Definition and Properties of the Household's Problem

Pascal Michaillat
www.pascalmichaillat.org

Household maximizes utility, subject to budget constraint, taking as given price of services p & market tightness x.

$$\max_{c,\, m} \quad u\left(c,\ m/p\right)$$

$$\text{s.t.} \quad m + p\left[1+\tau(x)\right] \cdot c - \mu - p\, f(x) \cdot k = 0$$

<u>Rewrite budget constraint:</u>

$$\frac{m}{p} = \frac{\mu}{p} + f(x) \cdot k - \left[1+\tau(x)\right] c$$

$$\hookrightarrow \quad \max_{c} \quad u\left(c,\ \frac{\mu}{p} + f \cdot k - (1+\tau) \cdot c\right)$$

$$\max_{c} \quad \underbrace{\frac{X}{1+X}\, c^{\frac{\varepsilon-1}{\varepsilon}}} + \underbrace{\frac{1}{1+X}\left[\frac{\mu}{p} + f \cdot k - (1+\tau)\, c\right]^{\frac{\varepsilon-1}{\varepsilon}}}$$

$\varepsilon > 1 \rightarrow$ <u>concave</u> inc

- $y \mapsto y^{\frac{\varepsilon-1}{\varepsilon}} / (1+X)$ is strictly concave in y, and increasing
- $c \mapsto \frac{\mu}{p} + fk - (1+\tau)c$ is linear so concave inc
- composition of concave & increasing function + concave function inc $\rightarrow$ <u>concave</u>

concave in $c \geq 0$