1.

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & -1 & 2 \\ -1 & 1 & 1 & -1 & -4 \\ 1 & -1 & -1 & -1 & -4 \end{array}\right] \xrightarrow{-L1+L2} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & -2 & 2 \\ -1 & 1 & 1 & -1 & -4 \\ 1 & -1 & -1 & -1 & -4 \end{array}\right] \xrightarrow{L1+L3}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & -2 & 2 \\ 0 & 2 & 0 & 0 & -4 \\ 1 & -1 & -1 & -1 & -4 \end{array}\right] \xrightarrow{-L1+L4} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & -2 & 2 \\ 0 & 2 & 0 & 0 & -4 \\ 0 & -2 & 0 & -2 & -4 \end{array}\right] \xrightarrow{L2+L3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & -2 & 2 \\ 0 & 0 & 2 & -2 & -2 \\ 0 & -2 & 0 & -2 & -4 \end{array}\right]$$

$$\xrightarrow{-L2+L4} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & -2 & 2 \\ 0 & 0 & 2 & -2 & -2 \\ 0 & 0 & -2 & 0 & -6 \end{array}\right] \xrightarrow{L3+L4} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & -2 & 2 \\ 0 & 0 & 2 & -2 & -2 \\ 0 & 0 & 0 & -2 & -8 \end{array}\right]$$

$\rightarrow T$

$-2T = -8$

$T = \frac{-8}{-2}$

$\boxed{T = 4}$

$\rightarrow Z$

$2z - 2T = -2$

$2z - 2.4 = -2$

$2z - 8 = -2$

$2z = -2 + 8$

$2z = 6$

$z = \frac{6}{2}$

$\boxed{z = 3}$

$\rightarrow y$

$-2y + 2z - 2T = 2$

$-2y + 2.3 - 2.4 = 2$

$-2y + 6 - 8 = 2$

$-2y + (-2) = 2$

$-2y - 2 = 2$

$-2y = 2 + 2$

$-2y = 4$

$y = \frac{4}{-2}$

$\boxed{y = -2}$

$\rightarrow x$

$x + y - z + T = 0$

$x + (-2) - 3 + 4 = 0$

$x + (-1) = 0$

$x - 1 = 0$

$x = 0 + 1 \rightarrow \boxed{x = 1}$

2 -
$$\begin{cases} x + y + z + t = 2 \\ x - y - 2z + 3t = 5 \\ 2x + y - 3z + t = -9 \\ 3x - y - z + t = -6 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 1 & -1 & -2 & -3 & 5 \\ 2 & 1 & -3 & 1 & -9 \\ 3 & -1 & -1 & 1 & -6 \end{array}\right] \xrightarrow{-L1+L2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & -3 & -4 & 3 \\ 2 & 1 & -3 & 1 & -9 \\ 3 & -1 & -1 & 1 & -6 \end{array}\right] \xrightarrow{-2L1+L3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & -3 & -4 & 3 \\ 0 & -1 & -5 & -1 & -13 \\ 3 & -1 & -1 & 1 & -6 \end{array}\right]$$

$$\xrightarrow{-3L1+L4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & -3 & -4 & 3 \\ 0 & -1 & -5 & -1 & -13 \\ 0 & -4 & -4 & -2 & -12 \end{array}\right] \xrightarrow{L2 \leftrightarrow L3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -1 & -13 \\ 0 & -2 & -3 & -4 & 3 \\ 0 & -4 & -4 & -2 & -12 \end{array}\right] \xrightarrow{-2L2+L3}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -1 & -13 \\ 0 & 0 & 7 & 2 & 29 \\ 0 & -4 & -4 & -2 & -12 \end{array}\right] \xrightarrow{-4L2+L4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -1 & -13 \\ 0 & 0 & 7 & -2 & 29 \\ 0 & 0 & 16 & 2 & 40 \end{array}\right] \xrightarrow{-L3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -1 & -13 \\ 0 & 0 & -7 & 2 & 29 \\ 0 & 0 & 16 & 2 & 40 \end{array}\right]$$

