1. a) $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & dx \\ \sin\theta & \cos\theta & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ euclidean

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta s^x & -\sin\theta s^x & dx \\ \sin\theta s^y & \cos\theta s^y & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ similarity

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ affine

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{11} \\ h_{10} & h_{11} & h_{12} \\ h_{21} & h_{22} & h_{23} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ projective

b) translation : 2

euclidean : 3

similarity : 4

affine : 6

projective: 8

for 2d

c) Because h_{23} serves as the scaling constant for x' and y'

d) $[3, 1, 1] \times [-1, 0, 1] = [1, -4, 1]$

$[1, 0, 1] \times [3, 0, 1] = [0, 2, 0]$

4. $A^TA = \begin{bmatrix} 74, & 5, & 7, & 0 \\ 5, & 1 & ,0, & 0 \\ 7, & 0, & 145, & 12 \\ 0, & 0, & 12, & 1 \end{bmatrix}$

$$\det(A^TA - \lambda I) = \det\begin{pmatrix} 74-\lambda & 5 & 7 & 0 \\ 5 & 1-\lambda & 0 & 0 \\ 7 & 0 & 145-\lambda & 12 \\ 0 & 0 & 12 & 1-\lambda \end{pmatrix}$$

$$= \lambda^4 - 221x^3 + 10951x^2 - 7131x$$

$\lambda_1 = 0$

$\lambda_2 = \sqrt{0.65994} = 0.812$

$\lambda_3 = \sqrt{73.676} = 8.\cancel{58}34$

$\lambda_4 = \sqrt{146.66} = 12.11$

$AA^T = \begin{bmatrix} 26 & 35 & 0 \\ 35 & 50 & 12 \\ 0 & 12 & 145 \end{bmatrix} = \omega$

$\det(\omega) = -x^3 + 221^2 - 10951x + 7131 = 0$

$x_1 = 0.65994$

$x_2 = 73.676$

$x_3 = 146.66$

$$\begin{bmatrix} 26-\lambda & 35 & 0 \\ 35 & 50-\lambda & 12 \\ 0 & 12 & 145 \end{bmatrix} x = 0$$

$SVs = \sqrt{x_1}, \sqrt{x_2}, \sqrt{x_3}$

$(26 - \sqrt{0.66})x_1 + 35x_2 = 0 \quad \Rightarrow x_2 = -0.719647x_1$

$35x_1 + (50 - \sqrt{.66})(-0.72x_2) + 12x_3 = 0 \quad \Rightarrow x_3 = 0.033144x_1$

... Too complex to calculate by hand

$$\Sigma = \begin{bmatrix} 12.11 & 0 & 0 \\ 0 & 8.58 & 0 \\ 0 & 0 & 0.812 \end{bmatrix}$$

np.linalg.svd(A) =

$$\begin{bmatrix} -0.04 & 0.58 & 0.8 \\ -0.13 & 0.79 & -0.58 \\ -0.98 & -0.13 & 0.04 \end{bmatrix} \begin{bmatrix} 12.11 \\ 8.58 \\ 0.81 \end{bmatrix} \begin{bmatrix} -0.09 & -0.003 & -0.99 & -0.08 \\ 0.99 & 0.068 & -0.09 & -0.015 \\ -0.06 & 0.99 & -0.001 & 0.05 \\ 0.01 & -0.05 & -0.08 & 0.99 \end{bmatrix}$$