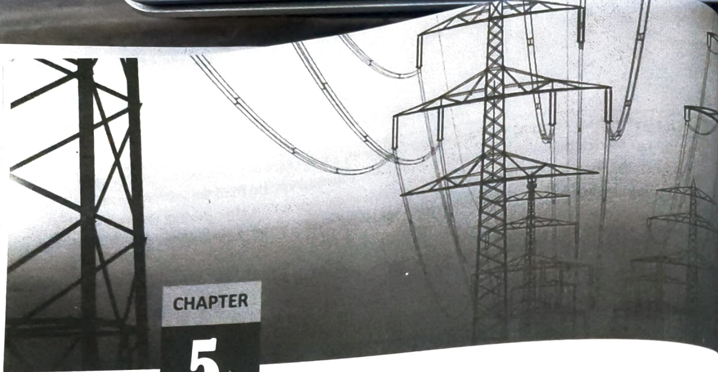

CHAPTER

5

LEARNING OBJECTIVES

The circuit that is excited using alternating source is called an AC circuit. In an AC circuit, the value of the magnitude and direction of the voltage is changes with time. In DC circuit, the opposition to the changes with time. In DC circuit, the opposition to the flow of current is the only resistance of the circuit whereas the opposition to the flow current in the AC circuit is because of the resistance, inductive reactance and capacitive reactance of the circuit.

Different forms of phasor representation of alternating quantities, steady state response of different electrical circuits consisting of R, L and /or C to single phase AC supply and concept of apparent and reactive power has been explained in this chapter.

AC CIRCUITS

After reading this chapter, the reader should be familiar with the following concepts.

- Phasor representation of sinusoidal quantities.
- Trigonometric, Rectangular, Polar and complex forms.
- Analysis of simple AC circuits: Purely resistive, inductive & capacitive circuits; Inductive and capacitive reactance,
- Concept of impedance. Average Power, Power factor.
- Analysis of RL, RC and RLC series circuits-
- Active, reactive and apparent power.

1. BASIC TERMINOLOGIES

1. PHASE

Phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

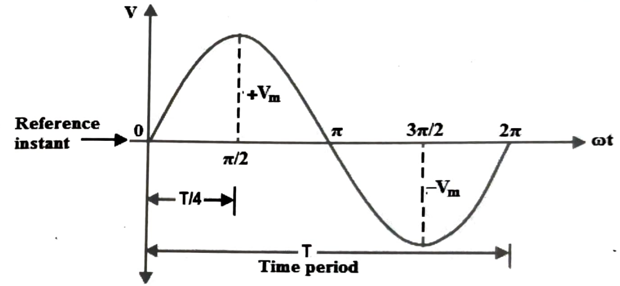

Figure 1: Concept Phase of an alternating quantity

Consider an alternating voltage wave of time period T second as shown in figure1. Note that the time is counted from the instant the voltage is zero. The maximum positive value ($+ V_m$) occurs at T/4 second or $\pi/2$ radians. We say that phase of maximum positive value is T/4 second or $\pi/2$ radians. It means that as the fresh cycle starts, $+ V_m$ will occur at T/4 second or $\pi/2$ radians. Similarly, the phase of negative peak ($-V_m$) is 3T/4 second or $3\pi/2$ radians.

2. PHASE DIFFRERENCE

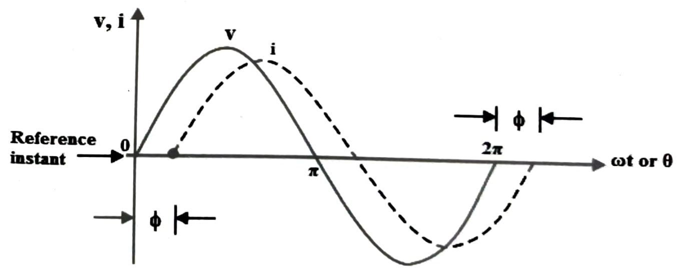

Figure 2: Waveforms showing phase difference

- ***When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference.***

The angle between zero points is the angle of phase difference ϕ. It is generally measured in degrees or radians. The quantity which passes through its zero point earlier is said to be leading while the other is said to be lagging.

Consider an ac circuit in which current i lags behind the voltage v by ϕ^o, we say that phase difference between voltage and current is ϕ^o. This phase relationship is shown by waves in figure 2.

The equations of voltage and current are

$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t - \phi)$$

2. PHASOR REPRESENTATION OF AN ALTERNATING QUANTITY

2.1 Need for phasor representation

- In the analysis of A.C. circuits, it is very difficult to deal with alternating quantities in terms of their waveforms and mathematical equations. The job of adding, subtracting, etc. of the two alternating quantities is tedious and time consuming in terms of their mathematical equations. Hence, it is necessary to study a method which gives an easier way of representing an alternating quantity. Such a representation is called phasor representation of an alternating quantity.
- ***The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow in this method. The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to a vector representation. Such a line is called a phasor.***

Consider an alternating quantity (current) as shown in figure 3 represented by the equation $i = I_m \sin \omega t$ Take a line OP to represent to scale the maximum value I_m. Imagine the line OP or Phasor is rotating in anticlockwise direction at an angular velocity of ω rad/sec about the point O. Measuring the time from the instant when OP is horizontal, let OP rotate through an angle ($\theta = \omega t$) in the anticlockwise direction. The projection of OP on Y axis is OM.

$$OM = OP \sin \theta$$
$$= I_m \sin \omega t$$
$$= i, \text{ the value of current at that instant}$$

Hence the projection of the phasor OP on Y axis at any instant gives the value of current at that instant. Thus when $\theta = 90^0$, the projection on Y axis is $OP = I_m$. If we plot the projections of the phasor on Y axis versus its angular position, point by point a sinusoidal alternating current is generated as shown in figure 3

- A phasor is rotating vector with constant magnitude and constant angular velocity. Magnitude of the vector peak value of a sine wave and its angular velocity $\omega = 2\pi f$. Here f is the frequency of the sine wave.
- ***As the magnitude of rotating vector is constant and only phase angle θ varies, rotating vector is a function of phase angle only and hence it is called phasor.***

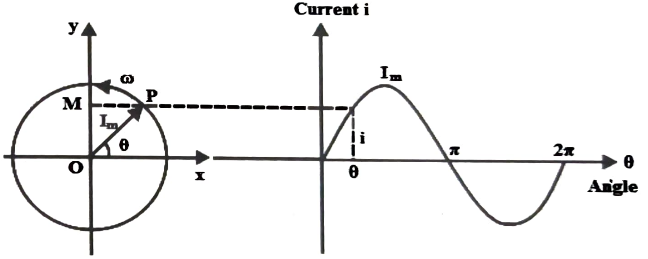

Figure 3: Phasor representation of alternating current

One complete cycle of a sine wave is represented by one complete rotation of a phasor. The anticlockwise direction of rotation is purely a conventional direction which has been universally adopted.

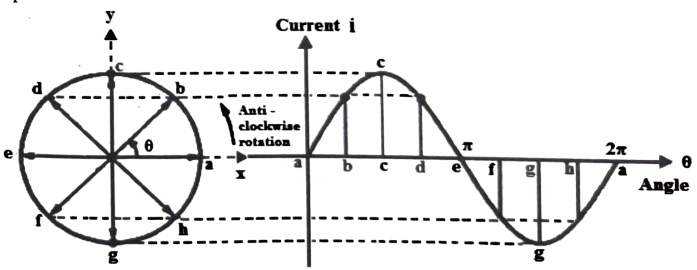

Figure 4: Phasor representation of a sine wave.

Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position '**a**' as shown in the figure 4. If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ', (or time as $\theta = \omega t$), we get a sine waveform. Consider the various positions shown in the figure 4.

2.2 Phasor diagram of voltage and current of same frequency

Consider a sinusoidal voltage wave v and sinusoidal current wave i of the same frequency. Suppose the current lags behind the voltage by ϕ^o. The two alternating quantities can be represented on the same phasor diagram because the phasors V_m and I_m (see figure 5 (b)) rotate at the same angular velocity ω and hence phase difference ϕ between them remains the same at all times. For all instant of time I_m lag V_m by ϕ.

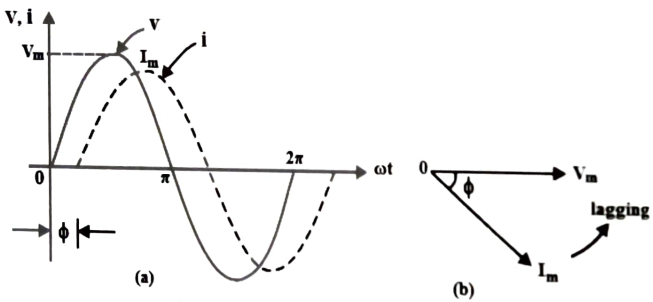

Figure 5: Voltage and current of same frequency

3. FORMS OF PHASOR REPRESENTATION

Any alternating quantity can be represented by a rotating phasor. When several alternating currents or voltages are evolved, there would be definite phase relationships between them. Phasors can be expressed mathematically in the following forms.

1. **Rectangular form**
2. **Trigonometric form**
3. **Exponential form**
4. **Polar form**

The j operator: The letter j is used to express operation of counter clock-wise rotation of the vector through 90^o. If this operation is done twice on a vector, the vector gets rotated counter clockwise through 180^o and reverse its sign. That is, it gets multiplied by -1

Thus $j \times j = j^2 = -1$

$j = \sqrt{-1}$

$j^2 V = -V$

3.1 RECATNGULAR FORM

In rectangular form, the phasor is represented in terms of its horizontal and vertical components with an operator j. So the phasor as shown in figure 6 (a) can be represented as

$\vec{v_1} = a + jb$ where j is equal to $\sqrt{-1}$.

$\vec{v_2} = \mathbf{a - jb}$

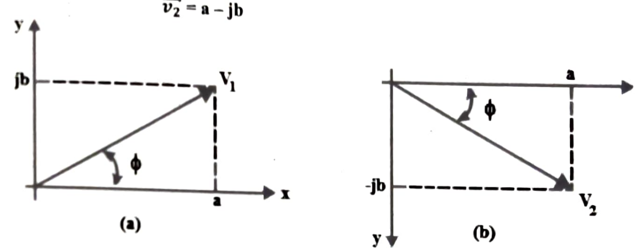

Figure 6: Phasor representation in rectangular form (a) $\vec{v_1} = \mathbf{a + jb}$ **(b)** $\vec{v_2} = \mathbf{a - jb}$

The magnitude of the phasor is given by $v_1 = \sqrt{a^2 + b^2}$

Its angle with x- axis is given by $\phi = \tan^{-1}\frac{b}{a}$

3.2 TRIGONOMETRIC FORM

Figure 7 shows the vector V and its X component is V$\cos\theta$ and its Y component is V$\sin\theta$

Hence we may write $\vec{v} = \mathbf{V(\cos\theta + j\sin\theta)}$

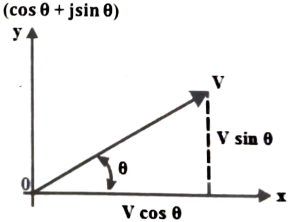

Figure 7: Phasor representation in trigonometric form

3.3 POLAR FORM

In polar form, the phasor is represented with its magnitude and the phase angle with respect to the reference axis (x axis).

For example, $\vec{v_1}$ shown in figure 6 (a) is represented in polar form as $\vec{v_1} = V_1 \angle \phi$

The magnitude of the phasor is given by $v_1 = \sqrt{a^2 + b^2}$

Its angle with x- axis is given by $\phi = \tan^{-1}\frac{b}{a}$

$\vec{v_2}$, shown in figure 6 (b) is represented in polar form as $\vec{v_2} = V_2 \angle -\phi$

The magnitude of the phasor is given by $v_2 = \sqrt{a^2 + b^2}$

Its angle with x- axis is given by $\phi = \tan^{-1}\left(\frac{-b}{a}\right)$

3.4 EXPONENTIAL FORM

The exponential representation is based on Euler's relation $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

The phasor in figure 6(a) can be represented in exponential form as $\vec{v_1} = V_1 \angle e^{j\phi}$

The phasor in figure 6(b) can be represented in exponential form as $\vec{v_2} = V_2 \angle e^{-j\phi}$

4. ADDITION AND SUBTRACTION OF TWO PHASORS

Addition and subtraction of alternating quantities can be easily done if they are represented in rectangular form.

Consider two voltages $\vec{v_1} = a_1 + jb_1$ and

$$\vec{v_2} = a_2 + jb_2$$

The sum of voltages $\vec{v} = \vec{v_1} + \vec{v_2} = (a_1 + a_2) + j(b_1 + b_2)$

The magnitude of $\vec{v} = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$

Phase angle of $\vec{v} = \tan^{-1}\frac{b_1 + b_2}{a_1 + a_2}$

Similarly, the difference between $\vec{v_1}$ and $\vec{v_2}$ is given by, $\vec{v_1} - \vec{v_2} = (a_1 - a_2) + j(b_1 - b_2)$

5. MULTIPLICATION AND DIVISION OF TWO PHASORS

The polar form of representing phasors is best suited for multiplication and division.

Consider two voltage phasors represented as

$$\vec{V_1} = a_1 + jb_1 = V_1 \angle \theta_1 \text{ where } \theta_1 = \tan^{-1}\frac{b_1}{a_1}$$

$$\vec{V_2} = a_2 + jb_2 = V_2 \angle \theta_2 \text{ where } \theta_2 = \tan^{-1}\frac{b_2}{a_2}$$

Multiplication

- ***When the phasor quantities are represented in polar form, while multiplying their magnitudes are multiplied and their angles added algebraically.***

$$\vec{V_1} \times \vec{V_2} = V_1 \angle \theta_1 \times V_2 \angle \theta_2$$
$$= V_1 V_2 \angle \theta_1 + \theta_2$$

Division

- ***In this case the magnitude of phasor quantities (expressed in polar form) are divided and their angles subtracted algebraically.***

$$\frac{\vec{V_1}}{\vec{V_2}} = \frac{V_1 \angle \theta_1}{V_2 \angle \theta_2} = \frac{V_1}{V_2} \angle \theta_1 - \theta_2$$

6. ANALYSIS OF SIMPLE AC CIRCUITS

A resistance, an inductance and a capacitance are the basic elements of an A.C circuit. In this section we will study the relationship of applied voltage and current in an A.C circuit involving only a resistance, an inductance and a capacitance.

