Module-2

Regular Expressions

1) ϕ is regular expression representing regular language $\{\}$

2) $\varepsilon \rightarrow \{\varepsilon\}$

3) if $a \in \Sigma$, $a \rightarrow \{a\}$

4) if $r \rightarrow L_r$, $s \rightarrow L_s$, then $r+s \rightarrow L_r \cup L_s$

$rs \rightarrow L_r . L_s$

$r^* \rightarrow L_r^*$

$r^+ \rightarrow L_r^+$

$(r) \rightarrow (L_r)$

$0+1$ $\{0,1\}$

$00+11$

$(a+b)^*$: any no. of a and b

$(a+b)^*abb$: ending with abb

$00(0+1)^*$: begin with 00

$(a+b)^*aa(a+b)^*$: contain the substring 'aa'

*) Write regular expression for

→ generating even no. of 1's : $(11)^*$

→ generating even no. of a's followed by odd no. of b's

: $(aa)^*(bb)^*b$ or $(aa)^*.b.(bb)^*$

→ any no. of a's followed by any no. of b's followed by no. of c's : $(a^*).(b^*).(c)^*$

→ $L = \{w \in (a,b)^*, |w| \bmod 3 = 0\}$

$((a+b)(a+b)(a+b))^*$

→ 10^{th} symbol from right is 1. Any no. of 0 and 1.

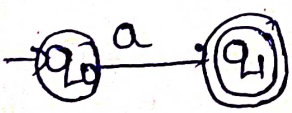
$(0+1)^* 1 (0+1)^9$

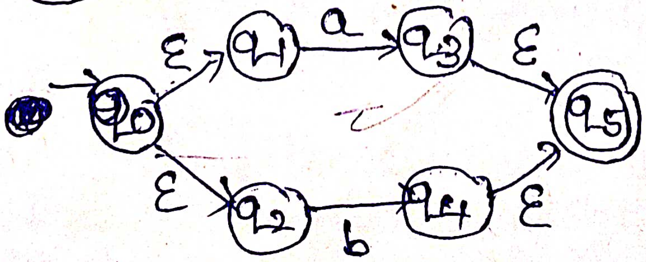
Equivalence of Regular expression and FA

Regular expression to FA (NFA-ε)

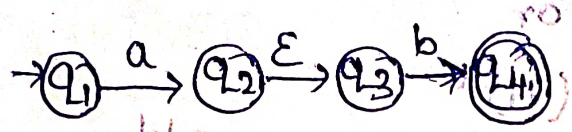
ϕ 

ε 

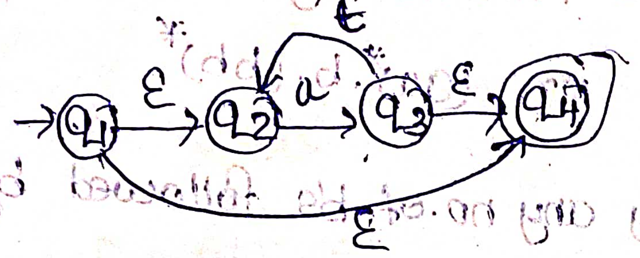
a 

$a+b$ 

ab:

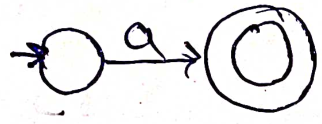


a* ;

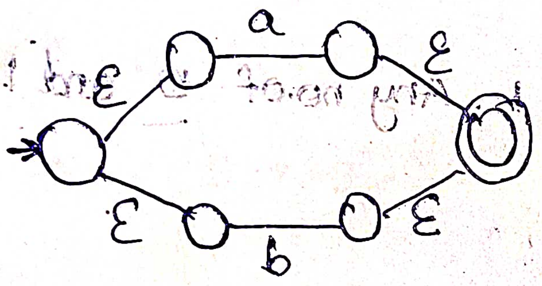


$a(a+b)^*bb$:

