Computing the Aggregate Demand Curve

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Aggregate demand: Amount of services that households purchase so as to maximize their utility, given price of service p and market tightness x.

Notation $y^d(x, p)$

To max. utility, household consumes

$$c = \left(\frac{x}{1+\tau(x)}\right)^{\varepsilon} \cdot \frac{m}{p}$$

To max utility, household purchases $c \cdot [1+\tau(x)]$.

$$y = \frac{x^{\varepsilon}}{[1+\tau(x)]^{\varepsilon-1}} \cdot \frac{m}{p}$$

Budget constraints of all households:

$$m + p \cdot [1+\tau(x)]\, c = \mu + p \cdot f(x) \cdot k$$

Through matching: # services sold = # services purchased

trades given by matching function

services sold $= f(x) \cdot k = m(k, v)$

services purchased $= q(x) \cdot v = m(k, v)$

$$= c \cdot [1 + \tau(x)] \quad \text{(by definition of } \tau(x)\text{)}$$

$$= y$$

$\rightarrow f(x) * k = [1 + \tau(x)] \cdot c$

Plug into budget constraint. $\boxed{m = \mu}$

Combining FOC from household problem w/ aggregate budget constraint.

$$\underbrace{y}_{\text{\# service purchased / demanded by households}} = \boxed{\frac{x^{\varepsilon}}{[1+\tau(x)]^{\varepsilon-1}} \cdot \frac{\mu}{p} = y^{d}(x, p)}$$

$y^{d}(x, p)$ is the AD curve