

# Module-5

## RG

$$G = (V, T, P, S)$$

$$P: \alpha \rightarrow \beta$$

$$\alpha \in V$$

$$\beta = T \mid T^* \mid VT^* \mid \epsilon$$

## CFG

$$G = (V, T, P, S)$$

$$P: \alpha \rightarrow \beta$$

$$\alpha \in V$$

$$\beta = (VUT)^*$$

## CSGP Type-1

context sensitive grammar

$$P: \alpha A \beta \rightarrow \alpha y \beta$$

$$\alpha, \beta = (VUT)^*$$

$$A \in V \\ y \in (VUT)^*$$

$$|LTS| \leq |RTS|$$

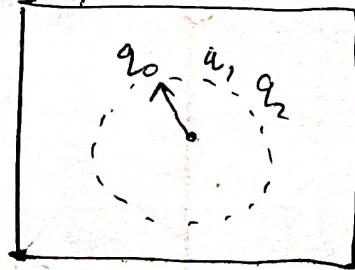
$$S \rightarrow \epsilon \text{ is allowed}$$

Recursively Enumerable Language

Entire blank space is filled with Blank symbol (B) in infinite tape.

$$B \mid B \mid B \mid B \mid x_1 \mid x_2 \mid \dots \mid \dots \mid \dots \mid \dots \mid \dots \mid x_n \mid B \mid B \mid B$$

read/write  
Head

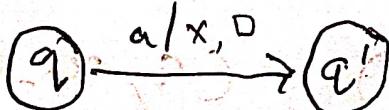


either left or right move

(q1) can move left or right

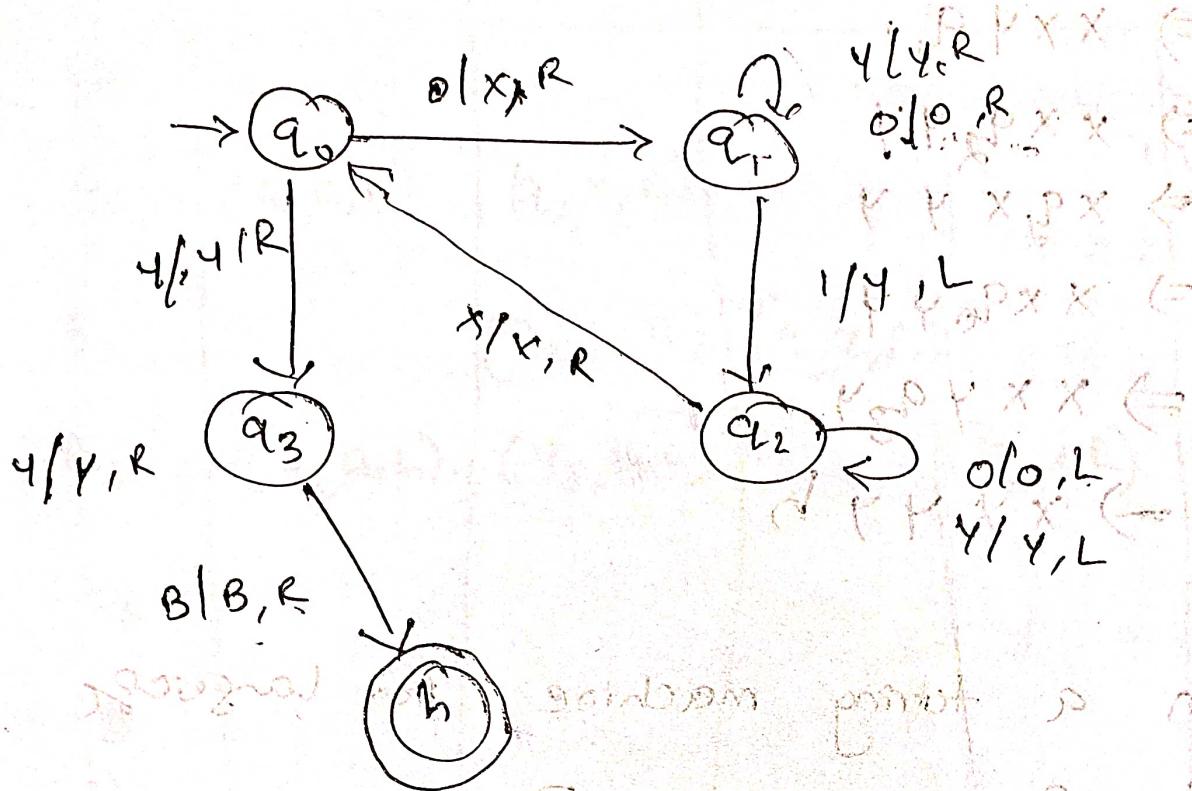
## Transition

$$\delta(q, a) = (q', x, \{L, R\})$$



States	Tape symbols		
	$x_1$	$x_2$	$\dots$
$q_0$	$a_1$	$a_2$	$\dots$
$q_1$	$b_1$	$b_2$	$\dots$
$q_2$	$c_1$	$c_2$	$\dots$
$q_3$	$d_1$	$d_2$	$\dots$
$q_4$	$e_1$	$e_2$	$\dots$
$q_5$	$f_1$	$f_2$	$\dots$
$q_6$	$g_1$	$g_2$	$\dots$
$q_7$	$h_1$	$h_2$	$\dots$
$q_8$	$i_1$	$i_2$	$\dots$
$q_9$	$j_1$	$j_2$	$\dots$
$q_{10}$	$k_1$	$k_2$	$\dots$
$q_{11}$	$l_1$	$l_2$	$\dots$
$q_{12}$	$m_1$	$m_2$	$\dots$
$q_{13}$	$n_1$	$n_2$	$\dots$
$q_{14}$	$o_1$	$o_2$	$\dots$
$q_{15}$	$p_1$	$p_2$	$\dots$
$q_{16}$	$q_1$	$q_2$	$\dots$
$q_{17}$	$q_3$	$q_4$	$\dots$
$q_{18}$	$q_6$	$q_7$	$\dots$
$q_{19}$	$q_9$	$q_{10}$	$\dots$
$q_{20}$	$q_{12}$	$q_{13}$	$\dots$
$q_{21}$	$q_{15}$	$q_{16}$	$\dots$
$q_{22}$	$q_{17}$	$q_{18}$	$\dots$
$q_{23}$	$q_{19}$	$q_{20}$	$\dots$
$q_{24}$	$q_{21}$	$q_{22}$	$\dots$
$q_{25}$	$q_{23}$	$q_{24}$	$\dots$
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$q_{30}$	$q_{29}$	$q_{30}$	$\dots$
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$q_{193}$	$q_{192}$	$q_{193}$	$\dots$
$q_{194}$	$q_{193}$	$q_{194}$	$\dots$
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$q_{196}$	$q_{195}$	$q_{196}$	$\dots$
$q_{197}$	$q_{196}$	$q_{197}$	$\dots$
$q_{198}$	$q_{197}$	$q_{198}$	$\dots$
$q_{199}$	$q_{198}$	$q_{199}$	$\dots$

