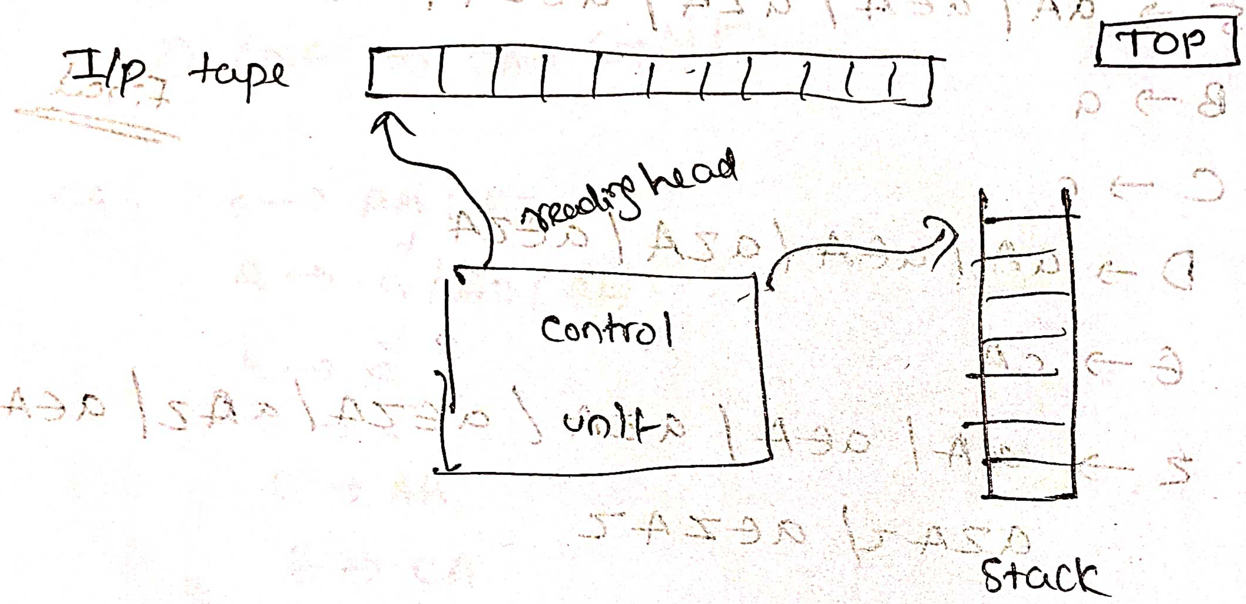
Module-4

Push-Down Automata (PDA)

* It is CFG

It is NFA-ϵ (FA) with memory (stack)

Push Pop



FA

I/p ⇒ current state, q
current input symbol, a

O/p → next state q'

$\delta(q,a) = q'$

PDA

I/p ⇒ q,
a
stack top x

O/p ⇒ next state, q'
stack operation

$\delta(q,a,x) = (q', \gamma)$

γ ⇒ push / pop / Nope

nop

$$\delta(q, a, x) = (q', x)$$

pop

$$\delta(q, a, x) = (q', \epsilon)$$

push

$$\delta(q, a, x) = (q', ax)$$

Formal definition

FA

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times \Sigma \rightarrow Q$$

PDA

stack start symbol

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Stack alphabet

$$\cancel{\delta : Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*}$$

$$\delta : Q \times \Sigma \cup \{\epsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$$

~~type~~ ~~accep~~ Language acceptance of PDA.

2 types

1) Acceptance by Final state.

