Module-2

Regular - Expression

is a declarative way to Express the regular language.

Primitive regular Expression

Regular Expression	Language
i) ϕ	$L(\phi) = \{\}$
ii) ϵ	$L(\epsilon) = \{\epsilon\}$
iii) a	$L(a) = \{a\}$

Basic operations on Regular Expression.

If r_1 and r_2 are regular Expression

i) Union :- $M_1 + M_2$

$L(M_1) \cup L(M_2)$

ii) concatenation :- $M_1 M_2$

$L(M_1) L(M_2)$

eg:- $L(M_1) = \{a, b\}$

$L(M_2) = \{c, d\}$

$L(M_1) L(M_2) = \{ac, ad, bc, bd$

iii) closure M_1^* $\Rightarrow L(M_1^*) = (L(M_1))^*$

eg:- $L(M_1) = \{a, b\}$

$L(M_1)^* = (L(M_1))^* = \{a, b$

$= \{\epsilon, a, b, aa, bb, aab, abb$

Identities

- $\phi + R = R + \phi = R$
- $\phi R = R\phi = R.$
- $\epsilon R = R\epsilon = R.$
- $R + S = S + R.$
- $R(St) = (RS)t$
- $R(S+t) = (RS)t$
- $R(S+t) = RS + Rt$
- $(S+t)R = SR + tr.$
- $R + R = R$
- $R^*R^* = R^*$
- $(RS)^*R = R(SR)^*$

eg:- $(ab^*)a = \{a, aba, ababa, \ldots\}$

$= a(ba)^*$

$(R+S0)$

- $(R+S)^* = (R^* + S^*)^* = (R^*S^*)^*$

$L(R) = \{a\}$

$L(S) = \{b\}$

$L(R+S) = \{a, b\}$

$L(R+S)^* = \{\epsilon, aa, bb, aabb \ldots\}$

$= \{\epsilon, a, aa, aaa, \ldots\}$

$= \{\epsilon, b, bb, bbb, \ldots\}$

• $\epsilon + xx^{+} = x^{*}$

RE	Language
$a+b$	$\{a, b\}$
$(a+b)(a+b)$	$\{aa, ab, ba, bb\}$
a^{*}	$\{\epsilon, a, aa, aaa \ldots\}$
$(a+b)^{*}$	$\{\epsilon, a, b, aa, bb, aabb, \ldots\}$
$(a^{*}b^{*})^{*}$	$\{\epsilon, a, b, aa, bb, aabb \ldots\}$
$a+a^{*}b$	$\{a, b, ab, aab \ldots\}$

Write regular expression for the following languages

a.) set of all strings of 0 & and 1 begining and ending with 1.

$1(0+1)^*1$

a.) set of all strings of a & b containing any no. of a followed by any no. of b

$a^* b^*$

a) any combination of a or any combination of b. followed by any combination of c.

$(a^*+b^*)c^*$ or $a^* c^* + b^* c^*$

Q) Set of all strings of 0 & 1 with even no. of 0's followed by odd no of 1's.

$$(00)^* (11)^* 1$$

Q) any combination of a & b begining with a and ending with b

Q) 2) any combination of a & b containing abb as a substrain

Q) 3) combination of a & b containing atleast 2a's.

Q) 4) $L = \{a^n b^m \mid m+n \text{ is even}\}$

Note:- any

Hints :- • any no of a

a^*

• any no of a&b

$(a+b)^*$

• Even no. of a

$(aa)^*$

$\{\epsilon, aa, aaaa \ldots\}$

• odd no. of a

$(aa)^* a$

Q) Combination of a & b having Exactly 1a.

ans- 1) $a(a+b)^*b$

ans:- 2) $(a+b)^* abb (a+b)^*$

ans:- 3) $(a+b)^* a(a+b)^* a(a$

ans:- 4) $H1 = (aa)^*(b)^*$ $H2 = (aa)^* a (bb)^* b$; $H = (aa)^*(bb)^* + (aa)^* a (bb)^* b$

$= (aa)^* [(bb)^* + a(bb)^* b]$.

ans 5:-) $b^* a b^*$

Q) Using identities of regular Expression prove the following

$$(1+100)^* + (1+100)^*(0+10)^*(0+10^*)^*$$

$$= 10^*(0+10)^*$$

$$\text{LHS} = (1+100)^* + (1+100)^*(0+10)^*(0+10^*)^*$$

$$= (1+100)^* [\epsilon + \underbrace{(0+10^*)}_{x}\underbrace{(0+10^*)^*}]$$

$[\epsilon + x^*]$

$= (1+100^*)(0+10^*)^*$

$= 1(\epsilon+00^*)(0+10^*)^*$

$= 10^*(0+10^*)^*$

Q) $\epsilon + 1^*(011)^*[1^*(011)^*]^* = (1+011)^*$

~~LHS =~~

$LHS = \epsilon + 1^*(011)^*(1^*(011)^*)^* =$

$= \epsilon + 1^*(011)^*($

$LHS = \epsilon + 1^*(011)^*[1^*(011)^*]^*$

$= \epsilon + \underline{1^*(011)^*}\ \underline{(1^*+011)^*}$ $\qquad \epsilon + xx^* = x^*$

