 1640. These theorems offer a simple way for computing the area of surface of revolution and the volume of bodies of revolution.

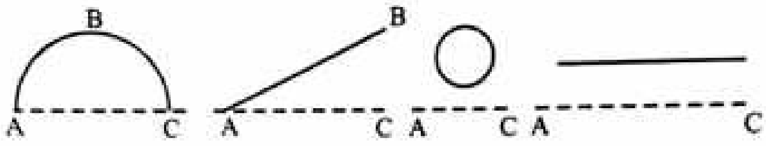


Fig. 3.55

A surface of revolution is a surface which may be generated by rotating a plane curve about a fixed axis. The surface of a sphere is obtained by rotating a semicircular Arc ABC about the axis AC, the surface of a cone by rotating inclined line AB about axis AC, the surface of a ring by rotating the circumference of a circle about the axis AC, the surface of a cylinder by rotating a horizontal line about the axis AC as shown in Fig 3.55.

A body of revolution is the body which is generated by rotating a plane area about a fixed axis. A solid sphere is generated by rotating a semicircular area, a cone by rotating a triangular area and a cylinder by rotating a rectangular area about axis AB shown in Fig. 3.56.

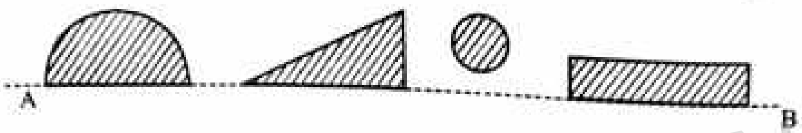


Fig. 3.56

Theorem 1.

The area of surface generated by revolving a plane curve about a non-intersecting axis in the plane of the curve is equal to the product of length of curve and the distance travelled by the centroid of the curve while the surface is being generated.

Theorem II

The volume of a body generated by revolving a plane area about a non-intersecting axis in the plane of the area is equal to the product of area and the distance travelled by the centroid of the plane area while the body is being generated.

volume generated by A, $V = \int 2\pi y\,dA = 2\pi\int y\,dA$

$= 2\pi\bar{y}A$ Where $2\pi\bar{y}$ is the distance travelled by the centroid of area A.

4.

1. The force of friction always acts in a direction opposite to the direction in which the body moves or tends to move
2. Till the limiting value is reached, the magnitude of friction is equal to the force which tends to move the body
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two contact surfaces.
4. The force of friction depends upon the roughness of the surfaces in contact.
5. The force of friction is independent of the area of contact between the two surfaces.
6. For low velocities, the frictional force is independent of magnitude of velocity. But generally the dynamic friction is less than the limiting friction.

5. $u=15m/s$

$V=0$

$t=5s$

$V=u+at$

$a= -3m/s^2$

$S=ut+1/2at^2$

$=37.5m$

6.

When any elastic system is displaced from its equilibrium position ... a periodic motion. This periodic motion is known as vibration. Hence a vibration is a periodic motion which repeats itself after a definite interval of time. If the periodic motion takes place without any external force, the vibration is called as free vibration. The free vibrations are of three types :

1. The longitudinal vibrations,
2. The transverse vibrations, and

A system is said to undergo forced vibration when a periodic disturbing force acts on the system. In forced vibration the system will vibrate at the frequency of the exciting force regardless of the initial conditions of the system.

7.

The motion of rotation and translation of a body ... that of pure rotation of the body about a point. This point about which the body can be assumed to be rotating at the given instant is called instantaneous centre of rotation. Since the velocity of this point at the given instant is zero, this point is also called instantaneous centre of zero velocity. This point is not a fixed point, and when the body changes its

position, the position of instantaneous centre also changes. The locus of the instantaneous ... on changing its position is called centrode.

...agnitude of velocity of any point on a body is proportional to its distance from the instantaneous centre and is equal to the angular velocity times the distance.

(ii) The direction of velocity of any point on a body is perpendicular to the line joining that point and the instantaneous centre.

8. $V_{max}=r\,\omega$

$a_{max}=\omega^2 r$

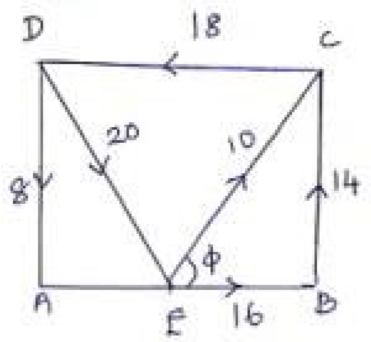
$\omega=2\pi f$

$\omega=10\pi$ rad/s

$V_{max}=3.768$ m/s

$a_{max}=118.32$ m/s^2

9



$\tan\phi = 20/15$

$\phi = \underline{53.13^\circ}$

$\Sigma F_x = 16 - 18 + 10\cos 53.13 + 20\cos 53.13$

$= 16\,N$

$\Sigma F_y = 14 - 8 + 10\sin(53.13) - 20\sin(53.13)$

$= -2\,N$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{16^2 + (-2)^2}$

$= \underline{16.124\ N}$

$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \underline{7.125^\circ}$

$\theta_R = 352.875^\circ$

Moment about A $= 20\cos(53.13) \times 20 + 10\cos(53.13) \times 20 - (14 \times 30) - (18 \times 20) - 30 \times 10\sin(53.13)$

$= \underline{660\ ccw}$

$\Sigma M_A = R \cdot x$

$x = \frac{\Sigma M_A}{R} = \frac{660}{16.12} = 40.92$ mm from A

10.

Sol. Given :

Weight of each sphere, $W = 1000$ N

Radius of each sphere, $R = 25$ cm

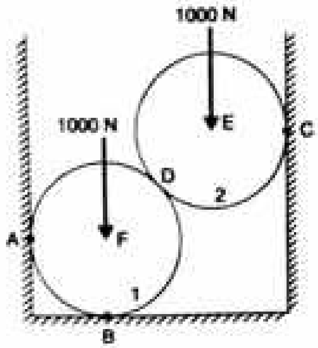
$\therefore$ $AF = BF = FD = DE = CE = 25$ cm

Width of horizontal channel = 90 cm

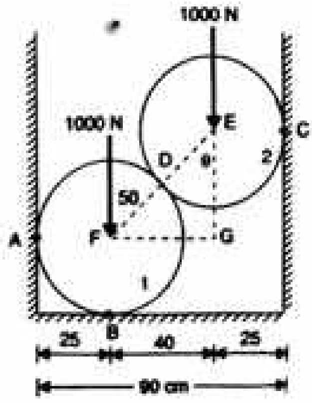
Join the centre E to centre F as shown in Fig. 2.18(b).

Now $EF = 25 + 25 = 50$ cm, $FG = 40$ cm

In ΔEFG, $EG = \sqrt{EF^2 - FG^2} = \sqrt{50^2 - 40^2} = \sqrt{2500 - 1600} = 30$



(a)



(b)

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$\therefore \quad \cos\theta = \frac{\quad}{EF} = \frac{\quad}{50} = \frac{\quad}{5}$ and $\sin\theta = \frac{\quad}{EF} = \frac{\quad}{50} = \frac{\quad}{5}$.

Equilibrium of Sphere No. 2. The sphere 2 has points of contact at C and D.

Let $\quad R_C$ = Reaction at C

and $\quad R_D$ = Reaction at D

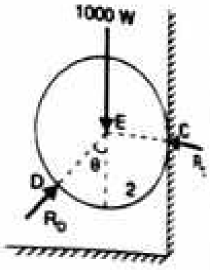


Fig. 4.18(c)

The free-body diagram of sphere No. 2 is shown in Fig. 4.18 (c).

The reaction R_D at point D, will pass through the centre E of the sphere No. 2, as any line normal to any point on the circumference of the circle will pass through the centre of circle. For the equilibrium of the sphere No. 2, the resultant force in x and y directions should be zero.

For $\quad \Sigma F_x = 0$, we have $R_D \sin\theta = R_C$ $\quad$ ...(i)

For $\quad \Sigma F_y = 0$, we have $R_D \cos\theta = 1000$

or
$$R_D = \frac{1000}{\cos\theta} = \frac{1000}{\left(\frac{3}{5}\right)} \qquad \left(\because \cos\theta = \frac{3}{5}\right)$$

$$= 1000 \times \frac{5}{3} = \frac{5000}{3} \text{ N} \qquad ...(ii)$$

Substituting the value of R_D in equation (i),

$$\frac{5000}{3} \times \sin\theta = R_C$$

or
$$\frac{5000}{3} \times \frac{4}{5} = R_C \qquad \left(\because \sin\theta = \frac{4}{5}\right)$$

or
$$1333.33 = R_C$$

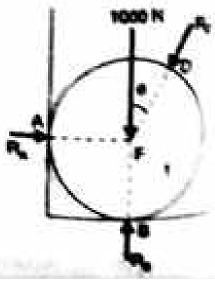
$\therefore \quad$ **R_C = 1333.33 N. Ans.**

Equilibrium of Sphere No. 1. The sphere 1 has points of contact at A, B and D.

Let $\quad R_A$ = Reaction at point A

$\quad R_B$ = Reaction at point B

The free-body diagram of sphere No. 1 is shown in Fig. 4.18(d). The reactions R_A, R_B and R_D will pass through the centre F of the sphere No. 1.



For $\quad \Sigma F_x = 0$, we have

or $\quad R_A - R_D \sin\theta = 0$

$$R_A = R_D \sin\theta$$

$$= \frac{5000}{3} \times \frac{4}{5}$$

$\left(\because R_D = \frac{5000}{\ldots}\right.$

For $\quad \Sigma F_y = 0$, we have

$$R_B - 1000 - R_D \cos\theta = 0$$

$\therefore$
$$R_B = 1000 + R_D \cos\theta$$

$$= 1000 + \frac{5000}{3} \times \frac{3}{5} \qquad \left(\because \cos\theta = \frac{3}{5}\right)$$

11. a)

$\alpha_x = 69.3^\circ$ $\qquad$ $\alpha_y = ?$

$\alpha_z = 57.9^\circ$ $\qquad$ $F = ?$

$F_y = -174\ N$ $\qquad$ $F_x = ?$

$\qquad\qquad\qquad\qquad$ $F_y = ?$

$\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z = 1$

$\cos^2\alpha_y = 0.5927$; $\cos\alpha_y = -0.769$ (Since F_y is negative, $-$ve value is taken)

$\alpha_y = \cancel{39.73^\circ}\ 140.264^\circ$

$F_y = F\cos\alpha_y$; $F = \dfrac{-174}{\cos(140.264)}$

$\therefore F = 226.26\ N$

$F_x = F\cos\alpha_x$

$= 79.97\ N$

~~Fy~~ ⇒ $F_z = F\cos\alpha_z$

$= 120.23\ N$

11(b)

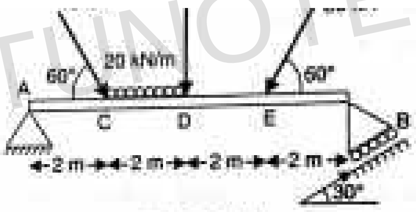


Fig. 5.28(a)

Sol. Given :

Length of beam $\qquad = 8$ m

Let $\qquad R_A$ = Reaction at A, and

$\qquad R_B$ = Reaction at B.

The reaction R_B will be normal to the support as the beam at B is supported on the rollers. The support at B makes an angle of 30° with the horizontal or 60° with the vertical. Hence the reaction R_B will make an angle of 30° with the vertical as shown in Fig. 5.28 (*b*).

The reaction at A will be inclined, as the end A is hinged and beam carries inclined load.

Let $\quad (R_A)_x$ = Horizontal component of R_A

$\quad (R_A)_y$ = Vertical component of R_A.

For equilibrium of the beam, the moments of all the forces about any point should be zero. Taking the moments about the point A, we get

$$(R_B \cos 30°) \times 8 - (50 \sin 68) \times 6 - 80 \times 4 - 40 \times 3 - (40 \sin 60°) \times 2 = 0$$

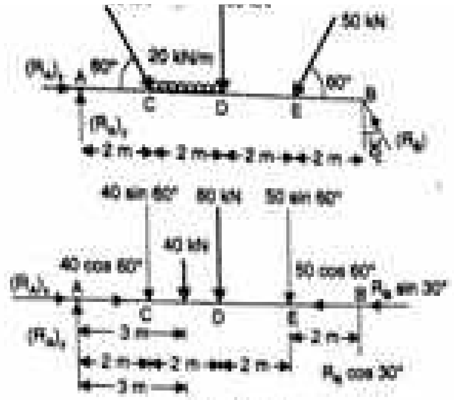


Fig. 5.28(b)

or $$6.928R_B - 259.8 - 320 - 120 - 69.28 = 0$$

or $$6.928R_B = 259.8 + 320 + 120 + 69.28 = 769.08$$

$\therefore$ $$R_B = \frac{769.08}{6.928} = 111 \text{ kN.} \quad \textbf{Ans.}$$

To find the reaction R_A, use the equilibrium equations

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

For $\Sigma F_x = 0$, we have $(R_A)_x + 40 \cos 60° - 50 \cos 60° - R_B \sin 30° = 0$

or $$(R_A)_x + 20 - 25 - 111 \times 0.5 = 0 \qquad (\because R_B = 111)$$

$\therefore$ $$(R_A)_x = -20 + 25 + 111 \times 0.5$$
$$= -20 + 25 + 55.5 = 60.5 \text{ kN}$$

For $\Sigma F_y = 0$, we have $(R_A)_y - 40 \sin 60° - 40 - 80 - 50 \sin 60° + R_B \cos 30° = 0$

or $$(R_A)_y = 40 \sin 60° + 120 + 50 \sin 60° - 111 \times 0.866 \qquad (\because R_B = 111)$$
$$= +120 + 90 \sin 60° - 111 \times 0.866$$
$$= +120 + 90 \times 0.866 - 111 \times 0.866$$
$$= +120 - 21 \times 0.866 = 101.8 \text{ kN}$$

$\therefore$ Reaction at A,

$$R_A = \sqrt{(R_A)_x^2 + (R_A)_y^2} = \sqrt{60.5^2 + 101.8^2} = \sqrt{3660.25 + 10363}$$
$$= \textbf{118.42 kN.} \quad \textbf{Ans.}$$