$$(q_0, w, z_0) \overset{*}{\Longrightarrow} (p, \epsilon, \surd)$$

irrelevant

where $p \in F$

2) Acceptance by empty stack

$(q_0, w, Z_0) \xrightarrow{*} (q, \epsilon, \epsilon)$

$\Downarrow$

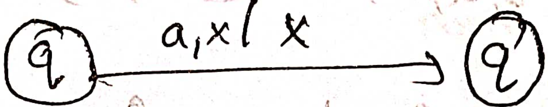
irrevelent

$q \in Q$

~~aaaab~~

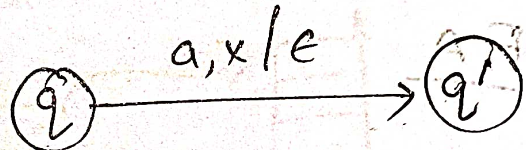
$\delta(q, a, x) = (q', x)$

$\Longrightarrow$



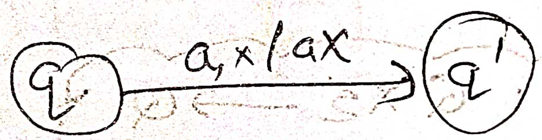
$\delta(q, a, x) = (q', \epsilon)$

$\Longrightarrow$

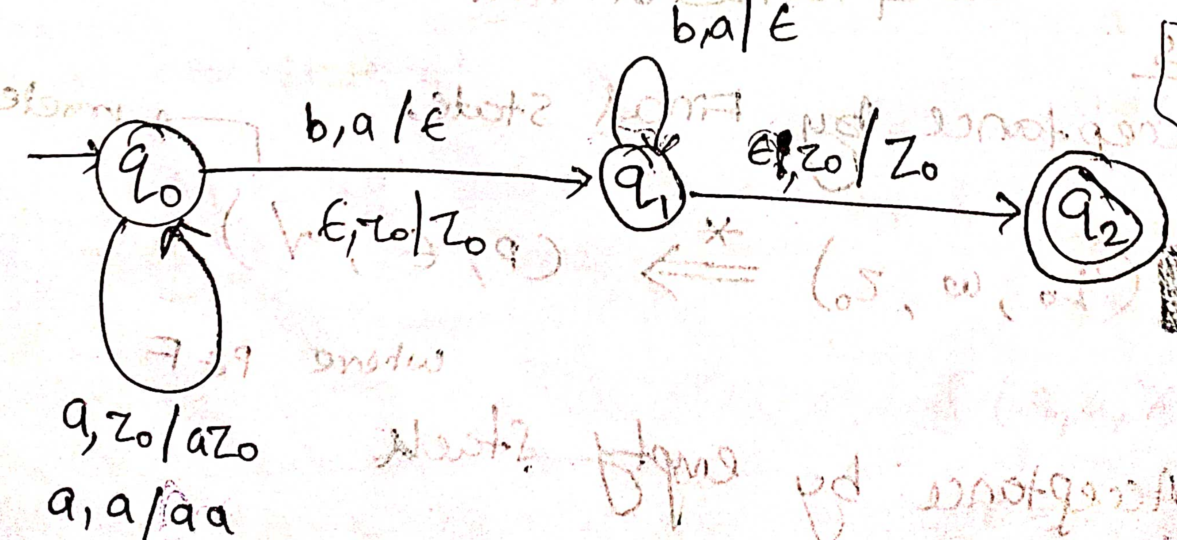


$\delta(q, a, x) = (q', ax)$

$\Longrightarrow$



Q. $L = \{a^n b^n ; n \geq 0\}$ Design a PDA for the given Language.



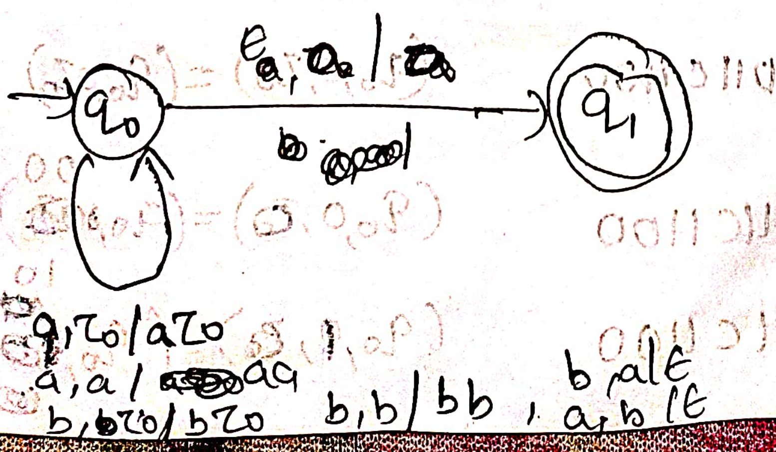
Sample String = aabb

State	input	transition	Stack
q_0	aabb		Z_0
q_0	abb	$(q_0,a,Z_0) = (q_0,aZ_0)$	aZ_0
q_0	bb	$(q_0,a,a) = (q_0,aa)$	aaZ_0
q_0	b	$(q_0,b,a) = (q_1,\epsilon)$	aZ_0
q_1	ϵ	$(q_0,b,a) = (q_1,\epsilon)$	Z_0
q_1	ϵ	$(q_0,\epsilon,Z_0) = (q_2,Z_0)$	Z_0

