Empirical Industrial Organization

Problem 1.1.

① Context : The company is a monopoly that can perfectly price discriminate. This means it can tailor its price to each type of consumer to extract their entire surplus while maximizing profit.

Consumer's surplus : $U_i = \ln(1 + w_i q) - T$

- q : quantity of service offered
- T : price by the consumer
- wi : consumer's willingness to pay

The profit per consumer : $\pi = T - cq$

- c : cost.

② Participation constraint : For a consumer to accept the offer, their surplus must be at least zero :

$$U_i = \ln(1 + w_i q) - T \geq 0 \Rightarrow T \leq \ln(1 + wq).$$

Since the company can perfectly discriminate, it sets T equal to the maximum the consumer is willing to pay to leave them with zero surplus,

$$\Rightarrow T = \ln(1 + w_i q).$$

③ Profit maximization : The company substitutes this participation constraint into its profit function to find the optimal quantity q.

$$\Rightarrow \pi = T - cq = \ln(1 + w_i q) - cq$$

It maximizes this profit with respect to q :

$$\frac{\partial \pi}{\partial q} = \frac{w_i}{1 + w_i q} - c = 0$$

- la valeur marginal to the firm $(w_i / 1 + w_i q)$, which decreases as q increases due to diminishing marginal satisfaction for the consumer.
- the marginal cost of production (c) is constant

$$\frac{\partial \pi}{\partial q} = \frac{w_i}{1 + w_i q} = c \Rightarrow q^* = \frac{1}{c} - \frac{1}{w_i}$$

Substituting q^* into $T^* = \ln(1 + w_i q^*)$
$$T^* = \ln(w_i/c).$$

④ Conclusion :

ⓐ For q^* : • The higher the consumer's valuation for the service (w_i), the greater the quantity q^* provided, as the firm wants to serve them more to maximize profit.
• If c increases, q^* decreases because providing more becomes costlier for the firm.

ⓑ For T^* : • The price depends on w_i and c. Consumers with higher willingness to pay (w_i) are charged a higher price.
• If the marginal cost c rises, the price also increases.

Problem 1. 2

① Hypotheses :

ⓐ Participation individuelle (IR) :
- $T_1 \leq \ln(1 + w_1 q_1)$
- $T_2 \leq \ln(1 + w_2 q_2)$

ⓑ Incitation individuelle (IC) :

w_1 • $\ln(1 + w_1 q_1) - T_1 \geq \ln(1 + w_1 q_2) - T_2$

w_2 • $\ln(1 + w_2 q_2) - T_2 \geq \ln(1 + w_2 q_1) - T_1$

ⓒ Profit maximization :
$$\max \Pi = T_1 - cq_1 + T_2 - cq_2$$
subject to : IR and IC

Problem 1. 3.

Given the IC constraints, it may be optimal to serve only one type of consumer.

