Module 2

Interference and Diffraction

Interference

The intensity variations obtained by the superposition of two or more light waves is called interference.

The intensity variations recieved on a screen is called interference pattern. such pattern consists of alternatively arranged dark and bright frindges or intensity minima and intensity maxima

The phenomenon of Interference can be explained on the basis of principle of superposition of waves.

Principle of super position of waves

It states that when two or more light waves interfere at a medium, the resultant displacement at any point in the medium is the vector sum of displacement due to individual waves.

Case 1

Bright frindge or constructive interference pattern

The crust and crust or trough and trough of waves interfere.

waves are in phase.

Dark frindge or destructive pattern

crust and trough of waves interfere and viceversa.

waves are out of phase.

A perfect interference pattern consists of all bright fringes with equal brightness and dark with equal and complete darkness. such pattern should be formed when two coherand wave super imposes with each other.

coherence

A predictable corelation of the amplitude and phase at any point of a wave with any other wave is called coherence.

There are two types of coherence:

1) Temporal coherence

It is the measure of corelation b/w the phases of a light wave at different points along direction of propogation. This tells us how monochromatic the source is.

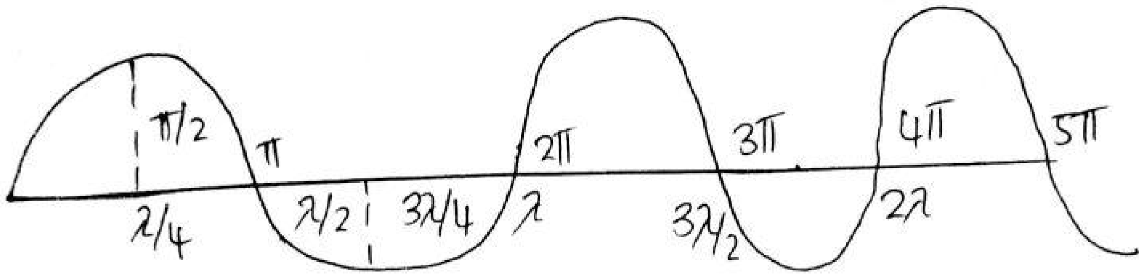
2) Spatial coherence

It is the measure of corelation b/w the phases of a light wave at different points traverse to direction of propogation. This tells us how uniform the phase of wave front is.

coherent sources

The source emitting waves of same amplitude, frequency, phase are called coherent sources.

Relation b/w path difference (x) and phase difference (δ)



$$\frac{\text{Path difference }(x)}{\text{phase difference }(\delta)} = \frac{\lambda}{2\pi}$$

$$\text{Phase difference}(\delta) = \left(\frac{2\pi}{\lambda}\right)\text{path difference}(x)$$

$$\text{path difference}(x) = \left(\frac{\lambda}{2\pi}\right) \times \text{phase difference}(\delta)$$

Condition of constructive and destructive interference in terms of path difference (x) and phase difference (δ)

constructive interference

a) In terms of path diff. (x)

If path difference b/w two waves is an integral multiple of λ constructive interference takes place

$\boxed{x = n\lambda}$ where $n = 0, 1, 2, 3 \cdots$

b) In terms of phase difference (δ)

Constructive interference occurs, only when the phase difference between two waves is an even integral multiple of π.

$\boxed{\delta = 2n\pi}$ where $n = 0, 1, 2, 3 \cdots$

~~Interference due to~~ Destructive interference

a) In terms of path difference(x)

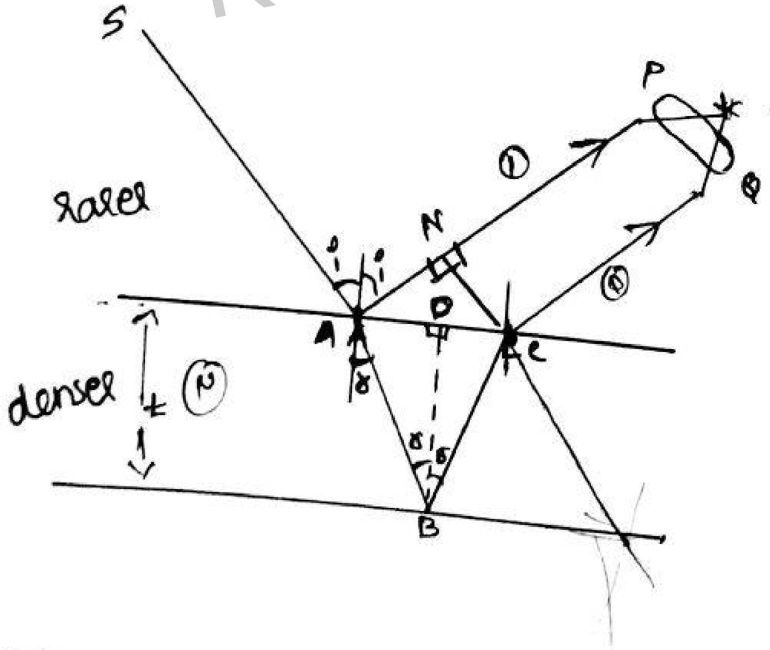
If the path difference b/w two waves is an odd integral multiple of $\lambda/2$. Destructive interference take place.

$\boxed{x = (2n \pm 1)\lambda/2}$ where $n = 0, 1, 2, 3 \ldots$

b) In terms of phase diff. (δ)

Destructive interference occurs only when the phase difference b/w two waves is an odd integral multiple of π.

$\boxed{\delta = (2n \pm 1)\pi}$ $n = 0, 1, 2, 3 \ldots$



Interference due to reflected light from a plane parellel thin film

Consider a transparent film of refractive index μ and thickness t. suppose a ray SA incident on the upper surface of the film, at the point A is partially reflected along AP and partially refracted along AB.

At B part of it is reflected along BC and finally emerges out along CQ.

To find optical path diff. b/w rays SAP and SABCQ, draw a line CN normal to AP, such that NP = CQ.

$x = (2) - (1)$

$= SA + [AB + BC]\mu + CQ - [SA + AP]$

$\therefore x = \cancel{SA} + [AB + BC] + CQ - [\cancel{SA} + AN + NP]$, $NP = CQ$

$\boxed{x = [AB + BC]\mu - AN}$ —— (1)

$\boxed{x = 2AB\mu - AN}$ — (2) $\left[\begin{array}{l} \Delta^{le} ABD = \Delta^{le} CBD \\ AB = BC \\ AD = DC \end{array} \right]$

Δ^{le} ABD,

$\cos\hat{r} = \frac{BD}{AB} = \frac{t}{AB}$

ie, $\boxed{AB = \frac{t}{\cos r}}$ — (3)

[To find the value of AB and AN in terms of μ, T, R considering Δ^{le} ABD as above]

sub (3) in (2) we get

$\boxed{x = \frac{2\mu t}{\cos r} - AN}$ — (4)

To find value of AN in terms of μ, T, R.

consider Δ^{le} ACN and find value of sin i,

$\sin i = \frac{AN}{AC} = \frac{AN}{AD+DC} = \frac{AN}{2AD}$

$\therefore \boxed{AN = 2AD \sin i}$ — (5)

Consider ΔABD and find tan r

$\tan r = \frac{AD}{BD} = \frac{AD}{t}$

$\therefore \boxed{AD = t \cdot \tan r}$ — (6)

Sub. 6 in 5

$AN = 2t \tan r \sin i \quad \left[\mu = \frac{\sin i}{\sin r}, \sin i = \mu \sin r\right]$

$$\boxed{AN = 2t \frac{\sin r}{\cos r} \mu \sin r} \quad (7)$$

Sub 7 in 4

$$x = \frac{2\mu t}{\cos r} - \frac{2t \sin^2 r \mu}{\cos r}$$

$$x = \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$\therefore \boxed{x = \frac{2\mu t \cos^2 r}{\cos r} = 2\mu t \cos r}$$

$$\Rightarrow \boxed{x = 2\mu t \cos r \pm \lambda/2}$$

[As the ray undergo reflection at rarer denser interface a phase change π or path diff. $\lambda/2$ exists b/w incident ray SA and the reflected ray AP.

