Beveridge Curve in the Dynamic Model

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$\begin{cases} \theta^* = 1 \\ u^* = \sqrt{uv} \end{cases}$ requires $\begin{cases} p = 1 & \text{(recruiting cost)} \\ \text{Beveridge curve is hyperbola.} \end{cases}$

Beveridge curve in dynamic model:

$$u(\theta) = \frac{\lambda}{\lambda + f(\theta)}$$

job-separation rate

matching elasticity

$$\Leftrightarrow \quad u = \frac{\lambda}{\lambda + \mu [v/u]^{1-\eta}}$$

$$\Leftrightarrow \quad \lambda u + \mu v^{1-\eta} u^{\eta} = \lambda$$

$$\Leftrightarrow \quad \mu v^{1-\eta} u^{\eta} = \lambda(1-u)$$

$$\Leftrightarrow \quad v^{1-\eta} = \frac{\lambda(1-u)}{\mu u^{\eta}}$$

$$\Leftrightarrow \quad \boxed{v(u) = \left[\frac{\lambda(1-u)}{\mu u^{\eta}}\right]^{1/1-\eta}}$$

$$\frac{d\ln v}{d\ln u} = \frac{1}{1-\eta}\left[-\frac{u}{1-u} - \eta\right] = -\frac{1}{1-\eta}\left[\eta + \frac{u}{1-u}\right]$$

$\eta \gg u/1-u$

$\Rightarrow$ Beveridge curve is almost isoelastic

$\begin{bmatrix} u \sim 5\% \\ 1-u \sim 0.95\% \end{bmatrix}$

$u/1-u \sim 5\%$

$\eta \sim 0.5$

If $\eta = 0.5$ (Petrongolo & Pissarides 2001)

then $\frac{d\ln\sigma}{d\ln u} = -2\left(0.5 + u/_{1-u}\right) = -\left(1 + u/_{1-u}\right)$

$$\boxed{\frac{d\ln\sigma}{d\ln u} \simeq -1}$$

To have hyperbola at efficiency point.

set η such that $\frac{\eta}{1-\eta} + \frac{u^*}{(1-\eta)(1-u^*)} = 1$

$u^* = 4\% \rightarrow \frac{\eta}{1-\eta} + \frac{0.04}{(1-\eta)\times 0.96} = 1$

$$\rightarrow \boxed{\eta \simeq 0.44}$$