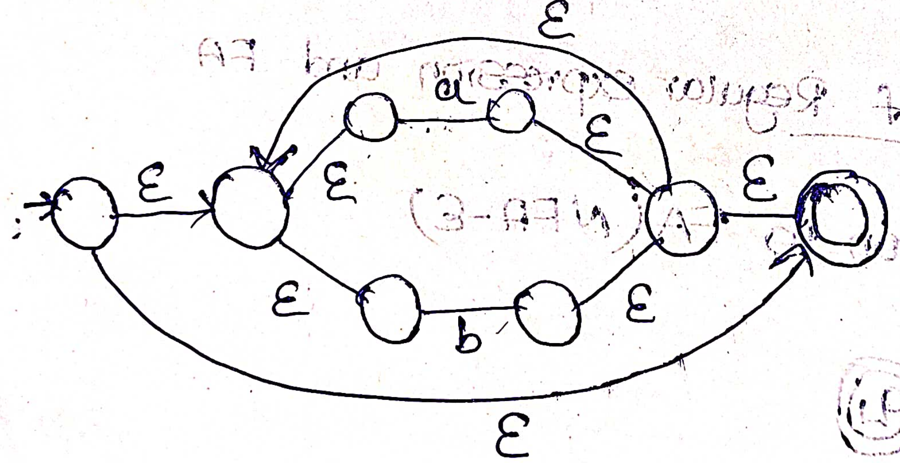
a :



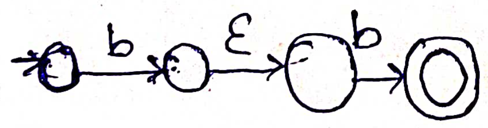
a+b

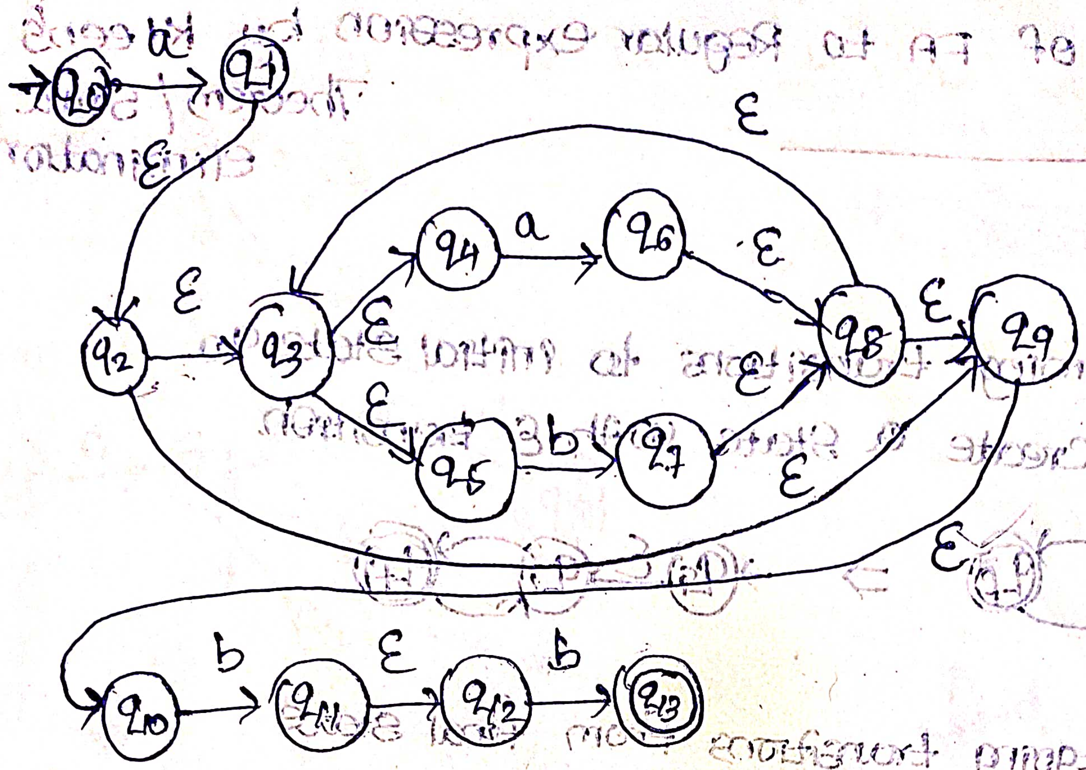


$(a+b)^*$



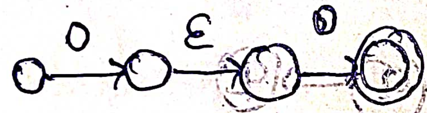
bb :