$$\xrightarrow{\frac{16}{7}L3+L4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -1 & -13 \\ 0 & 0 & -7 & 2 & -29 \\ 0 & 0 & 0 & \frac{46}{7} & \frac{-184}{7} \end{array}\right]$$

$$T \rightarrow \frac{46}{\cancel{7}}T = \frac{-184}{\cancel{7}}$$

$$T = \frac{-184}{7}$$

$$\boxed{T = -4}$$

S/L | T/M | Q/M | Q/J | S/V | S/S | D/D

$z \rightarrow -7z + 2t = -29$

$-7z + 2.(-4) = -29$

$-7z + (-8) = -29$

$-7z - 8 = -29$

$-7z = -29 + 8$

$-7z = -21$

$z = \frac{-21}{-7}$

$z = 3$

$y \rightarrow -y - 5z - t = -13$

$-y - 5.3 - (-4) = -13$

$-y - 15 + 4 = -13$

$-y - 11 = -13$

$-y = -13 + 11 \quad (*-1)$

$\cancel{-}y = \cancel{-}2$

$\boxed{y = 2}$

$x \rightarrow x + y + z + t = 2$

$x + 2 + 3 + (-4) = 2$

$x + 1 = 2$

$x = 2 - 1$

$\boxed{x = 1}$

$x_1 = 1$, $x_2 = -2$, $x_3 = 3$, $x_4 = 4$

2. produto:

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a = 1 . (-1) + 0 . 1 + 1 . 2 = 1$$

$$b = 1 . 0 + 0 . 1 + 1 . 2 = 2$$

$$c = 2 . (-1) + (-1) . 1 + 1 . 2 = -1 \rightarrow \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$d = 2 . 0 + (-1) . 1 + 1 . 2 = 1$$

S/L | T/M | Q/M | Q/J | S/V | S/S | D/D

3 – $A^T = \begin{bmatrix} 3 & 2 & 0 \\ 5 & 1 & -1 \end{bmatrix}$ $\qquad D = A^T + B.C$

$B.C$ $\qquad \begin{bmatrix} 4 \\ 3 \end{bmatrix} . \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} a & B & C \\ D & E & F \end{bmatrix}$

$a = 4.2 = 8$

$B = 4.1 = 4$

$C = 4.3 = 12 \rightarrow \begin{bmatrix} 8 & 4 & 12 \\ 6 & 3 & 9 \end{bmatrix}$

$D = 3.2 = 6$

$E = 3.1 = 3$

$F = 3.3 = 9$

$$D = \begin{bmatrix} 3 & 2 & 0 \\ 5 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 8 & 4 & 12 \\ 6 & 3 & 9 \end{bmatrix} = \begin{bmatrix} 3+8 & 2+4 & 0+12 \\ 5+6 & 1+3 & -1+9 \end{bmatrix}$$

$$D = \begin{bmatrix} 11 & 6 & 12 \\ 11 & 4 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \end{bmatrix}$$

$D_{12} + D_{22} = ?$ $\qquad$ Resposta: 10

$6 + 4 = 10$

4 – $a_{11} = (-1)^{1+1} . 1 . 1 = 1$

$a_{12} = (-1)^{1+2} . 1 . 2 = -2 \rightarrow \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$

$a_{21} = (-1)^{2+1} . 2 . 1 = -2$

$a_{22} = (-1)^{2+2} . 2 . 2 = 4$

Resposta
Letra A

5 - $A = \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} x & y \\ 3 & 2 \end{bmatrix}$

$$A.B = \begin{bmatrix} 0.x+1.3 & 0.y+1.2 \\ 3.x+1.3 & 3.y+1.2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 3x+3 & 3y+2 \end{bmatrix}$$

$$B.A = \begin{bmatrix} x.0+y.3 & x.1+y.1 \\ 3.0+2.3 & 3.1+2.1 \end{bmatrix} = \begin{bmatrix} 3y & x+y \\ 6 & 5 \end{bmatrix}$$

$$A.B = B.A \quad \begin{bmatrix} 3 & 2 \\ 3x+3 & 3y+2 \end{bmatrix} = \begin{bmatrix} 3y & x+y \\ 6 & 5 \end{bmatrix}$$

