# Module-5

## RG

$G = (V, T, P, S)$

$P: \alpha \rightarrow \beta$

$\alpha \in V$

$\beta = T \mid TV \mid VT \mid \epsilon$

## CFG

$G = (V, T, P, S)$

$P: \alpha \rightarrow \beta$

$\alpha \in V$

$\beta = (V \cup T)^*$

## CSG Type-1

context sensitive Grammer

$P: \alpha A \beta \rightarrow \alpha \gamma \beta$

$\alpha, \beta = (V \cup T)^*$

$A \in V$

$\gamma \in (V \cup T)^+$

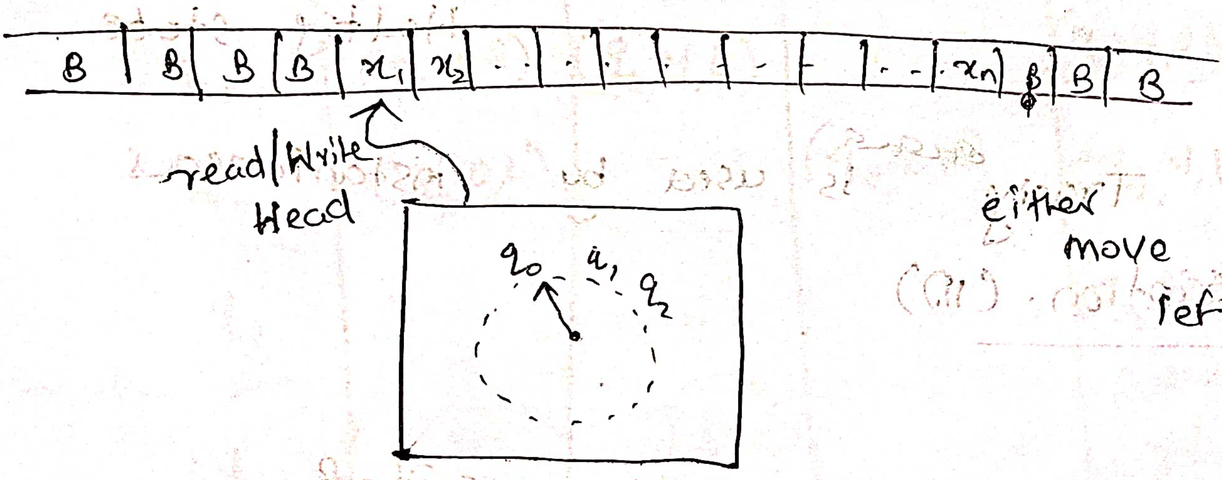
$|LHS| \leq |RHS|$

$S \rightarrow \epsilon$ is allowed

## Type (0) Turing Machine (TM)

★ Recursively Enumarable Language ★

Entire blank space is filled with Blank Symbol (B) in infinite tape.




either move left or right

## Transition

$\delta(q, a) = (q', x, \{L, R\})$

$$q \xrightarrow{a/x,\,D} q'$$

| States \ Tape symbols | $x_1$ | $x_2$ | ...... | B |
|---|---|---|---|---|
| $q_0$ | | | | |
| $q_1$ | | | | |
| ⋮ | | | | |

Turing machine (TM)

$$M = (Q, \Sigma, \tau, \delta, q_0, B, F)$$

$\tau$ → Tape symbols

$$\delta : Q \times \Sigma \rightarrow Q \times \tau \times \{L, R\}$$

F (Final state) ⇒ here ~~also~~ called Halting state

Here Tracing ~~area~~ is used by instantaneous discription. (ID)

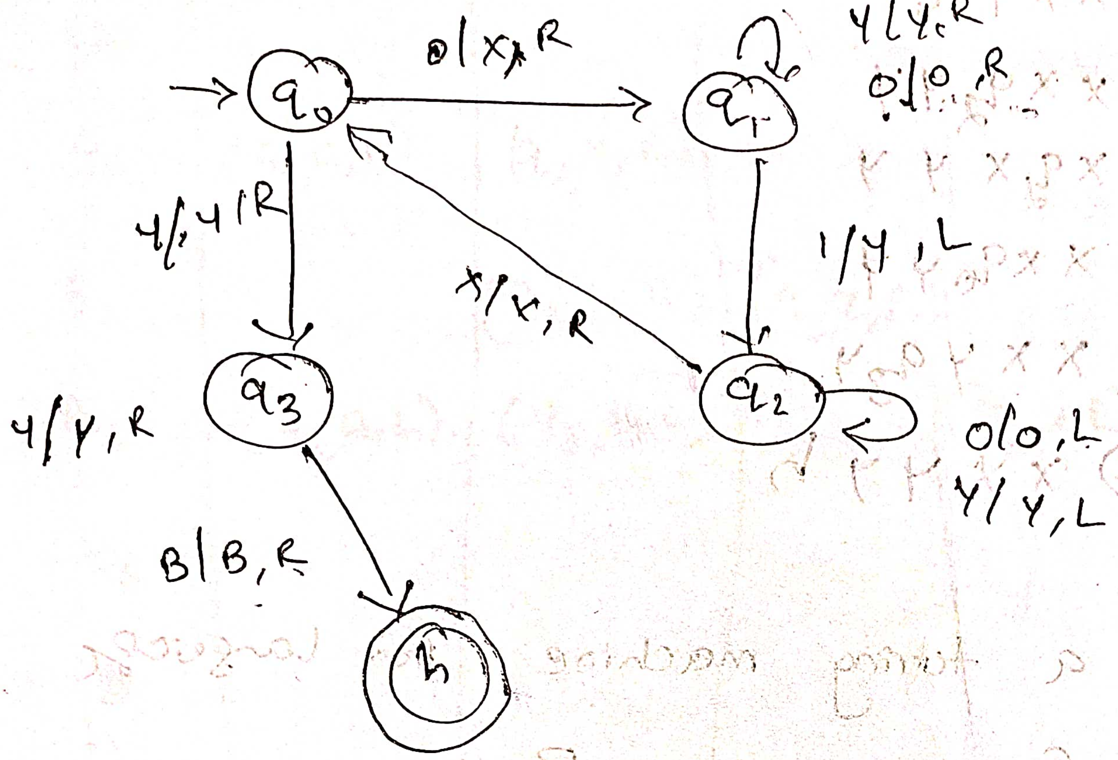
$ID_1$ : $B x_1 x_2 \ldots \ldots q x_i \ldots \ldots x_n B$