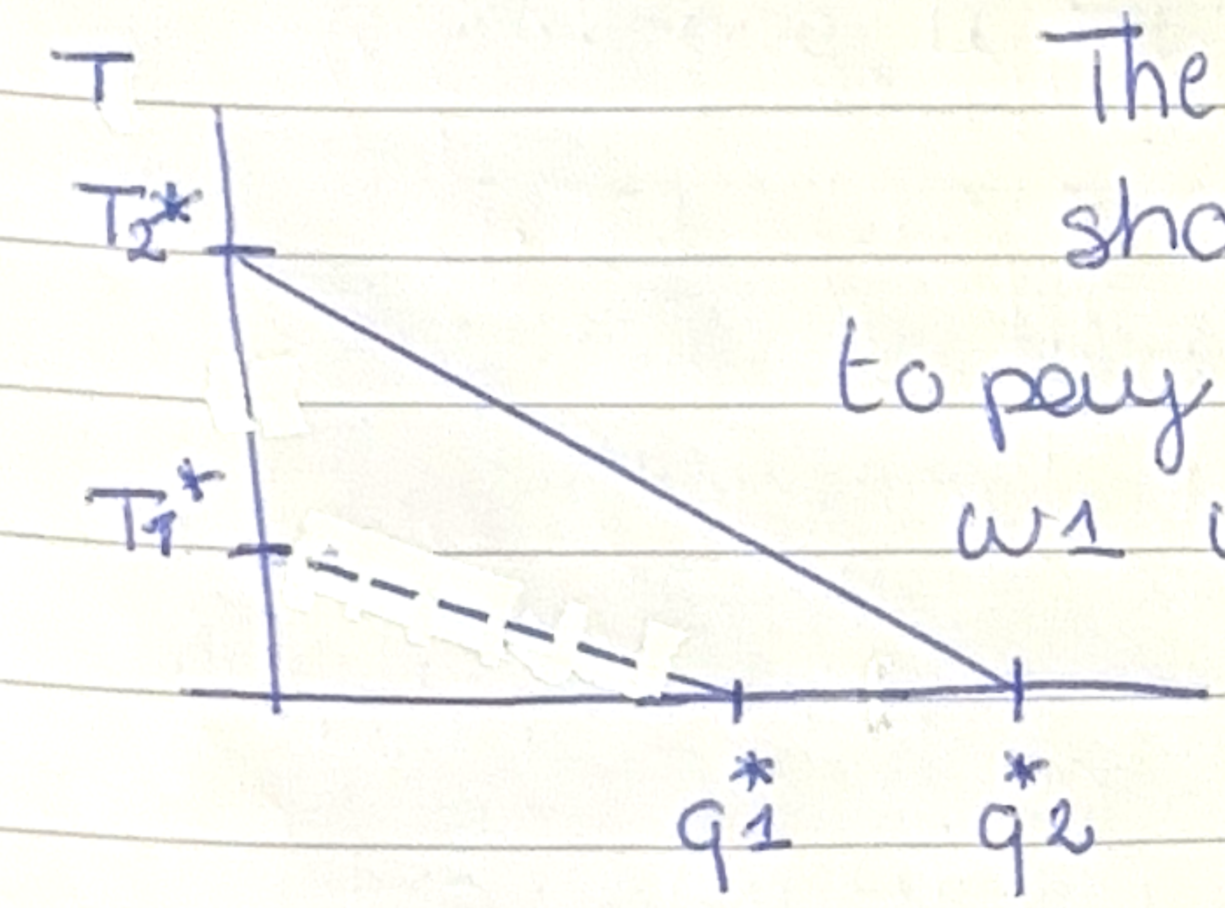
① Only type 2 consumers (w_2)
~ Problem 1.1 $\Rightarrow q_2^* = 1/c - 1/w_2$

② Only type 1 consumers (w_1)
$$\Rightarrow q_2^* = 1/c - 1/w_1$$

The statement indicates that $0 < w_1 < w_2$,
we deduce $q_2^* > q_1^*$
Comparing the profits $\Pi_1^* = \ln(1 + w_1 q_1^*) - c q_1^*$
$\Pi_2^* = \ln(1 + w_2 q^*_2) - c q^*_2$
It indicates that $c \in (0, w_1)$.
So $\Pi_2^* > \Pi_1^*$.

③ Conclusion :
The compagny may only serve type 2 consumers. Ignoring type 1 (wich less profitable) allows us to offer an optimal tariff to type 2, without having to limit its surplus extraction.

Problem 1.4.



The steeper slope of the curve for w_2 shows that this consumer is willing to pay more for a given qtt. The consumers w_1 will not be served, especially if it may lead to inefficiences

Problem 1.5.
① Hypothesys: We know assume a continuum of consumer type $\theta \in [\underline{\theta}, \bar{\theta}]$, where θ represents the consumer's willingness to pay per unit of service.
ⓐ IR: a consumer of type θ must receive non-negative utility $U(\theta) = \ln(1 + \theta q(\theta)) - T(\theta) \geq 0$.
ⓑ IC: a consumer of type θ must prefer their assigned contract $\{q(\theta), T(\theta)\}$ over any other.
$$U(\theta) = \ln(1 + \theta q(\theta)) - T(\theta) \geq \ln(1 + \theta q(\hat{\theta})) - T(\hat{\theta})$$
② IC and Utility: to satisfy the IC constraint, $U(\theta)$ must be increasing in θ, $dU(\theta)/d\theta \geq 0$.

$$\frac{dU(\theta)}{d\theta} = \frac{q(\theta)}{1+\theta q(\theta)} + \ln(1+\theta q(\theta))' - T'(\theta).$$

The IC implies that a higher θ should lead to a higher quantity $q(\theta)$, as $\ln(1+\theta q(\theta))$ increases in $q(\theta)$. Hence, we expect $q'(\theta) > 0$.

③ Profit maximization : $\pi = T(\theta)q(\theta) - cq(\theta)$

The monopolist will increase the quantity $q(\theta)$ for higher value of θ, as these consumers are willing to pay more for a greater quantity of service

④ Conclusion : The monopolist does this because higher-type consumers are willing to pay more and value the service more so offering them more is both efficient and profitable.