Markov Chain

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Introduction

The purpose of this report is model the economic transitions of countries through the use of a Finite Homogeneous Markov Chain model.

For individuals who pay attention to the state of the global economy, it is evident that there has been a shift in ideology from globalization towards national protectionism. National leaders and politicians are gaining widespread support by claiming to deport immigrants, close their borders, and in certain cases—return the nation back to its height of greatness. A well known example of this phenomenon is the rise of current US President Donald Trump, who was elected on the following platform:

- Make America great again
- Close immigration borders
- Reduce involvement in international trade agreements

Inherent in that first statement is a few key points:

I The status of the country has changed

II It is possible to return to this previous status. $\therefore$ $\exists$ a set S of statuses ≥ 2, such that $\{s1, s2\} \in$ S, for which it is possible to move from one to the next. Furthermore, if S1 is the current status of the US and S2 is the previous, then, S1$\Leftrightarrow$ S2 meaning that this movement is bidirectional for at least these 2 statuses.

III There is an ordinality to these statuses—the current status is not great, but the previous one is.

Because of the rise of technology and the internet, our access to current information is now instantaneous. For us, this means that we do not need to make certain assumptions in order to continue with our analysis. We can see from current and historic global events that the above statements are true not just for the US, but also generally. Examples include; the recently concluded elections in France where the runner up ran on a similar platform, the referendum leading to the exit of Britain from the EU, termed 'Brexit', and the ongoing events with North Korea and it's desire for vengeance against the US.

Questions

1. how are they defined?
2. how many statuses are there in our set S?
3. how are they ranked?
4. how often does a country change status?

We will use these questions to unpack and explore the preceding statements. Q1) The answer to that question varies depending on the source. There are several metrics used for classifying countries. The United Nations has the World Happiness Index— this attempts to measure the happiness of a country. The Organization for Economic Co-operation and Development (OECD) has a similar metric called subjective well-being index, World Health Organization(WHO) has its own metric which measures health of countries. The most widely used definitions however, involve the output of a country. This is usually measured in either GDP (Gross Domestic Output) or more recently GNI (Gross National Income). It is worth noting that these measurements are ultimately arbitrary and are created based simply on a relevant need at the time. This of course means that alteration of any of these measures is valid as long as there is still interpretation in context.In this report, the definition of choice came from the World Bank, their metric is based on the GNI of a country

Q2) It classifies countries into the following 4 groups:

1. H-High Income Countries
2. UM-Upper Middle Income Countries
3. LM-Lower Middle Income Countries
4. L-Low Income Countries

Q3) The rankings are defined according to the atlas methodology—a formula that reduces the impact of exchange rates in cross country comparisons and then classifies them based on certain standardized thresholds. A further addition to this point is the idea that not all countries are equal, while this is an obvious statement, it is relevant to our analysis. We can see from the classifications names the implicit ordinality of the groupings. A country categorized as L must be worse off than a country with the classification H. In this report we added an extra status, S-Superpower. This refers to the unofficial group of countries that have globally dominant positions characterized by a disproportionate ability to exert global influence. So our updated groups and ranking are now:

1. S-Super Power
2. H-High Income Countries
3. UM-Upper Middle Income Countries
4. LM-Lower Middle Income Countries
5. L-Low Income Countries

Q4) As stated earlier, not all countries are equal, so a movement from L to any other group is a positive movement, while a movement from H is considered a positive movement iff it goes directly to group S. The null hypothesis prior to analysis is that most countries will tend to remain within the same group. Furthermore, We can superficially apply the law of entropy here to state that the economic deterioration of a country (transition from a higher group to a lower group) is easier than the opposite. So under this assumption, we should expect countries to exhibit little economic movement, and we should see more negative transitions than positive.

So, the problem we are trying to model is the economic movement of countries between groups and the relevant questions.