$\therefore$ The total optical path diff. b/w the two rays is $\pm \lambda/2$]

$$\Rightarrow \boxed{x = 2\mu t \cos r \pm \lambda/2} \quad (8)$$

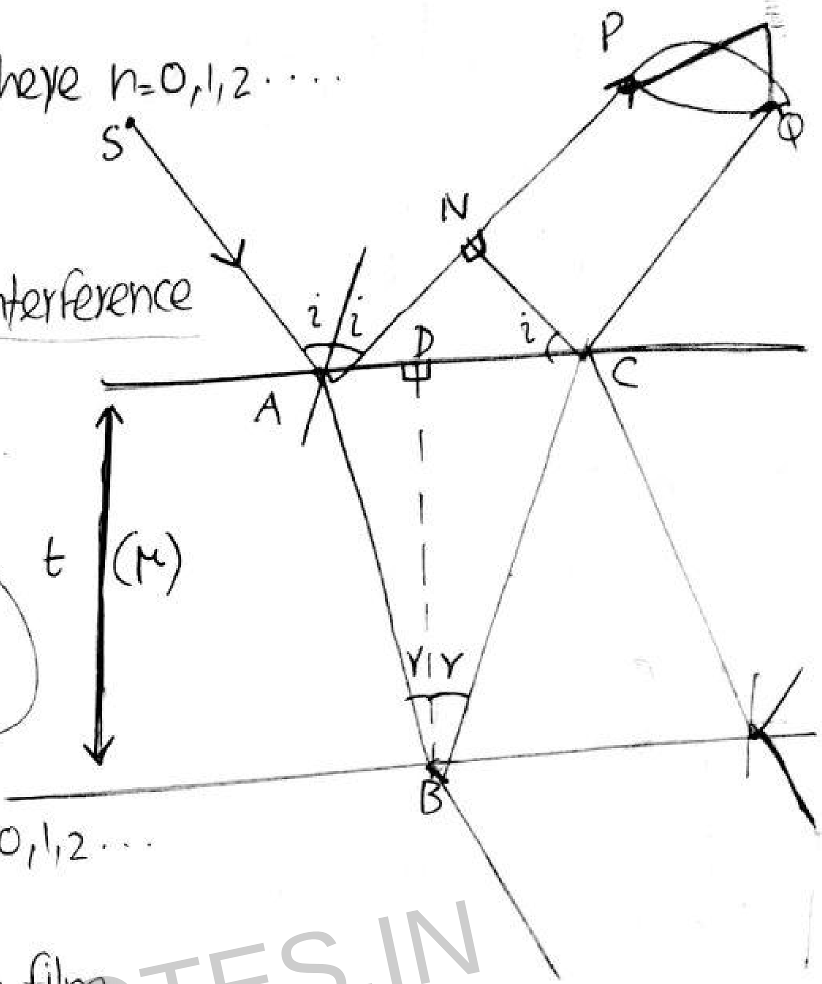
Case 1 [condition for constructive interference]

$x = n\lambda$ (General condition)

Sub in 8

$n\lambda = 2\mu t \cos r \pm \lambda/2$

$\Rightarrow \boxed{2\mu t \cos r = (2n+1)\lambda/2}$ where $n = 0, 1, 2 \cdots$



Case 2

condition for destructive interference

General condition,

$x = (2n+1)\lambda/2$

comparing,

$(2n+1)\lambda/2 = 2\mu t \cos r \pm \lambda/2$

$\therefore \boxed{2\mu t \cos r = n\lambda}$ where $n = 0, 1, 2 \cdots$

Reason for colours of thin film

when a beam of white light (sun light) incident normally on a thin film and seen in reflected light, coloured fringes are observed. These colours arised due to interference of light waves reflected from upper and lower surface of the film. In reflected system, $\boxed{2\mu t \cos r = (2n+1)\lambda/2}$ is the condition for constructive interference and $\boxed{2\mu t \cos r = n\lambda}$ is condition for destructive interference.

when white light is incident on a thin film, the light which come from any point will not include the colour whose wavelength satistify the eqn $\boxed{2\mu t \cos r = n\lambda}$ in reflected system.

$|||^{ly}$, the wavelength satisfy eqn $\boxed{2\mu t\cos r = (2n\pm1)\lambda/2}$ appear bright in the colour and also these colour depends on the thickness 't' of the film and angle of inclination 'r'.

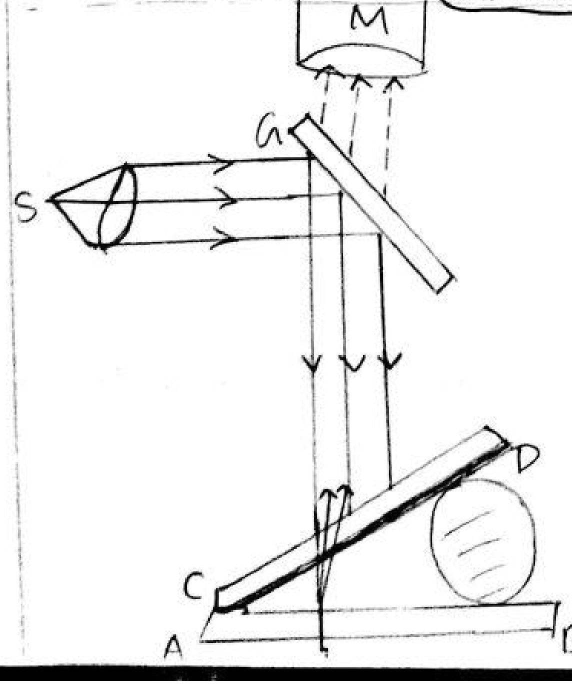
In case of oil spread over water, different colours are seen because its 'r' and 't' ~~and~~ values are varies. If 'r' and 't' are constant, colour of the film must be uniform.

Q) The colours exhibited by the reflected and transmitted systems are complementary with each other. why?

Ans:) In the case of reflected system,

we have, $2\mu t\cos r = (2n+1)\lambda/2$ for constructive interference and $2\mu t\cos r = n\lambda$ for destructive inteference. But in case of transmitting system we have $2\mu t\cos r = n\lambda$ for constructive and $2\mu t\cos r = (2n+1)\lambda/2$ for destructive inteference.

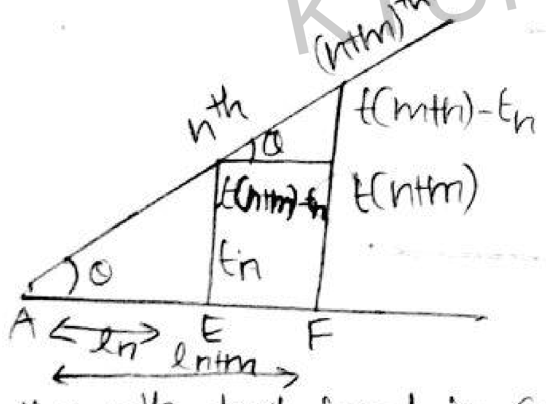
Interference due to thin wedge shaped air film



When two glass plates AB and CD are placed in such a way that the ends A and C are kept in contact with each other and a thin wire whose thickness is to be calculated is inserted in b/w B and D. This forms an wedge shaped air film as shown in the fig.

When a normally incident monochromatic light is incident on the wedge film from G, an alternatively arranged, equidistant fringes are formed llel to the line of intersection of the glass plates AB and CD.

Let 'θ' be the angle of the wedge



Let the n^{th} dark band is formed at E where the thickness of air film is t_n.