Since $q_2 \in F$ so this aabb string will be acceptable

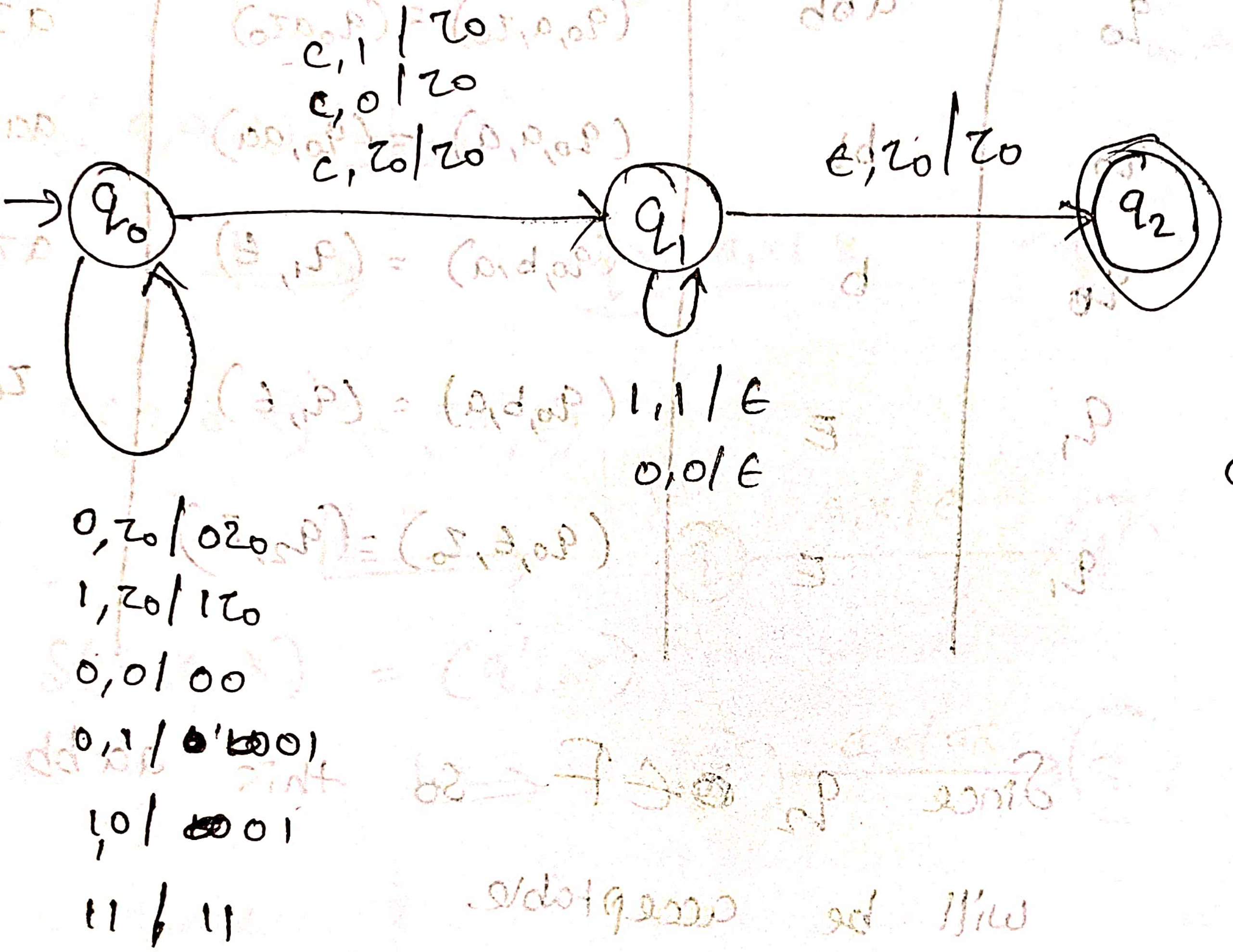
Q Desig a PDA for ~~accept~~ any the language

