THE 1 Solutions

Answer 1

a)

Table 1: Truth table for $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$q \to \neg p$	$p \leftrightarrow q$	$(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

b)

Table 2: Truth table for $((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$(p \vee q) \wedge (p \to r) \wedge (q \to r) \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

Thus, $((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$ is a tautology.

Answer 2

$\neg p \to (q \to r)$

$\equiv \neg p \to (\neg q \vee r)$ Implication (Table 7)

$\equiv \neg\neg p \vee (\neg q \vee r)$ Implication (Table 7)

$\equiv p \vee (\neg q \vee r)$ Double Negation Law (Table 6)

$\equiv (p \vee \neg q) \vee r$ Associative Law for Disjunction (Table 6)

$\equiv (\neg q \vee p) \vee r$ Commutative Law for Disjunction (Table 6)

$\equiv \neg q \vee (p \vee r)$ Associativity Law for Disjunction (Table 6)

$\equiv q \rightarrow (p \vee r)$ Implication (Table 7)

Thus, $\neg p \rightarrow (q \rightarrow r)$ is logically equivalent to $q \rightarrow (p \vee r)$.

Answer 3

a) $\forall x L(x, Burak)$

b) $\forall y L(Hazal, y)$

c) $\forall x \exists y L(x, y)$

d) $\neg \exists x \forall y L(x, y)$

e) $\forall y \exists x L(x, y)$

f) $\neg \exists x (L(x, Burak) \wedge L(x, Mustafa))$

g) $\exists x \exists y (L(Ceren, x) \wedge L(Ceren, y) \wedge (x \neq y) \wedge \forall z (L(Ceren, z) \rightarrow ((z = x) \vee (z = y)))$

h) $\exists y \forall x (L(x, y) \wedge \forall z (L(x, z) \rightarrow (y = z)))$

i) $\neg \exists x L(x, x)$

j) $\exists x \exists y (L(x, y) \wedge L(x, x) \wedge (x \neq y) \wedge \forall z (L(x, z) \rightarrow ((z = x) \vee (z = y)))$

Answer 4

1)	p	*premise*
2)	$p \rightarrow (r \rightarrow q)$	*premise*
3)	$q \rightarrow s$	*premise*
4)	$\neg q$	*assumption*
5)	$r \rightarrow q$	$\rightarrow e$ *2,1*
6)	r	*assumption*
7)	q	$\rightarrow e$ *5,6*
8)	$\perp$	$\neg e$ *7,4*
9)	$\neg r$	$\neg i$ *6-8*
10)	$s \vee \neg r$	$\vee i_2$ *9*
11)	$\neg q \rightarrow (s \vee \neg r)$	$\rightarrow i$ *4-10*

Answer 5

1)	$\forall x(\mathrm{p}(x) \to \mathrm{q}(x))$	*premise*
2)	$\neg\exists z \mathrm{r}(z)$	*premise*
3)	$\exists y \mathrm{p}(y) \vee \mathrm{r}(a)$	*premise*
4)	$\exists y \mathrm{p}(y)$	*assumption*
5)	$\mathrm{p}(c)$	*assumption*
6)	$\mathrm{p}(c) \to \mathrm{q}(c)$	$\forall$e 1
7)	$\mathrm{q}(c)$	$\to$e 5,6
8)	$\exists z \mathrm{q}(z)$	$\exists$i 7
9)	$\exists z \mathrm{q}(z)$	$\exists$e 4,5-8
10)	$\forall z \neg \mathrm{r}(z)$	**Lemma** 2
11)	$\mathrm{r}(a)$	*assumption*
12)	$\neg \mathrm{r}(a)$	$\forall$e 10
13)	$\perp$	$\neg$e 11,12
14)	$\mathrm{q}(a)$	$\perp$e 13
15)	$\exists z \mathrm{q}(z)$	$\exists$i 14
16)	$\exists z \mathrm{q}(z)$	$\vee$e 3,4-9,11-15

Proof of **Lemma** ($\neg\exists x \mathrm{p}(x) \vdash \forall x \neg \mathrm{p}(x)$):

1)	$\neg\exists x \mathrm{p}(x)$	*premise*
2)	x_0	$\forall$i constant
3)	$\mathrm{p}(x_0)$	*assumption*
4)	$\exists x \mathrm{p}(x)$	$\exists$i 3
5)	$\perp$	$\neg$e 4,1
6)	$\neg \mathrm{p}(x_0)$	$\neg$i 3-5
7)	$\forall x \neg \mathrm{p}(x)$	$\forall$i 2-6

Alternative solution (without using **Lemma**):

1)	$\forall x(\mathrm{p}(x) \to \mathrm{q}(x))$	*premise*
2)	$\neg\exists z\mathrm{r}(z)$	*premise*
3)	$\exists y\mathrm{p}(y) \vee \mathrm{r}(a)$	*premise*
4)	$\exists y\mathrm{p}(y)$	*assumption*
5)	$\mathrm{p}(c)$	*assumption*
6)	$\mathrm{p}(c) \to \mathrm{q}(c)$	$\forall$e 1
7)	$\mathrm{q}(c)$	$\to$e 5,6
8)	$\exists z\mathrm{q}(z)$	$\exists$i 7
9)	$\exists z\mathrm{q}(z)$	$\exists$e 4,5-8
10)	$\mathrm{r}(a)$	*assumption*
11)	$\exists z\mathrm{r}(z)$	$\exists$i 10
12)	$\perp$	$\neg$e 11,2
13)	$\mathrm{q}(a)$	$\perp$e 12
14)	$\exists z\mathrm{q}(z)$	$\exists$i 13
15)	$\exists z\mathrm{q}(z)$	$\vee$e 3,4-9,10-14