And $(n+m)^{th}$ dark fringe is formed at F where the thickness of air film is t_{n+m}

Assume,

$AE = l_n$, $A_F = l_{n+m}$

$$\tan\theta = \frac{t_n}{l_n} = \frac{t_{(n+m)}}{l_{(n+m)}} = \frac{t_{(n+m)} - t_n}{l_{(n+m)} - l_n}$$

Since θ is too small, $\tan\theta \approx \theta$

$$\therefore \boxed{\theta = \frac{t(n+m) - t_n}{l(n+m) - l_n}} \quad — (1)$$

Also, we have for n^{th} dark frindge in reflected system :

$$2\mu t_n \cos\gamma = n\lambda$$

In bridge film, $i = \gamma = 0$

$$\therefore \boxed{t_n = \frac{n\lambda}{2\mu}} \quad — (2)$$

|||ly, the condition of $(n+m)^{th}$ dark frindge

$$2\mu t_{(n+m)} \cos\gamma = (n+m)\lambda$$

$$\therefore t_{(n+m)} = \frac{(n+m)\lambda}{2\mu} \quad — (3) \quad (i = \gamma = 0)$$

From (2) and (3)

$$t_{n+m} - t_n = \frac{(n+m)\lambda}{2\mu} - \frac{n\lambda}{2\mu} = \frac{m\lambda}{2\mu} \quad — (4)$$

Sub (4) in (1)

$$\boxed{\theta = \frac{m\lambda}{2\mu(l_{(n+m)} - l_n)}} \quad \text{—} (5)$$

We know, fringe width $\beta = \frac{\text{distance b/w } n^{th} \text{ and } (n+m)^{th} \text{ dark band}}{\text{Total no. of fringes b/w } n \text{ and } (n+m)}$ ~~band~~

$$\beta = \frac{l_{(n+m)} - l_n}{m}$$

$$\frac{1}{\beta} = \frac{m}{l_{(n+m)} - l_n}$$

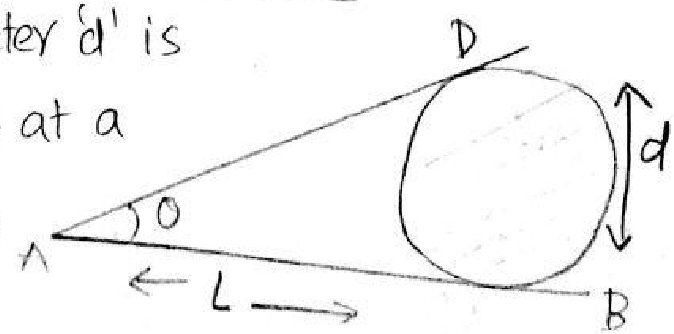
Sub in (5)

$$\boxed{\theta = \frac{\lambda}{2\mu\beta}}$$

It is the expression to findout the wedge angle θ. If two glass plates are optically plane then the fringde width β will be the same for all bands.

Equation to find out diameter of thin wire

Let a thin wire of diameter 'd' is placed in b/w the glass plates, at a distance 'L' from edge A. then,



$\tan\theta = \frac{d}{L}$

$\tan\theta \approx \theta$

$\theta = \frac{d}{L}$ —⑥

Sub ⑥ in ⑤

$\frac{d}{L} = \frac{\lambda}{2\mu\beta}$

$$\boxed{d = \frac{\lambda L}{2\mu\beta}} \quad —⑦$$

Using this equation thickness of a thin object can be determined.

Newton's ~~rays~~ Rings

A concentric circular interference pattern is formed due to the interference of monochromatic light waves, reflected from a thin symmetrically shaped air film, that enclosed b/w a plano-convex lens and plane glass plate is called newton's rings. This ring pattern were first discovered by newton and hence it is called newton's rings.

The fringes are circular because, the shape of the air film is symmetrical about the point of contact of the lens and the glass plate.

The centre of newton rings look dark in reflected system and bright in transmitted system.

Reason for central darkness of newton ring in reflected system

At the point of contact b/w ~~length~~ lens L and glass plate G, the thickness of air film is ≈ 0.

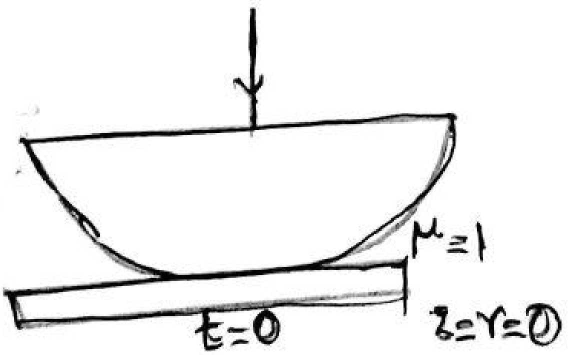
$\therefore$ The beam of light reflected from the air-glass interface undergoes an additional path diff. $\lambda/2$. ie, Two beams interfere destructively. Hence, the central spot is dark.

OR

The general condition for path diff. of light waves reflected from a thin film,

$\boxed{x = 2\mu t \cos r \pm \lambda/2}$

when it appeying in newton ring setup we consider a thin film of air is enclosed b/w glass plate and lens.



At centre, $t=0$

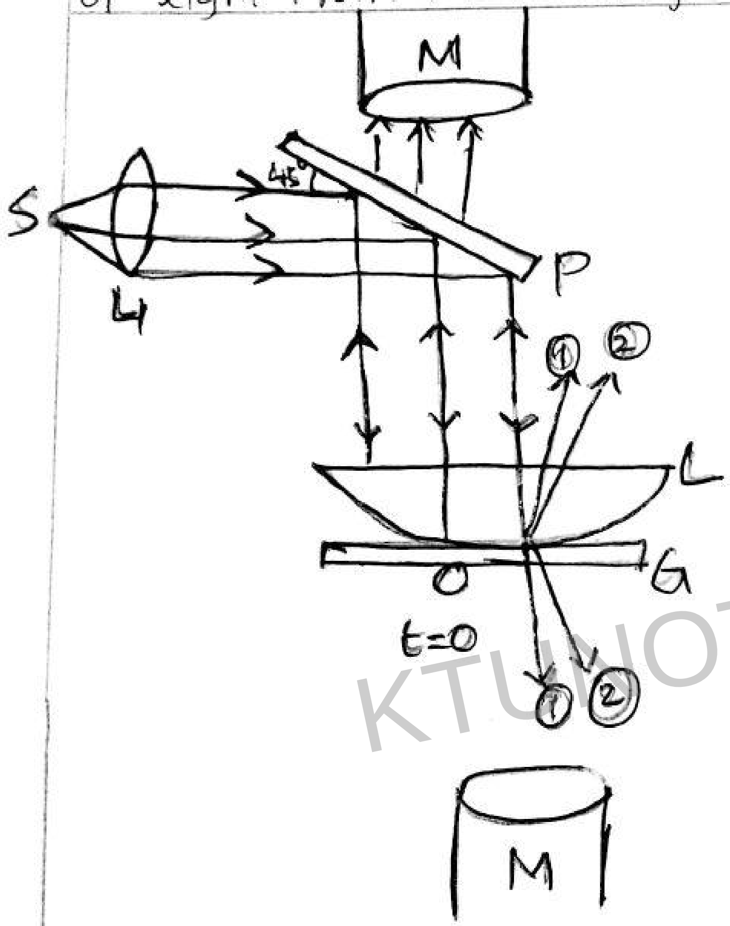
$\therefore i=r=0$

$\Rightarrow x = 2t \pm \lambda/2$

$\therefore \boxed{x = \pm \lambda/2}$ is the condition for destructive interference

Hence, the central spot looks ~~in~~ dark in reflected system.

Experimental setup and theory of obtaining wavelength of light from newton rings experiments



The apparatus are setup as shown in the fig.

when the normally incident monochromatic light reflected from the lower surface of the lens L and upper surface of glass plate G super imposes with each other, a concentric circular dark and bright fringes are formed and it can be viewed or focused in the field of view of travelling microscope m.