$$L = \{ w, w \in (a,b)^* \; n_a > n_b \}$$



Q Design a PDA for the Language

$L = \{ wcw^R, w \in (0,1)^* \}$

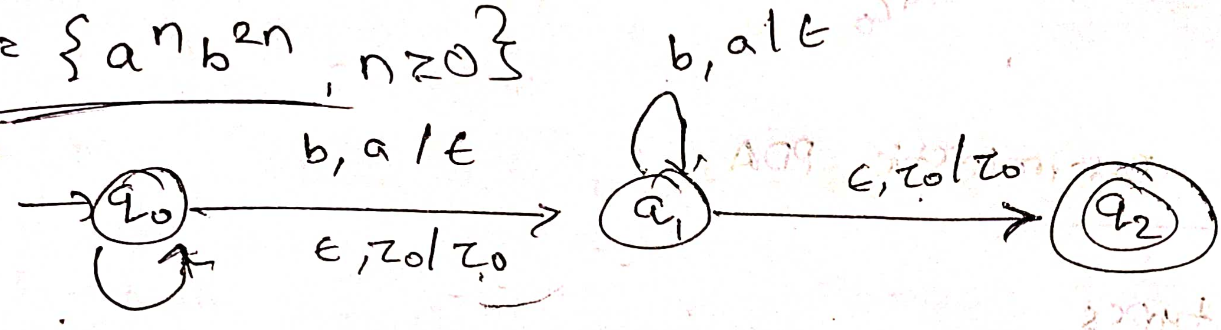


Sample String = 0011 c 1100

State	input	transition	Stack
q_0	0011c1100	←	Z_0
q_0	011c1100	$(q_0, 0, Z_0) = (q_0, 0Z_0)$	$0Z_0$
q_0	11c1100	$(q_0, 0, \cancel{Z_0}) = (q_0, 00)$	$00Z_0$
q_0	1c1100	$(q_0, \cancel{1}, \cancel{Z_0}) = (q_0, 10)$	100

q_0	c1100	$(q_0, 1, 1) = (q_0, 11)$	$1100 Z_0$
q_0	1100	$(q_0, c, 1) = (q_1, Z_0)$	$1100 Z_0$

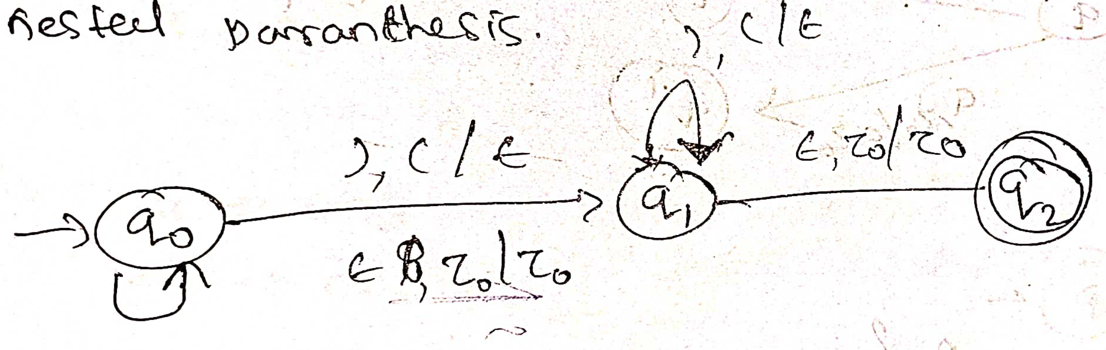
Q $L = \{a^n b^{2n}, n \geq 0\}$



$a, Z_0 / aaZ_0$

$a, a / aaa$

Q. Design a push down automata for balanced nested parranthesis.



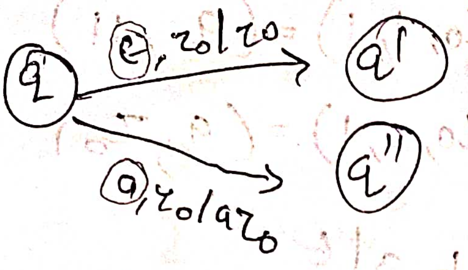
$(, Z_0 / cZ_0$

$(, c / cc$

Deterministic PDA

A PDA is deterministic i) if (q, a, z) has only 1 element

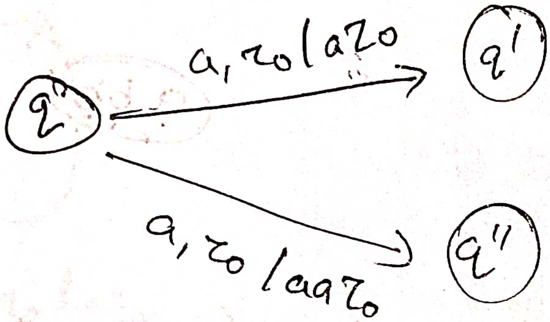
ii) if (q, ϵ, z) is not empty, then (q, a, z) for all alphabet a should be empty.