$\epsilon + (1^*(011)^*)^*$

$= (1+011)^*$

Prove that

$$10 + (1010)^* [\epsilon + \epsilon (1010)^*] = 10 + (1010)^*$$

$$LHS = 10 + (1010)^* (\epsilon + \epsilon (1010)^*]$$

$$10 + [\epsilon (1010)^* + 1010^* \epsilon (1010)^*]$$

$$10 + [(1010)^* + \epsilon (1010)^*]$$

$$10 + [(1010)^* + (1010)^*]$$

$$= 10 + (1010)^*$$

Equivalence of regular expression and finite automata

Regular expression and finite automata are equivalent

Kleene's construction theorem

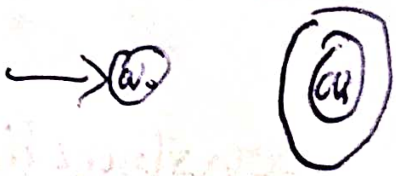
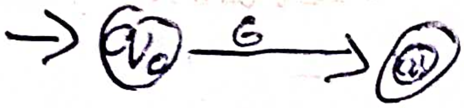
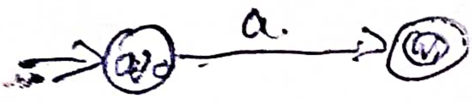
Every language defined by regular a finite automation.

corresponding to a regular Expression r their exist a finite automata M such that

$$L(r) = L(M)$$

__Basis__

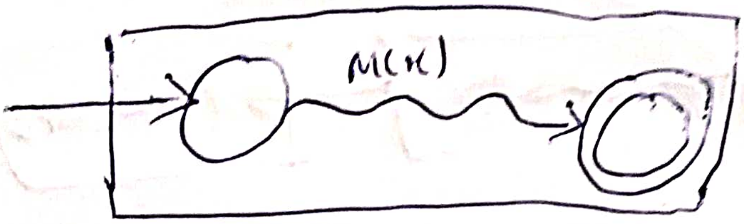
for the primitive regular Expression we can construct finite automata as follows

r	L(r)	M
ϕ	$\{\ \}$	
ϵ	$\{\epsilon\}$	
a	$\{a\}$	

If M is an NFA accepting the language

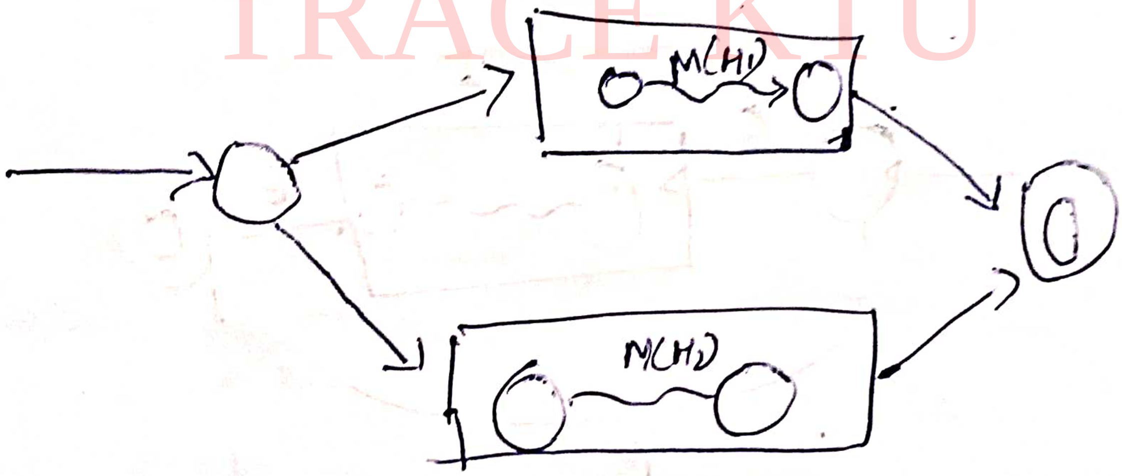
Let L(M)

