VISWAJYOTHI
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Module-4

ALGORITHM THINKING WITH PYTHON

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Brute-Force Approach to Problem Solving

- The brute-force approach represents simplicity and exhaustive computation.
- This approach is a fundamental method in problem-solving that involves systematically trying every possible solution to find the optimal one.
- It involves systematically checking every possible solution to find the optimal one, making it ideal for problems like cracking padlocks or guessing passwords.
- Despite its simplicity, brute-force methods can be computationally expensive, especially for problems with large solution spaces, leading to impractical execution times in real-world applications.
- The brute-force approach, also known as exhaustive search, operates by checking all possible solutions systematically, without employing any sophisticated strategies to narrow down the search space.
- It ensures finding a solution if it exists within the predefined constraints but can be computationally intensive and impractical for problems with large solution spaces.
- This section explores the concept, applications, advantages, and limitations of the brute-force approach through various examples across different domains.

Examples of Brute-Force Approach

- 1. **Padlock**

Imagine you encounter a padlock with a four-digit numeric code. The brute-force approach would involve sequentially trying every possible combination from ”0000” to ”9999” until the correct code unlocks the padlock.

- 2. **Password Guessing**

-In the realm of cybersecurity, brute-force attacks are used to crack passwords by systematically guessing every possible combination of characters until the correct password is identified.

-For instance, attacking a six-character password consisting of letters and digits would involve testing all 2.18 billion (366) possible combinations until the correct one is identified

- 3. **Cryptography: Cracking Codes**

-In cryptography, brute-force attacks are used to crack codes or encryption keys by systematically testing every possible combination until the correct one is found.

For example, breaking a simple substitution cipher involves trying every possible shift in the alphabet until the plain-text message is deciphered.

- 4. **Sudoku Solving**

-Brute-force methods can be applied to solve puzzles like Sudoku by systematically filling in each cell with possible values and backtracking when contradictions arise.

-This method guarantees finding a solution but may require significant computational resources, especially for complex puzzles.

Characteristics of Brute-Force Solutions

- 1. **Exhaustive Search**: Every possible solution is examined without any optimization.
- 2. **Simplicity**: Easy to understand and implement.
- 3. **Inefficiency**: Often slow and resource-intensive due to the large number of possibilities.
- 4. **Guaranteed Solution**: If a solution exists, the brute-force method will eventually find it.

Solving Computational Problems Using Brute-force Approach

- To solve computational problems using the brute-force approach, one must systematically explore and evaluate all possible solutions to identify the correct answer.
- This involves defining the problem clearly, generating every potential candidate solution, and checking each one against the problem's criteria to determine its validity.
- While this method ensures that all possible solutions are considered, it often results in high computational costs and inefficiencies, especially for large or complex problems.
- Despite these limitations, the brute-force approach provides a straightforward and reliable way to solve problems by exhaustively searching the solution space, offering a foundation for understanding and improving more advanced algorithms.

Problem solving Using Brute-force Approach

Problem-1 (String Matching)

- The brute-force string matching algorithm is a simple method for finding all occurrences of a pattern within a text. The idea is to slide the pattern over the text one character at a time and check if the pattern matches the substring of the text starting at the current position.

Here is a step-by-step explanation:

- 1. **Start at the beginning of the text**: Begin by aligning the pattern with the first character of the text.
- 2. **Check for a match**: Compare the pattern with the substring of the text starting at the current position. If the substring matches the pattern, record the position.
- 3. **Move to the next position**: Shift the pattern one character to the right and repeat the comparison until you reach the end of the text.
- 4. **Finish**: Continue until all possible positions in the text have been checked.
- This approach ensures that all possible starting positions in the text are considered, but it can be slow for large texts due to its time complexity.

Problem solving Using Brute-force Approach

Problem-1 (String Matching)

```
def brute_force_string_match(text, pattern):
    n = len(text)                    # Length of the text
for i in range(n - m + 1):
    substring = text[i:i + m]
    """ Loop over each possible starting index in the text,
Extracting the substring of the text from the current
position """
    # Compare the substring with the pattern
    if substring == pattern:
        print(f"Pattern found at index {i}")
# Example usage
text = "ABABDABACDABABCABAB"
pattern = "ABABCABAB"
brute_force_string_match(text, pattern)
```

Here is what the output of the function will be:

```
Pattern found at index 10
```

Problem solving Using Brute-force Approach

Problem-1 (String Matching)

Explanation:

1. **Function Definition:** The function **brute_force_string_match** takes two arguments: **text** and **pattern**.

2. **Length Calculation:** It calculates the lengths of both the **text** and **pattern** to determine how many possible starting positions there are.

3. **Loop Over Positions:** It uses a for loop to slide the pattern across the text. The loop runs from **0** to **n - m + 1**, where **n** is the length of the **text** and **m** is the length of the **pattern**.

4. **Substring Extraction:** At each position **i**, the code extracts a substring from the text that has the same length as the pattern (**text[i:i + m]**).

5. **Comparison:** It then compares this substring with the pattern. If they match, it prints the starting index where the pattern was found.

6. **Example Usage:** The example shows how to call the function with a sample text and pattern. In this case, the function prints the indices where the pattern occurs within the text.

This implementation is straightforward and guarantees finding all occurrences of the pattern, but it may not be efficient for large texts or patterns due to its **((n - m + 1) * m)** time complexity.

Problem solving Using Brute-force Approach

Problem-2 (Subset Sum Problem)

The Subset Sum Problem involves determining if there exists a subset of a given set of numbers that sums up to a specified target value. The brute-force approach to solve this problem involves generating all possible subsets of the set and checking if the sum of any subset equals the target value.

Here is how the brute-force approach works:

1. **Generate subsets:** Iterate over all possible subsets of the given set of numbers.
2. **Calculate sums:** For each subset, calculate the sum of its elements.
3. **Check target:** Compare the sum of each subset with the target value.
4. **Return result:** If a subset's sum matches the target, return that subset. Otherwise, conclude that no such subset exists.

This method guarantees finding a solution if one exists but can be inefficient for large sets due to its exponential time complexity.

Problem solving Using Brute-force Approach

Problem-2 (Subset Sum Problem)

Explanation:

1. **Function Definition:**`subset_sum_brute_force` takes a list of numbers (`nums`) and a target sum (`target`).
2. **Subset Generation:** The loop `for i in range(1 << n)` iterates over all possible subsets. Here, `1 << n` equals 2^n, which is the total number of subsets for `n` elements. Each subset is generated using a bitmask approach: for each bit in the integer `i`, if it is set, the corresponding element is included in the subset.
3. **Subset Construction:** The subset is constructed by including elements where the corresponding bit in `i` is set (`(i & (1 << j))`).
4. **Sum Calculation:** For each subset, the sum of its elements is calculated using `sum(subset)`.
5. **Target Check:** If the sum of the subset equals the target, the subset is returned.
6. **Return Result:** If no subset matches the target sum, the function returns `None`.
7. **Example Usage:** The example demonstrates finding a subset that sums up to `9` in the list `[3, 34, 4, 12, 5, 2]`. The output will either show the subset that matches the target or indicate that no such subset was found.

This brute-force approach is straightforward and guarantees finding a solution if one exists, but may not be efficient for large sets due to its exponential time complexity.

Problem solving Using Brute-force Approach

Problem-2 (Subset Sum Problem)

The function generates all possible subsets of the list **nums** and checks if any of them sum up to the target value **9**.

- **Subset Generation:** The function iterates through all possible subsets. For each subset, it calculates the sum and checks if it matches the target.

- **Subset Found:** In this case, the subset **[3, 4, 2]** sums up to **9**, which matches the target value. Therefore, this subset is returned and printed.

If no such subset were found, the function would print "No subset with the target sum found."

Problem solving Using Brute-force Approach

Problem-3 (Sudoku Solver)

- The Sudoku Solver using the brute-force approach is a method to solve a Sudoku puzzle by trying every possible number in each empty cell until the puzzle is solved.
- The brute-force algorithm systematically fills in each cell with numbers from 1 to 9 and checks if the puzzle remains valid after each placement.
- If a placement leads to a valid state, the algorithm proceeds to the next empty cell.
- If a placement leads to a contradiction, the algorithm backtracks and tries the next number.

Problem solving Using Brute-force Approach

Problem-3 (Sudoku Solver) step-by-step explanation:

1. **Find an Empty Cell:** Locate the first empty cell in the Sudoku grid.
2. **Try Numbers:** Attempt to place each number from 1 to 9 in the empty cell.
3. **Check Validity:** Verify that placing the number does not violate Sudoku rules:
    - No repeated numbers in the same row.
    - No repeated numbers in the same column.
    - No repeated numbers in the same 3x3 sub-grid.
4. **Move to Next Cell:** If the placement is valid, proceed to the next empty cell.
5. **Backtrack if Necessary:** If a placement leads to an invalid state later, undo the placement (backtrack) and try the next number.
6. **Complete:** Continue until the Sudoku puzzle is fully solved or all possibilities are exhausted.