Angle made by R_A with x-direction is given by

$$\tan \theta = \frac{(R_A)_y}{(R_A)_x} = \frac{101.8}{60.5} = 1.682$$

$\therefore$ $$\theta = \tan^{-1} 1.682 = \textbf{59.27°.} \quad \textbf{Ans.}$$

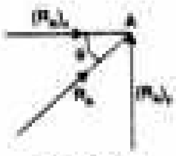


Fig. 5.28(c)

12 (a) Enclose the given cross section of culvert into rectangle of dimension 6x12 & two triangles of dimension 6x3 and a semi circle of dimension 4 cm (radius)

Moment of inertia of rectangle about A-A axis

$$= \frac{bd^3}{12} + Ax^2$$

$$= \frac{12 \times 6^3}{12} + 72 \times 1^2 = \underline{288 \text{ cm}^4}$$

Moment of inertia of two triangles about A-A

$$= 2\left[\frac{bh^3}{36} + A_1 x_1^2\right]$$

Here $A_1 = \frac{1}{2} \times 3 \times 6$

[$x_1 = 0$ centroid of triangle passes through A-A]

$$= 2\left[\frac{3 \times 6^3}{36} + 9 \times 0\right]$$

$$= \underline{36 \text{ cm}^4}$$

Moment of inertia of semi circle about A-A

$= $ M.O.I of semi circle about CG + Ax^2

$$= 0.11 R^4 + Ax^2$$

$$= 0.11 R^4 + \frac{\pi r^2}{2}\left[r - \frac{4r}{3\pi}\right]^2$$

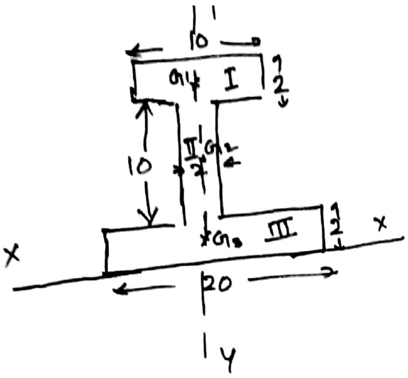
$$= 28.16 + 133.22 = \underline{161.38 \text{ cm}^4}$$

Moment of inertia of culvert about AA = Moment of inertia of rectangle − M.O.I. of two triangles − M.O.I of semi circle

$$= 288 - 36 - 161.38$$

$$= \underline{90.62 \text{ cm}^4}$$

(b)



$\bar{x} = 0$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

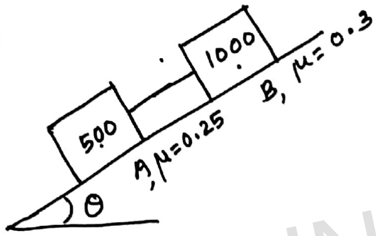
$a_1 = 10 \times 2$ $\quad y_1 = 13$

$a_2 = 10 \times 2$ $\quad y_2 = 7$

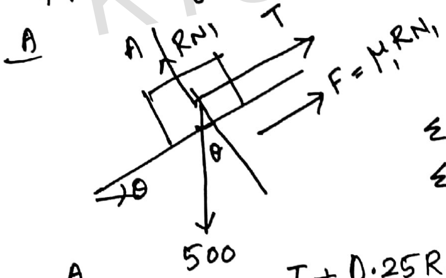
$a_3 = 20 \times 2$ $\quad y_3 = 1$

$\therefore \bar{y} = \underline{\underline{5.5}}$

(13)

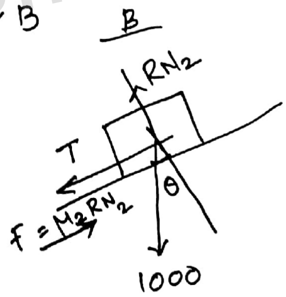


Free body diagrams of Block A & B



$\Sigma F_x = 0$

$\Sigma F_y = 0$



<u>A</u>

$T + 0.25\, RN_1 = 500 \sin\theta$

$RN_1 = 500 \cos\theta$

$T = 500 \sin\theta - 125 \cos\theta \rightarrow (1)$

<u>B</u>

$T + 1000 \sin\theta = \mu_2 RN_2$

$RN_2 = 1000 \cos\theta$

$T = \cancel{0.3}\ 300 \cos\theta - 1000 \sin\theta \rightarrow (2)$

From ① & ②

$$500\sin\theta - 125\cos\theta = 300\cos\theta - 1000\sin\theta$$

$$1500\sin\theta = 425\cos\theta$$

$$\tan\theta = 0.283$$

$$\theta = 15.8^\circ$$

$$T = 15.8N.$$

14)

Span of beam AB, $L = 10$ m

Point load at $C = 15$ kN

Point load at $D = 20$ kN

Distance $AC = 4$ m

Distance $AD = 6$ m

Let R_A = Reaction at A

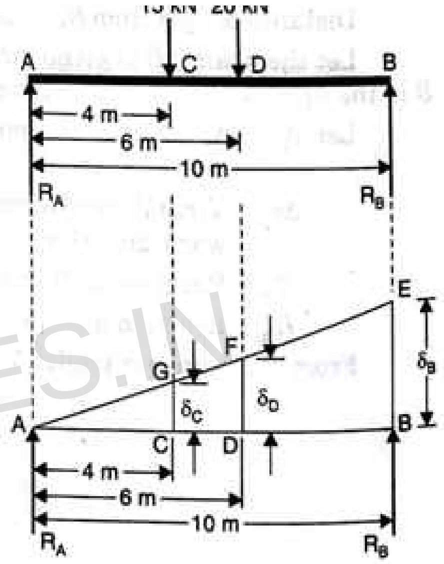
R_B = Reaction at B.

Let the beam AB is given a virtual displacement at B in the upward direction, keeping the end A intact.

Let δ_B = Virtual displacement at B in upward direction.

δ_D = Virtual displacement at D in upward direction.

δ_C = Virtual displacement at C in upward direction.



... of Fig. 10.5 are similar.

$$\therefore \quad \frac{GC}{AC} = \frac{FD}{AD} = \frac{BE}{AB}$$

or $$\frac{\delta_C}{4} = \frac{\delta_D}{6} = \frac{\delta_B}{10}.$$

Expressing δ_C and δ_D in terms of δ_B, we get

$$\delta_C = \frac{\delta_B}{10} \times 4 = 0.4\,\delta_B \qquad ...(i)$$

and $$\delta_D = \frac{\delta_B}{10} \times 6 = 0.6\,\delta_B \qquad ...(ii)$$

Virtual work done by reaction

$R_B = R_B \times$ Virtual displacement at B

$= R_B \times \delta_B$ kN m

(Work done is +ve as reaction R_B is in upward direction and also virtual displacement is in upward direction)

Virtual work done by point load at D

$= -$ Load at $D \times$ Virtual displacement at D.

(Work done is –ve as load at D is acting downward but virtual displacement is upward)

$= -20 \times \delta_D$

$= -20 \times 0.6\delta_B$ $\qquad$ ($\because$ From equation (ii), $\delta_D = 0.6\,\delta_B$)

$= -12\,\delta_B$ kN m.

Virtual work done by point load at C

$= -$ Load at $C \times$ Virtual displacement at C

(Load at C is acting downward but virtual displacement is upward and hence work is –ve)

$= -15 \times \delta_C$

$= -15 \times 0.4\,\delta_B$ $\qquad$ ($\because$ From equation (i), $\delta_C = 0.4\,\delta_B$)

$= -6\,\delta_B$ kN m

Virtual work done by reaction

$R_A = R_A \times$ Virtual displacement at A

$= R_A \times 0$

($\because$ End A is intact and hence virtual displacement is zero)

Total Virtual work done

$=$ Virtual work done by R_B + Virtual work done by point load at D + Virtual work done by point load at C + Virtual work done by R_A

$= R_B \times \delta_B + (-12\,\delta_B) + (-6\,\delta_B) + 0$

$= R_B \times \delta_B - 12\,\delta_B - 6\,\delta_B$

$= R_B \times \delta_B - 18\,\delta_B$

... the total virtual work done should be zero.

$$\therefore \quad R_B \times \delta_B - 18\,\delta_B = 0$$

or $$R_B = \frac{18\,\delta_B}{\delta_B} = 18 \text{ kN.} \quad \textbf{Ans.}$$

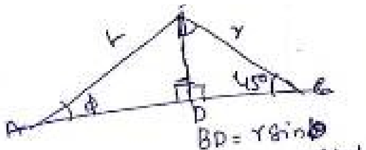
But $$R_A + R_B = 15 + 20 = 35$$

15.a) $N = 210$ rpm

$L = 100$ cm $= 1$m

$r = 20$cm $= 0.2$m

$\theta = 45^\circ$



$BD = r\sin\theta$
$= L\sin\phi$

$$V = L\sin\phi \cdot \frac{d\phi}{dt} + r\sin\theta \cdot \frac{d\theta}{dt} \rightarrow (1)$$

$\frac{d\theta}{dt}$ = Angular velocity of crank

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60}$$

$$= 7\pi \text{ rad/s}$$

$\frac{d\phi}{dt}$ = Angular velocity of connecting rod.

$$\sin\phi = (r/L)\sin\theta$$

$$= \left(\frac{0.2}{1}\right)\sin 45$$

$$= 0.1414$$

$$\phi = 8.129^\circ$$

$$\frac{d\phi}{dt} = \frac{d}{dt}\left(r/L \sin\theta\right) = \frac{d(\sin\phi)}{dt}$$

$$= r/L \cdot \cos\theta \times \frac{d\theta}{dt} = \cos\phi$$

$$\frac{d\phi}{dt} = r/L \cdot \frac{\cos\theta}{\cos\phi} \cdot \frac{d\theta}{dt}$$

$$= 0.2 \cdot \frac{\cos(45)}{} \cdot (7\pi) = 3.13 \text{ rad/s}$$

$(1) \Rightarrow$

$$\therefore V = 1 \times \sin(8.129) \cdot 3.13 + 0.2 \times \sin(45) \cdot 7\pi$$

$$= 0.443 + 3.108$$

$$V = 3.551 \text{ m/s}$$

15(b)

Acceleration of elevator, $a = 3\ m/s^2$

Weight of the operator, $W_2 = 700$ N

When the operator is standing on the scale, placed on the floor of the elevator, the reading of the scale will be equal to the reaction (R) offered by the floor on the operator.

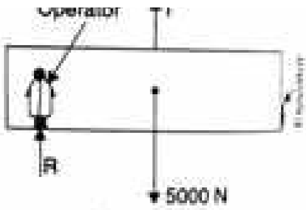


Fig. 15.15

Hence let R = Reaction offered by floor on operator. This is also equal to the reading of scale.

T = Total tension in the cables of elevator.

Consider the motion of operator. The operator is moving upwards alongwith the elevator with an acceleration, $a = 3\ m/s^2$. The net force on the operator is acting upwards.

$\therefore$ Net upward force on operator

= Reaction offered by floor on operator − Weight of operator

$= (R - 700)$

Mass of operator $= \dfrac{\text{Weight of operator}}{g} = \dfrac{700}{g}$

But, net force = mass × acceleration

$\therefore \quad (R - 700) = \dfrac{700}{9.8} \times 3 \qquad (\because \text{ Acceleration} = 3\ m/s^2)$

$\therefore \quad R = 700 + \dfrac{700}{9.8} \times 3 = 700 + 214.28 = \mathbf{914.28\ N.}$ **Ans.**

Total tension in the cables of elevator.

Let T = Total tension in the cables of elevator

W = Total weight (*i.e.*, weight of elevator + weight of operator)

$= 5000 + 700 = 5700$ N.

As the elevator with the operator is moving upwards with an acceleration $(= 3\ m/s^2)$ the

$\therefore$ Net upward force on elevator and operator

= Total tension in the cables–Total weight of elevator and operator

$= (T - 5700)$

Mass of elevator and operator

$= \dfrac{\text{Total weight}}{g} = \dfrac{5700}{9.80}$

But, net force = mass × acceleration

$\therefore \quad (T - 5700) = \dfrac{5700}{9.80} \times 3$

$\therefore \quad T = 5700 + \dfrac{5700}{9.80} \times 3 = 5700 + 1745 = \mathbf{7445\ N.}$ **Ans.**

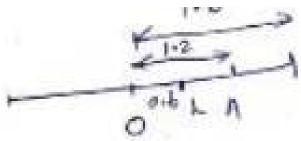
16(a)

$r = 1.6\,m$

$T = 4\,s.$

$x_1 = 1.2\,m$

$x_2 = 0.6\,m$



$$T = 2\pi/\omega$$

$$\therefore \omega = 2\pi/T = \underline{1.57\,rad/s}$$

$$x_1 = r\cos\omega t_1$$

$$1.2 = 1.6\cos\left(1.57\,t_1 \times \frac{180}{\pi}\right)$$

$$\cos\left(1.57\,t_1 \times \frac{180}{\pi}\right) = 0.75$$

$$t_1 = \underline{0.48\,s}$$

$$x_2 = r\cos\omega t_2$$

$$0.6 = 1.6\cos\left(1.57\,t_2 \times 180/\pi\right)$$

$$t_2 = \underline{0.79\,sec}$$

$$t = t_2 - t_1$$

$$= \underline{0.315\,sec.}$$

(b)

Stiffness of spring = $50/10$ = $5\ N/mm$

$$f = \frac{1}{100} \quad 100/60 =$$

$$T = 6/10 = \underline{\underline{0.6\ s}}.$$

$$T = 2\pi \sqrt{\frac{\text{Static Extension}}{g}}$$

$$0.6 = 2\pi \sqrt{e/g}$$

$$\left(\frac{0.6}{2\pi}\right)^2 = K/g$$

$$e = \left(\frac{0.6}{2\pi}\right)^2 \times 9.81$$

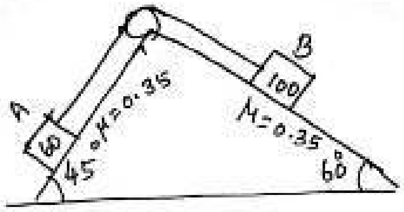
$$= \underline{0.089\ m}.$$

But stiffness; $= \dfrac{\text{weight attached}}{\text{static extension}}$

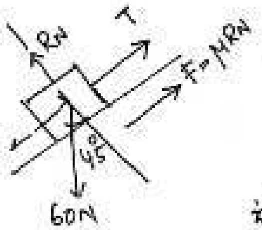
$$5000 = \frac{W}{0.089}$$

$$W = 447\ N$$

17)