Markov Chain Model

Since our goal is to model Economic transitions via a Finite Homogenous Markov Chain which from here on will be referred to as MC. The first step must be to express it in MC terms.

MC Assumptions

Def: A sequence $(\mathrm{X}_{(n)})\ n \in N, X_{(n)} : (\Omega, F, p) \rightarrow A$ is said to be MC with finite state spaces, initial distribution ρ and transition matrix P,iff:

(i) P is a non-negative stochastic matrix such that $p(i,j) \geq\ 0\ \forall\ i,j = 1,2..,n$.

(ii) a matrix is defined as stochastic *iff*: $\sum_{j \in S} p(i,j) = 1$

(iii) $p(X_{(0)} = i) = \rho(i)$

(iv) $p(X_{(n+1)} = i_{n+1}|X_n = i_{n+1}, X_{n-1} = i_{n-1}, ...X_0 = i_0) = p(X_{n+1} = i_{n+1}|X_n = i_n)\forall n \geq 0\, and\, i_0, ..., i_{n+1} \in A$ whenever our conditional probabilities exist

These assumptions are necessary for the generation of any MC.(i) and (ii) allow us to build a matrix of the probabilities of transitioning from one state to another within a single step. So the values must lie between $0 \leq p(i,j) \leq 1$ and the stochastic condition ensures that within each row, the probabilities are equal to the joint probability distribution as derived from the law of total probability. (iii) states that the probabilities of the stat we start in, are our initial distribution, and of course it must also sum to 1. (iv) is the markov property, it states that the probability of moving from on state to another is dependent only on the preceding state. It essentially states that transitions are memoryless.

We can now define $X_{(n)}$ to be the classification of a country at time n. If we do that, then we must also define our state spaces as the statuses or groups from earlier. So our states are $\{L, LM, UM, H, S\} \in$ A. and our transition matrix will be of the form:

$$\begin{array}{c c} & \begin{array}{ccccc} L & LM & UM & H & S \end{array} \\ \begin{array}{c} L \\ LM \\ UM \\ H \\ S \end{array} & \begin{pmatrix} p & p & p & p & p \\ p & p & p & p & p \\ p & p & p & p & p \\ p & p & p & p & p \\ p & p & p & p & p \end{pmatrix} \end{array}$$

where p just represents the probabilities.

With the assumptions above,we can now fully express the statements in our introduction with the proper terminology.

Recall from (I) that we had stated that a country can transition from status to class. This requires little explanation in terms of the states. From (II) we showed that the transition from state to state was bidirectional for at least 2 of these states. In markov terms that statement is equivalent to: state i leads to state j$(i \to j)$ iff $\exists n \geq 1$ such that P(n,i,j)>0. For example, we can go from L to H iff the probability of going from L to H is non negative $\forall$ n $\in$ N. We can then say that H is accessible from L. If the transition is bidirectional, then we say that L communicates with H, (H $\Leftrightarrow$ L) iff $i \to j \& i \leftarrow j$.

In this paper, we only address the following kinds of MC.

1. An Ergodic-Regular MC
2. An Ergodic-Cyclic MC

3. An Absorbing MC

The difference between them is based on 2 principles. The number of classes as well as the period of the MC. These principles are well documented and defined both in the course and in general literature, so in the interest of producing a paper of economical size, I will only define and explain general terms that are not easily accessed in common literature.

Estimation and Transition Matrix

The data collection and estimation were fairly labor intensive, so they require a bit of explanation. The data was collected from the World Bank, they have been collecting and classifying countries according to GNI income groups since the 1960s. Because this Markov Chain is not defined for a specific country, the procedure involved going to the website, and downloading and labelling all the countries from a time when the amount of missing data was 0. This ended up being 1987-2016. Once the data was collected, we then had to reclassify a few of the countries that have changed borders, i.e, USSR and present day Russia, a few of the smaller islands, yuogoslavia..etc. Once that was done the next step was estimation. The superpower state is an informal classification, and as such there is no formal classification method. So what we did was to source from 6 different online articles, the subset of countries which existed in all of them. These countries are:

1. US
2. UK
3. Germany
4. Russia
5. China
6. Japan
7. France

The classification for some of these countries was recent. i.e; China, and Russia—a reemergence after their decline post cold war. Others had been declining for a while and would no longer be classified in that category; France, UK. countries like Japan and Germany, were there because of the influence they posses over their respective continents. Lastly, the US is the only country that was always classified as a super power. We then altered some of the existing classes in each country to match their Super power status to the corresponding year. So for example; China as a super power is a fairly recent phenomenon (2015) so they only had 2 years in that state. Once these were estimated in a spreadsheet in excel, we then used the Markovchainfit function in r, to generate the transition matrix below:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} L & LM & UM & H & S \end{array} \\
\begin{array}{c} L \\ LM \\ UM \\ H \\ S \end{array} &
\begin{pmatrix}
0.9619 & 0.03738 & 0.0006 & 0.0000 & 0.0000 \\
0.0229 & 0.9326 & 0.0439 & 0.0006 & 0.0000 \\
0.0000 & 0.0258 & 0.9307 & 0.0426 & 0.0009 \\
0.0000 & 0.0000 & 0.0165 & 0.9809 & 0.0025 \\
0.0000 & 0.0000 & 0.0000 & 0.1707 & 0.8293
\end{pmatrix}
\end{array}
$$

The matrix satisfies all the properties of a transition matrix.

Economic Transience MC

We defined $X_{(n)}$ earlier to be the classification of a country at time n. We can see that this satisfies the Markov property(memoryless) because the probability of moving to another state depends only on the current state.

So now we want to know what kind of MC we have. We did this in r, with the function communicatingClasses in r, this returns all the classes in which states communicate with each other. It outputs a list of all the classes that have communicating states. IN our case it produced only one output which contained all our states, therefore, we know that the chain is irreducible and therefore ergodic. As a redundancy measure we also ran the is.irreducible function. this outputted a boolean result of True.

```
#type of MC
communicatingClasses(EconMC)
is.irreducible(EconMC)
```

Next we want to know the period of our MC, so that we can further classify it as regular or cyclic. The Period function in r was d=1. This tells us that the MC is aperiodic because one is the gcd of the number of steps for a state to return to itself. So despite the number of zeros in the transition matrix, we know that at some point p(i,j)>0 $\forall n \in N$.

Therefore, we know that our MC has 1 class, and it has a period of 1. So this is an Ergodic-Regular MC. We also have the graph associated with the MC.

