



AZUG

Azure User Group Belgium

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Microsoft Q# and Azure Quantum

involved

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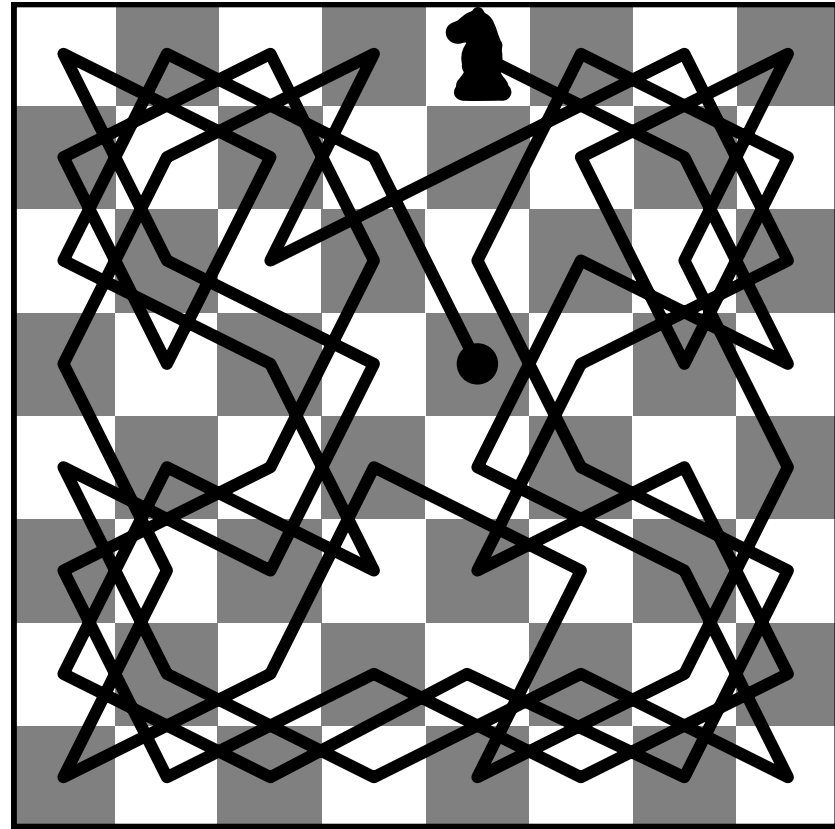
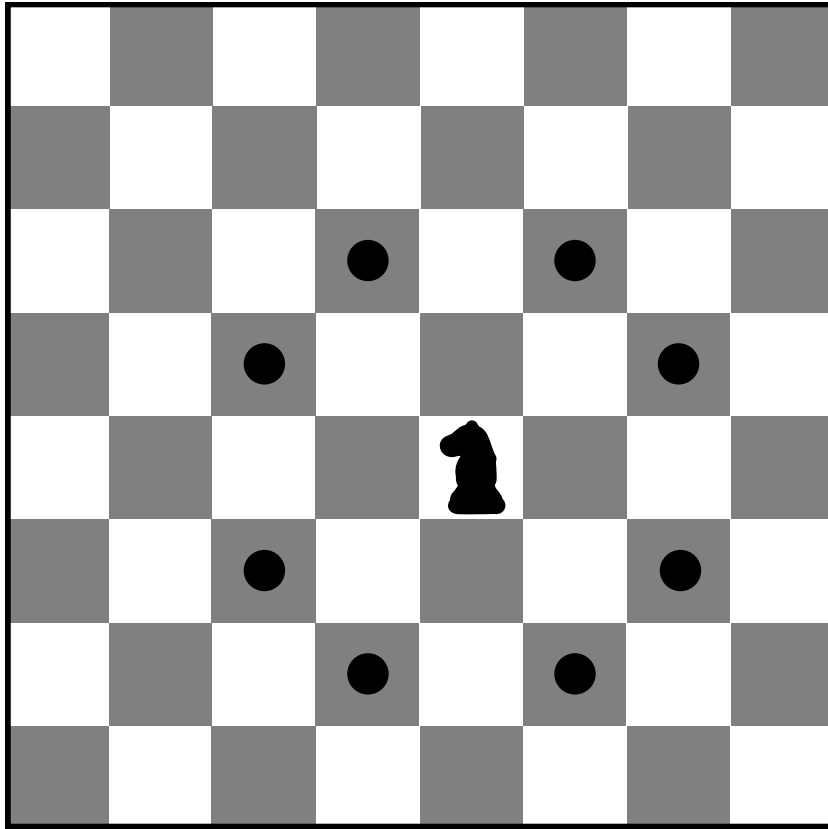
johnny.hooyberghs@involved-it.be

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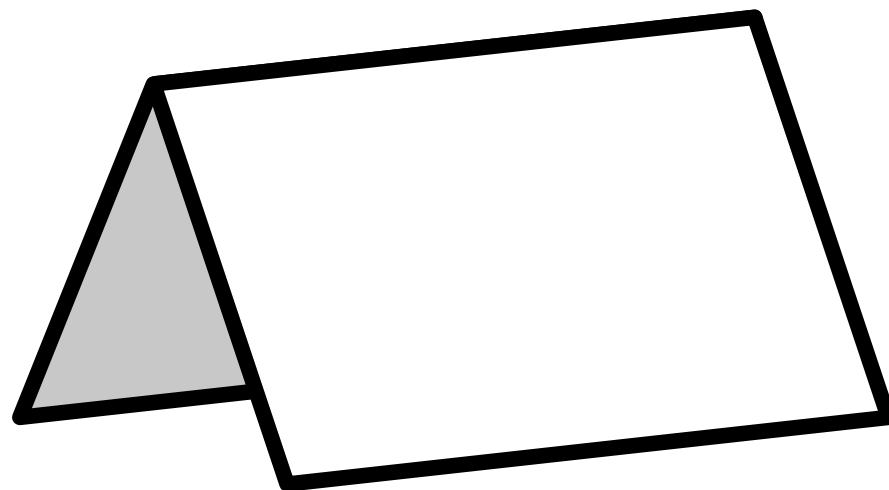
Why Quantum Computing?

- There are still a lot of problems that cannot be solved by computers
- CPU's have their physical limits
- Current classical computing architectures already have issues with unwanted quantum side effects because of their scale
- Why try to simulate a complex quantum world using classical computers?

Why Quantum Computing?

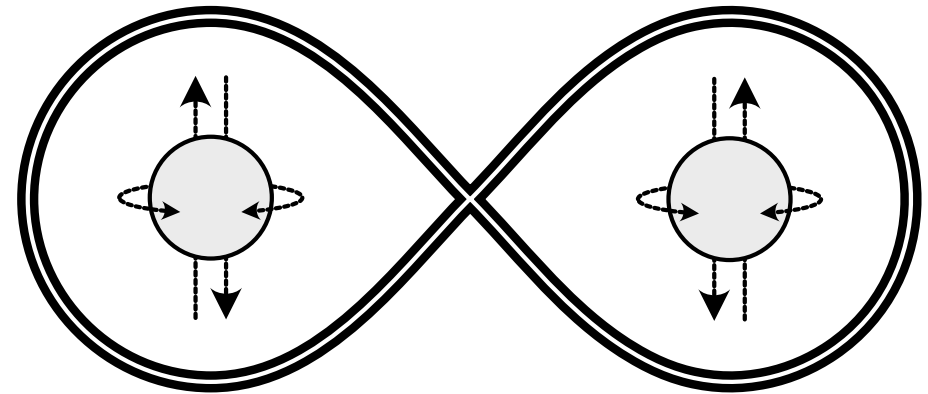
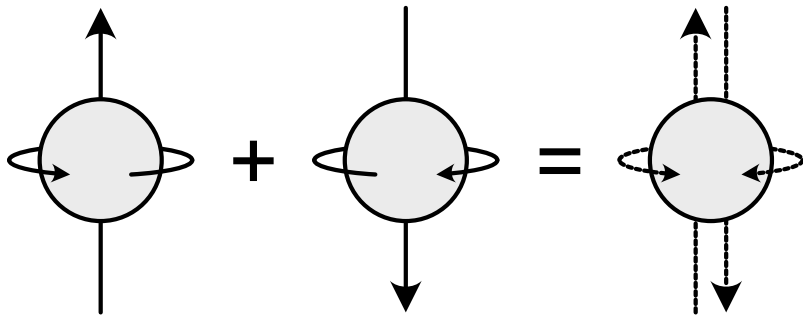


Why Quantum Computing?

