* Linear Algebra

* $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ 2row
3coloumn

order = no. of row x no of coloumn.

order = 2 x 3

* Square matrix ⇒ row = coloumn

2x2 Order = nxn $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

3x3 $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 8 \end{bmatrix}$ → off diagonal

→ main diagonal / Principal diag

* Diagonal elements

* Diagonal matrix = $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Diagonal element മാത്രമേ ഉണ്ട്

* Identity matrix:

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1 1 1

* Scalar matrix

$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

all no. same.

* Null matrix

$\phi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

* Upper Δlar matrix:

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

Main diag. ഉം മുകളിൽ
Δle formൽ.

* Lower Δlar matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

* Transpose of matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

* Symmetric matrix

$$\boxed{A = A_T} \quad A = \begin{bmatrix} 1 & -3 & -5 \\ -3 & 2 & 4 \\ -5 & 4 & 5 \end{bmatrix}$$

$$A_T = \begin{bmatrix} 1 & -3 & -5 \\ -3 & 2 & 4 \\ -5 & 4 & 5 \end{bmatrix}$$

* Skew Symmetric

$$\boxed{A = -(A^T)}$$

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \quad A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

* Conjugate of matrix

$$A = \begin{bmatrix} 2+i & -i \\ 3 & 2-4i \end{bmatrix} \quad \bar{A} = \begin{bmatrix} 2-i & i \\ 3 & 2+4i \end{bmatrix}$$

If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 4 & 5 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ -1 & -2 & -1 \end{bmatrix}$

Find $A+B$, $A-B$, AB, BA.

$$A+B = \begin{bmatrix} 3 & 6 & 6 \\ 3 & 0 & 6 \\ 3 & 3 & 2 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1 & 0 & 2 \\ -1 & -2 & -2 \\ 5 & 7 & 4 \end{bmatrix}$$

$$\cancel{AB = \begin{bmatrix} 2 & 9 & 8 \\ 2 & -3 & 8 \\ -4 & -10 & -3 \end{bmatrix}}$$

$$AB = \begin{bmatrix} (2\times1 + 3\times2 + 4\times-1) \Leftarrow \\ \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 4 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ -1 & -2 & -1 \end{bmatrix} \end{bmatrix}$$

$$AB = \begin{bmatrix} (2+6-4) & (6+2-4) & (4+8-2) \\ (3+6-3) & (& \\ & & \end{bmatrix}$$

First row × coloumm I

1) × IInd

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 4 & 5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ -1 & -2 & -1 \end{bmatrix}$$

$S_R \times C_1$

$$AB = \begin{array}{ccc} FR \times C1 & FR \times C2 & FR \times C3 \\ \left[\begin{array}{ccc} (2\times1 + 3\times2 + 4\times-1) & (2\times3 + 3\times1 + 4\times-2) & (2\times2 + 3\times4 + 4\times-1) \\ (1\times1 + -1\times2 + 2\times-1) & (1\times3 + (-1\times1) + (2\times-2) & (1\times2) + (-1\times4 + 2\times-1) \\ (4\times1 + 5\times2 + 3\times-1) & (4\times3 + 5\times1 + 3\times-2) & (4\times2 + 5\times4 + 3\times-1) \end{array}\right. \end{array}$$

$$AB = \begin{bmatrix} 4 & 1 & 12 \\ -3 & -2 & -4 \\ 11 & 11 & 25 \end{bmatrix}$$

2) $$B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ -1 & -2 & -1 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 4 & 5 & 3 \end{bmatrix}$$

$$BA = \left[\begin{array}{ccc} 13 & 10 & 16 \\ 21 & 25 & 2 \\ -8 & -6 & - \end{array}\right.$$

$$BA = \begin{bmatrix} (2+3+8) & (3+-3+10) & (4+6+6) \\ (4+1+16) & (6-1+20) & (8+2+12) \\ (-2-2-4) & (-3+2-5) & (-4-4-3) \end{bmatrix}$$

$(\cancel{-4-4-3})$

- **RANK OF A MATRIX** (ഉള്ള row യിലെ first non zero elem. (
- **Eclon Form** (i) First non zero element ആകണം 0.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) Second ലിലെ non zero elem ഉള്ള 1(0)

(iii) third " " ഉള്ള 2 zero

Pivot / Leading element (first non zero elem)

ROW-REDUCED FORM

→ If the leading element is not '1'.
→ no of zero before leading element ↑ than the preceding row

#

2 Q) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{bmatrix}$ $R_2 - 4R_1$

$\approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $R(A) = 3$

(3 row ലും elem ഉണ്ട്).

No of non zero row after reducing to echelon form or row reduced form.

3) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{bmatrix}$,

$\approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -1 & -2 \end{bmatrix}$ $R_2 - 4R_1$, $R_3 - R_1$

≈ ★ First row ന്റെ താഴെ ഉള്ളതും zero ayi

★ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -1 & -2 \end{bmatrix}$ Second row ന്റെ first non zero elem ന്റെ താഴെ Zero avanam

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -1 & -2 \end{bmatrix}$

$\approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$ $R_3 \Rightarrow 3R_3 - R_2$

(Row reduced form)

$\approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $R_2 \Rightarrow \frac{R_2}{-3}$.

