Solving the Heterogeneous-Agent Model

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2 key relationships.

AS. $y = y^S(x) = f(x)\,k$

AD. $y = \sigma(x) \times \left[y^S(x) + \mu/\rho \right]$

Need to solve a system of 2 equations:

$$\begin{cases} y = y^S(x) \\ y = \sigma(x)\left[y^S(x) + \mu/\rho \right] \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} y = y^S(x) \\ y = \sigma(x)\left[y + \mu/\rho \right] \end{cases}$$

(substitution)

$$\Leftrightarrow \quad \begin{cases} y = y^S(x) \\ y = \dfrac{\sigma(x)}{1-\sigma(x)} \left(\mu/\rho \right) \end{cases}$$

$$\frac{\sigma(x)}{1-\sigma(x)} = X^{\varepsilon}\left[1+\tau(x)\right]^{1-\varepsilon} \quad \text{b/c} \quad \sigma(x) = \frac{X^{\varepsilon}\left[1+\tau(x)\right]^{1-\varepsilon}}{1+X^{\varepsilon}\left[1+\tau(x)\right]^{1-\varepsilon}}$$

$$\text{so} \quad 1-\sigma(x) = \frac{1}{1+X^{\varepsilon}\left[1+\tau(x)\right]^{1-\varepsilon}}$$

The system that describes the model therefore is

$$\begin{cases} y = y^s(x) & \text{increasing in } x \\ \boxed{y = \dfrac{x^{\varepsilon}}{[1+\tau(x)]^{\varepsilon-1}} \left(\dfrac{\mu}{p}\right)} & \text{if } p \text{ is fixed, decreasing in } x \end{cases}$$

Define the aggregate demand curve

$$\boxed{y^d(x,p) = \frac{x^{\varepsilon}}{[1+\tau(x)]^{\varepsilon-1}} \cdot \frac{\mu}{p}}$$

Then the model is given by the following system:

$$\begin{cases} y = y^s(x) \\ y = y^d(x,p) \end{cases}$$

Market tightness is implicitly given by:

$$\boxed{y^s(x) = y^d(x,p)}$$

As in representative-agent model:

- Tightness equalizes AD & AS curves
- AD & AS curves have same expression, same properties

Once tightness is obtained: can compute aggregate variables:

- $p = p^n(x)$
- $y = y^s(x)$
- $c = y / [1 + \tau(x)]$
- $m = \mu$ (Walras's Law)
- $v = y / q(x)$

y_i, c_i, m_i, v_i $\rightarrow$ can be computed from μ_i, k_i and tightness x for all i

- $y_i = \sigma(x) [f(x) k_i + \mu_i / p]$
- $c_i = y_i / [1 + \tau(x)]$
- $\frac{m_i}{p} = [1 - \sigma(x)] [f(x) k_i + \mu_i / p]$
- $v_i = y_i / q(x)$