Linear Algebra

(1) Solving linear System of equations

Eg:
$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}$$
A X B

(i) Write Eqn as $AX = B$

(ii) Write augmented matrix $[AB]$

(iii) Reduce aug. matrix to row reduce form.

(iv) Find rank of AB, A

(v) Find no of unknown

(vi) If $R(AB) = R(A)$ System Consistent

$R(AB) = R(A) =$ no of unknown
Consistent
unique Solution

$R(AB) = R(A) \neq$ no of unknown
Consistent
∞ solⁿ

$R(AB) \neq R(A) \Rightarrow$ No solⁿ

(2) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

* Eigen Value :-

Charcterstic Eqn $|A-\lambda I| = 0$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = a_{11} + a_{22} + a_{33}$ (sum of main diag. element)

$S_3 = |A|$ (det. A)

$S_2 = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Eigen vector

$|A - \lambda I| x = 0$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1-\lambda & 1 & 2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & 3-\lambda \end{bmatrix}$$

(I) Substitute values of λ.

(II) choose two different row from matrix

(III) Find x_1

x_2

x_3.

Diagonalisation

1. Find eigen value
2. Find eigen vector
3. Form modal matrix $P = [x_1 \ \ x_2 \ \ x_3]$
4. $P^{-1} = \dfrac{\operatorname{adj} P}{|P|}$
5. $D = P^{-1} A P$.

Laplace Transform.

* $L(1) = 1/s$
  $L(t) = 1/s^2$
  $L(t^n) = \frac{n!}{s^{n+1}}$
  $L(e^{at}) = \frac{1}{s-a}$

- $L^{-1}(1/s) = 1$
- $L^{-1}(1/s^2) = t$
- $L^{-1}\left(\frac{1}{s^{n+1}}\right) = \frac{t^n}{n!}$
- $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

$L(\sin at) = \frac{a}{s^2+a^2}$ $\quad L(\sinh at) = \frac{a}{s^2-a^2}$

$L(\cos at) = \frac{s}{s^2+a^2}$ $\quad L(\cosh at) = \frac{s}{s^2-a^2}$

* $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$	$\cos^3\theta = \frac{1}{4}[3\cos\theta + \cos 3\theta]$ $\sin^3\theta = \frac{1}{4}[3\sin\theta + \sin 3\theta]$	$\cosh^2\theta = \frac{1+\cosh 2\theta}{2}$ $\sinh^2\theta = \frac{\cosh 2\theta - 1}{2}$

* $\sin A \cdot \cos B = 1/2 \,[\sin(A+B) + \sin(A-B)]$
  $\sin A \cdot \sin B = 1/2 \,[\cos(A-B) - \cos(A+B)]$
  $\cos A \cdot \sin B = 1/2 \,[\sin(A+B) - \sin(A-B)]$
  $\cos A \cdot \cos B = 1/2 \,[\cos(A+B) + \cos(A+B)]$

* $L[e^{at} \cdot f(t)] = f(s-a)$

* $L[t\, f(t)] = -\frac{d}{ds} f(s) \Rightarrow L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} L[f(t)]$

CONVOLUTION

$$(f * g)(t) = \int_0^t f(u) \cdot g(t-u) \cdot du .$$

LAPLACE USING ODE

$$y'' + 2y' + y = e^{2t}$$

$$y = CF + PI$$

$$L(y') = s\,L(y) - y(0)$$

$$L(y'') = s^2 L(y) - s\,y(0) - y'(0)$$

$$L(y''') = s^3 L(y) - s^2 y(0) - s\,y'(0) - y''(0)$$

Taylor Sereis

$$f(x) = f(x_0) + \frac{(x-x_0)}{1!} f'(x) + \frac{(x-x_0)^2}{2!} f''(x) + \frac{(x-x_0)^3}{3!} f'''(x) \cdots$$

Maclurian Sereis

$$x_0 = 0$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \cdots$$

Fourier Sereis

- $\int \sin x = -\cos x$
- $\int \cos x = \sin x$

$$\int \sin nx \, dx = -\frac{\cos nx}{n}$$

$$\int \cos nx \, dx = \frac{\sin nx}{n}$$

- $\int uv \, dx = uv_1 - u'v_2 + u''v_3$

Eg.

- $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{m} x + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{m} x$

$$a_0 = \frac{1}{m} \int_c^{c+2m} f(x) \, dx$$

$$a_n = \frac{1}{m} \int_c^{c+2m} f(x) \cdot \cos \frac{n\pi}{m} x \, dx$$

$$b_n = \frac{1}{m} \int_c^{c+2m} f(x) \sin \frac{n\pi}{m} x \, dx.$$

$$\int (ax+b)^n = \frac{(ax+b)^{n+1}}{(n+1)a}$$

$$\int e^{ax} \cdot \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$$