Dynamics of Aerial Robots — Equation Reference

Core equations with explanation of every term

1. Rotation Matrix and Angular Velocity

$$P \in SO(3), \qquad P^T P = I, \quad \det(P) = 1$$

Where:

- P : rotation matrix between two frames
- $SO(3)$: set of all 3D rotations
- I : identity matrix

$$\dot{P} = [\boldsymbol{\omega}]_\times P$$

Where:

- $\boldsymbol{\omega}$: angular velocity vector
- $[\boldsymbol{\omega}]_\times$: skew-symmetric matrix of $\boldsymbol{\omega}$

$$[\boldsymbol{\omega}]_\times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

2. Velocity Field of a Rigid Body

$$\boldsymbol{v}(M) = \boldsymbol{v}(P) + \boldsymbol{\omega} \times \overrightarrow{PM}$$

Where:

- $\boldsymbol{v}(M)$: velocity of point M
- $\boldsymbol{v}(P)$: velocity of reference point P
- $\boldsymbol{\omega}$: angular velocity
- $\overrightarrow{PM}$: position vector from P to M

3. Mass and Center of Mass

$$m = \int dm$$

Where:

- m : total mass
- dm : elementary mass

$$\int \overrightarrow{GM}\, dm = \mathbf{0}$$

Where:

- G : center of mass
- M : generic point of the body

4. Inertia Tensor

$$\boldsymbol{I}_G(\boldsymbol{\omega}) = \int GM \times (\boldsymbol{\omega} \times GM)\, dm$$

Where:

- $\boldsymbol{I}_G$: inertia operator at G
- GM : vector from center of mass to point M

$$\boldsymbol{H}_G = \boldsymbol{I}_G \boldsymbol{\omega}$$

Where:

- $\boldsymbol{H}_G$: angular momentum about G

5. Huygens (Parallel Axis) Theorem

$$\boldsymbol{I}_P = \boldsymbol{I}_G + m\, GP \times (\cdot\ \times GP)$$

Where:

- P : arbitrary point
- GP : vector from G to P

6. Kinetic Wrench

$$\mathcal{H} = \left\{ \begin{array}{l} \boldsymbol{R}_H = m\boldsymbol{v}_G \\ \boldsymbol{M}_H(G) = \boldsymbol{I}_G \boldsymbol{\omega} \end{array} \right\}$$

Where:

- $\boldsymbol{R}_H$: linear momentum
- $\boldsymbol{M}_H(G)$: angular momentum at G
- $\mathcal{H}$: kinetic wrench (torsor)

7. Kinetic Energy

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} \boldsymbol{\omega}^T \boldsymbol{I}_G \boldsymbol{\omega}$$

Where:

- T : kinetic energy
- v_G : speed of center of mass

8. Dynamic Wrench

$$\mathcal{K} = \frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t}$$

$$\mathcal{K} = \left\{ \begin{array}{l} \boldsymbol{R}_K = m\boldsymbol{a}_G \\ \boldsymbol{M}_K(G) = \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{I}_G \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\boldsymbol{I}_G \boldsymbol{\omega}) \end{array} \right\}$$

Where:

- $\boldsymbol{a}_G$: acceleration of center of mass

9. Newton–Euler Equations

$$\sum \boldsymbol{F}_{ext} = m\boldsymbol{a}_G$$

$$\sum \boldsymbol{M}_{G,ext} = \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{I}_G \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\boldsymbol{I}_G \boldsymbol{\omega})$$

Where:

- $\boldsymbol{F}_{ext}$: external forces
- $\boldsymbol{M}_{G,ext}$: external torques at G

10. Actions and Wrenches

$$\mathcal{T} = \left\{ \begin{array}{l} \boldsymbol{R} \\ \boldsymbol{M}_P \end{array} \right\}_P$$

Where:

- $\boldsymbol{R}$: resultant force
- $\boldsymbol{M}_P$: moment at point P
- $\mathcal{T}$: wrench (torsor)

Action types:

- Given actions: gravity, thrust
- Link actions: constraints
- Internal actions: cancel