6.1. AC THROUGH PURELY RESISTIVE CIRCUIT

- ***A resistance is connected across an ac supply is called purely resistive circuit. It is also called non- inductive circuit.***

A pure resistive or non- inductive circuit is a circuit which has inductance so small at normal frequency and its reactance is negligible as compared to its resistance. Ordinary filament lamps, water resistance etc. are the examples of non-inductive resistances.

When an alternating voltage is applied across pure resistance, then free electrons flow (i.e. current) in one direction for the first half-cycle of the supply and then flow in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit.

Consider a circuit containing a pure resistance of R Ω connected across an alternating voltage source.

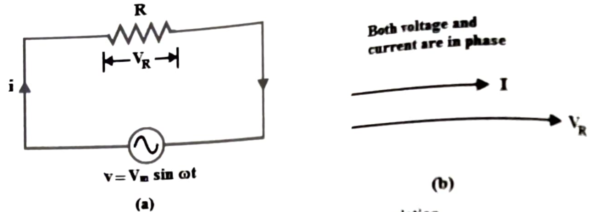

Figure 8: A resistive circuit and the phase relation

Let the alternating voltage be given by the equation: $v = V_m \sin\omega t$ ---------- (1)

Here v represents instantaneous value of applied voltage and $\omega = 2\pi f$ is the angular frequency.

By ohm's law the instantaneous current in the circuit

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

Current is maximum when $\sin \omega t$ is maximum. There for $i = I_m \sin \omega t$ ----------(2)

Here $I_m = \frac{V_m}{R}$, represents maximum value of current.

Comparing the voltage equation (1) and current equation (2), it is clear that voltage and current are in phase with each other. The phase difference between them is zero. This is indicated by a phasor diagram in figure 8 (b). Note that R.M.S. values have been used in drawing the phasor diagram

Power

Power is the product of voltage and current. The product P=V I has been calculated for all instants of time and has been shown in figure 9 (b).

Instantaneous power $p = vi$

$$= V_m \sin\omega t \times I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \frac{1-\cos 2\omega t}{2} = \frac{V_m I_m}{2}(1-\cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

Thus power consists of two parts viz. a constant part $\left[\frac{V_m I_m}{2}\right]$ and a fluctuating part $\left[\frac{V_m I_m}{2}\cos 2\omega t\right]$. Since power is a scalar quantity, average power over a complete cycle is to be considered. For a complete cycle, the average value of $\left[\frac{V_m I_m}{2}\cos 2\omega t\right]$ is zero.

Power consumed $P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} d(\omega t) + 0$

$$= \frac{1}{2\pi}\frac{V_m I_m}{2}[\omega t]_0^{2\pi}$$

$$= \frac{1}{2\pi}\frac{V_m I_m}{2}[2\pi - 0]$$

$$= \frac{V_m I_m}{2} = \frac{V_m I_m}{\sqrt{2}\sqrt{2}}$$

$$P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \quad \text{--------(3)}$$

We know that $V_{rms} = \frac{V_m}{\sqrt{2}} = V$ and $I_{rms} = \frac{I_m}{\sqrt{2}} = I$

Now the equation (3) becomes $P = V I$

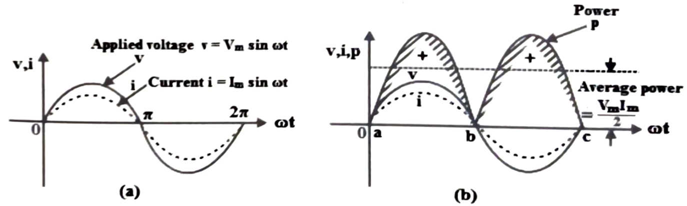

Figure 9: Voltage, current and power waveforms in a purely resistive circuit.

- ***Power factor is the cosine of the phase angle between voltage and current.***

In a resistive network, the phase difference between voltage and current is zero. That is, they are in phase. So the phase angle $\theta = 0$. Power factor, $p.f = \cos 0^o = 1$

Info Plus

- **Power in a pure resistive circuit is always positive.** This is because the instantaneous values of voltage and current are always either positive or negative and therefore the product is always positive. This means that the voltage source constantly delivers power to the circuit and the circuit consumes it.
- **Frequency of power is twice that of applied voltage or current.**
- **The power factor of a purely resistive circuit is unity.**

6.2 AC THROUGH PURELY INDUCTIVE CIRCUIT

- ***An inductance is connected across an ac supply is called purely inductive circuit.***

A pure inductance means that the resistance of the inductor coil is assumed to be zero. The coil has only inductance L. Such an inductor is connected across a sinusoidally varying voltage $v = V_m \sin\omega t$ as shown in figure 10 (a).

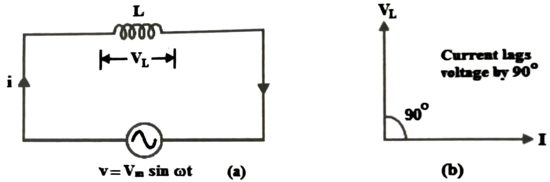

Figure 10: An Inductive circuit and the phase relation

Let the applied alternating voltage be given by the equation $v = V_m \sin\omega t$ ————(1)

This alternating voltage will cause an alternating current i to flow through the circuit. Due to the inductance of the coil, a self-induced emf ($-L\frac{di}{dt}$ volt) is induced in the coil which opposes the applied voltage at every instant.

Therefore, $v = -L\dfrac{di}{dt}$

By applying KVL, $V_m \sin\omega t - L\dfrac{di}{dt} = 0$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_m}{L}\sin\omega t$$

Integrating both sides we get $i = \int \dfrac{V_m}{L}\sin\omega t\, dt$

$$i = \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Current is maximum when $\sin\left(\omega t - \dfrac{\pi}{2}\right) = 1$

$$I_m = \frac{V_m}{\omega L}$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{--------(2)}$$

$I_m = \dfrac{V_m}{\omega L}$ is maximum value of current and ωL is equivalent to inductive reactance X_L

$$I_m = \frac{V_m}{X_L}$$

- The quantity ωL or X_L is called inductive reactance of the inductor and is expressed in ohm.
- $X_L = \omega L = 2\pi f L$. Here L is in Henry and ω is in radians/second.
- **Inductive reactance is the opposition offered to the flow of an ac by the inductance of an inductor.**

Thus from the equations (1) and (2) it is clear that current I lags behind the applied voltage V by $90°$ which is shown in figure 10 (b). The quantity ωL plays the same role as the resistance in a resistive circuit.

