Individual and Bilateral Surpluses from Trade

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Utility function.

$$u\left(c, \frac{m}{p}\right) = \frac{X}{1+X} c^{\frac{\xi-1}{\xi}} + \frac{1}{1+X}\left(\frac{m}{p}\right)^{\frac{\xi-1}{\xi}}$$

Marginal utility of services:

$$\frac{\partial u}{\partial c} = \frac{X}{1+X} \frac{\xi-1}{\xi} c^{-1/\xi}$$

→ buyer takes away from trade

Marginal utility of money.

$$\frac{\partial u}{\partial m} = \frac{1}{1+X} \cdot \frac{\xi-1}{\xi} \left(\frac{m}{p}\right)^{-1/\xi} \frac{1}{p}$$

← aggregate price level

For a transaction at price p^n (price norm), utility is

$$p^n \cdot \frac{\partial u}{\partial m} = \frac{p^n}{p} \cdot \frac{1}{1+X} \cdot \frac{\xi-1}{\xi} \cdot \left(\frac{m}{p}\right)^{-1/\xi}$$

All households hold μ units of money, so seller experience utility:

$$p^n \frac{\partial u}{\partial m} = \left(\frac{p^n}{p}\right) \frac{1}{1+X} \frac{\xi-1}{\xi} \cdot \left(\frac{\mu}{p}\right)^{-1/\xi}$$

↑ seller takes away from trade.

If there was no trade (required to compute surplus).

- seller gets 0.

- buyer gets p^m units of money, which provides utility:

$$p^m \frac{\partial u}{\partial m} = \left(\frac{p^m}{P}\right) \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{M}{P}\right)^{-1/\varepsilon}$$

- Seller enjoys surplus from trade:

$$S = \left(\frac{p^m}{P}\right) \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{M}{P}\right)^{-1/\varepsilon}$$

All prices are the same, given by price norm, so $p^m = P$.

$$S = \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{M}{P}\right)^{-1/\varepsilon} \qquad S > 0$$

- Buyer enjoys surplus from trade:

$$B = \frac{X}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot c^{-1/\varepsilon} - \left(\frac{p^m}{P}\right) \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{M}{P}\right)^{-1/\varepsilon}$$

$$B = \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left[X c^{-1/\varepsilon} - \left(\frac{p^m}{P}\right)\left(\frac{M}{P}\right)^{-1/\varepsilon} \right]$$

$$B = \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left[X c^{-1/\varepsilon} - \left(\frac{M}{P}\right)^{-1/\varepsilon} \right] \quad \text{(all prices are same)}$$

FOC from household's problem: (used to compute AD curve)

$$X c^{-1/\varepsilon} = [1+\tau(x)] (M/P)^{-1/\varepsilon}$$

$$B = \frac{1}{1+X} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot \left[(1+\tau(x)) (\nu/p)^{-1/\varepsilon} - (\nu/p)^{-1/\varepsilon} \right]$$

$$B = \frac{1}{1+X} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot \tau(x) \cdot (\nu/p)^{-1/\varepsilon} \qquad B > 0$$

Conclusion:

- seller surplus: $S > 0$
- buyer surplus: $B > 0$
- total surplus from match:

$$T = S + B > 0$$