<u>To find the radius of (r_n) and diameter (d_n) of n^{th} dark ring</u>

Let 'R' be the radius of curvature of plano convex lens L and 't' be the thickness of air film at 'A' corresponding to n^{th} dark ring.

Condition generally for nth dark fringe in reflected system is given by:

$$\boxed{2\mu t\cos r = n\lambda} \quad (1)$$

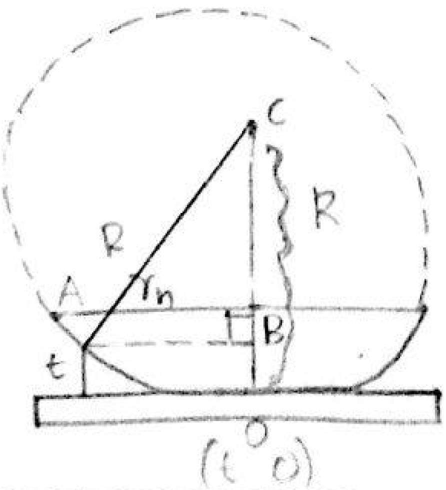
In newton ring setup,

$\mu = 1$ and $i = r = 0$ (light incident normally)

$\therefore 2t = n\lambda$

$$\boxed{t = \frac{n\lambda}{2}} \quad (2)$$

Also, we can find an eqn for 't' in terms of 'R' and 'r_n' by using geometry.



Consider the right angled Δle ABC,

$AB^2+BC^2=AC^2$

$\therefore r_n^2+(R-t)^2=R^2$

$\therefore r_n^2+R^2-2Rt+t^2=R^2$

$r_n^2-2Rt+t^2=0$

$\Rightarrow \boxed{r_n^2=2Rt-t^2}$

$\therefore r_n^2=2Rt$ [t^2 is neglected $t^2 \ll 2Rt$]

$$\boxed{t=\frac{r_n^2}{2R}} \quad \text{---} (3)$$

Comparing (2) and (3)

$$\frac{r_n^2}{2R}=\frac{n\lambda}{2}$$

$\therefore \boxed{r_n^2=n\lambda R}$

$\boxed{r_n=\sqrt{n\lambda R}}$

where, n = order of dark fringe, λ = wavelength of micro chromatic light, R = radius of curvature of lens.

For given wavelength λ and R

$\boxed{r_n \propto \sqrt{n}}$

we have the general condition for diameter of n^{th} dark ring $D_n = 2\sqrt{n\lambda R}$

Squaring,

$\boxed{D_n^2 = 4n\lambda R}$ — (5)

The square of diameter of $(n+k)^{th}$ dark ring $= (D_{n+k})^2$

$\Rightarrow \boxed{(D_{n+k})^2 = 4(n+k)\lambda R}$ — (6)

(6) − (5)

$(D_{n+k})^2 - D_n^2 = 4k\lambda R$

$\Rightarrow \boxed{\lambda = \frac{(D_{n+k})^2 - D_n^2}{4kR}}$

It is the expression for calculate wavelength λ from newton experiment, where k is an integer.

PROCEDURE

The apparatus are setup as shown in fig. when sodium paper lamp is switched on, a large no. of concentric dark and bright rings are formed. These are observed through a microscope 'm' arranged vertically above glass plate P.

The centre of cross wire of eye peice is adjusted at the centre of ring pattern and assigned order n=1 to the first clearly seen dark ring. Then, make the cross wire tangential to the first dark ring and horizontal scale reading is noted.

Again take the microscope reading for dark rings n+4, n+8, n+12 .... n+20 one left side and n=1, n+4, n+8 .... n+20 on right side. The values are tabulated as shown:

S.No	Order of ring	Microscope Reading		Diameter $D_n = L \sim R$	$D_{n+k}^2 - D_n^2$ with $k=4$
		Left	Right		
1	n=1	–	–	D_n	
2	n=4	–	–	D_{n+4}	
3.	n=8	–	–	D_{n+8}	
	⋮			⋮	
				D_{n+16}	
	n=20			D_{n+20}	

Mean $D_{n+k}^2 - D_n^2 =$

If 'f' is the focal length of lens, then the radius of curvature 'R' can be determined by boy's method.

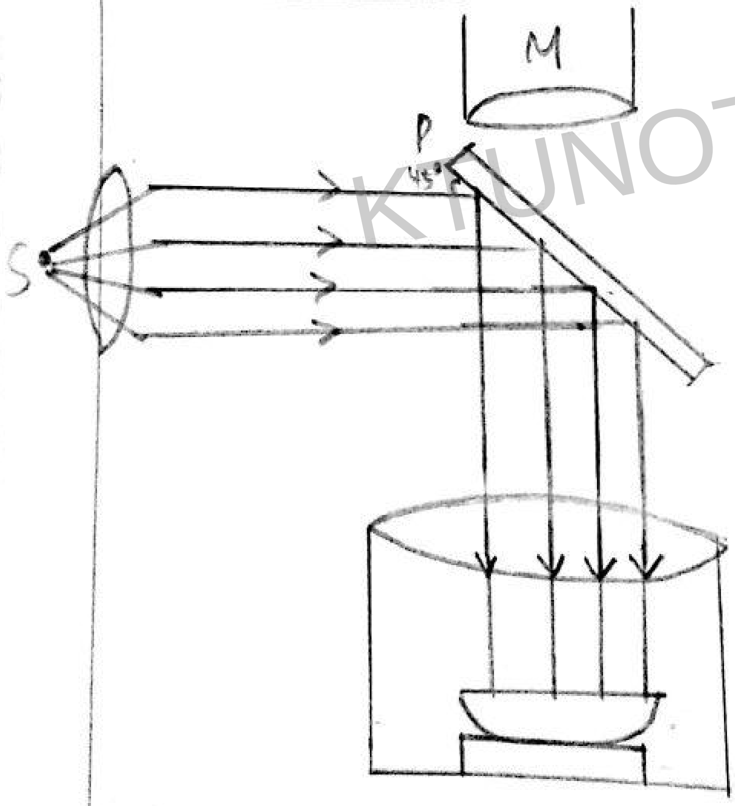
$$\boxed{R = \frac{fd}{f-d}}$$

where 'f' is the focal length and 'd' is the boy's distance.

Then, the wavelength of monochromatic light can be calculated as:

$$\boxed{\lambda = \frac{\text{Mean}\left[D_{n+k}^2 - D_n^2\right]}{4kR}}$$

<u>Expression to find out refractive index μ from newton rings experiment.</u>



- Plano convex lens, plane glass plate, liquid are ~~setup~~ arranged as shown in fig and get newton's ring pattern in the field of view of given microscope M.

Let 't' be the thickness of liquid film corresponding to n^{th} dark ring and 'μ' is the refractive index of liquid.

The general condition for n^{th} dark fringe in reflected system:

$2\mu t \cos r = n\lambda$

In this case, the light is incident normally on liquid film.

$\therefore\ i = r = 0$

$\Rightarrow$ sub in eqn

$2\mu t = n\lambda$

← Derivation if asked for higher marks

$\boxed{t = \frac{n\lambda}{2\mu}}$ — ①

Also we have 't' in terms of radius of n^{th} dark ring r_n and radius of curvature of lens R.

ie, $\boxed{t = \frac{r_n^2}{2R}}$ — ②

$\Rightarrow$ from ② and ①

$\frac{n\lambda}{2\mu} = \frac{r_n^2}{2R}$

$$\therefore r_n^2 = \frac{n\lambda R}{\mu}$$

$$r_n = \sqrt{\frac{n\lambda R}{\mu}}$$

$$\Rightarrow \boxed{D_n^2 = \frac{4n\lambda R}{\mu}}$$

$$\therefore \boxed{\mu = \frac{4n\lambda R}{D_n^2}}$$

$$\text{or } \boxed{\mu = \frac{4k\lambda R}{D_{(n+k)}^2 - D_n^2}}$$

where, r = radius of curvature of lens, λ = wavelength of light. D_n = Diameter of nth dark ring.