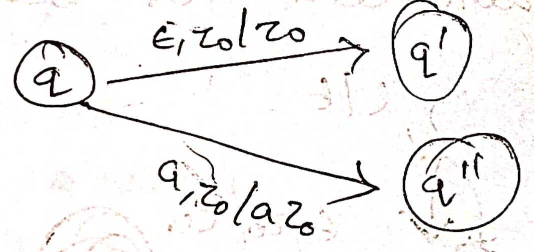
It is not possible

Non-Deterministic PDA

2 types

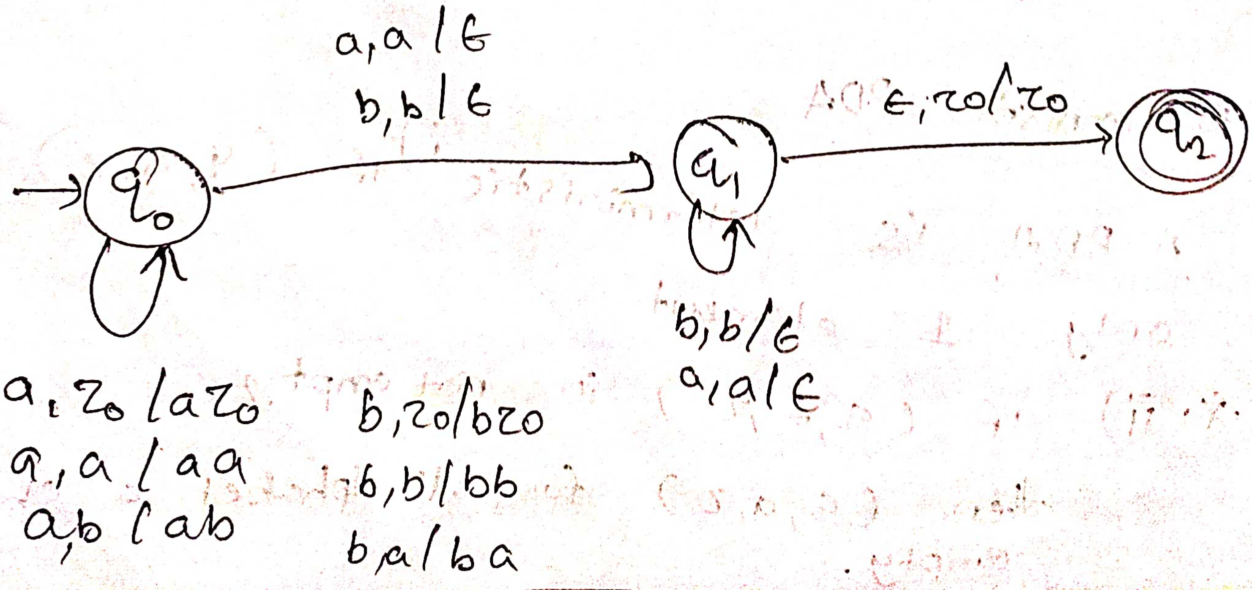


$(q, a, z_0) \rightarrow$ multiple element



It is possible.

