

## Consensus Control for T-S Fuzzy Multi-Agent Systems with Parametric Uncertainties

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**Abstract:** This paper investigates the application of a Takagi-Sugeno (T-S) fuzzy model-based consensus control strategy for multi-agent systems subject to uncertainties. Leveraging the advantages of T-S fuzzy models in handling uncertain nonlinear systems, each agent is represented using an uncertain T-S fuzzy model. The paper proposes stability conditions in terms of Linear Matrix Inequalities (LMIs) to achieve consensus in multi-agent systems, even in the presence of uncertainties. Simulation results are provided to demonstrate the effectiveness of the proposed approach.

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**Keywords:** Multi-agent systems, parametric uncertainties, T-S fuzzy model, Consensus control, LMI.

### 1. INTRODUCTION

Multi-agent systems (MASs) have attracted considerable interest from research teams across various industries due to their broad applicability in numerous domains. These systems comprise multiple autonomous agents that interact with each other to achieve specific objectives [Hokayem et al., (2010)], [Li et al., (2019)]. The versatility and flexibility of MASs make them well-suited for a wide range of applications. The inherent ability of MASs to enable collaboration, consensus, and distributed decision-making makes them valuable in many other domains as well, including robotics, swarm intelligence, smart grids, and social network analysis. The ongoing research and development in MASs continue to expand their capabilities and open new possibilities for practical implementation in various industries [Kada et al. (2020)], [Muslimov et al. (2021)], [Yang et al. (2021)], [Zuo et al. (2019)], [Ge et al. (2019)], [Chen et al. (2019)].

To effectively accomplish their missions or objectives on a global scale, a MAS should implement consensus strategies [Li et al. (2019)], [Rehan et al. (2020)]. Indeed, consensus strategies play a vital role in ensuring that the individual agents within the system can coordinate their actions and reach an agreement despite their autonomy and possibly differing perspectives or preferences. Several research studies have been conducted to investigate consensus in multi-agent systems, considering various issues such as robustness against uncertainties, rejection of disturbances [Utkin (2009)] [Ao et al. (2018)], and handling nonlinearities inherent in the system dynamics [Liu & Zhang (2017)]. For instance, in the works of [Restrepo (2021)] and [Amirkhani et al. (2021)], authors have studied uncertain multi-agent systems using communication and coordination mechanisms that enable agents to adapt to environment changes and uncertainty. Additionally, information fusion techniques are employed to combine the knowledge and observations from different agents. [Wang et al. (2023)] introduce two advanced dynamic event-triggered control frameworks for leader-following consensus in linear MASs. These frameworks use a moving average approach and

a fully distributed scheme in the Lyapunov sense, ensuring adaptable inter-event time.

On the other hand, most of available works mainly focus on addressing consensus within linear agent systems. However, it is essential to note that, in this scenario, the application of the developed methodologies in practical systems may be limited. Thus, in this paper, it is proposed to investigate a robust consensus control mechanism for nonlinear MASs using Takagi -Sugeno (T-S) fuzzy model [Takagi & al. 1980]. T-S fuzzy systems provide a systematic method for modeling the behavior of individual agents within a MAS. Each agent's behavior can be represented using fuzzy rules that capture its decision-making process in response to input variables or conditions. [Zhao (2015)]. Niemann et al. (1999)] [Sakthivel et al. (2023)] demonstrates that in T-S fuzzy MAS, consensus can be effectively achieved through nonlinear control methodologies, particularly with the distributed compensation (PDC) approach. Additionally, [Zhang et al. (2017)] introduced an innovative approach to distributed attitude control using T-S fuzzy modeling, significantly contributing to the investigation of satellite attitude coordination under challenging conditions. [Xiong et al. (2014)] addressed the issue of achieving consensus in multi-agent nonlinear systems characterized by variable structures. [Zhang et al. (2018)] proposed a sliding-mode controller for T-S fuzzy multi-agent systems, employing a tailored sliding-mode control strategy to ensure system stability and high trajectory tracking performance. Moreover, [Rehan et al. (2019)] presented a control methodology ensuring the convergence of consensus error to the origin within a specified region of initial conditions, incorporating a newly derived consensus protocol gain for leader-following consensus.

In this paper, a robust consensus control mechanism for nonlinear MASs is proposed. The T-S fuzzy model is used to address the nonlinearities of the system and exploit the advantages of control approaches derived from linear systems. Furthermore, to ensure the practical applicability of our proposed approach, this study is concentrated on a nonlinear

MAS subject to parametric uncertainties, a subject that has been underexplored in existing literature. Hence, this paper, provides a comprehensive understanding of how the proposed consensus control mechanism performs in realistic scenarios where uncertainties significantly influence MAS behavior. The proposed fuzzy controller design is derived using a state feedback control law based on the PDC-based approach, combined with quadratic Lyapunov function. The stabilizing conditions are given in term of Linear Matrix Inequality (LMI) [Boyd et al.(1994)], that can be solved using MATLAB software. Note that, this study not only extends the frontiers of consensus control but also addresses the practical challenges encountered by MASs operating in uncertain environments. Thus, this contribution makes with relevance not only in theoretical discussions but also in practical applications.