Explanation:

1. **is_valid Function:** This function checks if placing a number in a specific cell is valid according to Sudoku rules:
    - **Row Check:** Ensures the number is not already present in the same row.
    - **Column Check:** Ensures the number is not already present in the same column.
    - **Sub-grid Check:** Ensures the number is not already present in the 3x3 sub-grid.
2. **solve_sudoku Function:**
    - **Find Empty Cell:** Iterates through the grid to locate an empty cell (0).
    - **Try Numbers:** For each empty cell, try placing numbers from 1 to 9.
    - **Check Validity:** Uses **is_valid** to check if the placement is valid.
    - **Recursive Call:** Recursively attempts to solve the rest of the board with the current placement.
    - **Backtrack:** If no valid number can be placed, reset the cell and try the next number.
    - **Completion:** If all cells are filled validly, returns **True**. If no cells are left to fill, returns **True** indicating the board is solved.
3. **Example Usage:** Demonstrates solving a Sudoku puzzle with a given **sudoku_board**. If the puzzle is solved, the board is printed row by row. If no solution exists, a message is displayed.

This brute-force algorithm ensures a solution if one exists but can be slow due to its exhaustive search approach.

Here is what the output would look like if the provided **solve_sudoku** function successfully solves the given Sudoku puzzle:

```
[5, 3, 4, 6, 7, 8, 9, 1, 2]
[6, 7, 2, 1, 9, 5, 3, 4, 8]
[1, 9, 8, 3, 4, 2, 5, 6, 7]
[8, 5, 9, 7, 6, 1, 4, 2, 3]
[4, 2, 6, 8, 5, 3, 7, 9, 1]
[7, 1, 3, 9, 2, 4, 8, 5, 6]
[9, 6, 1, 5, 3, 7, 2, 8, 4]
[2, 8, 7, 4, 1, 9, 6, 3, 5]
[3, 4, 5, 2, 8, 6, 1, 7, 9]
```

Explanation of the Output:

1. **Completed Sudoku Grid**: Each row shows a valid configuration where all rows, columns, and 3x3 sub-grids contain the numbers 1 through 9 without repetition.

2. **Successful Solution**: The **solve_sudoku** function filled all empty cells (originally **0** values) with valid numbers, resulting in a fully completed Sudoku board.

If the **solve_sudoku** function did not find a solution, it would print:

```
No solution exists.
```

This approach guarantees to find a solution if one exists but might be slow for more complex or larger Sudoku puzzles due to its exhaustive nature.

Advantages of Brute-Force Approach

1. **Guaranteed Solution**: Brute-force methods ensure finding a solution if one exists within the predefined constraints.

2. **Simplicity**: The approach is straightforward to implement and understand, requiring minimal algorithmic complexity.

3. **Versatility**: Applicable across various domains where an exhaustive search is feasible, such as puzzle solving, cryptography, and optimization problems.

Limitations of Brute-Force Approach

1. **Computational Intensity**: It can be highly resource-intensive, especially for problems with large solution spaces or complex constraints.
2. **Time Complexity**: Depending on the problem size, brute-force approaches may require impractically long execution times to find solutions.
3. **Scalability Issues**: In scenarios with exponentially growing solution spaces, brute-force methods may become impractical or infeasible to execute within reasonable time constraints.

Optimizing Brute-force Solutions

- **Pruning**: Eliminate certain candidates early if they cannot possibly be a solution. For example, in a search tree, cutting off branches does not lead to feasible solutions.
- **Heuristics**: Use rules of thumb to guide the search and reduce the number of candidates.
- **Divide and Conquer**: Break the problem into smaller, more manageable parts, solve each part individually, and combine the results.
- **Dynamic Programming**: Store the results of subproblems to avoid redundant computations.

☞ While brute-force approaches can serve as a baseline for evaluating other algorithms, their inefficiency limits their practical use to small-scale problems or as a verification tool.

The brute-force approach is a fundamental but often inefficient problem-solving technique. While it guarantees finding a solution if one exists, its practical use is limited by computational constraints. Understanding brute-force methods is essential for developing more sophisticated algorithms and optimizing problem-solving strategies. This section provides an in-depth exploration of the brute-force approach for solving computational problems, highlighting its simplicity, applications, limitations, and potential optimizations.

Divide-and-conquer Approach to Problem Solving

- The divide-and-conquer approach simplifies complex tasks by breaking them down into smaller, manageable parts that can be solved independently and then integrated into a complete solution.
- For example, organizing a library becomes easier when books are grouped by genre, then categorized and alphabetized.
- In project management, construction projects are divided into distinct tasks, allowing for specialized teams to work efficiently in parallel.
- Similarly, in software development, e-commerce platforms are built in modular components like authentication, catalog, and payment systems.
- In healthcare, doctors diagnose complex conditions by categorizing symptoms and consulting specialists in each area.
- In logistics, supply chains are managed regionally to optimize efficiency.
- This method, essential in both practical and computational problem-solving, capitalizes on a problem’s recursive structure, enabling effective management of complex challenges.

Principles of Divide-and-Conquer

(a)Divide

- The first step involves breaking down the problem into smaller subproblems.
- This division should be done so that the subproblems are similar to the original problem.
- The key is to ensure that each subproblem is easier to solve than the original.

(b)Conquer

- Once the problem is divided, each subproblem is solved recursively.
- This step leverages the power of recursion, making the problem-solving process more manageable.
- For very small subproblems, a direct solution is applied without further division.

(c) Combine

- The final step involves combining the solutions of the subproblems to form the solution to the original problem.
- This step often requires merging results in a manner that maintains the problem's overall structure and requirements.

Merge Sort Algorithm

- Merge Sort is a classic example of the divide-and-conquer strategy used for sorting an array of elements.
- It operates by recursively breaking down the array into progressively smaller sections.
- The core idea is to split the array into two halves, sort each half, and then merge them back together.
- This process continues until the array is divided into individual elements, which are inherently sorted.
- The merging process relies on a straightforward principle: when combining two sorted halves, the smallest value of the entire array must be the smallest value from either of the two halves.
- By iteratively comparing the smallest elements from each half and appending the smaller one to the sorted array, we efficiently merge the halves into a fully sorted array.
- This approach is not only intuitive but also simplifies the coding of the recursive splits and the merging procedure.
- Here is how it works:
    1. **Divide**: Split the array into two halves.
    2. **Conquer**: Recursively sort both halves.
    3. **Combine**: Merge the two sorted halves to produce the sorted array.

Merge Sort Algorithm-Visualisation Steps

1. **Initial Split:** The array is divided into smaller chunks recursively.
2. **Recursive Sorting:** Each chunk is sorted individually.
3. **Merging:** The sorted chunks are merged back together in sorted order.

1. **Divide**: The array is recursively divided into two halves until each sub-array contains a single element.
2. **Merge**: The sub-arrays are then merged in a sorted order.

Let us use an example array: [**38, 27, 43, 3, 9, 82, 10**].

Step 1: Divide the Array

1. **Initial Array**: [**38, 27, 43, 3, 9, 82, 10**]
2. **Divide into Halves:**
   - Left Half: [**38, 27, 43**]
   - Right Half: [**3, 9, 82, 10**]
3. **Further Divide:**
   - Left Half: [**38, 27, 43**] becomes:
     - [**38**] and [**27, 43**]
     - [**27, 43**] becomes:
       - [**27**] and [**43**]
   - Right Half: [**3, 9, 82, 10**] becomes:
     - [**3, 9**] and [**82, 10**]
     - [**3, 9**] becomes:
       - [**3**] and [**9**]
     - [**82, 10**] becomes:
       - [**82**] and [**10**]

Step 2: Merge the Arrays

1. **Merge Smaller Arrays:**
   - [**27**] and [**43**] are merged to form [**27, 43**]
   - [**3**] and [**9**] are merged to form [**3, 9**]
   - [**82**] and [**10**] are merged to form [**10, 82**]
2. **Merge Larger Arrays:**
   - [**38**] and [**27, 43**] are merged to form [**27, 38, 43**]
   - [**3, 9**] and [**10, 82**] are merged to form [**3, 9, 10, 82**]
3. **Final Merge:**
   - [**27, 38, 43**] and [**3, 9, 10, 82**] are merged to form the sorted array [**3, 9, 10, 27, 38, 43, 82**]

