Household's Budget Constraint

Pascal Michaillat
https://www.pascalmichaillat.org/t5.html

Budget constraint:

Income

- Labor income.

$$p(t) \; a \; [1 - u(t)] \; h$$

- $p(t)$: price of a service
- a: productivity
- $[1 - u(t)]$: # workers w/ a job (# workers in labor force)
- h: # services sold by household per unit time

- Investment / saving income:

$$i(t) \; b(t)$$

- $i(t)$: nominal interest rate
- $b(t)$: # bonds held by household
- interest from bond holding

Expenditure

- expenditure on services

$$p(t) \; [1 + \tau(\theta(t))] \; c(t)$$

- $p(t)$: price of a service, given by a price norm
- $\tau(\theta(t))$: recruiting wedge
- $c(t)$: # services consumed by household
- $[1 + \tau(\theta(t))] \, c(t)$: # services purchased by household

- Lump-sum tax to finance interest payments $T(t)$

Nominal budget constraint.

$$\dot{b}(t) = i(t)\,b(t) + p(t)\,a\,[1-u(t)]\,h - p(t)\,[1+\tau(\Theta)]\,c(t) - T(t)$$

change in savings / nominal wealth at t.

Budget constraint in real terms:

real stock of bonds: $w(t) = \frac{b(t)}{p(t)}$

real interest rate $r(t) = i(t) - \pi(t)$

$i(t)$: nominal interest rate

$\pi(t)$: inflation rate $= \frac{\dot{p}(t)}{p(t)}$

d/dt $\ln(w(t)) = \ln(b(t)) - \ln(p(t))$

$$\frac{\dot{w}(t)}{w(t)} = \frac{\dot{b}(t)}{b(t)} - \frac{\dot{p}(t)}{p(t)} = \frac{\dot{b}(t)}{b(t)} - \pi(t)$$

$$\dot{w}(t) = w(t) \times \left[\frac{\dot{b}(t)}{b(t)}\right] - \pi(t)\cdot w(t)$$

$w(t)/b(t) = 1/p(t)$

$$\dot{w}(t) = \frac{\dot{b}(t)}{p(t)} - \pi(t)\cdot w(t)$$

Real budget constraint:

$$\dot{w}(t) = i(t)\, w(t) + a\,[1-u(t)]\,h - [1+\tau(\theta)]\,c(t) - \frac{T(t)}{p(t)} - T(t)\cdot w(t)$$

$$\dot{w}(t) = r(t)\, w(t) + a\,[1-u(t)]\,h - [1+\tau(\theta(t))]\,c(t) - \frac{T(t)}{p(t)}$$