Convergence to the Beveridge Curve

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Law of motion of unemployment rate u:

$$\dot{u}(t) = -[\lambda + f] u(t) + \lambda$$

job-finding rate   job-separation rate

Critical point. $\bar{u}$ such that $\dot{u} = 0$

$\dot{u} = 0 \implies -(\lambda + f) \bar{u} + \lambda = 0$

$$\implies \boxed{\bar{u} = \frac{\lambda}{\lambda + f}}$$

$$\implies \boxed{u(\theta) = \frac{\lambda}{\lambda + f(\theta)}}$$

Beveridge curve.

$$\lambda + f(\theta) = \frac{\lambda}{u}$$

$$f(\theta) = \frac{\lambda - \lambda u}{u}$$

$$\mu \, \theta^{1-\eta} = \frac{\lambda - \lambda u}{u}$$

$$\theta = \left[\frac{\lambda - \lambda u}{\mu \cdot u} \right]^{\frac{1}{1-\eta}} = \frac{v}{u}$$

$$v = \left[\frac{\lambda - \lambda u}{\mu \, u} \cdot u^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$v = \left[\frac{\lambda - \lambda u}{\mu \, u^{\eta}} \right]^{\frac{1}{1-\eta}}$$

Beveridge curve. $v(u)$ w/ $\frac{dv}{du} < 0$

Convergence of $u(t)$ to $u(\infty)$

$$\overset{o}{u}(t) + \left(\underset{gap}{\lambda + f} \right) u(t) = \lambda \quad \text{initial gap} \quad \overset{shrinking}{\nearrow}$$

Solution,

$$[u(t) - u(\infty)] = [u(0) - u(\infty)] e^{-(\lambda + f) t}$$

$$\Rightarrow u(t) = u(\infty) + [u(0) - u(\infty)] e^{-(\lambda + f) t}$$

$$\overset{o}{u}(t) = -(\lambda + f) [u(0) - u(\infty)] e^{-(\lambda + f) t}$$

$$\overset{o}{u}(t) = -(\lambda + f) \times [u(t) - u(\infty)]$$

$$\overset{o}{u}(t) + (\lambda + f) u(t) = (\lambda + f) u(\infty) = \lambda$$

⎛ differential equation is satisfied
⎝ + satisfy initial condition at $t=0$

Interpretation. Gap b/w $u(t)$ & Beveridge
curve shrinks at rate $(\lambda + f)$

$\Rightarrow$ $\lambda + f$ is speed at which unemployment
converge to Beveridge curve.

<u>In US</u> :   - $\lambda \approx 3\%$ per month

         - $f \approx 59\%$ per month

         $\Rightarrow \lambda + f \approx 62\%$ per month

<u>Half time</u>   time it takes for unemployment
rate to cover $1/2$ distance to
Beveridge curve.

$$\frac{\ln(2)}{\lambda + f} = \frac{0.69}{0.62} \approx 1.2 \text{ month}.$$

In a quarter, $(1/2)^3 = 1/8$ of initial
$\approx 10\%$
distance w/ Beveridge curve is left.

$\Rightarrow$ Convergence to Beveridge curve is

very fast b/c λ, f are large

$\Rightarrow$ Unemployment rate always on Beveridge
curve

$$u = u(\theta) = \frac{\lambda}{\lambda + f(\theta)}$$