Consider downward motion of 60N block



$a_A = a_B = a$

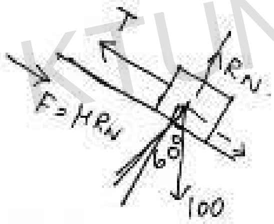
Net force $= m_A \cdot a$

$$60 \sin 45 - T - \mu R_N = \left(\frac{60}{9.81}\right) a \longrightarrow$$

$$60 \sin 45 - T - 0.35 \times 60 \cos 45 = \left(\frac{60}{9.81}\right) a$$

$$27.58 - T = 6.11 a \longrightarrow (1)$$

Consider upward motion of Block B



$$T - 100 \sin 60 - 0.35 \times 100 \cos 60 = \left(\frac{100}{9.81}\right) a$$

$$T - 104.10 = 10.194 a \longrightarrow (2)$$

$$6.11 a + T = 27.58$$
$$10.194 a - T = -104.10$$
$$16.304 a = -76.52$$
$$a = -4.69 \text{ m/s}$$

$T = \underline{56.235}$


KTUNOTES

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE MODEL EXAMINATION, CAPE, NOVEMBER 2017

BE 100 – ENGINEERING MECHANICS

Max. Marks: 100 **PART A** Time: 3 hrs

(Answer all questions. Each question carries 5 marks)

1. State and prove Varignon's theorem.
2. A simply supported beam AB of span 4m is carrying point loads 5kN, 2kN and 3kN at 1m, 2m and 3m respectively from the support A. It also carries a uniformly distributed load of 2kN/m over the entire span of the beam. Find the support reactions.
3. State and explain the theorems of Pappus and Guldinus.
4. State any five laws of friction.
5. Explain the term instantaneous centre of rotation. How can it be located for a body under combined motion of rotation and translation?
6. A lift is moving upward with an acceleration of $3m/s^2$. Find the reaction from the floor of the lift on which a man weighing 70kg is standing.
7. State D' Alembert's principle. Using this principle derive expression for the motion of two bodies connected over a smooth pulley.
8. What are the general conditions of simple harmonic motion?

PART B

(Answer two questions from each set)

SET 1

(Each question carries 10 marks)

9. ABCD is a rectangle where AB = 40mm and BC = 30mm. E is the middle point of AB. Forces of magnitude 18, 20, 14, 16, 10 and 8N act along AB, CB, CD, EC, DE and AD respectively. Find the magnitude, direction and position of the equilibrant. B is to the right of A and taken in the anticlockwise direction. (10)
10. Find the reactions at the points of contact of the cylinders placed in a trench as shown in **figure 1** (on page 3). (10)
11. a) State the conditions of equilibrium for a set of coplanar forces. (3)

    b) Determine the support reactions for the beam shown in **figure 2** (on page 3). (7)

SET 2

(Each question carries 10 marks)

12. Find the moment of inertia of the section shown in **figure 3** (on page 3) about the centroidal axes. (10)

13. A uniform ladder of weight 300N and 4m length rests against a vertical wall with which it makes an angle of 60˚. The coefficient of friction between the ladder and the wall is 0.3 and that between ladder and floor is 0.4. If a man, whose weight is 750N, ascends it, how high will he be when the ladder slips? (10)

14. a) An area A has the following properties. $Ix = 6.4x10^6$ mm^4, $Iy = 16x10^6$ mm^4 and $Ixy = 6.4x10^6$ mm^4. Calculate maximum and minimum principal moment of inertia. (5)

    b) A simply supported beam of span 5m is loaded with a concentrated load of 4kN at a distance of 1m from right end. The beam is also loaded with a uniformly distributed load of 2kN/m length over a distance of 2m from the left end of the beam. Find the reactions at the supports of the beam using principle of virtual work. (5)

SET 3

(Each question carries 10 marks)

15. For a reciprocating pump, crank rotates at a uniform speed of 300 rpm. The length of crank and connecting rod are 12cm and 50cm respectively. Find (i) the angular velocity of the connecting rod AB and (ii) the velocity of piston when the crank makes an angle 30˚ with the horizontal. (10)

16. Two blocks are placed on the inclined planes as shown in **figure 4** (on page 3). They are connected by a string passing over a frictionless pulley. If the coefficient of friction between planes and blocks is 0.3, find the resulting acceleration and tension in the string. (10)

17. a) A particle has simple harmonic motion. Its maximum velocity was 6m/s and the maximum acceleration was found to be $12m/s^2$. Determine the angular velocity and amplitude. Also determine its velocity and acceleration when displacement is half of the amplitude. (5)

    b) A spring stretches by 0.015m when a 1,75kg object is suspended from its end. How much mass should be attached to the spring so that its frequency of vibration is 3Hz? (5)

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE MODEL EXAM, CAPE, NOV.'17

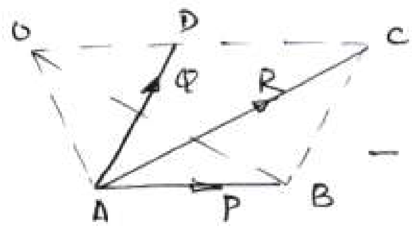
BE-100 - ENGINEERING MECHANICS

SCHEME OF EVALUATION AND ANSWER KEY.

PART A

1. Varignon's Theorem

- It states that if a no. of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point — $1\frac{1}{2}$ mark.



— $\frac{1}{2}$ mark

Proof: Consider two forces P & Q represented in mag. and distn. by AB & AC. Let O be the point about which the moments are to be taken. Through O draw a line parallel to AB to meet the line of action of Q at C. Now complete the parallelogram ABCD with AB & AC as adjacent sides. Then from parallelogram law of forces, the diagonal AD represents the resultant force in both mag. & distn.

Moment of P about O = 2 × area of ΔAOB = M_P

Moment of Q about O = 2 × area of ΔAOC. = M_Q

Moment of R about O = 2 × area of ΔAOD. = M_R

From geometry,

Ar. of ΔAOD = Ar. of ΔAOC + Ar. of ΔACD

But Ar. of ΔACD = Ar. of ΔADB = Ar. of ΔAOB

($\because$ ΔAOB & ΔADB are on the same base AB and between the same parallel lines).

$\therefore$ Ar. of ΔAOD = Ar. of ΔAOC + Ar. of ΔAOB

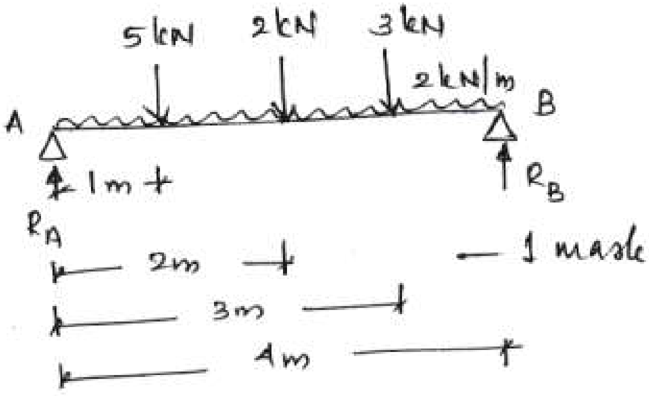
Multiplying both sides by 2

2 × Ar. of ΔAOD = 2 × Ar. of ΔAOC + 2 × Ar. of ΔAOB

ie, $M_R = M_P + M_Q$. — 3 marks.

2. Eqbm. eqns. are

$\Sigma V = 0, \Sigma H = 0, \Sigma M = 0$ — 1 mark.



— 1 mark

$\Sigma V = 0$,

$R_A + R_B - 5 - 2 - 3 - 2\times 4 = 0$.

$R_A + R_B = 18$ — ① — 1 mark.

$\Sigma M_A = 0$, $R_B \times 4 = 5\times 1 + 2\times 2 + 3\times 3 + 2\times 4\times 2$ — 1 mark.

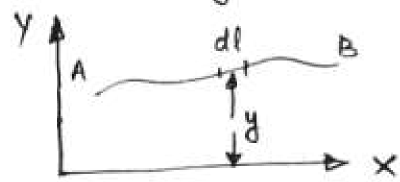
$\underline{R_B = 8.5\ kN}$ — ½ mark.

Substituting in ①, $\underline{R_A = 9.5\ kN}$ — ½ mark.

3. Theorems of Pappus - Guldinus.

Theorem 1 - the area of the surface generated by rotating any plane curve about a non-intersecting axis in its plane is equal to the product of the length of the curve and the distance travelled by its centroid.

— 1 mark



— ½ mark.

Let AB be any plane curve of length l that lies in the x-y plane and does not intersect the x-axis. Consider an elemental length dl. Let y-coordinate of its mid point be y and that of centroid of the entire curve be $\bar{y}$.

When this curve is rotated about the x-axis, then area generated by the elementary length $= 2\pi y\, dl$

Area generated by the entire line $= \int 2\pi y\, dl$

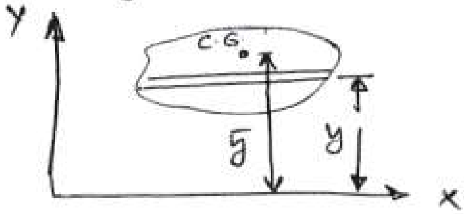
$\bar{y} = \dfrac{\int y\, dl}{\int dl} \Rightarrow \int y\, dl = \bar{y} l$ $\therefore$ Area generated $= l\, 2\pi \bar{y}$

$2\pi\bar{y}$ is the distance travelled by centroid of the curve

$\therefore$ Area generated = length of generating curve × distance travelled by centroid of the curve — 1 mark

Theorem 2 - The volume of the solid generated by rotating any plane figure about a non-intersecting axis in its plane is equal to the product of area of the figure and distance travelled by its centroid.

– 1 mark.



– ½ mark.

Consider area A of any plane figure divided into a large no. of very thin strips parallel to the x-axis. Let the y-cordinate of the middle of any strip of area dA be y and the centroid of the entire curve be $\bar{y}$.

When the plane of the figure is rotated about x-axis,

Volume generated by the elementary area $= 2\pi \bar{y} \cdot dA$.

Volume generated by the entire area $= \int 2\pi y \, dA$

$\int y \, dA = \bar{y} A$, $\therefore$ Volume generated $= A \cdot 2\pi \bar{y}$

$2\pi \bar{y}$ - distance travelled by the centroid of the area being rotated.

$\therefore$ Volume generated = area of the figure x distance travelled by the C.G.

– 1 mark.

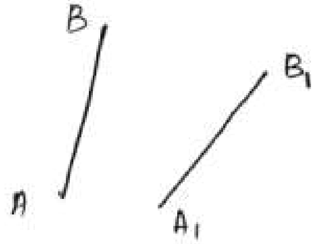
4. Laws of Friction

1. Force of friction acts in a direction opposite to the direction in which the body moves or tends to move.
2. Till the limiting value is reached, the magnitude of friction is equal to the force which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction.
4. Force of friction depends upon the roughness of the surfaces in contact.

5. Force of friction is independent of the shapes and areas of the surfaces in contact.

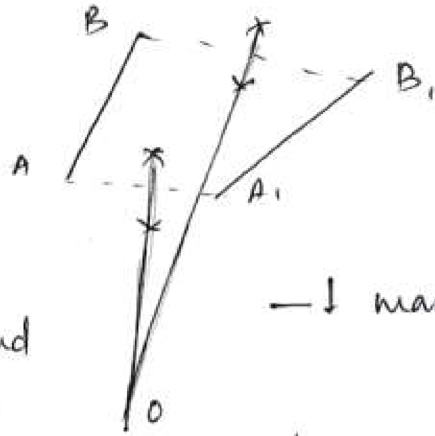
Five laws - 1 mark each - $1 \times 5 = 5$ marks.

5. Instantaneous centre



Consider a link AB which moves from its initial position AB to A_1B_1 in a short interval of time. The link has neither entirely motion of translation nor entirely motion of rotation, but a combination of the two. This combined motion of rotation and translation, may be assumed to be a motion of pure rotation about some centre. As the position of link AB goes on changing, the centre, about which the motion of rotation is assumed to take place, also goes on changing, from one instant to another, is known as Instantaneous centre. - 2 marks.

Position by graphical method



Consider the link AB which changed its position to A_1B_1 in a short interval of time.

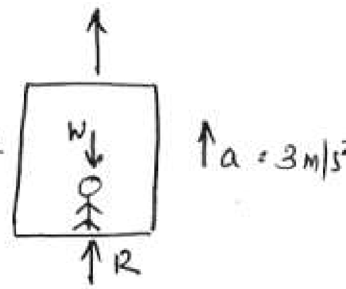
— 1 mark

Draw right bisectors of chord AA_1 and chord BB_1. Let these two bisectors meet at O. Then O is the instantaneous centre

— 2 mark

Velocity at A be V_A & at B be V_B

then $\frac{V_A}{OA} = \frac{V_B}{OB}$



6. Net force $=$ R - W, — 1 mark.

According to D'Alembert's principle R - W + (-ma) = 0 — 1 mark

R - W = ma

m = 70 kg, g = 9.81 m/s²

$R = W + ma = 70 \times 9.81 + 70 \times 3$ — 2 marks

$= \underline{\underline{896.7\ N}}$

$\therefore$ Reaction from floor $= \underline{896.7\ N}$ — 1 mark

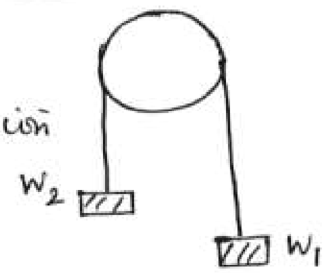
7. <u>D'Alembert's principle</u>

states that the net external force acting on the system and the resultant reversed effective force (or inertia force) are in dynamic equilibrium.

ie, Net force + reversed effective force = 0

ie, $F + (-ma) = 0$ — 2 marks.