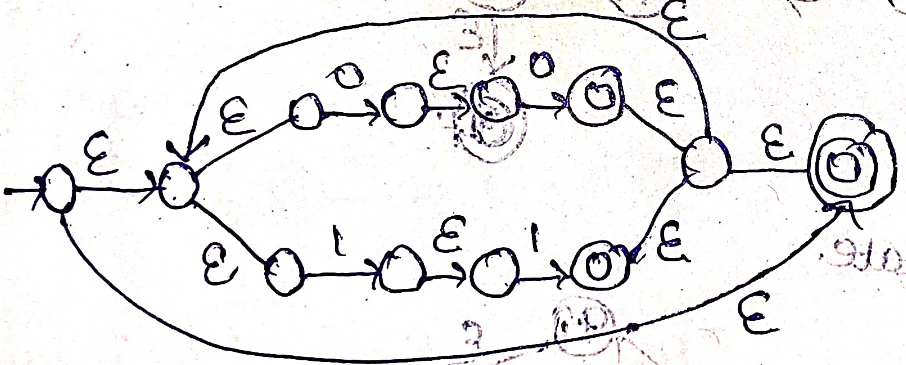


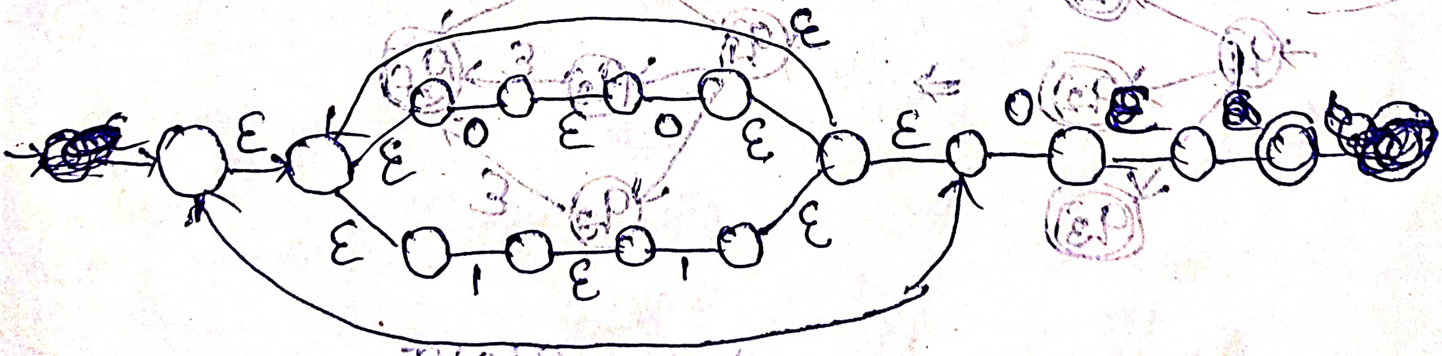
$(00+11)^*01$

00:



11:

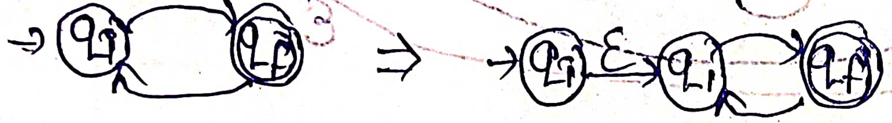




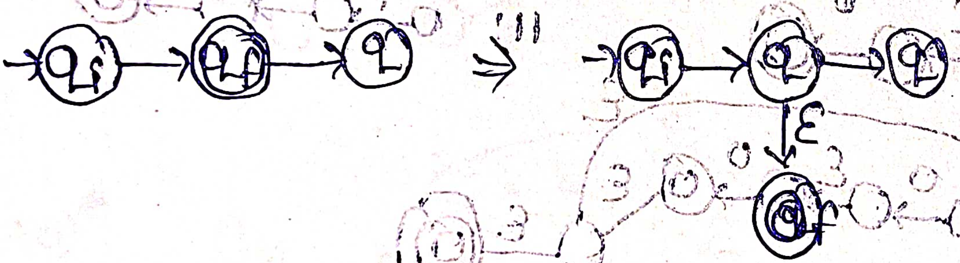
Conversion of FA to Regular expression by Kleen's Theorem / State elimination

Rules :-

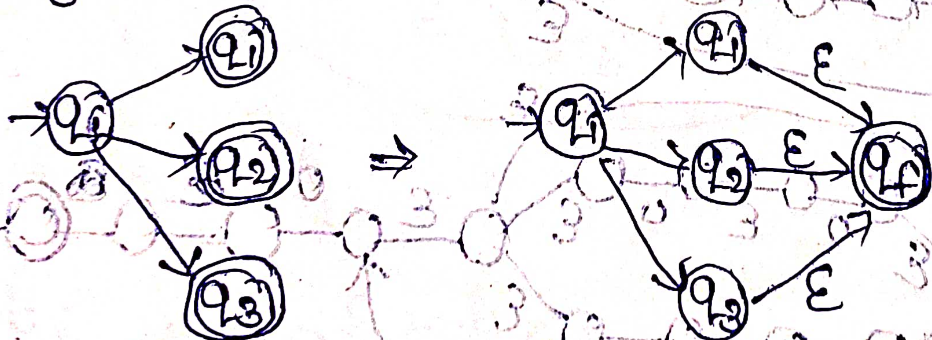
1) No incoming transitions to initial state q_0.
If yes, Create a state with ϵ transition.



