

# Definition and Properties of the Household's Problem

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Household maximizes utility, subject to budget constraint, taking as given price of services  $p$  & market tightness  $x$ .

$$\max_{c, m} u(c, m/p)$$

$$\text{s.t. } m + p[1 + \tau(x)] \cdot c - \mu - p f(x) \cdot h = 0$$

Rewrite budget constraint:

$$\frac{m}{p} = \frac{\mu}{p} + f(x) \cdot h - [1 + \tau(x)] c$$

$$\hookrightarrow \max_c u\left(c, \frac{\mu}{p} + f \cdot h - (1 + \tau) \cdot c\right)$$

$$\max_c \underbrace{\frac{x}{1+x}}_{\varepsilon > 1 \rightarrow \text{concave inc}} c^{\frac{\varepsilon-1}{\varepsilon}} + \underbrace{\frac{1}{1+x} \left[ \frac{\mu}{p} + f \cdot h - (1 + \tau) c \right]}_{\text{linear } p \text{ so concave inc}}^{\frac{\varepsilon-1}{\varepsilon}}$$

concave in  $c \geq 0$

$\varepsilon > 1 \rightarrow$  concave inc

- $y \mapsto y^{\frac{\varepsilon-1}{\varepsilon}} / (1+x)$  is strictly concave in  $y$ , and increasing
- $c \mapsto \frac{\mu}{p} + f \cdot h - (1 + \tau) c$  is linear  $p$  so concave inc
- composition of concave & increasing function + concave function  $\rightarrow$  concave inc