Tutorial Sheet 1

(1) (3) $O(N+N)$ time
$O(1)$ space

(2) $T(n) = O(n)$, space $O(1)$

(3) - $T(n) = O(\log_2 n)$, space $O(1)$

(4) -
```
int sum = 0, i;
for (i=0; i*i<n; i++)
{
    sum += i;
}
```

$= n + (n-1) + (n-4) + (n-9) + \dots (n-k)$
$= n + (n*k) - (1^2 + 2^2 + 3^2 + \dots k^2)$
$= \sqrt{n}$

$i^2 < n$
$i < \sqrt{n}$

$T(n) = O(\sqrt{n})$, space $O(1)$

(5) int j = 1, i = 0 ;
while (i <= n)
{
    i = i + j ;
    j++ ;
}

$0 <= n$ 1
$1 <= n$ 1
$3 <= n$

$(0, \underbrace{1, 3, 6, 10, 15, 21, ---- n}_{k \text{ term}})$

k^{th} term $= \frac{(k * (k+1))}{2}$

$n = \frac{k^2 + k}{2}$

$k^2 + k = 2n$

$k^2 + k - 2n = 0$

$k = \frac{-1 \pm \sqrt{1^2 + 8n}}{2}$

$k = \frac{\sqrt{8n+1}}{2} + 1$

$k = \frac{\sqrt{8n+1}}{2}$

$k = \frac{\sqrt{8n}}{2} = \sqrt{n}$

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$T(n) = \sqrt{n}$ space - $O(1)$

6- void Recursion (int n) $\longrightarrow T(n)$
{
if (n==1) return;
recursion (n-1) $\longrightarrow T(n-1)$
print (n); $\longrightarrow 1$
recursion (n-1); $\longrightarrow T(n-1)$
}

$$T(n) = \begin{cases} 1 & n=1 \\ 2T(n-1)+1 & n>1 \end{cases}$$

$T(n) = 2T(n-1) + 1$ — 1

$T(n-1) = 2T(n-2)+1$

$T(n) = 2(2T(n-2)+1)+1$

$T(n) = 4T(n-2) + (1+2)$ — 2

$T(n-2) = 2(T(n-3))+1$

$T(n) = 4(2T(n-3)+1) + (1+2)$

$T(n) = 8T(n-3) + (1+2+4)$ — 3

$T(n) = 8[2(T(n-4)+1)] + [1+2+4]$

$T(n) = 16T(n-4) + (1+2+4+8)$ — 4

$T(n) = 2^k T(n-k) + (1+2+4+8+ \text{———})$
(k times)

$T(n-K) = T(1)$

$K = n-1$

$T(n) = 2^{n-1}\, T(1) + \underbrace{(1+2+4+8 - - - -)}_{(n-1)\text{ times}}$

$T(n) = \frac{2^n}{2} + \underbrace{(1+2+4+8 - - - -)}_{(n-1)\text{ times}}$

$S_n = \frac{a(r^n - 1)}{r-1}$ $a=1, r=2, n=n-1$

$T(n) = \frac{2^n}{2} + \left(\frac{2^{n-1} - 1}{1}\right)$ | $T(n) = 2^{n-1} + \left(\frac{1(2^{n-1}-1)}{1}\right)$

$T(n) = \frac{2^n}{2} + \frac{2^n}{2} - 1$ | $T(n) = 2(2^{n-1}) - 1$

$T(n) = \cancel{2}\left(\frac{2^n}{\cancel{2}}\right) - 1$

$T(n) = 2^n - 1$

$T(n) = O(2^n)$

(7) - It is a Binary search Algorithm

$T(n) = \log_2 n$

$T(n) = T\left(\frac{n}{2}\right) + 1$

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by using masters method (can't be solved)

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

so $a = 1$
$b = 2$
$f(n) = 1$
$c = \log_0 0 = \log_2 1 = 0$

$0 \simeq 1$

$n^0 = f(n) = 1$

$n^c = f(n)$

$T(n) = O(\log_2 n)$

(8) - $T(1) = 1$

1 - $T(n) = T(n-1) + 1$ —— (1)

$T(n) = T(n-2) + 2$ —— (2)

$T(n) = T(n-3) + 3$ —— (3)

$T(n) = T(n-K) + K$ —— (4)

$n - K = 1$

$K = n - 1$

$T(n) = T(1) + n - 1$

$T(n) = n$

$T(n) = O(n)$

2- $T(n) = T(n-1) + n$ —— (1)

$T(n-1) = T(n-2) + (n-1)$

$T(n) = T(n-2) + (n + (n-1))$ —— (2)

$T(n) = T(n-3) + (n + (n-1) + (n-2))$ — (3)

$T(n) = T(n-k) + (n + (n-1) + (n-2) - - - (n-k-1))$

$T(n-k) = T(1)$

$n = k+1$

$k = n-1$

$T(n) = T(1) + (n + (n-1) + (n-2) - - (\cancel{n} - \cancel{n} + 1 - 1))$

$T(n) = 1 + (n + (n-1) + (n-2) + - - - 1)$

$T(n) = 1 + \frac{n(n+1)}{2}$

$= \frac{n^2+1}{2} + 1$

$T(n) = \frac{n^2+2}{2}$

$T(n) = O(n^2)$

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8 - 3 - $T(n) = T(n/2) + 1$ —— 1

