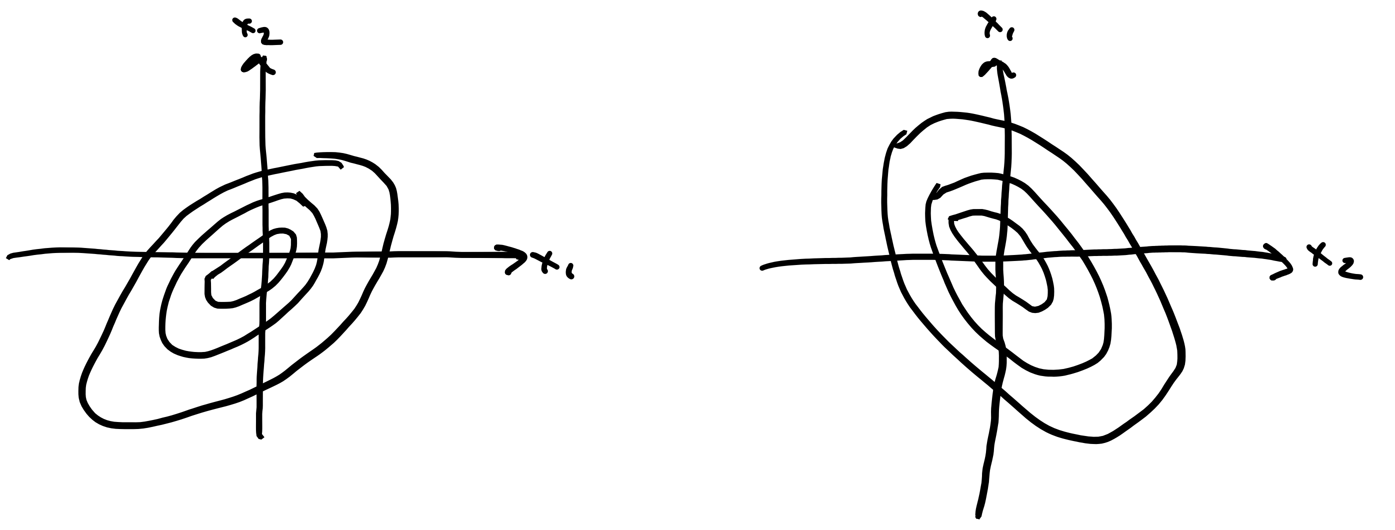
Numba 1

1. a) Since Σ is diagonal, there is no correlation between x_1 and x_2 i.e. they are independent so we can get the results with the marginal probabilities:

$$P(x_1 < \mu_1, \, x_2 < \mu_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(x_1 < \mu_1 \mid x_2 < \mu_2) = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{2}$$

b) If x_1 and x_2 are correlated it causes their pdf to become an ellipsoid that leans to the right if they are positively correlated or to the left if they are negatively correlated



The first probability would go up as correlation increases and down as the correlation decreases.

Second probability would go up if they are positively correlated and go down if they are negatively correlated

c)
$$\begin{aligned} \mathrm{var}(X+Y) &= E((X+Y)^2) - E(X+Y)^2 \\ &= E(X^2+Y^2+2XY) - E(X)^2 - E(Y)^2 - 2E(X)E(Y) \\ &= \underbrace{E(X^2)-E(X)^2}_{\mathrm{var}(X)} + \underbrace{E(Y^2)-E(Y)^2}_{\mathrm{var}(Y)} + \underbrace{2E(XY)-2E(X)E(Y)}_{2\mathrm{cov}(X,Y)} \end{aligned}$$

2. a)
$$\begin{aligned} p(\lambda \mid X=x) &\propto p(X=x \mid \lambda)\, p(\lambda) \\ &\propto e^{-\lambda} \frac{\lambda^x}{x!} \, p(\lambda) \\ &\propto e^{-\lambda} \lambda^x \lambda^{\alpha-1} e^{-\beta\lambda} \\ &= e^{\lambda^2\beta} \lambda^{\alpha+x-1} \end{aligned}$$

b) Let $y = \{x_1 \ldots x_N\}$

$\bar{y}$ = mean

$$\begin{aligned} p(\lambda \mid y) &\propto p(y \mid \lambda)\, p(\lambda) \\ &\propto \lambda^{N\bar{y}} e^{-N\lambda} \lambda^{\alpha-1} e^{-\beta\lambda} \\ &= \lambda^{\alpha+N\bar{y}-1} e^{-(\beta+N)\lambda} \end{aligned}$$

So the posterior distribution is a gamma distribution

$$\lambda \mid Y \sim \mathrm{Gamma}(\alpha + N\bar{y}, \, \beta + N)$$