Structure of the Solution of the Two-Market Model

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Variables: y, ~~c~~, x, ~~p~~, ~~k~~, ℓ, ~~n~~, θ, ~~w~~

9 variables → 9 conditions/equations

Once we know these variables:

- rate of idleness = $1 - f(x)$
- rate of unemployment = $1 - \hat{f}(\theta)$
- trading probabilities: $f(x)$, $q(x)$, $\hat{f}(\theta)$, $\hat{q}(\theta)$
- matching wedge: $\tau(x)$, $\hat{\tau}(\theta)$

Can simplify model from 9×9 description:

$c = y / [1 + \tau(x)]$ 8×8

$n = \ell / [1 + \hat{\tau}(\theta)]$ 7×7

$k = a \cdot n^{\alpha} = a \cdot [\ell / [1 + \hat{\tau}(\theta)]]^{\alpha}$ 6×6

$p = p^n(x, \theta)$
$w = w^n(x, \theta)$

w/ fixed price – fixed wage assumption p, w are parameters

↳ Model boiled down to 4×4 system.

4 variables: y, ℓ, x, θ

4 equations

① $\cdot \quad \ell = \ell^s(\theta) = \hat{f}(\theta) \cdot h$

② $\cdot \quad \ell = \ell^d(\theta, x) = \left[\frac{f(x) \cdot a \cdot \alpha}{w/p}\right]^{1/1-\alpha} \left[\frac{1}{1+\hat{\tau}(\theta)}\right]^{\alpha/1-\alpha}$

③ $\cdot \quad y = y^s(x, \theta, \ell) = f(x) \cdot a \cdot \frac{\ell^{\alpha}}{[1+\hat{\tau}(\theta)]^{\alpha}}$

$y = y^d = \sigma(x) \left[f(x) \cdot h + \frac{N}{p}\right]$

$\Leftrightarrow$ ④ $y = y^d = \sigma(x) \cdot \left[f(x)\, a \frac{\ell^{\alpha}}{[1+\hat{\tau}(\theta)]^{\alpha}} + \frac{N}{p}\right]$