# Student Information

Name : Batuhan Karaca

ID : 2310191

In order to conduct a *Monte Carlo study*, we first need to determine the number of simulations $N$. We are looking for the minimum number of simulations $N_{min}$, for efficiency. In the textbook page 116, there is a table showing the formulae for $N$, for two cases

$$(1) \quad N \geq p^*(1-p^*)(\frac{z_{\alpha/2}}{\epsilon})^2$$

where $p^*$ is a preliminary estimator of p, or

$$(2) \quad N \geq 0.25(\frac{z_{\alpha/2}}{\epsilon})^2$$

if no such estimator is available. For each case, $\epsilon$ is the *margin of error* of how accurate would be the result, whereas $1-\alpha$, is the probability of not exceeding that *margin of error*. Since, we are not given a preliminary estimator $p^*$, we could use the case (2), and

> For each option, with probability 0.98, [...] differ from the true value by no more than 0.03.

it is given in the problem text that $1-\alpha = 0.98$ ($\alpha = 0.02,\ \alpha/2 = 0.01$), and $\epsilon = 0.03$. Furthermore, the value of $z_{\alpha/2}$ can be attained such that

$$\Phi(-z_{\alpha/2}) = \alpha/2$$

$$z_{\alpha/2} = -\Phi^{-1}(\alpha/2)$$

$$z_{0.01} = -\Phi^{-1}(0.01)$$

In the table of the *Standard Normal Distribution* in the textbook page 417, corresponding to the row $-2.3$, and the columns $-0.02, -0.03$ it is seen that both $\Phi(-2.33) < 0.01 < \Phi(-2.32)$. With the aid of a calculator (MATLAB, `norminv`), we find $\Phi^{-1}(0.01) \sim -2.3263$ is a more precise result. Therefore, we find

$$z_{0.01} \sim -(-2.3263) = 2.3263$$

Substituting ($z_{0.01} = 2.3263, \epsilon = 0.03$) gives

$$N \geq 0.25(\frac{2.3263}{0.03})^2$$

$$N \geq 1503,2421$$

$$N_{min} = 1504$$

We repeat the simulation 1504 times. First, we need some sort of container (array) to contain the data of size 1504. We shall begin iterating. For each simulation; It is stated that

> The number of distinct goods types to be processed within a day is a Poisson random variable with $\lambda = 160$.

which is generated by the `poissrnd` function. It is also stated that

> The undirected graph is formed through Bernoulli trials with $p$ given as the probability that an edge is present.

Then for each edge $X$ of the graph

$$X_i = \begin{cases} 1 & if \quad U < p \; (edge\ exists) \\ 0 & otherwise \; (no\ edges) \end{cases}$$

Where $U$ is a randomly generated *Standard Uniform Random Variable*, generated by the `rand` function. We create a symmetric adjacency matrix `t`, by the rule above. By the way, it is given that the same type of goods are assumed incompatible, therefore no loops occur in the matrix (diagonal is zero). The number of triangles is calculated with the formula, number of triplets is $Combination(Number\ of\ goods, 3)$. The corresponding results are loaded in the arrays.

# Answers

## a-b-c)

In the textbook page 114

> For a random variable $X$, the probability $P\{X \in A\}$
>
> $$\hat{p} = \hat{P}\{X \in A\} = \frac{\sum_i^N X_i}{N} = \bar{X}$$
>
> where
>
> $$X_i = \begin{cases} 1 & if \quad X_i \in A \\ 0 & otherwise \end{cases}$$

$N$ is the size of a *Monte Carlo Experiment* and $X_1, X_2 ... X_N$ are generated random variables with the same distribution as $X$. Regarding the above expression, we estimate the probabilities for *part a* (with $p = 0.012$), and similarly *part b* (with $p = 0.79$); and $X, Y$ in *part c* (with both probabilities seperately).

### Results

#### for p=0.012

a)An estimator for the probability that at most 1 distinct choice can be made for the first shipment of the day is $0.6662234043 \sim 0.7$

c)An estimator for $X$ is 1.1894946809

An estimator for $Y$ is 0.0000017185

**for p=0.79**

b)An estimator for the probability that the ratio of the number of possible choices for the first shipment to the number of different triplets exceeds the threshold 0.5 in one day. is $0.1462765957 \sim 0.1$

c)An estimator for $X$ is 335193.3317819149

An estimator for $Y$ is 0.4926895054

## d)

As explained in the textbook in page 118, we can calculate the *standard deviations* using `std(estimatorX)`, for a random variable $X$.

### Results

**for p=0.012**

d)$Std(X) = 1.1738427763$

$Std(Y) = 0.0000016389$

**for p=0.79**

d)$Std(X) = 81140.0626731319$

$Std(Y) = 0.0069427860$