$L = \{ ww^R \}$



conversion of CFG to PDA

CFG, $G = (V, T, P, S)$

PDA $M = (\{q\}, T, V \cup T, \delta, q, S, \phi) \Rightarrow (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

1. For each NT, A

$(q, \epsilon, A) \rightarrow (q, B)$

$A \rightarrow B$ production

2. For each T, a

$(q, a, a) = (q, \epsilon)$

eg:

$S \rightarrow 0BB$

$B \rightarrow 0S \mid 1S \mid 0$

$\rightarrow (q)$

$\epsilon, S \mid 0BB$

$\epsilon, B \mid 0S$

$\epsilon, B \mid 1S$

$\epsilon, B \mid 0$

$0, 0 \mid \epsilon$

$1, 1 \mid \epsilon$

$(q, \epsilon, S) = (q, 0BB)$

$(q, \epsilon, B) = (q, 0S)$

$(q, \epsilon, B) = (q, 1S)$

$(q, \epsilon, B) = (q, 0)$

eg: $S \rightarrow 0BB$

$\rightarrow 01SB$

$\rightarrow 010BBB$

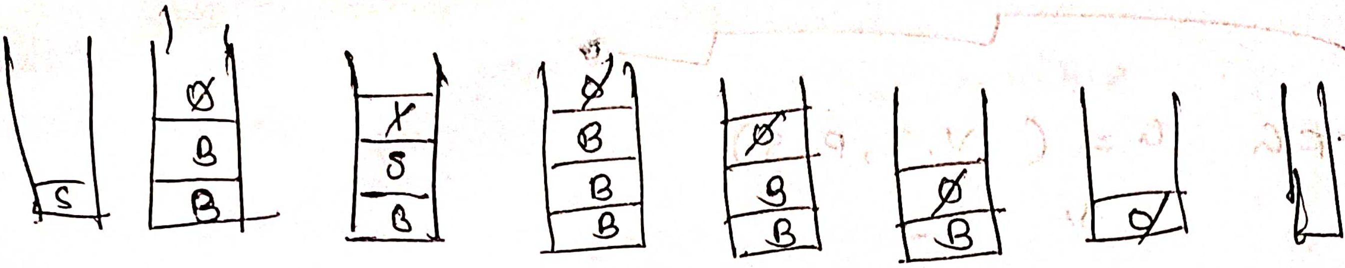
$\rightarrow 0100BB$

$\rightarrow 01000B$

$\rightarrow 010000$

left most derivative

String ⇒ ~~$01\phi\phi\phi\phi$~~



Empty Stack

So accept

Q. ~~S → AAB~~ CFG is $S \rightarrow aSb \mid a \mid b \mid \epsilon$ convert to PDA

$S \rightarrow aSb \mid a \mid b \mid \epsilon$

$(q, \epsilon, S) = (q, aSb)$

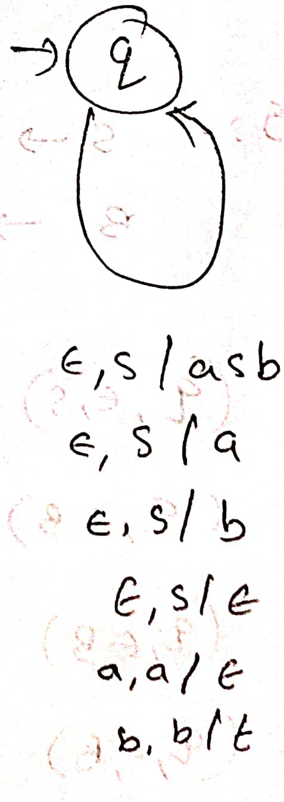
$(q, \epsilon, S) = (q, a)$

$(q, \epsilon, S) = (q, b)$

$(q, \epsilon, S) = (q, \epsilon)$

$(q, a, a) = (q, \epsilon)$

$(q, b, b) = (q, \epsilon)$

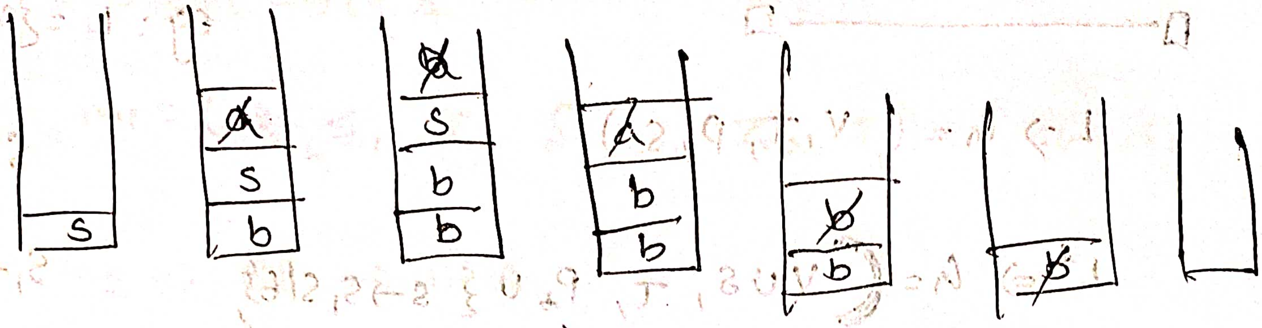


$S \rightarrow aSb$

$\rightarrow aaSbb$

$\rightarrow aaabb$

String = $\cancel{a}\cancel{a}\cancel{a}\cancel{b}\cancel{b}$



Closure properties of CFL

1. Union

$L_1 : G_1 = (V_1, T_1, P_1, S_1)$