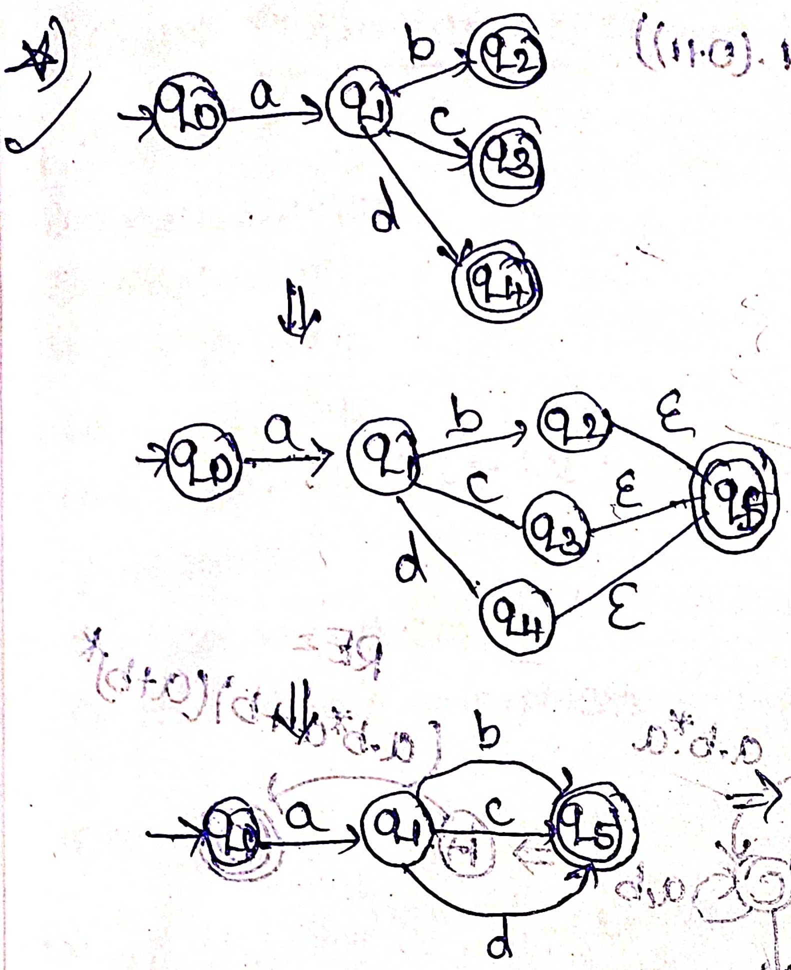
2) No outgoing transitions from final state
If yes, create ~~another~~ final state with ϵ transition, after removing current final state