(echlon form)

$R(A) = 2$

4 Q) By reducing to echlon form find the rank of.

$A = \begin{bmatrix} 1 & -1 & 0 & 2 & 1 \\ 3 & 1 & 1 & -1 & 2 \\ 4 & 0 & 1 & 0 & 3 \\ 9 & -1 & 2 & 3 & 7 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -1 & 0 & 2 & 1 \\ 0 & 4 & 1 & -7 & -1 \\ 0 & 4 & 1 & -8 & -1 \\ 0 & 8 & 2 & -15 & -2 \end{bmatrix}$ $R_2 \Rightarrow R_2 - 3R_1$, $R_3 \Rightarrow R_3 - 4R_1$, $R_4 \Rightarrow R_4 - 9R_1$

4, 8 ഉം Zero avanam

$A = \begin{bmatrix} 1 & -1 & 0 & 2 & 1 \\ 0 & 4 & 1 & -7 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \boxed{-1} & 0 \end{bmatrix}$ $R_3 \Rightarrow R_3 - R_2$, $R_4 \Rightarrow R_4 - 2R_2$

$$A = \begin{bmatrix} 1 & -1 & 0 & 2 & 1 \\ 0 & \boxed{4} & 1 & -7 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \Rightarrow R_4 - R_3$$

4 1 avanam echlon form avan.

$$A = \begin{bmatrix} 1 & -1 & 0 & 2 & 1 \\ 0 & 1 & 1/4 & -7/4 & -1/4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \Rightarrow \frac{R_2}{R_4} \quad R_3 \Rightarrow R_3(-1)$$

$R(A) = 3$

3) By reducing to echlon form find rank of

$$A = \begin{bmatrix} 0 & 1 & -2 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix} \quad R_2 \leftrightarrow R_1$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix} \quad \begin{matrix} R_3 = R_3 - R_1 \\ R_4 = R_4 + 2R_1 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & \boxed{-1} & 2 & -1 \\ 0 & \boxed{0} & 12 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 2 \end{bmatrix} \quad R_3 \Rightarrow R_3 + R_2$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 12 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \; R_3$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 1/6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R(A) = 3$

4) Find the rank of matrix.

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$

I

$$A = \begin{bmatrix} 1 & 6/5 & 7/5 & 8/5 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 6/5 & 7/5 & 8/5 \\ 0 & -1/5 & -2/5 & -3/5 \\ 0 & -6/5 & -12/5 & -18/5 \\ 0 & -11/5 & -32/5 & \end{bmatrix} \quad \begin{matrix} R_2 \\ R_3 \\ R_4 \end{matrix}$$

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & -1 & -2 & -3 \\ 0 & \boxed{-6} & -12 & -18 \\ 0 & -11 & -22 & -33 \end{bmatrix} \begin{array}{l} R_2 \Rightarrow 5R_2 - 6R_1 \\ R_3 \Rightarrow 5R_3 - 11R_1 \\ R_4 \Rightarrow 5R_4 - 16R_1 \end{array}$$

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_3 \Rightarrow R_3 - 6R_2 \\ R_4 = R_4 - 11R_2 \end{array}$$

$R(A) = 2$

Deduce to echlon form

$$A = \begin{bmatrix} 1 & 6/5 & 7/5 & 8/5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_1 = \frac{R_1}{5} \\ R_2 = (-R_2) \end{array}$$

$R(A) = 2$

Q) $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$

System of Linear Eqn

Homogeneous system of linear Eqn

$x + y + z = 0$
$3x - 4y = 0$

Non-hom. system lin. Eqn

$x + y + z = 3$
$x + 3y + 4z = -1$

SOLUTIONS :-

1. Write Eqn as $AX = B$
2. Write augmented matrix $[AB]$
3. Reduce " " AB to row reduced form.
4. Find rank(AB), R(A) no of unknowns.
5. $R(AB) = R(A)$

   System is consitent.

   (Eqn has sol)

6. $R(AB) \neq R(A)$ No solution.

7. $R(AB) = R(A) =$ no of unknown

   Consistent unique solution.

(ii) $R(AB) = R(A) \neq$ no of unkno

Consistent

∞ solution

$r(AB) \neq r(A)$ inconsistent

$R(AB) = 3$

$R(A) = 3$

$R(AB) = R(A)$

System consistent

no of unknown

Q) Using Gauss elimination method

Solve
$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

non-hom

$$\underset{A}{\begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ -8 \\ -20 \\ 0 \end{bmatrix}}$$

Ans (i)
$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

$$x + 2y - z = 3$$
$$-7y + 5z = -8$$
$$5z = -20$$

(ii)
$$[AB] = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$5z = -20$

$\underline{\underline{z = -4}}$

$-7y - 20 = -8$

$-7y = -12$

$y = 12/7$

$-7y + 5z = -8$

$-7y + 20 = -8$

$-7y = -28$

$\underline{\underline{y = 4}}$

(iii)
$$AB = \sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{bmatrix} \begin{matrix} R_2 \Rightarrow R_2 - 3R_1 \\ R_3 \Rightarrow R_3 - 2R_1 \\ R_4 \Rightarrow R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & -1 & -4 \end{bmatrix} \begin{matrix} R_3 \Rightarrow 7R_3 - 6R_2 \\ R_4 = 7R_4 - 3R_3 \end{matrix}$$

$x + 8 - 4 = 3$

$x + 4 = 3$

$\underline{\underline{x = -1}}$

$$\sim \begin{array}{cc} & \begin{matrix} A & & & B \end{matrix} \\ & \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} R_4 = 5R_4 + R_3$$

$r(A) = 3$

9Q) $x+y+z=6$

$3x+y+z=8$

$-x+y-2z=-5$

$-2x+2y-3z=-7$

Solve ?

