

## Student Information

Full Name : Batuhan Karaca

Id Number : 2310191

## Answer 1

a)

$p$	$q$	$(q \rightarrow \neg p)$	$(p \leftrightarrow q)$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
$T$	$T$	$F$	$T$	$F$
$F$	$F$	$T$	$T$	$T$
$T$	$F$	$T$	$F$	$F$
$F$	$T$	$T$	$F$	$F$

b)

$p$	$q$	$r$	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$	$((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$
$T$	$T$	$T$	$T$	$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$	$F$	$F$	$F$	$T$
$T$	$F$	$T$	$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$F$	$T$	$F$	$T$
$F$	$T$	$T$	$T$	$T$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$	$F$	$F$	$T$
$F$	$F$	$T$	$F$	$T$	$T$	$F$	$T$
$F$	$F$	$F$	$F$	$T$	$T$	$F$	$T$

Inferring by the truth table above, every possible truth combination of p, q and r gives the value T (true). Hence, we can say that the expression  $((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$  is a tautology.

## Answer 2

$$\begin{aligned}\neg p \rightarrow (q \rightarrow r) &\equiv \neg(\neg p) \vee (q \rightarrow r) \text{ TABLE 7} \\ &\equiv p \vee (q \rightarrow r) \text{ TABLE 6, by the double negation law} \\ &\equiv p \vee (\neg q \vee r) \text{ TABLE 7} \\ &\equiv (p \vee \neg q) \vee r \text{ TABLE 6, by the associative laws for disjunction} \\ &\equiv (\neg q \vee p) \vee r \text{ TABLE 6, by the commutative laws for disjunction} \\ &\equiv \neg q \vee (p \vee r) \text{ TABLE 6, by the associative laws for disjunction} \\ &\equiv q \rightarrow (p \vee r) \text{ TABLE 7}\end{aligned}$$

### Answer 3

- a)  $\forall x L(x, Burak)$   
b)  $\forall x L(Hazal, x)$   
c)  $\forall x \exists y L(x, y)$   
d)  $\forall x \exists y \neg L(x, y)$   
e)  $\forall x \exists y L(y, x)$   
f)  $\forall x \neg (L(x, Burak) \wedge L(x, Mustafa))$   
g)  $\exists x \exists y ((L(Ceren, x) \wedge L(Ceren, y) \wedge (x \neq y) \wedge \forall z (L(Ceren, z) \rightarrow ((z = x) \vee (z = y))))$   
h)  $\exists x ((\forall y (L(y, x)) \wedge \forall \omega \forall z (L(\omega, z))) \rightarrow (z = x))$   
i)  $\forall x \neg L(x, x)$   
j)  $\exists x \exists y (((x = y) \rightarrow L(x, y)) \wedge (((x \neq y) \wedge L(x, y) \wedge \forall z (L(x, z)))) \rightarrow ((z = x) \vee (z = y)))$

### Answer 4

1.	$p$	premise
2.	$p \rightarrow (r \rightarrow q)$	premise
3.	$q \rightarrow s$	premise
4.	$q \vee \neg q$	assumption
5.	$q$	assumption
6.	$q \vee \neg q$	$\vee i, 5$
7.	$q \rightarrow (q \vee \neg q)$	$\rightarrow i, 5-6$
8.	$\neg q$	assumption
9.	$q \vee \neg q$	$\vee i, 8$
10.	$\neg q \rightarrow (q \vee \neg q)$	$\rightarrow i, 8-9$
11.	$q \vee \neg q$	$\vee e, 4, 7, 10$
12.	$q$	assumption
13.	$s$	$\rightarrow e, 3, 12$
14.	$\neg q \vee s$	$\vee i, 13$
15.	$q \rightarrow (\neg q \vee s)$	$\rightarrow i, 12-14$
16.	$\neg q$	assumption
17.	$\neg q \vee s$	$\vee i, 16$
18.	$\neg q \rightarrow (\neg q \vee s)$	$\rightarrow i, 16-17$
19.	$\neg q \vee s$	$\vee e, 11, 15, 18$
20.	$\neg q$	assumption
21.	$q$	assumption
22.	$\perp$	$\neg e, 20, 21$
23.	$s$	$\perp e, 22$
24.	$s \vee \neg r$	$\vee i, 23$
25.	$\neg q \rightarrow (s \vee \neg r)$	$\rightarrow i, 20-24$

## Answer 5

1.	$\forall x(p(x) \rightarrow q(x))$	premise
2.	$\neg \exists z r(z)$	premise
3.	$\exists y p(y) \vee r(a)$	premise
4.	$(p(t) \rightarrow q(t))$	$\forall e, x=t, 1$
5.	$r(a)$	assumption
6.	$\exists z r(z)$	$\exists i, 5$
7.	$\perp$	$\perp i, 2, 6$
8.	$\neg r(a)$	$\neg i, 5, 7$
9.	$r(a)$	assumption
10.	$\perp$	$\neg e, 8, 9$
11.	$\exists y p(y)$	$\perp e, 10$
12.	$p(t)$	assumption
13.	$q(t)$	$\rightarrow e, 4, 12$
14.	$\exists z q(z)$	$\exists i, 13$
15.	$\exists z q(z)$	$\exists e, 11, 12-14$