Recasting the Model in Terms of Visits

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Recasting the model to focus on visits to shops

Household i chooses visits v_i to max. utility s.t. budget constraint, taking as given tightness x & price p.

Household's problem

- consumption: $c_i = y_i - p \cdot v_i = [q(x) - p] \cdot v_i$ ($q(x)$ v)
- spending/output: $y_i = q(x) \cdot v_i$

$$\max_{v_i} \quad \frac{x}{1+x} \left[(q(x) - p) v_i \right]^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x} \left(\frac{m_i}{p} \right)^{\frac{\varepsilon-1}{\varepsilon}}$$

$$\text{s.t.} \quad m_i + p \cdot q(x) v_i = \nu_i + p f(x) h_i$$

m_i

$$\hookrightarrow \max_{v_i} \quad \frac{x}{1+x} \left[(q(x) - p) v_i \right]^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x} \left[-q(x) v_i + \frac{\nu_i}{p} + f(x) h_i \right]^{\frac{\varepsilon-1}{\varepsilon}}$$

concave maximization problem

FOC: $\frac{x}{1+x} \cdot \frac{\varepsilon-1}{\varepsilon} \left[q(x) - p \right]^{\frac{\varepsilon-1}{\varepsilon}} v_i^{-1/\varepsilon} = \frac{q(x)}{1+x} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left[\quad \right]^{-1/\varepsilon} = 0$

$$v_i = \frac{x^{\varepsilon}}{q(x)^{\varepsilon}} \left[q(x) - p \right]^{\varepsilon-1} \left[f(x) h_i + \frac{\nu_i}{p} - q(x) v_i \right]$$

$$\left[1 + x^{\varepsilon} q(x)^{1-\varepsilon} \left[q(x) - p \right]^{\varepsilon-1} \right] \cdot v_i = \frac{x^{\varepsilon}}{q(x)^{\varepsilon}} \left[q(x) - p \right]^{\varepsilon-1} \left[f(x) h_i + \frac{\nu_i}{p} \right]$$

$$v_i(x,p) = \frac{x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{-\varepsilon}}{1 + x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{1-\varepsilon}} \left[f(x) k_i + \frac{w_i}{p} \right]$$

$$q(x)\, v_i(x,p) = \frac{x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{1-\varepsilon}}{1 + x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{1-\varepsilon}} \left[f(x) k_i + \frac{w_i}{p} \right]$$

$q(x)\, v_i(x,p)$ ↑ $y_i(x,p)$

$\sigma(x)$ = MPS

[initial wealth + income]

Aggregate number of orders $v(x,p) = \sum_i v_i(x,p)$

$$q(x)\, v(x,p) = \sigma(x) \left[f(x) \cdot k + \frac{w}{p} \right]$$

same as AD curve