$\Rightarrow \quad AX = B$

I $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ -1 & 1 & -2 \\ -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -5 \\ -7 \end{bmatrix}$

II AB $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & 1 & 1 & 8 \\ -1 & 1 & 1 & -2 \\ -2 & 2 & -3 & -7 \end{bmatrix}$

III $\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 2 & -1 & 1 \\ 0 & 4 & -1 & -5 \end{bmatrix}$ $R_2 \Rightarrow R_2 - 3R_1$, $R_3 \Rightarrow R_3 + R_1$, $R_4 \Rightarrow R_4 + 2R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & 0 & -6 \end{bmatrix}$ $R_3 = R_3 + R_2$

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ \boxed{0} & -2 & -2 & -10 \\ 0 & \boxed{0} & -3 & -9 \\ 0 & 0 & -5 & -15 \end{bmatrix}$ $\rightarrow R_3 + R_2$, $\rightarrow R_4 - 2R_2$

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_4 \Rightarrow 3R_4 - 5R_3$

$R(AB) = 3$

$R(A) = 3$

$AB = A$

Solution ✓

$x + y + z = 6$

$-2y - 2z = -10$

$-3z = -9$

(i) $z = 3$

(ii) $-2y - 6 = -10$

$-2y = -4$

$y = +2$

(iii) $x + 2 + 3 = 6$

$x + 5 = 6$

$x = 1$

$x \rightarrow 1$

$y \rightarrow 2$

$z \rightarrow 3$

Q) Examine for consistency and solve the equation :

$$x + y + 2z = 2$$
$$2x - y + 3z = 2$$
$$5x - y + 8z = 10$$

I $AX = B$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}$$

II $[A\,B]$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & -1 & 3 & 2 \\ 5 & -1 & 8 & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & \cancel{4}\ -6 & -2 & 0 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 5R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & 0 & 4 \end{bmatrix} = R_3 - 2R_2$$

$-6 - 2\times -3$
$-6 +$

$R(AB) = 3$

$R(A) = 2$ $AB \neq A$

Q) $R(AB) = R(A)$ = no. of u. unique

" consistent

$R(AB) \neq R(A)$

inconsistent

Q) Show that equation

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & \boxed{3} & 6 & 24 \end{bmatrix} \begin{matrix} \\ R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_3 - 3R_2$$

$R(AB) = 2$

$R(A) = 2$

no of unknown = 3 (x y z)

$AB = A \neq$ no of unknown

Consistent

Infinite solution

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$$

$x + y + z = 6$ — (1)

$y + 2z = 8$ — (2)

$\cancel{x} + \cancel{8} = \cancel{6}$

$\cancel{x} = \cancel{-2}$

no of unknowns − rank

$(3-2) = 1$

ഒരെണ്ണം substitution kodukanam

Put $z = a$

$y + 2a = 8$

$y = 8 - 2a$

$x + (8 - 2a) + a = 6$

$x + 8 - a = 6$

$x - a = -2$

$x = a - 2$

$$\begin{bmatrix} x = a-2 \\ y = 8-2a \\ z = a \end{bmatrix}$$

Q) $x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1$

$2x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 = 2$

$3x_1 + 2x_2 - 4x_3 - 3x_4 - 9x_5 = 3$

(5 unknowns)

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 2 & -1 & 2 & 2 & 6 \\ 3 & 2 & -4 & -3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$[AB] = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & 1 \\ 2 & -1 & 2 & 2 & 6 & 2 \\ 3 & 2 & -4 & -3 & -9 & 3 \end{bmatrix}$$

Row reduce.

$$\sim \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & -3 & 6 & 0 & 0 & 0 \\ 0 & -1 & 2 & -6 & -18 & 0 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & -3 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -18 & -54 & 0 \end{bmatrix}$$

$\Rightarrow R_3 \to 3R_3 - R_2$

$R(AB) = 3$

$R(A) = 3$

no of unknown = 5

[no of unknown − rank) = 2

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 0 & -3 & 6 & 0 & 0 \\ 0 & 0 & 0 & -18 & -54 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1$$

$$-3x_2 + 6x_3 = 0$$

$$-18x_4 - 54x_5 = 0$$

Substitution done for 2 variable

Put $x_5 = a$

$$-18x_4 - 54a = 0$$

$$-18x_4 = 54a$$

$$\boxed{x_4 = -3a}$$

Put $x_3 = b$

$$-3x_2 + 6b = 0$$

$$-3x_2 = -6b$$

$$\boxed{x_2 = 2b}$$

$$x_1 + 2b - 2b + (-3a) + 3a = 1$$

$$x_1 = 1$$

x_1	1
x_2	$2b$
x_3	b
x_4	$-3a$
x_5	a

Q) $y + z - 2w = 0$

13 $2x - 3y - 3z + 6w = 2$

$4x + y + z - 2w = 4$

I $AX = B$

$$\begin{bmatrix} 0 & 1 & 1 & -2 \\ 2 & -3 & -3 & 6 \\ 4 & 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$$