This paper starts with a section of preliminaries, where the notions of graph theory and fuzzy modelling are presented. The fuzzy-based consensus stability analysis for uncertain MAS methodology is then introduced in section 3. Stabilizing conditions, in the form of a set of LMI terms, are depicted in section 4. Finally, section 5 presents a numerical example to illustrate the efficiency of the proposed control strategy.

## 2. PRELIMINARIES

### 2.1. Algebraic Graph Theory

To represent the interaction topology among agents, a digraph denoted as  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$  is employed. Here,  $\mathcal{V} = \{v_1, \dots, v_N\}$  denotes the set of nodes, with each node  $v_i$  representing the  $i$ th agent. The edge set  $\mathcal{E}$  comprises edges  $(v_j, v_i)$  that indicate the ability of node  $v_j$  to transmit relative state information to node  $v_i$ . A path from node  $v_j$  to  $v_i$  is composed of a sequence of edges  $(v_j, v_{kl}), \dots, (v_{kl}, v_i)$ .

For a digraph to have a spanning tree, a node called the root exists, which possesses directed paths to all other nodes in the graph. The adjacency matrix  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$  describes the digraph  $\mathcal{G}$ , where  $a_{ij} = 1$  if  $(v_j, v_i) \in \mathcal{E}$  and 0 otherwise. It is assumed that there are no repeated edges or self-loops, implying  $a_{ii} = 0$ . Furthermore, let  $\mathcal{D} = \text{diag}\{d_1, \dots, d_N\}$  with  $\text{di} = \sum_{j=1}^N a_{ij}$ , and the Laplacian matrix  $\mathcal{L} = \mathcal{D} - \mathcal{A}$ .

The in-degree of node  $v_i$ , denoted as  $\deg_i^{in}$ , is the sum of the entries  $a_{ij}$  for all  $j$ , i.e.,  $\deg_i^{in} = \sum_{j=1}^N a_{ij}$ . Similarly, the out-degree of node  $v_i$ , denoted as  $\deg_i^{out}$ , is also the sum of the entries  $a_{ij}$  for all  $j$ , i.e.,  $\deg_i^{out} = \sum_{j=1}^N a_{ij}$ . The digraph  $\mathcal{G}$  is considered balanced if and only if  $\deg_i^{in} = \deg_i^{out}, \forall i = 1, \dots, N$ .

**Assumption 1:** The digraph  $\mathcal{G}$  is balanced and weakly connected with at least one loop.

**Lemma 1** [Ren et al. (2005)]: The Laplacian matrix  $\mathcal{L}$  associated with the digraph  $\mathcal{G}$  satisfies  $(\mathcal{L} + \mathcal{L}^T)/2 > 0$  if and only if  $\mathcal{G}$  is balanced and weakly connected with at least one loop.

Consequently, Lemma 1 implies the existence of a positive scalar  $\nu$  such that  $\nu \leq \lambda_{\min}((\mathcal{L} + \mathcal{L}^T)/2)$  if Assumption 1 is valid.

### 2.2. Fuzzy system modeling under parametric uncertainty

The control problem of nonlinear MASs is tackled through the implementation of a T-S fuzzy model. This model comprises several IF-THEN fuzzy rules and is employed to address the control challenges within the MASs. Each agent within the system possesses its own fuzzy system, with the  $m_i$ th rule serving as a description of the specific fuzzy logic utilized for the  $i$ th agent

**Rule  $m$ :** IF  $x_{i1}(t) = M_{m1}$  and ... and  $x_{ip}(t) = M_{mp}$ ,

THEN

$$\dot{x}_i(t) = (A_m + \Delta A_m)x_i(t) + B_m(u_i(t) + f_i(t)), \quad (1)$$

$$i = 1, \dots, N, m = 1, \dots, r.$$

where the states variables of each agent are set as  $x_i(t) = [x_{i1}(t), \dots, x_{ip}(t)]^T \in \mathbb{R}^{p \times 1}$ , and  $M_{m1}, \dots, M_{mp}$  are fuzzy sets; the  $u_i(t) \in \mathbb{R}^{q \times 1}$  and  $f_i(t) \in \mathbb{R}^{q \times 1}$  are the control input and external disturbances respectively;  $N$  and  $r$  denotes the total number of agents and the total number of rules IF-THEN rules; The matrices  $A_m \in \mathbb{R}^{p \times p}$  and  $B_m \in \mathbb{R}^{p \times q}$  are constant. The T-S fuzzy-based model can be expressed as follows:

$$\dot{x}_i(t) = A(x_i(t))x_i(t) + B(x_i(t))(u_i(t) + f_i(t)), \quad (2)$$

$$i = 1, \dots, N.$$

where

$$A(x_i(t)) = \sum_{m=1}^r \mu_m(x_i(t))(A_m + \Delta A_m),$$

$$B(x_i(t)) = \sum_{m=1}^r \mu_m(x_i(t))B_m.$$

Further  $\mu_m(x_i(t)) = \theta_m(x_i(t))/\sum_{m=1}^r \theta_m(x_i(t))$ , and  $\theta_m(x_i(t)) = \prod_{k=1}^p M_{mk}(x_i(t))$  is the weighting function, where  $M_{mk}(x_i(t))$  represents the grade membership of the states  $x_i(t)$  in the fuzzy set  $M_{mk}$ .