Power

Instantaneous power $p = vi = V_m \sin\omega t \times I_m \sin\left(\omega t - \dfrac{\pi}{2}\right)$

$$= V_m I_m \sin\omega t \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin\omega t \cos\omega t \qquad \text{Since } \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos\omega t$$

$$p = -\frac{V_m I_m}{2}\sin 2\omega t \qquad \text{Since } \sin 2A = 2\sin A\cos A$$

The waveforms for voltage, current and power for a purely inductive circuit are shown in figure 11. The average power P for complete cycle is obtained by averaging the above expression for instantaneous power over a complete cycle.

$$\text{Average power, } P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2}\sin 2\omega t \, d\omega t = -\frac{1}{2\pi}\frac{V_m I_m}{2\times 2}\left[-\cos 2\omega t\right]_0^{2\pi}$$

$$= -\frac{V_m I_m}{8\pi}\left[-\cos 4\pi + (\cos 0)\right] = -\frac{V_m I_m}{8\pi}[-1+1] = 0$$

Therefore, **average power considered in a purely inductive circuit is zero.**

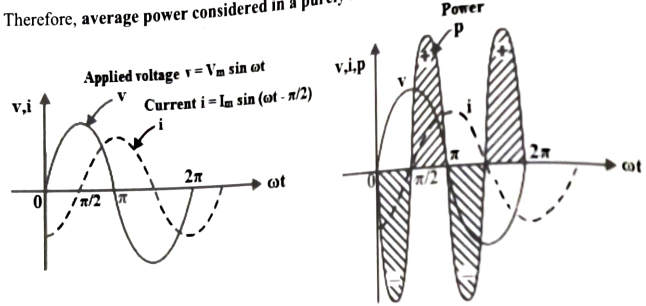

Figure 11: Voltage, current and power waveforms in a purely inductive circuit.

Info Plus

- **In a pure inductive circuit power absorbed is zero.**

Physically the above fact can be explained as below:

During the second quarter of a cycle the current and the magnetic flux of the coil increases and the coil draws power from the supply source to build up the magnetic field (the power drawn is positive and the energy drawn by the coil from the supply source is represented by the area between the curve p and the time axis). The energy stored in the magnetic field during build up is given as $W = \frac{1}{2}LI^2$

In the next quarter the current decreases. The emf of self-induction will, tend to oppose its decrease. The coil act as a generator of electrical energy, returning the stored energy in the magnetic field to supply source (now the power drawn by the coil is negative and the curve p lies below the time axis). The chain of events repeats itself during the next half cycles. Thus, a proportion of power is continuously exchanged between the field and the inductive circuit and the power consumed by a pure inductive coil is zero.

- **Frequency of inductive power is twice that of applied voltage or current.**

6.3 AC THROUGH PURELY CAPACITIVE CIRCUIT

- ***A capacitance is connected across an ac supply is called purely capacitive circuit.***

Consider a capacitor of capacitance C Farads connected to an AC circuit as shown in figure 12(a).

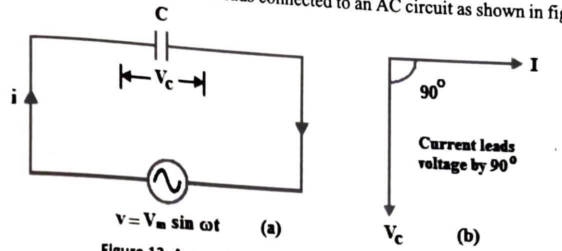

Figure 12: A capacitive circuit and the phase relation

Let the applied alternating voltage be given by the equation $v = V_m \sin\omega t$ ----------(1)

The charge across the capacitor is given by, $q = C\times v$ -----------(2)

Now the (2) can be rewritten as $q = C\times V_m \sin\omega t$

Current through the circuit, i =rate of flow of charge.

$$i = \frac{dq}{dt} = \frac{d}{dt} C\, V_m \sin\omega t$$

$$= \omega C . V_m \cos\omega t$$

$$= \frac{V_m}{\frac{1}{\omega C}}\sin\left(\omega t + \frac{\pi}{2}\right)$$

The term $\frac{1}{\omega C} = X_C$ = capacitive reactance

- **Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$**
- **Capacitive reactance is the opposition offered to the flow of AC by the capacitance of a capacitor.**

Now the expression for current can be written as $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$ --------(2)

where $I_m = \frac{V_m}{X_C}$

Thus from equation (1) and (2) it is clear that for or a purely capacitive circuit, current leads the applied voltage by 90^0 which is shown in figure 12 (b).

Power

Instantaneous power $p = vi = V_m \sin\omega t \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$

$$= V_m I_m \sin\omega t \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t \cos\omega t \qquad \text{Since } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t$$

$$p = \frac{V_m I_m}{2} \sin 2\omega t \qquad \text{Since } \sin 2A = 2 \sin A \cos A$$

The waveforms for voltage, current and power for a purely capacitive circuit are shown in figure 13.

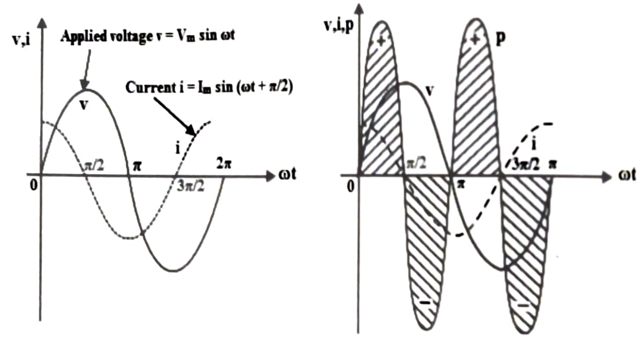

Figure 13: Voltage, current and power waveforms in a purely capacitive circuit.

The average power P in the circuit is obtained by taking the average of the above expression over a complete cycle.

$$\text{Average power, } P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d\omega t = \frac{1}{2\pi}\frac{V_m I_m}{2\times 2}\left[-\cos 2\omega t\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{8\pi}\left[-\cos 4\pi + (\cos 0)\right] = \frac{V_m I_m}{8\pi}[-1+1] = 0$$

- Therefore, **average power considered in a purely capacitive circuit is zero.**
- **Frequency of capacitive power is twice that of applied voltage or current.**

6.4 AC THROUGH SERIES R-L CIRCUIT

This is the most general case met in practice, nearly all circuits contain both resistance and inductance. Consider a RL series circuit connected across an AC voltage source as shown in figure 14.

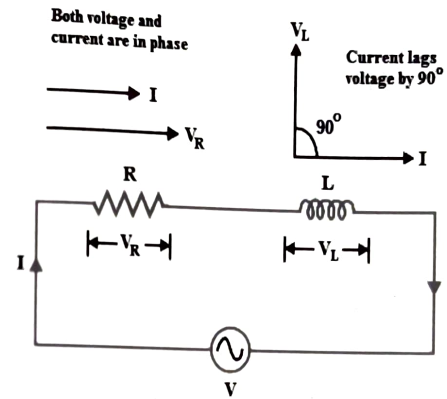

Figure 14: RL series circuit

Let V_R be the voltage drop across resistance R, V_L be the voltage across the inductance L and V be the applied AC voltage. The current I flowing through resistor and inductor are same since they are connected in series.

V = r.m.s value of the applied voltage and I = r.m.s value of resultant current

$V_R = IR$ ------ where V_R is in phase with I

$V_L = IX_L$ ------ where V_L leads I by 90° or I lags V_L by 90°

Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.

Following are the steps to draw the phasor diagram

1. Take current as a reference phasor.
2. In case of resistance, voltage and current are in phase, so V_R will be along current phasor.
3. In case of inductance, current lags voltage by 90°. But, as current is reference, V_L must be shown leading with respect to current by 90°.
4. The supply voltage being vector sum of these two vectors V_L and V_R.

