WAVE ENERGY CONVERTER

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Abstract. This paper describes a new physical model of a wave energy converters for optimising energy. A semi-submerged, spherical buoy with an internally suspended pendulum is used to study the constrained motion of wave energy converters. Translation motion of the spherical buoy is constrained at one point, and the link of the pendulum is attached internally at a diametrically opposite end. First, the dynamics are worked out using Lagrangian approach for a two-dimensional motion of the system and then it is modelled for a three-dimensional motion.

1. Introduction

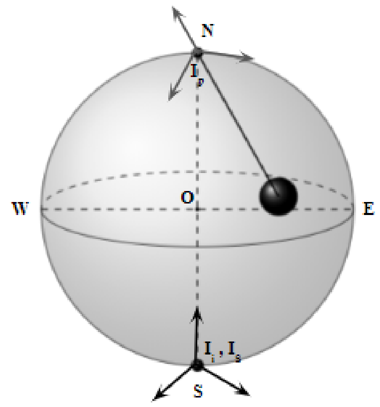
Ocean wave power is considered one of the most promising renewable source of energy. Wave energy converters (WECs) convert mechanical power of waves into electrical energy. Most of the WECs are oscillating systems excited by ocean waves. The power output can be increased by controlling the oscillations to approach a maximum interaction between the waves and the WEC. This paper describes a WEC consisting of a spherical buoy with an internally suspended pendulum. Moving pendulum due to the interaction of the waves with the buoy placed in magnetic field results in induced emf. So, the electrical output is maximum when the kinetic energy of the pendulum is maximised. This occurs when an optimally controlled motion of the pendulum is in resonance with the waves.

The system consists of a rigid spherical buoy partially submerged in a sea wave whose states are known with time. A spherical pendulum is suspended internally at the north pole of the sphere motion of which can be described by two angular coordinates with respect to the spherical body frame. The spherical buoy is constrained by a pivot at its south pole so that it does not translate but has three angular coordinates concerning to an inertial reference frame. Hence, the overall system can be described by five generalised coordinates. The dynamics of the system are modelled by Lagrangian approach, hence system is described in terms of generalised coordinates instead of Cartesian coordinates.

Initially it is assumed that the system is in known sea state and the kinetic energy is maximised for those states. Let $u(x, y, t) = c$ represents upper surface of sea waves. Under ideal conditions (uniform water density, no viscous forces, small wave height), assume that u satisfies the two-dimensional wave equation under certain boundary conditions.

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{1}$$

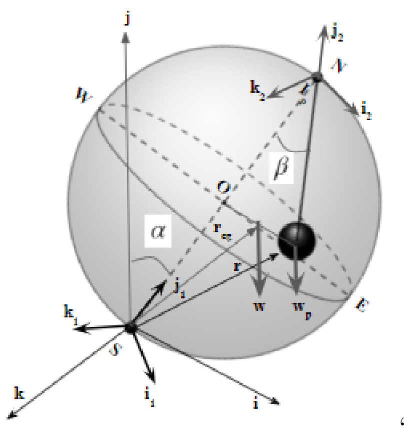
We aim to model the dynamics of such system and to control the motion of internal pendulum such that kinetic energy of the pendulum gets maximised.



In the above figure, I_s represents inertial reference frame, I_b represents body frame of the spherical buoy and I_p represents body frame of the bob. I_s and I_b are fixed at the same point at the south pole of the sphere. First, we will model the dynamics using the Lagrangian approach for the planar motion of the system and then proceed a step ahead to a 5 degree of freedom model.

2. Two DOF Dynamical Model

Assuming the system moves only in x-y plane with respect to frame I_i and z-axes of all the frames are aligned. Let α and β be the angles made by the diameter SN with y-axis of inertial frame and link of the pendulum with diameter SN as shown in the figure below. Assuming that link of the pendulum is massless for this part.