$L_2 : G_2 = (V_2, T_2, P_2, S_2)$

$L_1 \cup L_2 \rightarrow G_3$

$G_3 = (\{V_1 \cup V_2 \cup \{S\}\}, T_1 \cup T_2, P_1 \cup P_2 \cup \{S \rightarrow S_1 | S_2\}, S)$

$L_1 \cup L_2$

$S \rightarrow S_1 | S_2$

$S_1 \rightarrow aS_1b | \epsilon$

$S_2 \rightarrow bS_2c | \epsilon$

eg: $L_1 = a^n b^n$

$G_1 = (\{S_1\}, \{a,b\}, P_1, S_1)$

$S_1 \rightarrow aS_1b | \epsilon$

$L_2 = b^n c^n$

$G_2 = (\{S_2\}, \{b,c\}, P_2, S_2)$

$S_2 \rightarrow bS_2c | \epsilon$

2. Concatenation

$L_1 : G_1 = (V_1, T_1, P_1, S_1)$

$L_2 : G_2 = (V_2, T_2, P_2, S_2)$

$L_1 \cdot L_2 = G_3 = (V_1 \cup V_2 \cup \{S\}, T_1 \cup T_2, P_1 \cup P_2 \cup \{S \rightarrow S_1S_2\}, S)$

eg:

$L_1 = a^n b^n$.

$L_2 = b^n c^n$

$L_1 \cdot L_2$

$S \rightarrow S_1 \cdot S_2$

$S_1 \rightarrow aS_1b | \epsilon$

$S_2 \rightarrow bS_2c | \epsilon$

3. Kleen closure (*)

eg: $L = \{a^n b^n, n \geq 0\}$

$S \to S_1 S \mid \epsilon$

$S_1 \to a S_1 b \mid \epsilon$

$L \Rightarrow G = (V, T, P, S)$

$L^* \Rightarrow G = (V \cup S, T, P \cup \{S \to S_1 S \mid \epsilon\}, S)$

4) Homomorphism & Inverse Homomorphism

$h: T^* \to \Sigma^*$

$L: G = (V, T, P, S)$

eg: G: $S \to 0S0 \mid 1S1 \mid \epsilon$

$h(0) = aba$, $h(1) = bb$

$h(G) \Rightarrow S \to abaSaba \mid bbSbb \mid \epsilon$

5) Reversal

$L \to G$

$L^R \to G' = (V, T, P^R, S)$

Rule: $P: A \to \alpha$

$\Downarrow$

changed to

$P^R: A \to \alpha^R$

Q. Intersection of a regular language with a CFL context-free language is a regular language

Proof:-

$L_1 \to CFL \to PDA$.

