EST 130 : BEE

Module 2

Elementary Concepts of magnetic circuits, Electro-magnetic induction and AC fundamentals.

*) Basic Terminology

1) MMF (magneto motive force)

Similar to the way that emf drives a current in ~~m~~ electrical circuits, magneto motive force (mmf) drives magnetic flux through magnetic circuits.

Unit - Ampere Turns (AT).

2) Magnetic Flux (ϕ)

In any magnetic field, the total magnetic lines of force passing through a surface is called magnetic flux. Unit of magnetic flux is weber (Wb). It is represented by ϕ.

3) Magnetic flux Density (B)

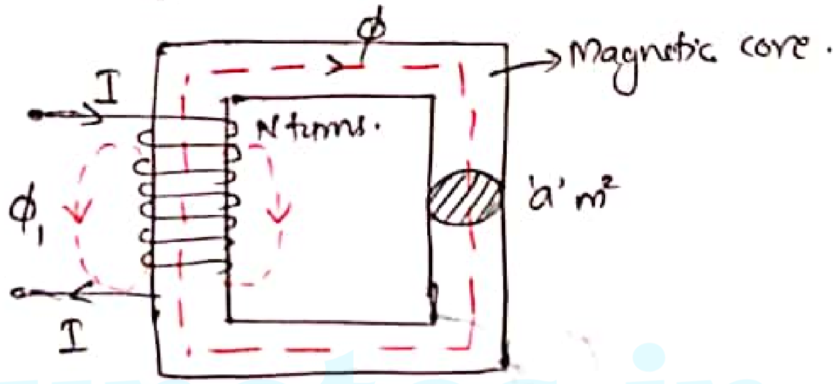
Flux density is defined as the flux passing through a unit area at right angles to the magnetic field lines.

ie, $\boxed{B = \frac{\text{Flux}}{\text{Area}} = \frac{\phi}{A}}$

Where, A = Area of cross section of magnetic path.

unit — $\frac{\text{Weber}}{m^2} = \frac{Wb}{m^2}$ or Tesla (T)

Eg: Consider a magnetic circuit as shown below



Let I = current passing through coil in Ampere

N = Number of turns in the coil

ϕ = Magnetic flux in weber

a = Cross sectional area of magnetic path.

$\boxed{\therefore \text{MMF} = F = NI}$ Ampere turns.

Flux density. $\boxed{B = \frac{\phi}{a}}$ $\frac{Wb}{m^2}$ or Tesla.

Note: A small amount of flux may take path through air. This flux is called leakage flux (ϕ_1).

4, Reluctance (S)

It is the opposition offered by the magnetic path to the flow of magnetic flux lines. It is analogous to resistance in electrical circuit.

It is represented by letter 'S'

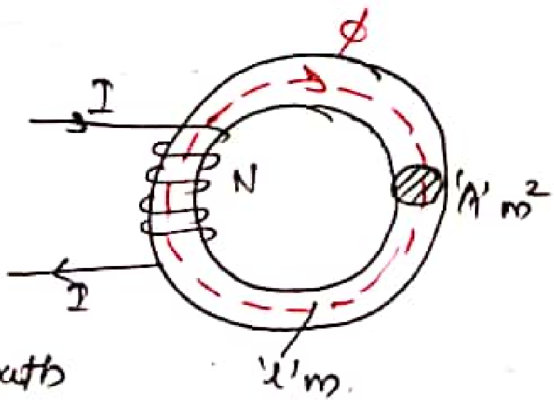
$$\boxed{\text{Reluctance } (S) = \frac{MMF}{Flux} = \frac{NI}{\phi}}$$

Unit — $\frac{AT}{Wb}$

Consider a ferromagnetic ring wound with 'N' turns of conductor carrying current 'I' ampere. Let mean circumference of ring is 'l' meter and the c.s.a is 'A' m^2, the flux of steel core is ϕ weber.

The reluctance of the ring.

$$\boxed{S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}} \frac{AT}{Wb}$$



$l \rightarrow$ length of magnetic path in meters.

μ = Permeability of magnetic material (Absolute permeability)

$\mu = \mu_0 \mu_r$

μ_0 = ~~perp~~ permeability of free space.

$\mu_0 = 4\pi \times 10^{-7}$ H/m

μ_r = Relative permeability of the material.

Note: For non magnetic materials, $\mu_r = 1$

5, Magnetic Field Strength (H)

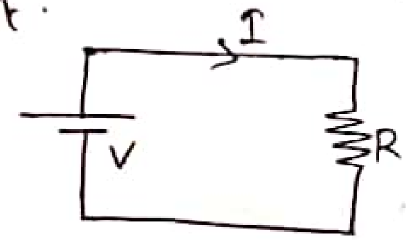
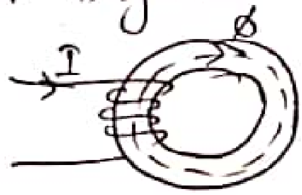
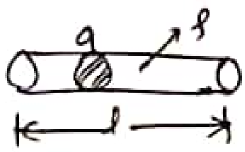
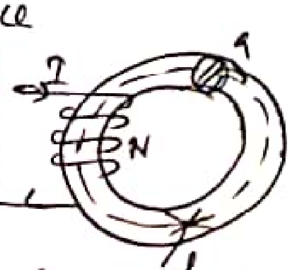
Magnetic flux density (B) is a function of μ, the permeability of the medium. Thus to calculate the force experienced by a conductor placed in a magnetic field, irrespective of the medium, a new term magnetic field intensity (H) is introduced. It is the ratio of the magnetic flux density and the absolute permeability of the medium μ.

ie, $\boxed{H = \frac{B}{\mu}}$ A/m

where $\mu = \mu_0 \mu_r$

Magnetic field strength or magnetic field intensity or magnetising force (H) is also defined as the magnetomotive force per meter length of the magnetic circuit. $\boxed{H = \frac{NI}{l}}$ AT/m

* Comparison between Electric and Magnetic circuits

Electric circuit	Magnetic circuit.
1, The closed path for electric current is called electric circuit. Eg: 	1, Closed path for build up of magnetic flux is called magnetic circuit. Eg: 
2, The cause of electric flow is EMF, Unit: Volt	2, The cause of build up of magnetic flux is MMF unit: Ampere turns
3, Opposition to the flow of electric current is called resistance $R = \frac{V}{I}$ (ohm)	3, Opposition to the flow of flux is called reluctance $S = \frac{NI}{\phi}$ (AT/wb)
4, Current density $\left(J = \frac{I}{A}\right)$ A/m^2	4, Flux density $\left(B = \frac{\phi}{A}\right)$ wb/m^2
5, Resistance. $\boxed{R = \frac{\rho l}{a} = \frac{l}{\sigma a}}$ 	5, Reluctance $\boxed{S = \frac{l}{\mu A}}$ 

6, Kirchhoff's current law and kirchhoff's voltage law are equally applicable to magnetic circuits.

↳ Dissimilarities.