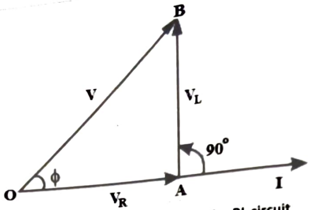

Figure 15: Phasor diagram of series RL circuit

From the phasor diagram shown in figure 15, the applied voltage V is the vector sum of V_R and V_L

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I \times \sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z}$$

The term $\sqrt{R^2 + X_L^2}$ is called impedance Z of the R-L circuit.

Voltage triangle

The triangle having V_R (voltage drop across resistor), V_L (voltage drop across inductor) and V(total supply voltage) as its sides is called voltage triangle for a series R-L circuit. It is shown in figure16.

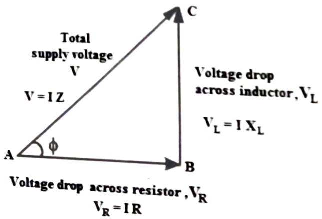

Figure 16: Voltage triangle of an R-L circuit

If the length of each side of the voltage triangle is divided by the current I the impedance triangle is obtained.

Impedance triangle

The voltage triangle of an R-L series circuit is shown in figure 16. Dividing each side of the triangle diagram by the same factor I, we get a triangle whose sides represent R, X_L, and Z. Such a triangle is known as impedance triangle which is shown in figure 17. Just as voltage triangle, the impedance triangle is also a right-angled triangle.

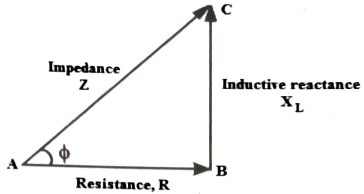

Figure 17: Impedance triangle

- ***The angle between Z and R sides of the impedance triangle is known as phase angle of the circuit and $\cos$ of this angle is known as power factor of the circuit.***

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

- ***The phase angle ϕ can be found either from the voltage triangle or impedance triangle.*** From the impedance triangle shown in figure 17

$$\tan\phi = \frac{X_L}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Benefits to draw impedance triangle

Impedance triangle is a useful concept in A.C. circuits as it enables us to calculate the following.

1. The impedance of the circuit, $Z = \sqrt{R^2 + X_L^2}$
2. Power factor of the circuit, $\cos\phi = \dfrac{R}{Z}$
3. Phase angle $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right)$
4. To understand whether current leads or lags the voltage.

Therefore, it is always gainful to draw the impedance triangle while analyzing an A.C. circuit

Power in R-L series circuit

Power consumed by resistor is called active power active power, $P = V_R I = VI\cos\phi$

Power stored inductor is called reactive power, $Q = V_L I = VI\sin\phi$

Total available power is apparent power, $S = VI$

In a series R-L circuit power consumed by the inductor is zero. Power is dissipated only in resistor.

6.5 AC THROUGH SERIES R-C CIRCUIT

Consider a RC series circuit connected across an AC voltage V. Let V_R be the voltage drop across resistance R, V_C be voltage drop across the capacitor and V be the supply voltage. As same current I flowing through resistor and capacitor in series RC circuit, current is taken as reference phasor. In a resistor current is in phase with voltage and in a capacitor current leads the voltage by 90°.

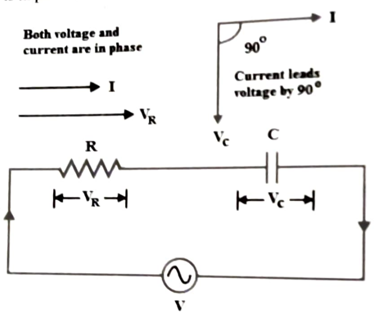

Figure 18: RC series circuit

V = r.m.s value of the applied voltage and I = r.m.s value of resultant current

$V_R = IR$ ------ where V_R is in phase with I

$V_C = IX_C$ ------ where V_C lags I by 90° or I leads V_C by 90°

$X_C = \frac{1}{\omega C}$, the reactance of the capacitor

Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.

Following are the steps to draw the phasor diagram

1. Take current as a reference phasor.
2. In case of resistance, voltage and current are in phase, so V_R will be along current phasor.
3. In case of pure capacitance, current leads voltage by 90° i.e. voltage lags current by 90° so V_C is shown downwards i.e. lagging current by 90°.
4. The supply voltage being vector sum of these two vectors V_R and V_C

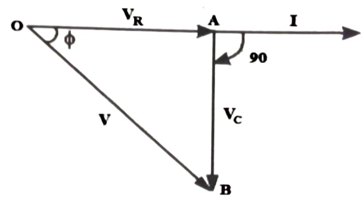

Figure 19: Phasor diagram of series RL circuit

From the phasor diagram shown in figure 19, the applied voltage V is the vector sum of V_R and V_C

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = I \times \sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

The term $\sqrt{R^2 + X_C^2}$ is called impedance Z of the R-C circuit.

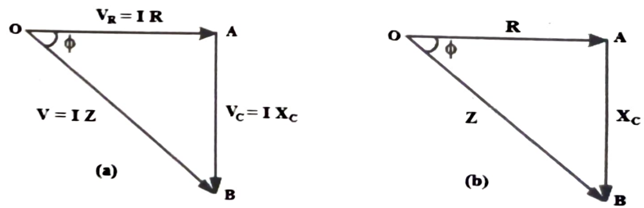

Figure 20: Voltage triangle and impedance triangles of an RC circuit.

The voltage triangle of an R-C series circuit is shown in figure 20(a). Dividing each side of the triangle diagram by the same factor I, we get a triangle whose sides represent R, X_C, and Z. Such a triangle is known as impedance triangle which is shown in figure 20(b). Just as voltage triangle, the impedance triangle is also a right-angled triangle.

- ***The phase angle ϕ can be found either from the voltage triangle or impedance triangle***

From the impedance triangle shown in figure 20(b)

$$\tan\phi = \frac{X_C}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

- ***The angle between Z and R sides of the impedance triangle is known as phase angle of the circuit and cos of this angle is known as power factor of the circuit.***

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

Power in R-C series circuit

Power consumed by resistor is called active power active power, $P = V_R I = VI\cos\phi$

Power stored capacitor is called reactive power, $Q = V_C I = VI\sin\phi$

Total available power is apparent power, $S = VI$

In a series R-C circuit power consumed by the capacitor is zero. Power is dissipated only in resistor.

6.6 AC THROUGH SERIES R-L-C CIRCUIT

Consider a RLC series circuit connected across an AC voltage V. Let V_R be the voltage drop across resistance R, V_L be voltage drop across the inductor, V_C be voltage drop across the capacitor and V be the supply voltage. As same current I flowing through resistor, inductor and capacitor in series RLC circuit, current is taken as reference phasor. In a resistor current is in phase with voltage, in an inductor current lags the voltage and in a capacitor current leads the voltage by 90°.