Note that  $\sum_{m=1}^r \mu_m(x_i(t)) = 1, \forall i = 1, \dots, N$  and it is assumed that the function  $\mu_m(x_i(t)) \geq 0, \forall m_i = 1, \dots, r$ .

## 3. FUZZY-BASED CONSENSUS SYSTEM MODELLING

In this section, the control design is developed to satisfy the leader-following problem for T-S fuzzy MASs with parametric uncertainties to ensure stability of the nonlinear systems.

Consider the system (2) without disturbances:

$$\dot{x}_i(t) = A(x_i(t))x_i(t) + B(x_i(t))u_i(t), \quad (3)$$

$$i = 1, \dots, N.$$

and assuming that the state variable  $x_i(t)$  is accessible to the  $i$ th agent, the controller can be described by the IF-THEN rules as follows:

**Rule  $n$ :** IF  $x_{i1}(t) = M_{n1}$  and ... and  $x_{ip}(t) = M_{np}$

THEN

$$u_i(t) = K(x_i(t)) \left( \sum_{j=1}^N a_{ij} (x_j(t) - x_i(t)) + a_{ii} x_i(t) \right), \quad (4)$$

$$i = 1, \dots, N.$$

where  $K(x_i(t)) = \sum_{n=1}^r \mu_n(x_i(t)) K_n$ ,  $K_n$  is the local stabilizing control gain matrix to be determined. Considering (3) and (4), the global system can be presented as follows:

$$\dot{X}(t) = \bar{A}X(t) + \bar{B}U(t), \quad (5)$$

where

$$X(t) = [x_1^T(t), \dots, x_N^T(t)]^T.$$

$$U(t) = [u_1^T(t), \dots, u_N^T(t)]^T.$$

$$\bar{A} = \text{diag}\{A(x_1(t), \dots, A(x_N(t)\}$$

$$\bar{B} = \text{diag}\{B(x_1(t), \dots, B(x_N(t)\}$$

In this paper, the stability of (5) is ensured in the Lyapunov sense, where the function denoted  $V$  is positive definite of the form:

$$V(t) = X^T(t)PX(t) > 0, \quad (6)$$

with  $P$  is symmetric positive definite matrix ( $P = P^T > 0$ ) the global asymptotic stability theorem is valid when the first derivative of the Lyapunov function  $\dot{V}(t)$  is strictly negative.

Unfortunately, due to matrixial commutativity property, it is not necessarily true that a symmetric definite positive matrix  $P$  that ensure the asymptotical stability based, exists.

To solve this problem, the control input form is rewritten as follows [Zhang et al. (2018)]:

$$u_i(t) = (\mathcal{L} \otimes K(x_i(t)))x_i(t), \quad (7)$$

where  $\mathcal{L}$  represents the global Laplacian matrix of the corresponding graph that considers edges pointing to the agent  $i$ . According to (3), (5) and (7) the closed loop system is given by the following equation:

$$\dot{X}(t) = (\bar{A} + \bar{B}(\bar{\mathcal{L}} \otimes \bar{K}))X(t), \quad (8)$$

where

$$X(t) = [x_1^T(t), \dots, x_N^T(t)]^T.$$

$$\bar{A} = \text{diag}\{A(x_1(t), \dots, A(x_N(t)\}$$

$$\bar{B} = \text{diag}\{B(x_1(t), \dots, B(x_N(t)\}$$

$$\bar{K} = \text{diag}\{K(x_1(t)), \dots, K(x_N(t)\}$$

$$\bar{\mathcal{L}} = \text{diag}\{\mathcal{L}, \dots, \mathcal{L}\}$$

The matrixial representation in (8) is developed into summation representation, this helps to study the global stability of the MAS.

$$\dot{X}(t) = (\sum_{i=1}^N \tilde{A}(x_i(t)) + \sum_{i=1}^N \tilde{B}(x_i(t)) \sum_{i=1}^N \mathcal{L} \otimes K(x_i(t))X(t), \quad (9)$$

where

$$\begin{aligned} \tilde{A}(x_i(t)) &= \begin{bmatrix} \tilde{A}_m + \Delta\tilde{A}_m & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tilde{A}_m + \Delta\tilde{A}_m \end{bmatrix} \\ \tilde{B}(x_i(t)) &= \begin{bmatrix} \tilde{B}_m + \Delta\tilde{B}_m & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tilde{B}_m + \Delta\tilde{B}_m \end{bmatrix} \end{aligned}$$

Furthermore, control term can be expressed as follows:

$$\begin{aligned} \sum_{i=1}^N \tilde{B}(x_i(t)) \sum_{i=1}^N \mathcal{L} \otimes K(x_i(t)) &= \sum_{i=1}^N \tilde{B}(x_i(t)) \\ &\times (\mathcal{L} \otimes K(x_i(t))). \end{aligned} \quad (10)$$