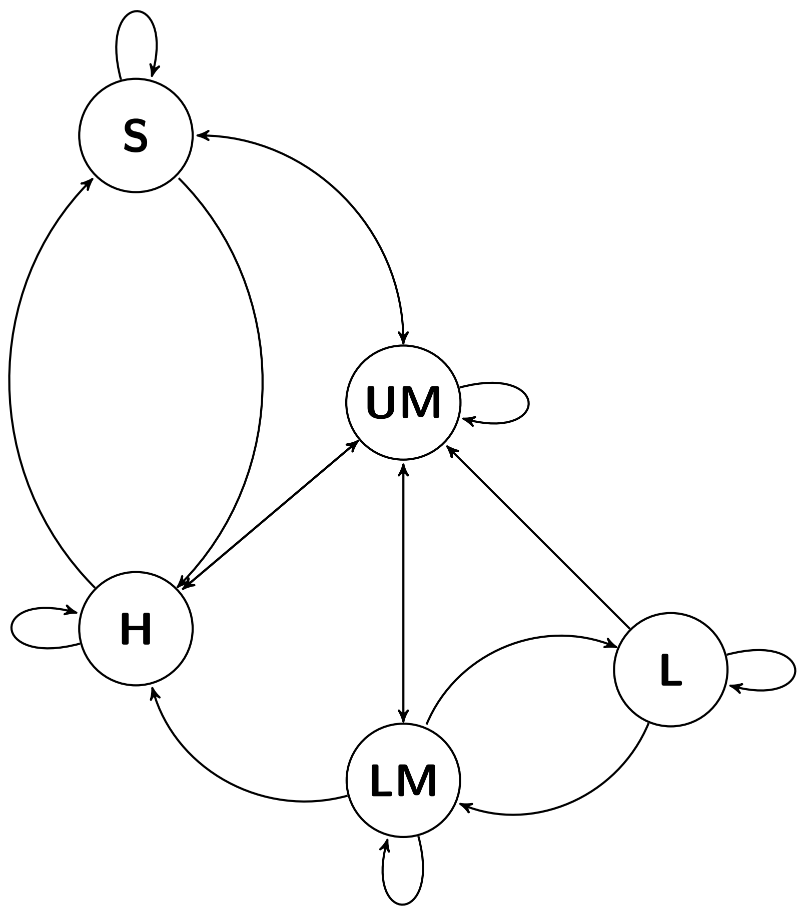

Figure 1: Graph of transition states

Calculations from r

With a regular matrix there a lot of questions we can ask, in our case, we can ask how many times we would expect to visit state S if we start in H, we can also calculate the mean number of times we will bounce back and forth between a subset of states. We might also be wondering what the mean recurrence time is for the superpower state. we can calculate the variance of this time as well. We also would want to know what the steady state distribution is for this matrix.

P^{10}	1	2	3	4	5	P^{50}	1	2	3	4	5
1	0.70	0.24	0.05	0.01	0.00	1	0.29	0.28	0.21	0.22	0.00
2	0.14	0.55	0.24	0.06	0.00	2	0.17	0.22	0.24	0.37	0.01
3	0.02	0.14	0.54	0.30	0.01	3	0.07	0.14	0.23	0.54	0.01
4	0.00	0.01	0.11	0.86	0.01	4	0.03	0.08	0.21	0.68	0.01
5	0.00	0.00	0.06	0.77	0.16	5	0.02	0.07	0.20	0.69	0.01

P^{100}	1	2	3	4	5	P^{150}	1	2	3	4	5
1	0.15	0.19	0.22	0.43	0.01	1	0.10	0.15	0.22	0.52	0.01
2	0.11	0.16	0.22	0.50	0.01	2	0.09	0.14	0.22	0.55	0.01
3	0.08	0.13	0.22	0.57	0.01	3	0.08	0.13	0.22	0.57	0.01
4	0.06	0.11	0.21	0.61	0.01	4	0.07	0.12	0.22	0.59	0.01
5	0.05	0.11	0.21	0.62	0.01	5	0.07	0.12	0.22	0.59	0.01

P^{200}	1	2	3	4	5	P^{500}	1	2	3	4	5
1	0.09	0.13	0.22	0.55	0.01	1	0.08	0.12	0.22	0.57	0.01
2	0.08	0.13	0.22	0.56	0.01	2	0.07	0.12	0.22	0.57	0.01
3	0.08	0.13	0.22	0.57	0.01	3	0.07	0.12	0.22	0.57	0.01
4	0.07	0.12	0.22	0.58	0.01	4	0.07	0.12	0.22	0.57	0.01
5	0.07	0.12	0.22	0.58	0.01	5	0.07	0.12	0.22	0.57	0.01

Table 1: P^n Matrices

We can see that the even at step n = 500, we are close to but not quite at the stationary distribution yet, but this gives a sense of the range of time before we converge on Π

To ensure that this is in fact the steady state we can check to see if π'(I-P)=0 or equivalently that; $\pi\prime P = \pi\prime$.

Now we can also calculate the eigenvectors for this MC.