II $[A B]$

$$\begin{bmatrix} 0 & 1 & 1 & -2 & 0 \\ 2 & -3 & -3 & 6 & 2 \\ 4 & 1 & 1 & -2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 1 & 1 & -2 & 0 \\ & & & & \\ & & & & \end{bmatrix} R_2$$

Interchange

$$\sim \begin{bmatrix} 2 & -3 & -3 & 6 & 2 \\ 0 & 1 & 1 & -2 & 0 \\ 4 & 1 & 1 & -2 & 4 \end{bmatrix} R_3 - 2R_1$$

$$\sim \begin{bmatrix} 2 & -3 & -3 & 6 & 2 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 7 & 7 & -14 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & -3 & 6 & 2 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_3 - 7R_2$$

$R(AB) = 2$

$R(A) = 2$ $\quad AB = A$

no of unk = 4.

∞ solution.

$(4 - 2 = 2)$ 2 substitution.

$2x - 3y - 3z + 6w = 2$

$y + z - 2w = 0$

Put $w = a$

$z = b$.

$y + b - 2a = 0$

$\underline{\underline{y = 2a-b}}$

$x - \cancel{2a-b} - \cancel{3b} + \cancel{6a} = 2$

$\cancel{x} - \cancel{2a + 6a - 4b} = 2$

$x - 3(2a-b) - 3b + 6a = 2$

$2x - \cancel{6a} + \cancel{3b} - \cancel{3b} + \cancel{6a} = 2$

$2x = 2$

$\underline{\underline{x = 1}}$

$x = 1$

$y = 2a - b$

$z = b$

$w = a$

Q) For what value of λ and μ do the system of equation $x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

have

a) no sol^n

b) unique sol^n

c) more than 1 sol

$\Rightarrow$ I $\quad AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

II $\quad [AB]$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

III

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & (\lambda-1) & (\mu-6) \end{bmatrix} \begin{matrix} R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow R_3 - R_1 \end{matrix}$$

IV

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & (\lambda-3) & (\mu-10) \end{bmatrix} R_3 \Rightarrow R_3 - R_2$$

$R(A) = 3$

$R(AB) = 3$

a) no solution

$R(AB) \neq R(A)$ System inconsistent

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & (\lambda-3) & (\mu-10) \end{bmatrix} \quad \lambda = 3, \ \mu \neq 10$$

If $\lambda - 3 = 0 \Rightarrow r(AB) = 3$

$\mu - 10 \neq 0 \qquad r(A) = 2$

b) unique solution

$r(AB) = r(A) =$ no of unknown

System consistent

$\lambda - 3 \neq 0$

$\mu - 10 =$ any value

c) more than 1 solution

∞ sol^n

$r(AB) = r(A) \neq$ no of unknown

$\lambda - 3 = 0 \qquad \lambda = 3$

$\mu - 10 = 0 \qquad \mu = 10$

$r(AB) = 2$

$r(A) = 2 \qquad$ unknown $= 3$

Q) $2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$$

$7R_1 - 2R_2$

$7\times9 - 2\times8$

$63 - 16$

$7\times2 - 2\times7$

$7\times3 - 2\times3 =$

$7\times5 - 2\times2$

35

$$\begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & 15 & 39 & 47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{bmatrix}$$

$7\times2 - 2\times7$

$7\times3 - 2\times3$

$7\times5 - 2\times-2$

$7\times9 - 2\times8$

a) $r(AB) \neq r(A)$

$\lambda - 5 = 0 \qquad AB = 3$

$\mu - 9 \neq 0 \qquad A = 2$

b) $\lambda - 5 \neq 0$

$\mu - 9 =$ any v

c) $\lambda = 5$

$\mu = 9$

Q) $x + y + z = 1$

$x + 2y + 4z = \lambda$

$x + 4y + 10z = \lambda^2$ consistent

I $AX = B$

II $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$

[AB] $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 3 & 9 & (\lambda^2-1) \end{bmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix} R_3 - 3R_2$

$(\lambda^2-1) - 3(\lambda-1)$

$\lambda^2-1, -3\lambda-1$

$\lambda^2 - 3\lambda + 2$

Given system of Eqn; consist

$R(AB) = R(A)$.

$r(A) = 2$

$\therefore r(AB) = 0$

$\Rightarrow \lambda^2 - 3\lambda + 2 = 0$

$\lambda = \dfrac{-(-3) \pm \sqrt{3^2 - 4\times1\times2}}{2\times1}$

$= \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3+1}{2} = 2$

$\dfrac{3-1}{2} = 1$

when $\lambda = 1$

$1 - 3 + 2 = 0$ $\quad \lambda = (1, 2)$.

$\lambda = 2$

$4 - 6 + 2 = 0$

$\therefore r(AB) = r(A)$.