<u>Expression for the motion of two bodies connected over a smooth pulley</u>



Let $W_1 > W_2$. Let 'a' be the acceleration

Net external force acting in the dirn. of motion $= W_1 - W_2$

Reversed effective force on $W_1 = -\frac{W_1}{g} a$

Reversed effective force on $W_2 = -\frac{W_2}{g} a$

$\therefore$ Resultant reversed effective force $= -\frac{a}{g}(W_1 + W_2)$

According to D'Alembert's principle,

Net external force + resultant reversed effective force = 0

$$(W_1 - W_2) - \frac{a}{g}(W_1 + W_2) = 0.$$

$$a = \frac{g(W_1 - W_2)}{W_1 + W_2}$$ — 3 marks.

8. <u>General conditions of SHM</u>

(1) the acceleration of the body performing periodic motion should be proportional to the distance of the

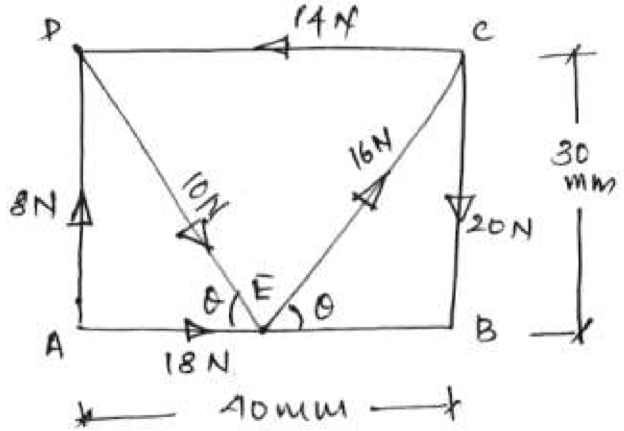
body from a fixed point called mean position of the body. — 2.5 marks.

(2) the acceleration of the body should be directed towards the mean position. — 2.5 marks.

PART B

SET 1

9.



— 2 marks.

$\tan\theta = \frac{30}{20}$

$\theta = 56.31°$ — 1 mark

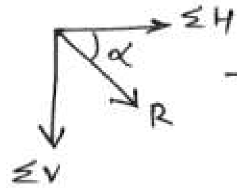
$R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$

$\Sigma V = -20 + 8 + 16 \sin 56.31 - 10 \sin 56.31 = -7.01\,N$ — 1 mark

$\Sigma H = 18 - 14 + 16 \cos 56.31 + 10 \cos 56.31 = 18.42\,N$ — 1 mark

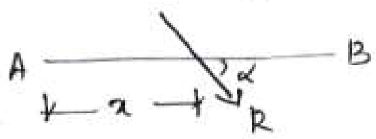
$R = \sqrt{(-7.01)^2 + (18.42)^2} = \underline{\underline{19.71\,N}}$ - 1 mark

$\tan\alpha = \frac{\Sigma V}{\Sigma H} = \frac{7.01}{18.42} \Rightarrow \alpha = \underline{\underline{20.84°}}$



— 1 mark

Let R be at a distance 'x' along AB as shown below

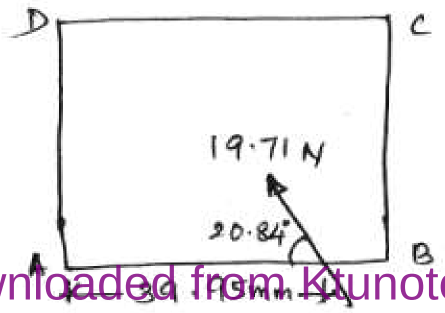


Applying Varignon's theorem,

$10 \sin 56.31 \times 20 - 16 \sin 56.31 \times 20 + 20 \times 40 - 14 \times 30 = 19.71 \sin 20.84 \times x$ — 1 mark

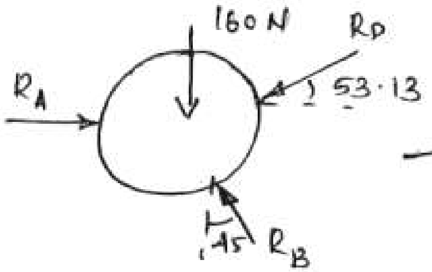
$\Rightarrow x = \underline{\underline{39.95\,mm}}$ - 1 mark.

∴ Equilibrant is as shown below

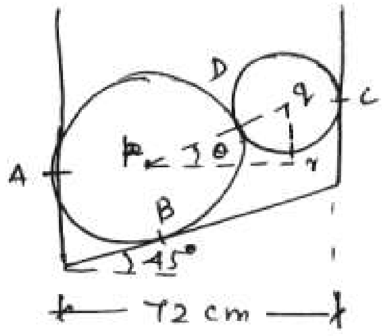


— 1 mark

10. FBD of 160 N



— 2 marks

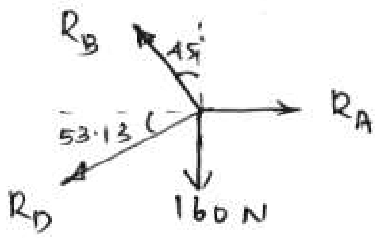


$pq = \frac{60}{2} + \frac{30}{2}$

$pr = 72 - \left(\frac{60}{2} + \frac{30}{2}\right)$

$\cos\theta = \frac{pr}{pq} = \frac{27}{45}$

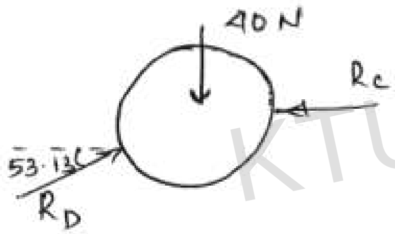
$\theta = 53.13^\circ$ — 1 mark

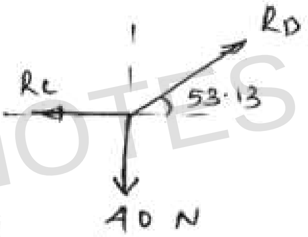


$\Sigma V = 0$, $R_B \cos 45 - R_D \sin 53.13 - 160 = 0$ — (1) — 1 mark

$\Sigma H = 0$, $R_A - R_D \cos 53.13 - R_B \sin 45 = 0$ — (2) — 1 mark

FBD of 40 N





— 2 marks

Applying Lami's Theorem

$$\frac{R_c}{\sin 143.13} = \frac{R_D}{\sin 90} = \frac{40}{\sin 126.87}$$ — 1 mark

$R_c = 30\,N$

$R_D = 50\,N$ } 1 mark

Substituting in eqns. (1) & (2)

$R_B = 282.84\,N$

$R_A = 230\,N$ } 1 mark

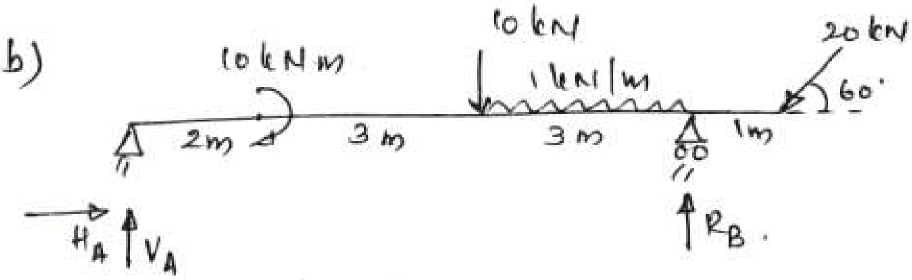
11 a) $\Sigma V = 0$ ie, algebraic sum of components of all the forces acting along vertical dirn. is zero

$\Sigma H = 0$ ie, algebraic sum of components of all the forces acting along horizontal dirn. is zero

$\Sigma M = 0$ ie, algebraic sum of moments of all these forces about any point in the body is zero

1 × 3 = 3 marks

11 b)



Eqbm. eqns. are $\Sigma V = 0$, $\Sigma H = 0$, $\Sigma M = 0$. — 1 mark

$\Sigma V = 0$, $V_A + R_B - 10 - 1\times 3 - 20\sin 60 = 0$ — 1 mark

$V_A + R_B = 30.32$

$\Sigma H = 0$, $H_A - 20\cos 60 = 0$

$H_A = 20\cos 60 = 10$

$\underline{\underline{H_A = 10\,kN}}$ ($\rightarrow$) — 1 mark

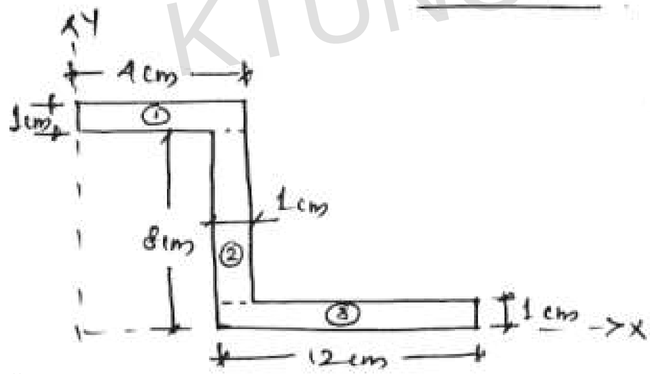
$\Sigma M_A = 0$, $10 + 10\times 5 + 1\times 3\times 6.5 - R_B\times 8 + 20\sin 60\times 9 = 0$ — 2 marks

$R_B = \underline{\underline{29.42\,kN}}$ ($\uparrow$) $\longrightarrow$ 1 mark

$\therefore V_A = \underline{\underline{0.9\,kN}}$ ($\uparrow$) $\longrightarrow$ 1 mark

SET 2

12.



$\bar{x} = \dfrac{\Sigma a x}{\Sigma a}$

$\bar{y} = \dfrac{\Sigma a y}{\Sigma a}$

Location of C.G.

Portion	a	x	y.
①	4×1	2	8.5
②	7×1	3.5	4.5
③	12×1	9	0.5

$\bar{x} = 6.11$ cm — 2 marks

$\bar{y} = 3.11$ cm. — 2 marks

Moment of Inertia

$I_{xx} = I_{G_{xx}} + A h_x^2$

Portion	$I_{G_{xx}}$	$h_x = y - \bar{y}$
1	$\frac{4 \times 1^3}{12}$	$8.5 - 3.11$
2	$\frac{1 \times 7^3}{12}$	$4.5 - 3.11$
3	$\frac{12 \times 1^3}{12}$	$0.5 - 3.11$

$I_{xx} = \underline{\underline{241.39 \text{ cm}^4}}$

— 3 marks

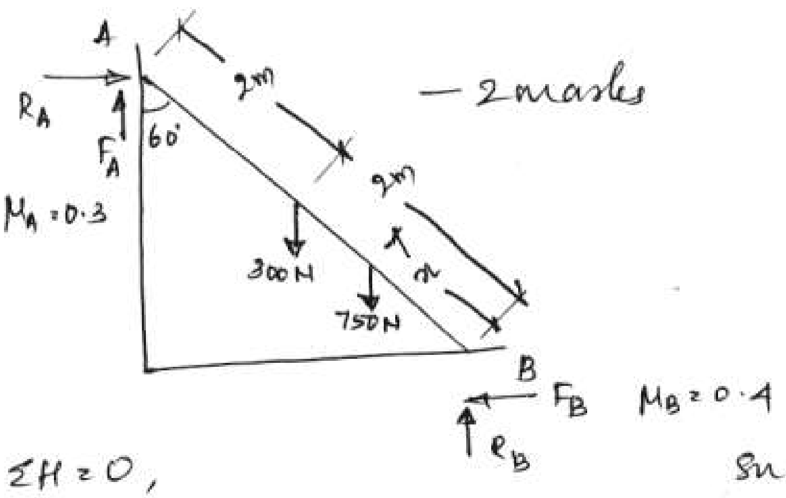
$I_{yy} = I_{G_{yy}} + A h_y^2$

Portion	$I_{G_{yy}}$	$h_y = x - \bar{x}$
1	$\frac{1 \times 4^3}{12}$	$2 - 6.11$
2	$\frac{7 \times 1^3}{12}$	$3.5 - 6.11$
3	$\frac{1 \times 12^3}{12}$	$9 - 6.11$

$I_{yy} = \underline{\underline{365.39 \text{ cm}^4}}$

— 3 marks.

13.



— 2 marks

$\Sigma V = 0$,

$F_A + R_B = 300 + 750$

$\mu_A R_A + R_B = 1050$ — (1)

— 1 mark

$\Sigma H = 0$,

$R_A - F_B = 0$

$R_A = \mu_B R_B$ — (2)

— 1 mark

Substituting (2) in (1)

$\mu_A \mu_B R_B + R_B = 1050$.

$R_B = \underline{\underline{937.5 \text{ N}}}$ — 1 mark

$\therefore R_A = \underline{\underline{375 \text{ N}}}$ — 1 mark

$\Sigma M_B = 0.$ ⌐ 2 marks

$$F_A \times 4 \sin 60 + R_A \times 4 \cos 60 - 300 \times 2 \cos 30 - 750 \times x \cos 30 = 0$$

i.e., $\mu_A R_A \times 4 \sin 60 + R_A \times 4 \cos 60 = 300 \times 2 \cos 30 + 750 \times x \cos 30$

$$(0.3 \times 4 \sin 60 + 4 \cos 60)\, 375 = 600 \cos 30 + 750 \cos 30 \times x.$$

$\underline{\underline{x = 0.955\,m}}$ — 2 marks.

14 a) Maximum principal moment of inertia

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + (I_{xy})^2}$$ — 1 mark

$$= \frac{6.4 \times 10^6 + 16 \times 10^6}{2} + \sqrt{\left(\frac{6.4 \times 10^6 - 16 \times 10^6}{2}\right)^2 + (6.4 \times 10^6)^2}$$

$$= 11.2 \times 10^6 + 8 \times 10^6 = \cancel{19.2 \times 10^6\, mm^4}$$

$$= \underline{\underline{19.2 \times 10^6\, mm^4}}$$ — 1½ marks.