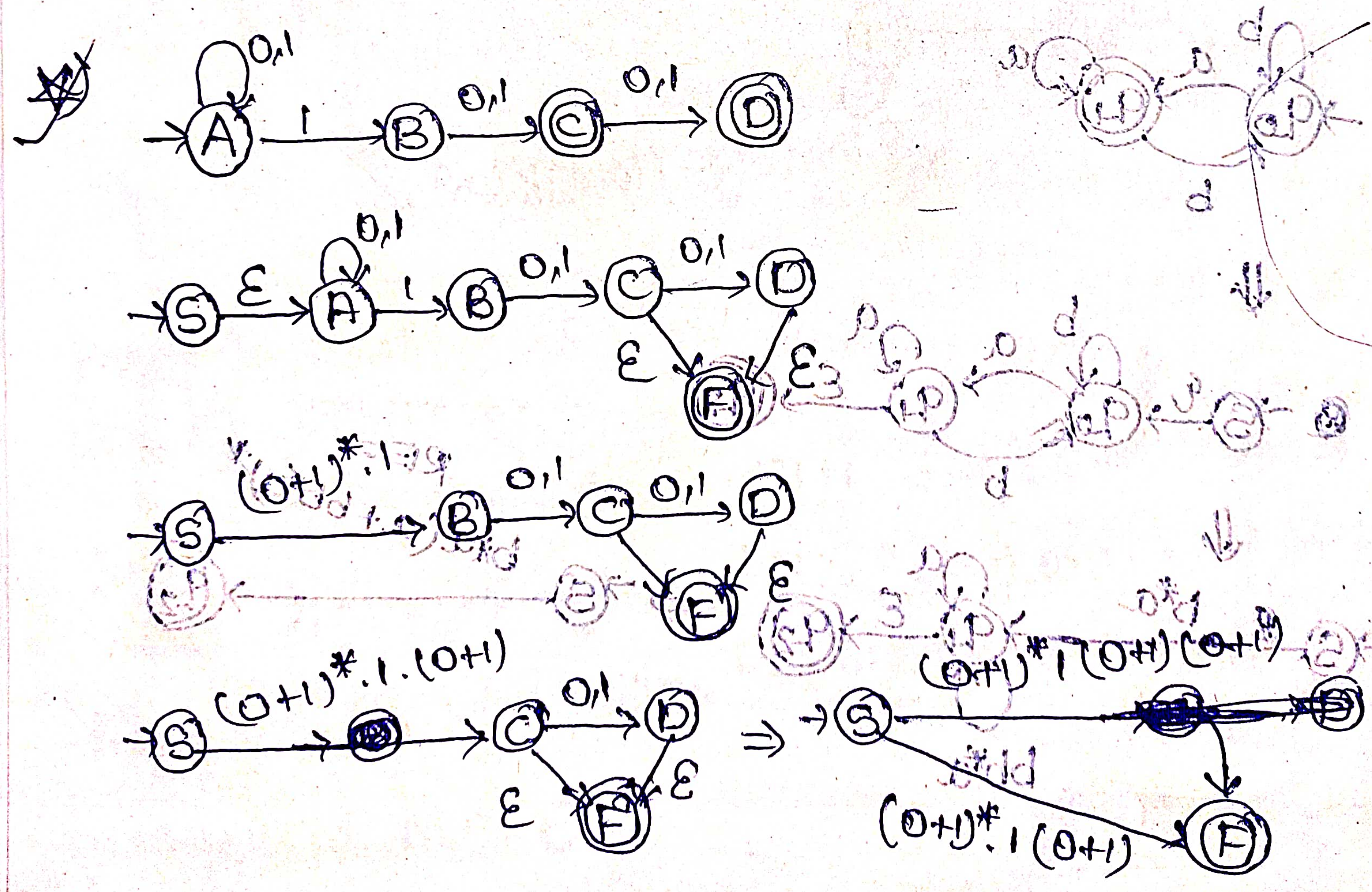
3) Only one final state.



4) Elimination of intermediate states.

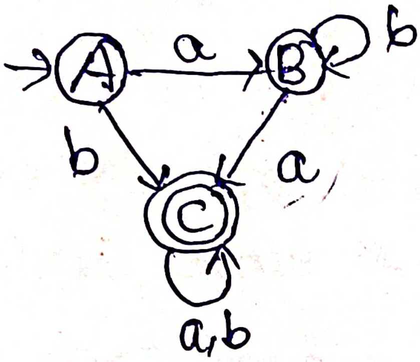


Regular expression : a.(b+c+d)

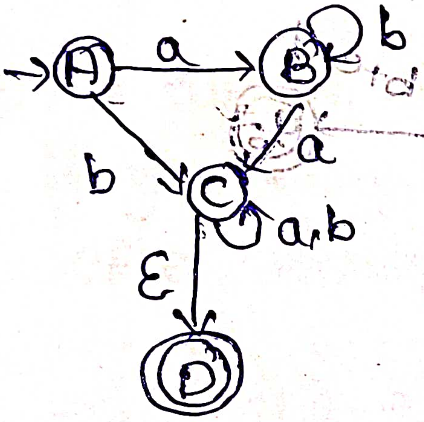


$RE = ((0+1)^* . 1 . (0+1)^2) + ((0+1)^* . 1 . (0+1))$

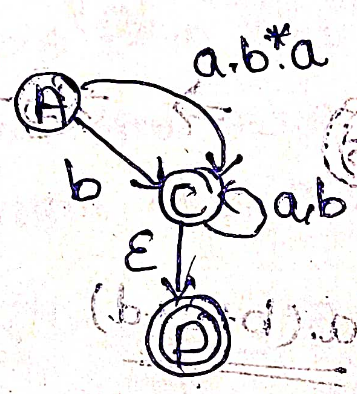
*)