Let $\vec{r}$ be the position vector of the bob with respect to the inertial frame. Let $(\hat{i}, \hat{j}, \hat{k})$, $(\hat{i_1}, \hat{j}_1, \hat{k_1})$ and $(\hat{i_2}, \hat{j}_2, \hat{k_2})$, represent unit vectors of frames I_i, I_s and I_p frames respectively. Then vector $\vec{r}$ is given by,

$$\vec{r} = D\hat{j}_1 - a\hat{j}_2 \tag{2}$$

where D is the diameter of the sphere and, a is the length of the link.Now differentiating above equation with respect to time,

$$\dot{\vec{r}} = D(\vec{\omega}_1 \times \hat{j}_1) - a(\vec{\omega}_2 \times \hat{j}_2) \tag{3}$$

Here $\vec{\omega}_1$ and $\vec{\omega}_2$ are angular velocities of vectors $\hat{j}_1$ and $\hat{j}_2$ with respect to I_s and I_b frames. For rotation about z-axes in both the frames $\vec{\omega}_1$ and $\vec{\omega}_2$ can be written as,

$$\vec{\omega}_1 = -\dot{\alpha}\hat{k}, \;\; \vec{\omega}_2 = \dot{\beta}\hat{k}_1 \tag{4}$$

Since all the z-axes are aligned, hence $\hat{k} = \hat{k}_1 = \hat{k}_2$. Now, the expression for $\dot{\vec{r}}$ becomes,

$$\dot{\vec{r}} = -D\dot{\alpha}(\hat{k}_1 \times \hat{j}_1) - a\dot{\beta}(\hat{k}_2 \times \hat{j}_2) = D\dot{\alpha}\hat{i}_1 + a\dot{\beta}\hat{i}_2 \tag{5}$$

or

$$\dot{r} = \begin{pmatrix} D\dot{\alpha} & a\dot{\beta} \end{pmatrix} \begin{pmatrix} \hat{i}_1 \\ \hat{i}_2 \end{pmatrix}$$

3. Kinetic Energy of the system

Now to model the dynamics using the Lagrangian approach, the kinetic energy of the sphere alone is given by,

$$T_1 = \frac{1}{2} I \dot{\alpha}^2 \tag{6}$$

where I is moment of inertia of the sphere about the z-axis and the kinetic energy of the bob is given by,

$$T_2 = \frac{1}{2} m \langle \dot{r}, \dot{r} \rangle \tag{7}$$

Hence, the total kinetic energy of the system is,

$$T = T_1 + T_2 = \frac{1}{2} I \dot{\alpha}^2 + \frac{1}{2} m \langle \dot{r}, \dot{r} \rangle \tag{8}$$

Here, the value of inner product $\langle \dot{r}, \dot{r} \rangle$ is equal to,

$$\langle \dot{r}, \dot{r} \rangle = \begin{pmatrix} \hat{i}_1 & \hat{i}_2 \end{pmatrix} \begin{pmatrix} D^2\dot{\alpha}^2 & Da\dot{\alpha}\dot{\beta} \\ Da\dot{\alpha}\dot{\beta} & a^2\dot{\beta}^2 \end{pmatrix} \begin{pmatrix} \hat{i}_1 \\ \hat{i}_2 \end{pmatrix} \tag{9}$$

Using rotational transformations to get $\hat{i}_1$ and $\hat{i}_2$ in terms of spatial frame unit vectors,

$$\hat{i}_1 = \cos\alpha \hat{i} - \sin\alpha \hat{j}, \;\; \hat{i}_2 = \cos(\alpha - \beta)\hat{i} - \sin(\alpha - \beta)\hat{j} \tag{10}$$

hence, the inner product becomes,

$$\langle \dot{r}, \dot{r} \rangle = D^2\dot{\alpha}^2 + 2Da\dot{\alpha}\dot{\beta}\cos\beta + a^2\dot{\beta}^2 \tag{11}$$