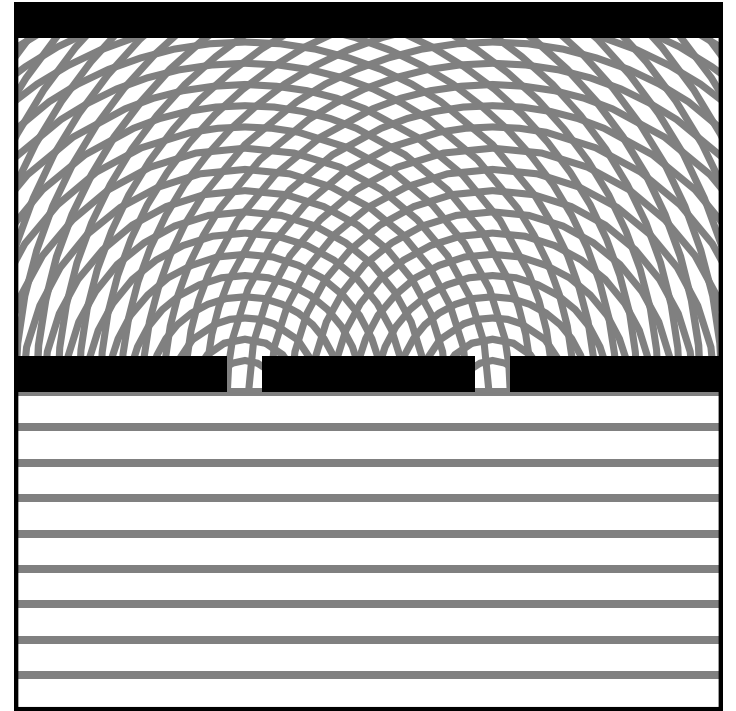
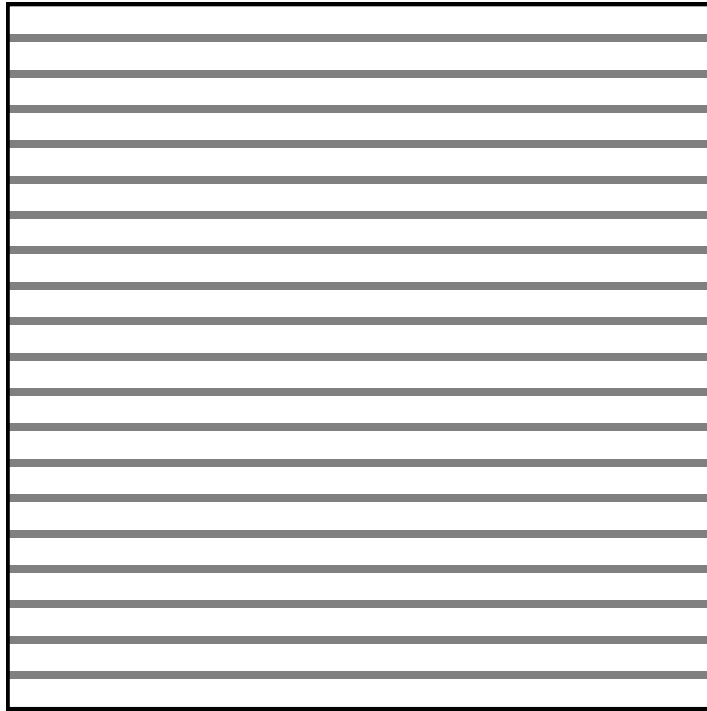
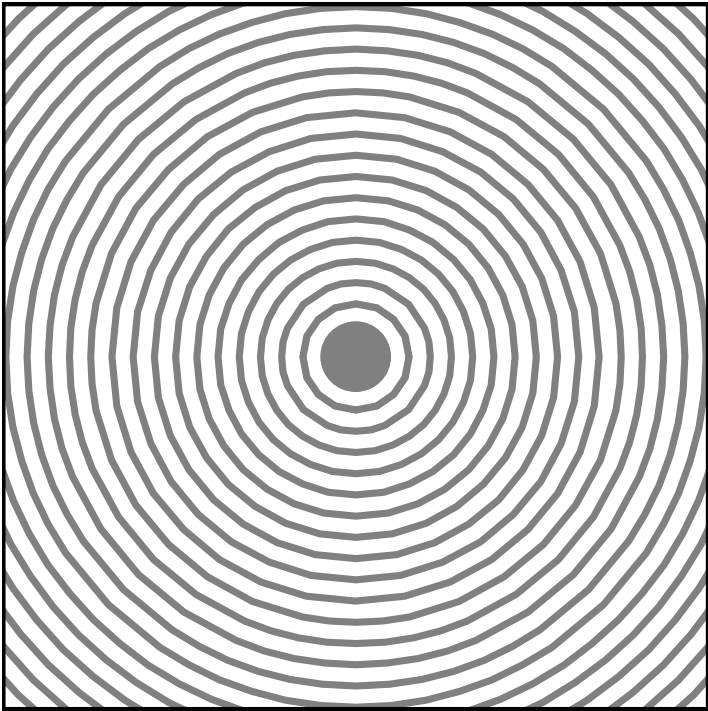


Superposition and Entanglement

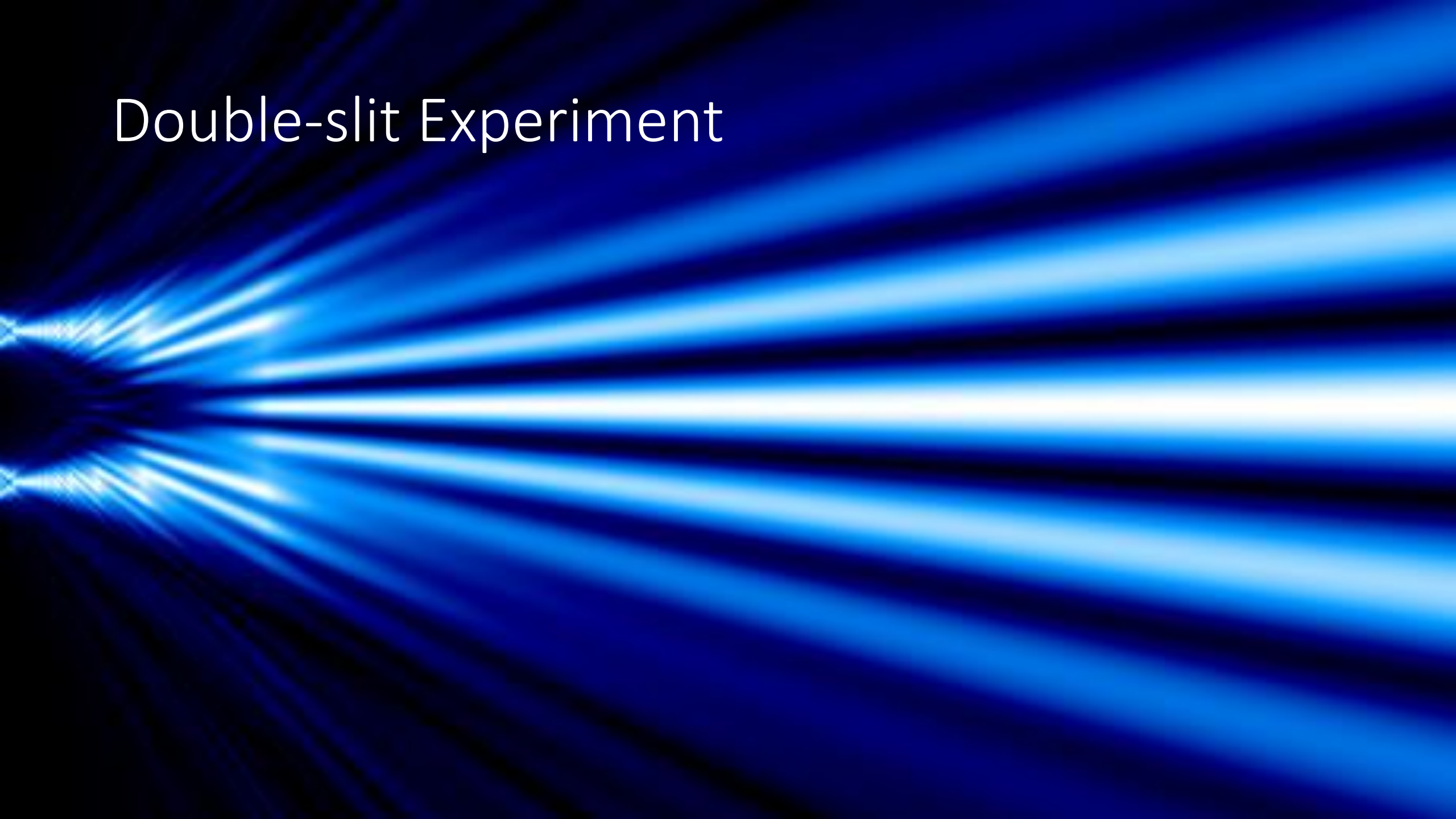
- Quantum mechanics describes superposition and entanglement of quantum particles
- Quantum Computing can use these phenomenon to its advantage