Therefore, the equation (6) can be expressed as.

$$\begin{aligned} \dot{X}(t) &= (\sum_{i=1}^N \tilde{A}(x_i(t)) + \sum_{i=1}^N \tilde{B}(x_i(t)) (\mathcal{L} \otimes K(x_i(t))))X(t) \\ &= \sum_{i=1}^N (\sum_{m=1}^r \mu_m(x_i(t))(\tilde{A}_m + \Delta\tilde{A}_m) + \\ &\quad \sum_{m=1}^r \mu_m(x_i(t))\tilde{B}_m \sum_{n=1}^r \mu_n(x_i(t))\mathcal{L} \otimes K_n)X(t), \end{aligned} \quad (11)$$

where the matrices under consideration  $\tilde{A}_m$ ,  $\Delta\tilde{A}_m$  and  $\tilde{B}_m$  are diagonal matrices, where each matrix consists of an  $i$ th main diagonal submatrix  $A_m$ ,  $\Delta A_m$  and  $B_m$ , respectively, while all other submatrices are zero.

By applying the property that  $\sum_{m=1}^r \mu_m(x_i(t)) = \sum_{n=1}^r \mu_n(x_i(t)) = 1, \forall i$ , we have:

$$\begin{aligned} \dot{X}(t) &= (\sum_{m=1}^r \mu_m(x_1(t))(\tilde{A}_m + \Delta\tilde{A}_m) + \cdots + \\ &\quad \sum_{m=1}^r \mu_m(x_N(t))(\tilde{A}_m + \Delta\tilde{A}_m) + \sum_{m=1}^r \mu_m(x_1(t)) \\ &\quad \times \tilde{B}_m \sum_{n=1}^r \mu_n(x_1(t))(\mathcal{L} \otimes K_n) + \cdots + \cdots \\ &\quad \sum_{m=1}^r \mu_m(x_N(t))\tilde{B}_m \sum_{n=1}^r \mu_n(x_N(t))(\mathcal{L} \otimes \\ &\quad K_n))X(t). \end{aligned} \quad (12)$$

Additionally, it can infer that:

$$\begin{aligned} \sum_{i=1}^N \tilde{B}_m (\mathcal{L} \otimes K_n) &= \sum_{i=1}^N \tilde{B}_m (\mathcal{L} \otimes I_p)(I_N \otimes K_n) \\ &= \sum_{i=1}^N \tilde{B}_m (I_N \otimes K_n)(\mathcal{L} \otimes I_p) \\ &= \sum_{i=1}^N \tilde{B}_m \tilde{K}_n (\mathcal{L} \otimes I_p), \end{aligned} \quad (13)$$

where  $\tilde{K}_n$  is a diagonal matrix, where the  $i$ th main diagonal submatrix  $K_n$ , while all other submatrices are zero.

By substitution of (13) into (11).

$$\begin{aligned} \dot{X}(t) &= \prod_{i=1}^N \sum_{m=1}^r \mu_m(x_i(t)) \prod_{i=1}^N \sum_{n=1}^s \mu_n(x_i(t)) \\ &\quad \left( \sum_{i=1}^N (\tilde{A}_m + \Delta\tilde{A}_m) \sum_{i=1}^N \tilde{B}_m \tilde{K}_n (\mathcal{L} \otimes I_p) \right) X(t) \\ &= \sum_{m=1}^r \sum_{n=1}^s \left( (\mu_m(x(t)) \mu_n(x(t))) \otimes I_N \right) [I_N \otimes (\tilde{A}_m \right. \\ &\quad \left. + \Delta\tilde{A}_m) + (I_N \otimes (\tilde{B}_m \tilde{K}_n)) (\mathcal{L} \otimes I_p)] X(t), \end{aligned} \quad (14)$$

where the sum is expressed as follows:

$$\begin{aligned} \sum_{i=1}^N (\tilde{A}_m + \Delta\tilde{A}_m) &= I_N \otimes (\tilde{A}_m + \Delta\tilde{A}_m), \\ \sum_{i=1}^N \tilde{B}_m \tilde{K}_n &= I_N \otimes (\tilde{B}_m \tilde{K}_n). \end{aligned}$$

In addition to the key concepts previously discussed, another pivotal notion integral to the foundation of this work is the characterization of uncertainties, particularly focusing on their types. The uncertainty matrices are presented as a structured parametric uncertainty related to the dynamic of the model [Chadli et al. (2013)] as follows:

$$\Delta\tilde{A}_m = \tilde{D}_m \tilde{F}_m(t) \tilde{E}_m, \quad (15)$$

where  $\tilde{D}_m$  and  $\tilde{E}_m$  are known diagonal matrices, where the  $i$ th main diagonal submatrix  $D_m$  and  $E_m$  respectively and  $\tilde{F}_m(t)$  is a

diagonal unknown matrix where the  $i_{th}$  main diagonal submatrix  $F_m(t)$  satisfies the condition  $F_m^T(t)F_m(t) < \varepsilon_m I_{n \times n}$ ,  $\varepsilon_m$  is the maximum norm boundary of the uncertainty. This leads to simpler expression of (14).

