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APMA 3080 - Worksheet Section 8.1

Date:

1. Consider the following vectors:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{u}_2 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

Compute (i). $\mathbf{u}_1 \cdot \mathbf{u}_2$ (ii). $2\,\mathbf{u}_1 \cdot \mathbf{u}_2$ (iii). $\|\mathbf{u}_1\|$

i) $1(3) - 2(2)\ 1(-1) =$

$3 - 4 - 1 = -2$

ii $2(u_1 \cdot u_2) =$

$2(-2) = -4$

iii) $\|u_1\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

$= \sqrt{1^2 + (-2)^2 + 1^2}$

$= \sqrt{6}$

2. Find $\|\mathbf{x}\|$ and $\|-5\mathbf{x}\|$ and dist($\mathbf{x}$, -5 $\mathbf{x}$) if $\mathbf{x} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$

$\|x\| = \sqrt{(-3)^2 + (1)^2 + (4)^2}$

$= \sqrt{26}$

$\|-5x\| = |-5|\,|x|$

$= 5\sqrt{26}$

dis $(x, -5x) = \sqrt{(-18)^2 + (6)^2 + 24^2}$

$= \sqrt{936}$

$= 6\sqrt{26}$

3. Find all values of a and b (if any) so that the given vectors form an orthogonal set:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -3 \\ 6 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ a \\ -5 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 0 \\ -4 \\ 3 \\ b \end{bmatrix}.$$

$a = 1$

$b = -30$

$$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 1 & -4 \\ 6 & a & 3 \\ 1 & -5 & b \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 7 & -4 \\ 0 & a-12 & 3 \\ 0 & -5 & b \end{bmatrix} \to [$$

$u_1 \cdot u_2$ $(1)(2) + (-3)(1) + (6)(a) + (1)(-5)$

$u_2 \cdot u_3$ $(2)(0) + (1)(-4) + (a)(3) + (-5)(b)$

$u_1 \cdot u_3$ $(0)(1) + (-3)(-4) + 6(3) + 1(b)$

$1(2)(0) + -3(1)(-4) + 6(3)(a) + 1(-5)(b) = 0$

$12 + 18a - 5b = 0$

$18a - 5b = -12$

$18a = -12$

$a = -2/3$

4. $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ -3 \\ 2 \\ -1 \\ 0 \end{bmatrix}$, use MATLAB command *dot(...)* and *norm(...)* to assistant you to find (i)$3\mathbf{u} \cdot 5\mathbf{w}$ and (ii) dist($2\mathbf{u}$, $3\mathbf{w}$)

i) $5u \cdot 5w = -150$

ii) dist $(2u, 3w) = 20.1494$

5. Find a basis for $S^{\perp}$ for the subspace S, $S = span\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} \}$

$$\left[\begin{array}{cc|c} -1 & 2 & 0 \\ 2 & -3 & 0 \\ 1 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

basis for $S^{\perp}$ $\{ \begin{bmatrix} -7 \\ -4 \\ 1 \end{bmatrix} \}$

$-a + 2b + c$

$2a - 3b + 2c$

$x_3 = x_3$

$x_2 = 1$

$x_1 = 1$

$$\begin{bmatrix} -1 & 2 & 1 \\ 2 & -3 & 2 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

6. Use MATLAB command *Null(...)* to assist you to to find a basis for $S^{\perp}$ for the subspace S, $S = span\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \\ -1 \\ 0 \end{bmatrix} \}$

$$S^{+} = \left\{ \begin{bmatrix} .1446 \\ 0.6267 \\ 0.6740 \\ -0.2428 \\ -0.2705 \end{bmatrix}, \begin{bmatrix} 0.7006 \\ -0.1280 \\ -0.2614 \\ 0.4946 \\ -0.4241 \end{bmatrix}, \begin{bmatrix} 0.5153 \\ 0.2942 \\ -0.2829 \\ -0.4179 \\ 0.6271 \end{bmatrix} \right\}$$