Q) Show that eqn $x + y + z = a$, $3x + 4y + 5z = b$, $2x + 3y + 4z = c$

(i) have no soln $(a = b = c = 1)$

(ii) have many soln $(a = b/2 = c - 1)$

$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & a \\ 3 & 4 & 5 & b \\ 2 & 3 & 4 & c \end{bmatrix} = [AB]$

$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & b-3a \\ 0 & 1 & 2 & c-2a \end{bmatrix} \begin{matrix} R_2 - 3R_1 \\ R_3 - 2R_1 \end{matrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & (b-3a) \\ 0 & 0 & 0 & \cancel{c-2a} \end{bmatrix} R_3 - R_2$

$a-b+c$

$c - 2a - (b - 3a)$

$c - 2a - b + 3a$

$a - b + c$

$\Rightarrow$ If $a = b = c = 1$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad r(AB) = 3$, $r(A) = 2$

No solution

(ii) $2a = b = c = 1$ $\qquad 2a = 1$, $a = 1/2$, $b/2 = 1$, $b = 2$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$r(AB) = 2$

$r(A) = 2$

unknown = 3.

∞ soln.

Homogeneous System. Lin Eqn

$AX = 0$

Trivial

$x=0$
$y=0$
$z=0$

$r(A)$ = no of unknown.

or

$|A| \neq 0$

{Square matrix 2×2, 3×3}

Non-trivial

$x=0$
$y=b$
$z=c$, (non-zero values)

R(A) < unknown

or

$|A| = 0$

Q) 18
$3x + 2y + z = 0$
$2x + 3z + \; = 0$
$x + 2y + 3z = 0$.

$AX = 0$

3 2 1
2 0 0

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 2 & 3 \end{vmatrix}$$

$|A| = 3[0-6] - 2[6-3] + 1[4-0]$
$= (3\times -6) - (2\times3) + 1\times4$
$= -18 - 6 + 4$
$= \underline{\underline{-20}}$.

$|A| \neq 0$

sol = trivial

$x=0$
$y=0$
$z=0$

Q) 19
$x + 3y + 2z = 0$
$2x - y + 3z = 0$
$3x - 5y + 4z = 0$
$x + 17y + 4z = 0$

$$AX = 0 \quad \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Ans:- $$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

Row Red

$$\sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix} R_2 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_3 - 2R_2 \\ R_4 + 2R_2 \end{matrix}$$

$r(A) = 2$

no of unknown = 3.

(no. un - rank = 3 - 2 = 1)

Substitution = 1

$Ax = 0$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + 3y + 2z = 0 \quad — ①$

$-7y - z = 0 \quad — ②$

Put $z = a$

$-7y - a = 0$

$-7y = a$

$y = -a/7$

$x + \left(3x - \frac{a}{7}\right) + 2a = 0$

$x - \frac{3a}{7} + 2a = 0$

$x = \frac{3a}{7} - 2a = -11a/7$

$x = -11a/7$

$y = -a/7$

$z = a$

Q) Solve Eqn

20

$4x + 2y + z + 3w = 0$

$6x + 3y + 4z + 7w = 0$

$2x + y + w = 0$

$Ax = 0$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$$\approx \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{matrix} \\ 4R_2 - 6R_1 \\ 2R_3 - R_1 \end{matrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_3 \Leftarrow R_3 + R_1 \\ 10R_3 + R_2 \end{matrix}$$

[$R(A) = 2$

~~R~~ unknown $= 4$.

Sub^s $= 2$

$R(A) <$ no of unk.

Non-trivial.

$4x + 2y + z + 3w = 0$

$10z + 10w = 0$

① Put $z = a$.

$w = b$

$10a + 10b =$

$10a + 10w = 0$

$10a = -10w$

$a = -w$

② $y = b$

$4x + 2b + a - 3a = 0$

$4x + 2b - 2a = 0.$

$wx = -a$

$w = a$

$10z + 10a = 0$

$10z = -10a$

$4m = 2a - 2b$

$m = \frac{2a - 2b}{4}$

$a = -z$

$z = -a$

$4x + 2b + a$

$4x + 2b - a + 3a = 0$

$4x + 2b + 2a = 0$

$4x = -2a - 2b$

$x = \frac{-2a - 2b}{4}$

Q) Find the value of λ,

Q) $x + y + 3z = 0$

$2x + 3y + \lambda z = 0$

$-3x - 4y + z = 0$

(Non-trivial sol$\hat{n}$),

Determine sol$\hat{n}$?

$3x + y - \lambda z = 0$

$4x - 2y - 3z = 0$

$2\lambda x + 4y + \lambda z = 0$

Has non-trivial solution?

$\Downarrow$

$R(A) <$ no of unknown / $|A| = 0$ $\Rightarrow$ $AX = 0$

$$\begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & \lambda \\ -3 & -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We know that, system of eqn has non-trivial sol$\hat{n}$ iff $|A| = 0$.

$\Rightarrow$ Non-trivial solution

$|A| = 0$.

$$|A| = 0 \quad \begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = 0$$

$$|A| = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & \lambda \\ -3 & -4 & 1 \end{vmatrix} = 0$$

$3(-2\lambda + 12) - 1(4\lambda + 6\lambda) + -\lambda(16 + 4\lambda)$

$-6\lambda + 36 - 10\lambda - 16\lambda - 4\lambda^2 = 0$

$-4\lambda^2 - 32\lambda + 36 = 0$

$-\lambda^2 - 8\lambda + 9 = 0$

$\lambda^2 + 8\lambda - 9 = 0$

$\lambda = -9, 1$

$|A| = 1(3 + 4\lambda) - 1(2 + 3\lambda) - 3$

$|A| = 3 + 4\lambda - 2 - 3\lambda + 2\ldots$

$|A| = \lambda - 2 = 0$

~~$\lambda = 2$~~

$|A| : (3 + 4\lambda) - 2 - 3\lambda - 3$

$= \lambda + 4$

$= \lambda = -4$

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & -4 \\ -3 & -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & -4 \\ -3 & -4 & 1 \end{bmatrix}$$

$-4 + 3\times 1$

$$\approx \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -10 \\ 0 & -1 & 10 \end{bmatrix} \quad \begin{matrix} R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 + 3R_1 \end{matrix}$$

$$\approx \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -10 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \Rightarrow R_3 + R_2$$

$\underline{\underline{R(A) = 2}}$. no of un = 3

$\underline{\underline{\text{Sub} = 1}}$

$x + y + 3z = 0$

$y - 10z = 0$

Put $y = a$.