1, In an electric circuit energy is released in the form of heat	1, In a magnetic circuit energy is stored in the magnetic field
2, Electric current is actually flows in a circuit	2, magnetic flux does not actually flow, it just build up
3, At a given temperature, electric resistance is constant	3, magnetic reluctance is variable ~~Store~~

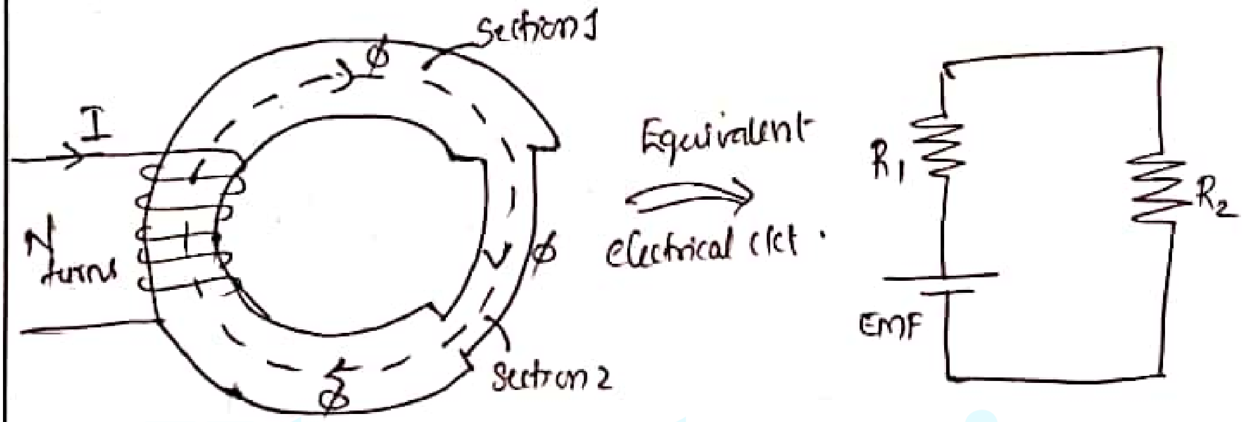
✗ Series and Parallel magnetic circuits.

In magnetic circuits eg: in electrical machines the flux passes through both stationary and rotating masses of ferromagnetic materials as well as very small air gap in between the two magnetic paths. The flux is confined to the ferromagnetic materials in the same way as a current flow is restricted in the conductor. Thus magnetic circuits consist of mainly iron paths with possibly air gaps in between.

Permeability of magnetic material >> Permeability of air.

* Series magnetic circuits

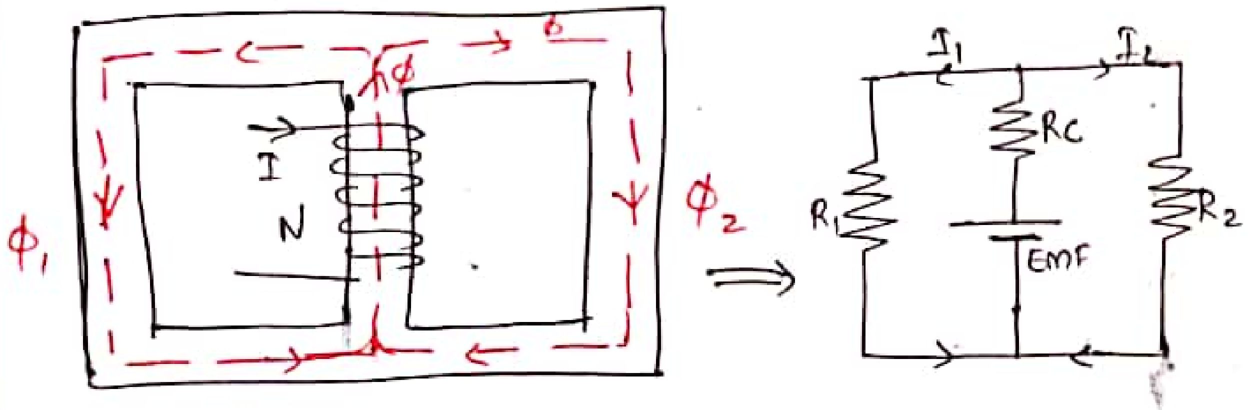
↳ A closed path for magnetic field of different sections of varying permeabilities and physical dimensions but having the same magnetic flux is called a series magnetic circuits.



Total Reluctance = Reluctance of section 1 + Reluctance of section 2.

* Parallel magnetic circuits.

The flux created by the mmf acting in the magnetic circuit gets devided in to two or more portions of the magnetic core. The fluxes in these portions may be different but the magnetic potential drops across the branches remains the same.



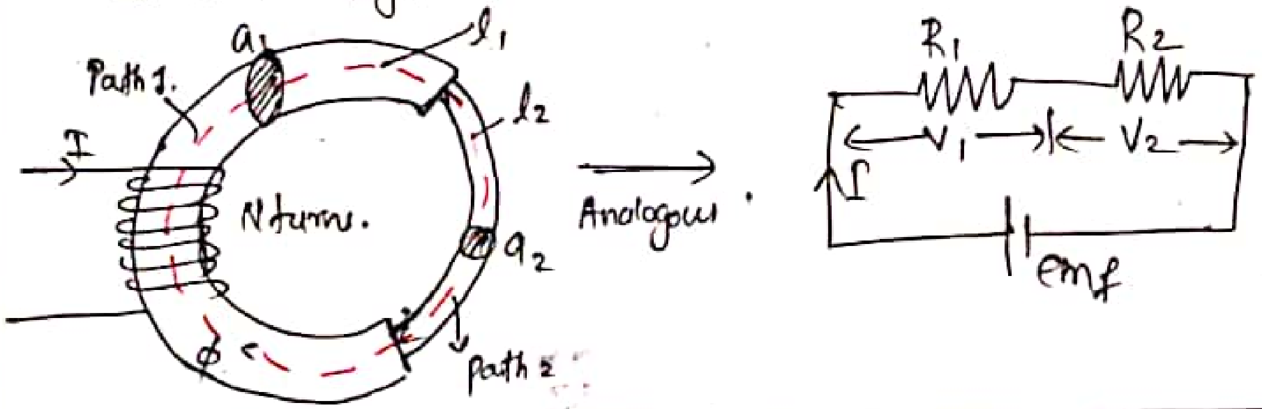
$\phi = \phi_1 + \phi_2$

Total reluctance = Reluctance of core (centre core) + Reluctance of path 1 || Reluctance of path 2.

* Series Magnetic circuits.

↳ A magnetic circuit consisting of a number of magnetic paths connected in series is referred as series magnetic circuit.

↳ Consider a series magnetic circuit consisting of 2 different magnetic core materials of different c.s.a as shown in figure.



Let l_1 = length of path 1, a_1 = c.s.a of path 1

l_2 = length of path 2, a_2 = c.s.a of path 2.

μ_{r1} = Relative permeability of path 1

μ_{r2} = Relative permeability of path 2.

We have in series magnetic circuit.

Total reluctance = Reluctance of path 1 + Reluctance of path 2

$$\boxed{S = \frac{l_1}{\mu_0 \mu_{r1} a_1} + \frac{l_2}{\mu_0 \mu_{r2} a_2}}$$

Total MMF required, MMF = MMF required by path 1 + MMF required by path 2.

$\therefore$ total MMF $= NI = H_1 l_1 + H_2 l_2$

$$\boxed{\underset{(\text{total})}{MMF} = \frac{B_1}{\mu_0 \mu_{r1}} l_1 + \frac{B_2}{\mu_0 \mu_{r2}} l_2}.$$

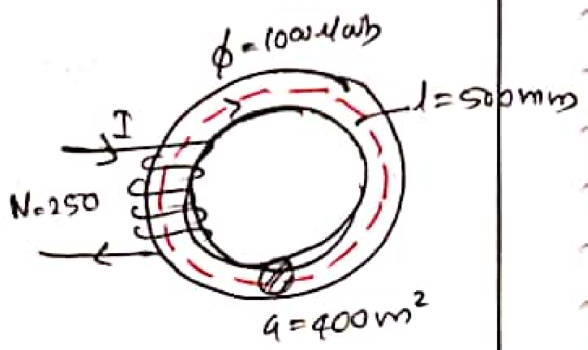
Q. Numerical

1) An iron ring having a c.s.a of $400 m^2$ and mean circumference of 500mm carries a coil of 250 turns wound uniformly around it, calculate.

