Firm's Problem

Pascal Michaillat
https://www.pascalmichaillat.org/t5.html

Firms maximize profits by choosing # of producers n, taking as given market tightnesses x, θ and prices p, w -

(Equivalent. - firm chooses # employees e

[b/c $e = [1+\hat{\tau}(\theta)]\, n$]

- firm chooses # vacancies $\hat{v}$

[b/c $\hat{v} = e / \hat{q}(\theta)$])

Profits : Revenue - Cost

= Revenue from sale of services - Wage bill

$$= \underbrace{p}_{\text{price of service}} \cdot \underbrace{f(x)}_{\text{selling proba}} \cdot \underbrace{k}_{\text{capacity}} - \underbrace{w}_{\text{nominal wage}} \cdot \underbrace{e}_{\text{employees}}$$

$$= p \cdot f(x) \cdot \underbrace{a}_{\text{technology}} \cdot n^{\alpha} - w\, \underbrace{[1+\hat{\tau}(\theta)]}_{\text{matching wedge}} \cdot \underbrace{n}_{\text{producers}}$$

function of n:

concave in n

($\alpha < 1$)

standard concave maximization problem

$$\max_{n>0} \quad p \cdot a \cdot f(x)\, n^{\alpha} - w\,[1+\hat{\tau}(\theta)]\, n$$

producer

wage for recruiter to get producer

↳ derivative . FOC

↳ sufficient condition for global maximum

$$p \cdot a \cdot f(x)\, \alpha\, n^{\alpha-1} = w\,[1+\hat{\tau}(\theta)]$$

MR from 1 producer : MPL × price × selling proba

MC of product : cost of 1 extra producer

$$\iff \quad n^{\alpha-1} = \frac{[1+\hat{\tau}(\theta)]}{a\,\alpha\, f(x)}\,\frac{w}{p}$$

$$\iff \quad \boxed{n = \left[\frac{a \cdot \alpha \cdot f(x)}{[1+\hat{\tau}(\theta)] \cdot (w/p)}\right]^{\frac{1}{1-\alpha}}}$$

Profit-maximizing # employees: $\ell = [1+\hat{\tau}(\theta)]\, n$

$$\ell = \left[\frac{a\,\alpha\, f(x)}{w/p}\right]^{1/1-\alpha} \cdot [1+\hat{\tau}(\theta)]^{\underbrace{1-1/1-\alpha}_{-\alpha/1-\alpha}}$$

$$\iff \quad \boxed{\ell = \left[\frac{a\,\alpha \cdot f(x)}{w/p}\right]^{1/1-\alpha} \cdot \left[\frac{1}{1+\hat{\tau}(\theta)}\right]^{\alpha/1-\alpha}}$$