

Stabilization Achieved by Optimal Stimulus Spending

Pascal Michailat

<https://www.pascalmichailat.org/t5.html>



Optimal public expenditure w/ initial gap $u_0 - u^*$:

$$\frac{g/c - g/c^*}{g/c^*} = \frac{2 \cdot \varepsilon \cdot m}{1 + 2 \cdot \varepsilon \cdot m^2} \cdot \frac{u_0 - u^*}{u^*}$$

resulting unemployment gap, after public expenditure:

$$\frac{u - u^*}{u^*} = \frac{u_0 - u^*}{u^*} - \frac{m \cdot z}{2} \cdot \frac{g/c - g/c^*}{g/c^*}$$

$$z = \left[\frac{1}{u^*} \cdot \frac{2}{\frac{1}{g^*} + \frac{1}{c^*}} \right]$$

Combine optimal stimulus w/ effect of stimulus on unemployment:

$$\frac{u - u^*}{u^*} = \frac{u_0 - u^*}{u^*} - \frac{z m}{2} \left(\frac{2 \cdot \varepsilon m}{1 + 2 \cdot \varepsilon m^2} \right) \frac{u_0 - u^*}{u^*}$$

$$\frac{u - u^*}{u^*} = \left[1 - \frac{z \varepsilon m^2}{1 + z \varepsilon m^2} \right] \frac{u_0 - u^*}{u^*}$$

$$\frac{u - u^*}{u^*} = \frac{1}{1 + z \varepsilon m^2} \cdot \frac{u_0 - u^*}{u^*}$$

final unemployment gap (w/ optimal stimulus)

initial gap

Final unemployment gap ($u - u^*$) is a share

$$\frac{1}{1 + \varepsilon + m}$$

of the initial unemployment gap.

- share $\neq 0$: stimulus spending never eliminates unemployment gap
- share $\in (0, 1)$: stimulus spending always reduces the initial unemployment gap.
- share is \downarrow w/ ε : final unemployment gap is smaller when public goods are more substitutable w/ private goods.
- share is \downarrow w/ m : final unemployment gap is smaller when multiplier is larger
- final unemployment gap $\rightarrow 0$ when $\begin{cases} \varepsilon \rightarrow \infty \\ m \rightarrow \infty \end{cases}$