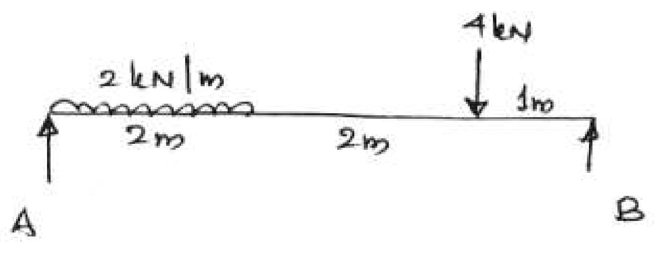
Minimum principal moment of inertia

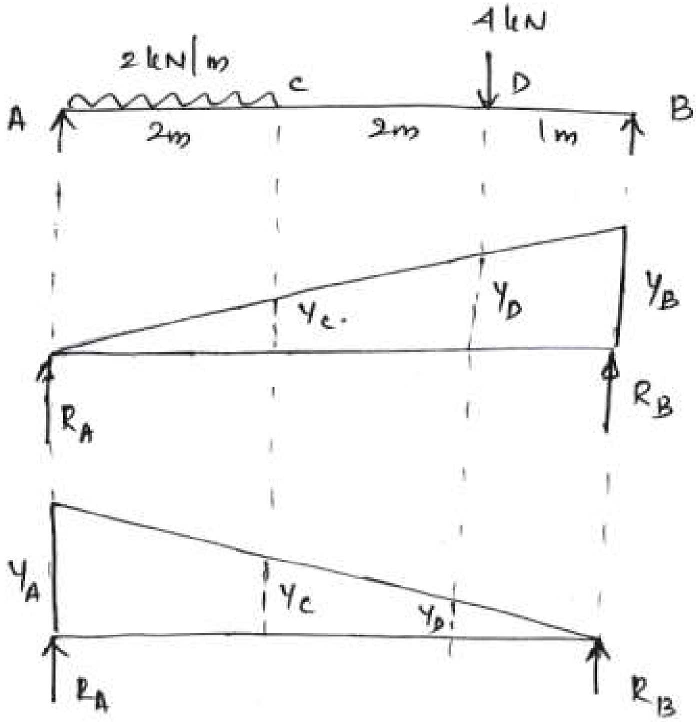
$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + (I_{xy})^2}$$ — 1 mark

$$= 11.2 \times 10^6 - 8 \times 10^6$$

$$= \underline{\underline{3.2 \times 10^6\, mm^4}}$$ — 1½ marks.

b)





(a) — ½ mark

(b) — ½ mark

To calculate R_B — Pivot at A and give virtual displacement at B. (figure (a))

Work done by $R_A = 0$. $\qquad y_A = 0$

Work done by UDL $= -\left[\frac{y_A + y_C}{2} \times 2 \times 2\right] = -\left[2(y_A + y_C)\right]$

$= -2y_C$.

Work done by 4 kN $= -4 \times y_D$.

Work done by $R_B = R_B y_B$.

From principle of virtual work, total virtual work done $= 0$

ie, $R_A \times 0 - 2y_C - 4y_D + R_B y_B = 0$. — 1 mark.

From similar Δles, $\frac{y_B}{5} = \frac{y_D}{4} = \frac{y_C}{2}$

$y_D = \frac{4}{5} y_B$ & $y_C = \frac{2}{5} y_B$.

ie, $-2 \times \frac{2}{5} y_B - 4 \times \frac{4}{5} y_B + R_B y_B = 0$.

$R_B = \frac{4}{5} + \frac{16}{5} = \underline{\underline{4 \text{ kN}}}$ — 1 mark.

Similarly to calculate R_A, give virtual displacement as shown in figure (b).

$$R_A Y_A - 2\left(\frac{Y_A + Y_C}{2}\right)\times 2 - 4 \times Y_D + R_B \times 0 = 0$$ — 1 mark.

From similar Δ^{les}, $\frac{Y_A}{5} = \frac{Y_C}{3} = \frac{Y_D}{1}$

$$Y_C = \frac{3}{5} Y_A \;,\; Y_D = \frac{Y_A}{5}$$

ie, $R_A Y_A - 2\left(Y_A + \frac{3}{5} Y_A\right) - 4 \times \frac{Y_A}{5} = 0.$

$$R_A - 2 - \frac{3}{5} - \frac{4}{5} = 0$$

$$\underline{\underline{R_A = 3.4\ kN}}$$ — 1 mark

SET 3

15.

— 1 mark

OA = 12 cm
AB = 50 cm

$$\omega_{OA} = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.4\ rad/s$$ — 1 mark

$$V_A = \omega_{OA} \times OA = 31.4 \times 0.12$$

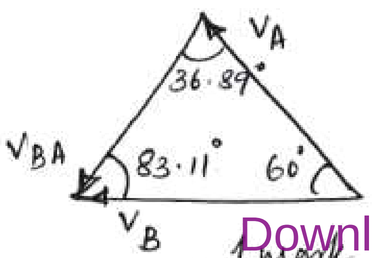
$= 3.77\ m/s$ ($\perp^r$ to OA, 60° inclined with horz.) — 1 mark

$$V_B = V_A + V_{BA}.$$

Let the inclination of AB with horizontal be ϕ, then

$OA \sin 30 = AB \sin\phi \Rightarrow \phi = 6.89°$ — 1 mark

V_{BA} is $\perp^r$ to AB or inclined $90 - 6.89 = 83.11°$ with horz.



$$\frac{V_A}{\sin 83.11} = \frac{V_B}{\sin 36.89} = \frac{V_{BA}}{\sin 60}$$ — 1 mark

$V_{BA} = 3.29\ m/s$. — 1 mark

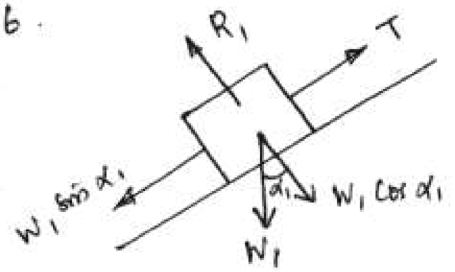
Velocity of piston $V_B = \underline{2.28\ m/s}$ — 1 mark

Angular velocity of connecting rod,

$$\omega_{AB} = \frac{V_{BA}}{AB}$$

$= \underline{\underline{6.58 \text{ rad/sec}}}$ — 2 marks

16.



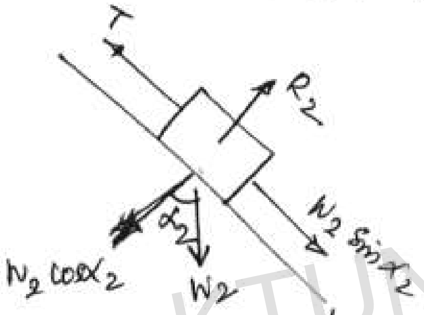
— 1 mark

$$W_1 \sin\alpha_1 - (T + F_1) = \frac{W_1}{g} a$$

$$W_1 \sin\alpha_1 - T - \mu W_1 \cos\alpha_1 = \frac{W_1}{g} a$$

ie, $15 \times 9.81 \times \sin 30 - T - 0.3 \times 15 \times 9.81 \times \cos 30 = 15a$

$35.34 - T = 15a$ —— (1). — 3 marks.



— 1 mark

$$T - (W_2 \sin\alpha_2 + F_2) = \frac{W_2}{g} a$$

$$T - 5 \times 9.81 \sin 15 - \mu W_2 \cos\alpha_2 = \frac{W_2}{g} a$$

ie, $T - 5 \times 9.81 \sin 15 - 0.3 \times 5 \times 9.81 \cos 15 = 5a$

$T - 26.91 = 5a$ —— (2) — 3 marks

Solving (1) & (2) $a = \underline{\underline{0.42 \text{ m/s}^2}}$ — 1 mark

$\underline{\underline{T = 29.01 \text{ N}}}$ — 1 mark.

17 a) $V_{max} = r\omega = 6 \text{ m/s}$ Let amplitude be X } — 1 mark

$a_{max} = -\omega^2 r = 12 \text{ m/s}^2$

$\omega X = 6.$

Angular velocity $= \frac{6}{X}$

$\omega^2 X = 12. \Rightarrow \omega(\omega X) = 12$

$\omega \times 6 = 12$ $\therefore \underline{\underline{X = 3 \text{ m}}}$

$\underline{\underline{\omega = 2 \text{ rad/sec}}}$ — 2 marks when $r = \frac{X}{2}$ ie $= 1.5\,m$,

$$V = \omega \times r = 2 \times 1.5 = \underline{\underline{3\,m/s}}$$

$$a = \omega^2 r = 2^2 \times 1.5 = \underline{\underline{6\,m/s^2}}$$

2 marks.

b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ — 1 mark

when $m = 1.75\,kg$, $\delta = 0.015\,m$

$mg = k\delta$ — 1 mark

$k = \frac{mg}{\delta} = \frac{1.75 \times 9.81}{0.015} = 1144.5\,N/m$ — 1 mark

$3 = \frac{1}{2\pi}\sqrt{\frac{1144.5}{M}} \Rightarrow (3 \times 2\pi)^2 = \frac{1144.5}{M}$ — 1 mark

$M = \frac{1144.5}{(6\pi)^2} = \underline{\underline{3.22\,kg}}$ — 1 mark.

∴ The mass to be attached to the spring $= \underline{\underline{3.22\,kg}}$


KTUNOTES

Name:_______________________________________ Roll No:________

COOPERATIVE ACADEMY OF PROFESSIONAL EDUCATION

FIRST SEMESTER B.TECH DEGREE MODEL EXAMINATION –NOV, 2017

Course Code: **BE100**

Course Name: **ENGINEERING MECHANICS**

Max.Marks: 100 Duration: 3 Hours

Part A

Answer all questions. 5 marks each

1. State the fundamental principles of Engineering Mechanics.
2. The resultant of two forces when they act at an angle of 60° is 14N. If the same forces are acting at right angles, their resultant is $\sqrt{(136)}$ N. Determine the magnitude of two forces.
3. State arid prove Parallel axis theorem
4. Explain Laws of Friction
5. State D'Alembertz Principle.
6. Explain the concept of instantaneous centre
7. Differentiate between free vibration and forced vibration of bodies
8. Explain the following terms with respect to a simple harmonic motion (a) amplitude (b) time period (c) frequency

Part B

Answer any two questions from each SET.

SET 1

Each question carries 10 marks

9. (i) Distinguish between composition and resolution of forces with suitable example.

   (ii) Two arbitrary axes OA and OB are at an angle 105° each other. A force of 500N is acting at 45° with axis OA. Find the components of given forces along axes OA and OB

10. Two smooth circular cylinders each of weight 150 N and radius 120 mm are connected at their centres by a string AB of length 340 mm and rest upon a .horizontal plane as shown in Figure-1. The cylinder above them has a weight 250 N and radius of 120 mm. Find the tension in the string AB and the reaction exerted by the floor at the points of contact D and E.

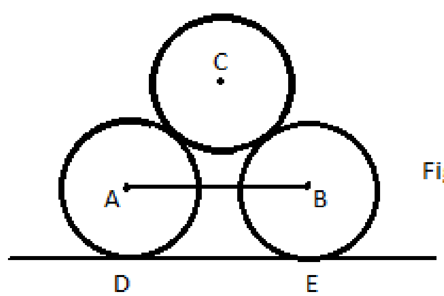


Figure-1

11. Replace the given system of forces to (i) a single force (ii) an equivalent force-couple system at B. (Figure–2)

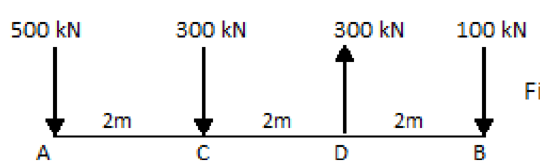
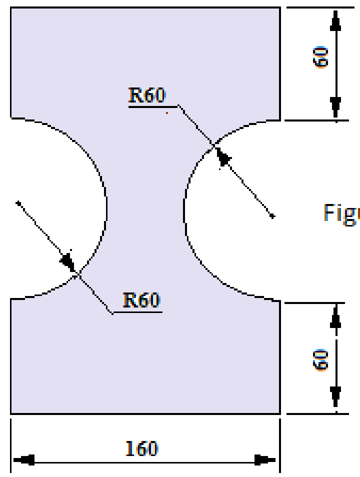


Figure-2

SET 2

Each question carries 10 marks.

12. Find second moment of area of cut section as shown in figure 3 about its centroidal axes



13. What is Principal axis and Principal moment of inertia? Given lamina of area 'A' has I_X = 6.4 x 10^6 mm^4, I_Y = 16 x 10^6 mm^4 and I_{XY} = 6.4 x 10^6 mm^4. Calculate Principal Moment of Inertia.
14. A uniform ladder of length 8m and weight W is leaning against a wall. It makes 45° with the horizontal. A man whose weight is 0.6 times that of ladder goes up the ladder. Determine the maximum distance he can climb before the ladder slips. Assume coefficient of friction between the ladder and wall to 0.25 and that between the ladder and floor to be 0.3

SET 3

Each question carries 10 marks.

15 A lift has an upward acceleration of 1.2 m/s^2. What force will a man weighing 750 N exert on the floor of the lift? What force would he exert if the lift had an acceleration of 1.2 m/s^2 downwards? What upward acceleration would cause his weight to exert a force of 900 N on the floor?

16. In the reciprocating engine mechanism, the crank OA rotates at a uniform speed of 350 rpm. The length of the crank and connecting rod are 120 mm and 500 mm respectively. Find the angular velocity of the connecting rod and velocity of the piston when the crank makes an angle of 30° with horizontal
17. A body is moving with simple harmonic motion and has velocities of 8m/s and 3m/s at a distance of 1.5m and 2.5m respectively from the centre. Find the amplitude and time period of the body.

COOPERATIVE ACADEMY OF PROFESSIONAL EDUCATION

ANSWER BOOK for

FIRST SEMESTER B.TECH DEGREE MODEL EXAMINATION –NOV, 2017

Course Code: **BE100**

Course Name: **ENGINEERING MECHANICS**

Max.Marks: 100 Duration: 3 Hours

Part A

Answer all questions. 5 marks each

1. *State the fundamental principles of Engineering Mechanics.*

Newton's First Law of Motion: A body continues in it's state of rest or of uniform motion unless it is compelled by an external force.

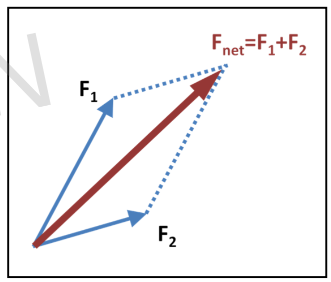
Newton's Second Law of Motion: The acceleration of an object as produced by a net force acting on it is directly proportional to the magnitude of the net force, and act in the same direction of the net force, and inversely proportional to the mass of the object.