$$(q, x_i) = (q', y_i, L)$$

$ID_2$ : $B x_1 x_2 \ldots \ldots q' x_{i-1} y_i \ldots \ldots x_n B$

$$(q', x_{i-1}) = (q, x_{i-1}, R)$$

$ID_3 : B x_1 x_2 \ldots\ldots x_{i-1} q y_i \ldots x_n B$




| | 0 | 1 | X | Y | B |
|---|---|---|---|---|---|
| $q_0$ | $(q_1, X, R)$ | | | $(q_3, Y, R)$ | |
| $q_1$ | $(q_1, 0, R)$ | $(q_2, Y, L)$ | | $(q_1, Y, R)$ | |
| $q_2$ | $(q_2, 0, L)$ | | $(q_0, X, R)$ | $(q_2, Y, L)$ | |
| $q_3$ | | | | $(q_3, Y, R)$ | $(h, B, R)$ |
| $h$ | | | | | |

Trace '0011'

$q_0 0011$

$\Rightarrow X q_1 011$

$\Rightarrow X 0 q_1 11$

$\Rightarrow X q_2 0 Y 1$

$\Rightarrow q_2 X 0 Y 1$

$\Rightarrow X q_0 0 Y 1$

$\Rightarrow x x q_1 y 1$

$\Rightarrow x x y q_1 1$

$\Rightarrow x x q_2 y y$

$\Rightarrow x q_2 x y y$

$\Rightarrow x x q_0 y y$

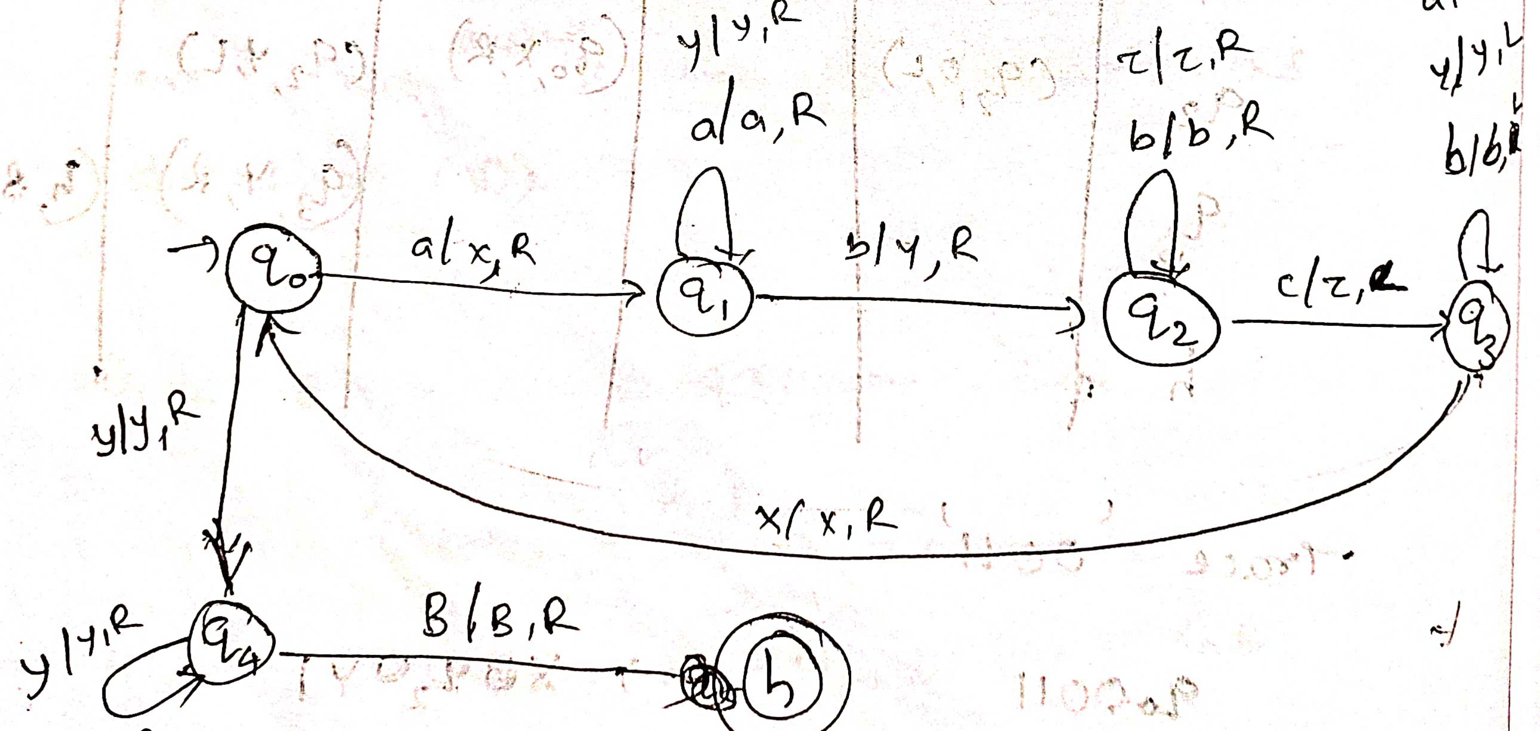
$\Rightarrow x x y q_3 y$

$\Rightarrow x x y y h$

Q. Design a turring machine for langurage

$L = \{a^n b^n c^n,\ n \geq 1\}$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| B | a | a | a | b | b | b | c | c | c | B |



| | a | b | c | X | Y |
|---|---|---|---|---|---|
| $q_0$ | $(q_1, X, R)$ | | | | $(q_4, Y, R)$ |
| $q_1$ | $(q_1, a, R)$ | $(q_2, Y, R)$ | | | $(q_1, Y, R)$ |
| $q_2$ | | $(q_2, b, R)$ | $(q_3, Z, L)$ | | |
| $q_3$ | $(q_3, a, L)$ | $(q_3, b, L)$ | | $(q_0, X, R)$ | $(q_3, Y, L)$ |
| $q_4$ | | | | | $(q_4, Y, R)$ |
| B | | | | | |
| h | | | | | |

| | Z | B |
|---|---|---|
| $q_0$ | | |
| $q_1$ | | |
| $q_2$ | $(q_2, Z, R)$ | |
| $q_3$ | $(q_3, Z, L)$ | |
| $q_4$ | $(q_4, Z, R)$ | $(h, B, R)$ |
| h | | |