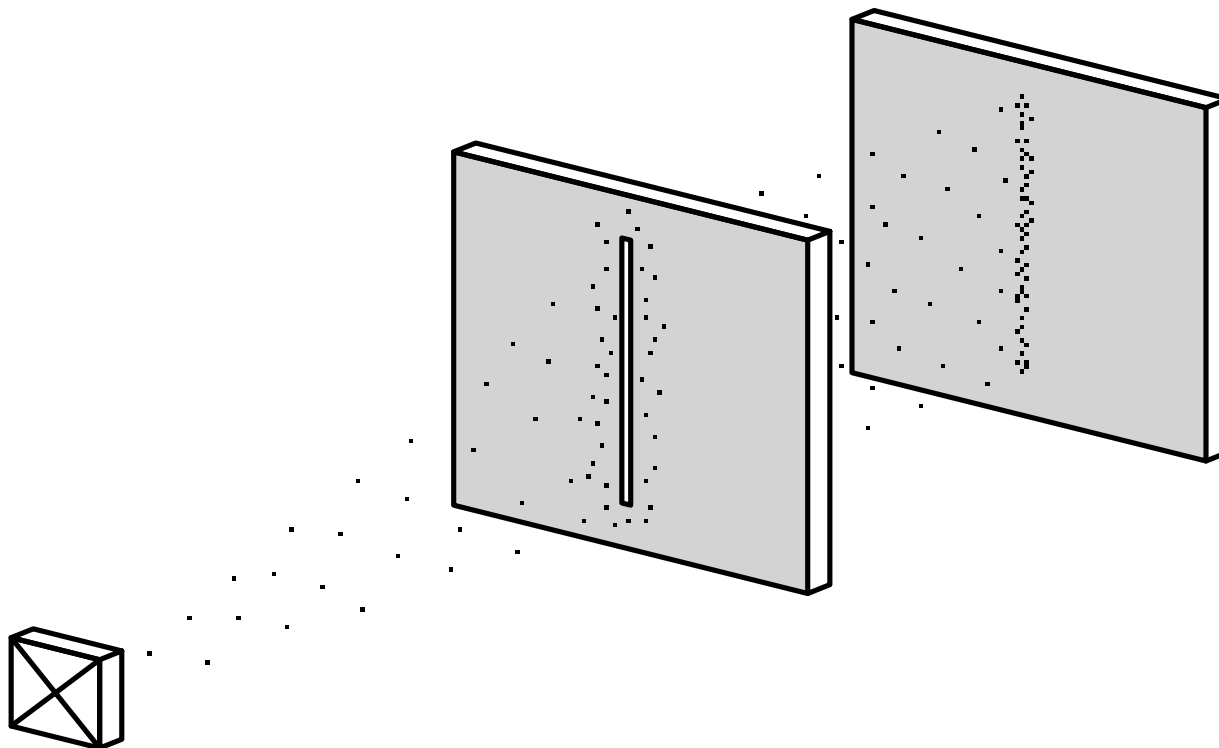
Double-slit Experiment



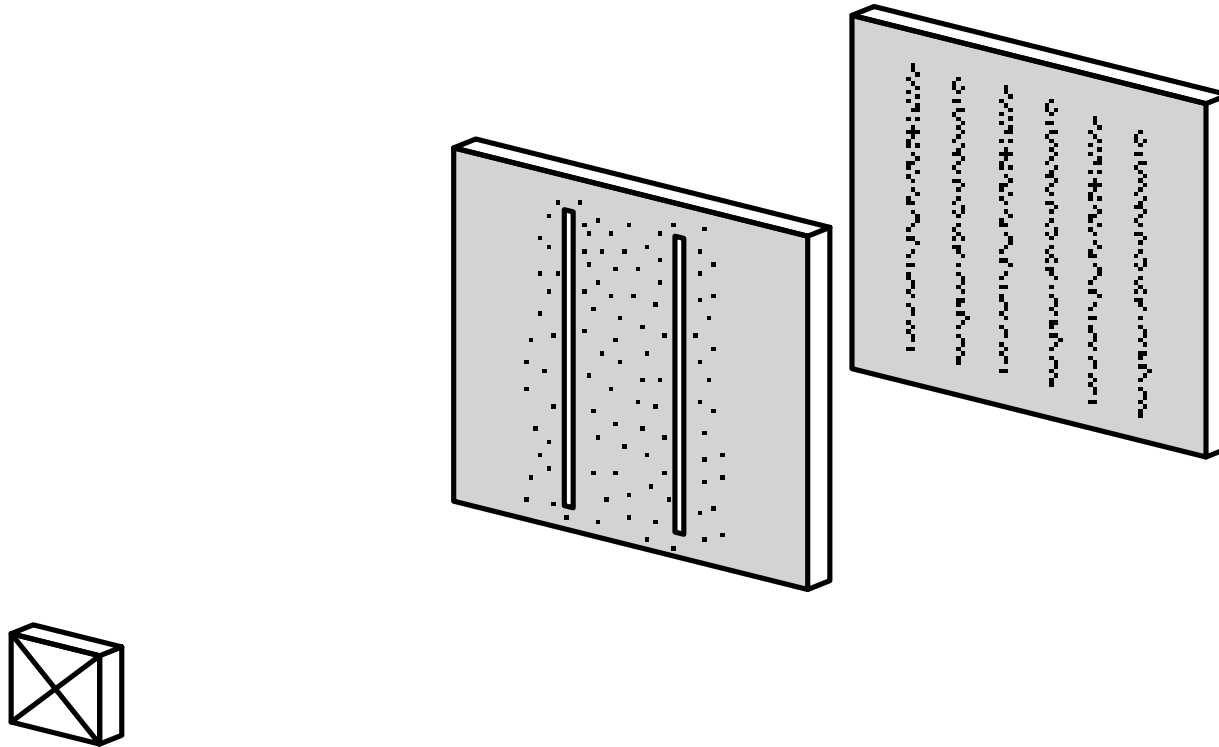
Double-slit Experiment



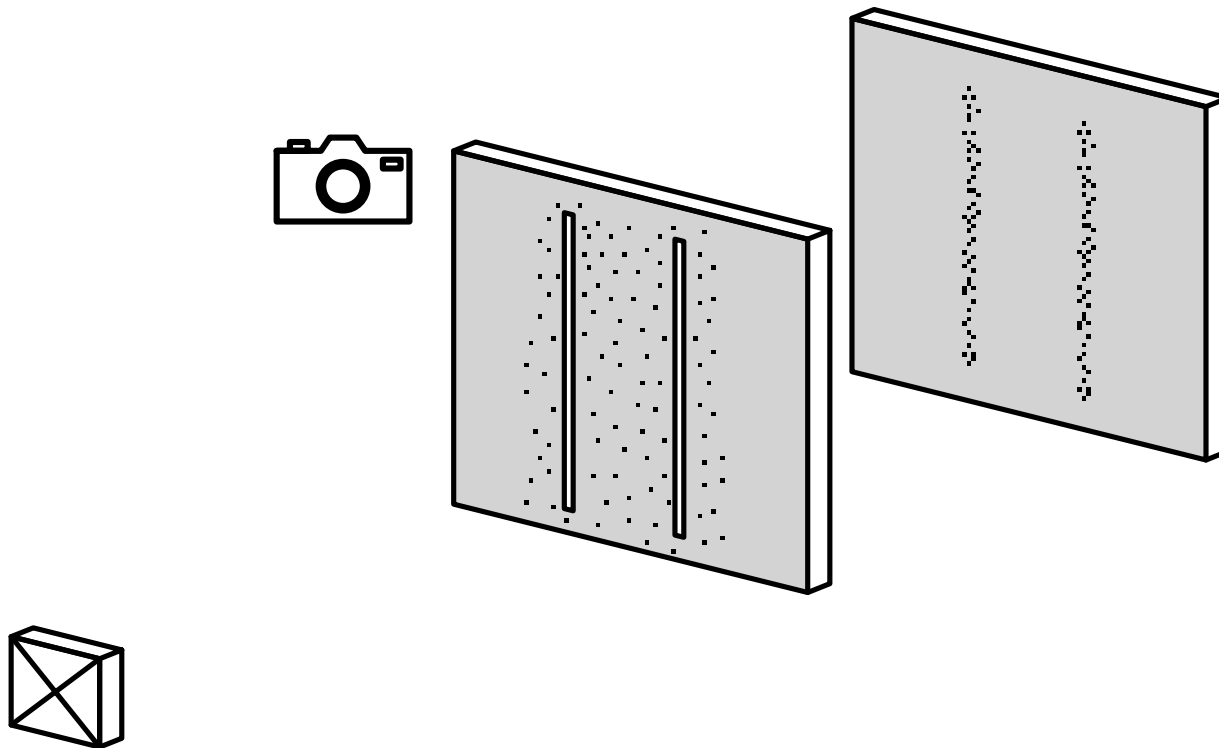
Double-slit Experiment



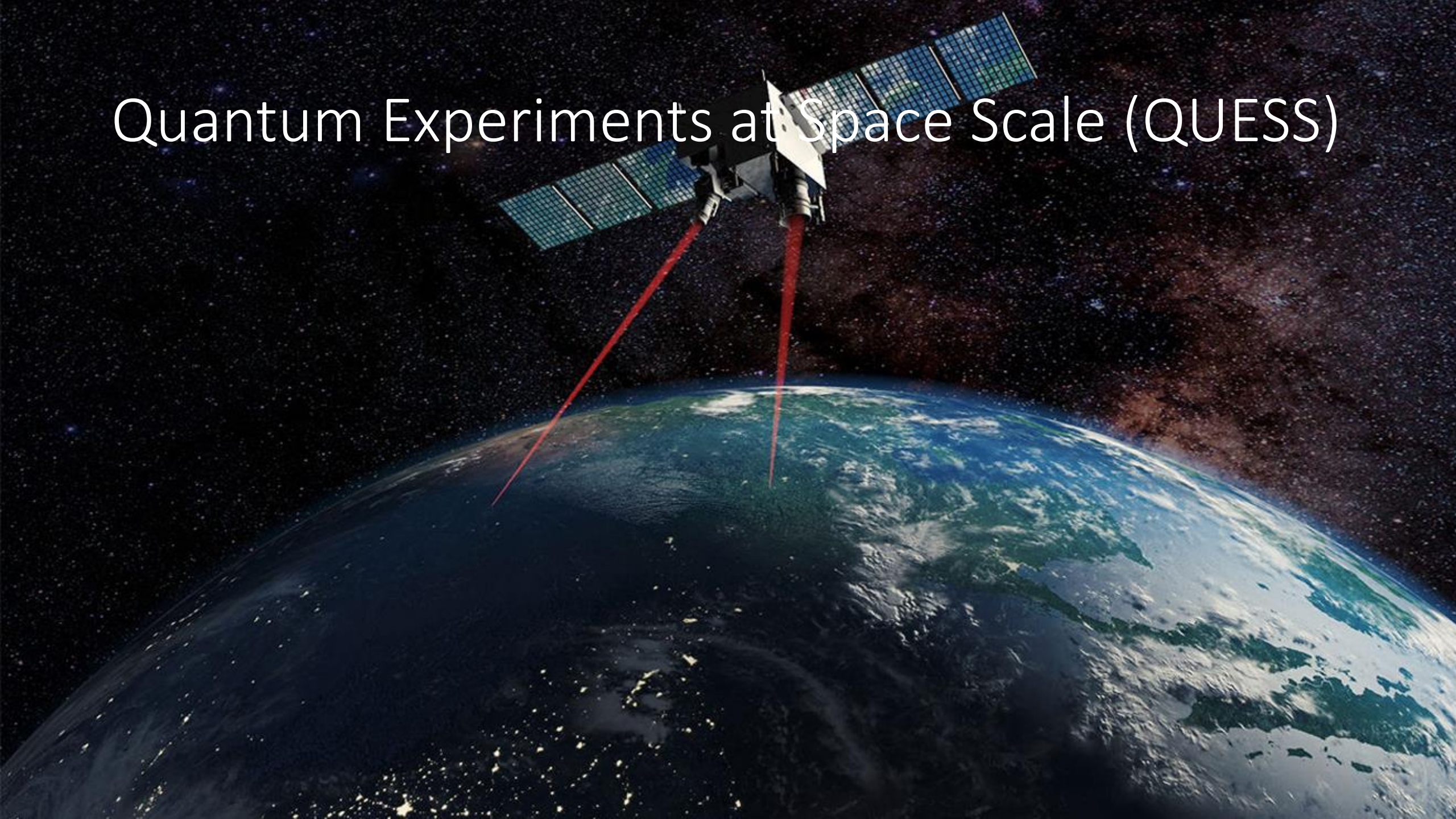
Double-slit Experiment



Double-slit Experiment



Quantum Experiments at Space Scale (QUESS)



Why Quantum Computing?

- Security
 - Public/private key encryption?
 - Could make current RSA encryption obsolete
 - QKD (Quantum Key Distribution)

$$3.167 \times 6.301 = 19.955.267$$

Why Quantum Computing?

- Drug development
 - It takes a quantum system to simulate a quantum system
 - Interactions between molecules
 - Gene sequencing
 - Protein folding

Why Quantum Computing?

- Machine Learning
 - Analyze large quantities of data
 - Fast feedback
 - Emulate human mind

?



CAN IT RUN CRYISIS?

