Formula for Efficient Unemployment: u* = √uv

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Social planner's problem

- Allocate labor across producing / recruiting / job seeking to maximize welfare.
- $\min \quad u + v \quad$ subject to $\quad v = v_0/u$
- $\min_u \quad u + v_0/u$
  - convex minimization problem
  - FOC is necessary & sufficient for global minimum.
- FOC: $\frac{\partial}{\partial u}\left(u + v_0/u\right) = 0$

$\Leftrightarrow \quad 1 - v_0/u^2 = 0$

$\Leftrightarrow \quad u^2 = v_0$

$\Leftrightarrow \quad u^* = \sqrt{v_0}$

↑ location of Beveridge curve: $uv = v_0$

$\Leftrightarrow \quad \boxed{u^* = \sqrt{uv}}$

Efficient unemployment rate is the geometric average of unemployment rate & vacancy rate