Solving the Household's Problem

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<u>Household's problem (concave maximization problem):</u>

$$\max_{c} \quad \frac{\chi}{1+\chi} \, c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \left[\frac{w}{p} + \rho \cdot k - (1+\pi)\, c \right]^{\frac{\varepsilon-1}{\varepsilon}}$$

Necessary & sufficient condition for the optimal c (c maximizing utility): derivative = 0

Here: $\cancel{\frac{X}{1+X}} \cdot \cancel{\frac{\varepsilon-1}{\varepsilon}} \cdot c^{-\frac{1}{\varepsilon}}$

$$- [1+\tau(x)] \cancel{\frac{1}{1+X}} \cancel{\frac{\varepsilon-1}{\varepsilon}} [\ldots]^{-1/\varepsilon} = 0$$

$\cancel{\frac{\mu}{p}} + \int k - (1+\tau) c = \frac{m}{p}$

$$\underbrace{X c^{-1/\varepsilon}}_{\propto \text{ MU of services}} = [1+\tau(x)] \underbrace{[m/p]^{-1/\varepsilon}}_{\propto \text{ MU of real money balances / real wealth}}$$

price of services relative to real money balances

$$c^{-1/\varepsilon} = \frac{1+\tau(x)}{X} (m/p)^{-1/\varepsilon}$$

$$\boxed{c = \left[\frac{X}{1+\tau(X)}\right]^{\varepsilon} \cdot \frac{m}{p}}$$

- c : consumption of service
- # services purchased by household : y

$$\boxed{y = [1+\tau(x)] \cdot c}$$

- # visits by household : v

$$\boxed{v = y / q(x) = c \, [1+\tau(x)] / q(x)}$$