V = r.m.s value of the applied voltage and I = r.m.s value of resultant current

$V_R = IR$ ----- where V_R is in phase with I

$V_L = IX_L$ ----- where V_L leads I by 90° or I lags V_L by 90°

$V_C = IX_C$ ----- where V_C lags I by 90° or I leads V_C by 90°

$X_C = \frac{1}{\omega C}$, the reactance of the capacitor and $X_L = \omega L$, the reactance of the inductor

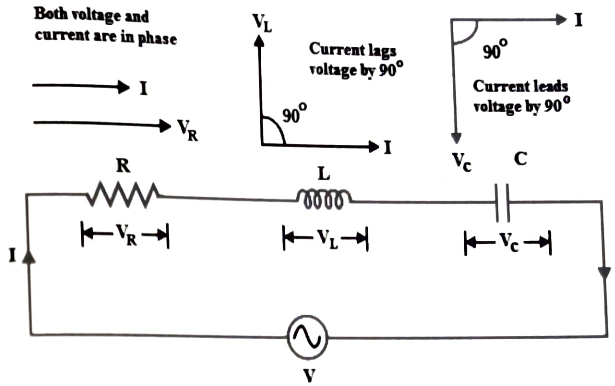

Figure 21: RLC series circuit

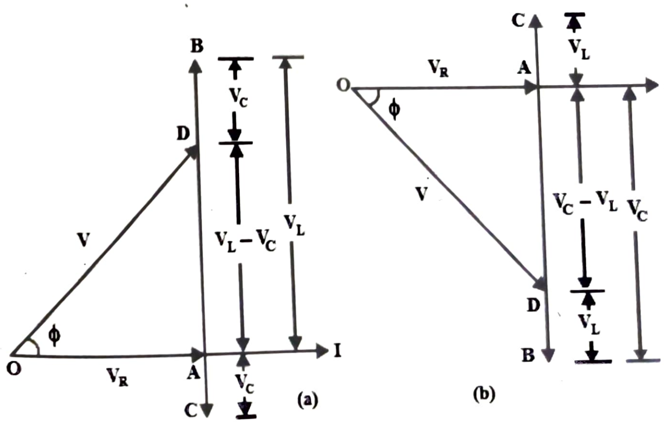

Figure 22: Phasor diagrams (a) $V_L > V_C$ (b) $V_C > V_L$

The applied voltage V is the vector sum of V_R, V_L and V_C. The current I is lagging or leading depending up on the voltage V_L and V_C

If $V_L > V_C$ current is lagging

If $V_L < V_C$ current is leading

Consider the case $V_L > V_C$

Net reactive drop $= V_L - V_C = IX_L - IX_C = I(X_L - X_C)$

The applied voltage V= $= \sqrt{V_R^2 + (V_L - V_C)^2}$

$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

$V = I\sqrt{R^2 + (X_L - X_C)^2}$

Therefore $I = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{V}{Z}$

- The term $\sqrt{R^2 + (X_L - X_C)^2}$ is the total impedance of the circuit (Z)

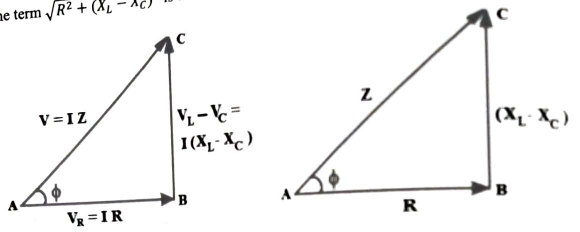

Figure 23: (a) Voltage triangle $V_L > V_C$ (b) Impedance triangle $V_L > V_C$

From the impedance triangle $\tan\phi = \frac{X_L - X_C}{R} = \frac{\text{Net reactance}}{\text{Resistance}}$

Voltage and current in the circuit are represented as

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t \pm \phi)$$

When current leads [i.e., $V_C > V_L$ or $X_C > X_L$], then (+) sign is used. When current lags [i.e., $V_L > V_C$ or $X_L > X_C$], then (-) sign is used.

The average power consumed by an RLC circuit is equal to power consumed by the resistor $P = V_R I = VI\cos\phi$

- Power factor = $\cos\phi = \frac{R}{Z}$

7. ACTIVE POWER, REACTIVE POWER AND APPARENT POWER

1) APPARENT POWER

- ***The total power that appears to be transferred between the source and load is called apparent power.***

It is equal to the product of applied voltage (V) and circuit current (I)
i.e. Apparent power, S = V × I = VI
It is measured in in Volt –Amperes (VA)

- **Apparent power has two components viz true power and reactive power.**

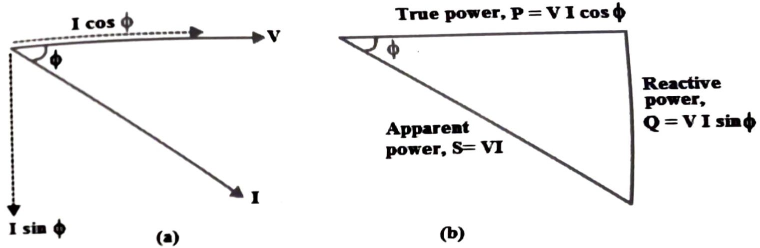

Figure 24(a) Components of apparent power (b) Power triangle

2) TRUE POWER OR ACTIVE POWER

- ***The power which is actually consumed in the circuit is called true power or active power.***

We know that power is consumed in resistance only since neither pure inductor (L) nor pure capacitor (C) consumes any active power. Now, current and voltage are in phase in a resistance. Therefore, current in phase with voltage produces true or active power. It is the useful component of apparent power.
The product of voltage (V) and component of total current in phase with voltage $[I\cos\phi]$ is equal to true power i.e.
True power, P = Voltage × Component of total current in-phase with voltage $= V \times I\cos\phi$

$$P = VI\cos\phi$$

It is measured in watts (W). The component $I\cos\phi$ is called in-phase component or watchful component because it is this component of total current which contributes to true power $(i.e., VI\cos\phi)$. It may be noted that it is the true power which is used for producing torque in motors and supply heat, light etc. It is used up in the circuit and cannot be recovered.

3) REACTIVE POWER

- ***The component of apparent power which is neither consumed nor does any useful work in the circuit is called reactive power.***

The power consumed (or true power) in L and C is zero because all the power received from the source in one quarter-cycle is returned to the source in the next quarter-cycle. This circulating power is called reactive power. Now, current and voltage in L or C are 90° out of phase. Therefore, current 90° out of phase with voltage contributes to reactive power.

The product of voltage (V) and component of total current 90° out of phase with voltage is equal to reactive power i.e.

Reactive power, Q = Voltage × Component of total current 90° out of phase with voltage

$$= V \times I \sin\phi$$

$$Q = VI\sin\phi$$

It is measured in volt-amperes reactive (VAR). The component $I\sin\phi$ is called the reactive component (or wattless component) and contributes to reactive power (i.e. $VI\sin\phi$). It does no useful work in the circuit and simply flows back and forth in both directions in the circuit. A wattmeter does not measure the reactive power.

4) POWER TRIANGLE

If we multiply each of the current phasors in figure 24(a) by V, we get the power triangle shown in figure 24(b).