<u>M</u>



then we can have for union

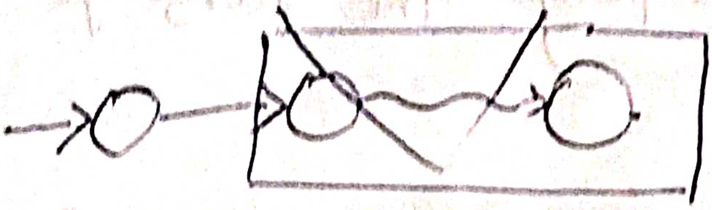
$$L(M_1+M_2) = L(M_1) \cup L(M_2)$$

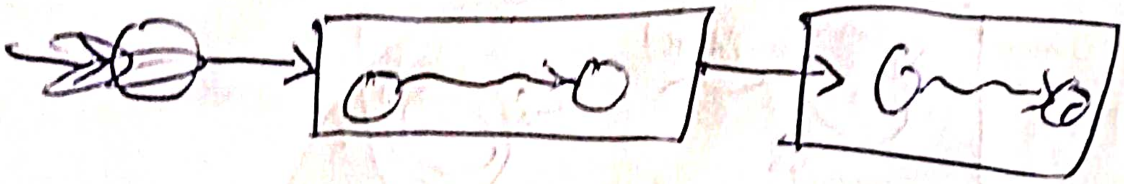


for contamination

$$L(M_1 M_2) = L(M_1) L(M_2)$$

$$M(M_1, M_2)$$

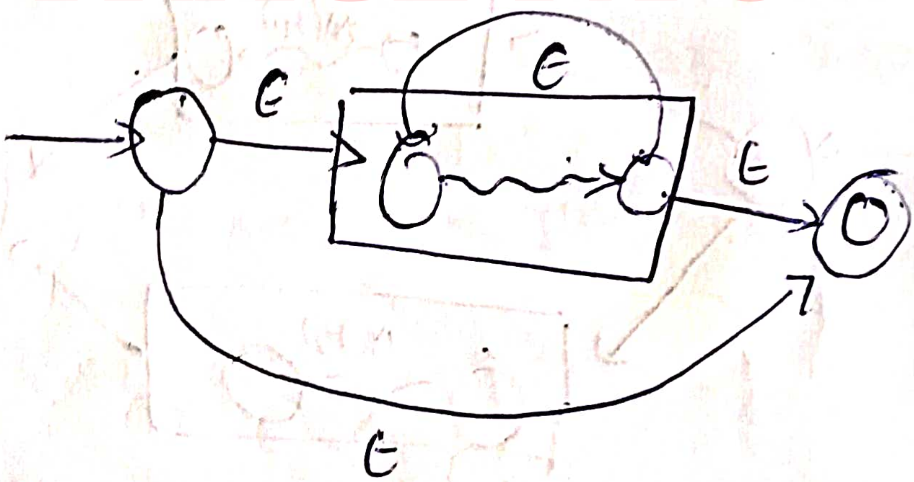




Closure

$L(M^*) = (L(M))^*$

$M(M^*)$

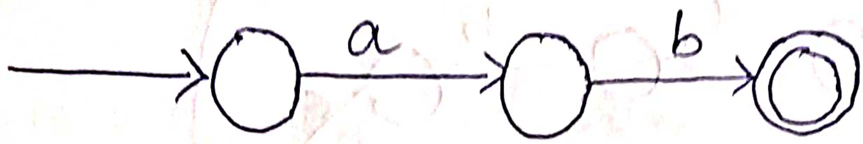


Hence for Every regular Expression M their Exists a finite automata

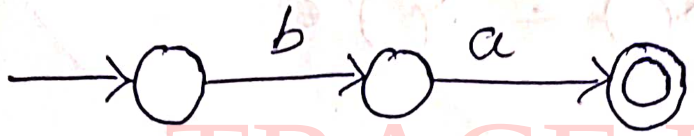
Q) Construct NFA for given regular Expression.

