Optimal Public Expenditure

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Optimal public expenditure maximizes $\mathcal{U}(c(g), g)$

Assumption: $g \mapsto \mathcal{U}(c(g), g)$

is well behaved: admits a unique extremum

& the extremum is an interior maximum.

[strictly concave function w/ interior extremum]

$\Rightarrow$ FOC is necessary & sufficient to find the solution of planner's problem

Take FOC. $\quad \frac{d\mathcal{U}}{dg} = 0$

$$\Rightarrow 0 = \frac{\partial \mathcal{U}}{\partial g} - \frac{\partial \mathcal{U}}{\partial c} + \frac{\partial \mathcal{U}}{\partial c} \times \left[-\overset{+}{u'}(g)\right] \times \left[1 - \left(-\overset{+}{v'}(u)\right)\right]$$

$$\Rightarrow \cancel{\frac{\partial \mathcal{U}}{\partial c}} = \frac{\partial \mathcal{U}}{\partial g} + \cancel{\frac{\partial \mathcal{U}}{\partial c}} \times \left[-u'(g)\right] \times \left[1 - \left(-v'(u)\right)\right]$$

$\left(\frac{\partial \mathcal{U}}{\partial g} / \frac{\partial \mathcal{U}}{\partial c}\right)$

$-\frac{du}{dg} = m$

$$\Rightarrow 1 = MRS_{gc} + \left[-u'(g)\right]\left[1 - \left(-v'(u)\right)\right]$$

$$\Rightarrow \boxed{1 = \underline{MRS_{gc}} + \underline{m \times \left[1 - \left(-v'(u)\right)\right]}}$$

Samuelson rule (neoclassical) $\qquad MRS_{gc} = 1 \iff \frac{\partial \mathcal{U}}{\partial c} = \frac{\partial \mathcal{U}}{\partial g}$

$m \times [1 - (-v'(u))] = m \times [1 - \text{Beveridge slope}]$

<u>correction</u> term that appears in a model w/ inefficient slack, w/ productive inefficiencies

↳ <u>stabilization</u> <u>term</u> → appears b/c economy is not stabilized at efficient unemployment u^*