$B\, q_0\, a a b b c c B$

$\Rightarrow B x\, q_1\, a b b c c B$

$\Rightarrow B x a\, q_1\, b b c c B$

$\Rightarrow B x a y\, q_2\, b c c B$

$\Rightarrow B x a y b\, q_2\, c c B$

$\Rightarrow B x a y b\, q_3\, z c B$

$\Rightarrow B x a y\, q_3\, b z c B$

$\Rightarrow B x a\, q_3\, y b z c B$

$\Rightarrow B x\, q_3\, a y b z c B$

$\Rightarrow B\, q_3\, x a y b z c B$

$\Rightarrow B x\, q_0\, a y b z c B$

$\Rightarrow B x x\, q_1\, y b z c B$

$\Rightarrow B x x y\, q_1\, b z c B$

$\Rightarrow B x x y y\, q_2\, z c B$

$\Rightarrow B x x y y z\, q_2\, c B$

$\Rightarrow B x x y y\, q_3\, z z B$

$\Rightarrow B x x y\, q_3\, y z z B$

$\Rightarrow B x x\, q_3\, y y z z B$

$\Rightarrow B x\, q_3\, x y y z z B$

$\Rightarrow B x x\, q_0\, y y z z B$

$\Rightarrow B x x y\, q_4\, y z z B$

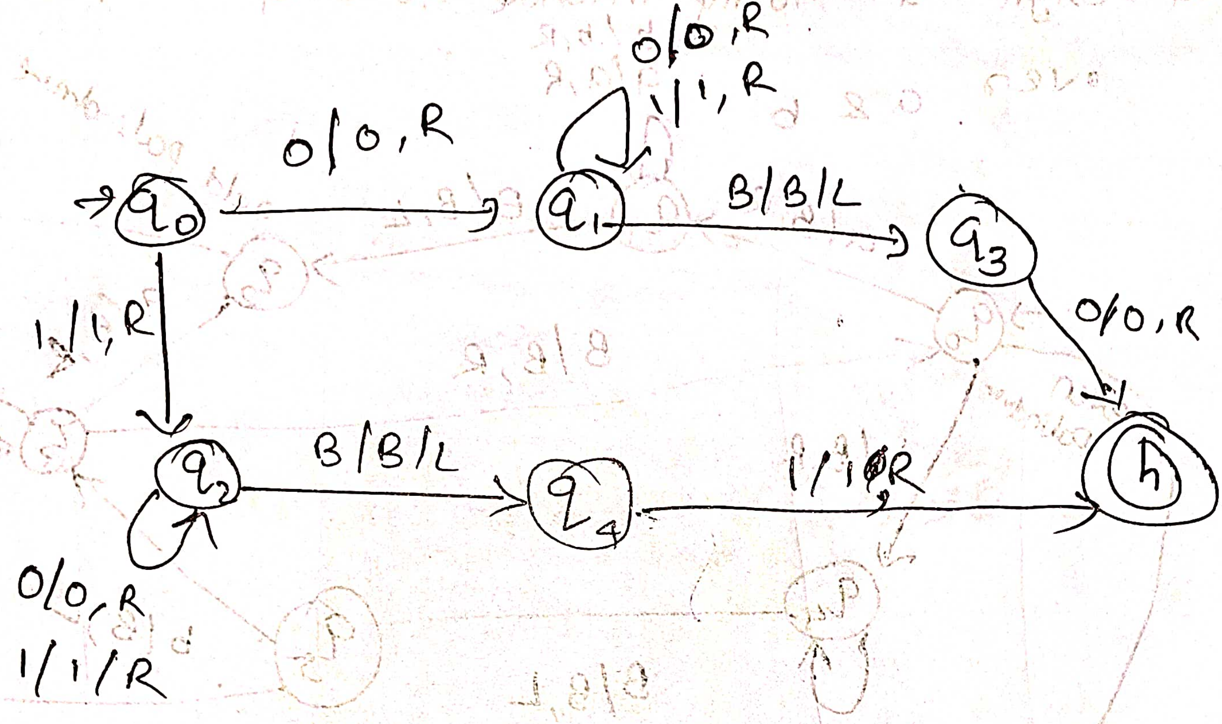
$\Rightarrow B x x y y\, q_4\, z z B$

$\Rightarrow B x x y y z\, q_4\, z B$

$\Rightarrow B x x y y z z\, q_4\, B$

$\Rightarrow B x x y y z z B h$

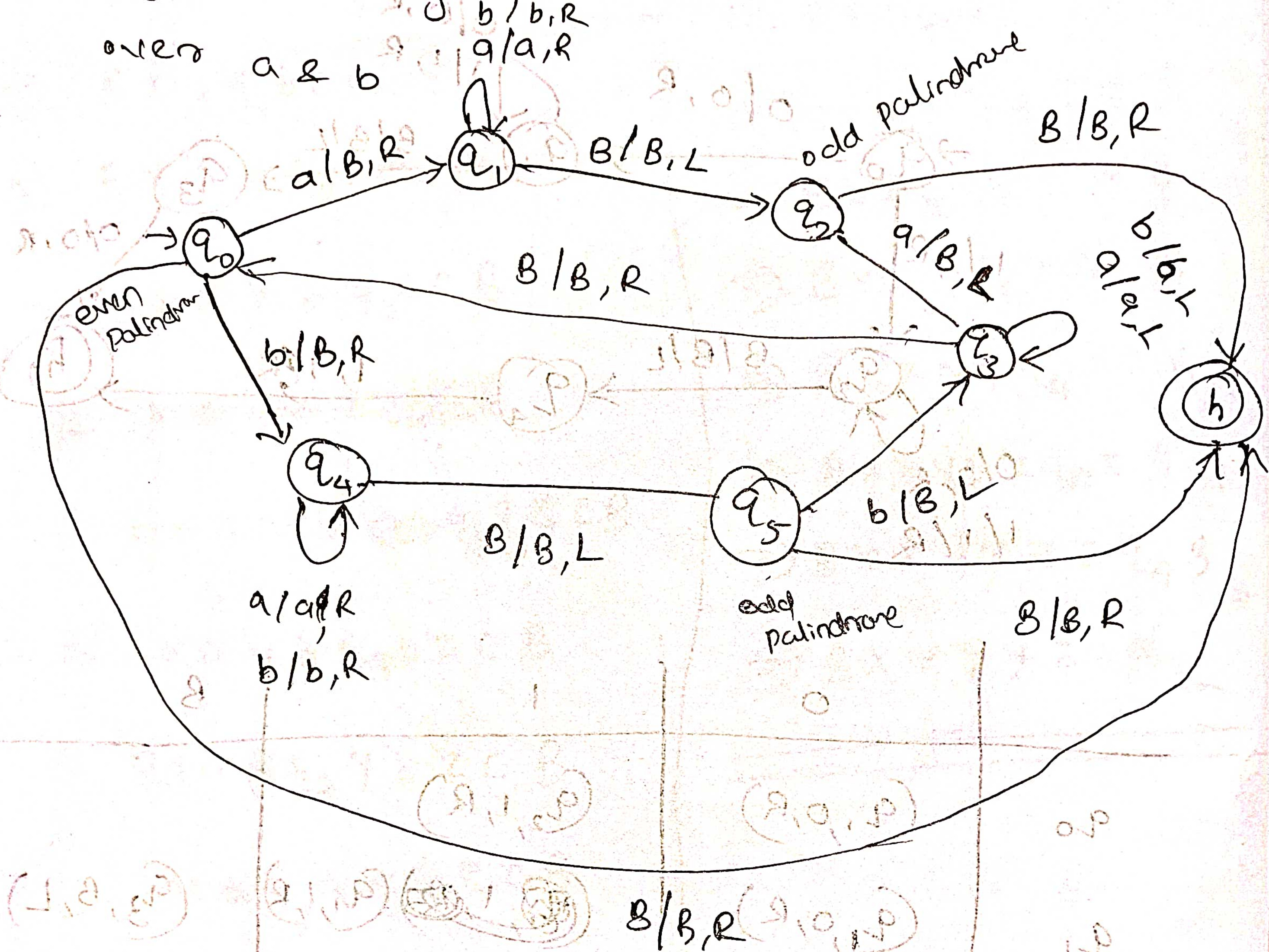
Q. Design a turning machine to accept set of all string with 0 and 1 that start and end with same number.




| | 0 | 1 | B |
|---|---|---|---|
| $q_0$ | $(q_1, 0, R)$ | $(q_2, 1, R)$ | |
| $q_1$ | $(q_1, 0, R)$ | $(q_1, 1, R)$ | $(q_3, B, L)$ |
| $q_2$ | $(q_2, 0, R)$ | $(q_2, 1, R)$ | $(q_4, B, L)$ |
| $q_3$ | $(h, 0, R)$ | | |
| $q_4$ | | $(h, 1, R)$ | |
| h | | | |