$1D_3 : B \otimes_1 n_2 \dots \otimes_{i-1} n_i \otimes_i \dots \otimes_{n-1} n_n B$



	0	1	x	y	z
$q_0$	$(q_0, x, R)$	$(q_1, 0, R)$	$(q_2, 1, L)$	$(q_3, y, R)$	$(h, z)$
$q_1$		$(q_0, 0, R)$		$(q_2, 1, L)$	$(q_3, y, R)$
$q_2$			$(q_0, x, R)$	$(q_1, 0, R)$	$(q_3, y, L)$
$q_3$				$(q_0, 1, L)$	$(h, z)$
$h$					

trace '0011'

$$\begin{aligned} & q_0 0 0 1 1 \\ \Rightarrow & x q_1 0 1 1 \\ \Rightarrow & x 0 q_2 1 1 \end{aligned}$$

$$\Rightarrow x @ q_2 0 y 1$$

$$\Rightarrow q_2 x 0 y 1$$

~~$\Rightarrow$~~   ~~$\Rightarrow$~~   $\Rightarrow x q_0 0 y 1$

$\Rightarrow x \times q_1 y$

$\Rightarrow x \times y q_1$

$\Rightarrow x \times q_2 y y$

$\Rightarrow x q_2 x y y$

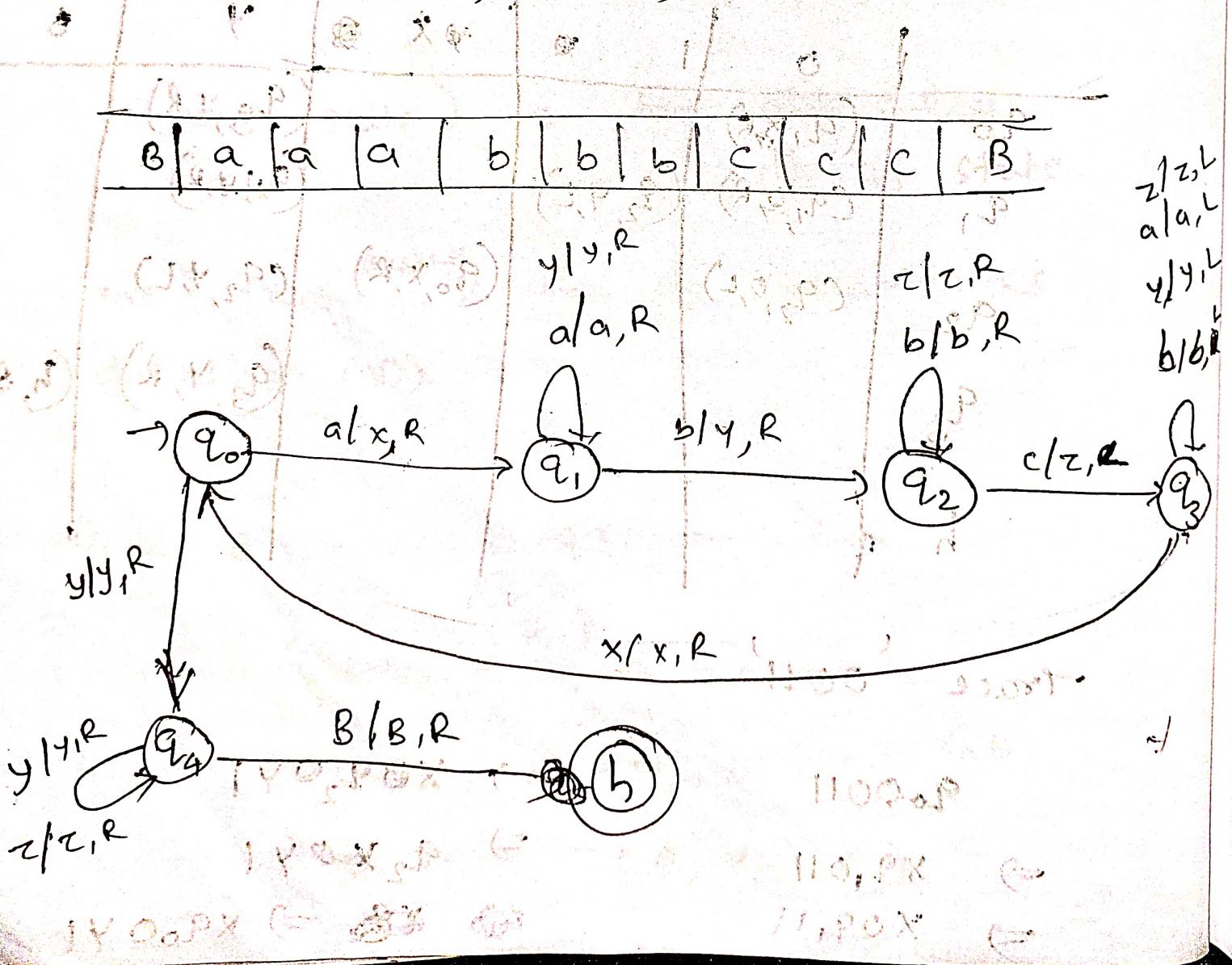
$\Rightarrow x \times q_3 y y$

$\Rightarrow x \times y q_3 y$

$\Rightarrow x \times y y b$

Q. Design a turing machine for language

$$L = \{a^n b^n c^n, n \geq 1\}$$



	a	b	c	x	y
$q_0$	$(q_1, x, R)$				$(q_4, y, R)$
$q_1$	$(q_1, a, R)$	$(q_2, y, R)$			$(q_1, y, R)$
$q_2$		$(q_2, b, R)$	$(q_3, z, R)$		
$q_3$	$(q_3, a, L)$	$(q_3, b, L)$		$(q_0, x, R)$	$(q_3, y, L)$
$q_4$					$(q_4, y, R)$
$h$					