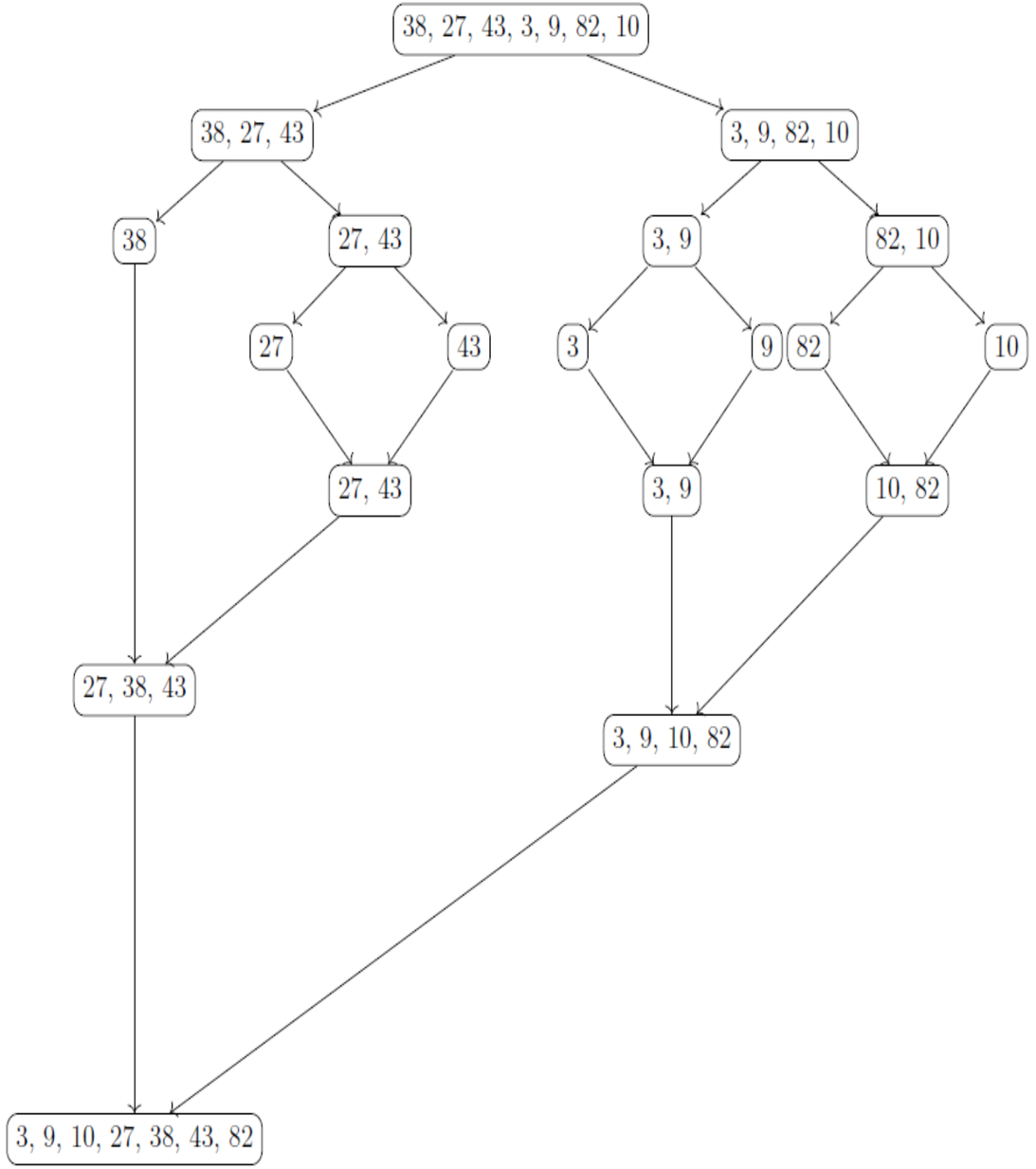
Merge Sort Algorithm-Visualisation Steps

1. **Function mergeSort**

   - Check if the array has one or zero elements. If true, return the array as it is already sorted.
   - Otherwise, find the middle index of the array.
   - Split the array into two halves: from the beginning to the middle and from the middle to the end.
   - Recursively apply mergeSort to the first half and the second half.
   - Merge the two sorted halves using the merge function.
   - Return the merged and sorted array.

2. **Function merge**

   - Create an empty list called **sorted_arr** to store the sorted elements.
   - While both halves have elements:
     - Compare the first element of the left half with the first element of the right half.
     - Remove the smaller element and append it to the **sorted_arr** list.
   - If the left half still has elements, append them all to the **sorted_arr** list.
   - If the right half still has elements, append them all to the **sorted_arr** list.
   - Return the **sorted_arr** list, which now contains the sorted elements from both halves.



Merge Sort Algorithm-Problems

- **Problem-1 (Finding the Maximum Element in an Array)**
- Given an array of integers, find the maximum value in the

1. Initial Setup:
   - The **find_max** function is called with the entire array and indices that cover the whole array. For example, **find_max(array, 0, len(array) - 1)**.
2. Divide:
   - Calculate the middle index **mid** of the current array segment. For the array [3, 6, 2, 8, 7, 5, 1], **mid** would be $(0 + 6) // 2 = 3$.
   - Split the array into two halves based on this **mid** index
     - Left half: [3, 6, 2, 8]
     - Right half: [7, 5, 1]
3. Conquer:
   - Recursively apply the **find_max** function to the left half [3, 6, 2, 8]:
     - Split into [3, 6] and [2, 8]
     - Further split [3, 6] into [3] and [6], finding 3 and 6, respectively.
     - Combine these to get the maximum 6.
     - Similarly, split [2, 8] into [2] and [8], finding 2 and 8, respectively.
     - Combine these to get the maximum 8.
     - Combine 6 and 8 from the two halves to get 8.
   - Apply the same process to the right half [7, 5, 1]:
     - Split into [7] and [5, 1]
     - Further split [5, 1] into [5] and [1], finding 5 and 1, respectively.
     - Combine these to get the maximum 5.
     - Combine 7 and 5 from the two halves to get 7.
4. Combine:
   - Finally, compare the maximums obtained from the left half (8) and the right half (7).
   - Return the larger value, which is 8.

This method efficiently finds the maximum element in the array by recursively dividing the problem, solving the subproblems, and combining the results.

Merge Sort Algorithm-Problems

- **Problem-2 (Finding the Maximum Subarray Sum)**
- Given an array of integers, which include both positive and negative numbers, find the contiguous subarray that has the maximum sum.

Step-by-Step Solution

1. **Initial Setup:**
   - If the array contains only one element, that element is the maximum subarray sum.
2. **Divide:**
   - Divide the array into two approximately equal halves. This involves finding the middle index of the array and separating the array into a left half and a right half.
3. **Conquer:**
   - Recursively find the maximum subarray sum for:
     - The left half of the array.
     - The right half of the array.
     - The subarray that crosses the middle boundary between the two halves.
4. **Combine:**
   - Combine the results from the three areas:
     - The maximum subarray sum in the left half.
     - The maximum subarray sum in the right half.
     - The maximum subarray sum that crosses the middle.
   - Return the largest of these three values.

Merge Sort Algorithm-Problems

- **Problem-2 (Finding the Maximum Subarray Sum)**
- Given an array of integers, which include both positive and negative numbers, find the contiguous subarray that has the maximum sum.

Consider the array: [2, 3, -4, 5, -1, 2, 3, -2, 4]

Let us find the maximum subarray sum that crosses the midpoint of the array.

Step-by-Step Explanation:

1. **Divide the Array:**
   - Suppose we are working with the whole array and the midpoint is calculated to be 4. So we divide the array into two halves:
     - Left half: [2, 3, -4, 5, -1]
     - Right half: [2, 3, -2, 4]
   - Our goal is to find the maximum sum of subarrays that might cross this midpoint between the two halves.
2. **Find the Maximum Crossing Subarray:**
   - To find the maximum subarray sum that crosses the midpoint, we need to consider two parts:
     - The part of the subarray that extends from the midpoint to the left end
     - The part of the subarray that extends from the midpoint to the right end.
   - **Calculate the Maximum Sum of the Left Part:**
     - Start from the midpoint and extend leftward.
     - Track the maximum sum while extending leftward from the midpoint.
     - In the example, the midpoint is 4, so we start from index 4 and move left.

       Array segment: [-1, 5, -4, 3, 2]
     - Compute maximum subarray sum ending at the midpoint:
       * Start from -1, sum is -1.
       * Extend to include 5: sum becomes 4.
       * Extend to include -4: sum becomes 0.
       * Extend to include 3: sum becomes 3.
       * Extend to include 2: sum becomes 5.

       The maximum subarray sum ending at the midpoint (and extending leftward) is 5.
   - **Calculate the Maximum Sum of the Right Part:**
     - Start from the midpoint + 1 and extend rightward.
     - Track the maximum sum while extending rightward from the midpoint

       Array segment: [2, 3, -2, 4]
     - Compute maximum subarray sum starting from the midpoint + 1:
       * Start from 2, sum is 2.
       * Extend to include 3: sum becomes 5.
       * Extend to include -2: sum becomes 3.
       * Extend to include 4: sum becomes 7.

       The maximum subarray sum starting at midpoint + 1 (and extending rightward) is 7.
   - **Combine the Results:**
     - The maximum sum of the subarray that crosses the midpoint is the sum of the maximum sum of the left part and the maximum sum of the right part.
     - From the above calculations, the maximum crossing sum is 5 (left) + 7 (right) = 12.

Advantages of Divide and Conquer Approach

- 1. **Simplicity in Problem Solving**: By breaking a problem into smaller subproblems, each subproblem is simpler to understand and solve, making the overall problem more manageable.
- 2. **Efficiency**: Many divide-and-conquer algorithms, such as merge sort and quicksort, have optimal or near-optimal time complexities.

  These algorithms often have lower time complexities compared to iterative approaches.
- 3. **Modularity**: Divide-and-conquer promotes a modular approach to problem-solving, where each subproblem can be handled by a separate function or module.

  This makes the code easier to maintain and extend.
- 4. **Reduction in Complexity**: By dividing the problem, the overall complexity is reduced, and solving smaller subproblems can lead to simpler and more efficient solutions.
- 5. **Parallelism**: The divide-and-conquer approach can easily be parallelized because the subproblems can be solved independently and simultaneously on different processors, leading to potential performance improvements.
- 6. **Better Use of Memory**: Some divide-and-conquer algorithms use memory more efficiently. For example, the merge sort algorithm works well with large data sets that do not fit into memory, as it can process subsets of data in chunks.

Disadvantages of Divide and Conquer Approach

- 1. **Overhead of Recursive Calls**: The recursive nature can lead to significant overhead due to function calls and maintaining the call stack.