If R and λ values are unknown, experimentally μ can be found out simply by knowing the diameter of ring with air as the film and then with the liquid as film. Then, their ratio will give the refractive index of medium

ie, $$\boxed{\frac{D_n^2(air)}{D_n^2(liq)} = \mu_{(liq)}}$$

HW
Imp. Eqns
From $x = n\lambda$ ....
to $\frac{D_n^2 air}{} = \mu_{(liq)}$

Interference filters

Interference filters are the optical instruments in which the wavelength that are not transmitted are removed by interference rather than by absorption and scattering.

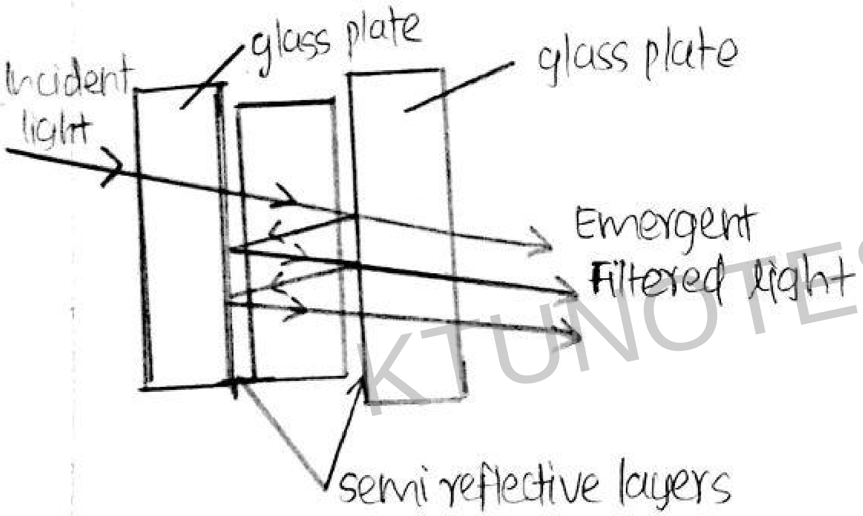
Usually it allows only one wavelength to be transmitted.

There are two types of filters:

1) Transmitted type

a) Construction and working

These filters are made by placing thin transparent spacer layer such as cryolite, Na_3AlF_6 etc.. in b/w two semi reflective coatings.



The thickness of the spacer layer will be usually half of the wavelength ($\lambda/2$) or integral multiple of $\lambda/2$. This thickness determines wavelength to be transmitted. ie, If the spacer is a half wavelength ($\lambda/2$) for the decided wavelength, then the remaining wavelength will be vanishes by destructive interference. For protecting semireflective coating on both side of spacer layer, a set of glass plates are cemented on its

For transmission filters, the general condition for max. transmission of light is:

$$\boxed{2\mu t \cos r = n\lambda}$$

In order to extract and transmit light of wavelength λ the thickness of layer required is:

$$\boxed{t = \frac{n\lambda}{2\mu \cos r}}$$

Usually, interference filters made for a particular wavelength λ and designated for its normal incidence. ie, as the angle of incidence increases different colours are filtered out due to change in 't'.

2) Reflection type | Dichroic mirror

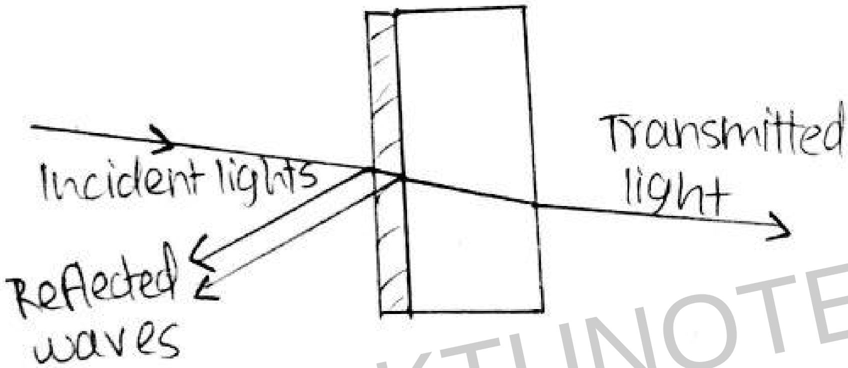
The interference filters are also made for reflection. They reflects only certain wavelength by total reflective black layer. Such filters are also called dichroic mirrors.

Antireflection coating

It is a type of optical coating applied to the surface of lenses and other optical instruments, to reduce degree of

A thin layer of transparent dielectric material, whose refractive index in b/w those of air & glass is coated on the surface of glass.

The thickness of coating is adjusted such that light reflected from top and bottom surfaces interfere destructive. Hence, no reflected light comes out.



we have, the condition for destructive interference in transmitted system:

$2\mu t \cos r = (2n \pm 1)\lambda/2$

For normal incidence $i = r = 0$ and thickness is min for $n=0$

$\therefore 2\mu t = \lambda/2$

or $\boxed{t = \frac{\lambda}{4\mu}}$

were, μ = refractive index of coating.

This is the min. thickness of coating for no reflection for a wavelength λ.

When the light of wavelength λ reflected from top and bottom surface of layer of thickness $t = \lambda/4$, will interfere out of phase which each other and vanishes due to destructive interference, hence entire light is transmitted.

The thickness of anti reflection coating is diff. for diff wavelength. Hence, the reflection of coating is chosen for yellow green region of visible spectra because eye is most sensitive in this region (this coating material should be insoluable and scratch resistant)

eg. Magnesium Flouride (MgF_2) & Cryolite.

Problems

Q. A parellel beam of light of wavelength 5890 A° is incident on a glass plate of $\mu = 1.5$, such that the $\angle$le of refraction ~~a~~ in to plate is 60°. calculate smallest thickness of plate, which will make it appear dark by reflection?

Ans: Given, $\lambda = 5890 A^\circ$, $\mu = 1.5$, $r = 60^\circ$, $n = 1$

we have, $2\mu t\cos r = n\lambda$

$\therefore t = \frac{1 \times 5890 \times 10^{-10}}{2 \times 1.5 \times \cos 60^\circ}$ $\qquad t = \frac{n\lambda}{2\mu\cos r}$

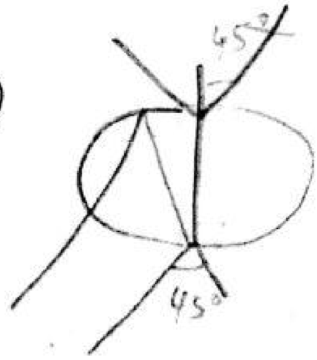
$= \frac{5890 \times 10^{-10}}{3 \times \cos 60^\circ}$

$= \frac{5890 \times 10^{-10}}{1.5}$

$= 3.9 \times 10^{-7} m$

2. A soap film of refractive index 1.33 is illuminated by white light incident at angle of 45°. The light ~~reflected~~ refracted by it is examined by a spectrometer and a bright band is found corresponding to wavelength of 6000 A°. Find thickness of film corresponding to Ist bright fringe?

S: $\mu = 1.33$, $i = 45°$, $\lambda = 6000 A° = 6000 \times 10^{-10} m$, $n = 1$



We have, ~~$2\mu t\cos r = (2n \pm 1)\lambda/2$~~

$2\mu t\cos r = n\lambda$

$2 \times 1.3 \times t \times \cos 32° = 6000 \times 10^{-10}$

Here, $i = 45°$

$\frac{\sin 45°}{\sin r} = 1.33$

$\sin r = \sin 45° \div 1.33 = ~~32~~\ 0.53$ //

$2.6 \times t \times 0.84 = 6000 \times 10^{-10}$

$t = \frac{6000 \times 10^{-10}}{2.6 \times 0.84}$

$= \underline{\underline{2.7 \times 10^{-7} m}}$

3. A thin transparent film of $\mu = 1.5$ is illuminated by sodium light of $\lambda = 5893 A^0$ by normal reflection it appears dark. Find min. thickness of film?

