Numerical Methods of Thermo-Fluid Dynamics I

Winter Semester 2020-2021

DELIVERABLE TASK 2:
Numerical Solution of Boundary Layer Equation

Given: Monday, 14/12/2020

Deadline: 11/01/2021

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Deliverable Task 2

The goal of the first deliverable task is to solve the boundary layer equation over a flat plate:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2}\,. \tag{2}$$

The Reynolds number is defined as $\mathrm{Re} = u_\infty L/\nu$, where ν is the kinematic viscosity, L is the plate length and u_∞ is the free-stream velocity of the outer flow. Here the velocity components u and v are dimensionless, $u = \bar{u}/u_\infty$ and $v = \bar{v}/u_\infty$, where $\bar{u}$ and $\bar{v}$ are the dimensional velocities. Lengths have been made dimensionless by using the plate length, e.g. $x = \bar{x}/L$. The boundary conditions are as follows:

$$y = 0: \qquad u = v = 0 \qquad \text{no-slip condition} \tag{3}$$

$$y \to \infty: \qquad u \to 1\ (\bar{u} \to u_\infty) \qquad \text{free outer flow.} \tag{4}$$

- (5p) Discretise equation (1). One possible scheme for $\frac{\partial u}{\partial x}$ is as follows:

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{1}{2}\left(\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + \frac{u_{i,j-1} - u_{i-1,j-1}}{\Delta x}\right) \tag{5}$$

  Discretise $\frac{\partial v}{\partial y}$, write the iterative formula for v based on your schemes, show the order of accuracy. Note that you could choose other schemes to approximate $\frac{\partial u}{\partial x}$.

- (10p) Discretise equation (2), write the iterative formula for u, give comments on your scheme (explicit/implicit, order of accuracy, stability constraint, etc.)

- (30p) Write a MATLAB/OCTAVE program that solves the boundary layer equation ($0 \le x \le 1$ and $0 \le y \le 2\delta$, where $\delta = \frac{5}{\sqrt{\mathrm{Re}}}$ is the boundary layer thickness at $x = 1$).

- (25p) Calculate the solution for $\mathrm{Re} = 10000$, generate 2 figures for the numerical results of u and v of the whole domain, explain your results.

- (30p) Consider $u_\infty = 10[m/s]$ and $L = 1[m]$, compare your u and v velocity profiles obtained above at $x = 0.0005$ and $x = 0.5$ with the results you obtained from the Blasius equation in Exercise Sheet 1. Is there good agreement between the two? Why? Explain the results and state your conclusions.

The Deliverable Task 2 must be submitted to *suharto.saha@fau.de* before the tutorial on **11.01.2021**