  This can be a problem for algorithms with deep recursion or large subproblem sizes.
- 2. **Increased Memory Usage**: Divide-and-conquer algorithms often require additional memory for storing intermediate results, which can be a drawback for memory-constrained environments.
- 3. **Complexity of Merging Results**: The merging step can be complex and may not always be straightforward. Efficient merging often requires additional algorithms and can add to the complexity of the overall solution.
- 4. **Not Always the Most Efficient**: For some problems, divide-and-conquer might not be the most efficient approach compared to iterative or dynamic programming methods.

  The choice of strategy depends on the specific problem and context.
- 5. **Difficulty in Implementation**: Implementing divide-and-conquer algorithms can be more challenging, especially for beginners.

  The recursive nature and merging steps require careful design to ensure correctness and efficiency.
- 6. **Stack Overflow Risk**: Deep recursion can lead to stack overflow errors if the recursion depth exceeds the system's stack capacity, particularly with large inputs or poorly designed algorithms.

Features of Divide and Conquer Approach

- The divide-and-conquer approach is a versatile and powerful problem-solving strategy that breaks down complex problems into simpler subproblems.
- Its applications span various fields, including

  sorting,

  searching, and

  Computational geometry.

  While it offers significant advantages in terms of efficiency and simplicity,
- it also comes with challenges such as recursion overhead and merge step complexity.
- Understanding and mastering this technique is essential for tackling a wide range of algorithmic problems.

Dynamic Programming Approach

- Dynamic programming (DP) is a method for solving problems by breaking them down into smaller overlapping subproblems, solving each subproblem just once, and storing their solutions.
- It is particularly useful for optimization problems where the problem can be divided into simpler subproblems that are solved independently and combined to form a solution to the original problem.
- Dynamic Programming was first introduced by Richard Bellman in the 1950s as part of his research in operations research and control theory.
- In this context, the term “programming” does not relate to coding but refers to the process of optimizing a series of decisions.
- Bellman chose the term ”dynamic programming” to avoid confusion and political issues, as ”programming” was strongly associated with computers at the time.
- At its core, Dynamic Programming (DP) involves breaking down a problem into smaller, more manageable subproblems and storing the solutions to these subproblems for future use.
- This approach is particularly effective for problems that exhibit two key properties: optimal substructure and overlapping subproblems.

Dynamic Programming Approach-example

•**Key Idea**: Solve the larger problem of finding the optimal route from Thiruvananthapuram to Ernakulam by breaking it into smaller sub-problems.

•**Approach**:

1.Find the **best route** from Thiruvananthapuram to Alappuzha.

2.Use the **known optimal route** from Alappuzha to Ernakulam (provided by your cousin).

•**Principle of Optimality**:

•An optimal route from Thiruvananthapuram to Ernakulam via Alappuzha must have **optimal sub-routes**:

- •Thiruvananthapuram → Alappuzha is optimal.
- •Alappuzha → Ernakulam is optimal.

•**Benefits**: Simplifies problem-solving by leveraging optimal solutions for smaller segments.

Dynamic Programming Approach

- This approach is particularly effective for problems that exhibit two key properties: optimal substructure and overlapping subproblems.

1. Optimal Substructure:

- The best solution to the overall problem is built from optimal solutions to smaller subproblems.
- Example: To find the shortest path to cell (i, j):
  - Use the shortest paths to cells **(i-1, j)** (above) and **(i, j-1)** (left).
  - Cost at (i, j) = **min(cost(i-1, j), cost(i, j-1)) + current cell cost**.
- **How It Works**:

1. Solve for **smallest subproblems** (e.g., top-left corner).
2. Build solutions incrementally for larger parts of the grid.
3. Combine results to reach the bottom-right corner.

Dynamic Programming Approach

- This approach is particularly effective for problems that exhibit two key properties: optimal substructure and overlapping subproblems.
- 2. **Overlapping Subproblems**:

-Many problems require solving the same subproblems multiple times.

- Dynamic Programming improves efficiency by storing the results of these subproblems in a table to avoid redundant calculations.
- By caching these results, the algorithm reduces the number of computations needed, leading to significant performance improvements.

- Dynamic programming breaks problems down into overlapping subproblems, storing solutions to avoid redundant calculations.

Dynamic Programming: Fibonacci Sequence Example

- **The Problem**:
- Fibonacci numbers: Each number = **sum of the two preceding ones**.
- Example: To compute **Fibonacci(5)**, you need **Fibonacci(4)** and **Fibonacci(3)**.
- **Without Dynamic Programming**:
  - **Redundancy**: Fibonacci(3) is computed multiple times, leading to repeated work.
- **Dynamic Programming Approach**:

1. **Compute**: Calculate Fibonacci numbers once and store results in an array.
2. **Reuse**: Retrieve stored results instead of recalculating when needed.

- **Benefits**:
- Avoids redundant calculations.
- Speeds up the process significantly by reducing the number of computations.
- **Key Insight**: Store and reuse previously computed results to optimize performance.

Dynamic Programming Vs Other Problem-solving Techniques

Dynamic Programming:

- Solves problems by **storing and reusing solutions** to overlapping subproblems.
- Guarantees an **optimal solution** through systematic evaluation of all choices.
- **Comparison with Divide and Conquer**:
- **Similarities**: Both break problems into smaller subproblems.
- **Key Difference**:
  - Divide and Conquer solves subproblems **independently**, often leading to redundant calculations.
  - Dynamic Programming **stores solutions** to avoid redundancy, improving efficiency.
- **Comparison with Greedy Algorithms**:

- **Greedy**: Greedy algorithms make a series of locally optimal choices with the hope of finding the global optimum. They are typically simpler to implement but may not always yield the best overall solution.
- **Dynamic Programming**: Dynamic Programming, on the other hand, guarantees an optimal solution by evaluating all possible choices and storing the best solutions for each subproblem, ensuring the most efficient overall result.

Fundamental Principles of Dynamic Programming

- **Overlapping Subproblems**:
- Some subproblems repeat during computation.
- **Solution**: Store results in a data structure
  (e.g., array or hash table) to avoid redundant calculations.
- **Benefit**: Significantly improves efficiency by reusing stored results.
- **Optimal Substructure**:
- The optimal solution to a larger problem is built from the
  optimal solutions to its smaller subproblems.
- *Optimal substructure* is central to Dynamic Programming's recursive nature.
- By solving subproblems optimally and combining their solutions,
  you ensure that the final solution is also optimal.

Dynamic Programming Approaches: Memoization vs Tabulation

(I) Memoization (Top-Down Approach):

- **Steps**:
  1. Define base cases and recursive relation.
  2. Store results of subproblems in a dictionary (e.g., memo).
  3. Reuse stored results to avoid redundant calculations.
- **Example**: Fibonacci Sequence

```python
def fib(n, memo={}):
    if n in memo: return memo[n]
    if n <= 1: return n
    memo[n] = fib(n-1, memo) + fib(n-2, memo)
    return memo[n]
```

- **Benefits**: Simplifies recursive problems by reducing time complexity from exponential to linear.
- **Drawback**: Overhead from recursive calls may impact efficiency.

Dynamic Programming Approaches: Memoization vs Tabulation

Example: Fibonacci Sequence

```python
def fib(n):
    if n <= 1: return n
    dp = [0] * (n + 1)
    dp[1] = 1
    for i in range(2, n + 1):
        dp[i] = dp[i-1] + dp[i-2]
    return dp[n]
```

(II)Tabulation (Bottom-Up Approach):

- **Steps**:
  1. Define base cases and iterative table (e.g., dp array).
  2. Fill the table iteratively using the recursive relation.
  3. Extract the solution from the table.
- **Example**: Fibonacci Sequence
- **Benefits**: Memory-efficient, avoids recursion, and faster due to iterative nature.
- **Drawback**: Requires careful planning for data structures and dependencies.
- **Key Takeaway**:
- Use **memoization** for recursive problems with overlapping subproblems.
- Use **tabulation** for iterative solutions with clear base cases and dependencies.

Steps for Solving Computational Problems Using Dynamic Programming

1. **Identify Subproblems**:
   - Break the problem into smaller subproblems that can be combined to solve the original problem.
2. **Define the Recurrence Relation**:
   - Express the solution in terms of smaller subproblem solutions using a recursive formula.
3. **Choose a Strategy**:
   - **Memoization (Top-Down)**:
     - Solve recursively and store results in a table to avoid redundancy.
   - **Tabulation (Bottom-Up)**:
     - Solve iteratively, starting with the smallest subproblems and building up.
4. **Implement the Solution**:
   - Write code to handle base cases and use a table to store/retrieve results of subproblems.
5. **Optimize Space Complexity**:
   - If possible, reduce memory usage by storing only necessary states (e.g., with sliding arrays).

Dynamic Programming Solution to the Knapsack Problem

- **Problem Definition**
- **Objective:** Maximize value in a knapsack of capacity W.
- **Given:**
  - n: Number of items.
  - w[i]: Weight of item i.
  - v[i]: Value of item i.
- **Constraint:** Each item can be included at most once.
- **Example:**

  W=50,n=3.

  Weights: [10, 20, 30].

  Values: [60, 100, 120].
- **Result:** Maximum value = 220.

