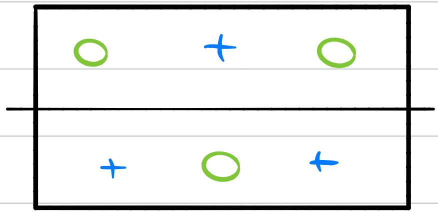
1. Initially, $w_i = \frac{1}{150}$ for all nodes.

After this first split,



$$err_1 = \frac{\sum_{i=1}^{150} w_i \, I(y_i \neq \phi_i(x_i))}{\sum_{i=1}^{150} w_i}$$

$$= \frac{50}{150} = \frac{1}{3}$$

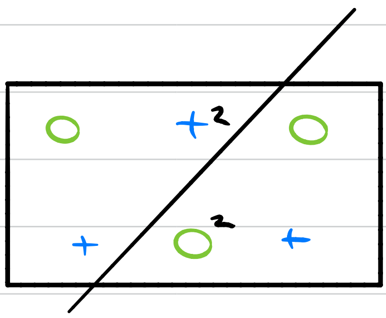
$$\alpha_1 = \log \frac{1 - err_1}{err_1} = \log 2$$

Weights are updated to

$$w_i = w_i \, e^{\alpha_1 I(\hat{y}_i \neq \phi_m(x_i))}$$

$$w_i = \begin{cases} w_i & \text{if } \hat{y}_i = \phi_m(x_i) \\ w_i 2 & \text{if } \tilde{y} \neq \phi_m(x_i) \end{cases}$$

Second split gives



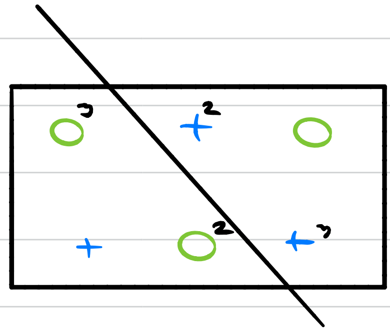
$$err_2 = \frac{1/3}{4/3} = \frac{1}{4}$$

$$\alpha_2 = \log 3$$

Now there are three groups of weights, normalized

$$g_1 = w_i = \frac{1}{300}$$
$$g_2 = 2w_i = \frac{2}{300}$$
$$g_3 = 3w_i = \frac{3}{300}$$

$$err_3 = \frac{50}{300} = \frac{1}{6}$$



$$\alpha_3 = \log 5$$

Let $\circ = 1$

$\times = -1$

$$f^{BL}(x) = \text{sign}(-\log 2 - \log 3 + \log 5) = -1$$
$$f^{TL}(x) = \text{sign}(\log 2 - \log 3 + \log 5) = 1$$
$$f^{TM}(x) = \text{sign}(\log 2 - \log 3 - \log 5) = -1$$
$$f^{BM}(x) = \text{sign}(-\log 2 + \log 3 + \log 5) = 1$$
$$f^{TR}(x) = \text{sign}(\log 2 + \log 3 - \log 5) = 1$$
$$f^{BR}(x) = \text{sign}(+\log 2 + \log 3 - \log 5) = -1$$

B = Bot
T = Top
L = Left
R = Right
M = Mid

It seems that this dataset can be split with three weak learners, so with these splits adding more weak learners would not help.

3. Computing derivatives w.r.t x_2

variable	computation	derivatives
w_1	e^{x_2}	$\frac{\partial w_1}{\partial x_2} = e^{x_2}$
w_2	$x_1^2 + w_1$	$\frac{\partial w_2}{\partial w_1} = 1$
w_3	$\sin(w_2)$	$\frac{\partial w_3}{\partial w_2} = \cos(w_2)$
w_4	$x_1^2 w_3$	$\frac{\partial w_4}{\partial w_3} = x_1^2$

$$\frac{\partial f}{\partial w_3} = \frac{\partial f}{\partial w_4}\frac{\partial w_4}{\partial w_3} = 1 \cdot x_1^2$$
$$\frac{\partial f}{\partial w_2} = \frac{\partial f}{\partial w_3}\frac{\partial w_3}{\partial w_2} = x_1^2 \cos(w_2)$$
$$\frac{\partial f}{\partial w_1} = \frac{\partial f}{\partial w_2}\frac{\partial w_2}{\partial w_1} = 1 \cdot x_1^2 \cos(w_2)$$

$$\frac{\partial f}{\partial x_2} = \frac{\partial f}{\partial w_1}\frac{\partial w_1}{\partial x_2} = x_1^2 \cos(w_2)\, e^{x_2} = x_1^2 \cos(x_1^2 + e^{x_2})\, e^{x_2}$$

$$= 1 \cos(1 + e^2) e^2 \approx -3.7677$$

Numerical approx ≈ -3.77

Derivatives w.r.t x_2

variables	computation	derivatives
w_1	x_1^2	$\frac{\partial w_1}{\partial x_1} = 2x_1$
w_2	$x_1^2 + e^{x_2}$	$\frac{\partial w_2}{\partial x_1} = 2x_1$
w_3	$\sin(w_2)$	$\frac{\partial w_3}{\partial w_2} = \cos(w_2)$
w_4	$w_1 w_3$	$\frac{\partial w_4}{\partial w_1} = w_3$, $\frac{\partial w_4}{\partial w_3} = w_1$

$$\frac{\partial f}{\partial w_3} = \frac{\partial f}{\partial w_4}\frac{\partial w_4}{\partial w_3} = w_1$$

$$\frac{\partial f}{\partial w_2} = \frac{\partial f}{\partial w_3}\frac{\partial w_3}{\partial w_2} = w_1 \cos(w_2)$$

$$\frac{\partial f}{\partial w_1} = \frac{\partial f}{\partial w_4}\frac{\partial w_4}{\partial w_1} = w_3$$

$$\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial w_1}\frac{\partial w_1}{\partial x_1} + \frac{\partial f}{\partial w_2}\frac{\partial w_2}{\partial x_1} = w_3 2x_1 + 2x_1 w_1 \cos(w_2)$$

$$= 2x_1 (w_3 + w_1 \cos(w_2))$$

$$= 2x_1 (\sin(e^{x_2} + x_1^2) + x_1^2 \cos(e^{x_2} + x_1^2))$$

$$= 2(\sin(e^2 + 1) + 1 \cdot \cos(e^2 + 1))$$

$$\approx 0.7$$

Numerical estimate ≈ 0.7