$$\begin{aligned} \dot{X}(t) &= (\sum_{m=1}^r \mu_m(x_i(t)) \sum_{n=1}^r \mu_n(x_i(t)) [\sum_{i=1}^N (\tilde{A}_m + \\ &\quad \tilde{D}_m \tilde{F}_m(t) \tilde{E}_m) + \sum_{i=1}^N \tilde{B}_m \sum_{l=1}^N \tilde{K}_l (\mathcal{L} \otimes I_p)]) X(t) \\ &= \sum_{m=1}^r \sum_{n=1}^r ((\mu_m(x(t)) \mu_n(x(t))) \otimes I_N) [\tilde{A}_m \\ &\quad + \tilde{D}_m \tilde{F}_m \tilde{E}_m] + \tilde{B}_m \tilde{K}_n (\mathcal{L} \otimes I_p) X(t). \end{aligned} \quad (16)$$

Based on Assumption 1, the Laplacian  $\mathcal{L}$  matrix could be transformed into a valid Laplacian matrix  $\tilde{\mathcal{L}}$  that represents an undirected graph, where  $\tilde{\mathcal{L}} = SLS$  (Lemme 1),  $S$  represents non-negative eigenvector, and  $\mathcal{L}$  has an eigenvalue 0 [Zhang & al. 2018].

#### 4. STABILITY ANALYSIS AND CONTROLLER DESIGN

In this section, we will present the controller synthesis approach for the transformed system described in (16). Our objective here is to establish a consensus among the chosen MASs by employing the protocol (4). The foundation for achieving consensus within MAS (2) is articulated in Theorem 1 assuming  $\Delta A_m = 0$ , by employing control protocol (4) [Sakthivel et al. (2023)].

**Lemma 1** [Sakthivel et al. (2023)]: Let  $\alpha > 0$  be a given scalar and suppose that  $\lambda_i (i = 2, 3, \dots, N)$  be the non-zero eigenvalues of  $\tilde{\mathcal{L}}$ . If there exists positive definite matrix  $\tilde{Q}$  and any appropriate dimensional matrices  $Y_n (n = 1, 2, \dots, r)$  such that the following LMIs hold.

$$\begin{cases} \tilde{x}_{mm} < 0, & m = n \\ \tilde{x}_{mn} + \tilde{x}_{nm} < 0, & m < n \end{cases} \quad (17)$$

where

$$\begin{aligned} \tilde{x}_{mm} &= \alpha \tilde{A}_m \tilde{Q} + \alpha \tilde{Q} \tilde{A}_m^T - \lambda_i \alpha \tilde{B}_m \tilde{Y}_m - \lambda_i \alpha \tilde{Y}_m^T \tilde{B}_m^T \\ \tilde{x}_{mn} &= \alpha \tilde{A}_m \tilde{Q} + \alpha \tilde{Q} \tilde{A}_m^T - \lambda_i \alpha \tilde{B}_m \tilde{Y}_n - \lambda_i \alpha \tilde{Y}_n^T \tilde{B}_m^T \\ \tilde{x}_{nm} &= \alpha \tilde{A}_n \tilde{Q} + \alpha \tilde{Q} \tilde{A}_n^T - \lambda_i \alpha \tilde{B}_n \tilde{Y}_m - \lambda_i \alpha \tilde{Y}_m^T \tilde{B}_n^T \end{aligned}$$

The control gain matrices are calculated by  $\tilde{K}_n = \tilde{Y}_n \tilde{Q}^{-1}$ .

*Proof:* The proof is based on the one given by [Sakthivel et al. (2023)] using the Lyapunov function (6).

According to the context of this paper, the following theorem extends the results obtained in Lemma 1 to encompass cases where the uncertainties are considered i.e.  $\Delta A_m \neq 0$ .

**Theorem 1:** Suppose that  $\lambda_i (i = 2, 3, \dots, N)$  be the non-zero eigenvalues of  $\tilde{\mathcal{L}}$ . For given positive scalar  $\alpha$ , the bipartite consensus of MAS is achieved via the non-fragile controller if there exists positive definite matrix  $\tilde{Q}$ , appropriate dimensional matrices  $\tilde{Y}_n (n = 1, 2, \dots, r)$  and scalars  $\varepsilon > 0$  such that the following LMI conditions hold:

$$\begin{cases} \Gamma_{mm} < 0, & m = n \\ \Gamma_{mn} + \Gamma_{nm} < 0, & m < n \end{cases} \quad (18)$$

with

$$\begin{aligned} \Gamma_{mm} &= \begin{bmatrix} \tilde{x}_{mm} + \varepsilon(\alpha \tilde{D}_m)(\alpha \tilde{D}_m^T) & \tilde{Q} \tilde{E}_m^T \\ \tilde{E}_m \tilde{Q} & -\varepsilon_m I \end{bmatrix} < 0, \\ \Gamma_{mn} &= \begin{bmatrix} \tilde{x}_{mn} + \varepsilon(\alpha \tilde{D}_m)(\alpha \tilde{D}_m^T) & \tilde{Q} \tilde{E}_m^T \\ \tilde{E}_m \tilde{Q} & -\varepsilon_m I \end{bmatrix} < 0, \\ \Gamma_{nm} &= \begin{bmatrix} \tilde{x}_{nm} + \varepsilon(\alpha \tilde{D}_n)(\alpha \tilde{D}_n^T) & \tilde{Q} \tilde{E}_n^T \\ \tilde{E}_n \tilde{Q} & -\varepsilon_m I \end{bmatrix} < 0, \end{aligned}$$