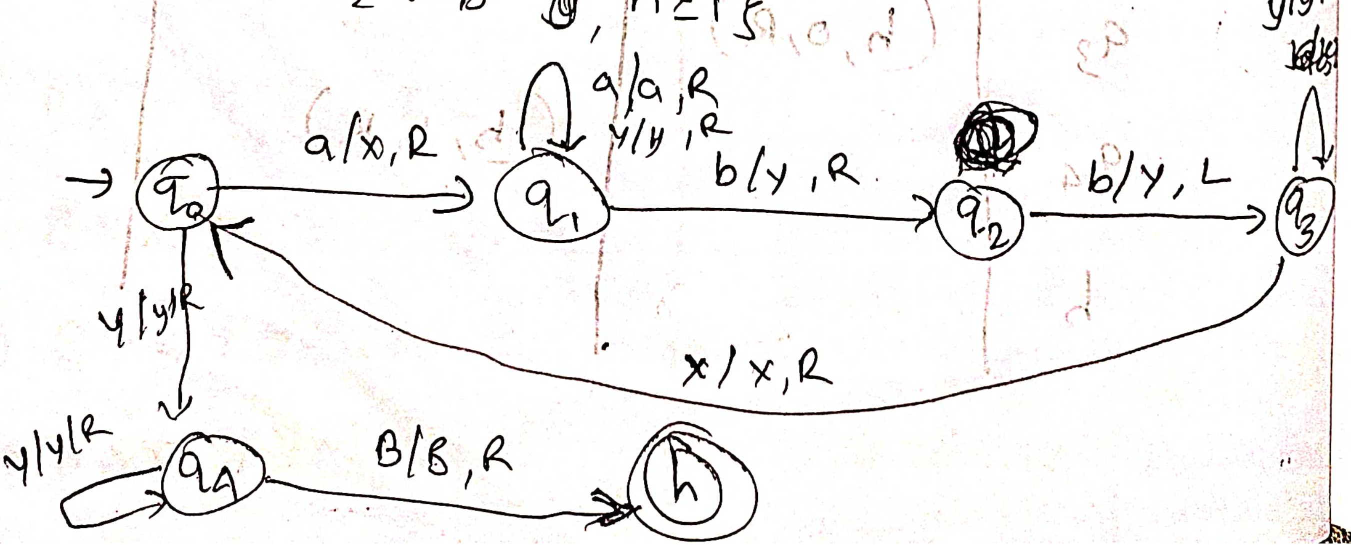
~~Q. Palindromes over a & b~~

Q Design a turing machine to accept palindromes over a & b



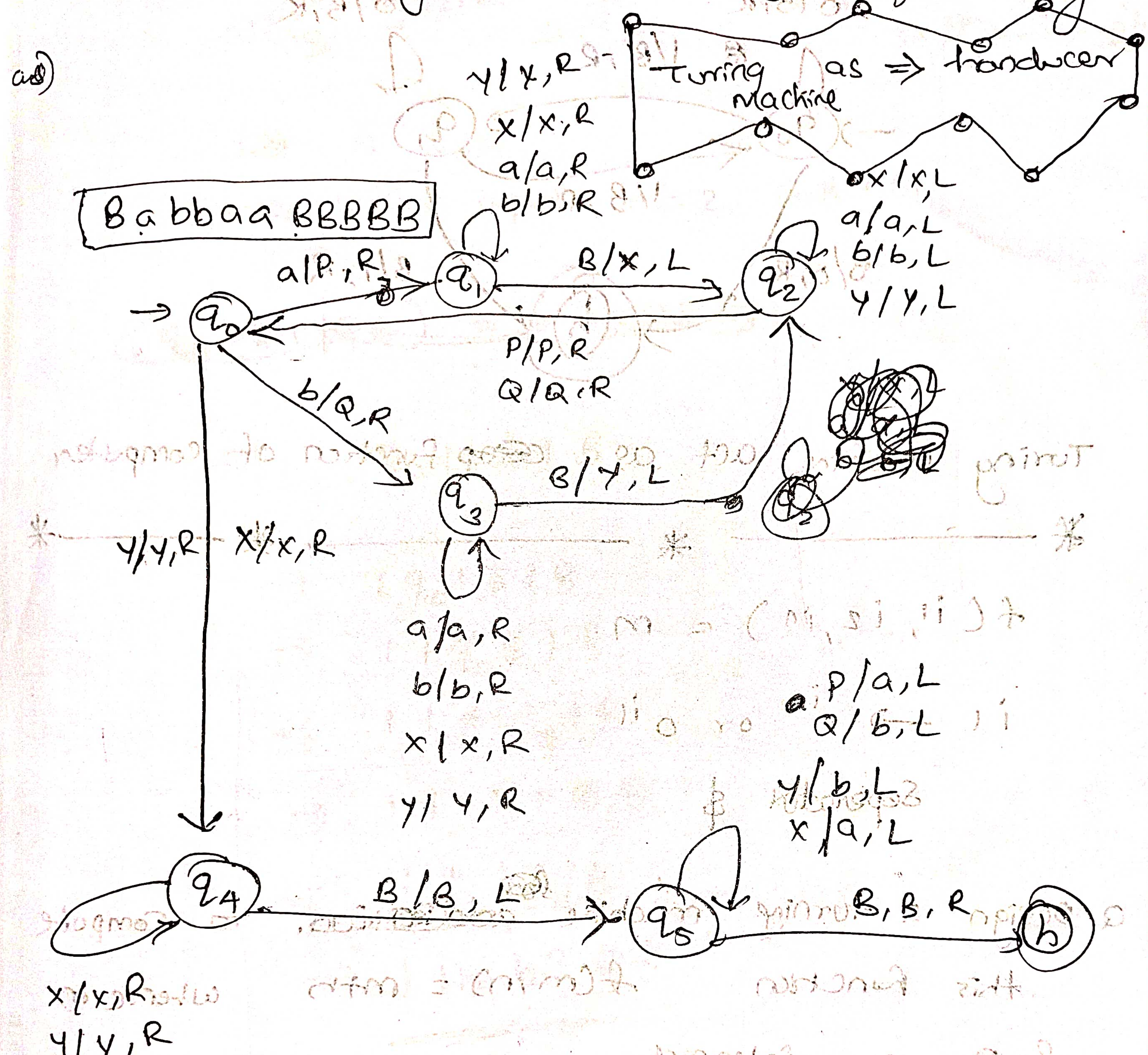

Q. Design a turing machine to accept ~~palindrome~~ ~~over~~

$L = \{a^n b^{2n}, n \geq 1\}$

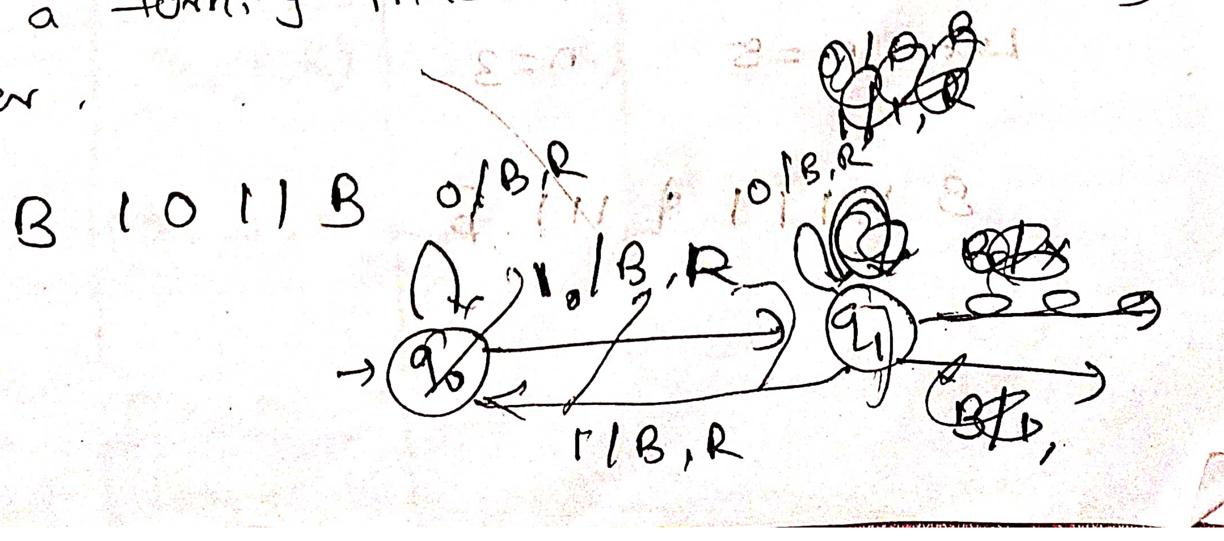


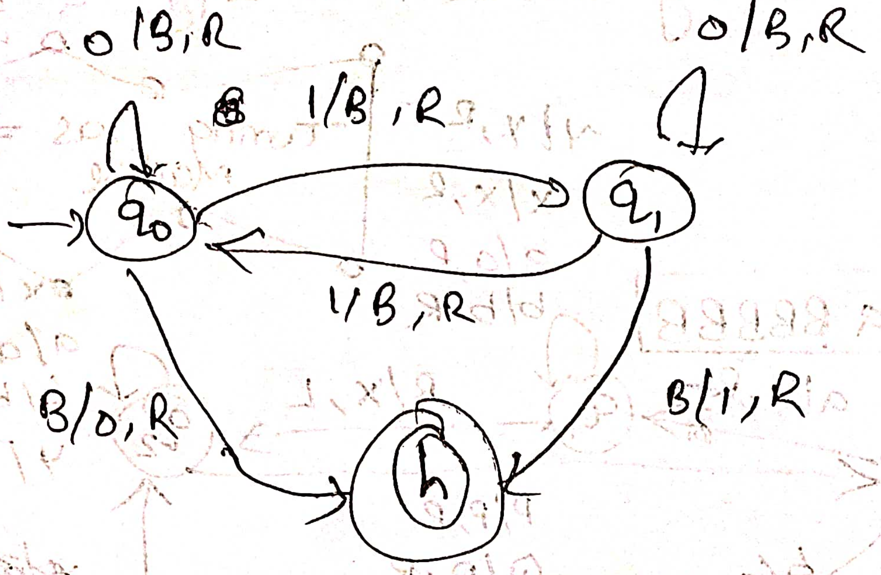
Q Design a turning machine to copy a given string

ans)




Q Design a turning machine to use ~~check~~ a parity checker.






Turing Machine act as ~~comp~~ function of computer

* ———— * ———— * ———— *

$f(i1, i2, in) = m$

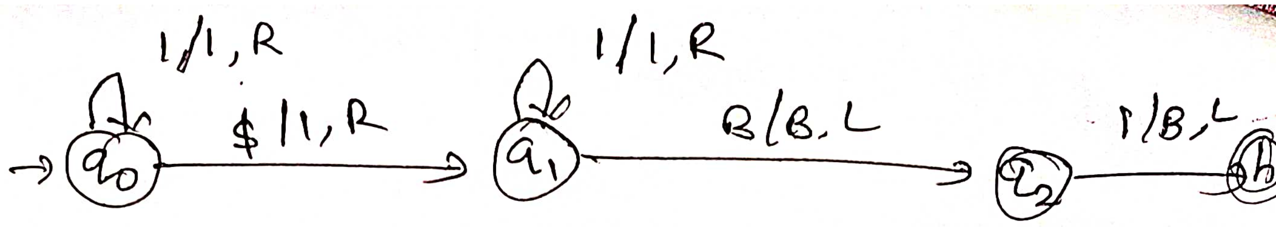
$i1 \rightarrow 1^{i1}$ or $0^{i1}$

Seperator $

Q. Design a turning machine ~~addition~~. to compute this function $f(m,n) = m+n$ where m & n are integers.

Let $m=5$ $n=3$

B 11111 $ 111 B




~~Let Q M=2~~

Trace for input m=2 and n=1

~~(q0, 11$1B) = q0 11$1B~~

$q_0 11\$1B$

$\rightarrow 1 q_0 1\$1B$

$\rightarrow 11 q_0 \$1B$

$\rightarrow 111 q_1 1B$

$\rightarrow 1111 q_1 B$

$\rightarrow 111 q_2 1B$

$\rightarrow 11 h 1BBB$

| | 1 | \$ | B |
|---|---|---|---|
| $q_0$ | $(q_0, 1, R)$ | $(q_1, 1, R)$ | |
| $q_1$ | $(q_1, 1, R)$ | | $(q_2, B, L)$ |
| $q_2$ | $(h, B, L)$ | | |
| $h$ | | | |

# Turing Machine as transducers

* Design an ~~automata~~ Turing machine for proper subtraction $f(m,n) = m-n$ if $m \geq n$
0 otherwise.