	a	b	c	x	y
$q_0$					
$q_1$					
$q_2$	$(q_2, z, R)$				
$q_3$		$(q_3, z, L)$			
$q_4$	$(q_4, z, R)$		$(h, B, R)$		
$h$					

$$\Rightarrow B q_0 a a b b c c B$$

$$\Rightarrow B x x y y q_3 z z B$$

$$\Rightarrow B x x y q_3 y z z B$$

$$\Rightarrow B x x q_3 y y z z B$$

$$\Rightarrow B x q_3 x y y z z B$$

$$\Rightarrow B x x q_0 y y z z B$$

$$\Rightarrow B x x y q_4 y z z B$$

$$\Rightarrow B x x y y q_4 z z B$$

$$\Rightarrow B x x y y z q_4 z B$$

$$\Rightarrow B x x y z z q_4 B$$

$$\Rightarrow B x x y y z z B h$$

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$$\Rightarrow B x a y b \cancel{q_0} \cancel{q_3} z c B$$

$$\Rightarrow B x a y \cancel{q_3} b z c B$$

$$\Rightarrow B x a q_3 a y b z c B$$

$$\Rightarrow B q_3 x a y b z c B$$

$$\Rightarrow B x q_0 a y b z c B$$

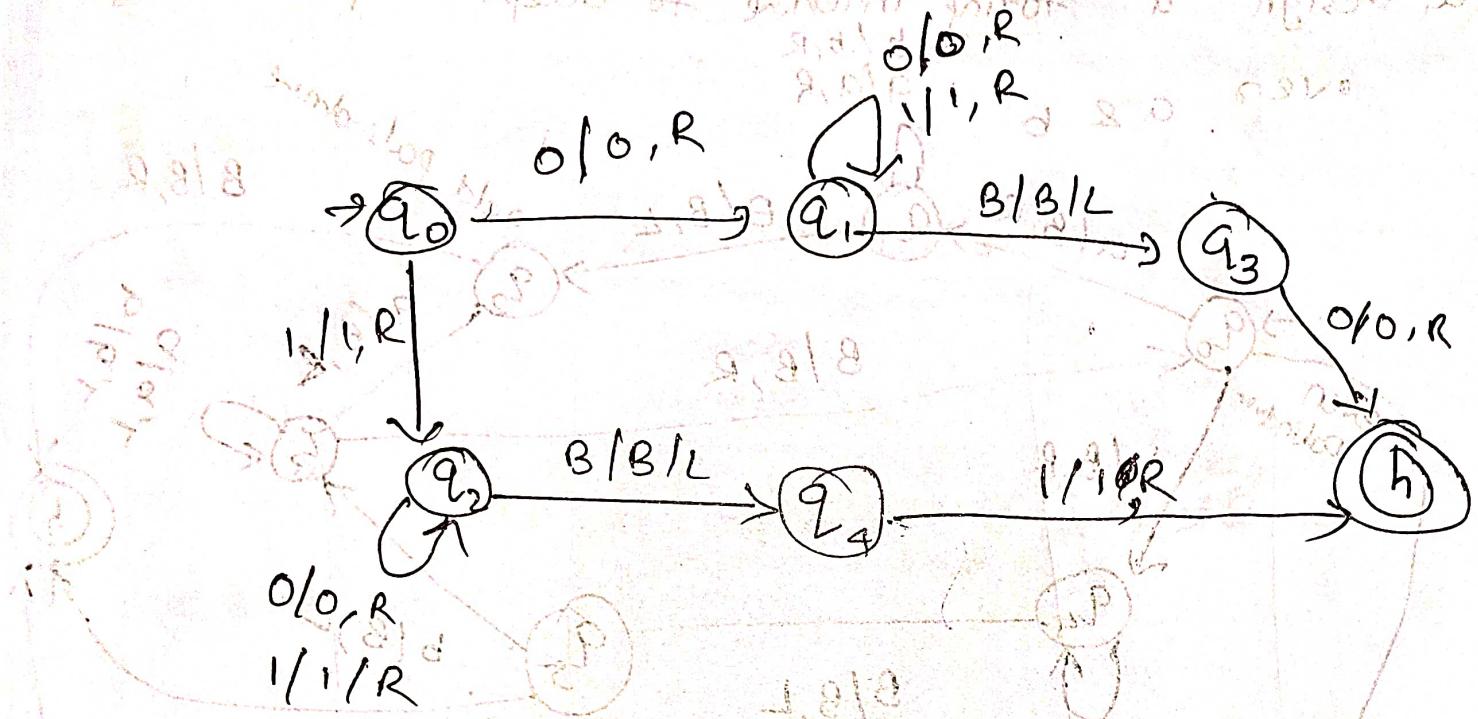
$$\Rightarrow B x x q_1 y b z c B$$

$$\Rightarrow B x x y q_1 b z c B$$

$$\Rightarrow B x x y z q_2 z c B$$

$$\Rightarrow B x x y y z q_2 c B$$

a. Design a turning machine to accept set of all string with 0 and 1 start and end with same number.



