Exercices 6

1 OLS RISK DECOMPOSITION

Read section 3.3.3 "Statistical analysis of OLS" (page 26) in lecture_notes.pdf.

1] Using the notations from this document, show that :

$$R_X(\theta^*) = \sigma^2 \tag{1}$$

2] Show the first part of proposition 15 ("Risk decomposition for OLS, linear model, fixed design").

$$R_X(\theta) - R_X(\theta^*) = \|\theta - \theta^*\|_{\hat{\Sigma}}^2 \tag{2}$$

where $R_X(\theta)$ is the fixed design risk, defined by

$$R_X(\theta) = E_y\left[\frac{1}{n}\|y - X\theta\|^2\right] \tag{3}$$