Dynamic Programming Approach-Knapsack problem

1. **Subproblems:**
   - Let $dp[i][w]$ = maximum value for knapsack capacity w using the first i items.
2. **Recurrence Relation:**
   - $dp[i][w] = \max(dp[i-1][w], v[i-1] + dp[i-1][w - w[i-1]])$
   - If item i fits: **Include it.**
   - Otherwise: **Exclude it** $dp[i][w] = dp[i-1][w]$.
3. **Base Cases:**
   - $dp[i][0] = 0$, $dp[0][w] = 0$.
4. **Tabulation:**
   - Build a 2D table $dp[i][w]$.

Longest Common Subsequence (LCS) using Dynamic Programming Approach

Problem Definition

- **Objective:** Find the longest sequence common to two strings while maintaining character order.
- **Key Points:**
  - Characters need **not be contiguous** (unlike substrings).
  - Useful in DNA sequencing, text comparison, etc.

Challenges:

- **Brute Force Approach:**
  - Examines all subsequences.
  - **Time Complexity:** Exponential ($O(2^n)$).

Dynamic Programming Solution:

1. **Subproblems:**
   - Define $dp[i][j]$: LCS length for substrings $s_1[0..i]$ and $s_2[0..j]$.
2. **Recurrence Relation:**
   - If $s_1[i] == s_2[j]$: $dp[i][j] = 1 + dp[i-1][j-1]$.
   - Otherwise: $dp[i][j] = \max(dp[i-1][j], dp[i][j-1])$.
3. **Base Case:**
   - $dp[i][0] = 0, dp[0][j] = 0$.
4. **Time Complexity:** $O(m \times n)$.

Rod Cutting Problem using Dynamic Programming Approach

Rod Cutting Problem

Problem Definition

- Objective: Maximize revenue from a rod of length n, given a price table for various lengths.
- Key Points:
  - Decision involves cutting the rod into pieces of different lengths.

Challenges:

- Brute Force Approach:
  - Evaluates all cutting combinations.
  - Time Complexity: Exponential ($O(2^n)$).

Dynamic Programming Solution:

1. Subproblems:
   - Define $dp[i]$: Maximum revenue for a rod of length i.
2. Recurrence Relation:
   - $dp[i] = \max(price[j] + dp[i-j])$ for $j = 1$ to i.
3. Base Case:
   - $dp[0] = 0$.
4. Time Complexity: $O(n^2)$.

Advantages and Disadvantages of the Dynamic Programming Approach

- **Advantages of the Dynamic Programming Approach**
- 1. **Efficiency**: DP reduces the time complexity of problems with overlapping subproblems by storing solutions to subproblems and reusing them.
- 2. **Optimal Solutions**: DP ensures that the solution to the problem is optimal by solving each subproblem optimally and combining their solutions.
- 3. **Versatility**: DP can be applied to a wide range of problems across different domains.
- **Disadvantages of the Dynamic Programming Approach**
- 1. **Space Complexity**: DP often requires additional memory to store the results of subproblems, which can be a limitation for problems with a large number of subproblems.
- 2. **Complexity of Formulation**: Developing a DP solution requires a deep understanding of the problem's structure and properties, which can be challenging.
- 3. **Overhead of Table Management**: Managing and maintaining the DP table or memoization structure can add overhead to the algorithm.

Recursion Vs Dynamic Programming

Features	Recursion	Dynamic Programming
Definition	By breaking a difficulty down into smaller problem of the same problem, a function calls itself to solve the problem until a specific condition is met.	A technique which breaks them into smaller problems and stores the results of theses sub problems to avoid repeated calculations
Approach	Recursion frequently employs a top-down method in which the primary problem is broken down into more manageable subproblems	Uses a bottom-up methodology, Dynamic programming starts resolving the smallest subproblems before moving onto the primary issue
Performance	Recursion might be slower due to the overhead of function calls and redundant calculations	It is often faster due to optimized subproblem solving memoization
Memory Usage	It does not require extra memory, only requires stack space	It requires additional memory to record intermediate results

Greedy Algorithm Approach

- **Definition**

-The greedy approach is often the most intuitive method in algorithm design.

-When faced with a problem that requires a series of decisions, a greedy algorithm makes the ”***best***” choice available at each step, focusing solely on the immediate situation without considering future consequences.

-This approach simplifies the problem by reducing it to a series of smaller subproblems, each requiring fewer decisions.

-Such approach quickly provide a solution, but don’t guarantee the best overall outcome for every problem.

-Greedy algorithms take all of the data in a particular problem, and then set a rule for which elements to add to the solution at each step of the algorithm.

-Greedy algorithms excel in problems with optimal substructure, where the problem can be broken down into smaller, solvable components.

Greedy Algorithm Approach-Implementation

- We often deal with problems where the solution involves a sequence of decisions or steps that must be taken to reach the optimal outcome.
- The greedy approach is a strategy that finds a solution by making the locally optimal choice at each step, based on the best available option at that particular stage.
- At a fundamental level, it shares a similar philosophy with dynamic programming and divide-and-conquer, which involves breaking down a large problem into smaller, more manageable components that are easier to solve.

Greedy Algorithm Approach-Properties

- Greedy solution could be implemented only if the problem statement follows 2 properties:
- ***Greedy Choice Property:***

-Choosing the best option at each phase can lead to a global (overall) optimal solution.

- ***Optimal Substructure:***

-If an optimal solution to the complex problem contains the optimal solutions to the subproblems, the problem has an optimal substructure.

Key Characteristics of the Greedy Approach

- 1. **Local Optimization**: At each step, the algorithm makes the best possible choice without considering the overall problem.

  -This choice is made with the hope that these local optimal decisions will lead to a globally optimal solution.

- 2. **Irrevocable Decisions**: Once a choice is made, it cannot be changed.

  -The algorithm proceeds to the next step, making another locally optimal choice.

- 3. **Efficiency**: Greedy algorithms are typically easy to implement and run quickly, as they make decisions based on local information and do not need to consider all possible solutions.

Motivations for the Greedy Approach

- 1. **Simplicity and Ease of Implementation**:

  Straightforward Logic: Greedy algorithms make the most optimal choice at each step based on local information, making them easy to understand and implement.

  Minimal Requirements: These algorithms do not require complex data structures or extensive bookkeeping, reducing the overall implementation complexity.
- 2. **Efficiency in Time and Space**:

  Fast Execution: Greedy algorithms typically run in linear or polynomial time, which is efficient for large input sizes.

  Low Memory Usage: Since they do not need to store large intermediate results, they have low memory overhead, making them suitable for memory-constrained environments.
- 3. **Optimal Solutions for Specific Problems**:

  Greedy-Choice Property: Problems with this property allow local optimal choices to lead to a global optimum.

  Optimal Substructure: Problems where an optimal solution to the whole problem can be constructed efficiently from optimal solutions to its sub-problems.
- 4. **Real-World Applicability**:

  Practical Applications: Greedy algorithms are useful in many real-world scenarios like scheduling, network routing, and resource allocation.

  Quick, Near-Optimal Solutions: In situations where an exact solution is not necessary, greedy algorithms provide quick and reasonably good solutions.

Characteristics of the Greedy Algorithm

1.Local Optimization

-Makes the **best possible choice** at each step based on the current problem state.

-Assumes local optimal choices lead to a **globally optimal solution**.

2.Irrevocable Decisions

-Decisions are **final**—no backtracking or reconsidering past choices.

3.Problem-Specific Heuristics

-Relies on heuristics tailored to the problem’s properties to guide decision-making.

4.Optimality

-Produces **optimal solutions** for specific problems (e.g., Coin Change, Huffman Coding, Kruskal's Algorithm).

-Not guaranteed for all problems; depends on the problem’s characteristics.

5.Efficiency

-Generally **time- and space-efficient**, as it does not explore all possible solutions.

Solving Computational Problems Using Greedy Approach

- To solve computational problems using the Greedy Approach,

1. identify if the problem can be decomposed into sub-problems with an optimal substructure and ensure it possesses the greedy-choice property.

2.Define a strategy to make the best local choice at each step, ensuring these local decisions lead to a globally optimal solution.

3. Design the algorithm by sorting the input data if needed and iterating through it, making the optimal local choice at each iteration while keeping track of the solution being constructed.

4.Finally, analyze the algorithm’s efficiency and correctness by testing it on various cases, including edge cases.

Problem-1 (Task Completion Problem)

- Given an array of positive integers each indicating the completion time for a task, find the maximum number of tasks that can be completed in the limited amount of time that you have.
- In the problem of finding the maximum number of tasks that can be completed within a limited amount of time, the optimal substructure can be identified by recognizing how smaller sub-problems relate to the overall problem.

Solution Approach (Task Completion Problem)

- 1. **Break Down the Problem**: Consider a subset of the tasks and determine the optimal solution for this subset. For example, given a certain time limit, find the maximum number of tasks that can be completed from the first *k* tasks in the array.
- 2. **Extend to Larger Sub-problems**: Extend the solution from smaller sub-problems to larger ones. If you can solve the problem for *k* tasks, you can then consider the (*k*+1)th task and decide if including this task leads to a better solution under the given time constraint.
- 3. **Recursive Nature**: The optimal solution for the first *k* tasks should help in finding the optimal solution for the first (*k* + 1) tasks. This recursive approach ensures that the overall solution is built from the solutions of smaller sub-problems.
- 4. **Greedy Choice**: At each step, make the greedy choice of selecting the task with the shortest completion time that fits within the remaining available time. This choice reduces the problem size and leads to a solution that maximizes the number of tasks completed.
- By iteratively applying this approach and making the best local choices (selecting the shortest tasks first), you can construct a globally optimal solution from optimal solutions to these smaller sub-problems, demonstrating the optimal substructure property.

