ON THE EFFICIENCY OF MATCHING AND RELATED MODELS OF SEARCH AND UNEMPLOYMENT

Arthur J. Hosios

UCSC – Feb 27th, 2024

RESEARCH QUESTION

Create a framework

to evaluate the allocative performance of economies with slackness

- so that at least we can compare the efficiency results come from different models
- and understand why different models give different idea about whether the current situation is efficient or not

POSITIONING

- Integrate the basic setup of the DMP model (bilateral matching-bargaining models)
  - Diamond(1981, 1982a, 1984a), Mortensen(1982a,b) and Pissarides(1984a,b)
- What does this paper do?
  - Derived the efficiency conditions for constrained Pareto efficiency in the DMP setup
  - showed that it can be used to analyze conventional natural rate models that lack an explicit matching-bargaining structure

MODEL SETUP

- n risk neutral workers, and k firms which provides k jobs.
- u unemployed workers, and v vacant jobs.
- number of employed workers equals the number of filled jobs $n - u = k - v$
- Job finding rate $p(s; \hat{s}, \hat{r}, u, v)$ and job filling rate $q(r; \hat{s}, \hat{r}, u, v)$ is dependent on the searching effort s and recruiting efforts r
- number of trades $m(s, r, u, v) = up(s; s, r, u, v) = vq(r; s, r, u, v)$
- fraction b of currently employed workers lose jobs ($b(n - u)$
- A worker-firm pair produce output y where y follows a distribution $F(y)$
- the pair that produce $y < y^*$ disengage and continue to find jobs/recruit

MODEL

- average wage is $\bar{w} = E[w(y)|y^*]$, and average output is $\bar{y} = E[y|y^*]$
- unemployed workers enjoy leisure z, and a vacant job cost firms c
- acceptable job finding rate is $a(y^*)p(s)$, and acceptable job filling rate is $a(y^*)q(r)$
- Expected present value of lifetime utility for an employed worker is $W_e(\bar{y}) = \frac{\bar{w}+bW_u(s)+(1-b)W_e(\bar{y})}{(1+\delta)}$, which gives us the benefit of being employed at the current moment given whatever future is $W_e(\bar{y}) - W_u(s) = \frac{\bar{w}-\delta W_u(s)}{\delta+b}$
- Define permanent average income of an unemployed as $Y_u = \delta W_u(s) = z - s + ap(s)\frac{\bar{w}-Y_u}{b}$

SOLUTION OF THE MODEL

- Wage as a result of negotiation is $w(y) = Y_u + \theta(y - Y_u - Y_v)$
- Average wage is solved from $\bar{w} = E[Y_u + \theta(y - Y_u - Y_v)|y^*]$
- which gives the expected net surplus from an acceptable match $y - Y_u - Y_v = S(\bar{y}) = \frac{\bar{y}+c+r-(z-s)}{b+ap+aq(1-\theta)}$
- Given that unemployed workers and vacant firms are maximizing their utilities by choosing the searching/recruiting effort, we get the relative effort between firm recruiting and unemployed searching

LOCAL EFFICIENCY

- Finding the sharing rule θ and the matching technology $m(\cdot, \cdot)$ such that the decentralized equilibrium matches the situation with a benevolent social planner
- Social planner solves $Y = (n - u)\bar{y} + u(z - s) - v(c + r)$ such that $a(y^*)m(s, r, u, v) = (n - u)b$ (with multiplier λ), and $v - k = n - u$ (with multiplier μ)
- λ gives the marginal social benefit of an additional worker-firm pair
- $\frac{\partial \mathcal{L}}{\partial n}$ is the marginal social benefit from an extra unemployed, and $\frac{\partial \mathcal{L}}{k}$ is the marginal social benefit from an extra vacancy.
- We get efficiency if private choices don't cause externalities

CONDITIONS FOR EFFICIENCY - ENTRY/EXIT

- When people's private benefit from being unemployed Y_u is equal to the marginal social benefit of having one extra unemployed $\frac{\partial \mathcal{L}}{\partial n}$,
- and firms' private benefit from being vacant Y_v is equal to the marginal social benefit of having one extra vacancy $\frac{\partial \mathcal{L}}{k}$
- there will be no externalities from the private choice of entry/exit
- $\theta = \frac{u m_u}{m}$
- $m_u = \theta \frac{m}{u}, m_v = (1 - \theta) \frac{m}{v}$

CONDITIONS FOR EFFICIENCY - JOB ACCEPTANCE

- When the private acceptable output cutoff y^* is equal to the social optimal acceptable output cutoff y^{**}
- there will be no externalities from job acceptance
- $m_u + m_v = \frac{m}{u} + (1 - \theta)\frac{m}{v}$
- conditions that remove the entry/exit externalities will guarantee the removal of job acceptance externalities.

CONDITIONS FOR EFFICIENCY - SEARCHING/RECRUITING EFFORTS

- When the social benefit of unemployed increasing 1 unit of effort $\frac{\partial \mathcal{L}}{\partial s}$ is equal to the sum of all unemployed individuals increase 1 unit of searching effort $u(\frac{\partial Y_u}{\partial s})$
- When the social benefit of vacant firms increasing 1 unit of effort $\frac{\partial \mathcal{L}}{\partial r}$ is equal to the sum of all vacant firms increase 1 unit of searching effort $u(\frac{\partial Y_v}{\partial r})$
- there will be no externalities from searching/recruiting efforts
- with $S(\bar{y}) = \lambda$, it will be satisfied (net surplus of having 1 more worker-firm pair equals the marginal social benefits of having 1 more pair)

DIAMOND'S MODEL

- Diamond's result:
- with e agents, the matching probability (=buying probability = selling probability) for agents to trade is $b(e)$
- so the totally matches is $eb(e)$, where $b'(e) \geq 0$
- Diamond shows that the resulting equilibrium is locally INEFFICIENT whenever $b' > 0$

APPLY IT ON THE DIAMOND'S MODEL

- Consider the symmetric equilibrium (n=k; u=v; share = 1/2): 2 types of trader that gain utilities only from consuming the type of goods that was made by the other type
- searching effort for all traders no matter the types are the same so it can be omitted like in the model we learned.
- $m(u, v) = m(e/2, e/2) = eb(e)$
- their conditions: $m_u = \theta \frac{m}{u} = \frac{1}{2} \frac{m}{u}$
- $b(e) + eb^{'}(e) = \frac{1}{2} m_u + \frac{1}{2} m_v = \frac{1}{2} m_u = \frac{1}{2} \frac{m}{u} = b(e)$
- so according to the paper's framework, the efficient condition is $b^{'}(e) = 0$, which matches Diamond's result