$(ab+ba)^* ab$

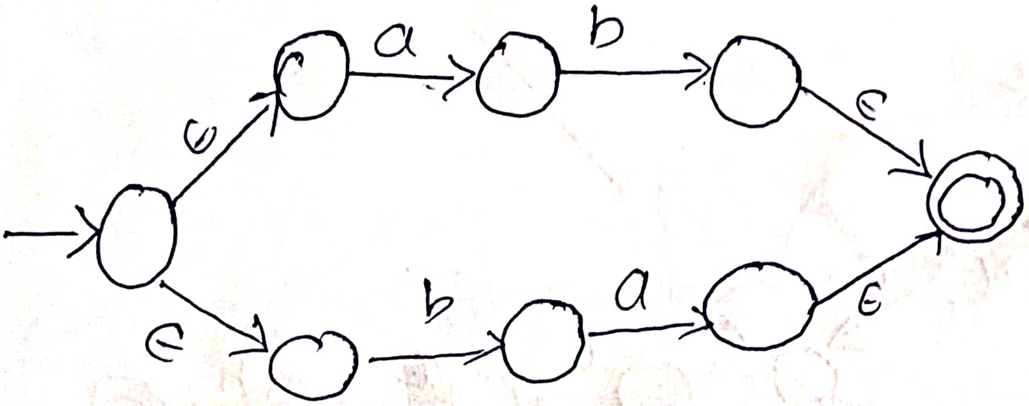
Step 1:- ab



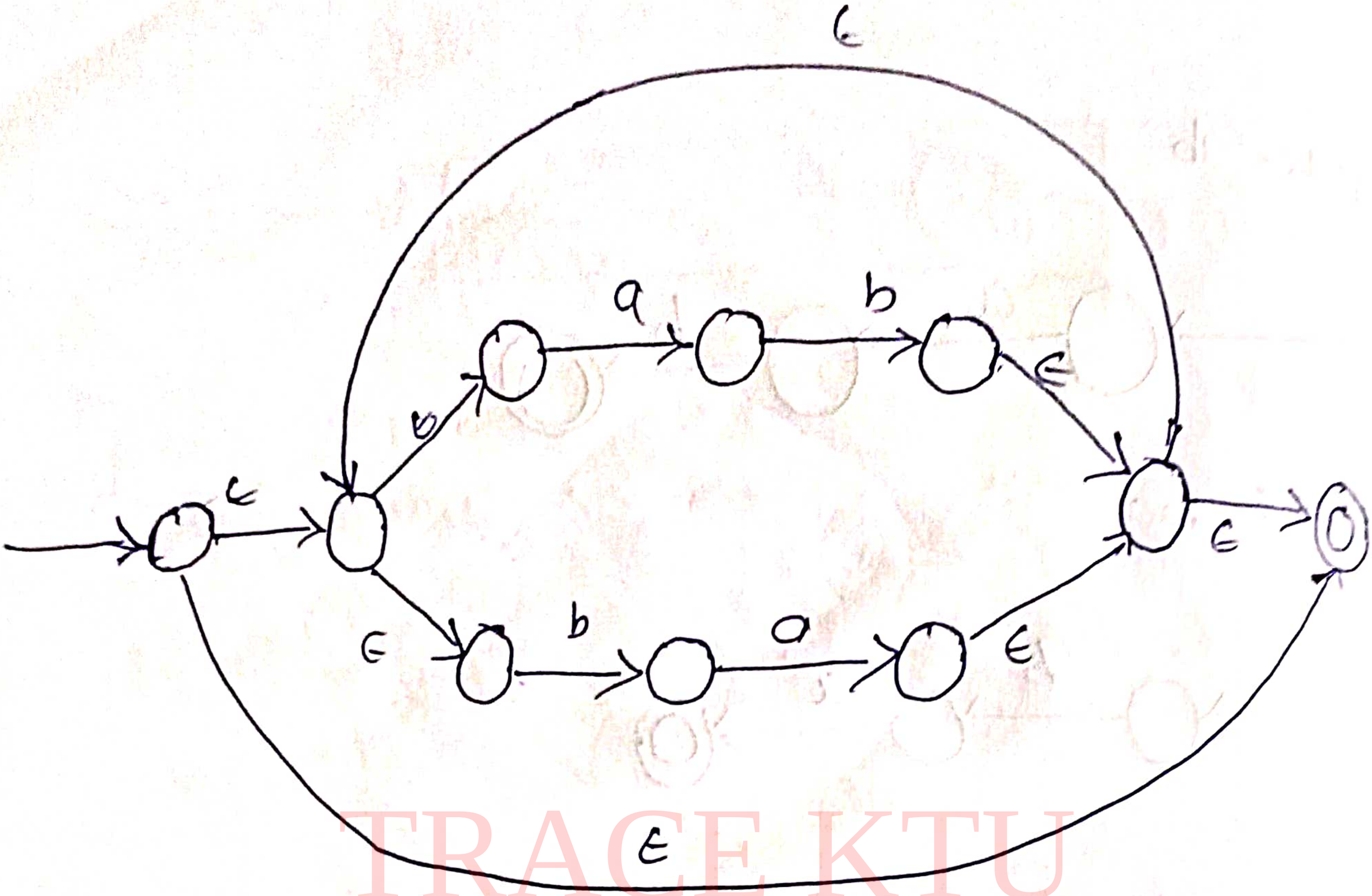
Step 2:- ba



Step 3:- ab + ba

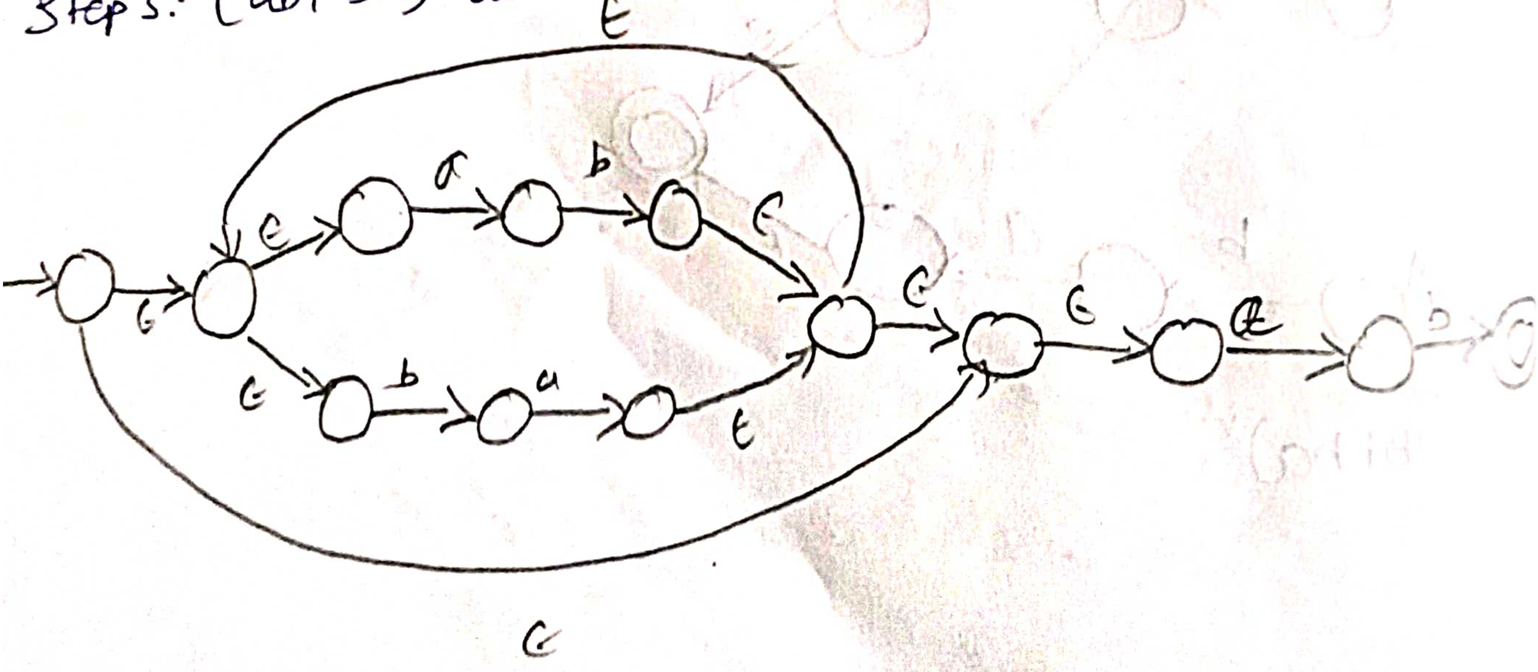


Step 4: $(ab+ba)^*$



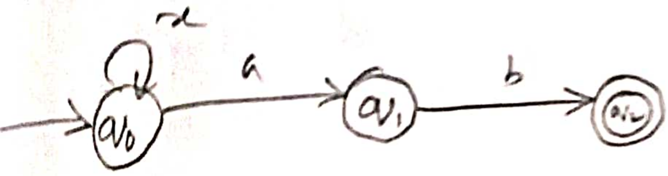
~~(ab+ba)~~

Step 5: $(ab+ba)^*ab$

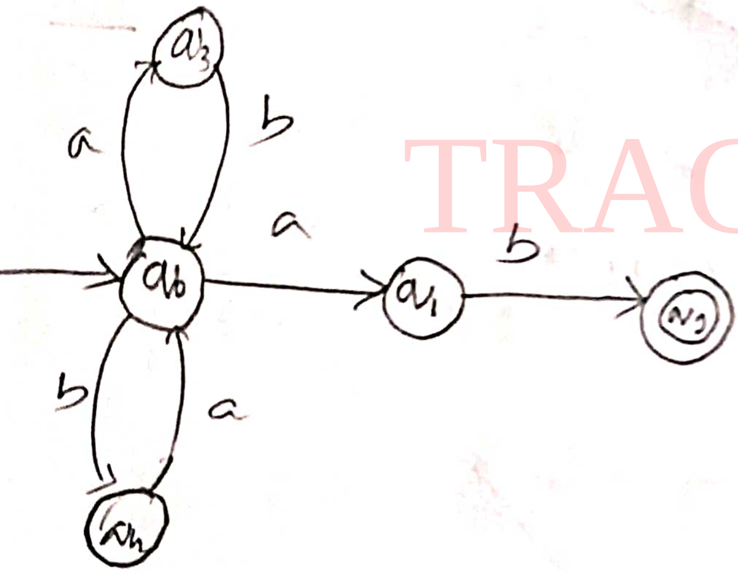


Step 1:- Let $ab+ba = x$

$RE = x^* ab$



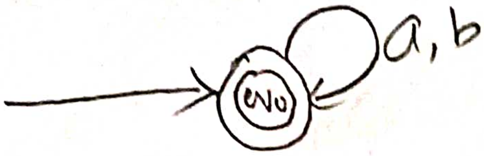