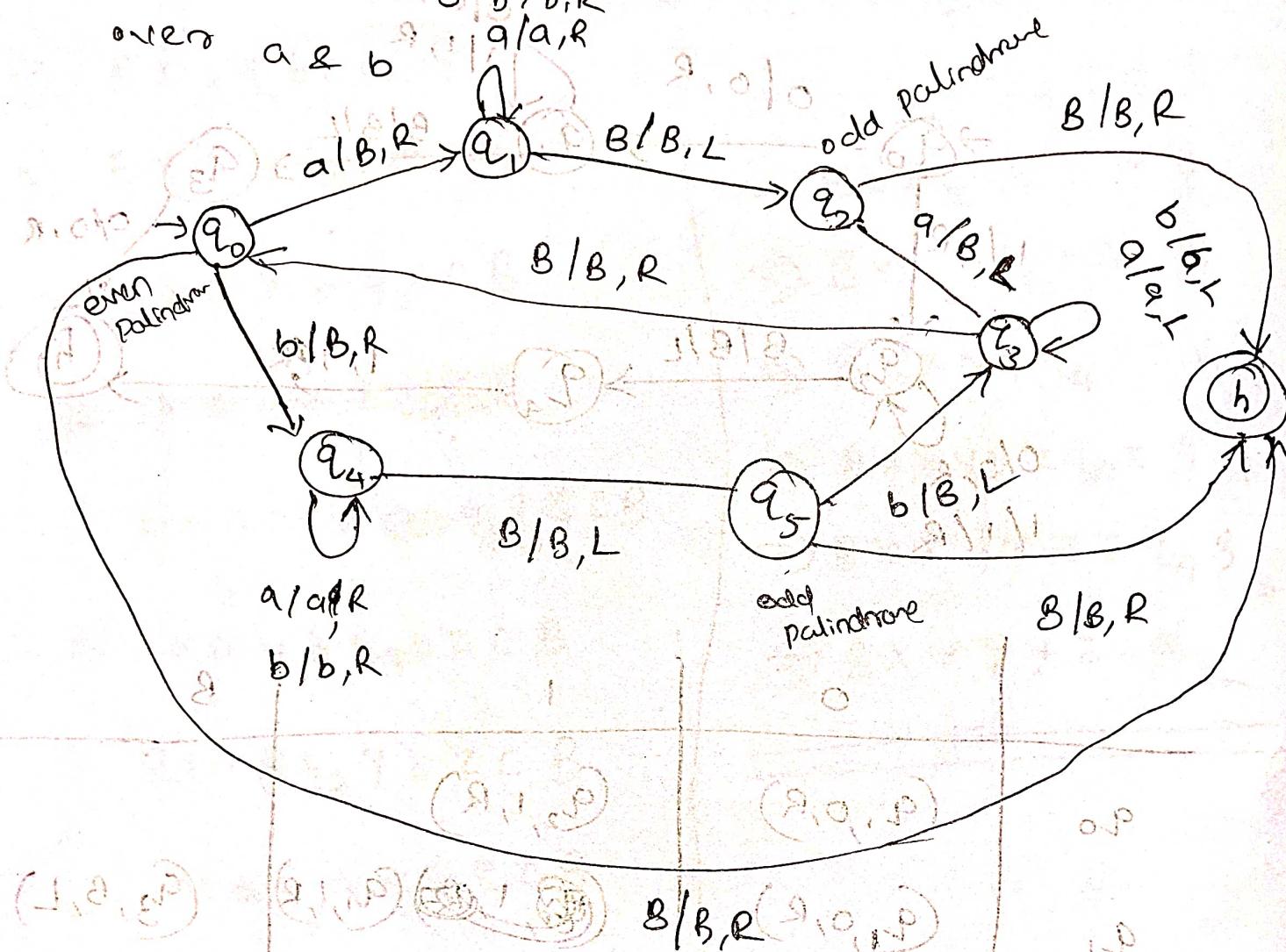
	0	1	B
q0	(q1, 0, R)	(q2, 1, R)	
q1	(q1, 0, R)	(q1, 1, R) (q1, 1, R)	(q3, B, L)
q2	(q2, 0, R)	(q2, 1, R)	(q4, B, L)
q3	(h, 0, R)		
q4		(h, 1, R)	
b			