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + 1$$

$$T(n) = T\left(\frac{n}{4}\right) + 2 \quad \text{—— 2}$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + 1$$

$$T(n) = T\left(\frac{n}{8}\right) + 3 \quad \text{—— 3}$$

$$T(n) = T\left(\frac{n}{2^K}\right) + K \quad \text{—— 4}$$

$$\frac{n}{2^K} = 1$$

$$2^K = n$$

$$K = \log_2 n$$

$$T(n) = T(1) + \log_2 n$$

$$T(n) = O(\log_2 n)$$

8 4 - $T(n) = 2T\left(\frac{n}{2}\right) + 1$

$$e = 1$$

$$n^e = n$$

$$f(n) = 1$$

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$n^c > f(n)$
$T(n) = \theta(n)$

(8) 5 - $T(n) = 2T(n-1)+1$
$T(n) = O(2^n)$

(8) 6 - $T(n) = 3T(n-1)$, $T(0) = 1$

$T(n) = 3T(n-1)$ — (1)
$T(n-1) = 3T(n-2)$
$T(n) = 9T(n-2)$
$T(n) = 3^3 T(n-3)$

$T(n) = 3^K T(n-K)$
for $n-K = 0$
$n = K$
$T(n) = 3^n T(0)$
$T(n) = 3^n$
$T(n) = O(3^n)$

(8) 7 - $T(n) = T(\sqrt{n})+1$ — (1)

$T(\sqrt{n}) = T(n^{1/4}) + 1$
$T(n) = T(n^{1/4}) + 2$ — (2)
$T(n) = T(n^{1/8}) + 3$ — (3)
$T(n) = T(n^{1/K}) + K$

for $T((\sqrt{n})^{1/K}) = T(2)$

$$n^{1/2^k} = 2$$

$$\frac{1}{2^k} \log n = 2$$

$$2^k = \log n$$

$$2^k = \log n$$

$$k = \log_2 (\log n)$$

$$T(n) = O(\log (\log (n)))$$

(8) 8 - $T(n) = T(\sqrt{n}) + n$

$$T(\sqrt{n}) = T(n^{1/4}) + \sqrt{n}$$

$$T(n) = T(n^{1/4}) + (n + \sqrt{n})$$

$$T(n) = T(n^{1/8}) + (n + \sqrt{n} + n^{1/4})$$

$$T(n) = T(n^{1/2^k}) + \underbrace{(n + n^{1/2} + n^{1/4} + \cdots)}_{k \text{ terms}}$$

for $n^{1/2^k} = 2$

$$\frac{1}{2^k} = 1 / \log(n)$$

$$2^k = \log(n)$$

$$k = \log(\log(n))$$

$$T(n) = 1 + (n + \sqrt{n} + \sqrt{n}\sqrt{n} + \cdots)$$

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$$T(n) = 1 + \begin{pmatrix} G.P \quad a = n \\ r = \sqrt{n} \\ \text{No. of terms} = K \end{pmatrix}$$

$$T(n) = 1 + \left[n \frac{\left((\sqrt{n})^K - 1 \right)}{(\sqrt{n} - 1)} \right]$$

$$T(n) = 1 + n \left(\frac{(\sqrt{n})^{\log\log(n)} - 1}{\log\log(n) - 1} \right)$$

$T(n) = n \cdot \log\log(n)$ {by neglecting other values

$T(n) = O(n \cdot \log(\log(n)))$

(9) -

```
int sum = 0, i;
for (i = 0; i < n; i++)
{
    sum += i;
}
```

0, 1, 2, ------ n

so $T(n) = O(n)$ // , space $O(1)$

(10) - $O(N * (N, N-1, \text{------} 1))$

$O\left(N * \left(\frac{N+1}{2}\right)\right)$

(4.) $O(N * N)$ //

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(11) $O\left(\frac{n}{2} * (\log_2 N)\right)$

$O(n \log n)$

(12) (2) X will always be a better choice for large input.

(13) (4) $O(\log N)$

(14) $T(n) = 7\left(T\left(\frac{n}{2}\right)\right) + (3n^2+2)$

$f(n) = 3n^2+2$

$a = 7$

$b = 2$

$c = \log_b a = \log_2 7 = 2.807$

$n^c = n^{2.8} \approx n^{2.8}$

$f(n) = 3n^2+2$

so $n^c > f(n)$

so $T(n) = \theta(n^2)$ or (c) $\theta(n^{2.8})$

(a) $O(n^{2.8})$

(d) $\theta(n^3)$

(15) $f_1(n) = n^{\sqrt{n}}$

$f_2(n) = 2^n$

$f_3(n) = (1.000,001)^n$

$f_4(n) = n^{(10 * 2^{n/2})}$

a) $f_2(n) > f_4(n) > f_3(n) > f_1(n)$

16 - $f(n) = 2^{2n}$

$\log f(n) = 2n \log_2 2$

$\log f(n) = 2n$

or

$f(n) = 2^n \cdot 2^n$

$\Omega(2^n)$

17 - $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

$c = 1$

$n^c = n$

$n^2 > n$

$f(n) > n^c$

$T(n) = \theta(n^2)$

18 - $O(\log N)$ [Its a G.C.D function wherein keeps on decreasing by $n/2$]

19 - $T(n) = O(N^2 + N)$

$T(n) = O(N^2)$