

# AN ECONOMICAL BUSINESS-CYCLE MODEL (CHAPTER 1-4)

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## MOTIVATION

- The New Keynesian model is the canonical model of business cycles
- But, the NK model has two problems
- First, the NK model is not tractable for day-to-day thinking about macroeconomic issues, while the IS-LM model captures the overview of the business dynamics keeping its structure simple
- Second, the NK model does not describe unemployment dynamics well, while it is one of the most crucial policy target
- This article develops a very tractable model of business cycles, while keeping the forward-looking behavior of the households
- The model tells the effects of several shocks and policies by comparative statics

## OVERVIEW OF THE MODEL: LABOR MARKET

- Households' labor participation:  $l$
- Aggregate output:  $y_t = an_t = [1 - u_t]al$
- Matching function:  $m_t = \mu v_t^{1-\eta} [l - n_t]^\eta$
- Labor market tightness:  $\theta_t = v_t / (l - n_t)$
- Unemployment dynamics:  $\dot{u}_t = \lambda[l - u_t] - f(\theta_t)u_t$
- Data of  $\lambda$  and  $f(\theta_t)$  suggests that the dynamics converges to the steady state very fast:  $u_t = \lambda / (\lambda + f(\theta_t))$
- AS curve:  $y = \frac{f(\theta_t)}{\lambda + f(\theta_t)} al$  (increasing function of  $\theta$ )

## OVERVIEW OF THE MODEL: HOUSEHOLD

- Households' budget constraint:

$$\dot{b}_t = i_t b_t + p_t[1 - u_t]al - p_t[1 + \tau(\theta_t)]c_t - T_t$$

- Households have to pay recruiting wedge:  $\tau(\theta_t)$
- By defining  $w_t \equiv b_t/p_t$ ,

$$\dot{w}_t = r_t w_t + [1 - u_t]al - [1 + \tau(\theta_t)]c_t - \frac{T_t}{p_t}$$

- Households' objective function:

$$\int_0^{\infty} e^{-\delta t} \left[ \frac{\sigma}{\sigma - 1} c_t^{(\sigma-1)/\sigma} + x(w_t - \bar{w}_t) \right] dt$$

- Households gain utility by comparing others' wealth level

## OVERVIEW OF THE MODEL: HOUSEHOLD

- Infinite horizon version of Pontryagin's maximum theorem (Halkin 1974, ECMA) leads to,

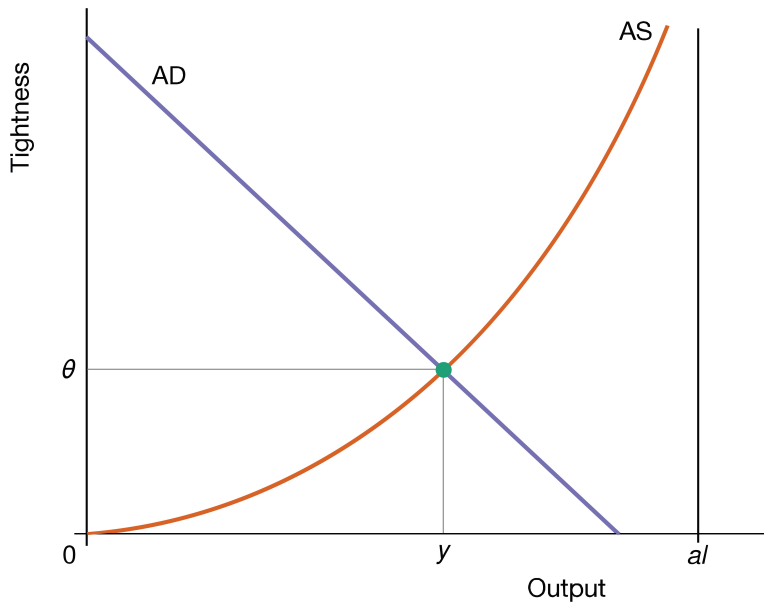
$$c_t^{-1/\sigma} = \gamma_t [1 + \tau(\theta_t)]$$
$$\dot{\gamma}_t = [\delta - r_t] \gamma_t - x'(w_t - \bar{w}_t)$$

- Assume central bank set nominal interest rate:  $i_t = i$
- Assume all the households are identical, so have the same wealth
- Then, the solution of the optimal control problem requires,

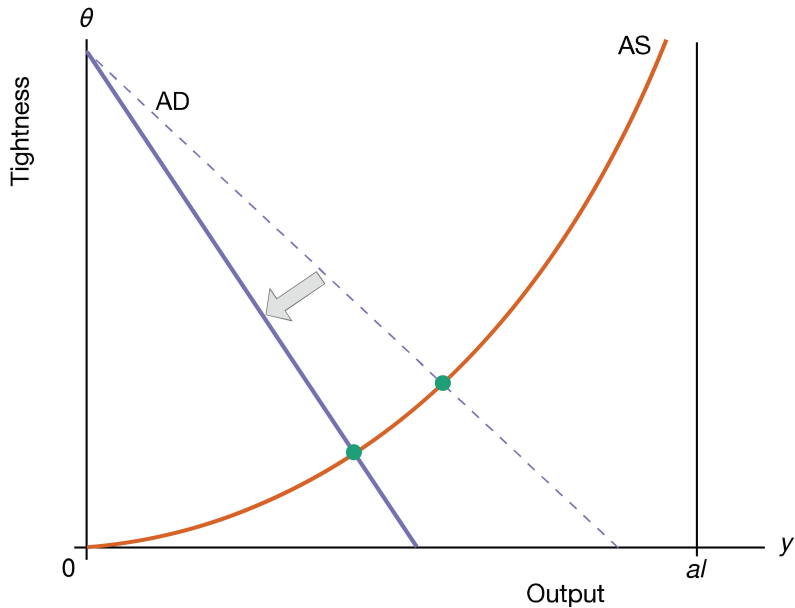
$$c_t = \left[ \frac{\delta - r}{x'(0)} \frac{1}{1 + \tau(\theta_t)} \right]$$

- Aggregate demand is given by  $y_t = [1 + \tau(\theta_t)] c_t$

## EQUILIBRIUM OF THE MODEL



## NEGATIVE SHOCK TO DISCOUNT FACTOR



# OKUN'S LAW IS CONSISTENT WITH AD SHOCKS

**Table 1.** Effects of business-cycle and policy shocks

Shock				Unemployment rate	
	Tightness	Output	Employment	Actual	Efficient
<b>AD shocks:</b>					
Decrease in discount rate	–	–	–	+	0
Increase in marginal utility of wealth	–	–	–	+	0
<b>AS shocks:</b>					
Decrease in labour productivity	+	–	+	–	0
Decrease in labour-force size	+	–	–	–	0
<b>Policy shocks:</b>					
Decrease in nominal interest rate	+	+	+	–	0
Increase in wealth tax rate	+	+	+	–	0