(a) Reluctance of the ring.

(b) Current required to produce a flux of 1000 μwb in the ring.

Let μ_r of ring is 400.



$l = 500 \times 10^{-3}$ m

$a = 400 \times 10^{-6} m^2$

$\mu_r = 400$

$$S = \frac{l}{\mu_0 \mu_r a} = \frac{500 \times 10^{-3}}{(4\pi \times 10^{-7}) \times 400 \times (400 \times 10^{-6})}$$

$$S = 2486795 \text{ AT/wb}$$

We have mmf = Flux × Reluctance

$$NI = \phi S$$

$$250 \times I = 1000 \times 10^{-6} \times 2486795$$

$$\therefore I = 9.94 A$$

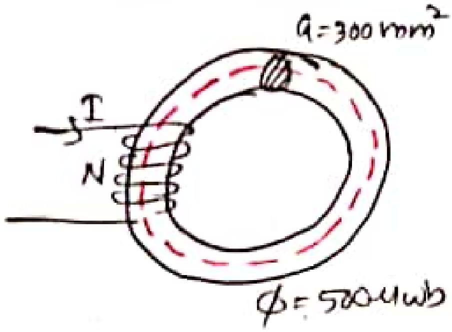
Q. 2, A mild steel ring has mean circumference of 500 mm and a uniform cs.a of 300 mm². Calculate the mmf required to produce a flux of 500 μWb. Assume $\mu_r = 1200$.

Ans: $l = 500 \times 10^{-3}$ m

$a = 300 \times 10^{-6} m^2$

$\phi = 500 \times 10^{-6} wb$

$\mu_r = 1200$



MMF = Flux × Reluctance

$= \phi \times S$

$= \phi \times \frac{l}{\mu_0 \mu_r a}$

$= (500 \times 10^{-6}) \times \frac{(500 \times 10^{-3})}{(4\pi \times 10^{-7} \times 1200 \times 300 \times 10^{-6})}$

$MMF = \underline{\underline{552.62 AT}}$

* Magnetic circuits with Air gaps.

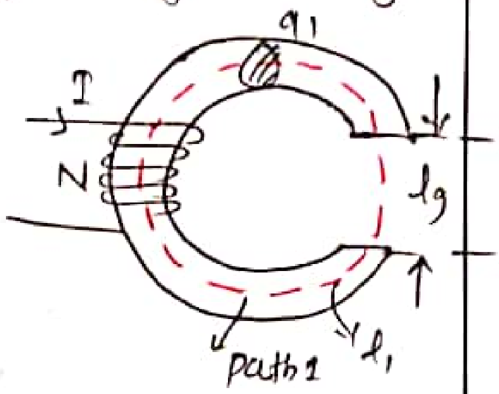
↳ Magnetic circuits may contain small airgaps in the magnetic field path.

↳ Energy conversion devices like motors which incorporate a moving element (rotor) have necessarily air gaps in its magnetic circuits.

↳ Air gaps between stator and rotor of a motor/generator will be very small few (millimeters)

↳ Air gap passes magnetic flux but offer some reluctance.

A magnetic circuit with air gap is given in figure.



l_1 = length of path 1

a_1 = C.S.A of path 1

μ_{r1} = Relative permeability of path 1.

<u>Air gap</u>

l_g = length of air gap

a_g = C.S.A of air gap

For air gap $\mu_r = 1$.

Total reluctance S = Reluctance of Path 1 + Reluctance of air gap

$$\boxed{S = \frac{l_1}{\mu_0 \mu_{r1} a_1} + \frac{l_g}{\mu_0 a_g}}$$

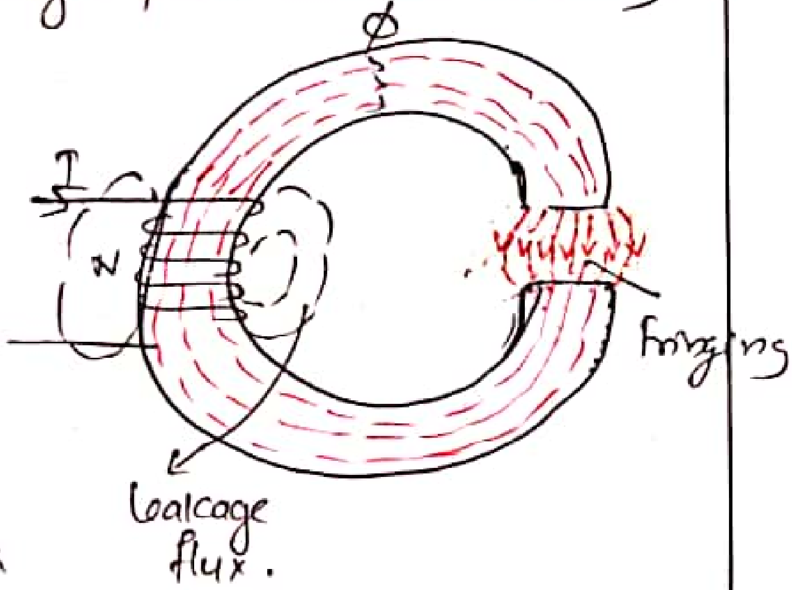
Total MMF required, MMF = NI = MMF required by path 1 + MMF required by air gap

Total MMF required

$$\boxed{MMF_{total} = \frac{B_1}{\mu_0 \mu_{r1}} l_1 + \frac{B_2}{\mu_0} l_g}$$

* Magnetic Leakage and Fringing.

Leakage flux => Flux which ~~flows~~ follows leakage path (undesired path)



↳ Fringing happens in air gap.

↳ The useful flux passing across air gap tends to bulge outwards thereby increasing effective area of cross section of air gap and reducing flux density in air gap. This effect is called as ~~Fri~~ 'Fringing'. The longer the air gap the greater is the fringing.

Note:

Area of cross section of air gap and flux density in the air gap can vary from the iron part, when fringing effect is considered.

* Useful Formulas for solving Numerical Problems.

① Reluctance $\boxed{S = \dfrac{l}{\mu_o \mu_r a}}$

② $\boxed{\text{Flux } \phi = \dfrac{\text{MMF}}{\text{Reluctance}} = \dfrac{NI}{S}}$

③ Flux density $\boxed{B = \dfrac{\text{Flux}}{\text{Area}} = \dfrac{\phi}{A}}$

④ Magnetic field intensity $\boxed{H = \dfrac{NI}{l} = \dfrac{\text{MMF}}{l}}$ or $\boxed{\text{MMF} = Hl}$

⑤ $B = \mu_o \mu_r H$

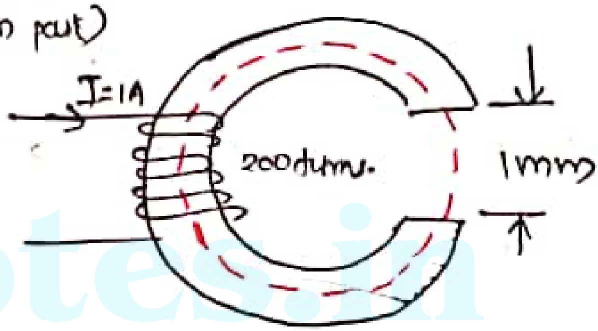
⑥ Inductance $\boxed{L = \dfrac{\mu_o \mu_r N^2 a}{l}}$

also $\boxed{L = \dfrac{N\phi}{I}}$

Numerical Problems

Q.Q. 3, An iron ring of mean length 50cm has an airgap of 1mm and a winding of 200 turns. If the relative permeability of iron is 300 when a current of 1A flows through the coil, find the flux density. Take permeability of air as $4\pi \times 10^{-7}$ A/m. Neglect leakage flux and fringing

