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Proof of Memoryless Property of the Exponential Distribution

Suppose that an Exponential variable T represents waiting time. Memoryless property means that the fact of having waited for t minutes gets *"forgotten"* , and it does not affect the future waiting time. Regardless of the event $T > t$, when the total waiting time exceeds t, the remaining waiting time still has Exponential distribution with the same parameter. Mathematically,

$$P\{T > t + x \mid T > t\} = P\{T > x\}$$

In this formula, t is the already elapsed portion of waiting time, and x is the additional, remaining time.

PROOF

We know that the *pdf* of Exponential Distribution

$$f(t) = \lambda e^{-\lambda t}$$

Then the probability

$$\begin{aligned} P\{T > x\} &= \int_x^\infty f(t)dt \\ &= \int_x^\infty \lambda e^{-\lambda t}dt \\ &= lim_{K\to\infty} \int_x^K \lambda e^{-\lambda t}dt \\ &= lim_{K\to\infty}[-e^{-\lambda t}]_x^K \\ &= lim_{K\to\infty}(e^{-\lambda x} - e^{-\lambda K}) \\ &= e^{-\lambda x} \end{aligned}$$

Using the formula for the conditional probability

$$\begin{aligned} P\{T > t + x \mid T > t\} &= \frac{P\{T>t+x \cap T>t\}}{P\{T>t\}} \\ &= \frac{P\{T>t+x\}}{P\{T>t\}} \\ &= \frac{e^{-\lambda(t+x)}}{e^{-\lambda t}} \\ &= e^{-\lambda x} \end{aligned}$$

The work includes modifications written in LaTex