Q. Design a turing machine to accept palindromes over {a, b}

over {a, b}

Q. Design a turing machine to accept palindromes over {a, b}

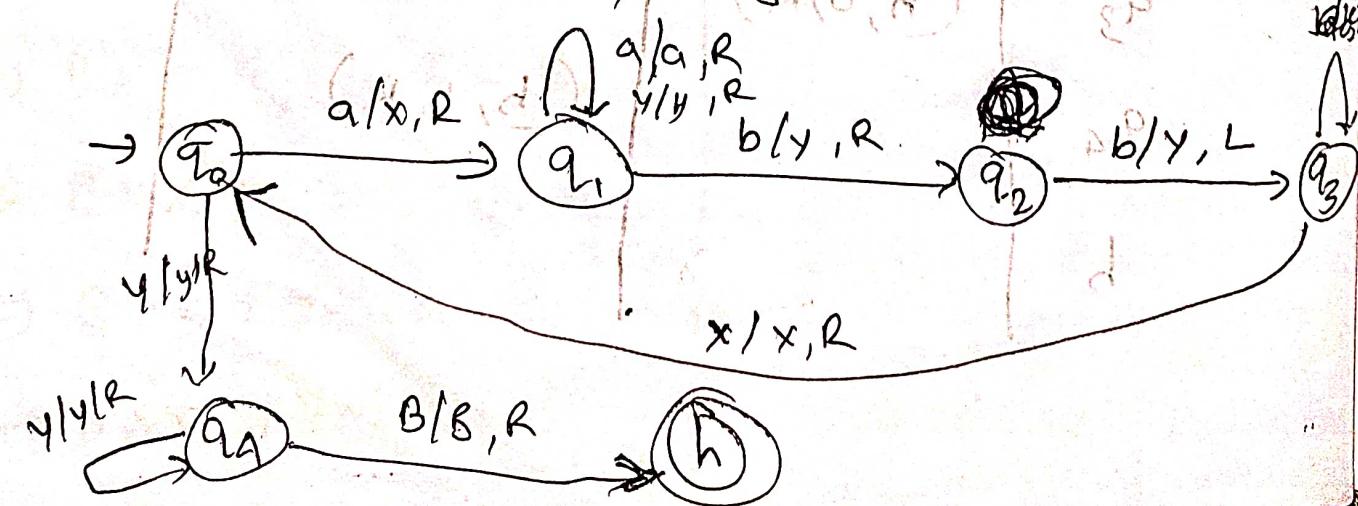
over {a, b}



Q. Design a turing machine to accept palindromes

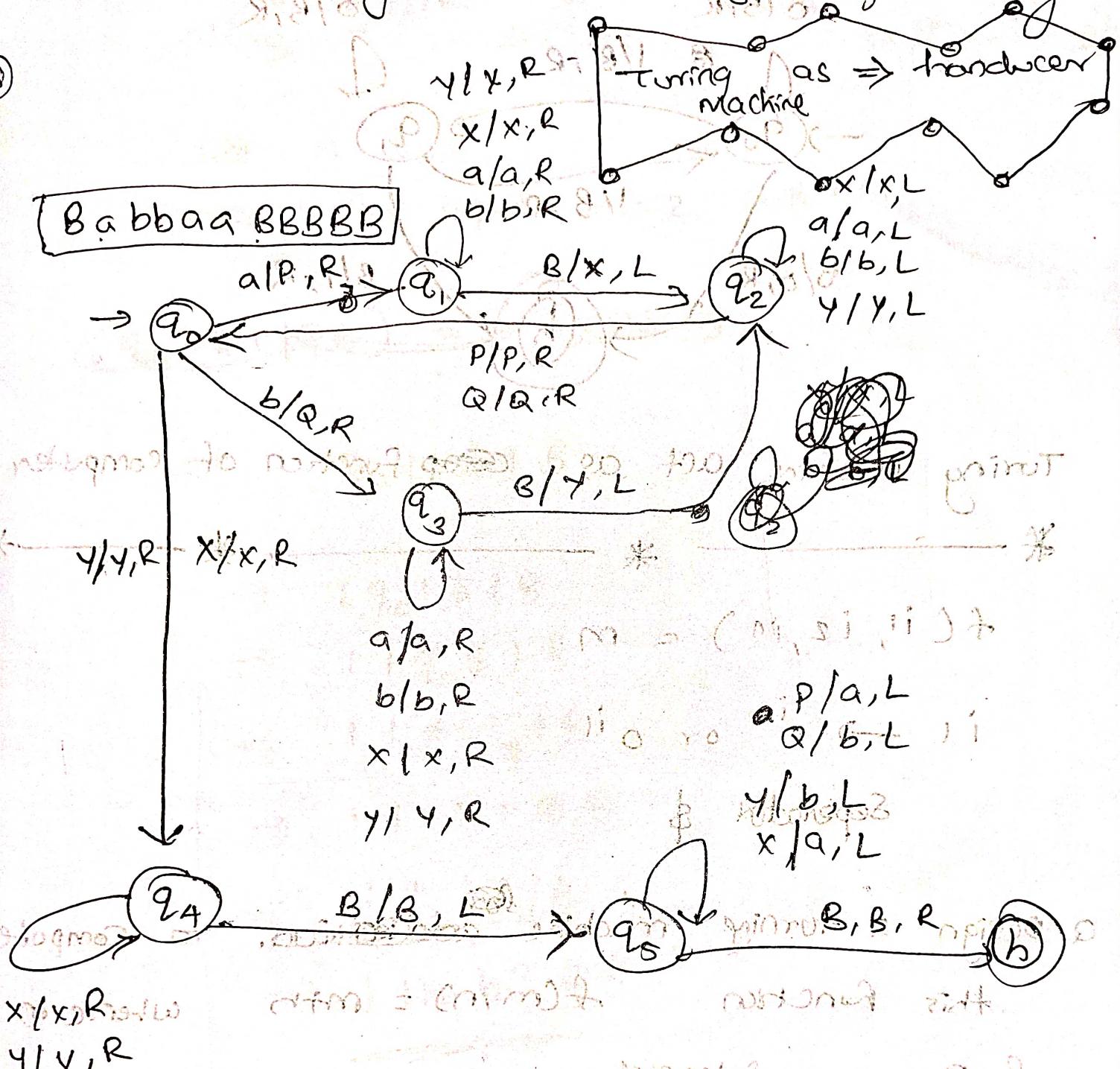
over

$$L = \{a^n b^{2n} a, n \geq 1\}$$

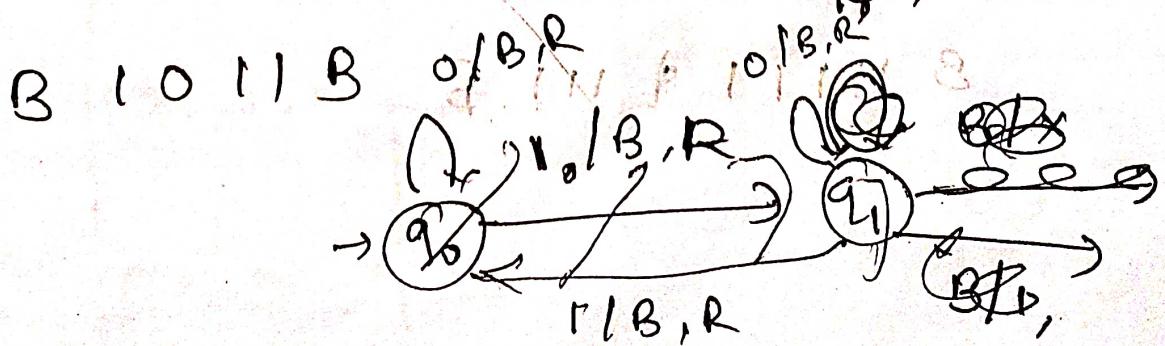


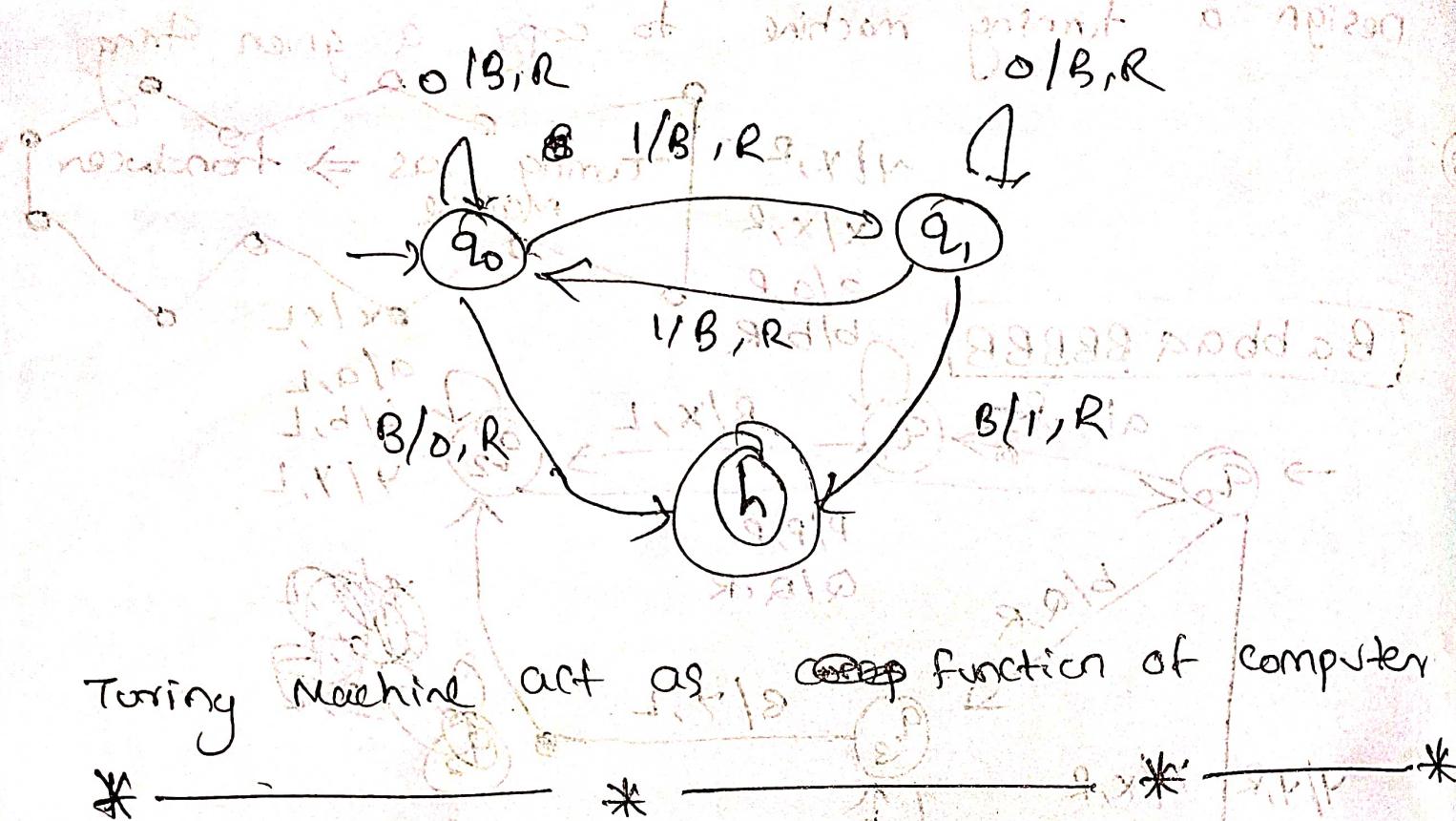
Q Design a turning machine to copy a given string

a)



Q Design a turning machine to check a parity checker.





Turing Machine act as ~~comp~~ function of computer

$$f(i_1, i_2, i_3) = m$$

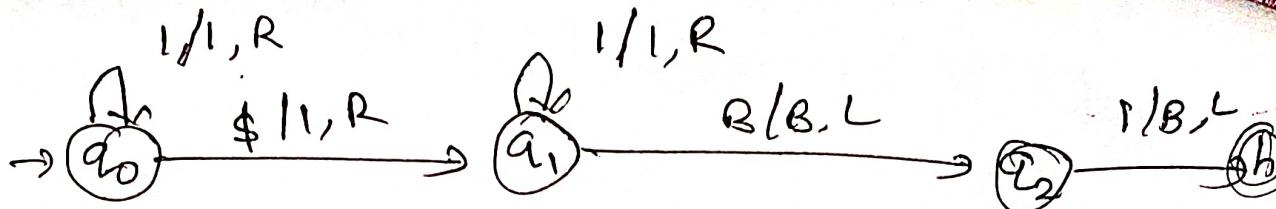
$$i_1 \rightarrow 1^m \text{ or } 0^n$$

Separator \$

Q. Design a Turing machine ~~addition~~ to compute  
this function  $f(m, n) = m + n$  where  $m$   
 $\& n$  are integers.

$$\text{Let } m = 5 \quad n = 3$$