So, the final kinetic energy equation is,

$$T = \frac{1}{2}\Big((I + mD^2)\dot{\alpha}^2 + m(2Da\dot{\alpha}\dot{\beta}\cos\beta + a^2\dot{\beta}^2) \Big) \tag{12}$$

4. Lagrange's Equation of motion for the system

Lagrangian mechanics is based on using the generalised coordinates of a system instead of using physical coordinates. Lagrangian equation of a motion for a system described by generalised coordinates $(q_1, q_2, ..., q_n)$ is given as,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q_k}} \right) - \frac{\partial T}{\partial q_k} = Q_k, \;\; k = 1, 2, ..., n \tag{13}$$

where T is the kinetic energy of the system and Q_k is the generalised force associated with the generalised coordinate q_k. Q_k is found by using the principal of virtual work. In our case system is described by two generalized coordinates $q_1 = \alpha$ and $q_2 = \beta$ and the kinetic energy of the system is given by eq.(12). For our system different terms of the eq.(13) are mentioned below,

$$\frac{\partial T}{\partial \dot{\alpha}} = (I + mD^2)\dot{\alpha} + mDa\dot{\beta}\cos\beta \tag{14}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) = (I + mD^2)\ddot{\alpha} + mDa\ddot{\beta}\cos\beta - mDa\dot{\beta}^2\sin\beta \tag{15}$$

$$\frac{\partial T}{\partial \dot{\beta}} = mDa\dot{\alpha}\cos\beta + ma^2\dot{\beta} \tag{16}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\beta}}\right) = mDa\ddot{\alpha}\cos\beta - mDa\dot{\alpha}\dot{\beta}\sin\beta + ma^2\ddot{\beta} \tag{17}$$

$$\frac{\partial T}{\partial \alpha} = 0 \tag{18}$$

$$\frac{\partial T}{\partial \beta} = -mDa\dot{\alpha}\dot{\beta}\sin\beta \tag{19}$$

Hence, the dynamical equation is given by,

$$(I + mD^2)\ddot{\alpha} + mDa\ddot{\beta}\cos\beta - mDa\dot{\beta}^2\sin\beta = Q_1 \tag{20}$$

$$mDa\ddot{\alpha}\cos\beta + ma^2\ddot{\beta} = Q_2 \tag{21}$$

Q_1 and Q_2 are the generalised forces associated with corresponding generalised coordinates. Q_1 causes change in α independent of β and similarly Q_2 causes change in β independent of α. When $\delta\beta = 0$, the moments causing a change in α are external torque due to sea waves and moment of weight of the system which acts at the centre of gravity of the whole system. Center of gravity of the system is at $\vec{r}_{cg}$ given by,

$$\vec{r}_{cg} = \frac{1}{w}\left((\frac{w_s}{2} + w_p)D\hat{j}_1 - w_p a\hat{j}_2 \right) \tag{22}$$

hence the moment Q_1 is given by,

$$\begin{aligned} Q_1 &= \vec{r}_{cg} \times w\hat{j} + \tau_{ext} \\ &= \big(\frac{w_s}{2} + w_p\big)D\big(\hat{j}_1 \times \hat{j}\big) + w_p a\big(\hat{j}_2 \times \hat{j}\big)\Big) + \tau_{ext} \\ &= \big(\frac{w_s}{2} + w_p\big)D\sin\alpha + w_p a\sin(\alpha - \beta) + \tau_{ext} \end{aligned} \tag{23}$$

Where w is the total weight of the system, w_p and w_s are the weights of pendulum and sphere respectively. Now when $\delta\alpha$=0, the moment causing a change in β is due to the weight of the pendulum. Hence, Q_2 which acts on the pendulum is given by,

$$Q_2 = w_p a\sin(\alpha - \beta) + \tau_{control} \tag{24}$$

Now, the dynamics of the system can be described by equations (20), (21), (22) and (23) together.