Steps Used(Task Completion Problem)

- To solve the problem of finding the maximum number of tasks that can be completed in a limited amount of time using a greedy algorithm, you can follow these steps:

1. **Sort the tasks by their completion times in ascending order**: This ensures that you always consider the shortest task that can fit into the remaining time, maximizing the number of tasks completed.

2. **Iterate through the sorted list of tasks and keep track of the total time and count of tasks completed**: For each task, if adding the task's completion time to the total time does not exceed the available time, add the task to the count and update the total time.

Step Explanation(Task Completion Problem)

- 1. **Sorting**: The list of completion times is sorted in ascending order.

  -This step ensures that we always consider the shortest tasks first, which helps in maximizing the number of tasks that can be completed within the given time.

- 2. **Iterating through sorted tasks**: The algorithm iterates through the sorted list and maintains two variables:
  - **total_time**: The cumulative time of tasks completed so far.
  - **task_count**: The count of tasks completed.
- 3. **Checking time constraint**: For each task, it checks if adding the task's completion time to **total_time** exceeds **available_time**.

  -If it does not exceed, the task is added to the count, and **total_time** is updated.

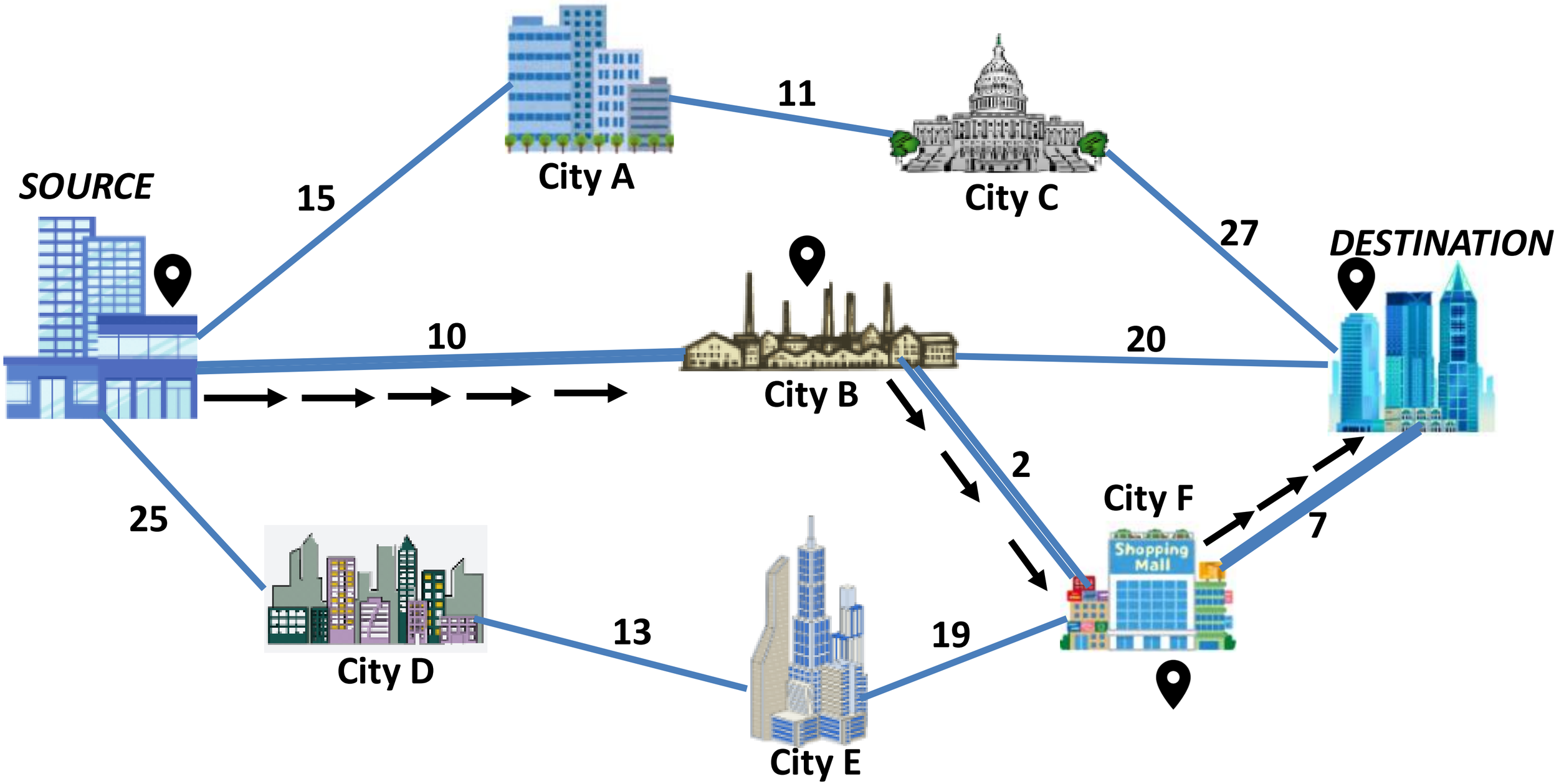
  -If it exceeds, the loop breaks because no more tasks can be completed without exceeding the available time.

Example (Task Completion Problem)

- Consider the example usage with **completion_times = [2, 3, 1, 4, 6]** and **available_time = 8**:
    - After sorting: [1, 2, 3, 4, 6]
    - Iterating:
        - Add task with time **1: total_time = 1, task_count = 1**
        - Add task with time **2: total_time = 3, task_count = 2**
        - Add task with time **3: total_time = 6, task_count = 3**
        - Next task with time **4** would exceed **available_time**, so the loop breaks.

- The maximum number of tasks that can be completed in 8 units of time is 3.

Problem-2 (Best Route from Source to Destination)



Steps involved (Best Route from Source to Destination)

- From starting location, choose the path to city B(10) as it has the smallest distance.
- From city B, choose the path to city F(2) because it has the smallest distance.
- From city F, choose the path to ***destination*** because it is the only path towards destination.
- The steps to generate the solution are given below:
    - Start from the source vertex
    - Pick one vertex at a time with a minimum edge weight(distance) from the source vertex.
    - Add the selected vertex to a tree structure if the connecting edge does not form a cycle.
    - Keep adding adjacent fringe vertices to the tree until you reach the destination vertex.

Greedy Algorithms vs. Dynamic Programming

Greedy Algorithms:

- **Approach**: Make the best possible choice at each step based on loca information, without reconsidering previous decisions.
- **Decision Process**: Makes decisions sequentially and irrevocably.
- **Optimality**: Guaranteed to produce optimal solutions only for cer· tain problems with the greedy-choice property and optimal substruc· ture.
- **Efficiency**: Typically faster and uses less memory due to the lack o extensive bookkeeping.
- **Example Problems**: Coin Change Problem (specific denomina· tions), Kruskal's Algorithm for Minimum Spanning Tree, Huffmar Coding.

Dynamic Programming:

- **Approach**: Breaks down a problem into overlapping sub-problems and solves each sub-problem only once, storing the results to avoid redundant computations.
- **Decision Process**: Considers all possible decisions and combines them to form an optimal solution, often using a bottom-up or top-down approach.
- **Optimality**: Always produces an optimal solution by considering all possible ways of solving sub-problems and combining them.
- **Efficiency**: Can be slower and use more memory due to storing results of all sub-problems (memoization or tabulation).
- **Example Problems**: Fibonacci Sequence, Longest Common Sub-sequence, Knapsack Problem.

Advantages and Disadvantages of the Greedy Approach

- **Advantages of the Greedy Approach**
- 1. **Simplicity**: Greedy algorithms are generally easy to understand and implement.
- 2. **Speed**: These algorithms typically run quickly, making them suitable for large input sizes.
- 3. **Optimal for Certain Problems**: For some problems, like the Coin Change Problem with certain denominations, greedy algorithms provide an optimal solution.

- **Disadvantages of the Greedy Approach**
- 1. **Suboptimal Solutions**: Greedy algorithms do not always produce the optimal solution for every problem.
  - They are most effective when the problem has the greedy-choice property, meaning a global optimum can be reached by making local optimal choices.
- 2. **Irrevocable Decisions**: Once a choice is made, it cannot be changed, which may lead to a suboptimal solution in some cases.
- 3. **Lack of Backtracking**: Greedy algorithms do not explore all possible solutions or backtracks, which means they can miss better solutions.

Randomized Approach

- The randomized approach plays a crucial role in modern problem-solving, leveraging probability and randomness to tackle challenges that defy straightforward deterministic solutions.
- By embracing uncertainty and probabilistic outcomes, randomized algorithms provide innovative solutions across various disciplines, highlighting their relevance and effectiveness in addressing complex, real-world problems.
- It explores the theoretical foundations, practical applications, and considerations of randomized approaches, equipping readers with insights into their strategic deployment in diverse problem-solving scenarios.

Randomized Approach to Problem Solving

- **What are Simulations?**

  -Utilize **randomness** to model, analyze, and solve complex problems.

  -Provide insights into systems under **uncertainty** and offer practical solutions.

  -Useful when **deterministic methods** are too complex or impractical.

