3. Ahora, lea el texto rápidamente, para comprobar si alguna de las ideas que escribió en el ejercicio 1 aparece en el mismo. En caso afirmativo, tilde ✓ la idea.

VELOCITY OF THE BALL

At time t, which direction is the ball going? Calculus watches the motion between *t* and *t* + *h*. For a ball on a string, we don't need calculus-just let go. *The direction of motion is tangent to the circle*. With no force to keep it on the circle, *the ball goes off on a tangent*. If the ball is the moon, the force is gravity. If it is a hammer swinging around on a chain, the force is from the center. When the thrower lets go, the hammer takes off -and it is an art to pick the right moment. (I once saw a friend hit by a hammer at **MIT**. He survived, but the thrower quit track.) Calculus will find that same tangent direction, when the points at t and $t + h$ come close.

The "***velocity triangle***" is in Figure 1.16b. It is the same as the position triangle, but rotated through 90°. The hypotenuse is tangent to the circle, in the direction the ball is moving. Its length equals 1 (the speed). The angle t still appears, but now it is the angle with the vertical. ***The upward component of velocity is*** $\cos t$, ***when the upward component of position is*** $\sin t$. That is our common sense calculation, based on a figure rather than a formula. The rest of this section depends on it -and we check $v = \cos t$ at special points.

At the starting time $t = 0$, the movement is all upward. The height is $\sin 0 = 0$ and the upward velocity is $\cos 0 = 1$. At time ¶/12, the ball reaches the top. The height is sin ¶/ 2 = 1 and the upward velocity is cos ¶/2 = 0. At that instant the ball is not moving up or down.

The horizontal velocity contains a minus sign. At first the ball travels to the *left*.

The value of x is $\cos t$, but *the speed in the x direction is* $-\sin t$. Half of trigonometry is in that figure (the good half), and you see how $\sin^2 t + \cos^2 t = 1$ is so basic. That equation applies to position and velocity, at every time.

Application of plane geometry: The right triangles in Figure 1.16 are the same size and shape. They look congruent and they are -the angle t above the ball equals the angle t at the center. That is because the three angles at the ball add to 180°.

Extraído de Strang, Gilbert. (2010). *Calculus. MIT Wellesley: Cambridge University Press.*

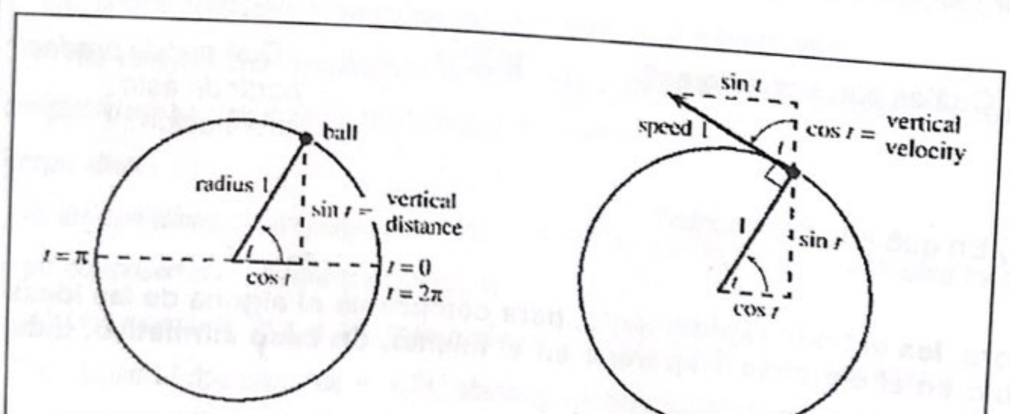

Fig. 1.16 Circular motion with speed 1, angle t, height $\sin t$, upward velocity $\cos t$.