$a - 10z = 0$

$\cancel{a = 10z}$

$\cancel{z = a/10}$

$y = a$.

$\underline{\underline{x = -13a/10}}$

Put $x = a$

$y - 10a = 0$

$y = 10a$

$\therefore x + a + \cancel{\frac{3a}{10}} = 0$

$x + \cancel{\frac{13a}{10}} = 0$

$x + 10a + 3a = 0$

$x = -13a$.

a)

Eigen Value (spectrum)

: Eigen value of 2x2 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Characterstic Eqn is

$|A - \lambda I| = 0$

$\lambda^2 - S_1\lambda + S_2 = 0$.

$\lambda^2 - S_1\lambda + S_2 = 0$

Eigen 3x3

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$|A - \lambda I| = 0$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$.

* S_1 = Sum of main daigonal elemen

$S_1 = a_{11} + a_{22}$

$S_2 = |A|$

Subtitude S_1 and S_2 in charect-

and solve check. Eqn.

Value of $\lambda \rightarrow$ Eign value

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

$*$ 3×3

$S_1 = a_{11} + a_{22} + a_{33}$ (major)

$S_2 =$ Sum of minor diag. elem

$S_2 = a_{22} + a_{23} + a_{32} + \cancel{a_{33}}$

$a_{11} + a_{13} + \cancel{a_{31}} + a_{33}$

$$S_2 = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} + \begin{bmatrix} a_{11} & +a_{13} \\ a_{31} & +a_{33} \end{bmatrix} + \begin{bmatrix} a_{21} & +a_{22} \\ a_{31} & +a_{32} \end{bmatrix}$$

$S_3 = |A|$

Q) Find Eigen value $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

23

(i) 2×2 matrix

(ii) characterstic Equation is $|A - \lambda I| = 0$

$\lambda^2 - S_1\lambda + S_2 = 0$

(iii) $S_1 = a_{11} + a_{22}$ (sum of main diag. ei)

$S_1 = 8 + 2$

$S_1 = \underline{\underline{10}}$

(iv) $S_2 = |A|$

$S_2 = [16 - -8]$

$= \underline{\underline{24}}$

$\Rightarrow \lambda^2 - 10\lambda + 24 = 0$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4\times1\times24}}{2\times1}$$

$$= \frac{10 \pm \sqrt{100 - 96}}{2} = \frac{10 \pm \sqrt{4}}{2}$$

$\frac{10-2}{2} = 4$ ||

$\frac{10+2}{2} = \underline{\underline{6}}$

$\lambda = (6, 4)$

Q) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$

24

(i) 2×2

(ii) Symmetric $A = A^T$

$A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$ $\quad A_T = \begin{bmatrix} 1 & 2 \\ 2 & - \end{bmatrix}$

Hence proved

$A = A_T$

(iii) $A - \lambda I = 0$.

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 1 + -2 = 1 - 2 = \underline{\underline{-1}}$

$S_2 = |A| = [-2 - 4] = \underline{\underline{-6}}$

$\lambda^2 - (-1\lambda) + (-6) = 0$

$\lambda^2 + \lambda - 6 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4\times-6}}{2\times1}$$

$$x = \frac{-1 \pm \sqrt{25}}{2}$$

$x = \frac{-1-5}{2}$

$x = \underline{\underline{-3, +2}}$

8) $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ Eigen value?

1. Characterstic Equation

$A - \lambda I = 0$

$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$

2. $s_1 = a_{11} + a_{22} + a_{33}$

$s_1 = 3 + (-3) + 7 =$

$s_1 = 7$

3. $s_2 = \begin{bmatrix} -3 & -4 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix}$ $\begin{bmatrix} 3 & 10 \\ -2 & -3 \end{bmatrix}$

$-9+20$

$s_2 = (-21+20) + (21-15) + (-10+9)$

$s_2 = 1+6-1 =$ $s_2 = 16$

4. $s_3 = |A|$

$= 3[-21+20] - 10(-14+12) + 5(-10+9)$

$= (3 \times -1) - (10 \times -2) - 10(-2) + 5(-1)$

$= -3 + 20 - 5$

$= 12$

$s_1 = 7$

$s_2 = 16$

$s_3 = 12$.

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$.

Put $\lambda = 1 \Rightarrow 1 - 7 + 16 - 12 \neq 0$

$\lambda = -1 \Rightarrow -1 - 7 - 16 - 12 \neq 0$

$\lambda = 2 \Rightarrow 8 - 28 + 32 - 12 = 0$.