$3 = 3y$ $\qquad$ $2 = x+y$

$y = \frac{3}{3}$ $\qquad$ $2 = x+1$, por tanto $y = 1$ e $x = 1$

$y = 1$ $\qquad$ $x = 2-1$

$x = 1$ $\qquad$ Resposta: Letra D

6 - $\begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{1} \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$

$$X = A^{-1}.I = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} (6.1)+(-1.0) & (6.0)+(-1.1) \\ (-5.1)+(1.0) & (-5.0)+(1.1) \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$$

$X = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$, portanto Resposta: Letra D

7. ⌈Resposta: Alternativa E⌋

8. ⌈Resposta: Alternativa D⌋

9.

$$M \cdot T = \begin{bmatrix} p & 1 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ q \end{bmatrix} = \begin{bmatrix} p \cdot 2 + 1 \cdot q \\ 3 \cdot 2 + (-1) \cdot q \end{bmatrix} = \begin{bmatrix} 2p + q \\ 6 - q \end{bmatrix}$$

$$\begin{bmatrix} 2p + q \\ 6 - q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{cases} 2p + q = 0 \\ 6 - q = 0 \end{cases} \rightarrow 6 - q = 0 \quad \ulcorner q = 6 \lrcorner \rightarrow 2p + 6 = 0$$

$$2p = -6$$

$$p = -\frac{6}{2}$$

portanto, $p \cdot q = (-3) \cdot (6) = -18$ $\quad \ulcorner p = -3 \lrcorner$

⌈Resposta: -18⌋

10.

$$A^T = \begin{bmatrix} 1 & x & x+1 \\ 2 & 1 & 0 \\ y & 0 & 1 \end{bmatrix}$$

Comparando,

$A_{11} = A^T_{11} \rightarrow 1 = 1$ (VERDADE) $\quad$ temos, então $x = 2$ e $y = x + 1 = 2 + 1 = 3$

$A_{12} = A^T_{21} \rightarrow 2 = x$

$A_{13} = A^T_{31} \rightarrow y = x + 1$

$A_{22} = A^T_{22} \rightarrow 1 = 1$ (Verdade) $\quad x^{-y} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$A_{23} = A^T_{32} \rightarrow 0 = 0$ (Verdade)

$A_{33} = A^T_{33} \rightarrow 1 = 1$ (Verdade) $\quad$ ⌈Resposta: $x^{-y} = \frac{1}{8}$⌋

S/L | T/M | Q/M | Q/J | S/V | S/S | D/D

13.

$$\left[\begin{array}{ccc|cc} 1 & 0 & -1 & 1 & 0 \\ k & 1 & 3 & k & 1 \\ 1 & k & 3 & 1 & k \end{array}\right] = 0$$

$$1 \cdot 1 \cdot 3 + 0 \cdot 3 \cdot 1 + (-1) \cdot k \cdot k - (0 \cdot k \cdot 3 + 1 \cdot 3 \cdot k + (-1) \cdot 1 \cdot 1$$

$$-3k^2 - (3k - 1) = 0$$

$$-k^2 - 3k + 4 = 0$$

$$k = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot (1) \cdot 4}}{2 \cdot (-1)}$$

$$k = \frac{3 \pm \sqrt{9 + 16}}{-2}$$

$$k = \frac{3 + 5}{-2} \quad k = \frac{8}{-2} \quad k_1 = -4$$

$$k = \frac{3 - 5}{-2}$$

$$k = \frac{3 - 5}{-2} \rightarrow \frac{2}{-2} \rightarrow k_2 = 1$$

11. $Det(A^{-1}) = \frac{1}{Det(A)}$

$Det(A) = 2 \cdot det\left(\begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix}\right) - 1 \cdot det\left(\begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix}\right) + 0 \cdot det\left(\begin{bmatrix} 6 & -1 \\ 2 & 0 \end{bmatrix}\right)$