Step 2 :- Replace x with $ab+b$.



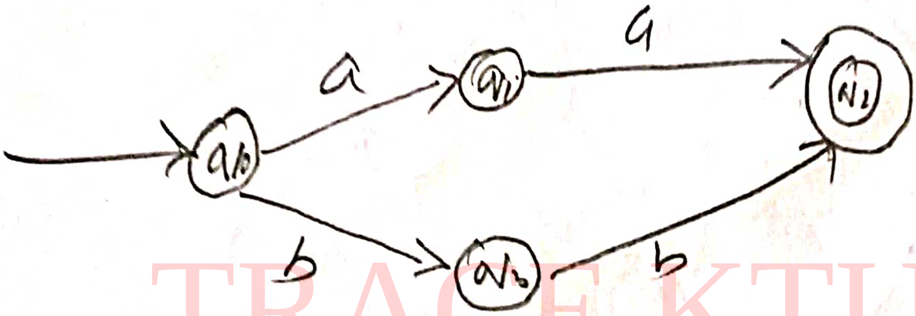
Construct finite automata for the following regular Expression.

$(a+b)^* (aa+bb) (a+b)^*$

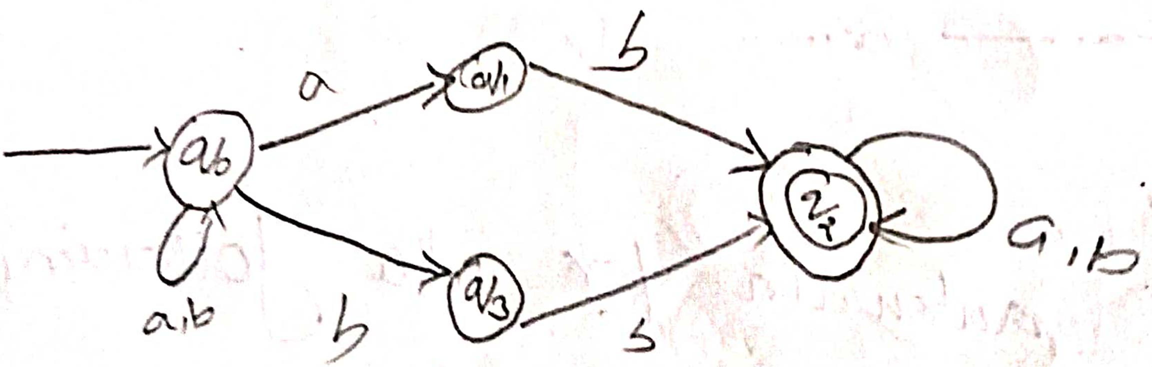
Step 1:- $(a+b)^*$



Step 2:- $aa+bb$



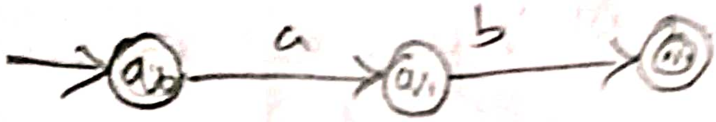
Step 3 :- $(a+b)^*(aa+bb)(a+b)^*$



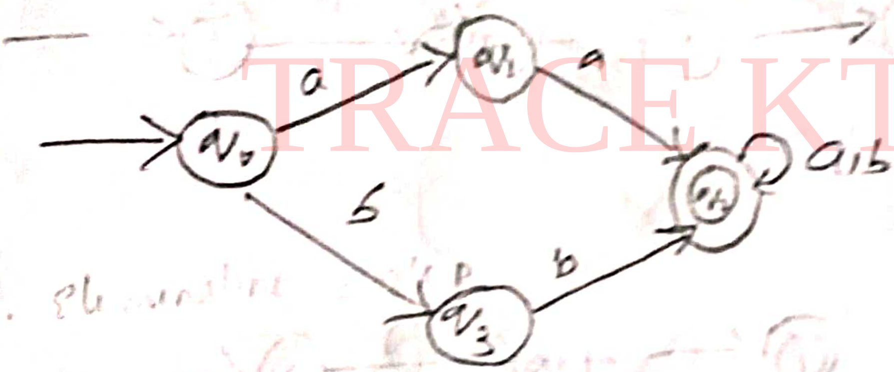
Q) construct finite automata for the regular expression $ab+(aa+bb)(a+b)^*$

