

# THE PURE THEORY OF PUBLIC EXPENDITURE

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## RESEARCH QUESTION

*Samuelson: "Boss is here, listen*

- I just want to generalize and summarize the ultimate framework of public spending
- Plus, I am qualified and capable of doing this"

## POSITIONING

- People neglected the theory of optimal public spending but focus on the theory of taxation
  - Except Sax, Wicksell, Lindahl, Musgrave, and Bowen
- This paper focus on the theory of optimal public spending

## MODEL SETUP - ASSUMPTION

- $s$  individuals
- $n$  private consumption goods  $X_j = \sum_i^s X_j^i$
- $m$  collective consumption goods  $X_{n+r} = X_{n+r}^i$
- Agent  $i$ 's utility function  $u^i = u^i(X_1^i, \dots, X_n^i, X_{n+1}^i, \dots, X_{n+m}^i)$ 
  - Among all of the  $X_i$ , there are input items where people just want to minimize, flip the signs for those input items
  - $u_j^i = \frac{\partial u^i}{\partial X_j^i} > 0$
- Production function:  $F(X_1, \dots, X_n, X_{n+1}, \dots, X_{n+m}) = 0$ 
  - suppose  $X_5, X_6$  are the inputs (capital and labor), the production function is like  $X_1 + \dots + X_{n+m} - aX_5 + bX_6 = 0$
  - $X_5, X_6$  will be flipped signs as they are inputs
  - $F_r = \frac{\partial F}{\partial X_r} > 0$

## MODEL SOCIAL WELFARE

- Assume a social welfare function  $U = U(u^1, \dots, u^s)$
- Social planner's problem:

$$\max_{X_1^1, \dots, X_n^s, X_{n+1}, \dots, X_{n+m}} W(U^1, \dots, U^s)$$

s.t

$$F\left(\sum_i^s X_1^i, \dots, X_{n+m}\right) = f$$

## OPTIMAL CONDITION - SOCIAL WELFARE

$$\frac{u^i_j}{u^i_r} = \frac{F_j}{F_r} \quad \begin{array}{l} (i = 1, 2, \dots, s; r, j = 1, \dots, n) \text{ or} \\ (i = 1, 2, \dots, s; r = 1; j = 2, \dots, n) \end{array} \quad (1)$$

$$\sum_{i=1}^s \frac{u^i_{n+j}}{u^i_r} = \frac{F_{n+j}}{F_r} \quad \begin{array}{l} (j = 1, \dots, m; r = 1, \dots, n) \text{ or} \\ (j = 1, \dots, m; r = 1) \end{array} \quad (2)$$

$$\frac{U_i u^i_k}{U_q u^q_k} = 1 \quad \begin{array}{l} (i, q = 1, \dots, s; k = 1, \dots, n) \text{ or} \\ (q = 1; i = 2, \dots, s; k = 1). \end{array} \quad (3)$$

## PRIVATE AGENTS' PROBLEM

$$\max_{X_1^i, \dots, X_n^i, X_{n+1}, \dots, X_{n+m}} u^i$$

s.t

$$F(X_1^i, \dots, X_n^i, \dots, X_{n+m}) = f$$

- optimal conditions for the collective goods will be different:

$$\frac{u_{n+j}^i}{u_r^i} = \frac{F_{n+j}}{F_r}$$

## CONCLUSION

- In general we can't expect a decentralized equilibrium gives us the best social allocation
- To achieve best public spending, theoretically we always need government intervention