$$11111 \$ 111 B \quad 11010$$



~~Q<sub>0</sub>~~ ~~Q<sub>1</sub>~~ ~~Q<sub>2</sub>~~

Trace for input m=2 and n=1

~~(q<sub>0</sub>, q<sub>1</sub> + q<sub>2</sub>) = q<sub>0</sub> q<sub>1</sub> q<sub>2</sub>~~

②  $q_0 \text{ } 1 \text{ } 1 \text{ } \$ \text{ } 1 \text{ } B$

$\rightarrow 1 \text{ } q_0 \text{ } 1 \text{ } \$ \text{ } 1 \text{ } B$

$\rightarrow 1 \text{ } 1 \text{ } q_0 \text{ } \$ \text{ } 1 \text{ } B$

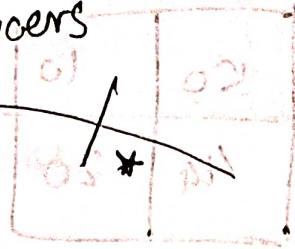
$\rightarrow 1 \text{ } 1 \text{ } 1 \text{ } q_1 \text{ } B$

$\rightarrow 1 \text{ } 1 \text{ } 1 \text{ } q_2 \text{ } B$

$\rightarrow 1 \text{ } 1 \text{ } b \text{ } 1 \text{ } b \text{ } B \text{ } B$

		1	\$	B
		(q <sub>0</sub> , I, R)	(q <sub>1</sub> , I, R)	
		(q <sub>1</sub> , I, R)		(q <sub>2</sub> , B, L)
q <sub>0</sub>				
q <sub>1</sub>				
q <sub>2</sub>				
b				