$\lambda = 2$ (needed 8 so $\hat{}$ due to λ^3)

λ^3	λ^2	λ	const
1	-7	16	-12

$2 \,|\, 1 \quad -7 \quad 16 \quad -12$

$2 \,|\, 1 \quad -7 \quad 16 \quad -12$

$0 \quad (2\times1) \quad (-10) \quad (2\times6)$

$(1+0) \quad (-7+2) \quad (16+-10)$

താഴേക്ക് member = add

മുകളിലേക്ക് = multiply

$2 \,|\, 1 \quad -7 \quad 16 \quad -12$

$0 \quad 2 \quad -10 \quad 12$

$1 \quad -5 \quad 6 \quad 0$

Value 0 anel - correct root

$\lambda^2 - 5\lambda + 6 = 0$

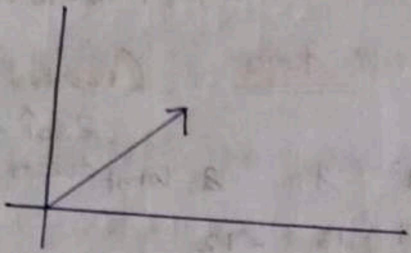
$x = \frac{-(-5) \pm \sqrt{5^2 - 4\times1\times6}}{2\times1}$

$x = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5\pm1}{2}$

$\lambda = 3, 2$

Eigen $= 3, 2, 2$

EIGEN VECTORS



$[A-\lambda I]X = 0$

) Find Eigen value & Eigen vector of

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

characteristic equation;

$$|A-\lambda I| = 0$$

$$\lambda^2 - S_1\lambda + S_2 = 0$$

$S_1 = 5+2 = 7$

$S_2 = [10-4] = 6$. $(|A|)$

$$\lambda^2 - 7\lambda + 6 = 0$$

igen value are $\Rightarrow (1, 6)$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

igen vector $\Rightarrow |A - \lambda I|x = 0$

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (5-\lambda) & (4-0) \\ (1-0) & (2-\lambda) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

When $\lambda = 1$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2 row same.

$x_1 + x_2 = 0$.

$4x_1 + 4x_2 = 0$

$4x_1 = -4x_2$

$x_1 = -x_2$

$\frac{x_1}{-1} = \frac{x_2}{1}$ (divided by opp coeff)

Eigen vector $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\Rightarrow$ When $\lambda = 6$

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2 row same.

$-x_1 + 4x_2 = 0$

$-x_1 = -4x_2$

$x_1 = 4x_2$

$\frac{x_1}{4} = \frac{x_2}{1}$ Eigen Vector $= \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

2) $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

$|A - \lambda I| = 0$

$\lambda^2 - s_1\lambda + s_2 = 0$

$s_1 = 8 + 2 = 10$

$s_2 = 16 + 8 = 24$

$\lambda^2 - 10\lambda + 24 = 0.$

$\lambda = \dfrac{-(-10) \pm \sqrt{100 - 4 \times 1 \times 24}}{2 \times 1}$

$x = \dfrac{10 \pm \sqrt{4}}{2} = \dfrac{10+4}{2} = \dfrac{10-4}{2}$ $\quad \dfrac{10+2}{2} = \dfrac{10-2}{2}$

$x = \cancel{7}, \cancel{6}\, 3$ $\qquad x = 7, 3.$

Eigen value $= (\cancel{7}, 3)$ $\quad \lambda = 4, 6$

* $|A - \lambda I|x = 0$

$$\left[\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$$

When $\lambda = 4.$

$$\begin{bmatrix} \cancel{1} & \cancel{-4} \\ \cancel{2} & \cancel{5} \end{bmatrix} \begin{bmatrix} \cancel{x_1} \\ x_2 \end{bmatrix}$$

$\lambda = 4$ $\quad \begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$4x_1 - 4x_2 = 0$ $\qquad 4x_1 = 4x_2$

$x_1 = x_2$

$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

When $\lambda = 6$

$\begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix} [\;] \Rightarrow$ $\quad 2x_1 - 4x_2 = 0$

$2x_1 = 4x_2$

$x_1 = 2x_2$

$\dfrac{x_1}{2} = \dfrac{x_2}{1}.$

$\Rightarrow x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Q) Find eigen value and eigen vector of $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

To find eigen value

$|A - \lambda I| = 0$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

(1) * S_1 = Sum of main daigonal el

$S_1 = 1 + 2 + 3 = 6$

* S_2 = sum minors of main diag

$S_2 = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}$

$S_2 = (6-1) + (3) + (2+1)$

$S_2 = 5 + 3 + 2 = \cancel{9}1$

* $S_3 = \begin{vmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{vmatrix}.$

det = $1(6-1) - 1(-3-0) + 2(-1-0)$

$= (1\times5) - (-3) + -2$

$= 5+3-2$

$= \underline{\underline{6}}$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\Rightarrow \cancel{-6}$ $\quad \underline{\underline{1,3,2}}$

(II) Eigen value are $\lambda = 1, 2, 3$

* To find eigen vector $[A-\lambda I]X = 0$

(III) $$\begin{bmatrix} 1-\lambda & 1 & 2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

* $\lambda = 1$

(IV) $$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

choose two different rows, (1, 2 rows).

x_1	x_2	x_3
0	1	2
-1	1	1

$$\frac{x_1}{(1\times1)-(2\times1)} = \frac{-x_2}{0-2} = \frac{x_3}{0=1}$$

$$\frac{x_1}{-1} = \frac{-x_2}{2} = \frac{x_3}{1}$$

$\therefore$ eigen vector $x_1 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