the control gain is computed using

$$K_n = \tilde{Y}_n \tilde{Q}^{-1}. \quad (19)$$

**Proof:**

The validity of Theorem 1 is established by firstly considering the following Lyapunov function:

$$V(t) = X^T(t) \tilde{P} X(t), \quad \tilde{P} = \tilde{P}^T > 0 \quad (20)$$

Then, the derivative of (20) leads to:

$$\begin{aligned} \dot{V}(t) &= 2X^T(t) \tilde{P} \dot{X}(t) \\ &= X^T(t) \left( \tilde{P} \times \sum_{m=1}^r \sum_{n=1}^r ((\mu_m(x(t)) \mu_n(x(t))) \otimes I_N) \right. \\ &\quad \left[ \tilde{A}_m + \tilde{D}_m \tilde{F}_m \tilde{E}_m + \tilde{B}_m \tilde{K}_n (\tilde{\mathcal{L}} \otimes I_p) \right] + \\ &\quad \sum_{m=1}^r \sum_{n=1}^r ((\mu_m(x(t)) \mu_n(x(t))) \otimes I_N) [\tilde{A}_m + \\ &\quad \left. \tilde{D}_m \tilde{F}_m \tilde{E}_m + \tilde{B}_m \tilde{K}_n (\tilde{\mathcal{L}} \otimes I_p) \right]^T \times \tilde{P} \Big) X(t) \\ &= \sum_{m=1}^r \sum_{n=1}^r ((\mu_m(x(t)) \mu_n(x(t))) \otimes I_N) X^T(t) \\ &\quad (\tilde{P} [\tilde{A}_m + \tilde{D}_m \tilde{F}_m \tilde{E}_m + \tilde{B}_m \tilde{K}_n (\tilde{\mathcal{L}} \otimes I_p)]) \\ &\quad + [\tilde{A}_m + \tilde{D}_m \tilde{F}_m \tilde{E}_m + \tilde{B}_m \tilde{K}_n (\tilde{\mathcal{L}} \otimes I_p)]^T \tilde{P} X(t). \\ &= \\ &\quad \sum_{m=1}^r \sum_{n=1}^r ((\mu_m(x(t)) \mu_n(x(t))) \otimes I_N) X^T(t) W_{mn}(t) X(t) \end{aligned} \quad (21)$$

where

$$\begin{aligned} W_{mn}(t) &= \tilde{P} [\tilde{A}_m + \tilde{D}_m \tilde{F}_m \tilde{E}_m + \tilde{B}_m \tilde{K}_n (\mathcal{L} \otimes I_p)] \\ &\quad + [\tilde{A}_m + \tilde{D}_m \tilde{F}_m \tilde{E}_m + \tilde{B}_m \tilde{K}_n (\tilde{\mathcal{L}} \otimes I_p)]^T \tilde{P} \\ &= \tilde{P} \tilde{A}_m + \tilde{A}_m^T \tilde{P} + \tilde{P} \tilde{B}_m \tilde{K}_n (\tilde{\mathcal{L}} \otimes I_p) + (\tilde{\mathcal{L}} \otimes I_p)^T \\ &\quad \times \tilde{K}_m^T \tilde{B}_n^T \tilde{P} + \tilde{P} \tilde{D}_m \tilde{F}_m \tilde{E}_m + \tilde{E}_m^T \tilde{F}_m^T \tilde{D}_m^T \tilde{P}. \end{aligned}$$

Pre- and post-multiplying  $W_{mn}(t)$  by  $\alpha \tilde{P}^{-1}$  and its transpose and letting  $\tilde{Q} = \tilde{P}^{-1}$  and integrating  $\tilde{x}_{mn}$  from (17), we can obtain the following expression of  $W_{mn}(t)$ , where:

$$\begin{aligned} W_{mn}(t) &= (\tilde{x}_{mn} + \alpha \tilde{D}_m \tilde{F}_m \tilde{E}_m \tilde{Q} + \alpha \tilde{Q} \tilde{E}_m^T \tilde{F}_m^T \tilde{D}_m^T) \\ &\leq X^T(t) (\tilde{x}_{mn} + \varepsilon_m (\alpha \tilde{D}_m)(\alpha \tilde{D}_m^T) + \varepsilon_m^{-1} \tilde{Q} \tilde{E}_m^T \tilde{E}_m \tilde{Q}) X(t). \end{aligned} \quad (22)$$