Ans: $l_1 = 50 \times 10^{-2}$ m (for iron part)



$\mu_{r1} = 300$

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$N = 200, \ I = 1A$

Air gap, $l_a = 1 \times 10^{-3}$ m

$\mu_r = 1$

To find B

We have total mmf required = MMF required by iron part + MMF required by air gap

$$NI = H_1 l_1 + H_2 l_a$$

$$NI = \frac{B}{\mu_0 \mu_{r1}} l_1 + \frac{B}{\mu_0} l_a$$

$$200 \times 1 = \frac{B}{(4\pi \times 10^{-7})300} + \frac{B}{(4\pi \times 10^{-7})} 1 \times 10^{-3}$$

Solving for B we get

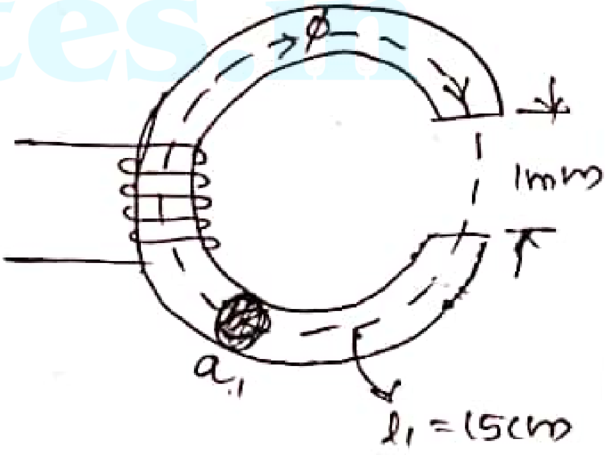
Flux density $B = 0.094$ wb/m² or Tesla.

Q.9 A circular ring has c.s.a of 12cm² and length 15cm in iron. An air gap of 1mm is made by a saw cut. Find the ampere turns needed to produce a flux of 24.84 μwb. Relative permeability of iron is 800. Neglect leakage and fringing.

$a_1 = 12 \times 10^{-4}$ m²

$l_1 = 15 \times 10^{-2}$ m

$\mu_{r_1} = 800$



$\therefore$ Air gap length $l_g = 1 \times 10^{-3}$ m

Flux $\phi = 24.84 \times 10^{-6}$ wb.

To find MMF required to produce this much flux in given magnetic circuit.

Total MMF required = MMF required by iron path + MMF required by air gap.

ie, Total MMF required $= \frac{B}{\mu_0 \mu_{r1}} l_1 + \frac{B}{\mu_0} l_g$.

We have flux density $B = \frac{\text{Flux}}{\text{Area}} = \frac{\phi}{a_1}$

$$= \frac{24.84 \times 10^{-6}}{12 \times 10^{-4}}$$

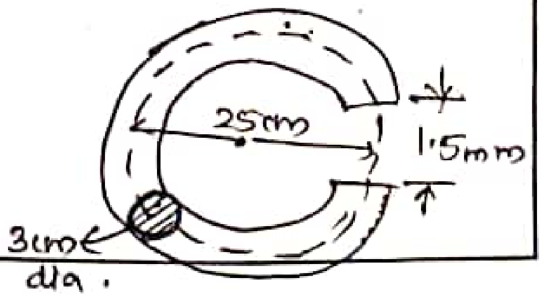
$$\therefore B = 0.0207 \text{ Tesla}$$

$\therefore$ Total MMF required $(A \cdot T) = \frac{0.0207}{4\pi \times 10^{-7} \times 800} \times 15 \times 10^{-2} + \frac{0.0207}{4\pi \times 10^{-7}} \times 1 \times 10^{-3}$

$$= 19.56 \text{ AT}$$

5, A steel ring of 25 cm diameter and of circular cross section 3cm in diameter has an airgap of 1.5 mm length. It is wound uniformly with 750 turns of wire carrying a current of 2.1 A. Calculate:
(i) MMF (ii) Flux density in air gap
(iii) Magnetic flux (iv) Relative permeability of steel ring.
Assume that iron part takes about 35% of total magneto motive force.

Ans: Given ring



$\therefore$ Area of cross section of steel part.

$$a_1 = \frac{\pi}{4} d^2$$

$$a_1 = \frac{\pi}{4}(3\times10^{-2})^2 = 7.06\times10^{-4}\,m^2$$

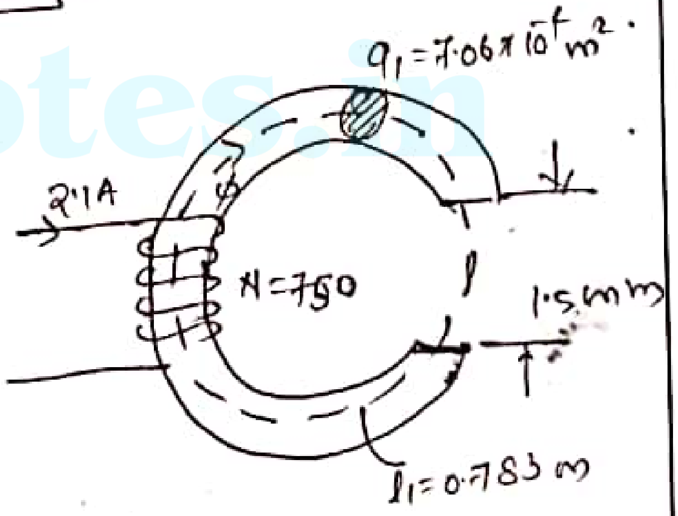
Length of iron part, l_1 = mean circumference of ring — Airgap length

$$l_1 = 2\pi R - (1.5\times10^{-3})$$

$$= 2\pi \times \left(\frac{25\times10^{-2}}{2}\right) - (1.5\times10^{-3})$$

$$l_1 = 0.783\,m$$

Given magnetic ckt can be redrawn as below.



Steel path

$a_1 = 7.06\times10^{-4}\,m^2$

$l_1 = 0.783\,m$

$l_g = 1.5\times10^{-3}$

$N = 750$, $I = 2.1A$

Total MMF produced $= NI = 750\times2.1$

$= 1575$ AT

MMF required by air gap = 65% total MMF

$$= \frac{65}{100} \times 1575 = 1023.75 \text{ AT}$$

$$\therefore \frac{B}{\mu_0} l_a = 1023.75$$

$$\frac{B}{4\pi \times 10^{-7}} \times (1.5 \times 10^{-3}) = 1023.75$$

$$\therefore B = 0.858 \text{ Wb/m}^2$$

Assume that B is same for air gap and steel part (No fringing)

$\therefore$ MMF required by steel part $= 35\%$ total MMF

$$= \frac{B}{\mu_0 \mu_{r1}} l_1$$

$$\therefore \frac{35}{100} \times 1575 = \frac{0.858}{(4\pi \times 10^{-7}) \times \mu_{r1}} \times 0.783$$