Newton's Third Law of Motion: For every action there is an equal and opposite reaction.

Newton's Law of Gravitation: Newton's law of universal gravitation states that a particle attracts every other particle in the universe using a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers

Principle of Transmissibility of Forces: states that the conditions of equilibrium or conditions of motion of a rigid body will remain unchanged if a force acting at a give point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action

Parallelogram Law to find Resultant Force (optional): If two forces acting simultaneously at a point are represented in magnitude and direction by the adjacent sides of a parallelogram, then the diagonal of the parallelogram passing through the point of concurrency represents the resultant of the two forces in both magnitude and direction.



2. *The resultant of two forces when they act at an angle of 60° is 14N. If the same forces are acting at right angles, their resultant is √(136) N. Determine the magnitude of two forces.*

We have by parallelogram law, $R^2 = P^2 + Q^2 + 2PQ\,Cos\theta$ where R is the resultant force of applied forces P and Q and θ is the angle between the forces P and Q.

Case-1: $14^2 = P^2 + Q^2 + 2PQ\,Cos60$

$196 = P^2 + Q^2 + 2PQ\,\frac{1}{2}$

$196 = P^2 + Q^2 + PQ$ ------------(1)

Case-2: $136 = P^2 + Q^2 + 2PQ\,Cos90$

$136 = P^2 + Q^2 + 2PQ$ x 0

$136 = P^2 + Q^2$-----------(2)

Subtracting, (1) – (2), gives

$PQ = 60$ So, $2PQ = 120$

Adding $2PQ$ =to eqn (2):

$136 + 120 = P^2 + Q^2 + 2PQ = (P + Q)^2$

$256 = (P + Q)^2$, So $(P + Q) = 16$--------------------------(3)

Subtracting $2PQ = 120$ from eqn (2):

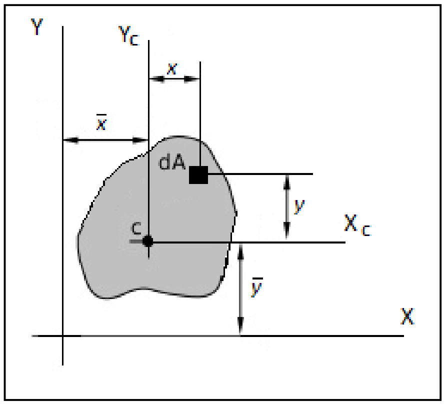
$136 – 120 = P^2 + Q^2 - 2PQ = (P - Q)^2$

$16 = (P - Q)^2$, So $(P - Q) = 4$-----------------------------(4)

Adding eqn (3) and (4), $2P = 20$, Hence P = 10N---------------------------(answer 1)

Subtracting eqn (4) from (3), $2Q = 12$, Hence Q = 6N----------------------(answer 2)
So, in both cases, the forces acting are 10N and 6N.

3. *State and prove Parallel axis theorem.*



Parallel axis theorem or transfer formula is used to determine the second moment of area of a lamina about any axis (say I_{XX} or I_{YY})(refer figure), given the second moment of area of lamina about a parallel axis through the object's center of gravity (I_{Xc} or I_{Yc}) and the perpendicular distance between the axes $\bar{x}$ *or* $\bar{y}$.

Referring figure, the second moment of area of the small element of area dA about the non-centroidal axes XX = $dI_{XX} = (y + \bar{y})^2 dA$

Hence $I_{XX} = \int (y + \bar{y})^2 \, dA = \int y^2 \, dA + \int \bar{y}^2 dA + \int 2y\bar{y} \, dA$

$= \quad I_{X_c} + \bar{y}^2 \int dA + 2\bar{y} \int y \, dA$, since $\bar{y}$ is a constant.

$= \quad I_{X_c} + \bar{y}^2 A + 2\bar{y}$ x 0, since $\int y \, dA$ is the first moment of area about centroidal Xc axis and is equal to zero.

i.e $\quad I_{XX} = I_{X_c} + \bar{y}^2 A$

similarly $\quad I_{YY} = I_{Y_c} + \bar{x}^2 A$

The parallel axis theorem can be stated as the second moment of area of a lamina about an axis in the plane of area is equal to the second moment of area of the lamina about its centroidal axis which is parallel to the given axis plus the product of area of the lamina and square of the distance between the parallel axes.

4. *Explain Laws of Friction*

The experiments by Coulombs give the following results known as laws/characteristics of dry friction:

- The total friction that can be developed is independent of the magnitude of area of contact.
- The total friction that can be developed is proportional to normal force (i.e. reactive force component normal to the surface of contact).
- For low velocities of sliding, the total friction that can be developed is practically independent of velocity. But the force necessary to start the motion is greater than that necessary to maintain the motion.

5. *State D'Alembert's Principle.*

The equation of rectilinear motion of aparticle (from Newton's second law of motion) can be written in the form: $\sum F_i - \sum m\ddot{x} = 0$ where $\sum F_i$ denotes resultant of all applied forces acting on the particle in x direction, *m* is the mass of particle and $\ddot{x}$ is the acceleration of the particle moving in *x* direction. This equation resembles like the equation of static equilibrium, and may be considered as an equation of dynamic equilibrium. In addition to the real forces (body forces and surface forces) acting on the particle, a fictitious force, $\ddot{x}$, equal to the product of mass and its acceleration, and directed oppositely to the acceleration, can be observed. This force is named as Inertia Force of the particle:

$$-\sum m\ddot{x} \;=\; -\ddot{x}\sum m \;=\; -\frac{W}{g}\ddot{x}$$, where W is the total weight of the body

D'Álembertz was the first to point out that equations of motion could be written as equilibrium equations simply by introducing inertia forces in addition to the real forces acting on the system. This idea is known as D'Alembert's Principle and can be expressed as:

$$\sum F_i - \frac{W}{g}\ddot{x} = 0$$

6. *Explain the concept of instantaneous centre.*

A general plane motion consists of translation plus rotation of the body. Eg: The connecting rod of a piston-crank mechanism possesses a general plane motion, in which piston is under pure translation and crank is under pure rotation. In a general plane motion, there exists a point on the body, who has an instantaneous velocity of zero, and this point is called instantaneous centre (IC) of velocity. The point of IC may lie outside the physical boundary of the body, but in the plane of motion of the body. If a rigid body is under pure translation, no point has zero velocity and IC lies in infinity in the direction perpendicular to direction of translation of the body. If a rigid body is under pure rotation, the centre of rotation is the zero velocity point. If a rigid body is under general plane motion, draw perpendiculars for velocity vectors through two points (minimum) in the body. The intersection of those perpendiculars will give a point of zero velocity at that instant of time, during its planar motion, which is IC of velocity of the body under general plane motion.

7. *Differentiate between free vibration and forced vibration of bodies.*

Different mechanical elements are assembled together to form a mechanical system or mechanism for a defined purpose. When such a system is disturbed by a force(s) causing displacement from its static equilibrium, vibrations are resulted, which are periodic movements in nature.

When a body vibrates under the action of force inherent in the system without any assistance of external force, or, even after the cause of original disturbance is removed, and if the periodic motion continues, it is said to have a free vibration. It will gradually cease due to loss of energy from the system.

When a body vibrates under the action of external force, the vibratory motion continues. Now the system is said to have forced vibration. If the external force is a periodic disturbing force, then vibration will also have the same frequency of periodicity of force. Upon removal of external force, it will possess free vibration till it damp. Objects which are free to vibrate will have one or more natural frequency at which they vibrate. In forced vibration system, if an object is forced to vibrate at its natural frequency, resonance will occur with large amplitude vibrations.

8. *Explain the following terms with respect to a simple harmonic motion (a) amplitude (b) time period (c) frequency*

Any motion that repeat itself with equal interval of time is called periodic motion. The projection of uniform circular motion of an object (P') to the diameter line of the circular path itself resembles a periodic motion and is termed as simple harmonic motion (SHM) in which certain physical quantity (say, displacement at any time) varies sinusoidally and can be expressed as a sine wave or cosine wave function. In the above case, the periodic motion along diameter line can be expressed as displacement (x) of projection of object (P') on diameter line at any time t.

ie x = r cos (ωt), where r = radius of circular path, ω = angular velocity of object along circular path.

Also velocity, $\dot{x} = -r\omega \sin(\omega t)$

and acceleration, $\ddot{x} = -\omega^2 x$ or $\ddot{x} \propto -x$

i.e. for a positive displacement x,the acceleration $\ddot{x}$ is negative for an SHM

Amplitude (A): It is the maximum displacement of the projection of object (P') from the mean position on diameter line (In the above case, it is equal to radius of circular path) (measured in meter or mm)

Time Period (T): The motion of projection of object (P') is periodic and repeats itself after a time period t given by, $t = \frac{2\pi}{\omega}$ where 2π is the angular displacement covered in one full cycle of the periodic motion, and ω is the angular displacement covered in one second. Thus, the ratio gives time required for one full cycle of rotation of object (P) in the circular path and termed as 'Time Period'. (Measured in Seconds)

Frequency (f): It is the number of oscillations of the point P' in one second and is given by,

$$f = \frac{1}{t} = \frac{2\pi}{\omega}. \; (\text{measured in Hz})$$

Part B

Answer any two questions from each SET.

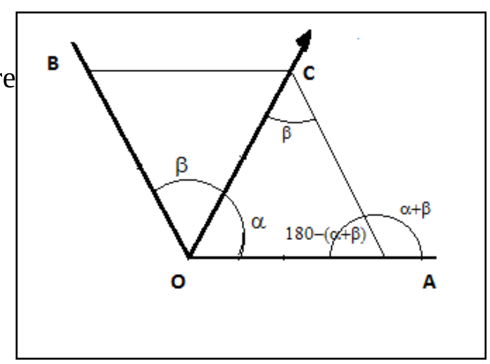
SET 1

Each question carries 10 marks

9. (*i*) *Distinguish between composition and resolution of forces with suitable example.*
(ii) Two arbitrary axes OA and OB are at an angle 105° each other. A force of 500N is acting at 45° with axis OA. Find the components of given forces along axes OA and OB

(i) The replacement of several concurrent forces with a single resultant force is called composition of forces. The parallelogram law of vector addition helps to find out the resultant of two concurrent forces separated by an angle θ, since the forces are vector quantities.

A single force acting on a rigid body can be replaced by two or more force components, which all together produce the same effect on the body as the single force is called resolution of forces. If a force F acts on a body at an angle θ with reference to X-axis, then two mutually perpendicular components of the force F along X and Y axes are F cosθ and F sinθ respectively (rectangular components).



(ii) The given forces can be represented as shown in figure

The angle between arbitrary axes OA and OB = α + β

= 105°

Given force = $\overrightarrow{OC}$ = 500N at angle α = 45° with OA

Applying Sine Rule to ΔOAC

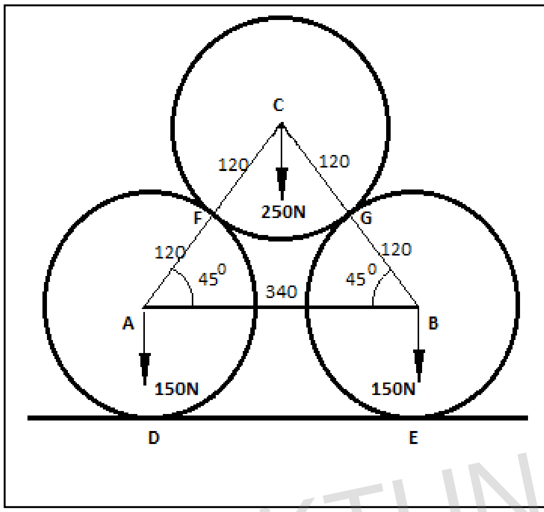
$$\frac{\overrightarrow{OA}}{\sin\beta} = \frac{\overrightarrow{AC}}{\sin\alpha} = \frac{\overrightarrow{OC}}{\sin(180-(\alpha+\beta))}$$

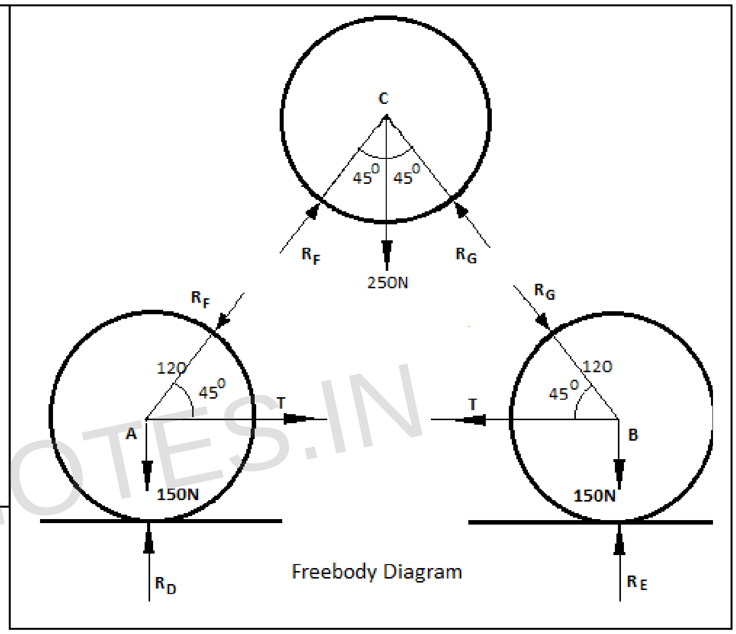
$$\frac{\overrightarrow{OA}}{sin\beta} = \frac{\overrightarrow{AC}}{sin\alpha} = \frac{\overrightarrow{OC}}{sin(\alpha+\beta)}$$

$$\therefore \quad \overrightarrow{OA} = \frac{\overrightarrow{OC}\,\sin\beta}{\sin(\alpha+\beta)} = \frac{500\,sin60}{sin(45+60)} = \text{448.29N, the component along OA axis}$$

$$\text{Similarly} \quad \overrightarrow{OB} = \frac{\overrightarrow{OC}\,\sin\alpha}{\sin(\alpha+\beta)} = \frac{500\,sin45}{sin(45+60)} = \text{366.03N, the component along OB axis}$$

10. *Two smooth circular cylinders each of weight 150 N and radius 120 mm are connected at their centres by a string AB of length 340 mm and rest upon a .horizontal plane as shown in Figure-1. The cylinder above them has a weight 250 N and radius of 120 mm. Find the tension in the string AB and the reaction exerted by the floor at the points of contact D and E.*





Freebody Diagram

Consider the freebody diagram of cylinder C. The active force is self weight of 250N and Reactive forces are R_F and R_G as shown infigure at an angle 45° with vertical. Each cylinder possess equilibrium condition. Resolving all forces and reactive forces into horizontal and vertical components and applying equilibrium conditions:

$$\sum F_X = R_F\,sin45 - R_G\,sin45 = 0 \quad \rightarrow \quad R_F = R_G \text{ ----------------------------(1)}$$

$$\sum F_Y = R_F\,cos45 - 250 + R_G\,cos45 = 0 \quad \rightarrow \quad 2R_F cos45 = 250$$

$$\rightarrow \quad R_F = R_G = 176.78\text{N} \text{ -------------------(2)}$$

Similarly, consider freebody diagram of Cylinder A and applying equilibrium conditions:

$$\sum F_X = -R_F\,cos45 + T = 0 \quad \rightarrow \quad T = 125\text{N} \text{ ----------------------------(3)}$$

$$\sum F_Y = -R_F\,sin45 - 150 + R_D = 0 \quad \rightarrow \quad R_D = 275\text{N} \text{ -------------------------(4)}$$

Also, consider freebody diagram of Cylinder B and applying equilibrium conditions:

$\sum F_X = R_G \cos 45 - T = 0 \quad \rightarrow \quad T = 125\text{N}$ ----------------------------(3)

$\sum F_Y = -R_G \sin 45 - 150 + R_E = 0 \quad \rightarrow \quad R_E = 275\text{N}$ -------------------------(5)

Hence, the reaction extertred by the floor at ponts of contact D and E are same and equal to 275N.