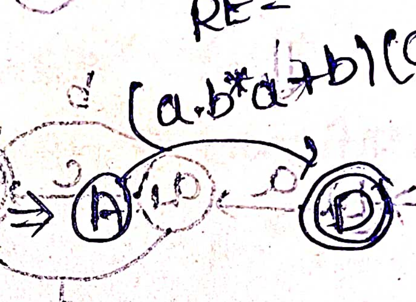
⇓



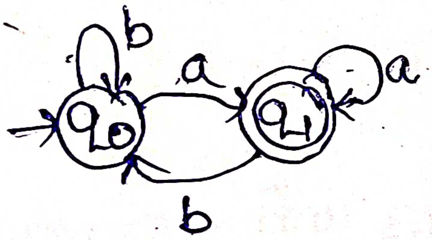
⇒



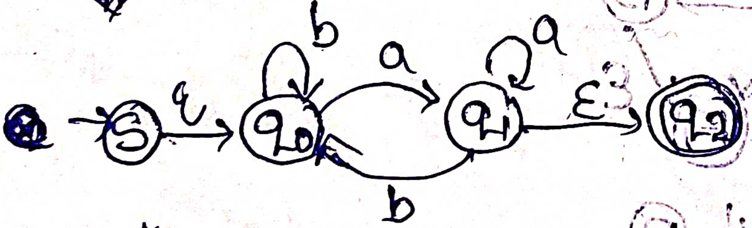
⇒



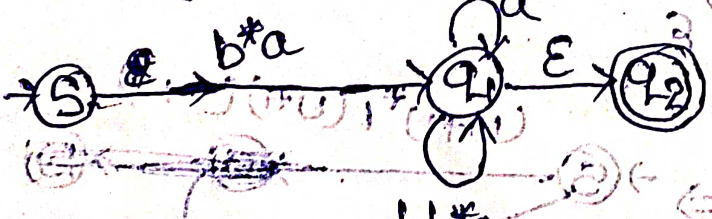
*)



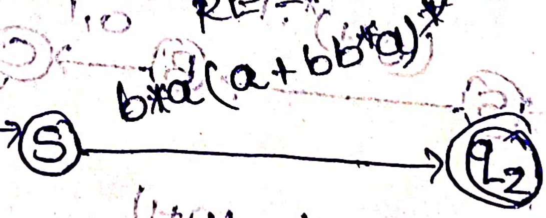
⇓



⇓



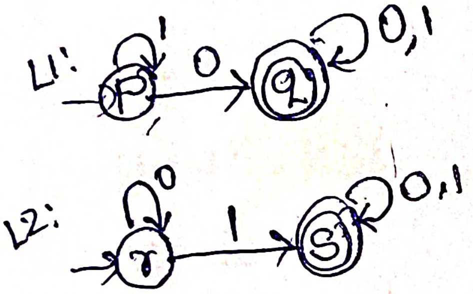
⇒



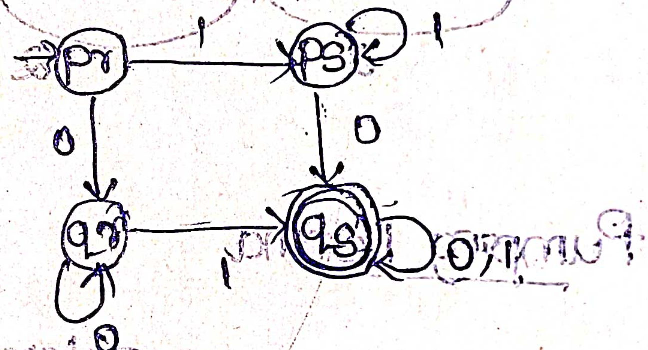
Closure Properties of Regular Language

1. Union
2. Concatenation
3. Kleen Star
4. Complement
5. Intersection
6. Difference $\Rightarrow L_1 - L_2 = L_1 \cap \overline{L_2}$
7. Reversal
8. Homomorphism
9. Inverse Homomorphism

Intersection



$L_1 \cap L_2 \Longrightarrow$



$\delta(pr, 0) = (\delta(p,0), \delta(r,0))$

Homomorphism

$h: \Sigma^* \rightarrow \Delta^*$ $\quad 0111 \Rightarrow ab\epsilon\epsilon\epsilon$

$L \rightarrow RL$ $\quad h(L) = \{h(w), w \in L\}$

$RE(L) = 01^* + 10^*$

$RE(h(L)) = ab\epsilon^* + \epsilon(ab)^*$

$= ab + (ab)^*$

$= (ab)^*$

$h: \{0,1\} \rightarrow \{a,b\}$

$h(0) = ab, \ h(1) = \epsilon$

Inverse homomorphism

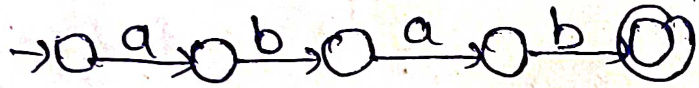
$h^{-1}(L) = \{ w \mid h(w) \in L \}$

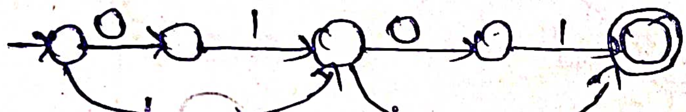
$h : h(0) = a,\ h(1) = b,\ h(2) = ab$

$\{0,1,2\} \rightarrow \{a,b\}$

$L = \{abab\}$

$h^{-1}(L) = \{0101, 012, 201, 22\}$

L 

$h^{-1}(L)$ 

Pumping Lemma,

⇒ Proving whether a ~~regu~~ language is not Regular ~~or not~~.

