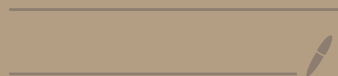


# Constant-Elasticity-of-Substitution Matching Function

---

Pascal Michailat

<https://www.pascalmichailat.org/t5.html>



## Cobb Douglas matching function

$$m(S, B) = \omega S^\eta B^{1-\eta}$$

$$m(S, B) > \min(S, B) \text{ if } S \text{ or } B \text{ large}$$

## CES matching function

$$m(S, B) = [S^{-r} + B^{-r}]^{-1/r}$$

$$r > 0$$

$$\bullet m(0, B) = m(S, 0) = 0$$

$$\bullet \frac{\partial m}{\partial S} > 0 \quad \frac{\partial m}{\partial B} > 0$$

$$\bullet \text{CRS}$$

$$\begin{aligned} m(\lambda S, \lambda B) &= [(\lambda S)^{-r} + (\lambda B)^{-r}]^{-1/r} \\ &= [(\lambda^{-r})(S^{-r} + B^{-r})]^{-1/r} \\ &= \lambda [S^{-r} + B^{-r}]^{-1/r} \\ &= \lambda m(S, B) \end{aligned}$$

$$\bullet \text{Check that } m(S, B) < \min(S, B)$$

$$S^{-r} + B^{-r} > S^{-r} \quad (B^{-r} > 0)$$

$$[S^{-\gamma} + B^{-\gamma}]^{-1/\gamma} < (S^{-\gamma})^{-1/\gamma}$$

$$\left. \begin{array}{l} m(S, B) < S \\ m(S, B) < B \end{array} \right\} m(S, B) < \min(S, B)$$

• Matching elasticity.

$$\eta \equiv \frac{\partial \ln m}{\partial \ln S}$$

$$\eta = \frac{\partial \ln m}{\partial \ln S} = \frac{\partial \ln [S^{-\gamma} + B^{-\gamma}]^{-1/\gamma}}{\partial \ln S}$$

$$= -\frac{1}{\gamma} \frac{\partial \ln (S^{-\gamma} + B^{-\gamma})}{\partial \ln S}$$

$$= -\frac{1}{\gamma} \cdot \left[ \frac{\partial \ln S^{-\gamma}}{\partial \ln S} \times \frac{S^{-\gamma}}{S^{-\gamma} + B^{-\gamma}} \right]$$

$$= -\frac{1}{\gamma} \frac{S^{-\gamma}}{S^{-\gamma} + B^{-\gamma}} \cdot (-\gamma)$$

$$\theta = B/S$$

$$\eta = \frac{S^{-\gamma}}{S^{-\gamma} + B^{-\gamma}} = \frac{1}{1 + \left(\frac{B}{S}\right)^{-\gamma}}$$

$$\eta(\theta) = \frac{1}{1 + \theta^{-\gamma}}$$

$$\cdot \quad \eta(0) = 0$$

$$\eta(1) = 1$$

$$\eta'(t) > 0$$