- ***Power triangle is a right-angled triangle and indicates the relation among apparent power, true power and reactive power.***

It tells the following details about the circuit:

- Power factor $= \cos\phi = \dfrac{\text{True power(P)}}{\text{Apparent power (S)}} = \dfrac{VI\cos\phi}{VI}$
- $(\text{Apparent power})^2 = (\text{True power})^2 + (\text{Reactive power})^2$ or $S^2 = P^2 + Q^2$
- True power, P = Apparent power × $\cos\phi$ = V I $\cos\phi$
  Reactive power, Q = Apparent power × $\sin\phi$ = V I $\sin\phi$

- **True power component should be as large as possible because it is this component which does useful work in the circuit.** This is possible only if the reactive power component is small.
- ***Power factor of a circuit is a measure of its effectiveness in utilizing the apparent power drawn by it.*** The greater the power factor of a circuit, the greater is its ability to utilize the apparent power. Thus 0·5 p.f. (i.e. 50% p.f.) of a circuit means that it will utilize only 50% of the apparent power whereas 0·8 p.f. would mean 80% utilization of apparent power. **For this reason, we wish that the power factor of the circuit to be as near to 1 as possible.**

8. SOLVED NUMERICAL PROBLEMS

1. A 10Ω resistor and 300 mH inductor are connected in series to a 230V sinusoidal supply. The circuit current is 4A. Calculate the supply frequency and phase angle between current and voltage. [KTU JANUARY 2017]

Solution: We know that $X_L = 2\pi fL$

Supply frequency $f = \dfrac{X_L}{2\pi L}$

$X_L^2 = Z^2 - R^2$ and $X_L = \sqrt{Z^2 - R^2}$

$Z = \frac{V}{I} = \frac{230}{4} = 57.5\Omega$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{57.5^2 - 10^2} = 56.6\Omega$

Now the supply frequency, $f = \frac{X_L}{2\pi L} = \frac{56.6}{2\pi \times 300 \times 10^{-3}} = 30.04$ Hz

Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{10}{57.5} = 0.174$

$$\phi = \cos^{-1}(0.174) = 80^{\circ}$$

2. A 220V, 50Hz single phase sinusoidal voltage produces a current of 2.2A in a purely inductive coil. Determine (1) Inductive reactance of the coil (2) Inductance (3) Power absorbed (4) Expression for applied voltage and current. [KTU DEC 2018]

Solution:

1) $I = \frac{V}{Z}$, In a purely inductive coil $Z = X_L$ .Hence $I = \frac{V}{X_L}$
   Inductive reactance, $X_L = \frac{V}{I} = \frac{220}{2.2} = 100\Omega$

2) $X_L = 100\Omega$ i.e. $2\pi fL = 100$
   Inductance, $L = \frac{100}{2\pi f} = 0.318$ H

3) Power absorbed in a purely inductive coil is zero

4) Expression for applied voltage and current
   $v = V_m \sin\omega t$ and $V_{rms} = \frac{V_m}{\sqrt{2}}$
   Hence $V_m = V_{rms} \times \sqrt{2} = 220 \times \sqrt{2} = 311.13$ V
   $\omega = 2\pi f = 314$
   $v = 311.13 \sin(314t)$
   $i = I_m \sin(\omega t - 90)$
   $I_m = I_{rms} \times \sqrt{2} = 2.2 \times \sqrt{2} = 3.11$A
   $i = 3.11 \sin(314t - 90)$

3. An inductor of 0.5 H is connected across a 230 V, 50 Hz supply. Write the equations for instantaneous values of voltage and current.

Solution: V=230 V, $V_m = \sqrt{2}V = 1.414\times230 = 325V$

$X_L = \omega L = 2\pi fL = 2\times\pi\times50\times0.5H = 157\Omega$

$I = \frac{V}{X_L} = \frac{230}{157} = 1.46A \qquad I_m = \sqrt{2}I = 1.414\times1.46 = 2.06A$

The equations are $v = V_m\sin\omega t = 325\sin\omega t = 325\sin2\pi ft = 325\sin314t$

$i = I_m\sin(\omega t - 90°) = 2.06\sin(314t - 90°)$

4. A 230 V, 50 Hz sinusoidal supply is connected across a (1) resistance of 25 Ω (2) inductance of 0.5 H (3) capacitance of 100μF. Write the expression for instantaneous current in each case.

Solution: V=230 V, $V_m = \sqrt{2}V = 1.414\times230 = 325.2V$

$\omega 2\pi f = 2\times\pi\times50 = 314 rad/sec$

Voltage equation is $v = V_m\sin\omega t = 325.2\sin\omega t = 325.2\ \sin2\pi ft = 325.2\ \sin314t$

Inductive reactance $X_L = \omega L = 2\pi fL = 2\times\pi\times50\times0.5H = 157\Omega$

Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{314\times100\times10^{-6}} = 31.84\Omega$

When the voltage is applied across a 25 Ω resistor, the current will be

$i = \frac{V_m}{R}\sin\omega t = \frac{325.2}{25}\sin314t = 13\sin314t\ A$

Current through the inductor is

$i = \frac{V_m}{X_L}(\sin\omega t - 90°) = \frac{325.2}{157}(\sin314t - 90°) = 2.07\ \sin(314t - 90°)A$

Current through the capacitor is

$i = \frac{V_m}{X_C}(\sin\omega t + 90°) = \frac{325.2}{32.2}(\sin314t + 90°) = 10.22\ \sin(314t + 90°)A$

5. A 100 Ω resistance is carrying a sinusoidal current given by $3\cos\omega t$. Determine (1) instantaneous power taken by the resistance (2) average power.

Solution:

1) Instantaneous power taken by the resistance

$p = v.i = iR\times i = 3\cos\omega t\times100\times3\cos\omega t = 900\cos^2\omega t = 450(1+\cos2\omega t)$

2) Average power = 450 watts

6. An alternating current of 1.5A flows in a circuit when applied voltage is 300 V. The power consumed is 225W. Find the resistance and reactance of the circuit.

Solution: Circuit resistance, $R = \frac{\text{Power consumed}}{I^2} = \frac{225}{(1.5)^2} = 100\Omega$

Circuit impedance $Z = \frac{V}{I} = \frac{300}{1.5} = 200\Omega$

Circuit reactance $X = \sqrt{Z^2 - R^2} = \sqrt{200^2 - 100^2} = 173.3\Omega$

7. A resistance and inductance are connected in series across a voltage $v = 283\sin314t$. An expression of current is found to be $i = 4\sin(314t - 45°)$. Find the values of resistance, inductance and power factor.

Solution: Supply frequency, $f = \frac{\text{coefficeinet of time,t}}{2\pi} = \frac{314}{2\pi} = 50\,Hz$

RMS value of applied, $V = \frac{V_{max}}{\sqrt{2}} = \frac{283}{\sqrt{2}} = 200.11$

RMS value of current, $V = \frac{I_{max}}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828A$ Phase angle, ϕ=45° (lagging)

Circuit impedance $Z = \frac{V}{I} = \frac{200.11}{2.828} = 70.75\ \Omega$

Resistance of the circuit, $R = Z\cos\phi = 70.75\cos45° = 50\Omega$

Reactance of the circuit, $X_L = Z\sin\phi = 70.75\sin45° = 50\Omega$

Inductance $L = \frac{X_L}{2\pi f} = \frac{50}{2\pi\times50} = 0.159H$

Power factor = $\cos\phi = \cos45^0 = 0.707$ (lagging)

8. Two impedances Z_1 and Z_2 when connected separately across a 220V, 50 Hz supply, consumes 300W and 150W at a power factor of 0.4 lagging and 0.7 leading respectively. When the two impedances are connected in series across the same supply, find total power consumed and overall power factor. [KTU DEC 2019]

Solution: The statement of the problem shows that Z_1 is inductive while Z_2 is capacitive.

