Sensor Fusion — Overview

Sensor fusion combines redundant and complementary sensor information to estimate the state of a system under uncertainty. Applications include robotics, tracking, autonomous driving, and navigation. Sensors are divided into proprioceptive (internal: odometry, IMU) and exteroceptive (external: lidar, sonar, camera, GPS).

Probability Basics

Random variables model states, controls, and measurements. Probabilities describe uncertainty.

- Discrete variables use probability mass functions
- Continuous variables use probability density functions (PDF)
- Joint, marginal, and conditional probabilities describe relationships

Independence implies joint distributions factorize. The theorem of total probability marginalizes hidden variables.

Bayes Rule

Bayes rule reverses conditioning and allows inference of hidden states from measurements. The posterior combines prior knowledge with sensor information. This is the mathematical foundation of all Bayesian filters.

Dynamic Systems and Time

System evolution is modeled over time using:

- State x_t: system configuration
- Control u_t: applied action
- Measurement z_t: sensor observation

Markov Assumptions

Bayesian filtering relies on:

- First-order Markov property: current state depends only on previous state and control
- Conditional independence of measurements given the current state

These assumptions define a Hidden Markov Model (HMM).

Belief Representation

The belief $bel(x_t)$ is the posterior distribution over the state given all past controls and measurements.

Bayesian Filtering Cycle

Each time step consists of:

- Prediction: propagate belief through motion model
- Correction: update belief using sensor measurements

Discrete Bayesian Filter

Used when the state space is finite. Beliefs are updated by summation and normalization. Suitable for low-dimensional problems.

Particle Filters

Nonparametric filters that approximate belief using weighted samples (particles).

- Handle non-linear, non-Gaussian systems
- Use Sampling–Importance–Resampling (SIR)
- Suffer from particle depletion if resampling is excessive

Effective Number of Particles

A metric used to decide when to resample. Low effective sample size indicates weight degeneration.

Probabilistic Motion Models

Motion models describe how control actions affect the state with uncertainty.

- Odometry-based models (wheel encoders)
- Velocity-based models (dead reckoning)

Motion uncertainty accumulates over time and produces characteristic distributions.

Sensor Models

Sensor models compute the likelihood of a measurement given a state.

- Proximity sensors: sonar, lidar
- Beam-based models assume independent rays
- Measurements modeled as mixture distributions

Mapping

Mapping estimates the environment representation from sensor data.

- Geometric maps: metric, dense
- Topological maps: semantic, compact
- Hybrid maps combine both

Occupancy Grid Maps

The environment is discretized into cells storing occupancy probabilities. Each cell is updated using a binary Bayes filter.

SLAM Problem

Simultaneous Localization and Mapping estimates robot pose and map together.

- Strong correlation between pose and map uncertainty
- Data association is a critical challenge

FastSLAM

A particle-filter-based SLAM approach using Rao–Blackwellization.

- Particles represent robot trajectories
- Landmarks estimated using EKFs
- Data association performed per particle

Grid-based FastSLAM

Each particle maintains its own occupancy grid map. Requires few particles but high memory.

EKF-SLAM

SLAM approach assuming Gaussian distributions.

- State includes robot pose and landmarks
- Correlations stored in covariance matrix
- Computational complexity grows quadratically