Ans: Given, $\mu = 1.5$, $\lambda = 5893 A^0$, $i = r = 0$

$2\mu t \cos r = n\lambda$

$2 \times 1.5 \times t \times \cos 0° = 5893 \times 10^{-10}$

$t = \frac{5893 \times 10^{-10}}{3 \times 1}$

$= \underline{\underline{1.9 \times 10^{-7} m}}$

Transmitted

$2\mu t \cos r = (2n \pm 1)\lambda/2$

$2 \times 1.5 \times t \times 1 = \lambda/2$ (lower)

$3t = 2.9 \times 10^{-7}$

$\rightarrow t = \frac{2.9 \times 10^{-7}}{3}$

$= 9.6 \times 10^{-8} m$

$2.6 \times t \times 0.84 = 6000 \times 10^{-10}$

$$t = \frac{6000 \times 10^{-10}}{2.6 \times 0.84}$$

$$= \underline{\underline{2.7 \times 10^{-7} m}}$$

A thin transparent film of $\mu = 1.5$ is illuminated by sodium light of $\lambda = 5893 A^0$ by normal reflection it appears dark. Find min. thickness of film?

Given, $\mu = 1.5$, $\lambda = 5893 A^0$, $i = r = 0$

$2\mu t \cos r = n\lambda$

$2 \times 1.5 \times t \times \cos 0° = 5893 \times 10^{-10}$

$$t = \frac{5893 \times 10^{-10}}{3 \times 1}$$

$$= \underline{\underline{1.9 \times 10^{-7} m}}$$

Transmitted

$2\mu t \cos r = (2n \pm 1) \lambda/2$

$2 \times 1.5 \times t \times 1 = \lambda/2$ (lower)

$3t = 2.9 \times 10^{-7}$

$$\rightarrow t = \frac{2.9 \times 10^{-7}}{3}$$

$$= \underline{9.6 \times 10^{-8} m}$$

Q. A newton ring's arrangement is used with sources emitting two wavelengths 6000 A° and 4500 A°, and is found that the n^{th} dark ring due to λ_1 coincides with $(n+1)_{th}$ due to λ_2. If the radius of curvature of lens is 90 cm, find the diameter of nth ring due to λ_1?

Ans: we have $n\lambda_1 = (n+1)\lambda_2$ - (1)

~~From (1)~~ $2\mu t\cos r = n\lambda_1$, $2\mu t\cos r = (n+1)\lambda_2$

$\frac{n\lambda_1}{(n+1)\lambda_2} = 1 \Rightarrow n\lambda_1 = (n+1)\lambda_2$

$\Rightarrow n \times 6000 \times 10^{-10} = n \times 6500 \times 10^{-10} + 4500 \times 10^{-10}$

$6000 \times 10^{-10} n - 4500 \times 10^{-10} n = 4500 \times 10^{-10}$

$n(6000 \times 10^{-10} - 4500 \times 10^{-10}) = 4500 \times 10^{-10}$

$n(1500) = 4500$

$n = \frac{4500}{1500} = 3$ $\Rightarrow D_3 = 2\sqrt{3 \times 6000 \times 10^{-10} \times 90 \times 10^{-2}} = \underline{0.25cm}$

An air wedge is formed using two glass plates each of length 5cm and thin wire. Wavelength of light used is 589 Nm. If 200 Fringes are formed, find the radius of wire?

$\lambda = 589 \times 10^{-7}$, $L = 5$, $\beta = \frac{5}{200} = \frac{1}{40}$

Here, the medium is air

$\therefore \mu = 1$

we have $D = \frac{\lambda L}{2\mu\beta}$

$\therefore r = \frac{\frac{\lambda L}{2\mu\beta}}{2} = \frac{\lambda L}{\mu\beta} = \frac{589 \times 10^{-7} \times 5}{4 \times \cancel{200}\ 1/40}$

$\Rightarrow r = \frac{2945 \times 10^{-7}}{4 \times \cancel{200}\ 1/\cancel{40}\ 10} = \cancel{14.72 \times 10^{-7} m}$ $\frac{2945 \times 10^{-7}}{\cancel{0.025}}$ $\underline{29.4 \times 10^{-6} m}$

Q. Two peices of plane glass are placed together with a piece of paper b/w the two at one edge. Find the angle in secs of wedge shaped air film b/w plates, if on viewin film normally with monochromatic light of $\lambda = 4800 A^\circ$. There are 18 fringes per cm?

Ans: $\lambda = 4800 \times 10^{-10} m$, $\beta = \frac{1cm}{18}$, $\theta = ?$

we have, $\theta = \frac{\lambda}{2\mu\beta}$

$= \frac{4800 \times 10^{-10}}{2 \times 1 \times \frac{1cm}{18}}$

$= \cancel{\frac{4800 \times 10^{-10}}{36}}$

$= \cancel{133.3 \times 10}$

$1^\circ = 60'$

$1' = 60''$

$1^\circ = 3600''$

180 deg $= \pi$ rad

1 deg $= \frac{\pi}{180}$ rad

1 rad $= \frac{180}{\pi}$ degree

Ans = 89 secs

Diffraction of light

The phenomenon of bending of light around the sharp edges of an obstacle towards the geometrical shadow region is called diffraction.

OR

The enchroachment of light waves in to the geometrical shadow of an obstacle placed in the path of light is called diffraction.

The resultant intensity distribution obtained on a screen is called diffraction pattern.

Diffraction pattern consists of alternative, dark and bright bands. These bands are produced by the superposition of secondary wave lengths originating from the diffracted wave front.

The phenomenon of diffraction can explained by huygen's principle.

Huygen's Principle

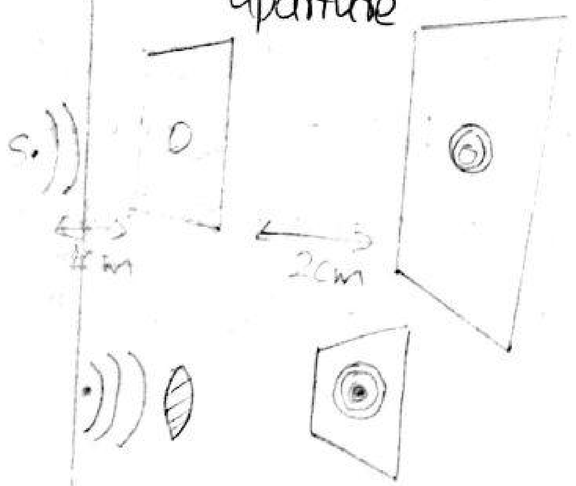
When a light is incident on an obstacle, each point on the primary wave front acts as a source of secondary wavelengths. ~~These~~ The superposition of these secondary wavelengths results dark and bright diffraction bands.

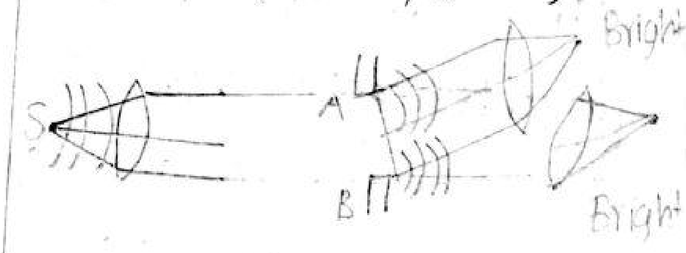
The diffraction phenomenon can be classified in to two general classes on the basis of relative positions of the source and screen with respect to the obstacle causing diffraction.