$det\left(\begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix}\right) = (-1) \cdot 1 - 3 \cdot 0 = -1$

$det\left(\begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix}\right) = 6 \cdot 1 - 3 \cdot 2 = 6 - 6 = 0$

$det\left(\begin{bmatrix} 6 & -1 \\ 2 & 0 \end{bmatrix}\right) = 6 \cdot 0 - (-1) \cdot 2 = 2$

$Det(A) = 2 \cdot (-1) - 1 \cdot 0 + 0 \cdot 2 = -2$

por tanto:

$det(A^{-1}) = \frac{1}{det(a)} = \frac{1}{-2} = -\frac{1}{2}$ Resposta: $-\frac{1}{2}$

12.

$Det\ A = 4 + 2 - 4 = 2$

$-\left(-\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 \cdot 1 + \frac{-1}{4} \quad -1 + \frac{-1-4}{4} + \frac{-5}{4}$

Resposta $-\frac{5}{4}$

|S/L|T/M|Q/M|Q/J|S/V|S/S|D/D|

13.
$$\left[\begin{array}{ccc|cc} 1 & 0 & -1 & 1 & 0 \\ K & 1 & 3 & K & 1 \\ 1 & K & 3 & 1 & K \end{array}\right] = 0$$

$$1 \cdot 1 \cdot 3 + 0 \cdot 3 \cdot 1 + (-1) \cdot K \cdot K - (0 \cdot K \cdot 3 + 1 \cdot 3 \cdot K + (-1) \cdot 1 \cdot 1$$

$$-3K^2 - (3K - 1) = 0$$

$$-K^2 - 3K + 4 = 0$$

$$K = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot (1) \cdot 4}}{2 \cdot (-1)}$$

$$K = \frac{3 \pm \sqrt{9 + 16}}{-2}$$

$$K = \frac{3+5}{-2} \qquad K = \frac{8}{-2} \qquad K_1 = -4$$

$$K = \frac{3-5}{-2}$$

$$K = \frac{3-5}{-2} \rightarrow \frac{2}{-2} \rightarrow K_2 = 1$$

14 – $\begin{bmatrix} (x \cdot x + (-4) \cdot y) & (x \cdot 2 + (-4) \cdot 1) \\ (x^2 \cdot x + y \cdot y) & (x^2 \cdot 2 + y \cdot 1) \end{bmatrix}$

$= \begin{bmatrix} x^2 - 4y & 2x - 4 \\ x^3 + y^2 & 2x^2 + y \end{bmatrix} = \begin{bmatrix} 13 & 2x - 4 \\ x^3 + y^2 & 8 \end{bmatrix}$

$x^2 - 4y = 13$

$2x - 4 = 2x - 4$ (sempre verdade)

$x^3 + y^2 = x^3 + y^2$ (sempre verdade)

$2x^2 + y = 8$

$x^2 - 4y = 13$

$y = \dfrac{x^2 - 13}{4}$

$2x^2 + \dfrac{x^2 - 13}{4} = 8$ →

$8x^2 + x^2 - 13 = 32$

$9x^2 - 13 = 32$

$9x^2 = 45$

$x^2 = \dfrac{45}{9}$

$x^2 = 5$

$x = \pm\sqrt{5}$

$y = \dfrac{x^2 - 13}{4}$ Se $x = \sqrt{5}$;

$y = \dfrac{5 - 13}{4} = \dfrac{-8}{4} = -2$

Se $x = -\sqrt{5}$;

$y = \dfrac{5 - 13}{4} = \dfrac{-8}{4} = -2$

Resposta $x = \pm\sqrt{5}$ e $y = -2$

15. Alternativa E