$\therefore$ Relative permeability of steel ring $\mu_{r1} = \underline{969.8}$

We have flux produced $\phi = B a_1$

$$= 0.858 \times (7.06 \times 10^{-4})$$

$$= 6.06 \times 10^{-4} \text{ wb}$$

Results

(i) MMF = 1575 AT

(ii) Flux density $B = 0.858\,T$

(iii) Flux $\phi \sim 6.06 \times 10^{-5}$ wb

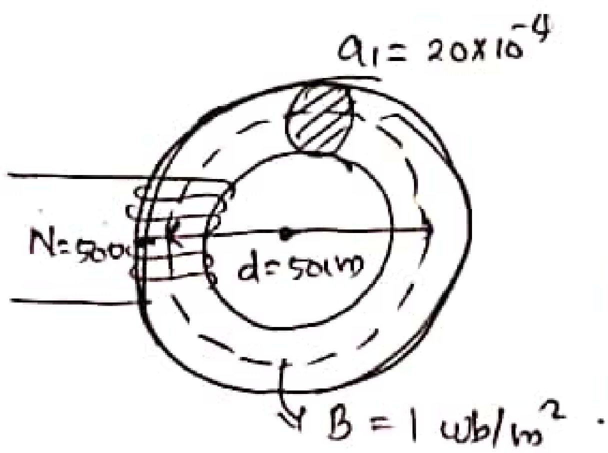
(iv) Relative permeability of steel, $\mu_r = 969.8$

Q.5 A steel ring of $20\,cm^2$ cross section having a mean diameter of 50 cm is wound uniformly with 500 turns. Flux density of $1\,wb/m^2$ is produced by 4000 AT per metre. Calculate

(i) the inductance (ii) the exciting current and

(iii) the inductance when a gap of 1mm long is cut in the ring, the flux density being $1.0\,wb/m^2$. Neglect leakage and fringing.

Ans: for (i) and (ii)



Given $a_1 = 20 \times 10^{-4}\,m^2$

diameter $= 50 \times 10^{-2}$

$\therefore$ length of path $l_1 = \pi D$

$= 1.57\,m$

$B = 1 wb/m^2, \quad H = 4000\ AT/m$

Let μ_r = Relative permeability of steel

we have $B = \mu_0 \mu_r H$

$\therefore 1 = \mu_0 \mu_r 4000$

$\therefore \mu_0 \mu_r = \frac{1}{4000} = 2.5 \times 10^{-4}$

(i) Inductance $L = \frac{\mu_0 \mu_r N^2 a}{l}$

$= \frac{(2.5 \times 10^{-4}) \times 500^2 \times 20 \times 10^{-4}}{1.57}$

$= 0.08$ Henry

(ii) We have

$H = \frac{NI}{l} \rightarrow 4000 = \frac{500 \times I}{1.57}$

$\therefore$ exciting current $I = 12.56\ A$

(iii) Magnetic circuit is modified by adding an airgap of 1mm

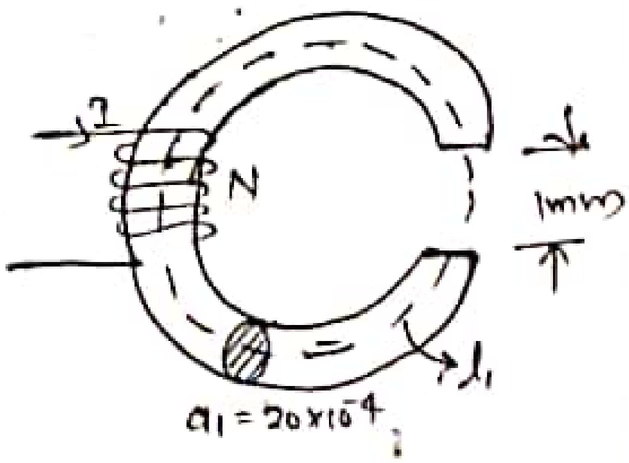
$l_g = 1mm, B = 1\ wb/m^2$

$l_1 = (1.57 - 1 \times 10^{-3})$

$= 1.569\ m$

$a_1 = 20 \times 10^{-4}$

$\mu_0 \mu_{r1} = 2.5 \times 10^{-4}$



Total mmf required = MMF required by Steel part + MMF required by air gap

$$= \frac{B}{\mu_0 \mu_{r_1}} l_1 + \frac{B}{\mu_0} l_g$$

$$= \left(\frac{1}{2.5\times10^{-4}} \times 1.569\right) + \left(\frac{1}{4\pi\times10^{-7}} \times 1\times10^{-3}\right)$$

$$= 6276 + 795.77 = \underline{\underline{7072\,AT}}$$

MMF = NI

$$\therefore I = \frac{mmf}{N} = \frac{7072}{500} = \underline{\underline{14.14A}}$$

We have,

Inductance $\boxed{L = \frac{N\phi}{I}}$

where, $\phi = B a_1 = 1\times20\times10^{-4}$

$$= \underline{\underline{20\times10^{-4}\,wb}}$$

$$\therefore L = \frac{500\times(20\times10^{-4})}{14.14}$$

$$L = \underline{\underline{0.07\text{ Henry}}}$$

* # Electromagnetic Induction

(i) When a conductor is moved in a stationary magnetic field, an emf is induced between the ends of the conductor and if the ends of this conductor are joined by a wire an induced current flows through it.

(ii) Alternatively a stationary conductor when placed in a magnetic field that changes with time develops an emf across it.

This phenomenon is known as electromagnetic induction

Faraday developed the following Laws based on this,

1) Faraday's First Law

This law states that whenever the magnetic flux linked with a conductor or a coil changes, an emf is induced in the coil or conductor.

2, Faraday's Second Law

This law states that the magnitude of the induced emf in the conductor by electromagnetic induction, is equal to the rate of change of flux linkages. The term flux linkage mean the product of flux in weber and the number of turns with which the flux is linked (ie, $N\phi$)

ie, Induced emf,

$$e = \text{Rate of change of flux linkage.}$$

$$e = \frac{d(N\phi)}{dt}$$

$$\boxed{e = N\frac{d\phi}{dt}} \text{ Volts.}$$

Eg: Consider a magnet moving inside a coil as shown below.

Fig.1 : Initial position

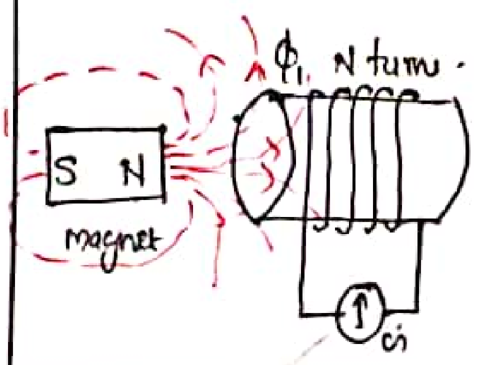
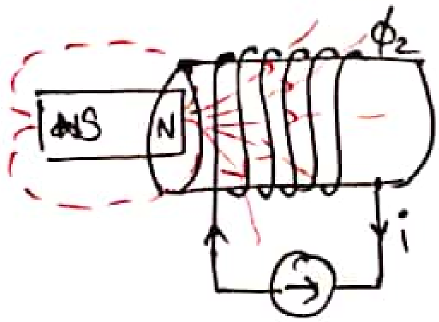


Fig 2. Magnet moving inside coil



ϕ_1 = Initial flux linking with coil

G → Galvanometer to detect current.

ϕ_2 = Final flux linking when magnet is moved.

