Model Solution with Bargained Prices

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Surplus sharing in transaction i:

$$p_i = (1-\beta)(1+\tau(x))\,p$$

All transactions are same: $p_i = p$ for all i

Surplus sharing becomes:

$$1 = (1-\beta)[1+\tau(x)]$$

$$\tau(x) = \frac{1}{1-\beta} - 1 = \frac{\beta}{1-\beta}$$

$$\boxed{x = \tau^{-1}\left(\beta/(1-\beta)\right)}$$ ← Tightness under surplus sharing

We know that x is given by $AD = AS$

$$y^d(x, p) = y^s(x)$$

Price p under surplus sharing is such

$$y^d\left(\tau^{-1}(\beta/1-\beta), p\right) = y^s\left(\tau^{-1}(\beta/1-\beta)\right)$$

→ unique p that satisfies this condition

→ <u>aggregate bargained price</u>

$$y^d(x, p) = \frac{x^\varepsilon}{[1+\tau(x)]^{\varepsilon-1}}\,\frac{\mu}{p}$$

$$\Rightarrow y^d\left(\tau^{-1}\left(\beta/1-\beta\right), p\right) = \frac{x^{\varepsilon}}{\left[1+\beta/1-\beta\right]^{\varepsilon-1}} \cdot \frac{\nu}{p}$$

$$= \frac{x^{\varepsilon}}{\left[1/1-\beta\right]^{\varepsilon-1}} \cdot \frac{\nu}{p}$$

$$y^d\left(\tau^{-1}\left(\frac{\beta}{1-\beta}\right), p\right) = x^{\varepsilon}(1-\beta)^{\varepsilon-1} \cdot \frac{\nu}{p}$$

$$y^s\left(\tau^{-1}\left(\beta/1-\beta\right)\right) = f\left(\tau^{-1}\left(\beta/1-\beta\right)\right) \cdot k$$

Bargained price satisfies:

$$p = \frac{x^{\varepsilon}(1-\beta)^{\varepsilon-1} \cdot \nu}{f\left(\tau^{-1}\left(\beta/1-\beta\right)\right) \cdot k}$$

AD

bargaining power

$$p = \frac{x^{\varepsilon} \cdot \nu}{k} \times \frac{(1-\beta)^{\varepsilon-1}}{f\left(\tau^{-1}\left(\beta/1-\beta\right)\right)}$$

AS

$p^n = [\quad]$ Price norm capturing surplus sharing.