Bits vs. Qubits

0

1

Bits vs. Qubits

1000110

Bits vs. Qubits

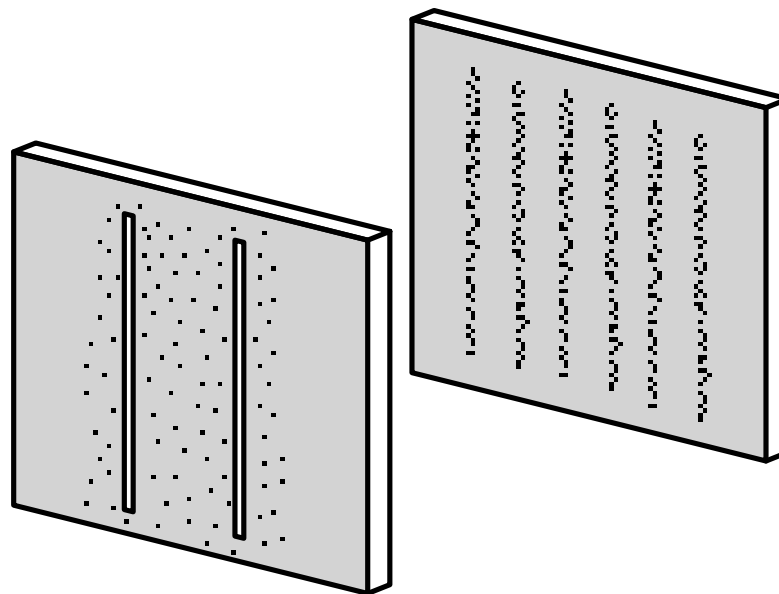
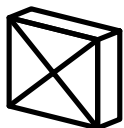
$|0\rangle$

$|1\rangle$

Bits vs. Qubits

$|100110\rangle$

Quantum state



Quantum state

$$\alpha|0\rangle + \beta|1\rangle$$

Quantum state

$$\alpha |0\rangle + \beta |1\rangle$$
$$|\alpha|^2 + |\beta|^2 = 1$$

Quantum state

$$\alpha |0\rangle + \beta |1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\alpha = a + bi$$

$$\beta = c + di$$

Quantum state

$$\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

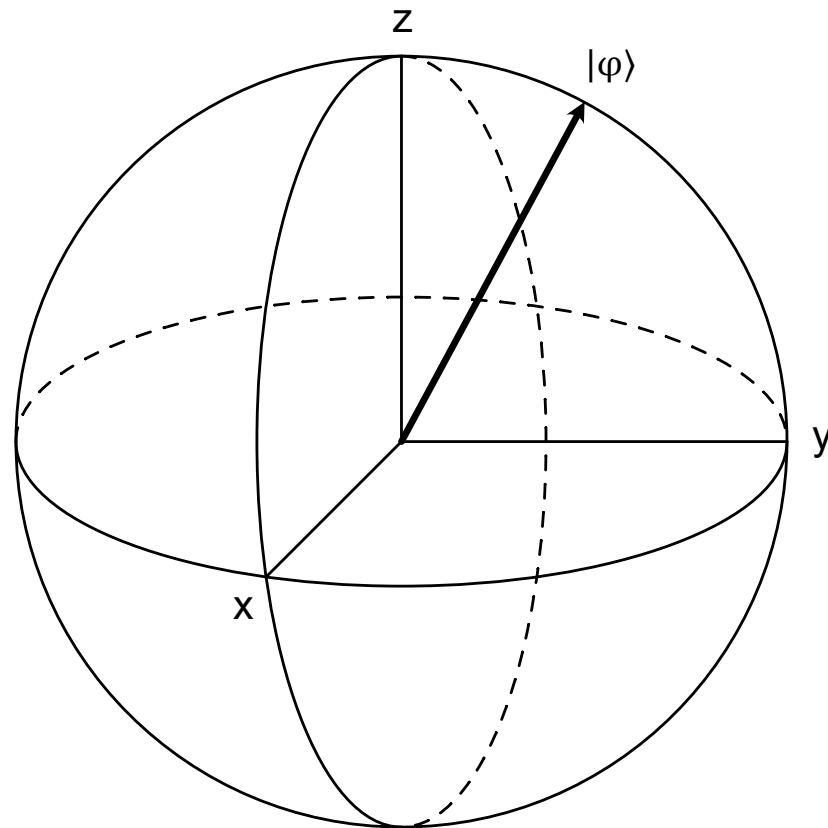
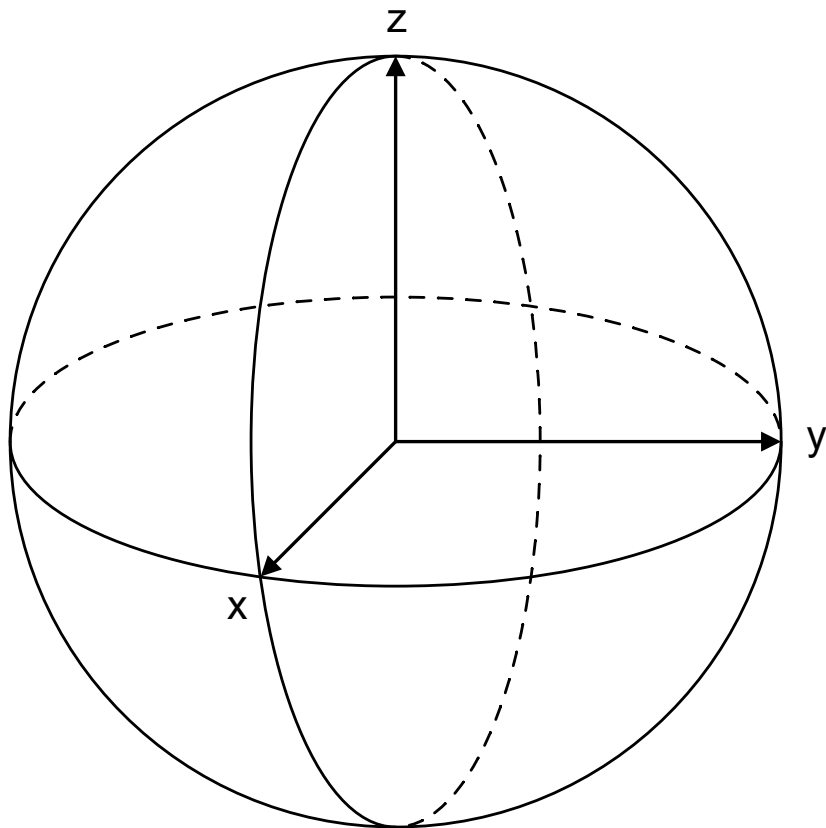
Quantum state

$$\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

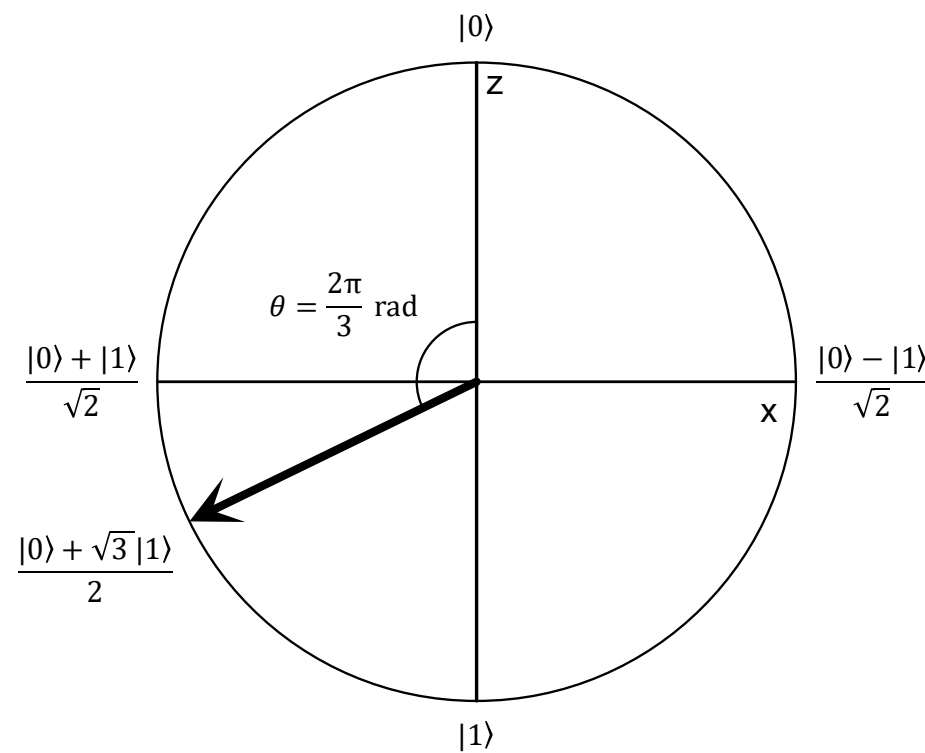
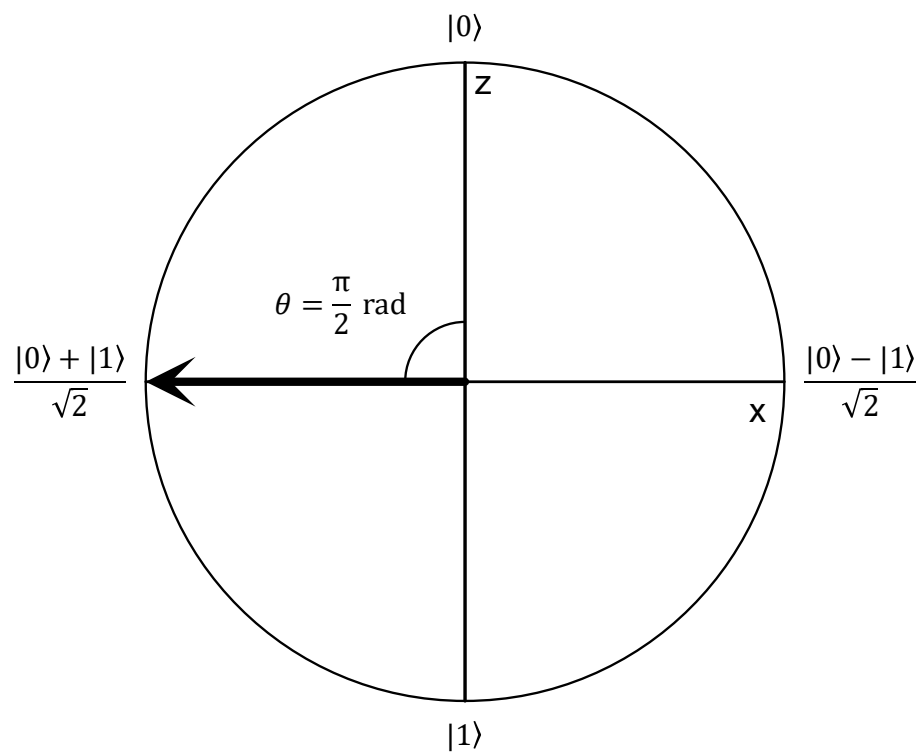
Quantum state