11. *Replace the given system of forces to (i) a single force (ii) an equivalent force-couple system at B. (Figure–2)*

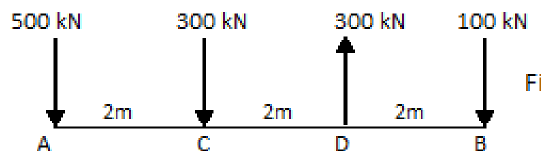


Figure-2

(i) Single force = Resultant of the parallel force system acting at beam ACDB

i.e. R = – 500kN – 300kN + 300kN – 100 = –600kN (downwards)

Taking total moment of forces about A, and equating to moment of resultant, R

$\sum M_A = (500 \times 0) - (300 \times 2) + (300 \times 4) - (100 \times 6) = 0 =$ R x moment arm

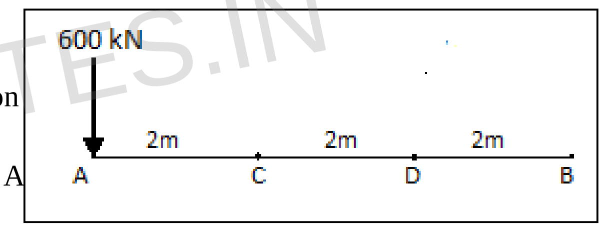
Since R $\neq 0$, moment arm of resultant force is zero, i.e. R is acting at point A

For the verification of this result, take total moment of forces about B and equate to moment of resultant R:

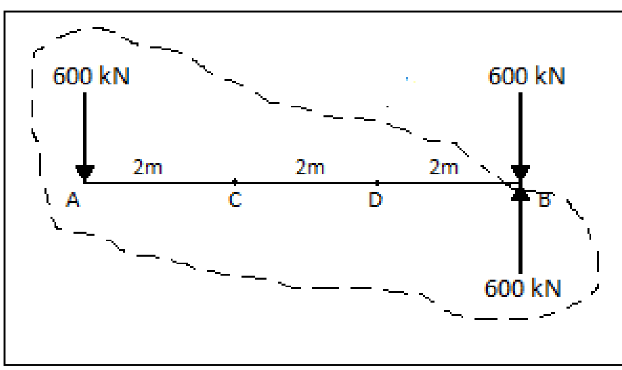
$\sum M_B = (500 \times 6) + (300 \times 4) - (300 \times 2) + (100 \times 0) = 3600$ (anti-clockwise)

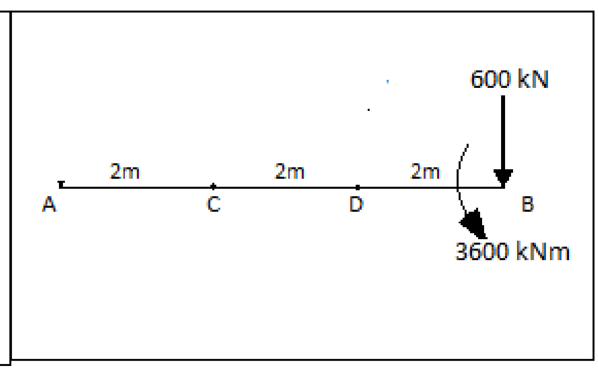
$=$ R $\times$ moment arm = 600 $\times$ 6

So, to obtain anti-clockwise moment with a downward resultant force, the point of application of R will be 6m away towards left from moment centre B. Hence, Resultant force R acts at point A (result confirmed)



(ii) To obtain a force-couple system at B, add 2 point loads of 600kN which are equal and opposite at point B:





The two selected forces form a couple of arm 6m, its moment = 600 x 6 = 3600 kNm. (anti-clockwise)

Additionally, there will be a force of 600kN (downward) at point B.

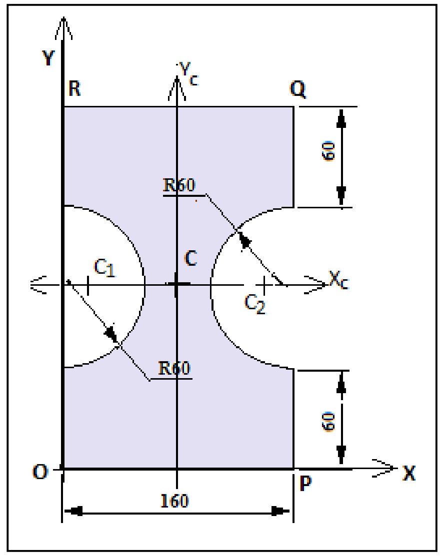
The final result of force-couple system at B is shown in next figure.

SET 2

Each question carries 10 marks.

12. *Find second moment of area of cut section as shown in figure 3 about its centroidal axes*

The cut-section has a horizontal and a vertical axis of symmetry. So, the intersection of both axes of symmetry will give the common centroid of the cut-section. The coordinates of common centroid with reference to X-axis and Y-Axis passing through the base and left extreme edge of the section is marked in the figure is C (80, 120)



Point C is also the centroid of rectangle OPQR. C_1 and C_2 are centroids of half circular area removed from the left and right edges of rectangle OPQR.

The x-coordinate of C_1 = 4r/3π = 25.45mm

The y-coordinate of C_1 = 60+60 = 120mm

i.e. C_1 is at (25.45, 120)

Similary, C_2 is at (134.55, 120)

<u>Second moment of area of rectangle OPQR about common centroidal axes are;</u>

$$I_{Xc} = \frac{bh^3}{12} = \frac{160 \times 240^3}{12} = 18432 \times 10^4\, mm^4 \text{---------(1)}$$

$$I_{Yc} = \frac{hb^3}{12} = \frac{240 \times 160^3}{12} = 8192 \times 10^4\, mm^4 \text{---------(2)}$$

<u>Second moment of area of half circle-1 about common centroidal X_C axis</u>:

$$I_{Xc} = \frac{\pi r^4}{8} = \frac{\pi \times 60^4}{8} = 509.14 \times 10^4\, mm^4 \text{---------------------------------------(3)}$$

Since Y-axis through the centroid C_1 is d_1 = 54.55 mm away from common centroidal Y_C axis, transfer formula is applied to obtain I_{Yc} of half circle C_1 (with area $A_1 = \frac{1}{2}\pi r^2$ = 5657.14 mm^2)

$$I_{Yc} = I_{Y1} + d_1^2 A_1 = 0.11R^4 + d_1^2 A_1 = (0.11 \times 60^4) + (54.55^2 \times 5657.14)$$
$$= 1825.96 \times 10^4\, mm^4 \text{--------------------------(4)}$$

<u>Second moment of area of half circle-2 about common centroidal X_C axis</u>:

$$I_{Xc} = \frac{\pi r^4}{8} = \frac{\pi \times 60^4}{8} = 509.14 \times 10^4\, mm^4 \text{---------------------------------------(5)}$$

Since Y-axis through the centroid C_2 is d_2 = 54.55 mm away from common centroidal Y_C axis, transfer formula is applied to obtain I_{Yc} of half circle C_2 (with area $A_2 = \frac{1}{2}\pi r^2$ = 5657.14 mm^2)

$$I_{Yc} = I_{Y2} + d_2^2 A_2 = 0.11R^4 + d_2^2 A_2 = (0.11 \times 60^4) + (54.55^2 \times 5657.14)$$
$$= 1825.96 \times 10^4\, mm^4 \text{-------------------------(6)}$$

<u>Second moment of area of entire cut-section about common centroidal X_C and Y_C axes</u>:

I_{Xc} of entire cut-section = (1) − (3) − (5) = (18432 − 509.14 − 509.14) × 10^4

= <u>17413.72 × 10^4 mm^4</u>

I_{Yc} of entire cut-section = (2) − (4) − (6) = (8192 − 1825.96 − 1825.96) × 10^4

= <u>4540.08 × 10^4 mm^4</u>

13. *What is Principal axis and Principal moment of inertia? Given lamina of area 'A' has $I_X = 6.4 \times 10^6\ mm^4$, $I_Y = 16 \times 10^6\ mm^4$ and $I_{XY} = 6.4 \times 10^6\ mm^4$. Calculate Principal Moment of Inertia.*

We know that, about a frame of reference of mutually perpendicular axes of X and Y, the second moment of area are given by:

$I_X = \int y^2 dA$, $\quad I_Y = \int x^2 dA \quad$ and $\quad I_{XY} = \int xy\, dA$

By rotating both axes of reference through 0° to 90° about origin O in anti-clockwise direction, we can determine second moment of area of same lamina about respective axes and using respective coordinates of dA with reference to those axes. In such a case, it can be observed that second moment of area varies as the axes of reference rotate and they reach a maximum and minimum value at a particular inclination of reference axes (say, one of the reference axis is at an inclination of θ, while the other axis at 90+θ). When second moment of area about one of the axis reaches a maximum value, second moment of area about the other mutually perpendicular axis reaches a minimum value. Same values will be obtained for the rotation of reference axes beyond 90°.

The axes about which second moment of area/ moment of inertia is maximum or minimum is termed as Principal axes and corresponding value of moment of inertia about those principal axes are termed as Principal Moment of Inertia

The Principal Moment of inertia is given by:

$$I_{MAX} = \left(\frac{I_X+I_Y}{2}\right) + \sqrt{\left(\frac{I_X-I_Y}{2}\right)^2 + (I_{XY})^2} \quad \text{and}$$

$$I_{MIN} = \left(\frac{I_X+I_Y}{2}\right) - \sqrt{\left(\frac{I_X-I_Y}{2}\right)^2 + (I_{XY})^2}$$

For the given lamina of area 'Á', following data are available:

$I_X = 6.4 \times 10^6\ mm^4$, $\quad I_Y = 16 \times 10^6\ mm^4 \quad$ and $\quad I_{XY} = 6.4 \times 10^6\ mm^4$

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$$\therefore \quad I_{MAX} = \left(\frac{I_X+I_Y}{2}\right) + \sqrt{\left(\frac{I_X-I_Y}{2}\right)^2 + (I_{XY})^2}$$

$$I_{MAX} = \left(\frac{6.4+16}{2}\right)10^6 + \sqrt{\left(\left(\frac{6.4-16}{2}\right)10^6\right)^2 + (6.4 \times 10^6)^2}$$

$$I_{MAX} = \underline{19.2 \times 10^6\ mm^4}$$

Also,
$$I_{MIN} = \left(\frac{I_X+I_Y}{2}\right) - \sqrt{\left(\frac{I_X-I_Y}{2}\right)^2 + (I_{XY})^2}$$

$$I_{MIN} = \left(\frac{I_X+I_Y}{2}\right) - \sqrt{\left(\frac{I_X-I_Y}{2}\right)^2 + (I_{XY})^2}$$

$$I_{MIN} = \underline{3.2 \times 10^6\ mm^4}$$

14. *A uniform ladder of length 8m and weight W is leaning against a wall. It makes 45° with the horizontal. A man whose weight is 0.6 times that of ladder goes up the ladder. Determine the maximum distance he can climb before the ladder slips. Assume coefficient of friction between the ladder and wall to 0.25 and that between the ladder and floor to be 0.3*

Given Data:

Weight of ladder = w_l

Weight of man, w_m = $0.6 \times w_l$

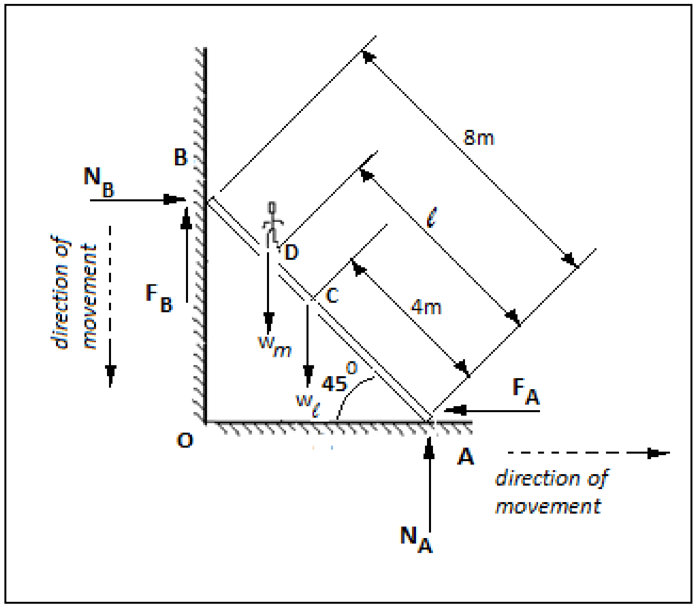
Length of ladder, L = 8m

Coefficient of friction between ladder and wall, $\mu_{l/w}$ = 0.25

$\therefore \quad F_B = 0.25\, N_B$------------------(1)

Coefficient of friction between ladder and floor, $\mu_{l/f}$ = 0.30

$\therefore \quad F_A = 0.3\, N_A$--------------------(2)



To find distance climbed by man before slipping of ladder = l

Solution: At the time of impending motion/slipping of ladder, the system is under equilibrium condition. Refer the figure to observe all acting and reacting forces and friction forces as per the direction of slipping of ladder to take place.

