COMPETITIVE SEARCH EQUILIBRIUM

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MOTIVATION

- Is the unemployment generated in the market efficient?
- This paper proposes the theory of efficient unemployment extending the DMP framework

BASIC SETTING

- There are unit 1 workers and firms
- There are many sub-markets. In each sub-market, the unemployed and vacancy jobs meet. Say, $i = 1, ...m$
- The unemployed and job vacancies can choose sub-markets to join
- Each sub-market is held by a market maker. Each market maker earns from the payment by the firms which join the sub-market
- Importantly, each sub-market maker can choose its wage and tightness simultaneously
- Assume free-entry of the market makers. Thus, it's competitive and the market makers earn zero profits
- We want to see the market equilibrium

BASIC SETTING

- Matching function: $x(u_i, v_i)$ (concave and homogeneous to degree 1)
- $p(\theta_i) = x(u_i, v_i)/u_i$ and $q(\theta_i) = x(u_i, v_i)/v_i$
- Job vacancies draw its productivity from F. Each point of F is $(y_1, ..., y_n)$
- Each jobs have to pay entry-cost, k, to enter a sub-market

WORKERS

- Workers Bellman equation,

$$U_i = \frac{1}{1+r}\left[z + p(\theta_i)E_i + (1 - p(\theta_i))U_i\right]$$
$$E_i = \frac{1}{1+r}\left[w_i + sU_i + (1 - s)E_i\right]$$

- For a fixed U, the labor market tightness is given by,

$$p(\theta_i) = \frac{rU - z}{w_i - rU}(r + s)$$

- $\theta(w_i, U)$ is decreasing in w and increasing in U

JOBS

- If separation occurs, the job vanishes
- Vacancy jobs Bellman equation,

$$V_i(y_i, w, \theta) = \frac{1}{1+r}\left[-c + q(\theta_i)J(y_i, w) + (1 - q(\theta_i))V_i(y_i, w, \theta)\right]$$

$$J(y_i, w) = \frac{1}{1+r}\left[y_i - w + (1 - s)J(y_i, w)\right]$$

- The value of the vacancy is given by

$$(r + q)V_i(y_i, w, \theta) = q(\theta)\frac{y_i - w}{r + s} - c$$

- Each vacancy enters a sub-market that maximizes its asset value V

EQUILIBRIUM

- Remember the competitiveness for market makers
- Let W^* be the equilibrium vector of the wages
- Suppose, for same i, there exists a wage $w^{'}$ such that $V(y_i, w^{'}, \theta(w^{'})) > V(y_i, w^*, \theta(w^*))$ for all w^*
- Then, new market maker can earn positive surplus by putting $w^{'}$ because the type i jobs will come
- This is prevented by assuming that each sub-market solves the following problem,

$$\max_{w} V(y_i, w, \theta(w; U))$$

for some i (there should exist at least one sub-market that corresponds to for each entrant-type i)