- Classical bit 0, Quantum bit $|0\rangle$
- Classical bit 1, Quantum bit $|1\rangle$
- Quantum bit in superposition
- $\alpha|0\rangle + \beta|1\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$
- α and β are complex numbers ($ai + b$)
- Value known after measurement
- Collapses to $|0\rangle$ with probability $|\alpha|^2$ or $|1\rangle$ with probability $|\beta|^2$

Quantum state



Quantum state



Quantum state

- 2 Qubit system (4 probabilities):

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

Quantum state

- 2 Qubit system (4 probabilities):

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

- 3 Qubit system (8 probabilities):

$$\alpha|000\rangle + \beta|001\rangle + \gamma|010\rangle + \delta|011\rangle + \epsilon|100\rangle + \zeta|101\rangle + \eta|110\rangle + \theta|111\rangle$$

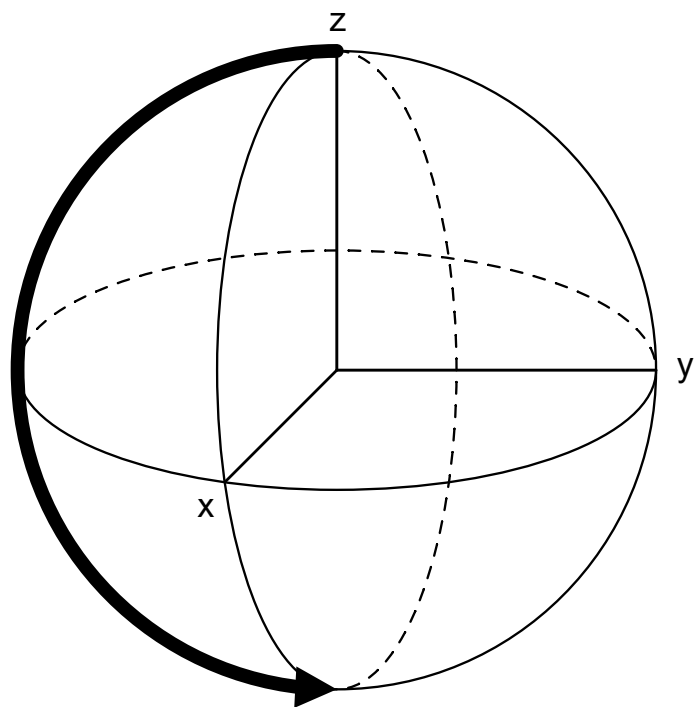
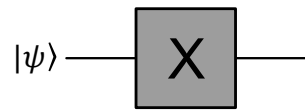
- 4 Qubit system (16 probabilities):

...

Quantum state

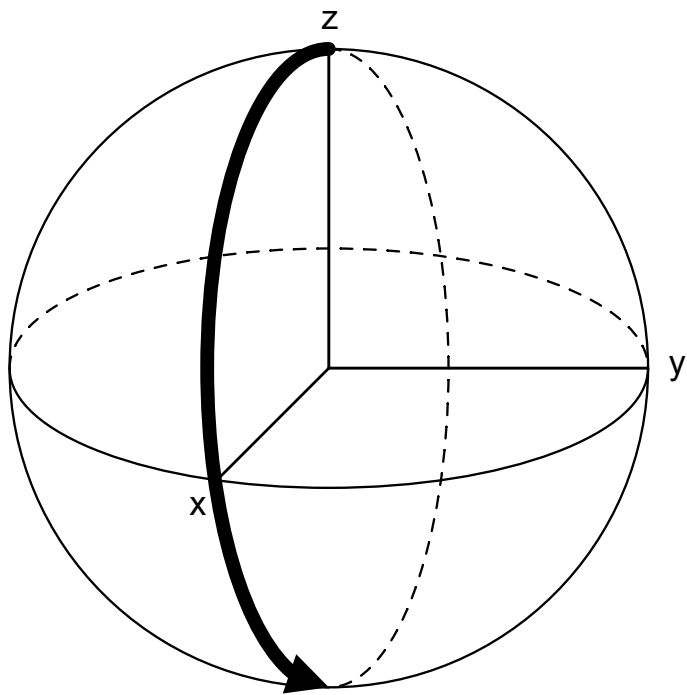
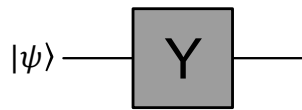
 $|0\rangle$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1\rangle$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

X-gate



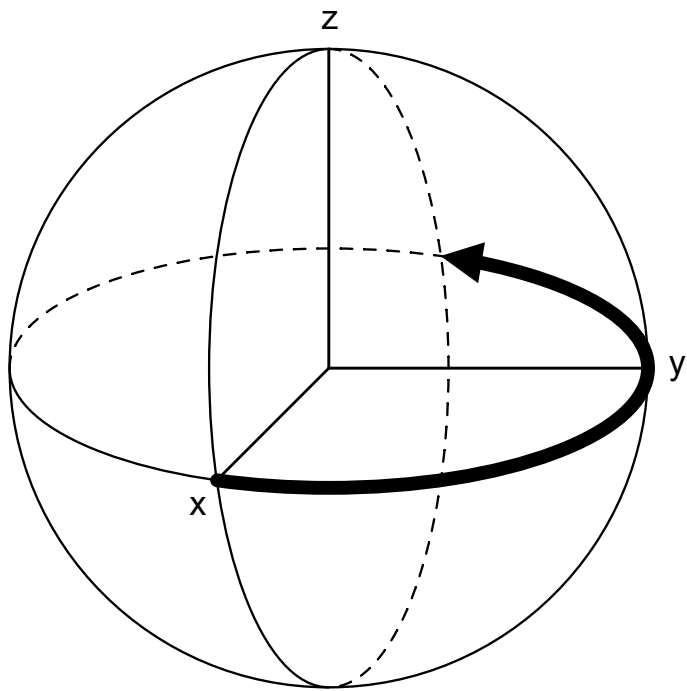
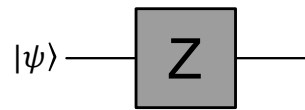
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Y-gate



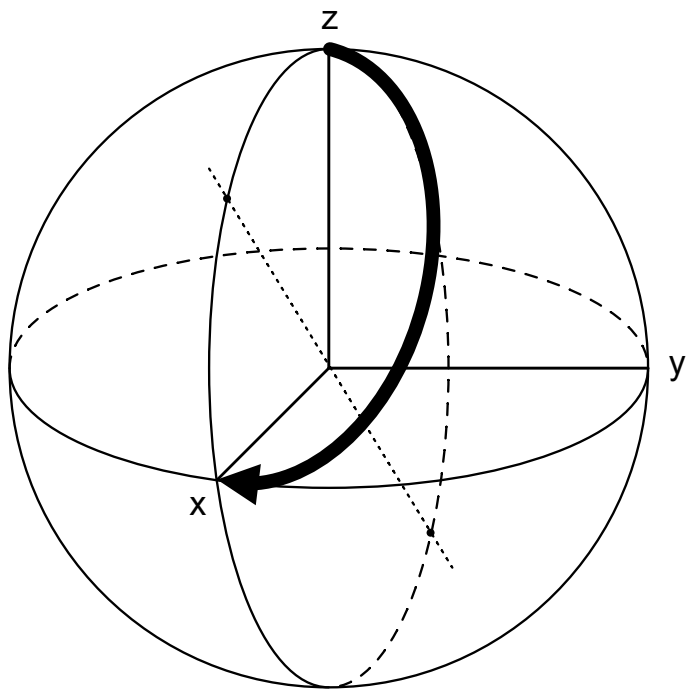
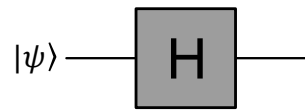
$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Z-gate



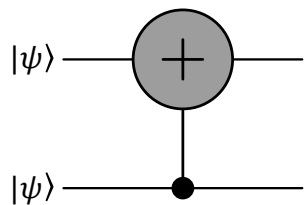
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

H-gate



$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

CNOT-gate



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

IBM Q Experience

<https://quantum-computing.ibm.com>



Microsoft Q#

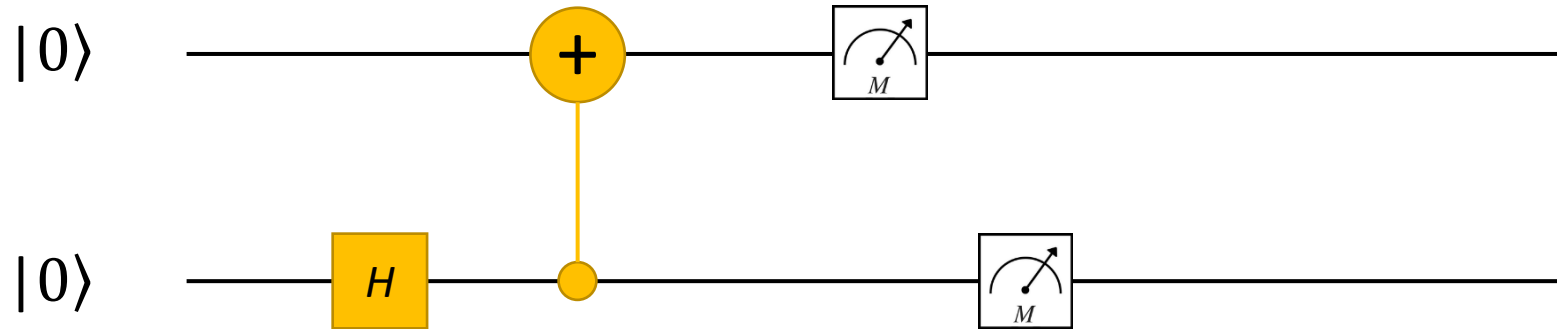
<https://www.microsoft.com/en-us/quantum/development-kit>