$$M_1 = (Q_1, \Sigma_1, \tau, \delta_1, q_{01}, Z_0, F_1)$$

$L_2 \to RL \to DFA$

$$M_2 = (Q_2, \Sigma_2, \delta_2, q_{02}, F_2)$$

$L_1 \cap L_2 \to CFL \to PDA$

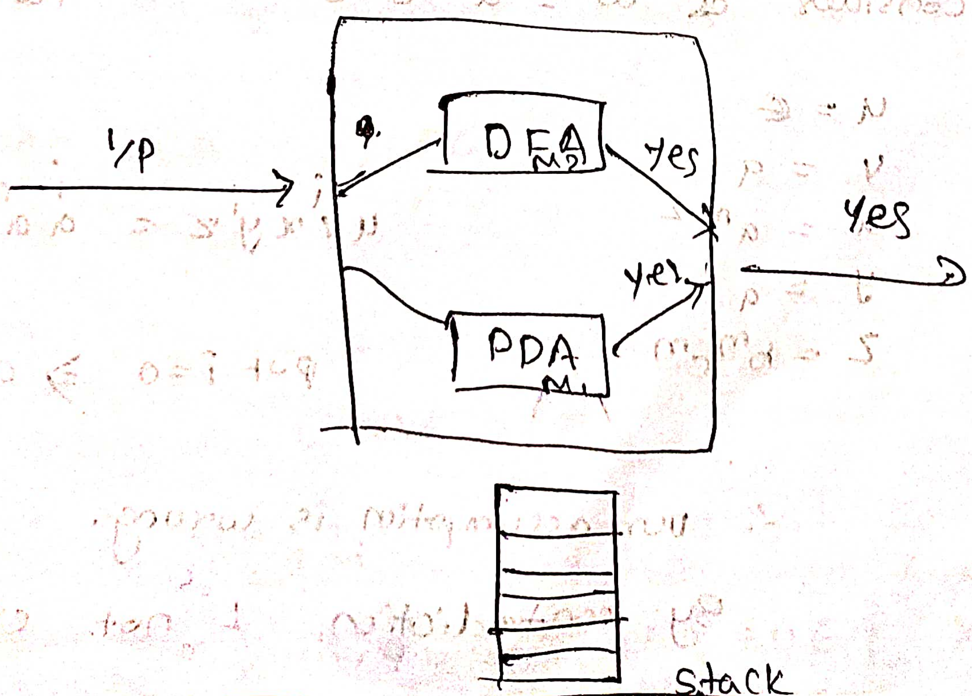
$$M = (Q_1 \times Q_2, \Sigma_1 \cup \Sigma_2, \tau, \delta, (q_{01}, q_{02}), Z_0, F_1 \times F_2)$$

$$\delta((q,p), a, \beta) = ((q', p'), \gamma)$$

Iff

$(q, a, \beta) = (q', \gamma)$ is a transition in M_1

$(p, a) = (p')$ is a transition in M_2



Pumping Lemma for CFL

Let L be a CFL then there is a pumping Lemma constant m which can be assumed as no. of states in PDA. ~~Let take~~ For any string $w \in L$, ~~with length~~ $|w| \geq m$

w can be divide in to 5 such that

$w \Rightarrow uvxyz$

$|vy| \geq 1$

$|vxy| \leq m$

then $uv^i x y^i z \;\forall_i \in L$

eg:- $L = \{a^n b^n c^n\}$ is not a CFL

Let L be a CFL.

consider $w = a^m b^m c^m$ $\qquad |w| = 3m \geq m$

$u = \epsilon$

$v = a$

$x = a^{m-2}$

$y = a$

$z = b^m c^m$

$uv^i x y^i z = a^i a^{m-2} a^i b^m c^m$

put $i=0 \Rightarrow a^{m-2} b^m c^m \notin L$

∴ our assumption is wrong.

By contradiction L is not CFL

Q. $L = \{ww, w \in (0,1)^*\}$

Let L be a CFL

Consider a $S = ww$

$w = 0^m 1^m$

$S = 0^m 1^m 0^m 1^m$

$|S| = 4m \geq m$

$u = \epsilon$

$v = 0$

$x = 0^{m-2}$

$y = 0$

$z = 1^m 0^m 1^m$

$uv^i xy^i z = 0^i 0^{m-2} 0^i 1^m 0^m 1^m$

Put $i = 0$

$= 0^{m-2} 1^m 0^m 1^m \notin L$

∴ our assumption is wrong

By contradiction L is not CFL.

Convert PDA to CFG

3 rules.

1) For every $q \in Q$ add a P

$S \to q_0 Z_0 q_i$

3) For every $q, r \in Q$, $a \in \{\Sigma \cup \epsilon\}$, $X \in \Gamma$

If $\delta(q, a, x) = (r, \epsilon)$ is a δ

add $[q, x, r] \rightarrow a$

3) For every $q, r \in Q$, $a \in \{\Sigma \cup \epsilon\}$ $x \in \Gamma$ if

$\delta(q, a, x) = (r, x_1 x_2 \ldots x_k)$ then for every

choice of $q_1 q_2 \ldots q_k \in Q$

add

$$[q x q_k] \rightarrow a[r x_1 q_1][q_1 x_2 q_2] \ldots [q_{k-1} x_k q_k]$$