EQUILIBRIUM OF THE MODEL

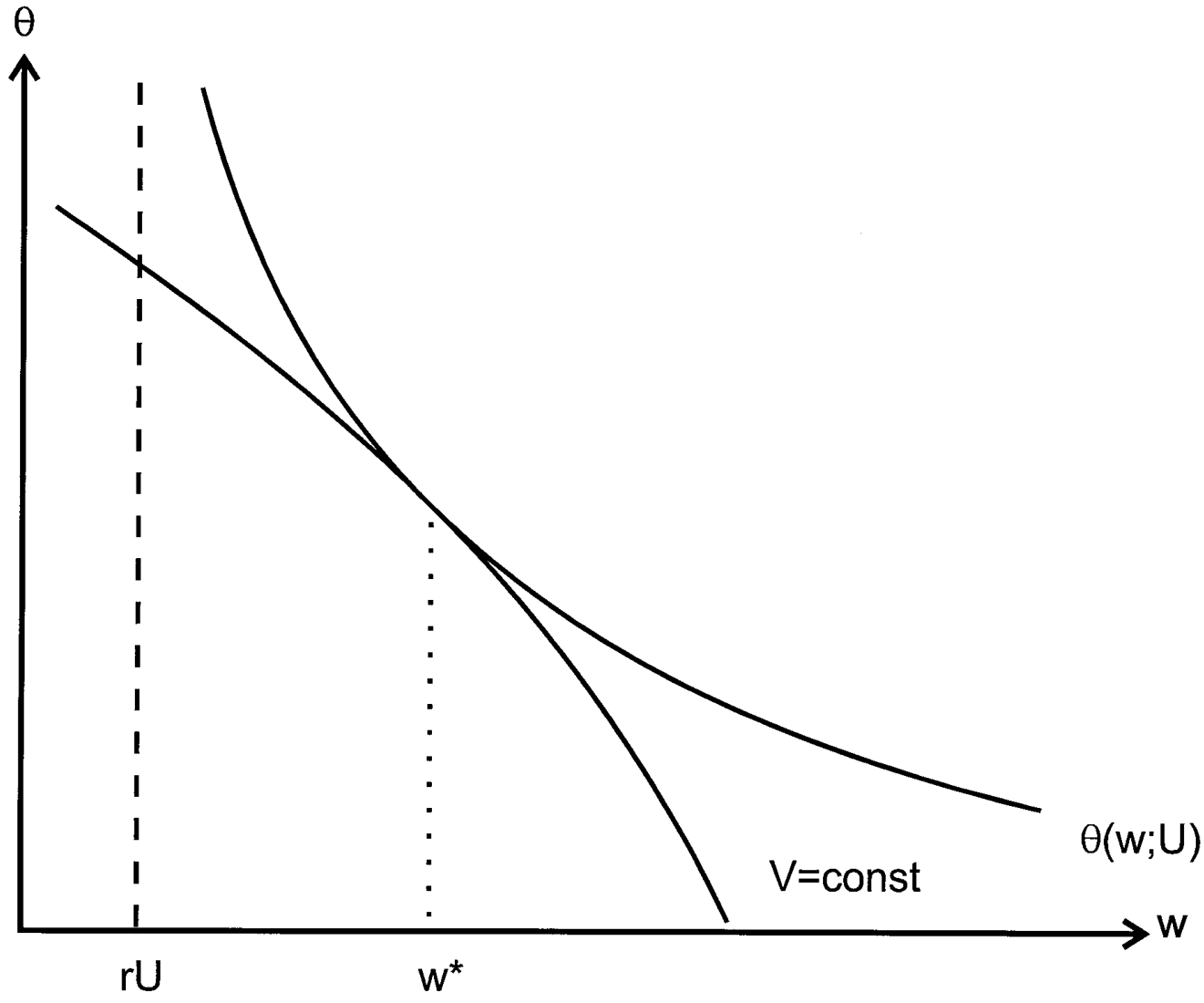
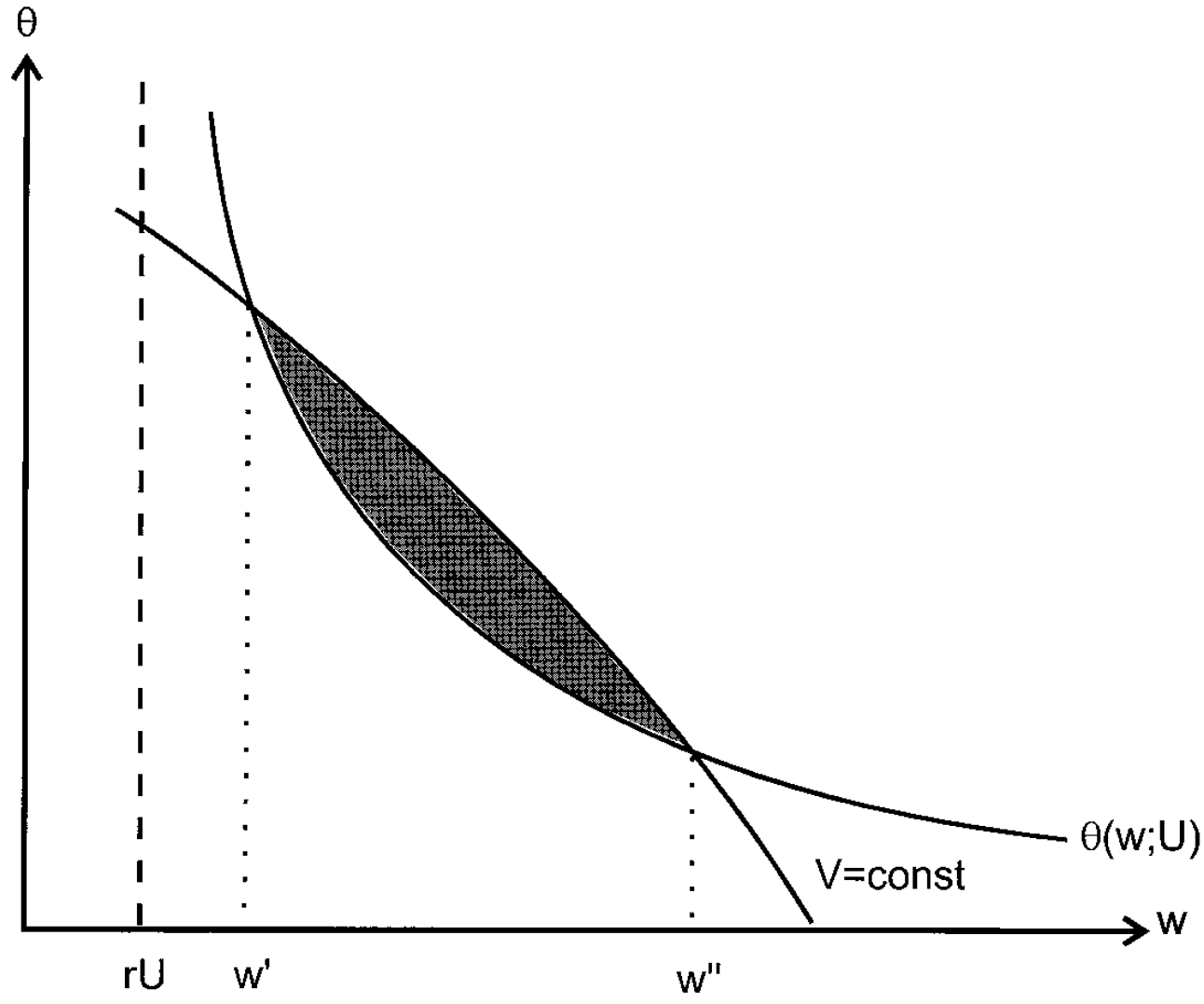


FIG. 2.—Equilibrium

EQUILIBRIUM OF THE MODEL



OPTIMALITY AND ANNOUNCEMENT

- Impose free-entry condition of the jobs and the condition that the inflow of vacancies is equal to the outflow in each sub-market
- Then, we can say (1) the existence of the equilibrium and (2) the equilibrium maximizes social welfare
- Suppose a set $W^a = w_1, ..., w_m$ of wages is announced by a measure $v_1, ..., v_m$ of vacancies
- This is like m sub-markets with $x(u_j, v_j)$
- This paper discusses that the equilibria in this setting are equivalent to the competitive equilibria