(V) When $\lambda = 2$

$$\begin{bmatrix} -1 & 1 & 2 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

choose any 2 diff row

x_1	x_2	x_3
-1	0	1
0	1	1

$$\frac{x_1}{(0-1)} = -\frac{x_2}{-1-0} = \frac{x_3}{-1}$$

$$\frac{x_1}{-1} = x_2 = \frac{x_3}{-1}$$

$x_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

(VI) When $\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 2 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

x_1	x_2	x_3
-1	-1	1
0	1	0

$$\frac{x_1}{0-1} = \frac{x_2}{0} = \frac{x_3}{-1}$$

$$\frac{x}{-1} = \frac{-x_2}{0} = -$$

$x_3 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$

20) Find eigen value and eigen vector of a $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

1. characterstic Equation $|A-\lambda I| = 0$

ie, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

2. $S_1 = 2+3+2 = 7$

3. $S_2 = \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$S_2 = [6-2] + [4-1] + [6-2]$

$S_2 = 4+2+4 = \underline{\underline{10}}$

4. $S_3 = 2[6-2] - 2[2-1] + 1[2-3]$

$= (2\times4) - 2(1) + 1\times-1$

$= \underline{\underline{5}}$

$\Rightarrow \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\lambda = 5, 1, 1$

Eigen value are $\lambda = 5, 1, 1$.

$\Rightarrow |A-\lambda I| X = 0$

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ When $\lambda = 5$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

all row diff

= choose 2 row

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ -3 & 2 & 1 \\ 1 & -2 & 1 \end{array}$$

$$\frac{x_1}{2-2} = \frac{-x_2}{-3-1} = \frac{x_3}{6-2}$$

$$\frac{x_1}{4} = \frac{x_2}{4} = \frac{x_3}{4}$$

$X_1 = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

when $\lambda = 1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

all rows are same.

$x_1 + 2x_2 + x_3 = 0$

Sub 0 for any one $(x_1\, x_2\, x_3)$

$x_1 + 2x_2 = 0$

$x_1 = -2x_2$

$\frac{x_1}{-2} = \frac{x_2}{1}$ $\quad X_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

Next eigen value is repeating

Put $x_2 = 0$

$x_1 + x_3 = 0$

$x_1 = -x_3$

$\frac{x_1}{-1} = \frac{x_3}{1}$ $\quad X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

30) $A = \begin{bmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{bmatrix}$

$|A - \lambda I| = 0$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 6 - 13 + 4 = -3$

$S_2 = (-52+60) + (24-35) + (-78+84)$

$S_2 = 3$

$S_3 = \begin{vmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{vmatrix} = 6(-52+60) - (-6)[56-70] + 5(-34+91)$

$\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$

$\lambda = -1, -1, -1$

Eigen value are $\lambda = -1, -1, -1$

all are same

$|A - \lambda I|x = 0$

$\begin{vmatrix} 6-\lambda & -6 & 5 \\ 14 & -13-\lambda & 10 \\ 7 & -6 & 4-\lambda \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$

When $\lambda = 1$

$\begin{bmatrix} 5 & -6 & 5 \\ 14 & -14 & 10 \\ -1 & -6 & 3 \end{bmatrix}$

$\begin{bmatrix} 7 & -6 & 5 \\ 14 & -12 & 10 \\ 7 & -6 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ 31)

✓ all row are same

$7x - 6x_2 + 5x_3 = 0.$

Put $x_3 = 0$

$7x_1 = 6x_2.$

$\frac{x_1}{6} = \frac{x_2}{7}$ $X_1 = \begin{bmatrix} 6 \\ 7 \\ 0 \end{bmatrix}$

* $7x_1 + 5x_3 = 0$

$7x_1 = -5x_3$

$\frac{x_1}{-5} = \frac{x_3}{7}$ $\begin{bmatrix} -5 \\ 0 \\ 7 \end{bmatrix}$

* $x_1 = 0$

$-6x_2 = -5x_3$

$6x_2 = 5x_3$

$\frac{x_2}{5} = \frac{x_3}{6}$ $\begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}$

31) Find eigen value and eigen Vector

$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$

$|A - \lambda I| = 0$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 2 + 1 + 4 = 7$

$S_2 = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ ✓

$= (4+2) + (8) + (2)$

$= \underline{\underline{16}}$

$S_3 = 2\,|4+2| - 1\,|0| + 0[\ ]$

$S_3 = \underline{\underline{12}}$

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

$\lambda = 3, 2, 2$

To Find eigen vector

$|A - \lambda I|x = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \lambda = 3$

$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix}$ } same row $\Rightarrow$

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ -1 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$\frac{x_1}{-1} = -\frac{x_2}{1} = \frac{x_3}{-2}$

$\frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{2}$ $\quad X_1 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

$\lambda = 2$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

3 row diff

$\begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$ $\quad X_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$\frac{x_1}{-1} = -\frac{x_2}{0} = \frac{x_3}{0}$

$\frac{x_1}{1} = x_2$

✓ $\lambda = 2$. λ repeating but row is diff

$x_1 + x_2 + x$

If eigen value are repea and rows are diff ; then there is only one eigen vector