Applying equilibrium condition-1 & 2, $\sum F_X = 0$ and $\sum F_Y = 0$

$\sum F_X = N_B - F_A = 0 \quad \rightarrow \quad N_B = F_A$

Applying eqn (2) $\quad N_B = 0.3N_A$--(3)

$\sum F_Y = F_B - W_m - W_l + N_A = 0 \quad \rightarrow$ Applying eqn(1):

$0.25N_B - 0.6\, W_l - W_l + N_A = 0 \quad \rightarrow$ Applying eqn(3):

$0.25 \times 0.3N_A - 0.6\, W_l - W_l + N_A = 0$

$1.075N_A - 1.6\, W_l = 0 \quad \rightarrow \quad N_A = \frac{1.6}{1.075} W_l$---(4)

Applying equilibrium condition-3, $\sum M_A = 0$

$\sum M_A = -N_B\, OB - F_B\, OA + 0.6\, W_l\ l\ cos45 + W_l\ \frac{L}{2}\ cos45 = 0$

Applying eqn(1):

$\sum M_A = -N_B\, 8\, sin45 - 0.25N_B\, 8\, cos45 + 0.6\, W_l\, l\, cos45 + W_l\ 4\ cos45 = 0$

Applying eqn(3):

$\sum M_A = -0.3\, N_A\, 8\, sin45 - 0.25 \times 0.3N_A\, 8\, cos45 + 0.6\, W_l\, l\, cos45 + W_l\ 4\ cos45 = 0$

Applying eqn(4):

$\sum M_A = -0.3\frac{1.6}{1.075}W_l\ 8\ sin45 - 0.25 \times 0.3\frac{1.6}{1.075}W_l\ 8\ cos45 + 0.6\, W_l\, l\, cos45 + W_l\ 4\ cos45 = 0$

$\sum M_A = -2.53W_l - 0.63W_l + 0.424\, W_l\ l + W_l\ 2.83 = 0$, cancelling W_l:

$-0.33 + 0.424\ l = 0$

$l = \frac{0.33}{0.424} = 0.778m$

Result:

The man will fall due to slipping of the ladder after he climbs up the ladder a distance of 0.778m.

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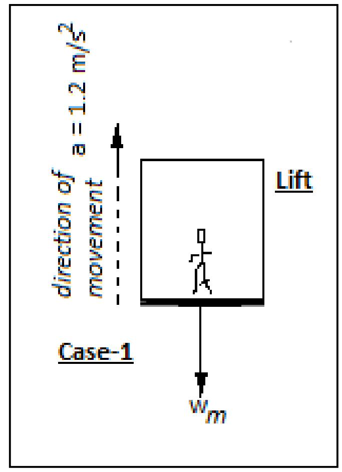
SET 3

Each question carries 10 marks.

15 *A lift has an upward acceleration of 1.2 m/s². What force will a man weighing 750 N exert on the floor of the lift? What force would he exert if the lift had an acceleration of 1.2 m/s² downwards? What upward acceleration would cause his weight to exert a force of 900 N on the floor?*



Case-1: Force exerted by the man on the floor of the lift while lift moves upwards with a = 1.2m/s²:

By applying D'Álembertz principle, the sum of net force acting and inertial forces developed is zero (F-ma = 0). Following a +ve sign convention for forces in the direction of motion of lift (upwards):

$(R_f - W_m) - \left(\frac{W_m}{g}\right) a = 0$, where

R_f = reactive force by the floor of the lift on the mass,

W_m = weight of given mass = 750 N

a = acceleration (upwards) of the lift = $1.2\, m/s^2$

g = acceleration due to gravity = $9.81\, m/s^2$

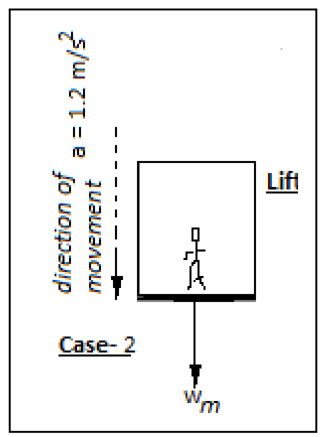
i.e $(R_f - 750) - \left(\frac{750}{9.81}\right)1.2 = 0$

$(R_f - 750) = 91.74$

$R_f = 91.74 + 750$

$R_f = 841.74$ N

Result: While the lift moves and accelerates upwards, the man of body weight 750N exerts a force of 841.74N on the floor of the lift, which is equal to the reactive force exerted by the floor, and is more than his weight,.



Case-2: Force exerted by the man on the floor of the lift while lift moves downwards with a = 1.2m/s²:

Following a +ve sign convention for forces in the direction of motion of lift (downwards) and applying D'Álenbertz principle:

$(-R_f + W_m) - \left(\frac{W_m}{g}\right) a = 0$

i.e $(-R_f - 750) - \left(\frac{750}{9.81}\right)1.2 = 0$

$(-R_f - 750) = 91.74$

$R_f = 750 - 91.74$

$R_f = 658.26N$

Result: While the lift moves and accelerates downwards, the man of body weight 750N exerts a force of 658.26N on the floor of the lift, which is equal to the reactive force exerted by the floor, and the man feels less body weight.

Case-3: Force exerted by the man on the floor of the lift while lift moves upwards is 900N., and to find acceleration of the lift:

Following a +ve sign convention for forces in the direction of motion of lift (upwards) and applying D'Álenbertz principle:

$$(R_f + W_m) - \left(\frac{W_m}{g}\right) a \quad = 0$$

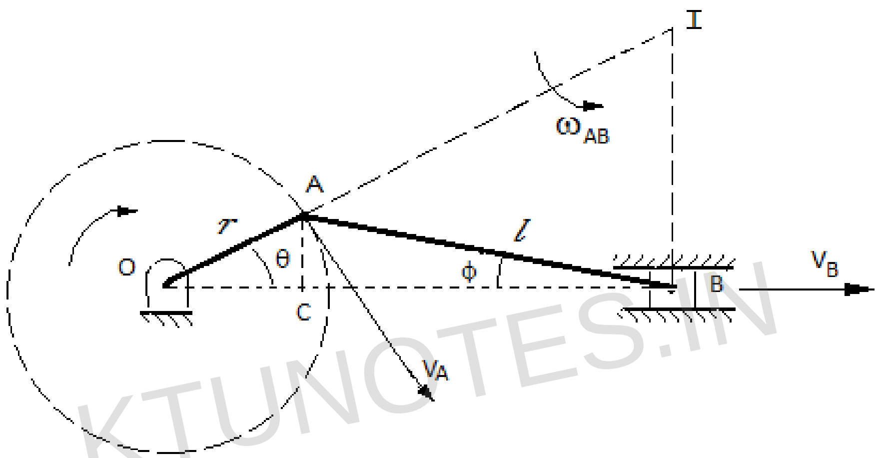
i.e $(900 - 750) - \left(\frac{750}{9.81}\right) a \quad = 0$

$$150 = 76.45\, a$$

$$a = \frac{150}{76.45} = 1.96\, m/s^2$$

Result: While the lift moves and accelerates upwards at a rate of $1.96 m/s^2$ the man of body weight 750N exerts a force of 900N on the floor of the lift,

16. *In the reciprocating engine mechanism, the crank OA rotates at a uniform speed of 350 rpm. The length of the crank and connecting rod are 120 mm and 500 mm respectively. Find the angular velocity of the connecting rod and velocity of the piston when the crank makes an angle of 30° with horizontal.*



The length of connecting rod, l = 500mm = 0.5m
The length of crank, r = 120mm = 0.12m
When crank makes an angle, θ = 30°, connecting rod makes an angle, φ

$$\frac{l}{\sin 30} = \frac{r}{\sin\varphi} \qquad \sin\varphi = \frac{r\ \sin 30}{l} = \frac{0.12 \times \sin 30}{0.5} = 0.12$$

$$\therefore \qquad \varphi = 6.89^0$$

The velocity vector V_A of the crank OA is tangential to rotation of the crank. The instantaneous centre I is located by drawing IA normal to velocity vector V_A of crank and IB normal to velocity vector V_B of piston. Their point of intersection I locates the instantaneous centre of rotation of the connecting rod AB as shown in figure.

Let angular velocity of crank about O = ω_{OA} = angular velocity of any point on crank OA at that instant of time about O
Angular velocity of connecting rod about I = ω_{AB} = angular velocity of any point on connecting rod AB at that instant of time about I.

We have Linear velocity ,v = r ω, and V_A is linear velocity for both OA and AI

∴ $V_A = IA\ \omega_{AB}$ ---------------(1) Also $V_B = IB\ \omega_{AB}$---------------------(2)

Hence $\omega_{AB} = \frac{V_A}{IA}$ ∴ $V_B = IB\ \frac{V_A}{IA}$-----------------------(3)

Also $V_A = r\ \omega_{OA}$-----------------(4) $\quad\therefore\quad V_B = IB\ \frac{r\ \omega_{OA}}{IA}$----------------(5)

From eqns (1) & (4) $\quad r\ \omega_{OA} = IA\ \omega_{AB} \quad \therefore \quad \omega_{AB} = \frac{r\ \omega_{OA}}{IA}$---------------------(6)

We have, $\omega_{OA} = \frac{2\pi N}{60} = \frac{2\pi\ 350}{60} = 36.65$ rad/s

To find angular velocity of connecting rod, as per eqn (1) or (6) we need value for *IA*. Also to find velocity of piston, as per eqn (2) or (5) we need value of *IB*.

<u>To find value of *IA* and *IB:*</u>

Refer ΔABC and ΔOAC in the figure:

$BC = l\ cos\varphi$ and $OC = r\ cos\theta$

$OB = OC + BC = (r\ cos\theta + l\ cos\varphi)$

Refer ΔIOB, $\quad tan\theta = IB/OB \qquad IB = OB\ tan\theta = (r\ cos\theta + l\ cos\varphi)\ tan\theta$

$$IB = (r\ cos\theta\ tan\theta + l\ cos\varphi\ tan\theta)$$
$$IB = (r\ sin\theta + l\ cos\varphi\ tan\theta)$$
$$IB = (0.12\ sin30 + 0.5\ cos6.89\ tan30)$$
$$IB = 0.3466m$$

Refer ΔIOB, $\quad cos\theta = OB/OI \qquad OI = OB\ sec\theta$

$IA = OI - r = OB\ sec\theta - r = (r\ cos\theta + l\ cos\varphi)\ sec\theta - r = l\ cos\varphi\ sec\theta$

$IA = 0.5\ cos6.89\ sec30 = 0.5732m$

<u>Result- angular velocity of connecting rod and velocity of piston:</u>

From eqn (6), $\quad \omega_{AB} = \frac{0.12 \times 36.65}{0.5732} = 7.673$ rad/s

i.e. Angular velocity of connecting rod, ω_{AB} = 7.673 rad/s---------------(Answer-1)

From eqn (5), $\quad V_B = \frac{0.3466 \times 0.12 \times 36.65}{0.5732} = 2.659$ m/s

i.e. Velocity of piston, V_B = 2.659 m/s ---------------------------------(Answer-2)

17. *A body is moving with simple harmonic motion and has velocities of 8m/s and 3m/s at a distance of 1.5m and 2.5m respectively from the centre. Find the amplitude and time period of the body.*

For a simple harmonic motion (SHM), the displacement *x* (along X-axis) at any time *t* from mean position is given by:

$x = r\ cos\ \omega t$--(1)

where r = maximum displacement from mean position observed by the movement of projection of an object on the diameter line of its circular path through which the object is moving, (called amplitude)

ωt = angular displacement in time *t* of the object moving in circular path with an angular velocity ω

Hence, velocity at a distance from *x* from mean position:

$\dot{x} = -r\omega\ \sin\omega t$-----------------------------------(2)

From eqn(1), $\cos \omega t = \frac{x}{r}$ $\therefore$ $\sin \omega t = \sqrt{1 - \cos^2 \omega t} = \sqrt{1 - \frac{x^2}{r^2}}$

$\therefore$ eqn (2) $\rightarrow$ $\dot{x} = -r\omega \sqrt{1 - \frac{x^2}{r^2}}$

$\therefore$ $\dot{x} = -\omega\sqrt{r^2 - x^2}$---------------------------------(3)

<u>To find amplitude and time period:</u>

From eqn (3), when $x = 1.5$m, $\dot{x} = 8$m/s

$8 = \omega\sqrt{r^2 - 1.5^2}$ ----------------------------------(4)

From eqn (3), when $x = 2.5$m, $\dot{x} = 3$m/s

$3 = \omega\sqrt{r^2 - 2.5^2}$ ----------------------------------(5)

Dividing eqn (4) by eqn (5) $\rightarrow$ $\frac{8}{3} = \frac{\sqrt{r^2 - 1.5^2}}{\sqrt{r^2 - 2.5^2}}$

i.e. $\left(\frac{8}{3}\right)^2 = \frac{r^2 - 1.5^2}{r^2 - 2.5^2}$

i.e. $64\, r^2 - 400 = 9\, r^2 - 20.25$

i.e. $35\, r^2 = 379.75$

i.e. $r = 3.294\, m$

i.e. <u>Amplitude of oscillation, $r = 3.294\, m$</u>---------------------------------Answer (1)

Substituting amplitude r in eqn (4) $\rightarrow$ $8 = \omega\sqrt{3.294^2 - 1.5^2}$

$\therefore$ $\omega = 8/\sqrt{8.6}$

$\therefore$ $\omega = 2.728\, rad/s$

We also have, Time Period $\tau = \frac{2\pi}{\omega}$

i.e. $\tau = \frac{2\pi}{2.728} = 2.3\, s$

i.e. <u>Time Period of oscillation, $\tau = 2.3$ seconds</u>----------------------------Answer (2)

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