Impedance Z_1

We have $V_1 = 220V; P_1 = 300W; \cos\phi = 0.4$(lagging)

$P_1 = V_1 I_1\cos\phi_1$

$I_1 = \frac{P_1}{V_1\cos\phi_1} = \frac{300}{220\times0.4} = 3.409\,A$

$P_1 = I_1^2 R_1$ and $R_1 = \frac{P_1}{I_1^2} = \frac{300}{3.40^2} = 25.81\Omega$

$Z_1 = \frac{V_1}{I_1} = \frac{220}{3.40} = 64.53\Omega$

Inductive reactance $X_L = \sqrt{Z_1^2 - R_1^2} = \sqrt{64.53^2 - 25.81^2} = 59.14\Omega$

Impedance Z_2

We have $V_2 = 220V; P_1 = 150W; \cos\phi = 0.7$ (leading)

$P_2 = V_2 I_2 \cos\phi_2$

$I_2 = \frac{P_2}{V_2 \cos\phi_2} = \frac{150}{220 \times 0.7} = 0.974\,A$

$P_2 = I_2^2 R_2$ and $R_2 = \frac{P_2}{I_2^2} = \frac{150}{0.974^2} = 158.12\Omega$

$Z_2 = \frac{V_2}{I_2} = \frac{220}{0.974} = 225.87\Omega$

Capacitive reactance $X_C = \sqrt{Z_2^2 - R_2^2} = \sqrt{225.87^2 - 158.12^2} = 161.29\Omega$

When Impedance Z_1 and Impedance Z_2 connected in series

Total resistance $R = R_1 + R_2 = 25.81 + 158.12 = 184.02\Omega$

Total reactance $X = X_C - X_L = 161.29 - 59.14 = 102.15\Omega$

Total impedance $Z = \sqrt{R^2 + X^2} = \sqrt{184.02^2 + 102.15^2} = 210.45\Omega$

Total current $I = \frac{V}{Z} = \frac{220}{210.43} = 1.045\,A$

Total power consumed $P = I^2 R = (1.045)^2 \times 184.02 = 200.95 \approx 201\,W$

Over all power factor $\cos\phi = \frac{R}{Z} = \frac{184.02}{210.43} = 0.874$ (leading)

9. A series RC circuit takes a power of 7000W when connected to 200V, 50Hz supply. The voltage across the resistor is 130V. Calculate (1) Current) (2) Resistance (3) Impedance (4) Capacitance (5) Power factor (6) Equation for instantaneous values of voltage and current. [KTU JUNE 2016]

Solution: Given- Voltage across the resistor, $V_R = IR = 130V$ and Power, $I^2R = 7000W$

1) Current $I = \frac{P}{V_R} = \frac{7000}{130} = 53.84\,A$

2) Resistance, $R = \frac{V_R}{I} = \frac{130}{53.84} = 2.41\Omega$

3) Impedance $Z = \frac{V}{I} = \frac{200}{53.84} = 3.71\ \Omega$

4) Capacitance :

Voltage across capacitor $V_C = \sqrt{V^2 - V_R^2} = \sqrt{200^2 - 130^2} = 151.98\ V$

Capacitive reactance $X_C = \frac{V_C}{I} = \frac{151.98}{53.98} = 2.81\ \Omega$

Capacitance $C = \frac{1}{2\pi f X_C} = \frac{1}{2 \times \pi \times 50 \times 2.81} = 1.133 \times 10^{-3}\ F$

5) Power factor $\cos\phi = \frac{R}{Z} = \frac{2.414}{3.71} = 0.65$

6) Equation for voltage and current

We have $\cos\phi = 0.65$ then $\phi = \cos^{-1}(0.65) = 49.46^0$

$v = V_m \sin\omega t = 200 \times \sqrt{2}\ \sin 314t = 282.84 \sin 314t$

$i = I_m \sin(\omega t + \phi) = 53.84 \times \sqrt{2} \sin(314t + 49.46) = 76.14 \sin(314t + 49.46)$

10. A 230V, 50Hz AC supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8μF capacitor. Calculate (1) Impedance (2) Current (3) Phase angel between current and voltage (4) Power factor and (5) Power consumed.

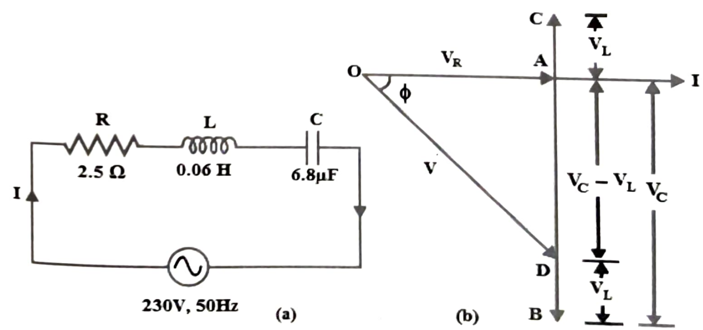

Figure 25

Solution: $X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.06 = 18.85\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 6.8 \times 10^{-6}} = 468\ \Omega$

1) Circuit impedance $Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{2.5^2+(18.85-468)^2}=449.2\Omega$

2) Circuit current $I=\frac{V}{Z}=\frac{230}{449.2}=0.512A$

3) Phase angel between current and voltage

$$\tan\phi=\left(\frac{X_L-X_C}{R}\right)$$

$$\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)=\tan^{-1}\left(\frac{18.85-468}{2.5}\right)=\tan^{-1}(-179.66)=-89.7^0=89.7^0\text{lead}$$

4) Power factor $\cos\phi=\frac{R}{Z}=\frac{2.5}{449.2}=0.00557\,\text{lead}$

5) Total power consumed $P=VI\cos\phi=230\times0.512\times0.00557=0.656W$

11. An alternating voltage $(80+j60)V$ is applied to a circuit and the current flowing is $(-4+j10)A$ Find (a) impedance of the circuit (b) power consumed and (c) phase angle. **[KTU JUNE 2017, KTU DEC 2019]**

Solution: Given $V=80+j60$ which is in rectangular form

When we converted this rectangular form in to polar form we get

$V=80+j60=100\angle 36.86^0$

Similarly $I=-4+j10=10.77\angle 111.80^0\,A$

$$Z=\frac{V}{I}=\frac{100\angle 36.87}{10.77\angle 111.80}=9.28\angle-74.93$$

Rectangular to polar conversion

$V=\sqrt{80^2+60^2}=100V$

$\theta=\tan^{-1}\left(\frac{60}{80}\right)=36.86^0$

Convert the polar form of impedance to rectangular form we get $Z=(2.413-j8.96)\Omega$

Polar to rectangular conversion

$A\cos\theta+jA\sin\theta=9.285\cos(-74.93)+j\,9.285\sin(-74.93)$

$=2.414-j8.966$

The obtained impedance is in the form of $R-jX$ and can observe that the impedance is capacitive in nature. It is an RC series circuit where $R=2.414\Omega$ and $X_C=8.966\Omega$

$$Z=\sqrt{R^2+X_C^2}=\sqrt{2.413^2+(-8.96)^2}=9.28\Omega$$

(b) Power consumed $P=VI\cos\phi$

$$\cos\phi=\frac{R}{Z}=\frac{2.413}{9.28}=0.26$$

$P=VI\cos\phi=100\times10.77\times0.26=280.02W$

(c) Phase angle $=\phi=\cos^{-1}(0.26)=74.92\approx 75$