# Turing Machine as transducers



\* Design an ~~universal~~ Turing machine for proper subtraction

$$f(m, n) = m - n \text{ if } m \geq n \\ \text{otherwise.}$$

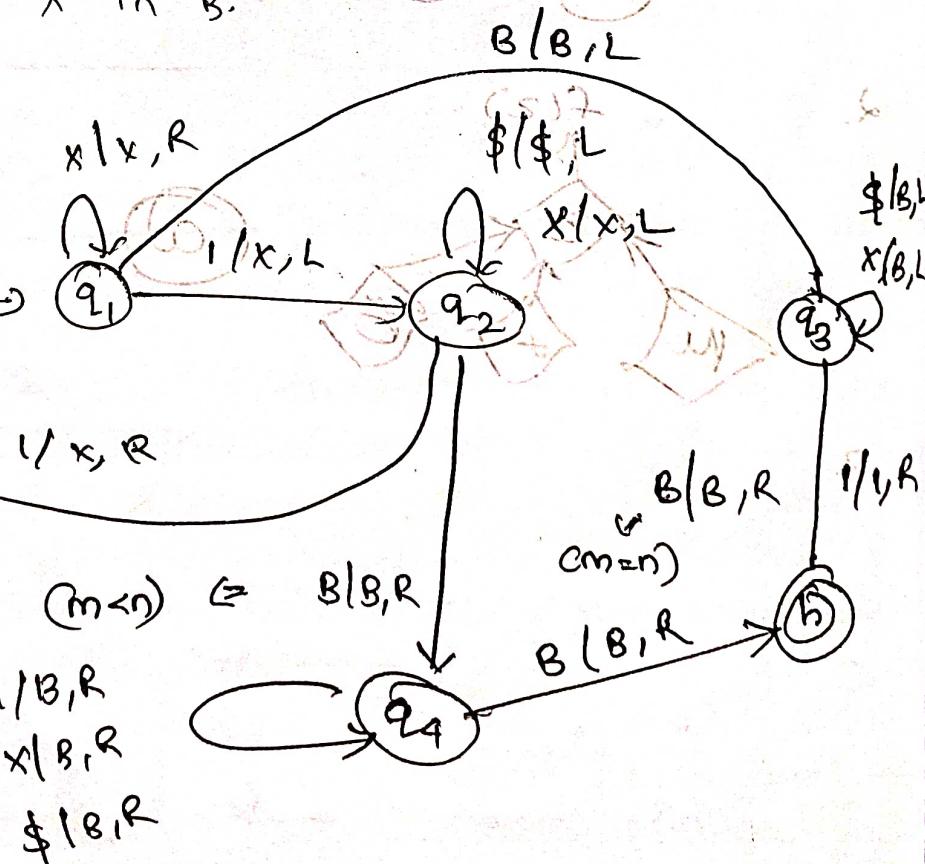
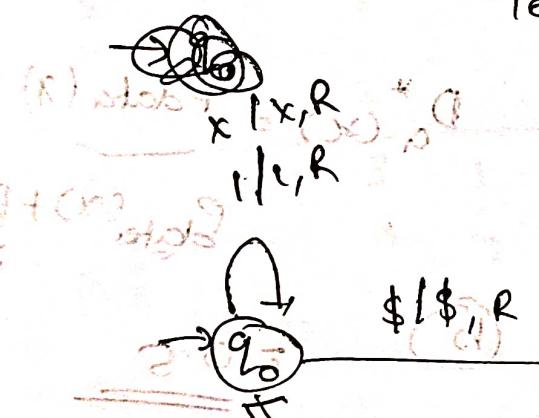
$$m=5 \quad n=2$$

$$m=2 \quad n=3$$

B 1 1 1 1 \$ 11 B

B 1 1 1 B

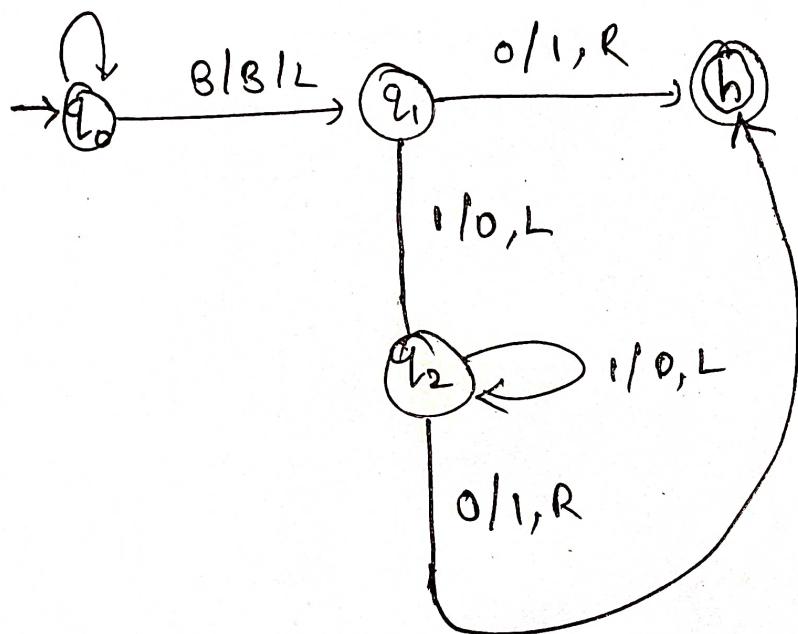
first right of \$ make to x and find parity in left and change to  $\bar{x}$ . last change all x in B.



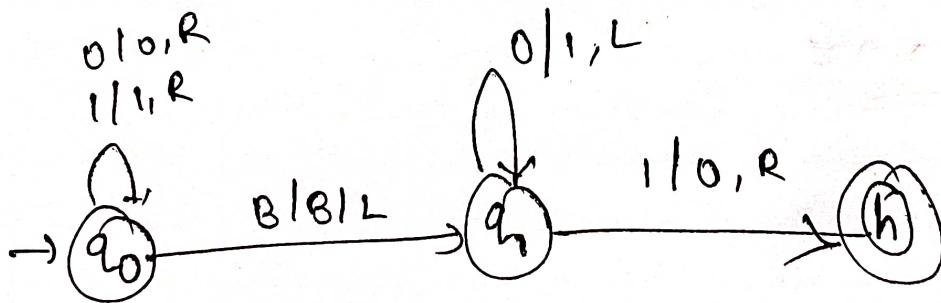
a. Design a turning machine to increment a binary number

- 1) If last bit '0' then flip into 1
- 2) If last bit '1' then move left all consecutive group of 1 changes to 0 and first 0 into 1.