Here change in flux linkage = $N(\phi_2 - \phi_1)$

By Faraday's Laws

$$e = \frac{d}{dt} N(\phi_2 - \phi_1)$$

$$e = N \frac{d}{dt}(\phi_2 - \phi_1).$$

Let $\phi = \phi_2 - \phi_1$

$$\boxed{e = N \frac{d\phi}{dt}}$$

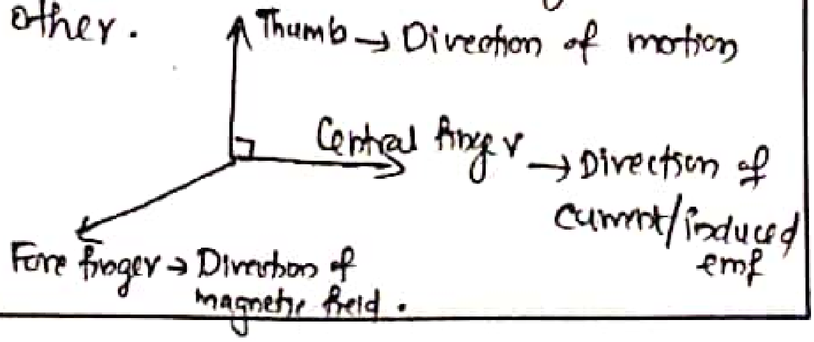
The direction of induced emf can be determined in two ways

(i) Fleming's right hand rule and

(ii) Lenz's law

* Fleming's Right hand Rule :-

Hold thumb, forefinger and central finger at right angles to each other.



* Lenz's Law :

It states that the direction of induced emf is such that it will establish a current which will oppose the change of flux responsible for inducing that emf. Therefore in accordance with Lenz's law a negative sign is assigned to the expression of emf

ie, Induced emf $\boxed{e = -N \frac{d\phi}{dt}}$

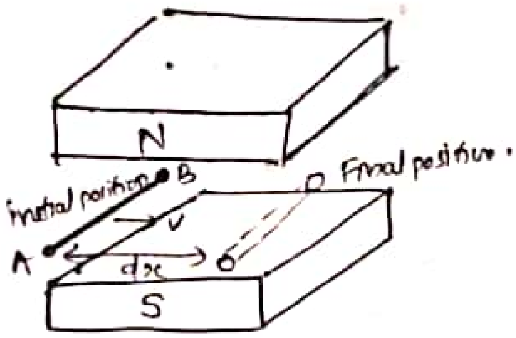
-Ve sign because of Lenz's law.

* Dynamically induced emf

↳ Emf induced due to relative motion of a conductor with respect to magnetic field

↳ Magnitude of this induced emf can be obtained by flux cutting rule.

Consider a stationary magnetic field with a flux density of 'B' wb/m². Let a single conductor of length 'l' is moving in that field with a velocity of 'v' m/s.



Let the conductor moves a small distance 'dx' in 'dt' time in perpendicular to field.

ie, Area swept by the conductor $= l \times dx$.
when it moves 'dx' distance

$\therefore$ Flux cut by the conductor during its motion $d\phi = B\, l\, dx$

By Faraday's laws of induction for a single conductor $e = \dfrac{d\phi}{dt} = B\, l \dfrac{dx}{dt}$

($\because$ B and l are constant values)

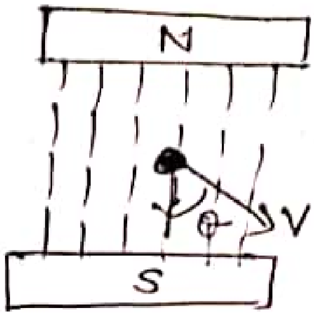
We have $\dfrac{dx}{dt}$ = Velocity (V)

$\therefore \boxed{e = BlV}$ volts.

Note: If conductor is moving at an angle 'θ' with the direction of magnetic field then,

induced emf $\boxed{e = Blv \sin\theta}$ Volts

The direction of this dynamically induced emf can be obtained by Fleming's Right hand rule.



Q. Numerical Problems

A straight conductor 100 cm long and moving in a direction perpendicular to the magnetic field of density 1.5 wb/m² If the velocity of motion is ~~at~~ 5 m/sec find the induced emf.

Given $l = 100\ cm = 1m$

$\theta = 90°$

$B = 1.5\ Wb/m^2$

$V = 5\ m/sec.$

$E = Blv \sin\theta = 1.5 \times 1 \times 5 \times \sin 90$

$= 7.5V$

Note: Emf induced will be maximum when conductor moves perpendicular to magnetic field (ie, sin 90 = 1) max value)

* Statically induced emf

↳ Emf induced in a conductor when it is placed in a time varying magnetic field. So there is no need for any physical relative movement.

↳ Magnitude and direction of statically induced emf is obtained by Lenz's law and Faraday's laws of EMI

ie. Statically induced emf $\boxed{e = -N\frac{d\phi}{dt}}$

Here ϕ will be time varying flux or alternating flux Eg: $\phi = \phi_m \sin\omega t$

Statically induced emf is further classified in to two, (i) self induced emf and

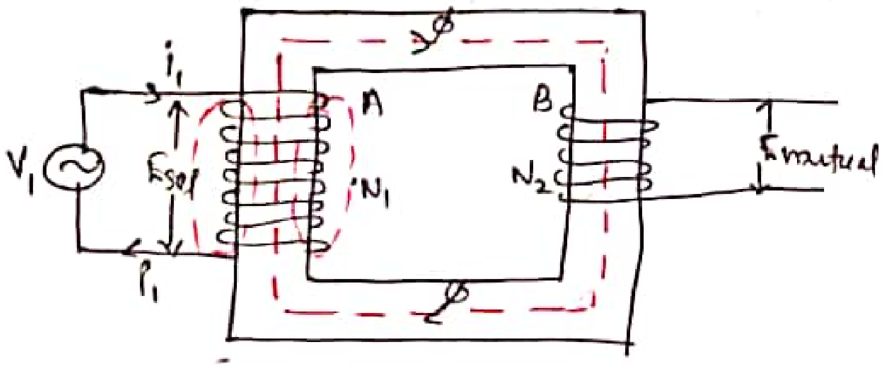
(ii) Mutually induced emf

Consider two coils A and B which are magnetically coupled (with a magnetic core).

Let N_1 = No. of turns in coil A

N_2 = No. of turns in coil B

Let an alternating voltage of 'V_1' is applied to coil A and an ac current 'i_1' flows through it



Let 'ϕ' is alternating or time varying flux produced in coil A

↳ Emf induced in a coil due to the time varying nature of current passing through the same coil itself is called self induced emf (In coil A)

↳ Emf induced in a coil due to time varying nature of current passing through another coil which is magnetically coupled to it is known as mutually induced emf (in coil B) due to current in coil A.

Self induced emf $e_s = -\frac{d(N_1\phi)}{dt}$

$$e_s = -N_1 \frac{d\phi}{dt}$$

$$e_s = -\left(N_1 \frac{d\phi}{di}\right) \times \frac{di}{dt} = -\frac{d(N_1\phi)}{di} \times \frac{di}{dt}$$

$$\boxed{e_s = -\frac{d(N_1\phi)}{di}\frac{di}{dt} = -L\frac{di}{dt}}$$

L is the self inductance of coil A.

and $L = \frac{d(N_1\phi)}{di}$ = Rate of change of flux linkage w.r.t change in current in same coil

Mutually induced emf $e_m = \frac{-d(N_2\phi)}{dt} = -N_2\frac{d\phi}{dt}$

$$e_m = -\left(N_2 \frac{d\phi}{di}\right) \cdot \frac{di}{dt} = -\frac{d(N_2\phi)}{di} \cdot \frac{di}{dt}$$

$$\boxed{e_m = -M\frac{di}{dt}}$$

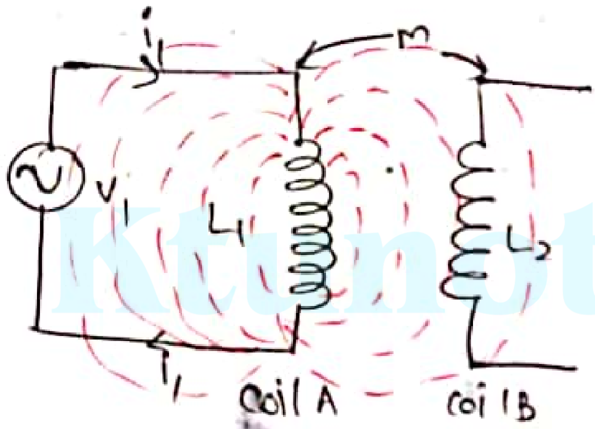
$M \rightarrow$ Mutual inductance between coil A and coil B

$M = \frac{-d(N_2\phi)}{di} \rightarrow$ Rate of change of flux linkage of a coil due to time varying current in another coil.