Proof by contradiction

→ Let L be a regular grammar then there exist a constant 'n' that depends on ~~L~~ such that ~~L be regular~~ ~~langua~~ for every string w in L with length of w greater than or equal to n ($w \in L, |w| \geq n$).

We can divide the w into 3 strings x, y, z such that ~~there are to~~

y cannot be NULL

$|xy| \leq n$

~~[illegible]~~ For all i greater than or equal to Zero

$xy^iz \in L$

*) Prove that $L = \{a^n b^n, n \geq 0\}$ is not regular.

Assume that L is regular. Then there exist a DFA with 'm' States.

So m be our pumping Lemma constant.

$w \in L, \ |w| \geq m$

$\therefore w = a^m b^m \quad , |w| = 2m > m$

$x = a^{m-k}$

$y = a^k, \ k \geq 1$

$z = b^m$

$xy^iz = a^{m-k}(a^k)^i b^m = a^{m-k+ki} b^m$

If L is regular, $xy^iz \in L \ \forall i$

$i = 0$, $xy^iz = a^{m-k} b^m \notin L, \ k \geq 1$

*) Prove that $L = \{ww^R, w \in (a,b)^*\}$ is not regular.

$w = \underbrace{a^m b^m}_{w} \underbrace{b^m a^m}_{w} \quad |w| = 4m > m$

$x = a^{m-k}$

$y = a^k, \ k \geq 1$

$z = b^m b^m a^m$

$$xy^iz = a^{m-k}(a^k)^i b^m b^m a^m$$

$$= a^{m-k+ki+m} b^{2m} = a^{2m-k+ki} b^{2m}$$

$i=0$, $xy^iz = a^{2m-k} b^{2m} \notin L$, $k \geq 1$

$\therefore$ L is not regular.

*) $L = \{a^{n!}, n \geq 1\}$ is not regular

1)

$\omega = a^{m!}$ $\quad |\omega| = m! > m$

$$\overbrace{a.a \ldots\ldots a}^{m!}$$

$x = a^{m-k}$

$y = a^k, k \geq 1$

$z = a^{m!-m}$

$$xy^iz = a^{m-k}(a^k)^i a^{m!-m}$$

$$= a^{\cancel{m}-k+ki+m!-\cancel{m}} = a^{k(i-1)+m!}$$

$m! \to (m+1)!$

$= m!m + m!$

$i=2$, $xy^2z = a^{k+m!}$

$m! + k \leq m! + m < (m+1)!$

$\therefore xy^2z = a^{k+m!} \notin L$

$\therefore$ L is not regular.

*) $L = \{0^k, k$ is a perfect square$\}$

Constant $\to m$

$w = 0^{m^2}$, $|w| = m^2 > m$

$\underbrace{0.0.0 \cdots 0}_{m^2}$

$x = 0^{m-k}$

$y = 0^{k}, k \geq 1$

$z = 0^{m^2-m}$

$xy^i z = 0^{m-k}\, 0^{ki}\, 0^{m^2-m} = 0^{\cancel{m}-k+ki+m^2-\cancel{m}}$

$= 0^{m^2+k(i-1)}$

$m^2 \to (m+1)^2$

m^2+2m+1

put $i=2$

$xy^2z = 0^{m^2+k} \notin L$

$m^2+k \leq m^2+m < (m+1)^2$

$\therefore$ L is not regular.