2) Construct finite automata for the regular expression $((\epsilon+a)b^*)^*$

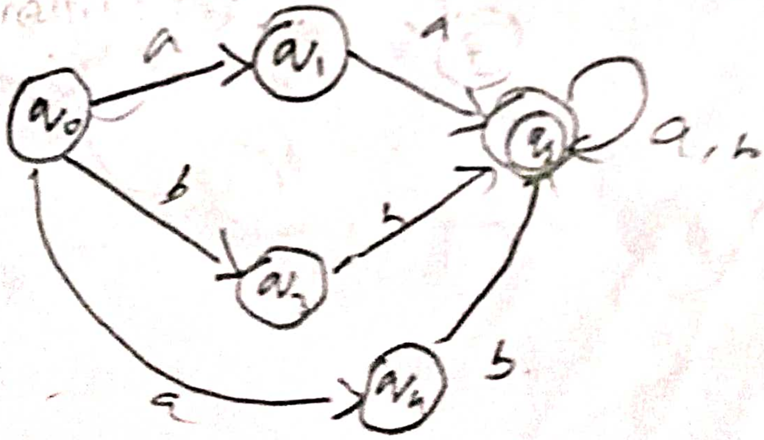
Ans: 1)



2)



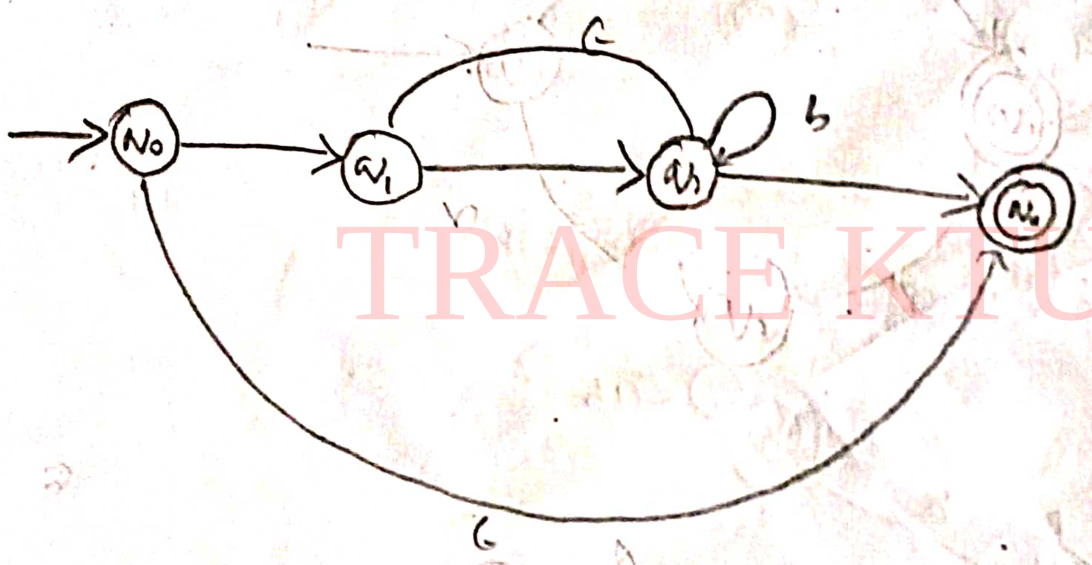
3) $ab+(aa+bb)(a+b)^*$



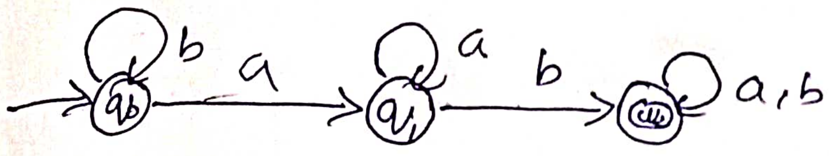
Hw

2) $((\epsilon + a))b^*)^*$

Ans.



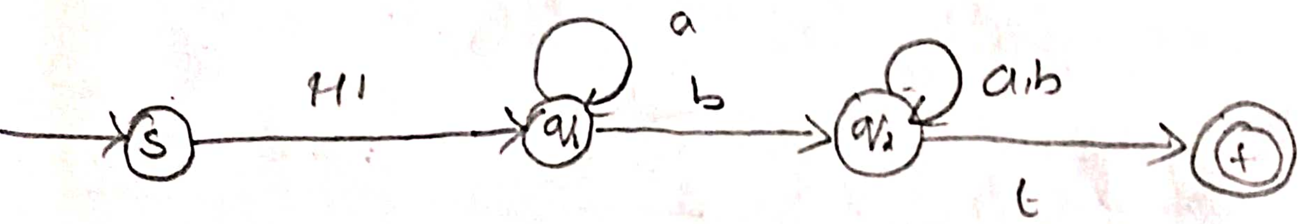
Find regular Expression of the following DFA



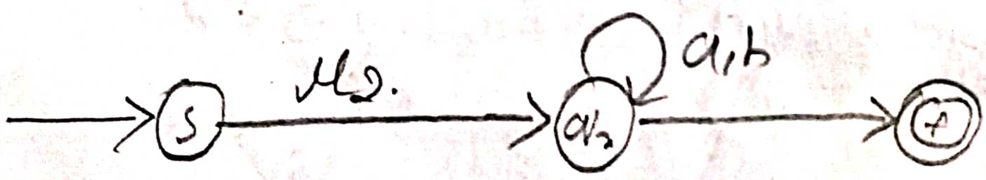
State Elimination method

Step 1:- adding two new states S & F

Step 2:- Eliminating State q_0



Step 3:- eliminating state q_1



$r_1 = r_1 a^* b$

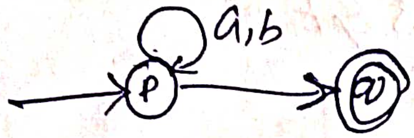
$= b^* a a^* b$

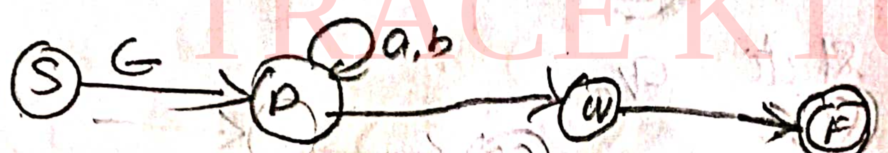
Eliminating State q_2

$$M_3 = M_2(a+b)^*\epsilon$$

$$= b^*aa^*b(a+b)^*$$

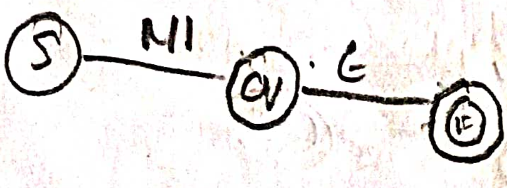
Hw.