$1/1, R$   
 $0/0, R$



Decrement a binary number.



Q. Binary addition

$$m = 3 \quad n = 2$$

011      010

$$B\ 011 + 010\ B \Rightarrow B\ 011 + 010\ B$$

↓

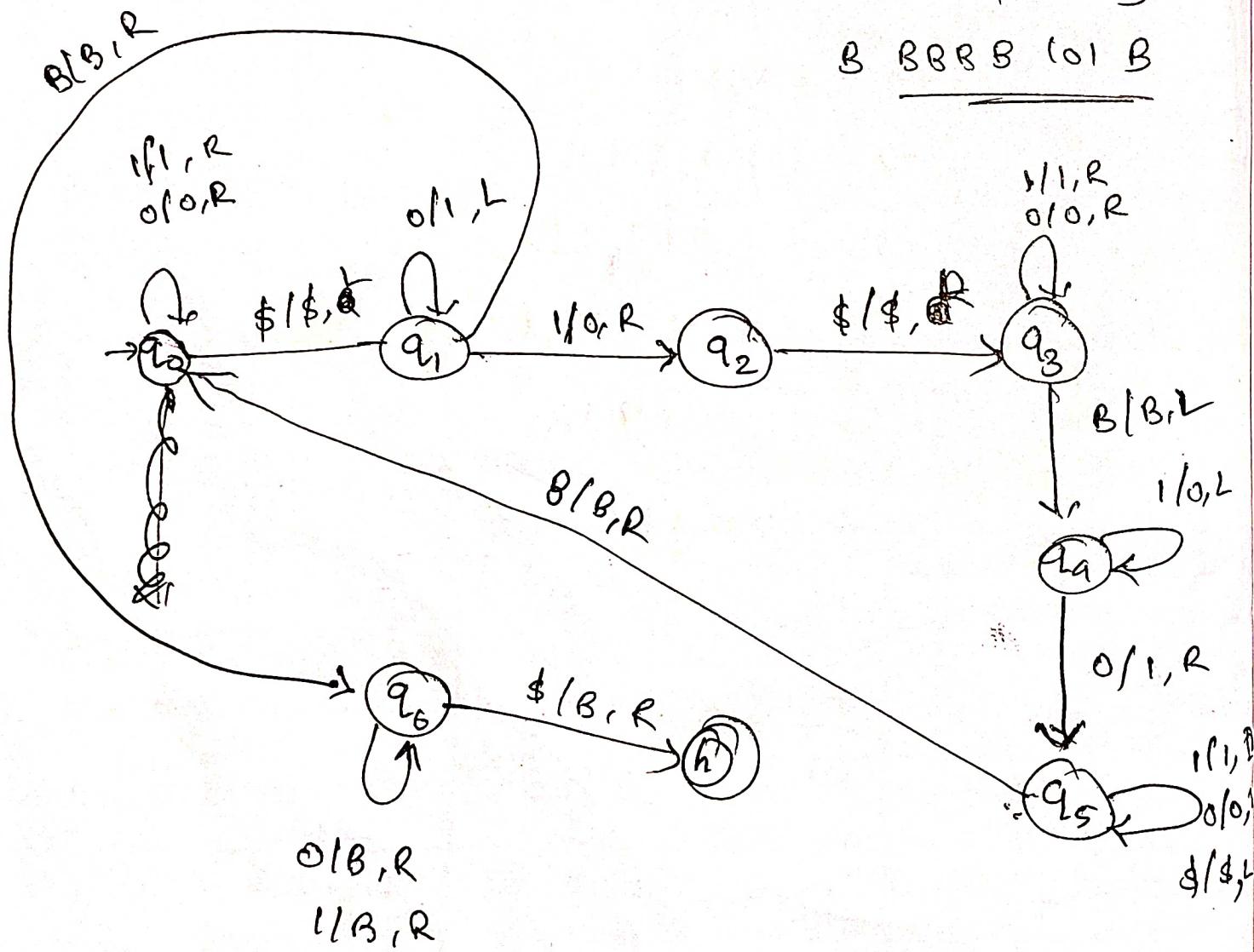
$$B\ 101\ B$$

$$B\ 010 + 011\ B$$

$$B\ 001 + 100\ B$$

$$B\ 000 + 101\ B$$

$$\underline{B\ B B B\ B\ 101\ B}$$



# Recursively Enumerable Language (RE)

1) not a type 3 grammar / unrestricted

$q \leftarrow p$

$\beta \alpha$

$A \beta$

open

nonmon

$\alpha \rightarrow \beta$

multiple

ESG

$\alpha\beta, \beta \in (VFT)^*$

$\alpha \notin E$

1.7.2

ESG

①

$L \in RE$ ,  $m \in TM$  is

Turing recognizable

\* if  $w \in L$ , TM halt at a final state

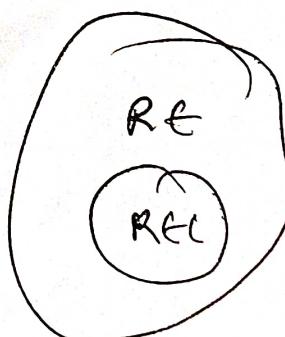
+ if  $w \notin L$ , TM halt in a non-final state

or  
loop forever.

LE REC

if  $w \in L$  → final state  
halt

if  $w \notin L$  → non final



# (38) Chomsky - hierarchy

Grammar	Language	Automata	Example	Rules for G
Type 3	Regular	FA	$a^*$	$\alpha \rightarrow \beta$ $\alpha \in V, \beta \in L$
Type 2	CFL	PDA	$a^n b^n$	$\alpha \rightarrow \beta$ $\alpha \rightarrow V, \beta \in L$
Type 1	CSL (MT)	LBA	$a^n b^n c^n$	$\alpha \times \beta \rightarrow \alpha \gamma \beta$ $\alpha, \beta \in (VUT)^*$
Type 0	RE	TM	$a^n b^n c^n$	$\alpha \rightarrow \beta$ $\alpha, \beta \in (VUT)^*$ $\alpha \neq \epsilon$