Azure Quantum

- Quantum in the cloud
 - Optimization
 - Machine Learning
 - Quantum Simulation
- Access to quantum hardware
 - Microsoft (Topological)
 - IonQ & Honeywell (Ion Traps)
 - QCI (Superconducting)
- Q# & QDK
 - Quantum Intermediate Representation (QIR)

Entanglement



$$\begin{aligned}
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{H} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 1 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \xrightarrow{CNOT} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 1 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = ?
 \end{aligned}$$

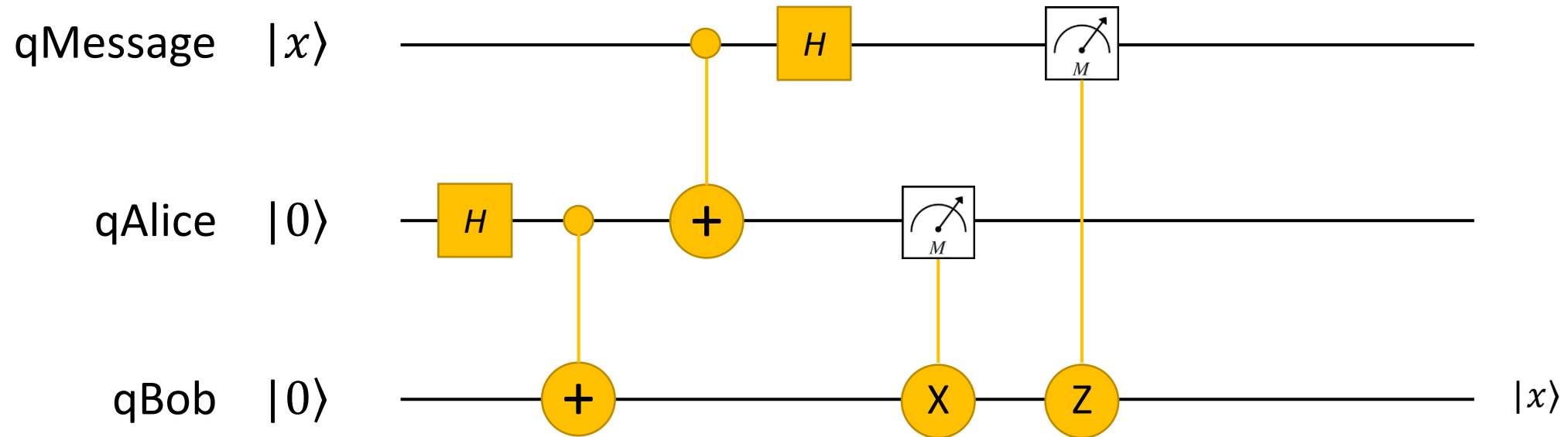
Entanglement

If the product state of two qubits cannot be factored, they are entangled

$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} \rightarrow \begin{aligned} ac &= \frac{1}{\sqrt{2}} \\ ad &= 0 \\ bc &= 0 \\ bd &= \frac{1}{\sqrt{2}} \end{aligned}$$

This set of two qubits has a 50% chance of collapsing to $|00\rangle$ and a 50% chance of collapsing to $|11\rangle$

Teleportation



Quantum Algorithms

- Deutch (1985)
 - Is there a problem that a Quantum Computer can solve faster than a Classical Computer?
 - Deterministic!

Quantum Algorithms

- Deutsch–Jozsa (1992)
 - Based on Deutch (for 1 bit), but applicable for n -bits
 - Deterministic!

Quantum Algorithms

- Grover's algorithm (1996)
 - “Searching a database”
 - Probabilistic!

Quantum Algorithms

- Shor's algorithm (1994)
 - Prime factorization of integers
 - Combination of classical and quantum algorithm
 - Probabilistic!

Deutsch's algorithm

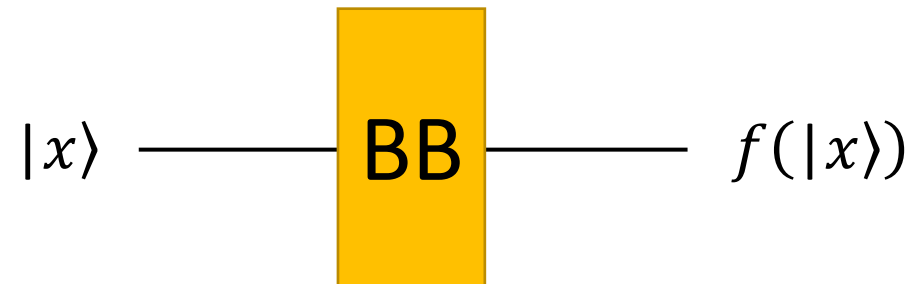
Can a Quantum Computer be quicker than a Classical Computer?

A Black-Box containing a function on one bit

How many operations do you need to figure out the function if input and output is known?

On a Classical Computer?

On a Quantum Computer?



Deutsch's algorithm

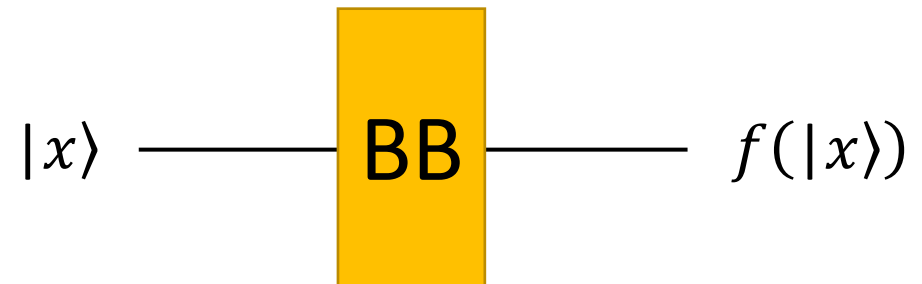
It is important to ask the right question!

A Black-Box containing a function on one bit

How many operations do you need to figure out if the function is
CONSTANT or VARIABLE if input and output is know?

On a Classical Computer?

On a Quantum Computer?



Deutsch's algorithm

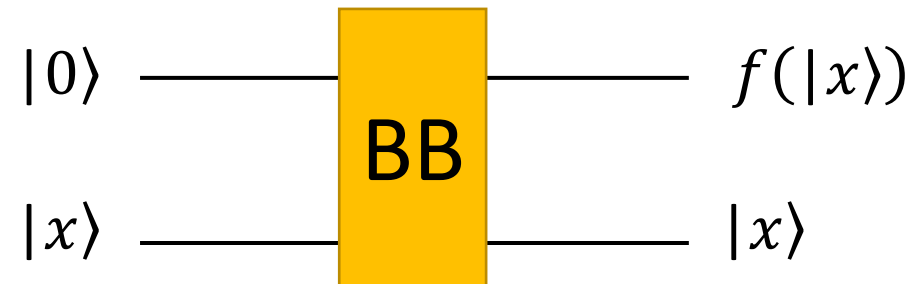
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A Black-Box containing a function on one bit

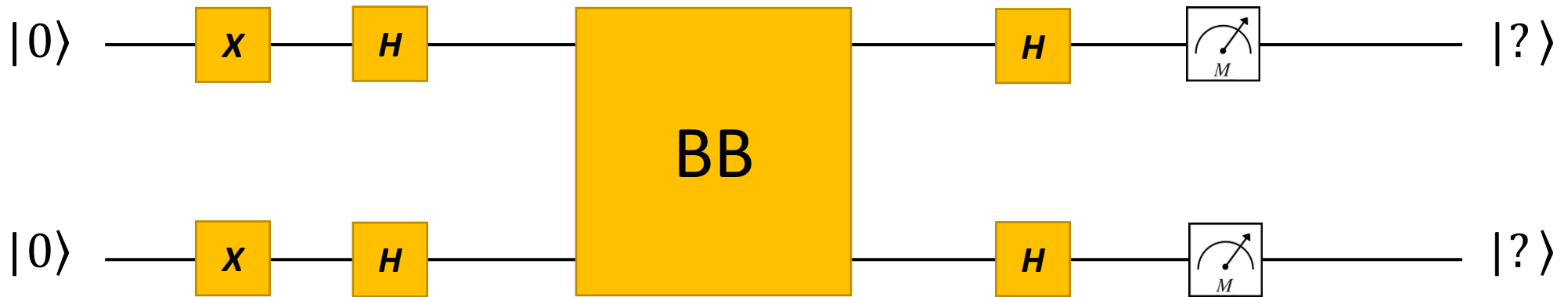
How many operations do you need to figure out if the function is
CONSTANT or VARIABLE if input and output is know?

On a Classical Computer?

On a Quantum Computer?