Now, combining (21) and (22) gives:

$$\begin{aligned} \dot{V}(t) &\leq \sum_{m=1}^r \sum_{n=1}^r ((\mu_m(x(t)) \mu_n(x(t))) \otimes I_N) X^T(t) \\ &\quad \times (\tilde{x}_{mn} + \varepsilon_m (\alpha \tilde{D}_m)(\alpha \tilde{D}_m^T) + \varepsilon_m^{-1} \tilde{Q} \tilde{E}_m^T \tilde{E}_m \tilde{Q}) X(t) \end{aligned}$$

$$\begin{aligned} \dot{V}(t) &\leq \sum_{m=1}^r \left( (\mu_m(x(t))^2) \otimes I_N \right) X^T(t) \\ &\times (\tilde{\chi}_{mm} + \varepsilon_m(\alpha \tilde{D}_m)(\alpha \tilde{D}_m^T) + \varepsilon_m^{-1} \tilde{Q} \tilde{E}_m^T \tilde{E}_m \tilde{Q}) X(t) + \\ &\sum_{m < n}^r \sum_{n=1}^r \left( (\mu_m(x(t)) \mu_n(x(t))) \otimes I_N \right) X^T(t) \\ &\times (\tilde{\chi}_{mn} + \varepsilon_m(\alpha \tilde{D}_m)(\alpha \tilde{D}_m^T) + \varepsilon_m^{-1} \tilde{Q} \tilde{E}_m^T \tilde{E}_m \tilde{Q} + \tilde{\chi}_{nm} \\ &+ \varepsilon_m(\alpha \tilde{D}_n)(\alpha \tilde{D}_n^T) + \varepsilon_m^{-1} \tilde{Q} \tilde{E}_n^T \tilde{E}_n \tilde{Q}) X(t) < 0, \quad (23) \end{aligned}$$

By considering

$$\Gamma_{mn} = \tilde{\chi}_{mn} + \varepsilon_m(\alpha \tilde{D}_m)(\alpha \tilde{D}_m^T) + \varepsilon_m^{-1} \tilde{Q} \tilde{E}_m^T \tilde{E}_m \tilde{Q}, \quad (24)$$

We rewrite (23) as follows:

$$\begin{aligned} \dot{V}(t) &\leq \sum_{m=1}^r \left( (\mu_m(x(t))^2) \otimes I_N \right) X^T(t) (\Gamma_{mm}) X(t) + \\ &\sum_{m < n}^r \sum_{n=1}^r \left( (\mu_m(x(t)) \mu_n(x(t))) \otimes I_N \right) X^T(t) \\ &\times (\Gamma_{mn} + \Gamma_{nm}) X(t). \quad (25) \end{aligned}$$

To ensure the strict negativity of (21), it is imperative that condition (18) extracted from (25) is met. Consequently, (18) establishes the foundation for guaranteeing the asymptotic stability of the MAS in the presence of structured parametric uncertainties.

## 5. NUMERICAL EXAMPLE

In this section, a numerical example that illustrates the efficiency of the proposed approach is investigated. The example is extracted from [Zhang et al. (2018)] and assumes that there are three agents subject to uncertainties. The communication topology among these agents is depicted in Fig. 1.

The global T-S fuzzy model for each uncertain agent is derived by interpolating  $r$  local LTI (Linear Time Invariant) models using nonlinear membership functions  $\mu_m(x_i(t))$  [Takagi & al. 1980]. It is presented as follows:

$$\dot{x}_i(t) = \sum_m \mu_m(x_i(t)) (A_m + \Delta A_m) x_i(t) + B u_i(t), \quad (26)$$

where

$$\begin{aligned} x_i(t) &= \begin{bmatrix} \omega_i(t) \\ q_i(t) \end{bmatrix}, B = \begin{bmatrix} J^{-1} \\ 0_{3 \times 3} \end{bmatrix}, \\ A_m &= \begin{bmatrix} -J^{-1} \omega_i^x(t) J & 0_{3 \times 3} \\ 0.5 \sqrt{1 - q_i^T(t) q_i(t)} & -0.5 \omega_i^x(t) \end{bmatrix}_m \end{aligned}$$

The corresponding T-S model is obtained through the combination of membership functions and the linear subsystems matrices. In addition, the matrices  $E_m$  and  $D_m$  are chosen as follows:

$$\begin{aligned} E_1 &= 0.1 A_1, E_2 = 0.1 A_2, E_3 = 0.1 A_3, E_4 = 0.1 A_4, \\ D_1 &= D_2 = D_3 = D_4 = 0.1 I_{n \times n}. \end{aligned}$$

The membership function  $M_{ml}(x_{il}(t))$ , where  $m = 1, \dots, 4$ ,  $l = 1, \dots, 6$ , are defined as in Fig. 2. The Laplacian matrix of the chosen network is computed as follow:

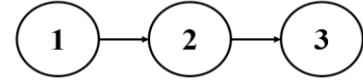
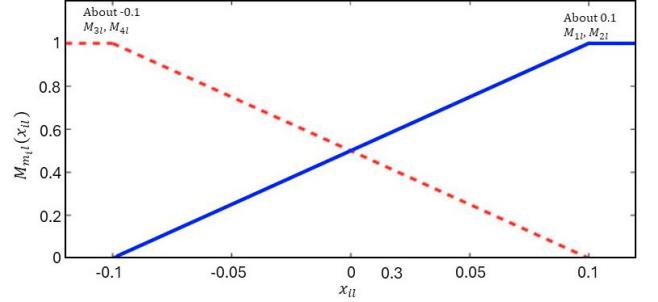
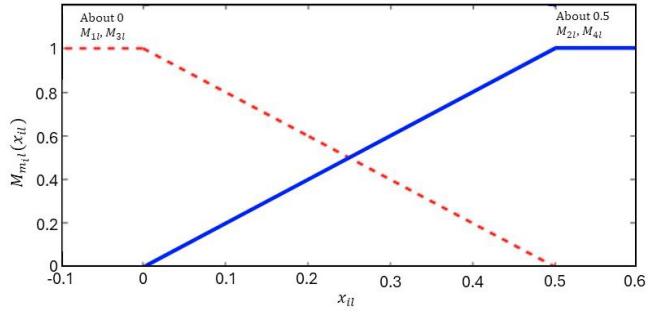


Figure 1. Communication topology among three agents.



$M_{ml}(x_{il}), m = 1, \dots, 4, l = 1, 2, 3.$



$M_{ml}(x_{il}), m = 1, \dots, 4, l = 4, 5, 6.$

Figure 2. Membership functions of fuzzy set.

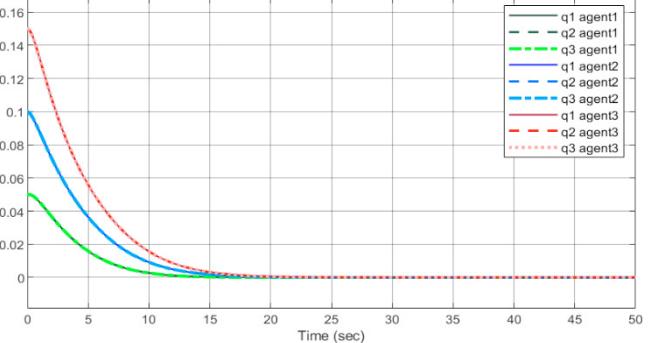


Figure 3. Agent's attitude trajectory in presence of uncertainties.

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

The simulation of system (26), is carried out using the following initial conditions of each agents:  $x_1(0) = [0 \ 0 \ 0.05 \ 0.05 \ 0.05]^T$ ,  $x_2(0) = [0 \ 0 \ 0 \ 0.1 \ 0.1 \ 0.1]^T$ ,  $x_3(0) = [0 \ 0 \ 0 \ 0.15 \ 0.15 \ 0.15]^T$ . By solving the LMI optimization problem using “sdpt3” solver, the control gains  $K_n$  are obtained with the uncertainty boundary of each subsystem  $\varepsilon_m$ , where  $\varepsilon_m \approx 15.386$ ,  $m = 1, 2, 3, 4$  and with  $\alpha = 0.25$ . The controller gains are as follows:

$$K_1 = - \begin{bmatrix} 14.76 & 0.515 & -0.29 & 14.94 & 0.753 & -0.45 \\ -0.29 & 14.77 & 0.516 & -0.45 & 14.94 & 0.753 \\ 0.515 & -0.29 & 14.76 & 0.752 & -0.45 & 14.94 \end{bmatrix}$$

$$K_2 = - \begin{bmatrix} 11.74 & 0.506 & -0.30 & 10.41 & 0.744 & -0.46 \\ -0.30 & 11.74 & 0.506 & -0.46 & 10.42 & 0.744 \\ 0.505 & -0.30 & 11.74 & 0.743 & -0.46 & 10.41 \end{bmatrix}$$

$$K_3 = - \begin{bmatrix} 14.76 & -0.29 & 0.515 & 14.93 & -0.45 & 0.752 \\ 0.516 & 14.77 & -0.29 & 0.753 & 14.94 & -0.45 \\ -0.29 & 0.515 & 14.76 & -0.45 & 0.753 & 14.94 \end{bmatrix}$$

$$K_4 = - \begin{bmatrix} 11.74 & -0.30 & 0.505 & 10.41 & -0.46 & 0.744 \\ 0.506 & 11.74 & -0.30 & 0.743 & 10.42 & -0.46 \\ -0.30 & 0.506 & 11.74 & -0.46 & 0.744 & 10.41 \end{bmatrix}$$

Comparing with the results of [Zhang et al. (2018)], the proposed study proposes a robust consensus for uncertain nonlinear MAS as shown in Fig. 3.

## 6. CONCLUSION

This paper discussed a consensus control approach for MASs employing T-S fuzzy models with bounded parametric uncertainties implementing a state feedback controller PDC-based approach. The stability analysis has been studied using a quadratic positive definite Lyapunov function. The obtained design LMI conditions show good performance of the proposed method, demonstrating the effectiveness of the proposed result.

The findings presented in this paper shed light on the complexities of this task and provide valuable insights into potential areas for further research and improvement. Future works could explore additional uncertainties, unknown inputs and considering fuzzy Lyapunov functions.

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