Key Examples of Simulations

1. **Estimating Circle Area:**
   1. Randomly place points in a square and count how many fall inside an inscribed circle.
   2. The ratio of points inside the circle to total points estimates the circle's area.
2. **Birthday Problem:**
   1. Simulate classrooms with random birthdays to estimate the probability of shared birthdays.
   2. Simplifies combinatorial mathematics and aids intuitive understanding.
3. **Random Walk:**
   1. Simulate a person taking random steps on a grid to calculate the average time to return to the starting point.
   2. Provides insights into **stochastic systems** and random processes thus provide valuable data about their dynamics
4. **Random Coin Flips:**
   1. Simulate multiple trials of coin flips to estimate the probability distribution of heads.
   2. Useful for analyzing probabilistic events empirically.

Advantages of Simulations

- Randomized approach introduces randomness into the decision-making process, often yielding simple and efficient solutions to complex problems.

- **Simplifies Complex Problems:**

  Provides intuitive solutions to challenging problems.

- **Flexibility:**

  Models a wide range of scenarios.

- **Insight into Uncertainty:**

  Explores system behavior under probabilistic conditions.

Motivations for the Randomized Approach

(I)Complexity Reduction:

1.Simplifies large or complex problems by introducing probabilistic choices.

- Example: organizing a community health screening event in a large city
- Constraint: You need to decide on the number of screening stations and their locations to maximize coverage and efficiency

1.Randomly select station locations instead of analyzing all possible setups.

2.Identifies effective solutions efficiently without exhaustive exploration.

-By evaluating a sample of these random setups, you can identify patterns or clusters of locations that work well.

-This method simplifies the complex problem of optimizing station placement by reducing the number of scenarios you need to explore in detail.

(II)Versatility:

1.Effective in diverse domains like optimization, simulations, and usability testing.

2.Example: **App Usability Testing**

-Randomly test features with a diverse user group instead of all combinations.

-This approach allows the company to obtain useful feedback and make improvements without needing to test every possible combination of user and feature.

Motivations for the Randomized Approach

(III)Performance Enhancement:

1. Offers significant time and resource savings, especially for large datasets.

- Example: **Library Book Sampling(**estimate how often books are checked out.)

-Randomly sample books from different genres and record their check-out rates over a period of time and estimate check-out frequency.

-Provides actionable insights without tracking the entire collection.

Highlight of Randomized algorithms

Randomized algorithms offer several significant advantages over deterministic ones:

1. Simplicity: Randomized algorithms are often simpler.

-For instance, finding the kth smallest element in an unordered list can he complex using deterministic methods.

-However, using a randomized approach, where a random element is picked to partition the problem, results in a much simpler algorithm.

2. Efficiency: Randomized algorithms can have better asymptotic running times.

For some problems, deterministic algorithms run in exponential time, while randomized algorithms run in polynomial time.

3. Lack of Information: Randomization is useful when dealing with incomplete information.

-For example, consider two parties, Alice and Bob, each with an n-bit number.

-They need to determine if their numbers are the same with minimal communication.

-A deterministic protocol requires exchanging n bits, but a randomized protocol based on fingerprinting can achieve this with high probability using only O(logn) bits.

Highlight of Randomized algorithms

4. Symmetry Breaking: Randomization helps in designing contention resolution mechanisms in distributed protocols.

-For example, the Ethernet protocol uses random back-off duration to manage simultaneous transmission attempts effectively, without prior communication.

5. Counting via Sampling: Randomization can help estimate the size of large spaces or sets through sampling.

-For instance, computing the integral of a multivariate function over a region can be approximated by sampling points from a bounding box and determining the fraction that lies below the function.

6. Searching for Witnesses: Randomized algorithms are effective in finding witnesses for verifying properties when the density of witnesses is high.

-In Polynomial Identity Testing, for example, random points are likely to identify non-zero evaluations if the polynomial is not identically zero.

Characteristics of Randomized Approach

- **Key Features of Randomized Approaches**

1.Probabilistic Choices:

1.Decisions based on random sampling or probabilistic events.

2.Example:

1. **Vending Machine Placement:** Randomly test locations and analyze results to find the most effective spots.

3.Outcome: Variable but statistically predictable results.

2.Efficiency:

-They often achieve efficiency by sacrificing deterministic guarantees for probabilistic correctness, optimizing performance in scenarios where exhaustive computation is impractical.

1.Example:

1. **Popular Menu Items Analysis:** Randomly survey a subset of customers to identify trends efficiently without exhaustive data collection.

3.Complexity Analysis:

- Evaluating the performance of randomized approaches involves analyzing their average-case behavior or expected outcomes over multiple iterations, rather than deterministic worst-case scenarios.
- Example:
  1. **Online Store Wait Times:** Randomly sample transactions to estimate average customer wait times, offering realistic insights into typical system performance.

Randomized Approach vs Deterministic Methods

- **Key Takeaway:**
- **Randomized Approaches:** Efficient and adaptable; suited for uncertain or large-scale problems.
- **Deterministic Methods:** Precise and reliable; best for tasks requiring accuracy and repeatability.

Randomized Approach

- **Key Features:**
  - Incorporates elements of chance, focusing on efficiency and adaptability.
  - Suitable for problems with large or variable datasets.
- **Examples:**
  - **Customer Satisfaction Surveys:** Randomly select branches and customers for efficient data collection.
  - **Exercise Habits Analysis:** Randomized sampling to estimate average weekly exercise time.
- **Advantages:**
  - Simplifies complex problems.
  - Efficient for scenarios where exhaustive analysis is impractical.

Randomized Approach vs Deterministic Methods

Deterministic Methods

- **Key Features:**
  - Based on fixed, predictable processes.
  - Delivers consistent and accurate results.
- **Examples:**
  - **Cost Calculation:** Adds item prices for an exact total.
  - **Engineering Design:** Precise calculations to ensure safety and performance.
- **Advantages:**
  - Ideal for precision and reliability.
  - Essential for problems requiring exact solutions.

Solving Computational Problems Using Randomization

- **Monte Carlo Method to Estimate π**

    1. Generate random points within a unit square (1×1).

    2. Count how many points fall inside a quarter circle of radius 1.

    3. The ratio of points inside the quarter circle to the total points approximates the area of the quarter circle ($\pi/4$).

    4. Multiply this ratio by 4 to estimate π.

- As you increase the number of samples, the estimate becomes more accurate.
- This demonstrates the power of the Monte Carlo method in approximating mathematical constants through randomized simulations.
- By leveraging randomness, we can simplify the analysis of algorithms, often focusing on expected performance rather than worst-case scenarios.

Here is a Python code to estimate π using the Monte Carlo method:

```
import random

def estimate_pi(num_samples):
    inside_circle = 0

    for _ in range(num_samples):
        x = random.random()
        y = random.random()
        if x**2 + y**2 <= 1:
            inside_circle += 1

    pi_estimate = (inside_circle / num_samples) * 4
    return pi_estimate

# Example usage:
num_samples = 1000000  # Number of random points to generate
pi_estimate = estimate_pi(num_samples)
print(f"Estimated value of pi with {num_samples} samples:
    {pi_estimate}")
```

Problem-1 (Coupon Problem)

- A company selling jeans gives a coupon for each pair of jeans. There are *n* different coupons. Collecting *n* different coupons would give you free jeans.
- How many jeans do you expect to buy before getting a free one?
- Let us start with an algorithmic solution in plain English to determine how many pairs of jeans you might need to buy before collecting all *n* different coupons and getting a free pair of jeans:

Algorithmic Solution

1. **Initialize Variables**:

   Total Jeans Bought: Start with a counter set to zero to track how many pairs of jeans you have bought.

   Coupons Collected: Use a set to keep track of the different types of coupons you have received.

   Number of Coupons: The total number of different coupon types is *n.*

2. **Buying Process**:

   Loop Until All Coupons Are Collected: Continue buying jeans until you have one of each type of coupon in your set.

   -Each time you buy a pair of jeans, increase the counter for the total jeans bought by one.

   -When you buy a pair of jeans, you get a coupon. Add this coupon to your set of collected coupons.

   -Check if you have collected all *n* different types of coupons by comparing the size of your set to *n.*

3. **Repeat for Accuracy**:

   -To get a reliable estimate, repeat the entire buying process many times (e.g., 100,000 times).

   -Keep a running total of the number of jeans bought across all these repetitions.

4. **Calculate the Average**:

   -After completing all repetitions, calculate the average number of jeans bought by dividing the total number of jeans bought by the number of repetitions.

5. **Output the Result**:

   -The average number of jeans bought from the repeated simulations gives you a good estimate of how many pairs of jeans you would typically need to buy before collecting all *n* coupons and getting a free pair.

walk through an example

1. **Setup and Initialization Step**:
    - Imagine there are 10 different types of coupons.
    - Start with **total_jeans = 0** and an empty set **coupons_collected**.
2. **Buying Jeans**:
    - You buy a pair of jeans and get a coupon. Add the coupon to your set.
    - Increase **total_jeans** by 1.
    - Check if your set now contains all 10 different coupons.
3. **Continue Until Complete**:
    - Repeat the buying process, each time adding the coupon to your set and increasing the total jeans count.
    - Once you have all 10 types in your set, note the total number of jeans bought for this repetition.
4. **Repeat Many Times**:
    - To ensure accuracy, repeat this entire process (buying jeans, collecting coupons) 100,000 times.
    - Sum the total number of jeans bought over all repetitions.
5. **Calculate Average**:
    - Divide the sum of all jeans bought by 100,000 to get the average number of jeans you need to buy to collect all coupons.