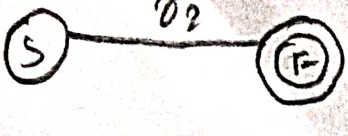


) remove p

$$M_1 = (a+b)^* b$$



3) remove q



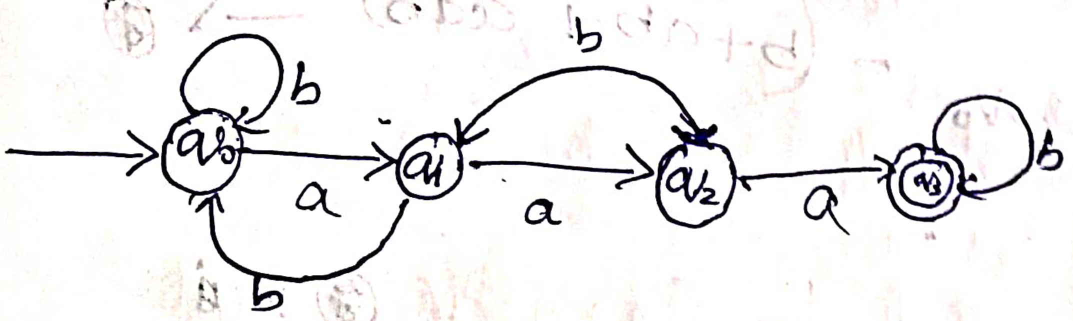
$r_2 = r_1 G$

$= (a+b)^* b$

Q) <u>Arden's Rule</u> :-

Let p & q are two regular Expression over Σ then the R.Equation $R = Q + RP$ has a unique Solution given by $R = QP^*$

find Regular Expression for the following DFA



$q_0 = \epsilon + q_0 b + q_1 b + q_2 b \rightarrow ①$

$q_1 = q_0 a \rightarrow ②$

$q_2 = q_1 a \rightarrow ③$

$q_3 = q_2 a + q_3 b$

Applying Ardens rule in eq Eq ④

$q_3 = q_2 ab^* \longrightarrow$ ⑤

$= q_1 aab^*$ (2) in (5)

$= q_0 aab^* \longrightarrow$ ⑥

$q_0 = \epsilon + q_0 b + q_1 b + q_2 b$; (3) in ①

$= \epsilon + q_0 b + q_0 ab + q_0 aab$

$q_0 = \epsilon + q_0(b + ab + aab) \longrightarrow$ ⑦

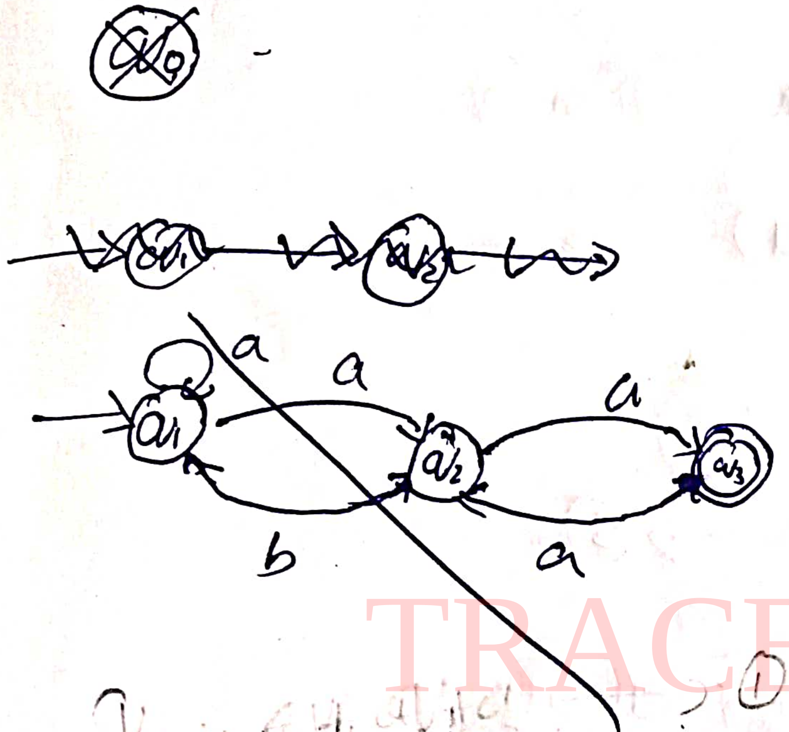
$= \epsilon (b + ab + aab)^*$ (Arden's rule)

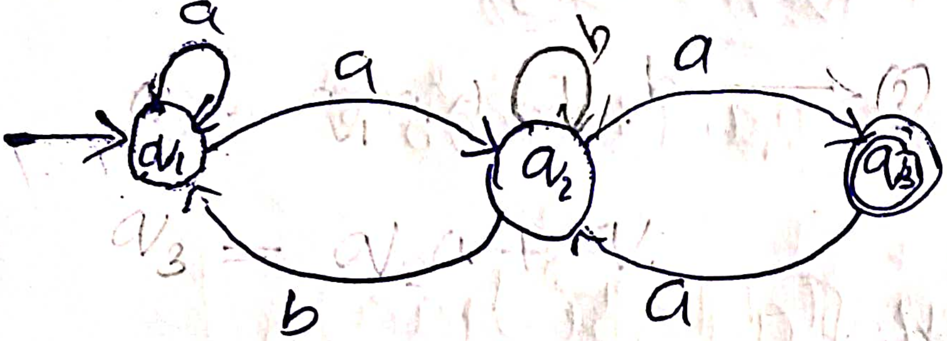
$q_0 = (b + ab + aab)^* \longrightarrow$ ⑧

