Properties of Optimal Stimulus Spending

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Formula for optimal stimulus spending

$$\frac{g/c - g/c^*}{g/c^*} = \frac{2 \varepsilon m}{1 + z \cdot \varepsilon m^2} \cdot \frac{u_0 - u^*}{u^*}$$

(+ in numerator, + in denominator)

Role of initial unemployment gap, $u_0 - u^*$

Stimulus spending is larger when unemployment gap is larger

Role of elasticity of substitution, ε

$\varepsilon = 0$ (digging holes) → no stimulus spending $(g/c - g/c^* = 0)$

stimulus spending is ↑ in ε → higher substitutability, stimulus package is larger.

$\varepsilon \to \infty$ → $\frac{g/c - g/c^*}{g/c^*} = \frac{2}{z m} \cdot \frac{u_0 - u^*}{u^*}$

Role of unemployment multiplier

$m = 0$: no stimulus spending $(g/c = g/c^*)$

small m: stimulus spending is ↗ in m

medium m: stimulus spending peaks

$$\frac{\partial}{\partial m}\left[\frac{2\varepsilon m}{1 + z\varepsilon m^2}\right] = 0 \Rightarrow \frac{2\varepsilon}{[\,]} - \frac{2\varepsilon m \times 2z\varepsilon m}{[\,]^2} = 0$$

$$\Rightarrow 2\varepsilon[1 + z\varepsilon m^2] - 4\varepsilon^2 z m^2 = 0$$

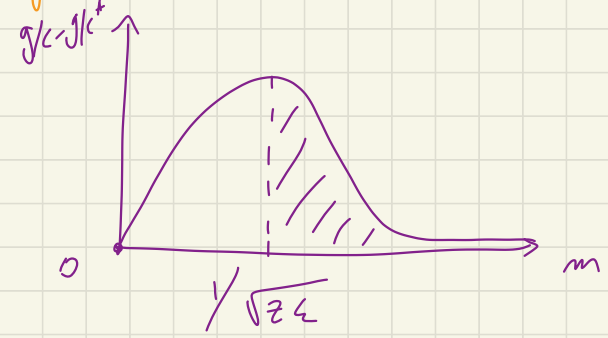
$$\Rightarrow 2\varepsilon + 2z\varepsilon^2 m^2 - 4\varepsilon^2 z m^2 = 0$$

$$\Rightarrow \quad 2\xi = \left[4\pi - 2\pi\right]\xi^2 m^2 = 2\pi\xi^2 m^2$$

$$\Rightarrow \quad m^2 = \frac{2\xi}{2\pi\xi^2} = \frac{1}{\pi\xi}$$

$$\Rightarrow \quad \boxed{m^* = 1/\sqrt{\pi\xi}}$$

After $m > m^*$ · stimulus spending ↓ w/ m



Ramey (2013) → evidence on $m = -du/dg > 0$

median estimate · $m \approx 0.5$