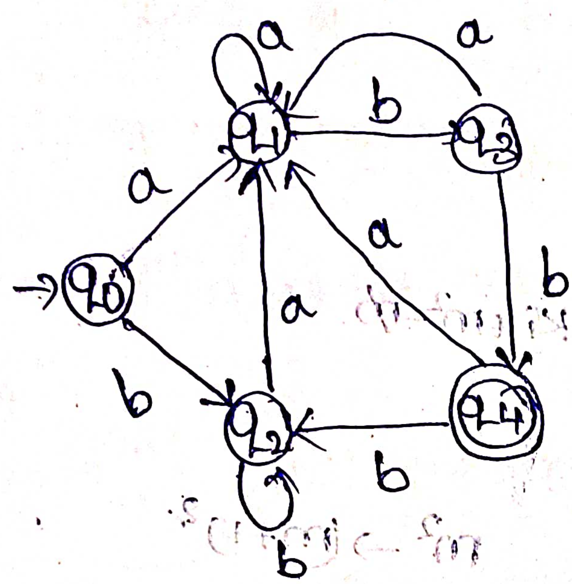
Minimization of DFA

Quotient Construction Algorithm

1. Eliminate inaccessible states & dead/trap states.
2. Initialize $k=0$
3. Partition your set of available states into final & non-final states $\leftarrow P_0$
4. Increment k
5. For every pair of states in set of P_0, identify each pair is indistinguishable or not. $\xrightarrow{} P_k$
6. Repeat the steps 4,5 until P_k and P_{k-1} are same.

2 States q_1, q_2 are distinguishable for any i/p symbol 'a' $\delta(q_1, a)$ & $\delta(q_2, a)$ goes to diff. sets in the partition.

✡)



There are no inaccessable States & trap states.

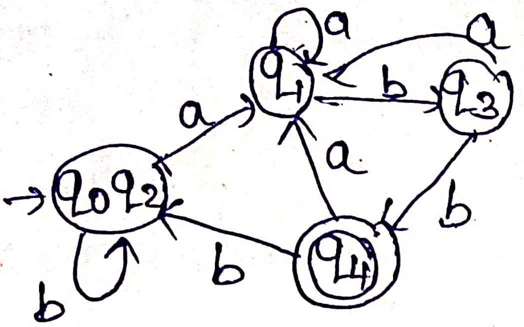
	a	b
q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
* q_4	q_1	q_2

P_0: $\{q_0, q_1, q_2, q_3\}$ $\{q_4\}$

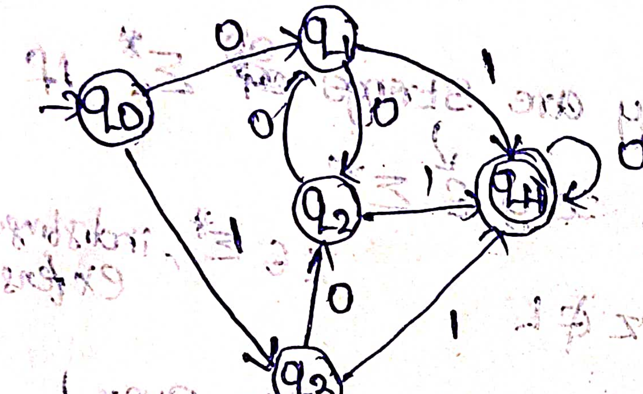
P_1: $\{q_0, q_1, q_2\}$ $\{q_3\}$ $\{q_4\}$

P_2: $\{q_0, q_2\}$ $\{q_1\}$ $\{q_3\}$ $\{q_4\}$

P_3 ~~P3: {q0} {q2} {q1} {q3} {q4}~~



*)

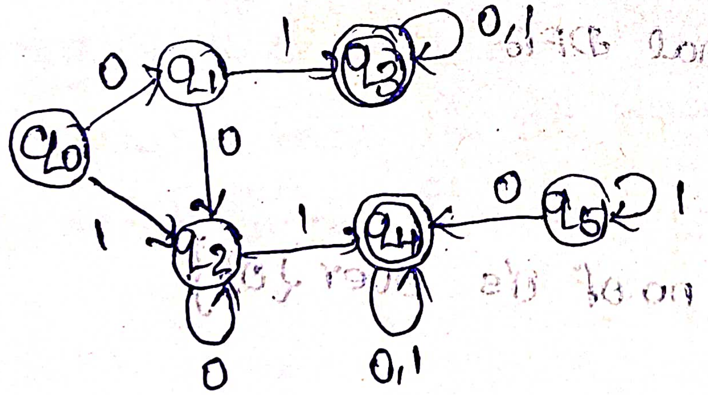


	0	1
q_0	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
$*q_4$	q_4	q_4

$P0 : \{q_0, q_1, q_2, q_3\}, \boxed{\{q_4\}}$

$P1 : \{q_1, q_3\}, \{q_0\}, \{q_2\}, \{q_4\}$

*)



	0	1
q_0	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_4
q_3	q_3	q_3
q_4	q_4	q_4
q_5		q_5

Remove q_5 because it is a inaccessible state

$P_0 = \{q_0, q_1, q_2\}, \{q_3, q_4\}$

$P_1 = \{q_0\}, \{q_1, q_2\}, \{q_3, q_4\}$

$P_2 = \{q_0\}, \{q_1, q_2\}, \{q_3, q_4\}$