* Coefficient of coupling between two coils (K)

The coefficient of coupling (K) between two coils is defined as the fraction of magnetic flux produced by current in one coil that links the other coil.

Consider two coils, coil A and coil B having self inductance L_1 and L_2 respectively. Let ~~M~~, 'M' is the value of mutual inductance b/w two coils.



Let coil A produces ϕ_1 flux when current passes through it. A part of this total flux will link only coil A and will not reach coil B (called leakage flux) while the balance flux will link coil B also (called mutual flux)

ie, $\phi_1 = \phi_{11} + \phi_{12}$

Where ϕ_{11} = Flux produced by coil A linking coil A only (leakage flux)

ϕ_{12} = Flux produced by coil A which also links coil B (mutual flux) or (common flux)

We have $\boxed{\phi_{12} = K\phi_1}$

Where 'K' is known as coefficient of coupling b/w these coils. Range of $0 \leq K \leq 1$

If $K = 1$ (means coils are perfectly coupled (All flux links coil B).)

$K = 0$ (No coupling b/w two coils)

$K > 0.5$ (coils are tightly coupled more than 50% flux links coil B)

Relationship between L_1, L_2 and M

$$\boxed{M = K\sqrt{L_1 L_2}}$$

Since range of K is 0 to 1. Maximum value of mutual inductance possible $M = \sqrt{L_1 L_2}$ when $K = 1$

Q. An ideal mutual inductor is made from a primary
UQ. coil of inductance 5mH and a secondary coil of inductance 10mH. Find the value of mutual inductance

$L_1 = 5mH, \quad L_2 = 10mH$

$$M = K\sqrt{L_1 L_2}$$

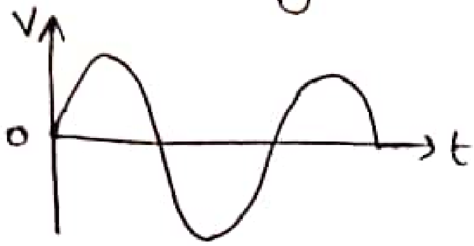
Given ideal mutual inductor means perfectly coupled coil, ie, $K = 1$

$$\therefore M = \sqrt{L_1 L_2}$$

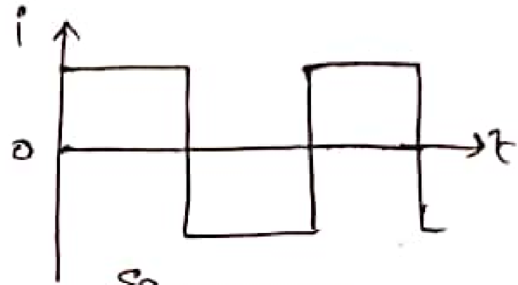
$$= \sqrt{5 \times 10} = 7.07mH$$

* Alternating current fundamentals.

An alternating current or voltage reverses its direction at regular repeated intervals.



Sin wave



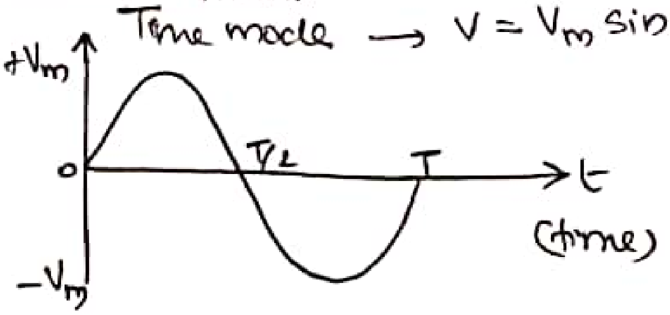
Square wave

In electrical Engineering the generated voltage and current waveforms are sinusoidal waves of frequency 50 Hz (in India)

* Representation of sinusoidal waveforms.

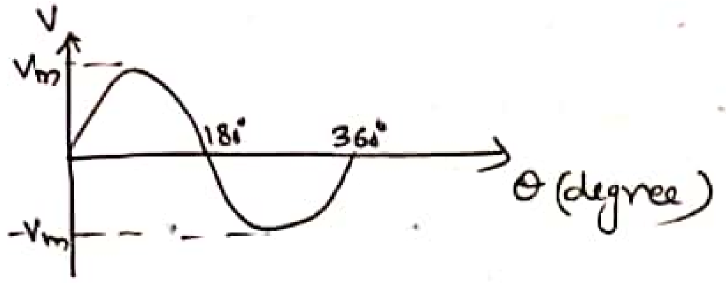
A sine wave can be represented in mainly three different forms.

(i) Time mode $\rightarrow V = V_m \sin(\frac{2\pi}{T} t)$

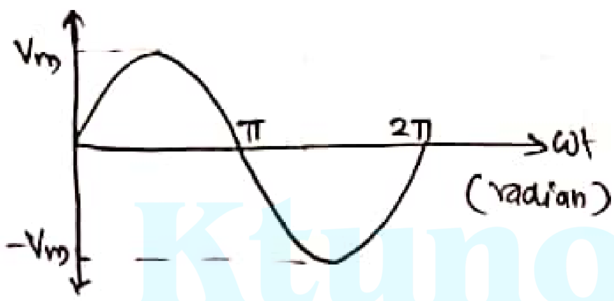


Where T = Time period (Time for 1 full cycle).

(ii) <u>Degree mode</u> → $V = V_m \sin\theta$



(iii) <u>Radian mode</u> → $V = V_m \sin(\omega t)$



where ω = Angular frequency in (radian/second)

$\omega = 2\pi f$

f = frequency in hertz.

ie. 2π radian $= 360°$

↳ Radian mode is most commonly used in electrical Engineering.

* <u>Basic Terminology</u>

1) <u>Time period (T)</u>

It is the time required to complete one complete cycle (complete set of +ve and −ve values)

2, Frequency (f)

It is the number of cycles per second.

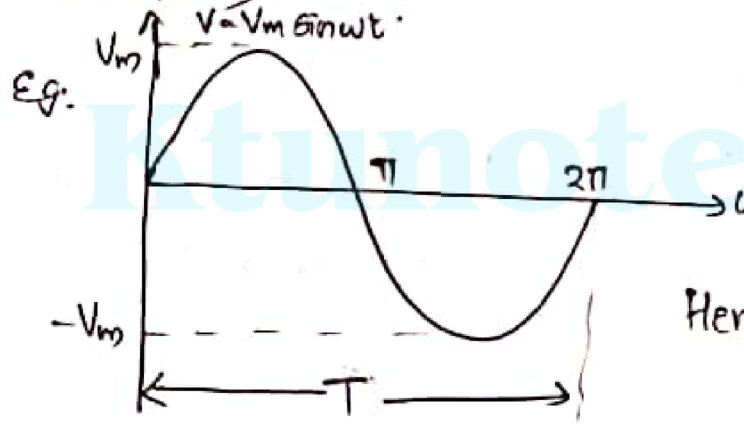
↳ Unit is Hertz (Hz)

$\left(f = \frac{1}{T}\right)$ Hz. where T = Time period in seconds.

