Solving the Two-Market Model

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Solve the two-market model (x, θ) such that

$$\begin{cases} y^d(x) = y^s(x, \theta) & (AD = AS) \\ l^d(x, \theta) = l^s(\theta) & (LD = LS) \end{cases}$$

- $y^d(x) = y^s(x, \theta)$

$$\Leftrightarrow \frac{X^\varepsilon}{[1+\tau(x)]^{\varepsilon-1}} \cdot \frac{\mu}{p} = f(x) \cdot a \cdot \left[\frac{\hat{f}(\theta)\, h}{1+\hat{\tau}(\theta)} \right]^\alpha$$

$$\Leftrightarrow \boxed{f(x)\,[1+\tau(x)]^{\varepsilon-1} = \frac{X^\varepsilon \mu}{p\, a\, h^\alpha} \cdot \left[\frac{1+\hat{\tau}(\theta)}{\hat{f}(\theta)} \right]^\alpha} \qquad (P)$$

- $l^d(x, \theta) = l^s(\theta)$

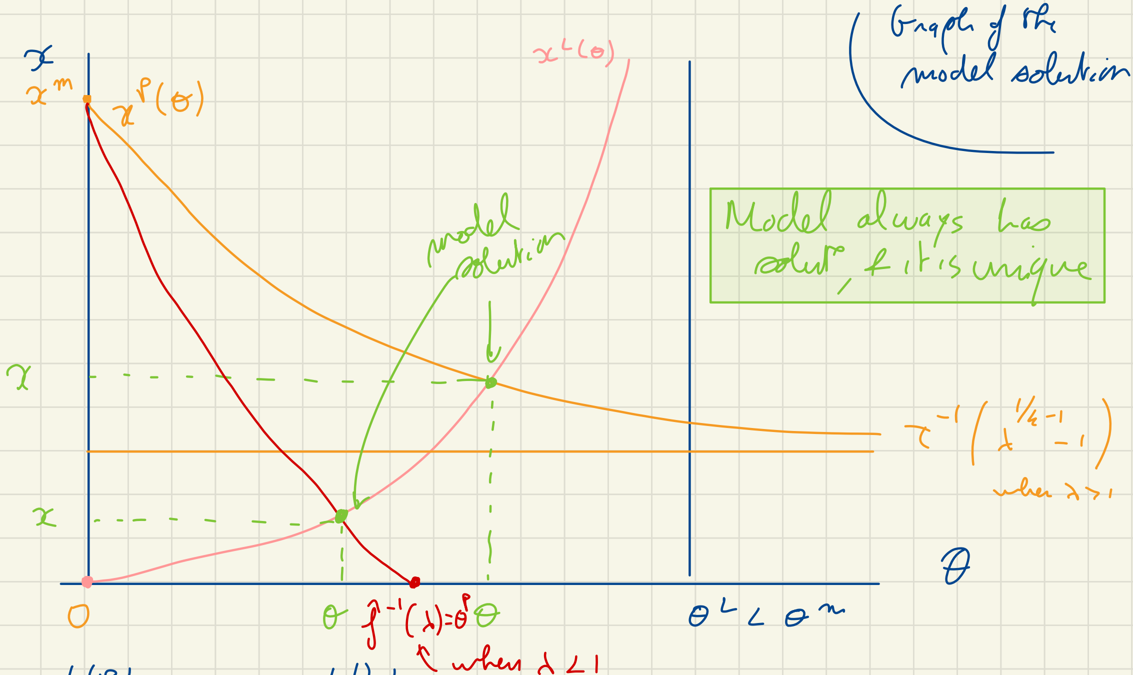
$$\Leftrightarrow \left[\frac{f(x)\, a\, \alpha}{w/p} \right]^{1/1-\alpha} \left[\frac{1}{1+\hat{\tau}(\theta)} \right]^{\alpha/1-\alpha} = \hat{f}(\theta)\, h$$

$$\Leftrightarrow \boxed{f(x) = \frac{w/p}{a\, \alpha} \cdot h^{1-\alpha}\, \hat{f}(\theta)^{1-\alpha} \left[1+\hat{\tau}(\theta)\right]^\alpha} \qquad (L)$$

What do we learn from (L)

$$x = x^L(\theta) = f^{-1}\left(\underbrace{\frac{w/p}{a\,\alpha}\, h^{1-\alpha}\, \hat{f}(\theta)^{1-\alpha} \left[1+\hat{\tau}(\theta)\right]^\alpha}_{\text{at } \theta^L \,=1} \right)$$

- x^L is strictly $\nearrow$ in θ
- $x^L(0) = f^{-1}(0) = 0$
- θ^L st $x^L(\theta^L) = f^{-1}(1) = +\infty$ with $\theta^L < \theta^m$



$$\begin{cases}(P)\\(L)\end{cases} \iff \begin{cases}(L)\\(P)/(L)\end{cases}$$

$$(P)/(L) \iff [1+\tau(x)]^{\varepsilon-1} = \frac{X^\varepsilon \nu \cancel{a}\,\alpha}{\cancel{p}\,\cancel{a}\cdot\cancel{\ell^\alpha}\,\cancel{\frac{w}{p}}\,h\,\cancel{\ldots}}\cdot\frac{1}{\bar{f}(\theta)}$$

$$\iff \tau(x) = \left[\frac{X^\varepsilon \nu\alpha}{w\cdot h}\cdot\frac{1}{\bar{f}(\theta)}\right]^{\frac{1}{\varepsilon-1}} - 1$$

$$\iff x = x^P(\theta) = \tau^{-1}\left[\left(\frac{X^\varepsilon \nu\alpha}{w\cdot h}\cdot\frac{1}{\bar{f}(\theta)}\right)^{\frac{1}{\varepsilon-1}} - 1\right]$$

- x^P is strictly $\downarrow$ in θ
- $x^P(0) = x^m$ b/c $\tau(x^m) = +\infty$
- Depending on whether $\left(\frac{X^\varepsilon \nu\alpha}{w\,h}\right)^{1/\varepsilon-1} - 1$ is >0 or <0.

I. $\left(\dfrac{x^{\varepsilon}\mu\alpha}{w \cdot h}\right)^{1/\varepsilon-1} - 1 > 0$ $\Leftarrow$ $\lambda > 1$

Then $\lim_{\theta \to \infty} x^P(\theta) = x^P$ st

$\tau(x^P) = \lambda^{1/\varepsilon-1} - 1$ a $\quad \boxed{x^P = \tau^{-1}\left(\lambda^{1/\varepsilon-1} - 1\right)}$

II. $\lambda < 1$. There is θ^P such that

$$\left(\frac{x^{\varepsilon}\mu\alpha}{w \cdot h}\,\frac{1}{\hat{f}(\theta^P)}\right)^{1/\varepsilon-1} = 1$$

$$\Leftrightarrow \hat{f}(\theta^P) = \frac{x^{\varepsilon}\mu\alpha}{w\; h} = \lambda \leftarrow \frac{x^{\varepsilon}\mu\alpha}{w \cdot h}$$

$$\Leftrightarrow \boxed{\theta^P = \hat{f}^{-1}(\lambda)}$$

Then $x^P(\theta^P) = \tau^{-1}(0) = 0$