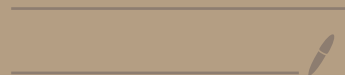


Solution of the Model in a Special Case Without Matching Cost

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<https://www.pascalmichailat.org/t5.html>



Special case:

- No matching cost: $p = 0$, $\tau = 0$
 - Matching function: $\gamma = 1$, $f(x) = \frac{x}{1+x}$, $q(x) = \frac{1}{1+x}$
- $$m = \left[\frac{1}{v} + \frac{1}{k} \right]^{-1} = \frac{1}{\frac{1}{v} + \frac{1}{k}}$$

Solution of the model: Find (v, x) such that

$$\begin{cases} v = x \cdot k \\ v = \frac{\sigma(x)}{q(x)} \left[f(x) k + \frac{\mu}{p} \right] \end{cases}$$

$$\sigma(x) = \frac{x^\varepsilon}{1+x^2} \quad \text{b/c} \quad \tau(x) = 0$$

$$\begin{cases} v = x \cdot k \\ v = \sigma \cdot \left[\frac{f(x)}{q(x)} k + \frac{\mu}{p \cdot q(x)} \right] \end{cases}$$

$$\begin{aligned} & \frac{1}{q(x)} = 1 + x \\ \Rightarrow & \begin{cases} v = k \cdot x \\ v = \sigma \left[k \cdot x + \frac{\mu}{p} (1+x) \right] \end{cases} \end{aligned}$$

$$\Rightarrow \begin{cases} v = h \cdot x \\ v = \underbrace{\sigma \cdot \left(h + \frac{\mu}{p} \right)}_{\lambda} x + \frac{\sigma \mu}{p} \end{cases}$$

$$\Rightarrow \begin{cases} v = \underline{h} \cdot x \\ v = \underline{\lambda} \cdot x + \frac{\sigma \mu}{p} \end{cases}$$

Is λ smaller or larger than h ?

To have a solution, amount of services demanded is less than capacity $\rightarrow y^d = \frac{\sigma}{1-\sigma} \frac{\mu}{p} < h$

$$\Rightarrow \sigma \frac{\mu}{p} < (1-\sigma) h$$

$$\Rightarrow \frac{\mu}{p} < \left(\frac{1-\sigma}{\sigma} \cdot h \right)$$

$$\Rightarrow \lambda < \sigma \left(h + \frac{1-\sigma}{\sigma} h \right)$$

$$\Rightarrow \lambda < \sigma h + (1-\sigma) h = h$$

$$\Rightarrow \boxed{\lambda < h}$$

Solution of model