3, Amplitude / maximum value / peak value

↳ Maximum +ve or −ve value of an alternating waveform

Eg.



Here T = 2π radian

4) Root mean Square (RMS) value / Effective value /

↳ RMS value is the most important value of an alternating voltage or current.

Simply an AC current 'i = 5A' means it is RMS value of current.

Definition : "Rms value is that steady value of a time varying waveform which could also develop same amount of heat as given by the original waveform when passed through a certain resistance for a certain time"

Equation $\boxed{V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t) \cdot dt}}$

5, Average Value

The average value of an alternating quantity is defined as the average over the positive half cycle of the waveform

a, For symmetrical waveforms → +ve area = -ve area

V_{avg} → 0 for one full cycle.

V_{avg} → $\boxed{V_{avg} = \frac{1}{T/2}\int_0^{T/2} v(t) \cdot dt}$ → For half cycle.

b, For unsymmetrical waveform → +ve area ≠ -ve area.

$$\boxed{V_{avg} = \frac{1}{T}\int_0^{T} v(t) \cdot dt}$$

6, Peak factor

Peak factor for a given waveform is the ratio of its peak value to the RMS value.

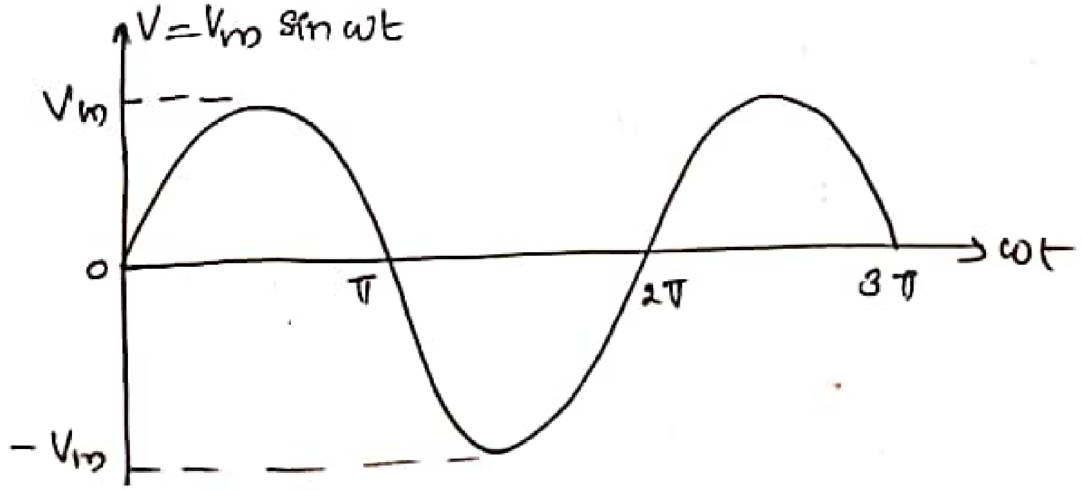
$$\text{Peak factor} = \frac{V_{peak}}{V_{rms}} = \frac{V_m}{V_{rms}}.$$

7) Form factor (F.F)

Form factor is the ratio of its rms value to the average value for an alternating waveform

$$\text{Form factor} \quad F.F = \frac{V_{rms}}{V_{avg}}.$$

* Derivation of Vrms, Vavg, form factor and peak factor for a pure sinusoidal waveform



Average Value

$V_{avg} = 0$ for 1 full cycle (since +ve area = −ve area)

For half cycle $V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t) \cdot d(\omega t)$

$$V_{avg} = \frac{V_m}{\pi}\int_0^{\pi} \sin \omega t \, d(\omega t)$$

$$V_{avg} = \frac{V_m}{\pi}\left[-\cos(\omega t)\right]_0^{\pi}$$

$$= \frac{V_m}{\pi}\left[-(\cos\pi - \cos 0)\right]$$

$$V_{avg} = \frac{V_m}{\pi}(-(-1-1))$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$

Rms value

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2(\omega t) \cdot d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2(\omega t) \cdot d(\omega t)}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \frac{1-\cos 2(\omega t)}{2} \cdot d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\int_0^{2\pi} 1-\cos 2(\omega t) \cdot d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[(\omega t) - \frac{\sin 2(\omega t)}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\left(2\pi - \frac{\sin 4\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi} \times 2\pi} = \frac{V_m}{\sqrt{2}}$$

$$\therefore \boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

$$\text{Peak factor} = \frac{V_m}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = \underline{\underline{\sqrt{2}}}.$$

$$\text{Form factor} = \frac{V_{rms}}{V_{avg}} = \frac{(V_m/\sqrt{2})}{(2V_m/\pi)} = \frac{\pi}{2\sqrt{2}} = \underline{\underline{1.11}}.$$

Q. <u>Numerical problems</u>

No. 1, 4 marks. Calculate the Rms and average values of a purely sinusoidal current having peak value 15A.

Given $I_m = 15A$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{15}{\sqrt{2}} = \underline{10.6\,A}$$

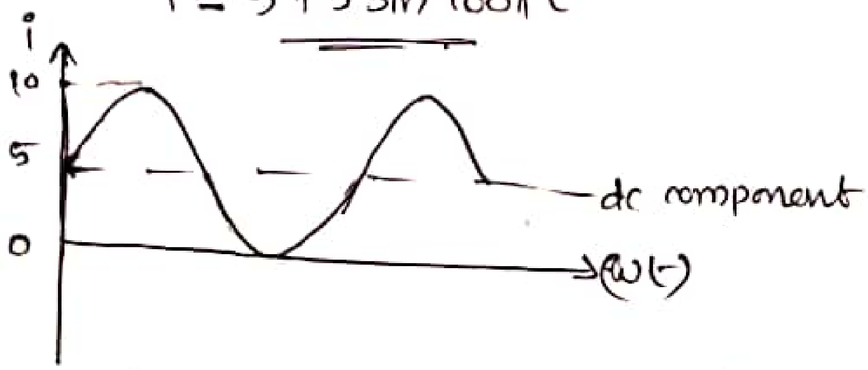
$$I_{avg} = \frac{2I_m}{\pi} = \frac{2\times15}{\pi} = \underline{9.55A}$$

2) A current wave is made up of two components a 5A DC component and a 50Hz AC component which is sinusoidal wave with a peak value of 5A. Sketch the resultant waveform and determine its RMS and average values.

Ans Given waveform $i = 5 + 5\sin\omega t$
(dc) (ac)

where $\omega = 2\pi \times f = 2\pi\times50 = 100\pi$ rad/sec.

$i = 5 + 5\sin 100\pi t$



Average value of i = 5A (for sinusoidal part Avg = 0).

RMS value $i_{rms} = \sqrt{5^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{25+12.5}$

$i_{rms} = \underline{6.1A}$

8) An alternating voltage is expressed as $V = 14.14 \sin 314t$

Determine a) rms voltage b) Frequency

c) form factor d) Instantaneous voltage when $t = 2$ msec.

Ans: Given $V = V_m \sin \omega t = 14.14 \sin 314t$

$\therefore V_m = 14.14 V$, $\omega = 314$ rad/sec.

a) $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{14.14}{\sqrt{2}} = \underline{\underline{10A}}$.

b) $\omega = 314 = 2\pi f \Longrightarrow f = \frac{314}{2\pi} = \underline{\underline{50 Hz}}$.

c) Form factor $FF = \frac{V_{rms}}{V_{avg}} = 1.11$

d) When $t = 2 ms = 2 \times 10^{-3} s$

$$V = 14.14 \sin (314 \times 2 \times 10^{-3})$$

$$= 14.14 \times 0.587 = \underline{\underline{8.30 V}}$$