Numerical Methods of Thermo-Fluid Dynamics I

Winter Semester 2020-2021

Deliverable Task 1:
Numerical Solution of 2D Heat Equation

Given: Monday, 9/11/2020
Deadline: 14/12/2020

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Deliverable Task I

Consider the dimensionless 2D heat equation:

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}, \tag{1}$$

$$[x, y] \in [0, 1] \times [0, 1],\ t \in [0, 0.16] \tag{2}$$

The initial and boundary conditions are:

$$w(x, y, 0) = 0, \tag{3}$$

$$w(0, y, t) = 1 - y^3, \quad w(1, y, t) = 1 - sin(\pi/2 * y), \quad w(x, 0, t) = 1, \quad w(x, 1, t) = 0. \tag{4}$$

- Discretize the equation by the CDS scheme in space and the Crank-Nicolson method in time:
  - (**5pts**) Show under what conditions the Crank-Nicolson scheme is stable.
  - (**5pts**) Show that the numerical results will converge when the grid is refined.
- (**50pts**) Write MATLAB/OCTAVE programs for the Crank-Nicolson and explicit Euler schemes to solve the heat equation. The programs should automatically generate all plots required.
- Use $h = 1/40$ in x and y directions and Δt with different values: 0.01, 0.001 and 0.0001.
  - (**10pts**) Generate two plots (one for Explicit Euler, and one for Crank-Nicolson) by using the stable Δt, show the time evolution of the temperature at $=y=0.4$, explain the results.
  - (**10pts**) Use astable Δt, generate two plots (one for Explicit Euler, another one for Crank-Nicolson) showing the vertical temperature profile at t=0.16 and x=0.4, explain the results.
  - (**10pts**) Compare the performance of the two methods.
- (**10pts**) Choose an appropriate scheme and time step and generate 5 plots of the numerical solution for the whole domain from t=0.01, 0.02, 0.04, 0.08, 0.16. Explain the physical phenomenon. You can use the command `conturf` or any other MATLAB/ OCTAVE output commands to generate proper figures.

The Deliverable Task I must be submitted to suharto.saha@fau.de as a .ZIP file containing codes and a combined report before **December 14th, 2020** (before the Tutorial session begins)