⑧ in 6

$q = (b + ab + aab)^* aab^*$

find regular expression for the given automata





$q_1 = \epsilon + q_1 a + q_2 b \longrightarrow (1)$

$q_2 = q_1 a + q_2 b + q_3 a \longrightarrow (2)$

$q_3 = q_2 a \longrightarrow (3)$

eq ③

$q_3 = q_2 a$

$q_2 = q_1 a + q_2 b + q_2 aa$ ③ in ②

$q_2 = q_1 a + q_2 (b + aa) \longrightarrow 4$

applying Arden's rule

$q_2 = q_1 a (b+aa)^* \longrightarrow ⑤$

$q_1 = \epsilon + q_1 a + q_2 b \longrightarrow ⑥$

$q_1 = \epsilon + q_1 a + q_1 a (b+aa)^* b$ ⑤ in ⑥

$= \epsilon + q_1 [a + ab (b+aa)^* b]$

Arden's rule

$q_1 = \epsilon [q_1 [a + ab (b+aa)^* b]$

$q_1 = [a + ab (b + aa)^*]^* \longrightarrow ⑦$

$= (b+aa)^* \longrightarrow ⑧$

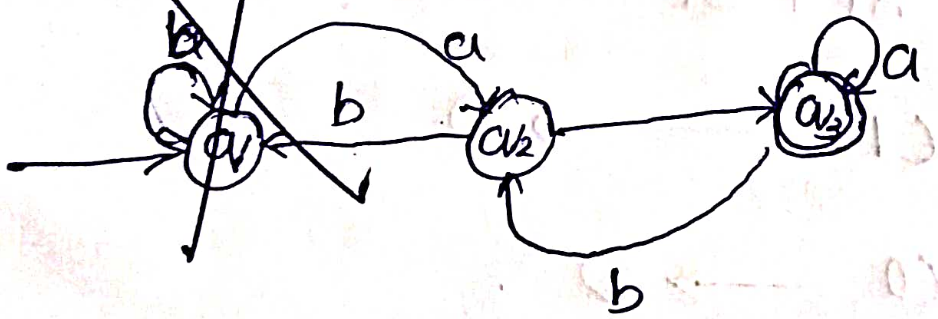
$$q_2 = [a + a(b+a)^* b]^* a [b+aa]^*$$ ⑦ in ⑤

$$q_3 = [a + a(b+a)^* b^*]^* a [b+aa]^* a$$

Q)

~~Let 0 = a~~

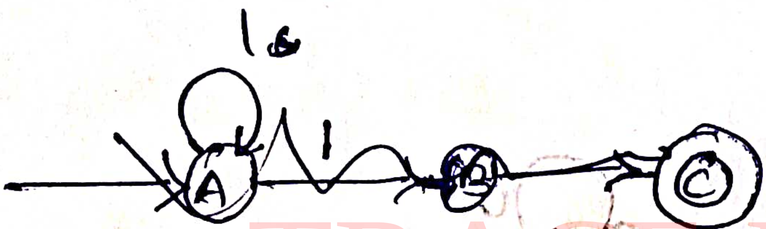
~~Let 1 = b.~~

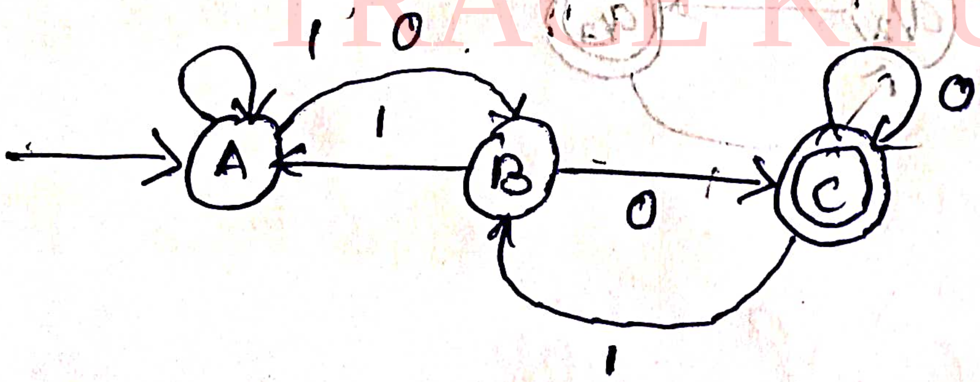


$q_1 = A$

$q_2 = B$

$q_3 = C$





$A = \epsilon + A1 + B1 \longrightarrow ①$

$B = A0 + C1 \longrightarrow ②$

$C = B0 + C0 \longrightarrow ③$

Applying argand rule

$C = B00^* \longrightarrow (4)$

$B = A0 + B00^*1 \longrightarrow (5)$ (4) in (2)

Applying argan's rule in (5)

$B = A0((00^*)1)^* \longrightarrow (6)$

$A = \epsilon + A1 + A0((00^*)1)^*$ (6) in (1)

$A = \epsilon + (A(1 + 0((00^*)1)^*)$

Applying argan's rule.

$= \epsilon(1 + 0((00^*)1)^*)$

$= (1 + 0((00^*)1)^*)^* \longrightarrow (7)$

$C = [1 + 0((00^*)1)^*]^* 0((00^*)1)^* \to (8)$ (7) in (6)

$= [1 + 0((00^*)1)^*]^* 0((00^*)1)^* 00^*$ (8) in (4)