Coupon Problem-Python Code

```
import random

def expected_jeans(n, num_simulations=100000):
    total_jeans = 0

    for _ in range(num_simulations):
        coupons_collected = set()
        jeans_bought = 0

        while len(coupons_collected) < n:
            jeans_bought += 1
            coupon = random.randint(1, n)  # simulate getting a
    random coupon
            coupons_collected.add(coupon)  # add the coupon to
    the set

        total_jeans += jeans_bought

    expected_jeans = total_jeans / num_simulations

    return expected_jeans


# Example usage:
n = 10  # number of different coupons
expected_num_jeans = expected_jeans(n)
print(f"Expected number of jeans before getting a free one with
    {n} coupons: {expected_num_jeans}")
```

Checking reliability of Simulation approach

Let us run the simulation three times and compare the results with the theoretical expectation for the Coupon problem. The theoretical expected number of purchases required to collect all n coupons is given by: $H_n = n\left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}\right)$, where H_n is the n-th harmonic number. Here are the results from three runs of the given simulation code for 10 coupons:

Run 1: Expected number of jeans before getting a free one with 10 coupons: 29.2446

Run 2: Expected number of jeans before getting a free one with 10 coupons: 29.3227

Run 3: Expected number of jeans before getting a free one with 10 coupons: 29.3175

The theoretically expected number of jeans needed is approximately 29.29. The results from the three runs are all very close to this theoretical value.

- **Run 1**: 29.2446
- **Run 2**: 29.3227
- **Run 3**: 29.3175

This demonstrates that the simulation results are consistent with the theoretical expectation for the Coupon problem.

The approach we used to solve the Coupon problem is an instance of Monte Carlo simulation to estimate the expected value, making it suitable for scenarios where an analytical solution is complex or impractical. Do more simulations by adjusting **num_simulations** to balance between computation time and accuracy of the estimated value. More simulations generally provide a more precise estimation of the expected value.

Aliter approach example

- Suppose a company is giving out coupons with every pair of jeans sold, and there are 5 different coupons. Once you collect all 5 different coupons, you get a free pair of jeans. To calculate the expected number of jeans you need to buy to collect all 5 different coupons.
- ***Step-by-step break down***
- 1st coupon: You buy 1st pair of jeans and get your 1st coupon. This is always new since you don't have any coupon yet.

  -Expected number of jeans bought so far: 1

- 2nd coupon: The probability of getting a new coupon is 4/5 because 4 out of the 5 possible coupons are still new to you.
  - The expected number of jeans to get a new coupon is 5/4
  - Expected number of jeans bought so far: 1+ 5/4=2.25
- 3rd coupon: The probability of getting a new coupon is 3/5 because 3 out of the 5 possible coupons are still new to you.
  - The expected number of jeans to get a new coupon is 5/3
  - Expected number of jeans bought so far: 2.25+ 5/3=2.25+1.67=3.92
- 4th coupon: The probability of getting a new coupon is 2/5 because 2 out of the 5 possible coupons are still new to you.
  - The expected number of jeans to get a new coupon is 5/2
  - Expected number of jeans bought so far: 3.92+ 5/2=3.92+2.5=6.42
- 5th coupon: The probability of getting a new coupon is 1/5 because 1 out of the 5 possible coupons are still new to you.
  - The expected number of jeans to get a new coupon is 5/1
  - Expected number of jeans bought so far: 6.42+ 5/1=6.42+5=11.42

General Formula Application

- To verify the formula

$E(n)=n.H_n$

For n=5, the Harmonic number H_5 is:

$H_5 = 1+1/2+1/3+1/4+1/5 =2.283$

Then

$E(5)=5*2.283$

$= 11.42$

Problem-2 (Hat Problem)

- *n* people go to a party and drop off their hats to a hat-check person.
- When the party is over, a different hat-check person is on duty and returns the n hats randomly back to each person.
- What is the expected number of people who get back their hats?
- To solve this problem, we can simulate the process of randomly distributing hats and count how many people get their own hats back.
- By running the simulation many times, we can calculate the expected number of people who receive their own hats. Let us start with an algorithmic solution in plain English.

Algorithmic Solution

- 1. **Initialization**:

  - Set up variables to count the total number of correct matches across all simulations.

  -Define the number of simulations to ensure statistical reliability.

  -Define the number of people *n.*

- 2. **Simulate the Process**:

  For each simulation:

  – Create a list of hats representing each person.

  – Shuffle the list to simulate random distribution.

  – Count how many people receive their own hat.

  – Add this count to the total number of correct matches.

- 3. **Calculate the Expected Value**:

  -Divide the total number of correct matches by the number of simulations to get the average.

- 4. **Output the Result**:

  -Print the expected number of people who get their own hats back.

walk through an example

- 1. **Setup and Initialization Step**:

  -Suppose there are 5 people at the party.

  -Initialize **total_correct** to 0, which will keep track of the total number of people who receive their own hat across multiple simulations and **num_simulations** to 100,000.

2. **Simulate the Process**:

   For each simulation:

   – Create a list of hats [1, 2, 3, 4, 5].

   – Shuffle the list, e.g., [3, 1, 5, 2, 4].

   – Initialize **correct** to 0.

   – Check each person:

   ∗ Person 1 (hat 3) - not correct.

   ∗ Person 2 (hat 1) - not correct.

   ∗ Person 3 (hat 5) - not correct.

   ∗ Person 4 (hat 2) - not correct.

   ∗ Person 5 (hat 4) - not correct.

   – Add **correct** (which is 0 for this run) to **total_correct**.

3. **Repeat Many Times**:

   -Repeat the simulation 100,000 times, each time shuffling the hats and counting how many people get their own hats back.

   -Sum the number of correct matches across all simulations.

4. **Calculate Average**:

   -Divide **total_correct** by 100,000 to get the average number of people who receive their own hat.

Example-Inference

- By following these steps, you can determine the expected number of people who will get their own hats back after the hats are randomly redistributed.

-In the case of this specific problem, the expected number will be approximately 1, meaning on average, one person will get their own hat back.

-This is due to the nature of permutations and the expectation of fixed points in a random permutation.

-This solution uses a Monte Carlo simulation approach to estimate the expected number of people who get their own hats back, which is a practical way to solve problems involving randomness and expectations.

Python Execution-Hat Problem

```
import random

def simulate_hat_problem(n, num_simulations):
    total_correct = 0

    for _ in range(num_simulations):
        hats = list(range(n))
        random.shuffle(hats)
        correct = sum(1 for i in range(n) if hats[i] == i)
        total_correct += correct



    expected_value = total_correct / num_simulations
    return expected_value

# Example usage
n = 10  # Number of people at the party
num_simulations = 100000  # Number of simulations to run
expected_hats_back = simulate_hat_problem(n, num_simulations)

print(f"The expected number of people who get their own hats
    back is approximately: {expected_hats_back}")
```

Checking reliability of Simulation approach

- Here are the results from three runs of the given simulation code:
- **Run 1**: The expected number of people who get their own hats back is approximately: 1.00039
- **Run 2**: The expected number of people who get their own hats back is approximately: 1.00051
- **Run 3**: The expected number of people who get their own hats back is approximately: 0.99972
- Across these three runs, the expected number of people who get their own hats back consistently revolves around 1.
- This aligns with the theoretical expectation that, on average, one person out of *n* will receive their own hat back in such a random distribution scenario.
- This outcome demonstrates the robustness of the Monte Carlo simulation approach for estimating expected values in problems involving randomness.

Aliter approach example

- Let X be the random variable representing the number of people who get their own hat back.

-For n=3, find E[X] by first computing the Probability mass function Px, and then applying the definition of expectation.

- Find a general formula for E[X], for any positive integer n

Step-by-Step Calculation

1. Enumerate all possible Permutations:

   For n=3, there are 3!=6 possible permuations of hats.

   ie { (1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2) and (3,2,1) }

2. Count fixed points in each permutation

   (1,2,3)-All 3 people get their own hat back

   (1,3,2)-Only person 1 gets their hat back(1 fixed point)

   (2,1,3) -Only person 3 gets their hat back(1 fixed point)

   (2,3,1)-No person gets their hat back(0 fixed point)

   (3,1,2)-No person gets their hat back(0 fixed point)

   (3,2,1)-Only person 2 gets their hat back(1 fixed point)

Aliter approach example

Step-by-Step Calculation

3. Compute the probability Mass Function P(X=k):

-P(X=0): Number of permutations with 0 fixed points / Total permutations = 2/6= 1/3

-P(X=1): Number of permutations with 1 fixed points / Total permutations = 3/6= 1/2

-P(X=0): Number of permutations with 2 fixed points / Total permutations = 0/6= 0

-P(X=0): Number of permutations with 3 fixed points / Total permutations = 1/6= 1/6

4. Definition of Expectation

$$E[X]=\sum_{k=0}^{n} P \quad (X=k)$$

5. Compute Expectation for n=3

E[X]=0*P(X=0)+1*P(X=1)+2*P(X=2)+3*P(X=3)

=0*(1/3)+1*(1/2)+2*0+3*(1/6)

=0+1/2+0+1/2

= 1