Deutsch's algorithm

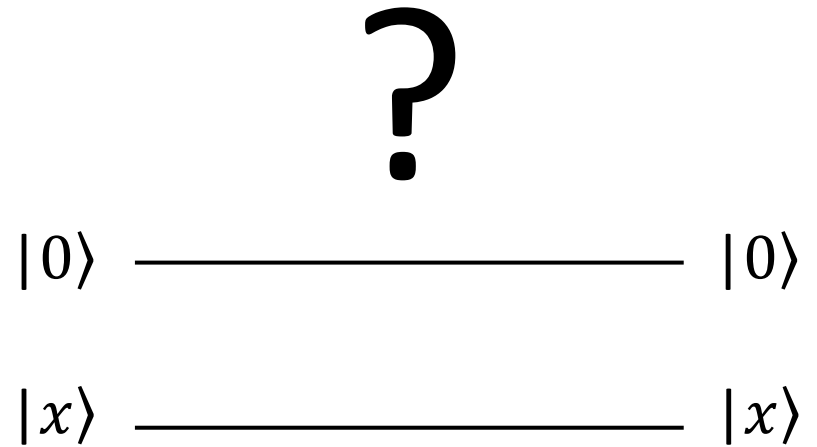
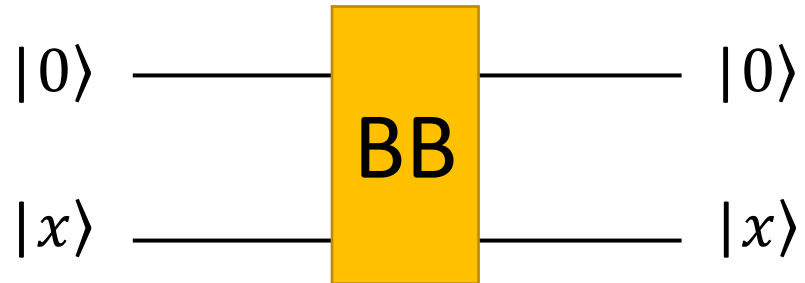


If BB is a constant function \rightarrow Quantum state will always measure to $|11\rangle$

If BB is a variable function \rightarrow Quantum state will always measure to $|01\rangle$

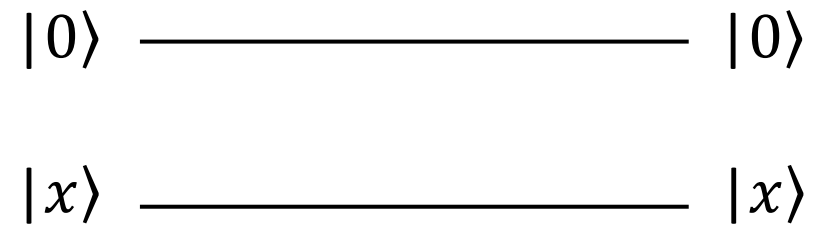
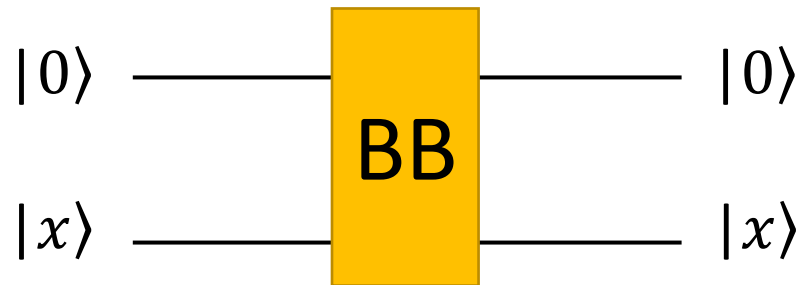
Deutsch's algorithm

Constant-0



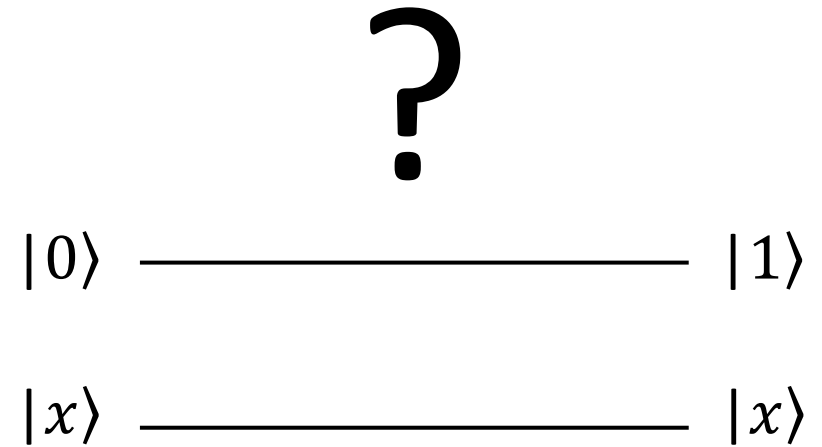
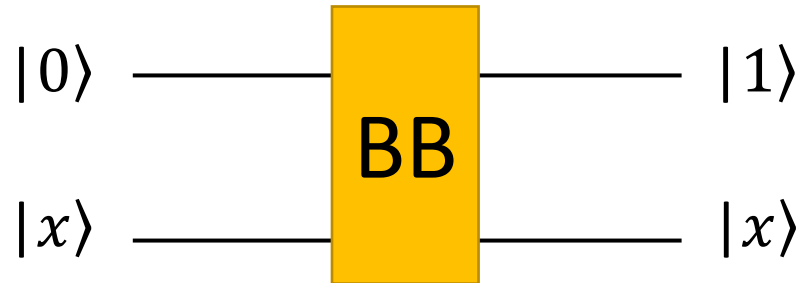
Deutsch's algorithm

Constant-0



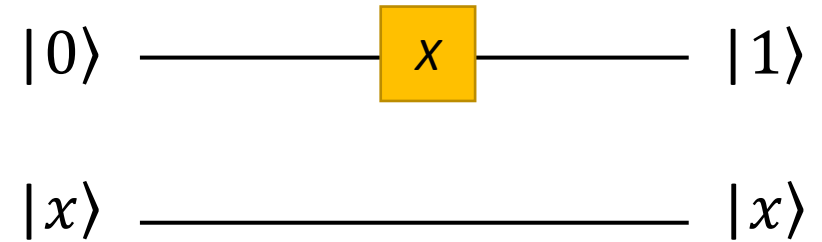
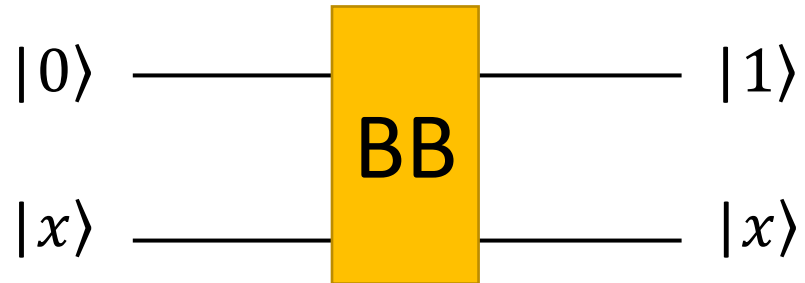
Deutsch's algorithm

Constant-1



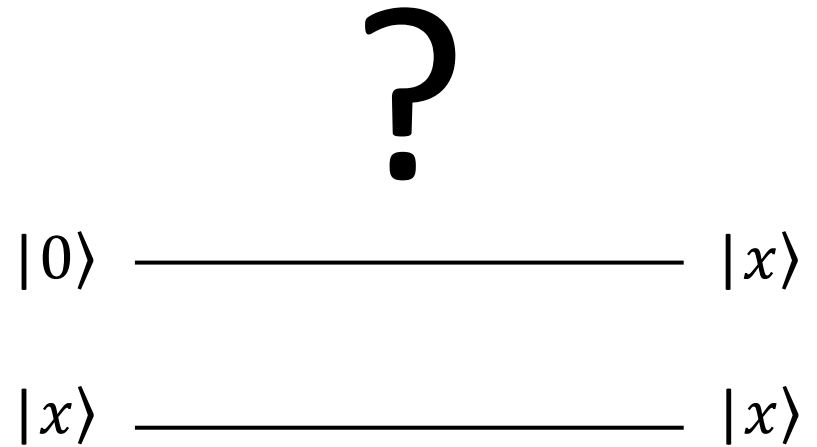
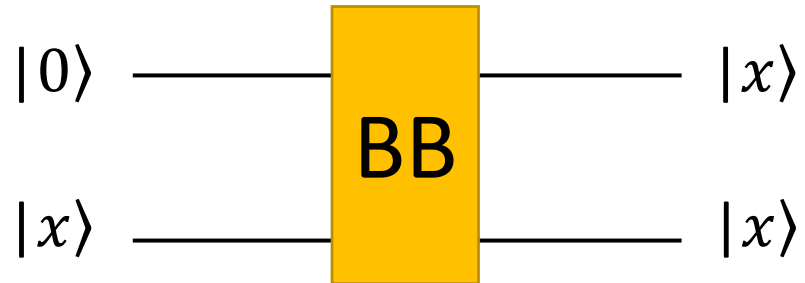
Deutsch's algorithm

Constant-1



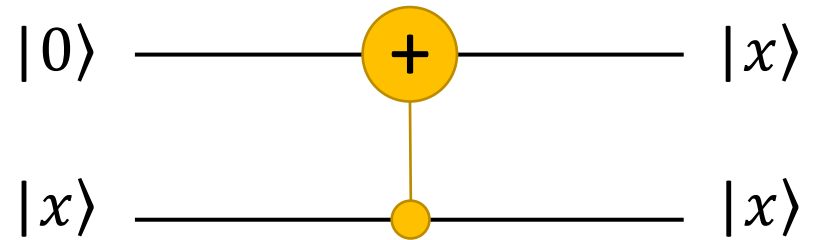
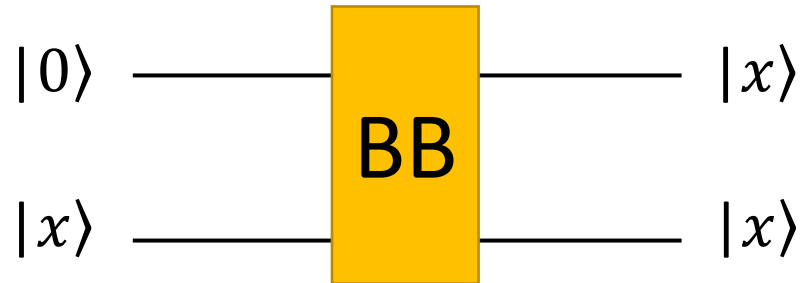
Deutsch's algorithm