Distinction between Fresnel and Fraunhoffer diffraction

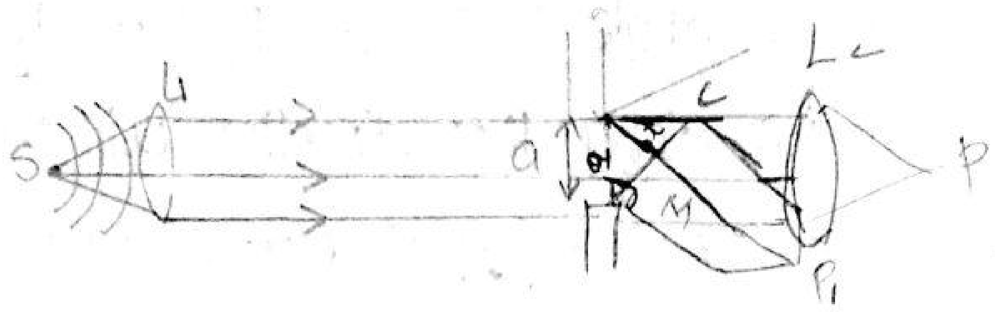
Fresnel	Fraunhoffer
1) The diffraction phenomenon in which either source or screen or both are at finite distance from obstacle causing diffraction.	1) The diffraction phenomenon in which both source or screen are effectively at infinite distance from obstacle causing diffraction.
2) Inside wave front used is spherical or cylindrical.	2) Inside wave front used is plane.
3) centre of diffraction pattern may be ~~bright or dark~~	3) Centre of diffraction pattern is ~~dark~~ always bright.

4) No lenses are necessary for observing pattern	4) Converging lenses are needed for observing pattern
5) Diffraction pattern can be projected on the screen.	5) pattern is formed at principal focus of convex lens.
6) optical or theoretical treatement is more complex	6) optical and theoretical treatment is more simpler
7) It is near field pattern from the obstacle.	7) It is far field pattern from obstacle.
8) Eg: Diffraction at a circular aparture	8) Eg: Diffraction due to single/double/grating slit.





<u>Fraunhoffer diffraction at a single slit</u>



Let 'S' be a point source of monochromatic light and L_1 is a convex lens of focal length F. Since the source S is placed at the principle focus of lens L_1, the emerger beam incident normally on slit AB of width 'a', is plan wave front.

By huygen's principle, we have each point on primary wavefront will acts as a source of secondary wavelength and these travels in all possible directions after A,C,B.

First of all, let us consider the wavelengths travels normal to the slits. ie, from the point A,C,B the waves travels straight and focus at P. Since, all these waves travels in same direction path diff. $x=0$. Hence they produce constructive interference and point P is bright with max. intensity.

Now let us consider the secondary waves moving along

~~A~~ ~~a~~ ARP' inclined at an angle θ to the horizontal.

All the waves traveling along this direction will reach at P' on the screen. The intensity at P' depends upon the path diff. b/w secondary waves originating from corresponding points AC and CB.

To calculate path diff. draw AN normal to BQ. Let that cut BQ at N.

From Δ_{le} ABN,

$$\sin\theta = \frac{BN}{AB}$$

ie, $BN = AB \sin\theta$

$$\Rightarrow \boxed{BN = a\sin\theta = x} \quad \text{—(1)}$$

If the path difference $BN = x = \lambda$

Then the pattern obtained at P' will be min. intensity. It is because path diff. b/w two halves ~~OA~~ AC and CB can be considered as $\lambda/2$. Therefore, each point on the upper half of wavefront will interfere destructively with corresponding point on lower half.

∴ In general,

For min. intensity $\boxed{a \sin \theta_n = n\lambda}$ — ②

were θ_n = corresponds to direction of nth minima

$n = 1, 2, 3 \ldots$, a = slit width.

In case, if the path difference is an integral multiple of $\lambda/2$ ~~$x = BN = (2n \pm 1)\lambda/2$~~ we get condition for secondary maxima.

$$\boxed{x = BN = (2n \pm 1)\lambda/2}$$

The diffraction pattern due to single slit consists of secondary maximas and minimas are arranged on either side of central bright maxima symmetrically.

Width of central maxima and width of slit

If the lens 'l' is placed very near to the slit. Then distance b/w slit and screen = focal length F of lens.

Also, x is the distance b/w central maxima and the Ist minima.

Then, $\boxed{\sin\theta = \theta = \tan\theta = \frac{x}{f}}$ — ①

For Ist minima general condition is: $a \sin\theta = \lambda$

$\therefore \boxed{\sin\theta = \lambda/a}$ — ②

comparing ① and ②

$$\boxed{\frac{\lambda}{a} = \frac{x}{f}}$$

OR $\boxed{x = \frac{f\lambda}{a}}$

$\Rightarrow$ width of central maxima = $\boxed{2x = \frac{2f\lambda}{a}}$

Also, the expression for slit width $\boxed{a = \frac{f\lambda}{x}}$

* This means fringe width is inversely $\propto$ to slit width 'a' and directly $\propto$ to wavelength λ.

The intensity of diffracted wave $\boxed{I(\theta) = I_0\left[\frac{\sin\beta}{\beta}\right]^2}$

were, $\boxed{\beta = (ka/2)\sin\theta}$, a = slit width, I_0 = intensity of incident light, θ = $\angle$le of diffraction.

Plane transmission grating

Diffraction grating is an arrangement of large no. of narrow ||el slits seperated by opaque spaces. when a plane wavefront is incident on grating surface light is transmitted through slits and obstructed by opaque spaces.

such grating is called transmission grating.

The secondary waves from the positions of all the slits superimposes each other and diffraction pattern is formed. These diffracted beams can be focused by a converging lens (usually telescope in spectrometer recieve pattern). When a polychromatic light is incident on a grating it can produce spectrum due to phenomenon of diffraction.

<u>Construction of grating</u>

Transmission grating is a plane sheet of transparent material on which opaque rulings are made with a diamond point. The space b/w rulings are equal & transparent and constitute the ||el slits. The rulings are opaque and of equal thickness. The combined width of a ruling and slit is called grating element.

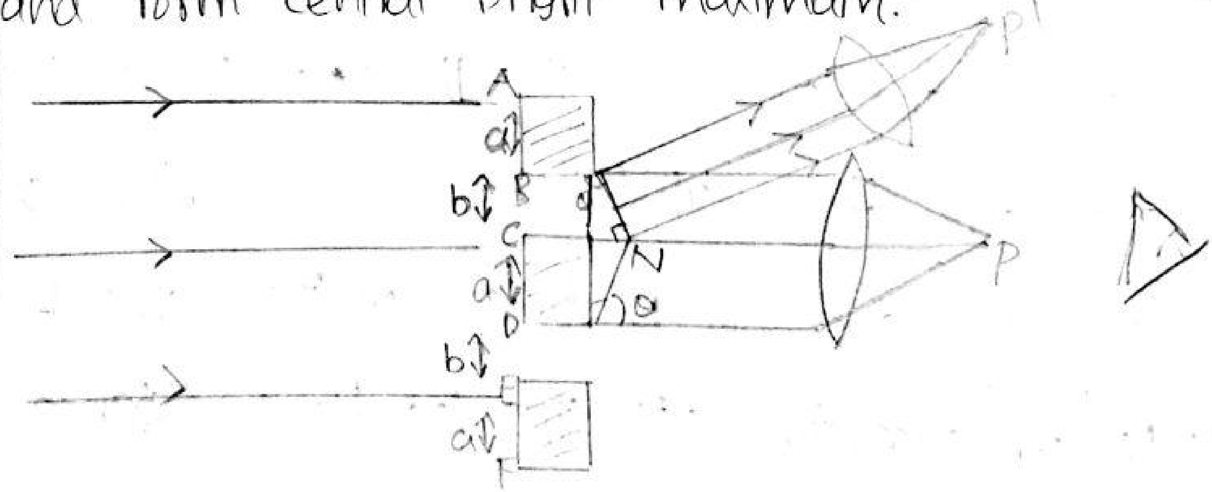
Points on sucessive slits seperated by a distance equal to grating element are called <u>corresponding points.</u>

<u>Theory of transmission grating / grating equation</u>

Let A, B, C, D, E, F represents section of a grating normal to the plane of paper. AB, CD, EF represents opaque rulings of width 'a' each and BC, DE... represents the slits of width ~~d~~ 'b' each.

