

Urn-Ball Matching Function

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<https://www.pascalmichailat.org/t5.html>



Market with B buyers & S sellers

Example. restaurants & lunch customers

S restaurants (one table, one meal)

B lunch customers (one meal at lunch)

One restaurant. $\frac{1}{S}$ proba to get one specific customer

$1 - \frac{1}{S}$ proba to NOT get customer

$\left(1 - \frac{1}{S}\right)^B$ proba to get NO customer

$1 - \left(1 - \frac{1}{S}\right)^B$ = proba to get AT LEAST ONE customer

= proba to sell a meal

= expected # meals sold by one restaurant

Expected # meals sold by all restaurants

$$S \times \left[1 - \left(1 - \frac{1}{S}\right)^B\right] = \# \text{ trades / meals sold}$$

Simplification $\left(1 - \frac{1}{S}\right)^B = \exp\left(B \cdot \ln\left(1 - \frac{1}{S}\right)\right)$

$$\ln(1-x) \approx -x \quad \text{when } x \approx 0$$

$$\left(1 - \frac{1}{s}\right)^B \sim \exp(B \times -(1/s)) \\ \sim \exp(-B/s)$$

$$\# \text{ meals} = \underline{S} \times \left(1 - \exp(-B/s)\right) = M(s, B)$$

mm-ball mat [↑] during function (clearly CPS)

$$- M(0, B) = 0 \quad M(s, 0) = 0$$

$$- \frac{\partial M}{\partial B} > 0 \quad - \frac{\partial M}{\partial s} \rightarrow \text{need to check}$$

$$\begin{aligned} \frac{\partial M}{\partial s} &= 1 - \exp(-B/s) + \left[-\frac{B}{s^2} \exp(-B/s) \right] S \\ &= 1 - e^{-B/s} - \frac{B}{s} e^{-B/s} \\ &= 1 - \left[1 + \frac{B}{s} \right] e^{-B/s} \end{aligned}$$

$$\frac{\partial M}{\partial s} = 1 - \frac{1 + B/s}{\exp(B/s)}$$

less than 1

$$e^x > 1 + x$$

$$\frac{\partial M}{\partial s} > 0$$