$m=5$ $n=2$

B 11111 $ 11 B

⇓

B 111 B

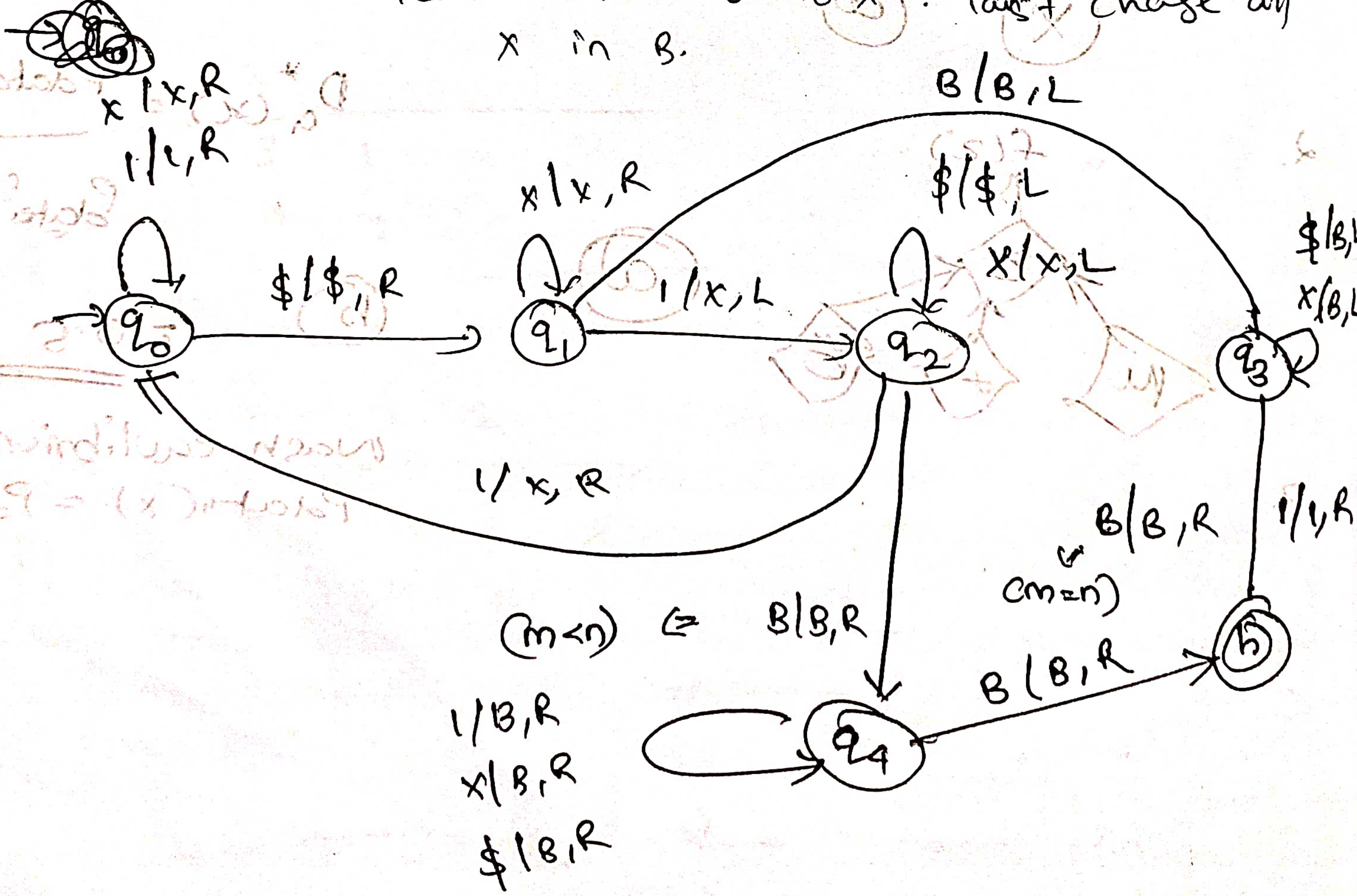
$m=2$ $n=3$

B11$111B

↓

B B B

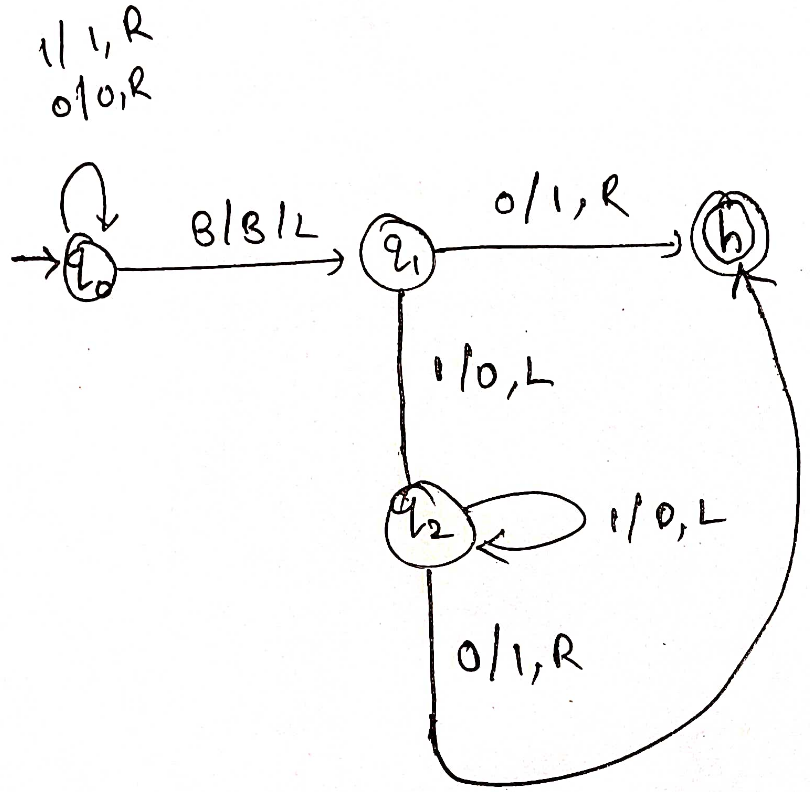
first right of $ make to x and find pair in left and chage to X. last chage all X in B.



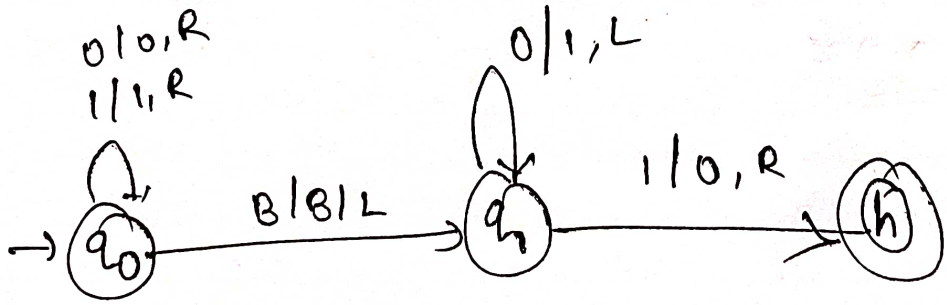
Q. Design a turning machine to increament a binary number

1) If last bit '0' then flip into 1

2) If last bit '1' then move left all consecutive group of 1 changes to 0 and first 0 into 1.




