Submission for Deep Learning Exercise 1

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Pen and Paper task: Probability Theory: Bertrand's Box Paradox

Note that the variables BB, WW and BW denote the event for the card drawn, with two black sides, two white sides, and with one black and one white side respectively. Furthermore, $showB$ denotes when the drawn card's side facing up is black. Similarly white for $showW$.

1)

$$
\begin{aligned}
p(showB) &= p(showB|BB)p(BB) + p(showB|WW)p(WW) + p(showB|BW)p(BW) \\
&= 1\frac{1}{3} + 0 + \frac{1}{2}\frac{1}{3} \\
&= \frac{1}{2} \\
p(showW) &= p(showW|BB)p(BB) + p(showW|WW)p(WW) + p(showW|BW)p(BW) \\
&= 0 + 1\frac{1}{3} + \frac{1}{2}\frac{1}{3} \\
&= \frac{1}{2}
\end{aligned}
$$

2)

$$
\begin{aligned}
p(BB|showB) &= \frac{p(showB|BB)p(BB)}{p(showB)} \\
&= \frac{1\frac{1}{3}}{\frac{1}{2}} \\
&= \frac{2}{3}
\end{aligned}
$$

3)

$$
\begin{aligned}
p(BW|showW) &= \frac{p(showW|BW)p(BW)}{p(showW)} \\
&= \frac{\frac{1}{2}\frac{1}{3}}{\frac{1}{2}} \\
&= \frac{1}{3}
\end{aligned}
$$

Question in BLR: How do the prior and posterior distributions differ? Why?

One can see the plot contours in figure 1. It is easier to interpret a 2x2 covariance matrix Σ. Σ_{11} corresponds to the variance of ω_0, whereas Σ_{22} to the variance of ω_1. Furthermore, Σ_{12} and Σ_{21} are equal to the covariance between ω_0 and ω_1. Width of the contours are determined by the variance of ω_0, whereas height of the contours are determined by the variance of ω_1. The sign of the slope $\frac{d\omega_1}{d\omega_0}$ of the semi-major axis (see here) of the elliptic contours is the sign of the covariance (or linear dependence) between ω_0 and ω_1. The prior distribution is $N(\mathbf{0}, \mathbf{I})$, hence symmetrical along all axes with zero mean. However, the fitted weights are negatively correlated (slope is negative) with smaller variances (less uncertanty); hence, we can say that the models estimations became more correlated, and less random.

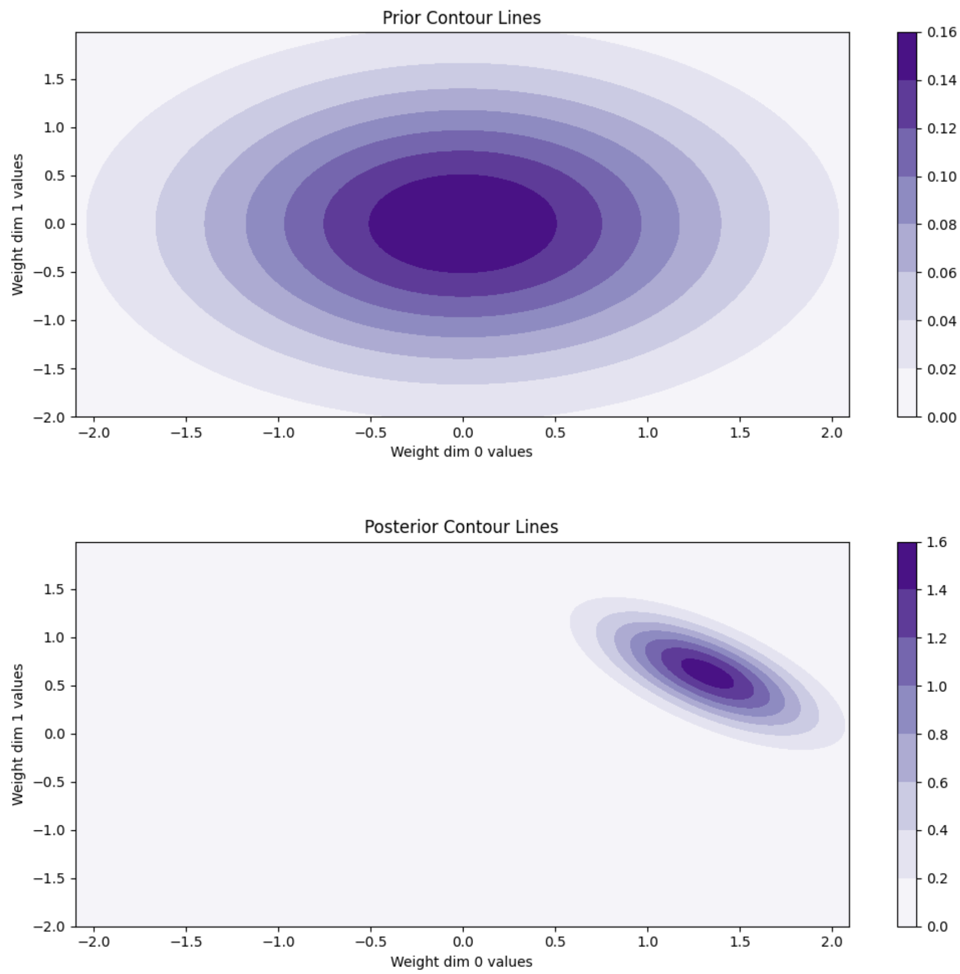

Figure 1: Contour lines of prior and post distributions