Identity



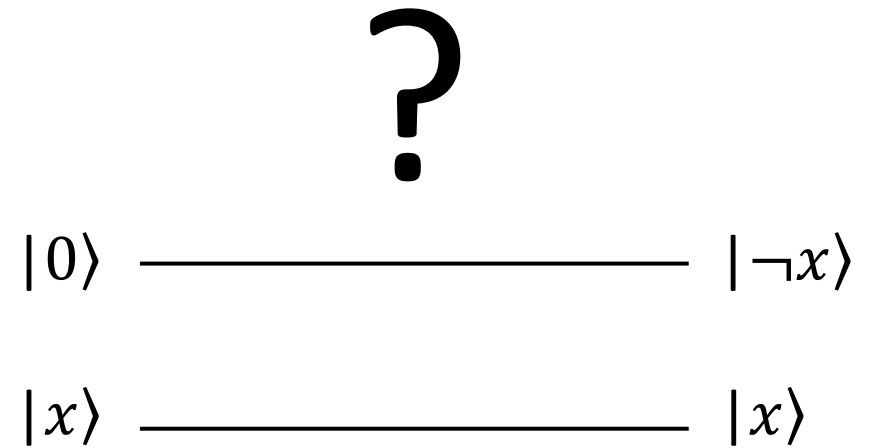
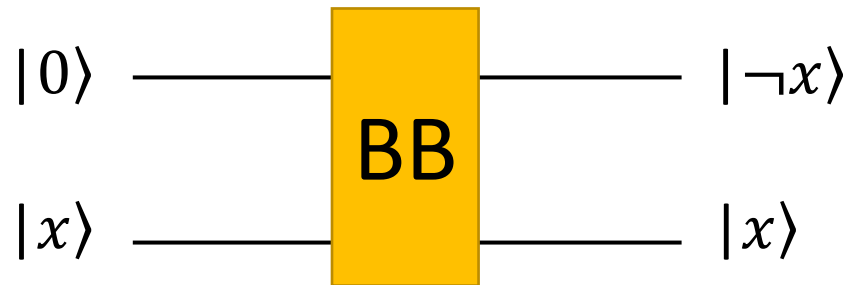
Deutsch's algorithm

Identity



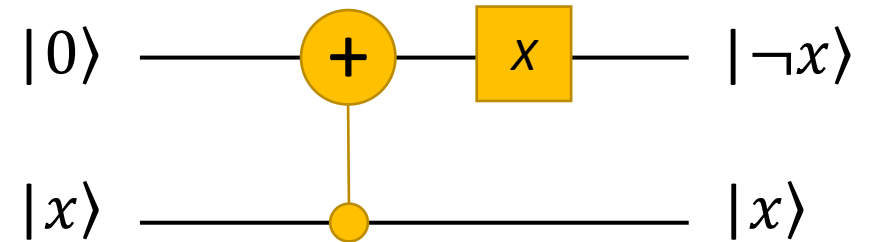
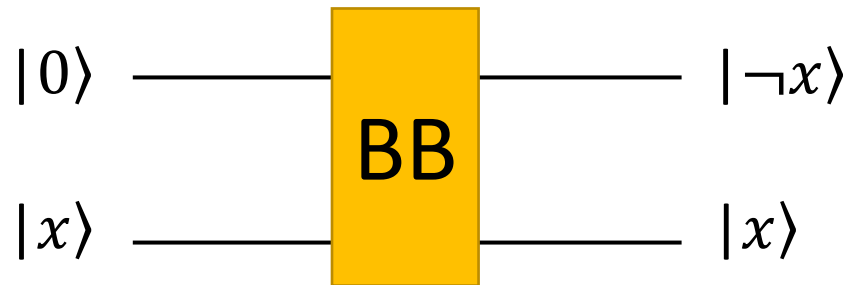
Deutsch's algorithm

Negation



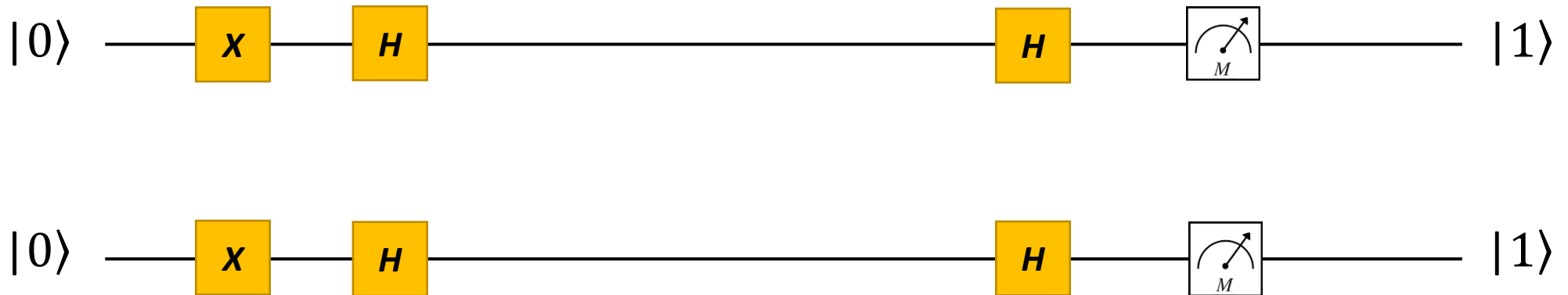
Deutsch's algorithm

Negation



Deutsch's algorithm

Constant-0 (circuit overview)



Deutsch's algorithm

Constant-0 (calculated proof – part 1)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

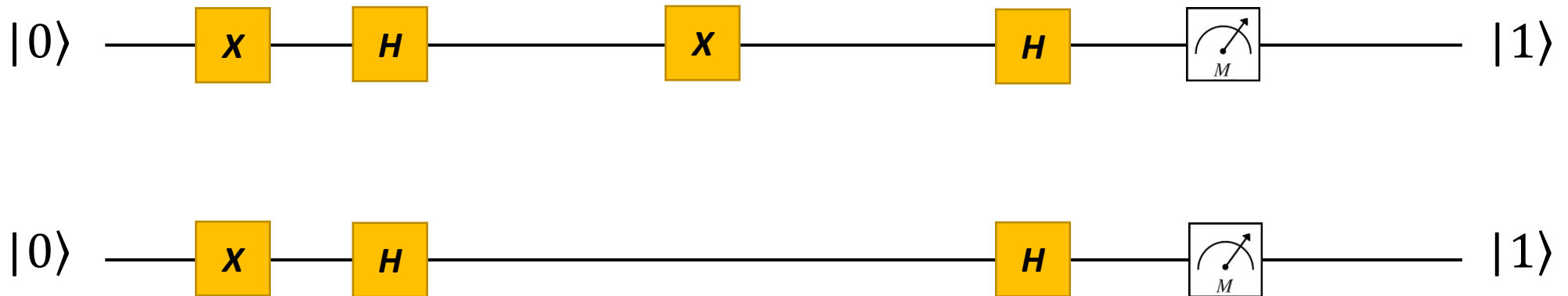
Deutsch's algorithm

Constant-0 (calculated proof – part 2)

$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$
$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

Deutsch's algorithm

Constant-1 (circuit overview)



Deutsch's algorithm

Constant-1 (calculated proof – part 1)

$$\begin{aligned} |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\ |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \end{aligned}$$

Deutsch's algorithm

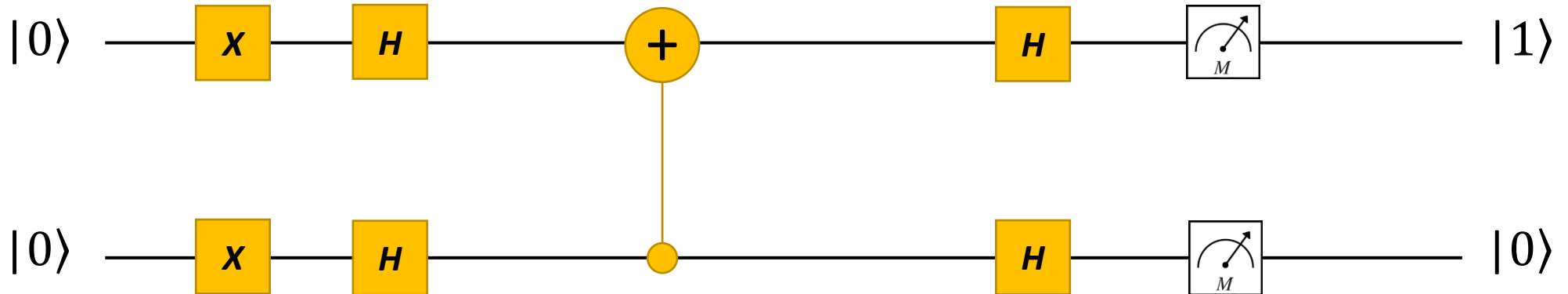
Constant-1 (calculated proof – part 2)

$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = |1\rangle$$

$$\begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{Id}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

Deutsch's algorithm

Identity (circuit overview)



Deutsch's algorithm

Identity (calculated proof – part 1)

$$\begin{aligned}
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}
 \end{aligned}$$

Deutsch's algorithm

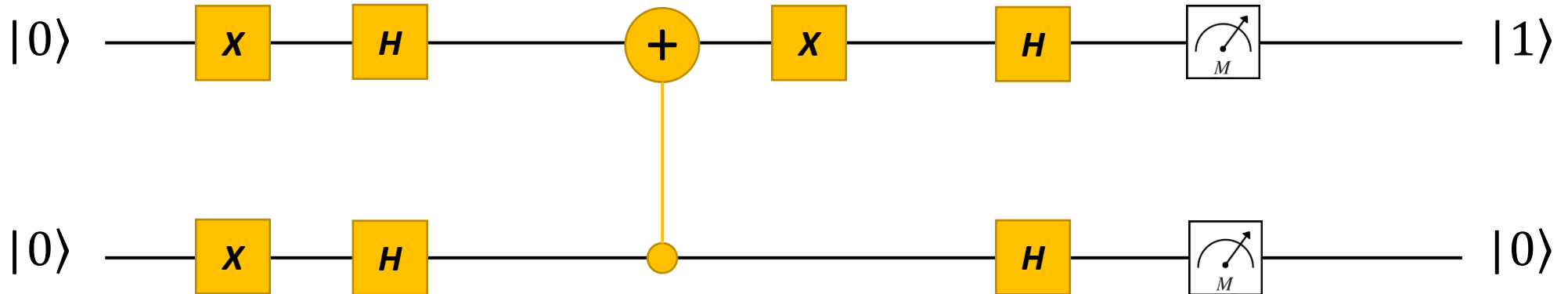
Identity (calculated proof – part 2)

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \xrightarrow{\mathbf{CNOT}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

Deutsch's algorithm

Negation (circuit overview)



Deutsch's algorithm

Negation (calculated proof – part 1)

$$\begin{aligned}
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\
 |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}
 \end{aligned}$$

Deutsch's algorithm

Negation (calculated proof – part 2)

$$\begin{aligned}
 & \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \xrightarrow{\mathbf{CNOT}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{X}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = |1\rangle \\
 & \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{Id}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \xrightarrow{\mathbf{H}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle
 \end{aligned}$$