Decrement a binary number.



Q. Binary addition

m = 3    n = 2

011    010

B 011 \$ 010 B ⇒ B 011 \$ 010 B

↓

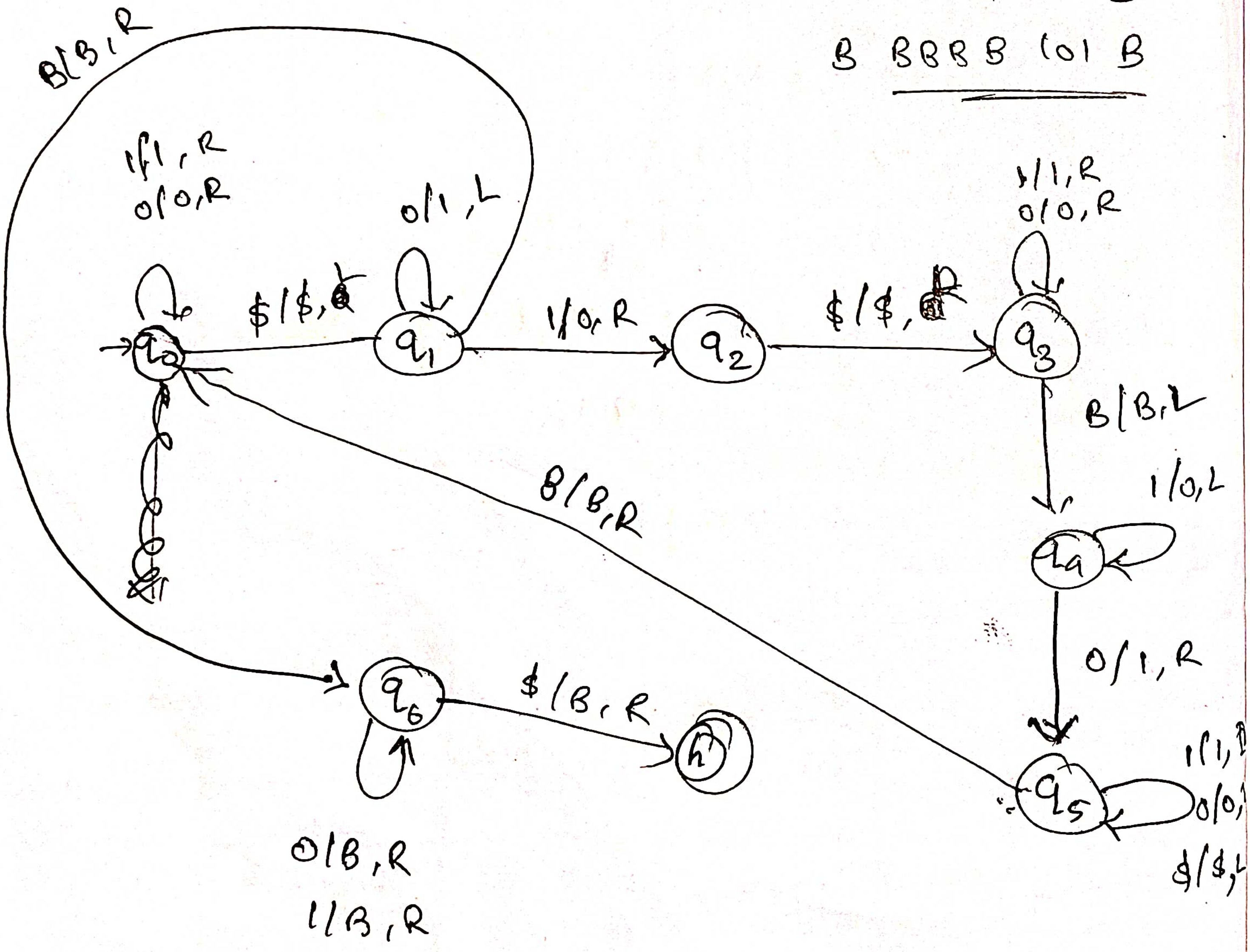
B 101 B

B 010 \$ 011 B

B 001 \$ 100 B

B 000 \$ 101 B

B BBBB 101 B



# Recursively Enumerable Language (RE) :

Type 0 / unrestricted

$\alpha \rightarrow \beta$

$\alpha, \beta \in (V \cup T)^*$

$\alpha \notin \epsilon$

$L \in RE$ , M is TM / Turing recognizable

if $w \in L$ , TM halt at a final state

if $w \notin L$ , TM halt in a non-final state

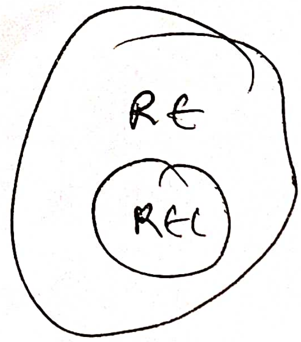
or

loop forever.

$L \in REC$

If $w \in L$ – halt final state

If $w \notin L$ → halt non final



# Chomsky - hierarchy

| Grammar | Language | Automata | Example | Rules for G |
| --- | --- | --- | --- | --- |
| Type 3 | Regular | FA | $a^*$ | $\alpha \to \beta$<br>$\alpha \in V,\ \beta \to \epsilon/T$<br>$T$ |
| Type 2 | CFL | PDA | $a^n b^n$ | $\alpha \to \beta$<br>$\alpha \to V,\ \beta \to (V \cup T)^*$ |
| Type 1 | CSL | LBA | $a^n b^n c^n$ | $\alpha X \beta \to \alpha Y \beta$<br>$\alpha, \beta \in (V \cup T)^*$<br>$X \in V,$<br>$Y \in (V \cup T)^+$ |
| Type 0 | RE | TM | $a^n b^n c^n$ | $\alpha \to \beta$<br>$\alpha, \beta \in (V \cup T)^*$<br>$\alpha \notin \epsilon$ |