Now, a+b = d represents the combined width of a ruling and slit is called grating element.

Let a plane wavefront incident normally on the grating surface. Then all the secondary wavelengths produced from each slits spread in all direction in all side of grating. All the secondary wavelength travelling in direction as that of the incident light will come to focus at point P on screen and form central bright maximum.



Now consider secondary wavelength travelling in direction inclined at i.e θ with direction of incident light come to focus at point P' on screen. The intensity at P' depends on path difference b/w secondary waves originating from the edge of neighbouring slits.

From Δ^{le} BDN,

$$\sin\theta = \frac{\text{opp. side}}{\text{hypothenuese}} = \frac{DN}{BD} = \frac{DN}{a+b}$$

$DN = (a+b)\sin\theta$

$\Rightarrow DN = d\sin\theta$ — ①

~~DN = x = nλ~~ If an n^{th} order bright maxima is obtained at P'. The path diff. $DN = n\lambda$

$\Rightarrow$ From ①

$d\sin\theta = n\lambda$

$\therefore \sin\theta = \frac{n\lambda}{d}$

ie, $\boxed{\sin\theta = nN\lambda}$ is called grating equation.

were, θ = $\angle^{le}$ of diffraction.

n = order of diffraction band.

$N = \frac{1}{d}$ is the no. of slits in unit length of grating

λ = wavelength of light.

Grating spectrum consists of a central maxima at $n=0$ and principle maxima of various orders ($n = 1, 2, 3 \cdots$) are produced on either side of it symmetrically. It can be seen that there are N-1 secondary minima and N-2 secondery maxima b/w two sucessive principle maxima, but they are of negligible intensity.

Here, N = no. of slits on a grating.

Advantages of grating spectra over prism spectra

1) In grating spectrum there is no overlapping or mixing up of spectral lines takes place, whereas in prism spectrum diff. spectral lines are seems to be overlapped.

2) Two different grating with same 'd' value will produce same dispersion and hence they are identical. whereas the dispersion by diff. prism cannot be identical due to the change of glass material.

3) The most deviated light in a grating spectrum is Red. whereas in prism spectrum most deviated light is violet.

4) The resolving power of a grating is very high as compared to prism.

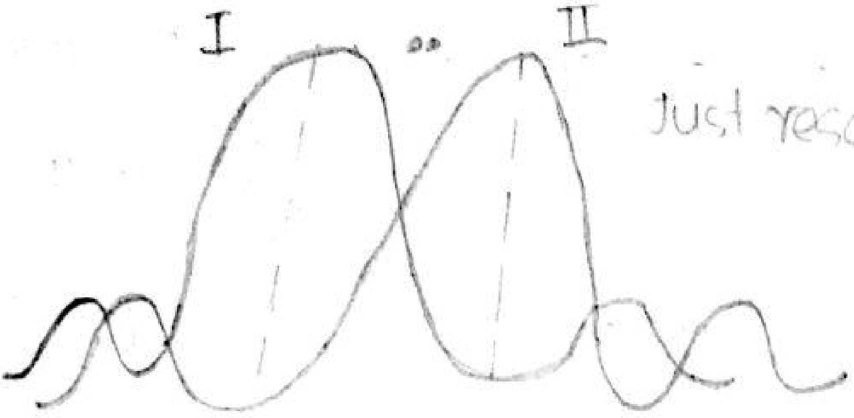
Rayleigh Criterion for resolving power

When two objects or images are very close to each other, they may appear as one as seen by naked eye. This is because the angle subtended by these objects at the eyes is very small. The optical instruments like telescope, microscope, grating etc.. can be used to increase the ∠le subtended at eye. So that objects are seen as seperate.

The method of observing two closer objects as distinctly seperate. by using some optical instrument is called resolution. and the capacity of the optical instruments to produce two seperate images of very close objects is called its resolving power.

To express the resolving power of an optical instrument Lord RayLeigh proposed an arbitirary criterion according to him:

"The minimum requirement for resolution of two objects or images are obtained when the central maximum of intensity curve of one image coincides with the first minimum of the other and vice versa.



∴ Minimum angle of resolution provided by an aperture or slit of diameter/width 'D' and at a wavelength 'λ' is:

$$\boxed{\theta_{min} = 1.22 \frac{\lambda}{D}}$$

were, 1.22 → Minimum distance of first order minima from central maxima.

λ → wavelength of light used.

D → width of the slit.

Dispersive power of a grating

when a polychromatic light is incident on a grating surface, different constituent wavelengths are diffracted through diff angles. The despersive power is defined as:

Ratio of change in angle of diffraction to corresponding change in wavelength.

Let the grating equation for nth order maxima,

$\boxed{d \sin\theta = n\lambda}$ which shows that for a given bright band of n^{th} order and grating element 'd', the angle of diffraction θ, changes with variation of 'λ'. ie, If the wavelength λ changes to (λ+dλ), corresponding angle of diffraction 'θ' also changes to (θ+dθ) then the factor

$\frac{d\theta}{d\lambda}$ is called dispersive power of grating.

Differentiating eqn $d\sin\theta = n\lambda$

$\therefore d\cos\theta \, d\theta = n \, d\lambda$

$\therefore \frac{d\theta}{d\lambda} = \frac{n}{d\cos\theta} = \frac{nN}{\cos\theta}$

$\Rightarrow \frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$ is called eqn of dispersive power of grating.

N = no. of slits on grating.

From the eqn, it is clear that despersive power of a grating is directly $\propto$ to 'N' and order of spectrum 'n'.

Resolving power of a grating

It is defined as the ratio of wavelength of spectral line to the difference in wavelength b/w this line and neighbouring line, such that two lines can just seen as seperate.

consider two wavelengths λ, $\lambda + d\lambda$ to be incident normally on the grating surface. The two wavelengths will give their own diffraction pattern. ie, If we consider two very close n^{th} order spectral lines of wavelength λ, $\lambda + d\lambda$ its spectral resolution is given by $\frac{d\theta}{d\lambda}$ $\frac{\lambda}{d\lambda}$

ie, $\boxed{\text{Resolving power} = \frac{\lambda}{d\lambda}}$

The direction of n^{th} order maximum for a wavelength

$\boxed{d \sin\theta = n\lambda}$ - ①

Also, the condition for n^{th} order maximum for the wavelength "$\lambda+d\lambda$" is given by:

$$\boxed{d\sin(\theta+d\theta)=n(\lambda+d\lambda)} - ②$$

According to rayleigh criteria, the two pattern can be just resolved, if the principle maxima of one falls on the first minima of other.

∴ condition for first minimum after n^{th} order maximum is given by:

$$\boxed{d\sin\theta(\theta+d\theta)=n\lambda+\frac{\lambda}{N}} - ③$$

were, N = Total no. of slits on grating.

d = Grating element.

The n^{th} order maximum corresponding to $\lambda+d\lambda$ has the same angle $\theta+d\theta$ as that of the first minimum after n^{th} order maximum due to λ.

Comparing ② and ③,

$n(\lambda+d\lambda)=n\lambda+\frac{\lambda}{N}$ $\quad$ $\cancel{n\lambda}+nd\lambda=\cancel{n\lambda}+\frac{\lambda}{N}$

$\therefore nd\lambda=\frac{\lambda}{N}$

$\Rightarrow \boxed{\frac{\lambda}{d\lambda}=nN}$ is the eqn for resolving power of grating.