	π
1	0.07
2	0.12
3	0.22
4	0.57
5	0.01

Table 2: stationary vector

L	LM	UM	H	S
0.074968217	0.124728780	0.217166994	0.573590453	0.009545556

Table 3: eigenvector

The calculation of the fundamental matrix Z is as follows:

$$Z = I + \sum_{n \geq 1} (P^n - \Pi) \tag{1}$$

we calculated this using the solve function in r. Here is the result:

	L	LM	UM	H	S
L	33.55	15.42	-3.50	-43.75	-0.72
LM	9.47	19.02	2.05	-29.07	-0.47
UM	-1.18	1.31	9.97	-8.96	-0.13
H	-5.76	-6.31	-3.31	16.22	0.16
S	-6.20	-7.04	-4.58	12.86	5.97

Table 4: Z matrix

Z is critical but as we can see it is not a stochastic matrix, so on its own it gives us no information about the MC, instead we need to use Z to solve for a new matrix M, this will give us our expectations.

$$M = (I - Z + EDiag(Z)(Diag\Pi))^{-1} \tag{2}$$

	L	LM	UM	H	S
L	13.34	28.86	62.03	104.56	699.98
LM	321.25	8.02	36.46	78.96	674.38
UM	463.27	142.02	4.60	43.90	639.16
H	524.39	203.14	61.12	1.74	607.85
S	530.25	209.00	66.98	5.86	104.76

Table 5: M matrix for expectations

Questions

So these are some problems relating to the relevant questions:

1. How long can a super power remain in power?
2. For a country in state L, how long would it take to get to a positive state? which we define as UMUM,H,S
3. was our superficial application of entropy right? do countries tend to gravitate towards decline? or should our definition of entropy state that countries tend towards improvement?

With the calculations above, we have all we need to answer these questions.

Q1) This is rewritten as:

$$m(S,S) = E_S(\tau_S) = \frac{1}{\pi(S)} = 104.76 \tag{3}$$

Therefore we expect that the mean recurrence time for a country to become a Super power if it currently one is 105 years. Which seems a bit low to me. Of course that could also be due to estimation reliability and given that most countries have not been able to remain in that state except for the US, it might be somewhat accurate.

Q2) Here we are asking the mean length of time to be reclassified as an economically developed country.

$$E_L(\tau_{UM}) \cup E_L(\tau_H) \cup E_L(\tau_S) = \min_{E_L(\tau_{UM} \cup \tau_H \cup \tau_S)} = 62.03 \tag{4}$$

So the mean length of time before a Low income country can become Upper Middle or higher is 62 years. This is in step with historic data, the only countries that have been able to accelerate at faster rates than that have been most recently China and India who did it in under 50. most countries do not possess the capabilities for that kind of rapid economic growth.

Q3) We can see from the M matrix table 6, that m(i,i) is relatively low for all states except S. This means that most countries can revisit their current state again relatively quickly. If our original hypothesis about a decline in growth is true, the we should expect to see that the mean recurrence time for states below the current one should be lower than the stat above where it is. With the exception of state S, It seems not to be the case. For example, we can see that if we start in state LM, the mean time to get to UM is 36.46 while the mean time for L is 321.25. This holds for all the states. We can also inspect the steady state to see where the greatest proportion of countries lie. It shows that 57% of countries will end up in the UM state, this could be due to a variety of factors including technological advancement, increase in trade between countries and the sharing of information. Of course, if countries do decide to pursue the protectionist ideology that so many seem to be embracing now, these estimates could be off.

conclusion

Practically speaking, a model like this could have a huge impact on trade agreements, future partnerships between countries. This is because if we have some predictive power and can tell when a country is close to approaching decline, then redistribution of global aid and resources can be better calculated. Secondly, We can use this model to test for any shocks to the system i.e; war, dynamic technological change, scarcity of resources etc. Of course, as the model currently is, there a myriad of shortfalls in the estimation methodology, but with a longer time frame, and more research put into estimation. This model could prove extremely useful in policy planning and analysis.